Theorem 1 (Compositional Subadditivity). *For N agents with losses $L_1, \ldots, L_N$, the aggregate risk is less than or equal to the sum of individual risks:*

$$\rho_\alpha\Big(\sum_{i=1}^{N} L_i\Big) \leq \sum_{i=1}^{N} \rho_\alpha(L_i).$$

Proof. **Step 1: Identify the Risk Measure.**
DAIS-10 uses Conditional Value-at-Risk (CVaR) as its coherent risk measure ρ_α.

Step 2: Coherence Properties of CVaR.
By definition, a coherent risk measure satisfies monotonicity, translation invariance, positive homogeneity, and subadditivity.

Step 3: Apply the Subadditivity Axiom.
For any two random variables X and Y:

$$\rho(X + Y) \leq \rho(X) + \rho(Y).$$

Step 4: Extension to N Agents.
Applying subadditivity iteratively across N agents gives:

$$\rho_\alpha\Big(\sum_{i=1}^{N} L_i\Big) \leq \sum_{i=1}^{N} \rho_\alpha(L_i).$$

Step 5: Conclusion.
This guarantees that multi-agent safety coordination in DAIS-10 does not produce unaccounted aggregate risk, supporting compositional safety. $\square$