Theorem 1 (Wasserstein Robustness). *Let $\mathcal{P} = \{Q : W_p(Q, P) \leq \epsilon\}$ be a Wasserstein ball of distributions. Then the DAIS-10 policy provides a lower worst-case expected loss than a threshold policy:*

$$\sup_{Q \in \mathcal{P}} \mathbb{E}_Q[L(\pi_{DAIS})] \leq \sup_{Q \in \mathcal{P}} \mathbb{E}_Q[L(\pi_\tau)].$$

Proof. **Step 1: Kantorovich Duality Application.**
Using Distributionally Robust Optimization (DRO), the worst-case expectation over a Wasserstein ball can be rewritten via Kantorovich duality:

$$\sup_{Q \in \mathcal{P}} \mathbb{E}_Q[L(\pi)] = \inf_{\lambda \geq 0} \left\{ \lambda\epsilon + \mathbb{E}_P[\varphi_\lambda(x)] \right\}.$$

Step 2: Define the Conjugate Function.
The robustified loss $\varphi_\lambda(x)$ is defined as:

$$\varphi_\lambda(x) = \sup_y \{L(\pi, y) - \lambda c(x, y)\},$$

where $c(x, y)$ is the transport cost from P to Q.

Step 3: Comparative Analysis of Policy Constraints.
By the DAIS-10 Decision Operator, π_{DAIS} triggers "safe" under a broader set of conditions (risk-mass decay, timeout T_s^*, CVaR) than the single-condition probability threshold π_τ. For any distribution shift, DAIS-10 is more likely to enter zero-loss "safe" states in high-risk regions.

Step 4: Conclusion.
Since $L(\pi_{DAIS}, y) \leq L(\pi_\tau, y)$ in safety-dominant regions, the inner optimization yields a lower value for DAIS-10. Therefore, the global supremum over the Wasserstein ball satisfies:

$$\sup_{Q \in \mathcal{P}} \mathbb{E}_Q[L(\pi_{DAIS})] \leq \sup_{Q \in \mathcal{P}} \mathbb{E}_Q[L(\pi_\tau)].$$

□