

Theorem 1 (Ambiguity-Forcing). *If ambiguity persists beyond a predefined timeout period (T_s^*), the DAIS-10 framework mandates a safe action:*

$$t \geq T_s^* \Rightarrow \pi_{DAIS} = \text{safe}.$$

Proof. **Step 1: Identify the Decision Operator.**

The DAIS-10 policy π_{DAIS} is a piecewise logical operator:

$$\pi_{DAIS} = \begin{cases} \text{safe}, & \text{if } \rho_\alpha(L|\mathcal{F}_t) \geq \tau_s^{eff}, \\ \text{safe}, & \text{if } t \geq T_s^*, \\ \text{continue}, & \text{otherwise.} \end{cases}$$

Step 2: Apply the Temporal Condition.

If $t \geq T_s^*$, the temporal condition is satisfied.

Step 3: Evaluate the Logical Implication.

By Step 1, this condition is sufficient to trigger the "safe" state:

$$t \geq T_s^* \Rightarrow \pi_{DAIS} = \text{safe}.$$

Step 4: Conclusion.

Based on Axiom 4 (Ambiguity Timeout), T_s^* is a stopping time with

$$\mathbb{E}[T_s^*] < \infty,$$

ensuring that once the threshold is crossed, the policy will enforce a safe action, regardless of the current risk mass. \square