Theorem 1 (Monotonicity)**.** *For all $t \geq 0$, the risk mass θ is non-increasing:*

$$\theta_{t+1} \leq \theta_t.$$

Proof. **Step 1: Recursive Definition.**
The risk mass at time $t+1$ is defined by the Fading Operator $F(\theta_t, d_t)$:

$$\theta_{t+1} = \theta_{\min} + (\theta_t - \theta_{\min}) \exp(-\lambda_s d_t).$$

Step 2: Exponential Bounds.
Since $\lambda_s > 0$ and $d_t \geq 0$, the exponential term satisfies:

$$0 \leq \exp(-\lambda_s d_t) \leq 1.$$

Step 3: Inequality Substitution.
By the Boundedness Axiom, $(\theta_t - \theta_{\min}) \geq 0$. Multiplying by the exponential term:

$$(\theta_t - \theta_{\min}) \exp(-\lambda_s d_t) \leq (\theta_t - \theta_{\min}).$$

Step 4: Final Comparison.
Adding $\theta_{\min}$ to both sides:

$$\theta_{\min} + (\theta_t - \theta_{\min}) \exp(-\lambda_s d_t) \leq \theta_{\min} + (\theta_t - \theta_{\min}) = \theta_t.$$

Hence:

$$\theta_{t+1} \leq \theta_t.$$

□