Theorem 1 (Convergence)**.** *If ambiguity persists ($d_t \geq c > 0$), then the risk mass converges to the minimum floor:*

$$\lim_{t \to \infty} \theta_t = \theta_{\min}.$$

Proof. **Step 1: General Term Expression.**
From the recursive fading operator, the risk mass at time t is:

$$\theta_t = \theta_{\min} + (\theta_0 - \theta_{\min}) \exp\Big(- \lambda_s \sum_{i=0}^{t-1} d_i \Big).$$

Step 2: Persistent Ambiguity Condition.
Since $d_t \geq c > 0$, the summation satisfies:

$$\sum_{i=0}^{t-1} d_i \geq ct.$$

Thus:

$$\theta_t \leq \theta_{\min} + (\theta_0 - \theta_{\min}) \exp(-\lambda_s ct).$$

Step 3: Evaluate the Limit.
As $t \to \infty$, $\exp(-\lambda_s ct) \to 0$ because $\lambda_s, c > 0$:

$$\lim_{t \to \infty} \exp(-\lambda_s ct) = 0.$$

Step 4: Conclusion.
Substituting the limit back:

$$\lim_{t \to \infty} \theta_t = \theta_{\min} + (\theta_0 - \theta_{\min}) \cdot 0 = \theta_{\min}.$$

□