Theorem 2: Deterministic Divergence Under Persistent Outward Drift

Theorem 1 (Persistent Outward Projection Theorem). *Let $D(t), A(t) \in \mathbb{R}^n$ be continuously differentiable. Define the misalignment vector*

$$E(t) = D(t) - A(t).$$

Assume there exists $\varepsilon > 0$ such that for all $t \geq 0$ with $E(t) \neq 0$,

$$\frac{E(t)}{\|E(t)\|} \cdot \left(\dot{D}(t) - \dot{A}(t) \right) \geq \varepsilon.$$

Then

$$\|E(t)\| \geq \|E(0)\| + \varepsilon t,$$

and therefore

$$\|E(t)\| \to \infty \quad as\ t \to \infty.$$

Proof. From Theorem 1 (Dual Failure Law), for all t such that $E(t) \neq 0$,

$$\frac{d}{dt} \|E(t)\| = \frac{E(t)}{\|E(t)\|} \cdot \left(\dot{D}(t) - \dot{A}(t) \right).$$

By hypothesis,

$$\frac{d}{dt} \|E(t)\| \geq \varepsilon.$$

This yields the differential inequality

$$\frac{d}{dt} \|E(t)\| \geq \varepsilon.$$

Integrating from 0 to t,

$$\|E(t)\| - \|E(0)\| \geq \int_0^t \varepsilon\, ds.$$

Since ε is constant,

$$\|E(t)\| - \|E(0)\| \geq \varepsilon t.$$

Thus

$$\|E(t)\| \geq \|E(0)\| + \varepsilon t.$$

Therefore $\|E(t)\|$ grows at least linearly and diverges as $t \to \infty$.

$\square$