Theorem 5: Unified Alignment Theorem

Theorem 1 (Unified Alignment Instability)**.** *Let $D(t), A(t) \in \mathbb{R}^n$ be continuously differentiable.*

Let P be a linear projection operator representing visibility. Define

$$D(t) = \tilde{D}(t) + D_{\text{dark}}(t), \quad \tilde{D}(t) = PD(t).$$

Let the misalignment vector be

$$E(t) = D(t) - A(t).$$

Assume:

1. *(Bounded Adaptation) There exists $M > 0$ such that*
$$\|\dot{A}(t)\| \le M \quad \text{for all } t.$$

2. *(Persistent Outward Effective Drift) There exist $\varepsilon > 0$ and $T \ge 0$ such that for all $t \ge T$,*
$$\frac{E(t)}{\|E(t)\|} \cdot \dot{D}(t) \ge M + \varepsilon.$$

Then there exists $T^* \ge T$ such that for all $t \ge T^*$,

$$\frac{d}{dt}\|E(t)\| \ge \varepsilon,$$

and therefore

$$\|E(t)\| \to \infty \quad \text{as } t \to \infty.$$

Proof. From the Dual Failure Law,

$$\frac{d}{dt}\|E(t)\| = \frac{E(t)}{\|E(t)\|} \cdot (\dot{D}(t) - \dot{A}(t)).$$

Rewrite:

$$\frac{d}{dt}\|E(t)\| = \frac{E(t)}{\|E(t)\|} \cdot \dot{D}(t) - \frac{E(t)}{\|E(t)\|} \cdot \dot{A}(t).$$

Using Cauchy–Schwarz,

$$\left| \frac{E(t)}{\|E(t)\|} \cdot \dot{A}(t) \right| \le \|\dot{A}(t)\| \le M.$$

By hypothesis (2),

$$\frac{E(t)}{\|E(t)\|} \cdot \dot{D}(t) \geq M + \varepsilon.$$

Therefore,

$$\frac{d}{dt}\|E(t)\| \geq (M + \varepsilon) - M = \varepsilon.$$

Integrating from T to t,

$$\|E(t)\| \geq \|E(T)\| + \varepsilon(t - T).$$

Thus $\|E(t)\|$ grows at least linearly and diverges as $t \to \infty$.

$\square$