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UNIVERSITY of GUJRAT

Department of Mathematics

Quiz 1 (b): BS Mathematics, Complex Analysis, FALL-2023 Marks: 10, Dated: 25.10.23

Roll no. . . . . Name . . . .

Q 1. Compute and draw four roots of $z = -1 + \sqrt{3}\,i$

Q 2. Let $A = \{(x, y) \in \mathbb{R}^2 : 2|\cancel{z} - 2| - |\cancel{z} + 1| = 1\}$. Then draw A. Also write domain and range of A. Is the given set is a function?

Q1:- $z^4 = -1 + \sqrt{3}\,i$

$\sqrt{r^2} = \sqrt{a^2+b^2}$

$\sqrt{r^2} = \sqrt{(-1)^2 + (\sqrt{3})^2}$

$\boxed{r = 2}$

$\theta = \tan^{-1}(y/x)$

$= \tan^{-1}(-\sqrt{3})$

$\Rightarrow -60 = -\pi/3$

$\boxed{\theta = -\pi/3}$

$z_k = (r)^{1/n} e^{i\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)}$

$z_k = (2)^{1/4} e^{i\left(-\pi/12 + \frac{2k\pi}{4}\right)}$

Put $\boxed{k=-1}$

$z_{-1} = (2)^{1/4} e^{i\left(\frac{-\pi}{12} - \frac{2\pi}{4}\right)}$

$= (2)^{1/4} e^{i\left(\frac{-\pi - 6\pi}{12}\right)}$

$= (2)^{1/4} e^{i\left(\frac{-7\pi}{12}\right)}$

$= (2)^{1/4}\left[\cos\left(-\frac{7\pi}{12}\right) + i\sin\left(-\frac{7\pi}{12}\right)\right]$

$= 2^{1/4}\left[\left(\frac{-\sqrt{6}+\sqrt{2}}{4}\right) + i\left(\frac{-\sqrt{6}+\sqrt{2}}{4}\right)\right]$

$\cancel{= 2^{1/4}}$

$= -0.3\cancel{0} - 1.148i$

roots $(-0.3, -1.148)$

Put $\cancel{k}=0$

$z_0 = (2)^{1/4} e^{i(-\pi/12)} \Rightarrow (2)^{1/4}\left[\cos\left(\frac{-\pi}{12}\right) + i\sin\left(\frac{-\pi}{12}\right)\right]$

$\Rightarrow (2)^{1/4}\left[\frac{\sqrt{6}+\sqrt{2}}{4} + i\,\frac{\sqrt{6}-\sqrt{2}}{4}\right]$

$\Rightarrow 1.189(0.96 + 0.25i) \Rightarrow 1.14 + 0.297i$ roots $(1.14, 0.297)$

Put $k=1$

$z_1 = (2)^{1/4} e^{i\left(-\frac{\pi}{12} + \frac{2\pi}{4}\right)} \Rightarrow 2^{1/4} e^{i\left(\frac{-\pi+6\pi}{12}\right)} \Rightarrow (2)^{1/4} e^{i\left(\frac{5\pi}{12}\right)}$

$= (2)^{1/4}\left[\cos\left(\frac{5\pi}{12}\right) - i\sin\left(\frac{5\pi}{12}\right)\right]$ roots $(0.308, -1.14)$

$= (1.189)\left[\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) - i\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)\right] \Rightarrow 0.308 - 1.148i$

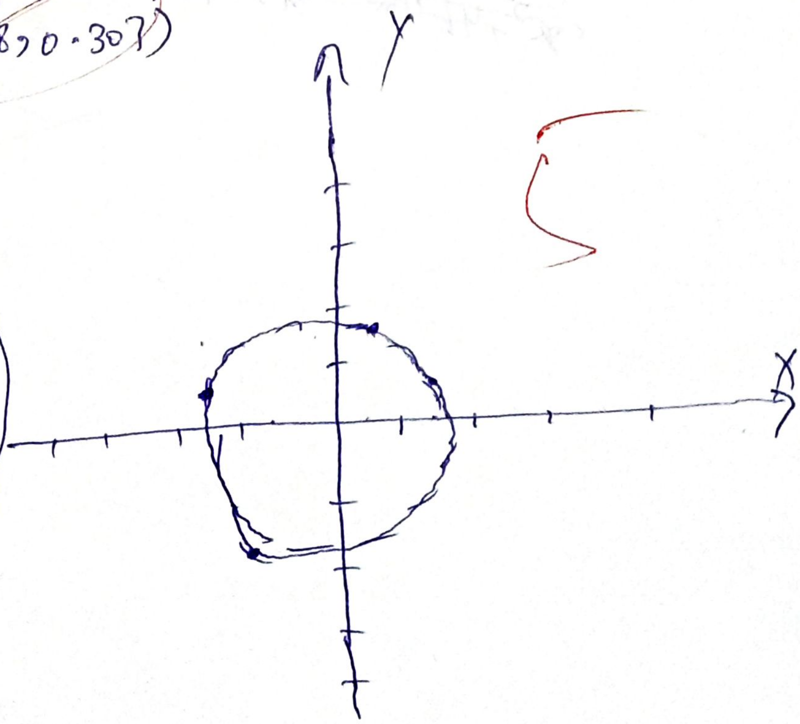
Put $k=2$

$z_2 = (2)^{1/4} e^{i\left(-\frac{\pi}{12} + \frac{4\pi}{4}\right)} \Rightarrow (2)^{1/4} e^{i\left(-\frac{\pi}{12} + \pi\right)} \Rightarrow (2)^{1/4} e^{i\left(\frac{-\pi+12\pi}{12}\right)}$

$= (2)^{1/4} e^{i(11\pi/12)} \Rightarrow (2)^{1/4}\left[\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right]$

$= -1.148 + 0.307i$

$(-1.148, 0.307)$ roots



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