Tutorial: Non-linear equations of a single variable

1. Use the Newton-Raphson method to find the root of the function below.

   $f(x) = x^2 - 5 = 0$

   Use the starting point $x_0 = 2$. Stop when $|f(x)| < 0.000001$

   What is the meaning of the root you have just found?

   How many iterations do you think it will take for the Interval Bisection method to find the same answer?

   Try this again but this time with the Interval Bisection method for about 5 iterations, how close do you think you are you to the solution?

2. Using the formulae for the Newton-Raphson and Secant methods, discuss the possible instabilities which could result due to numerical computation.

3. Show that the function $f(x) = e^x + 2x$ has a negative root in the interval [-1,0]. Use the Secant method to find a negative root for the function using a stopping criterion of $|f(x)| < 0.001$.

4. Use the Secant method to find either of the two roots of the following function $f(x) = 2 - 3x + x^3$ and stop when $|f(x)| < 0.00001$
   Hint: Use starting points between $x = -2.6$ *and* $x = -2.4$ to find one root or $x = 1.2$ *and* $x = 1.4$ to find the other. Draw the graph for *f(x)* and what do you observe?

5. Use the Newton-Raphson method to find one of the roots of the function below. Explain what you observe.

   $f(x) = x^3 - 0.03x^2 + 2.4 \times 10^{-6}$

   Using a stopping criterion of $|f(x)| < 0.0001$ and starting point of
   (a) $x_0 = 0.01999$
   (b) $x_0 = 0.0$

6. Write Java programs that implement the different methods (Interval Bisection, Newton-Raphson and Secant) to solve all of the above problems.