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UNIVERSITY *of* GREENWICH

EXAMINATION PAPER: ACADEMIC SESSION 2013/2014

Campus	Maritime Greenwich
School	Computing and Mathematical Sciences
Department	Computer Science
Level	5
TITLE OF PAPER	Computer Algorithms and Modelling
COURSE CODE	COMP 1555
Date and Time	January 2014
Duration	2 hours + 15 minutes reading time

Answer any **TWO** of the following THREE questions.
Each question is worth 50 marks.
If you answer more than two questions, marks will **ONLY** be awarded for your TWO best answers.

A formulae sheet and graph paper are attached to this paper.
Approved calculators may be used.

1. (a) (i) Explain with the aid of a diagram what the simple least squares regression line attempts to represent for a given set of pairs of (x,y) data.

[5 marks]

(ii) A car hire company is interested to find out if there is a relationship between the mileage of their cars and the cost of any repairs to their cars. The sample data below was gathered to undertake this study. The cost of repairs for each car is denoted by y and the mileage of the car, in thousands of miles, is denoted by x.

Mileage/'000s (x)	Cost of repairs/£ (y)
87	7210
65	4690
25	2800
24	3900
78	6010
15	3100
40	4600
34	3590
59	4590
21	2910

Using the graph paper provided, draw a scatter diagram for the data above and comment on any noticeable pattern.

[5 marks]

(iii) Obtain the equation of the least squares regression line of y on x.

[20 marks]

(iv) Plot the regression line on your scatter diagram and comment about the suitability of the regression line in representing the data provided.

[5 marks]

Question 1 continues on next page

Question 1 continued:

(b) (i) The following code performs the update of a temperature array TNEW as a weighted average of the current temperature vales T. Note that each matrix has the same number of rows and columns defined by the variable n.

```
// use for-i loop to process rows
for (i = 0, i < n; i++) {
     //special case when j=0
     TNEW[i][0] = T[i][0];
    // use for-j loop to process columns
    for (j = 1; j < n; j++) {
         // solutions are stored into array TNEW
         TNEW[i][j] = 0.25*(T[i][j-1] + T[i][j+1]);
     }
  }
```

Determine the computational count in terms of n for the code above and explain its performance characteristics. You should include the cost of the loop process. You may assume that all operations have the same cost.

[8 marks]

(ii) Describe how a computer language such as Java stores and manipulates floating point numbers. Include in your discussion cases where the floating point number is either too big or too small.

[7 marks]

2. (a) You are required to find the numerical solution to the following definite integral

$$\int_2^4 (4 + 3x - 5x^2 - 10x^3 + 5x^4)\, dx$$

(i) Use the trapezoidal rule with FOUR strips to compute numerically the definite integral given above. Work to 5 decimal places and show all your working in your solution.

[12 marks]

(ii) Use Simpson's rule with FOUR strips to compute numerically the definite integral given above. Work to 5 decimal places and show all your working in your solution.

[12 marks]

(iii) The analytical solution to the above problem is given as 324.66667 (5 decimal places). Determine the absolute true errors for both the trapezoidal rule and Simpson's rule.

[4 marks]

(iv) Comment on which method you consider to be the best in this case. Give reasons for you answer.

[4 marks]

(b) (i) Let $f(x) = x^5 - 3x^3 + 6x - 2$. Show this function has at least **one positive root.**

[2 marks]

(ii) Find one such root using the Newton-Raphson method with a starting estimate at $x=0$ and the stopping criteria of $|f(x)| < 0.001$

[12 marks]

(iii) The Newton-Raphson method can be potentially numerically unstable when trying to find the root of an equation. Use the formula for the method to explain how this may happen.

[4 marks]

3. (a) (i) Explain with the aid of an example how you would use a linked list to store information in a binary search tree.

[7 marks]

(ii) Write pseudo code (NOT the program code) to allow a node to be **inserted** into a binary search tree. Explain clearly the steps taken.

[6 marks]

(iii) Describe the Inorder, Preorder and Postorder traversal algorithms. Your descriptions should include an outline for each of the algorithms as well as suitable examples to illustrate the use of these traversal algorithms in practise.

[12 marks]

(b) Given a hash table of size $n = 10$ and the hash function $h(x)$ defined by

$h(x)$ = (sum of the values of the last TWO letters of x) modulus n

where the value of a letter is determined by its position in the alphabet (e.g. value of a is 1, value of b is 2, etc)

(i) compute the hash values $h(x)$ for the following names of trees:
Lime, Spruce, Birch, Willow, Redwood, Beech, Pine, Ash, Rubber, Maple.

[5 marks]

(ii) Using the techniques of linear probing and chaining when collisions occur together with the hash function values $h(x)$ obtained in 3(b)(i) above, construct the resulting hash tables after inserting, in order, the following names of trees:

Lime, Spruce, Birch, Willow, Redwood, Beech, Pine, Ash, Rubber, Maple

[**Note**: your solution should include two separate hash tables, one based on linear probing and one based on chaining].

[20 marks]

FORMULAE SHEET

Simple Linear Regression Model:

$y = \beta_1 x + \beta_0$ where the coefficients β_0 and β_1 are given by

$$\beta_0 = \bar{y} - \beta_1 \bar{x} \qquad \text{and} \qquad \beta_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$\bar{x}$ denotes the mean of the x (independent) values
$\bar{y}$ denotes the mean of the y (dependent) values

Correlation coefficient:

$$R^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}}$$

Differentition:

For a given function $f(x) = ax^n$ where a is a constant, then the derivative is given by $f'(x) = anx^{n-1}$ *e.g.* if $f(x) = 4x^3$ then $f'(x) = 12x^2$

Numerical quadrature:
For a given function $f(x)$ that is continuous over the interval $[a,b]$, the interval is subdivided into n equal number of intervals of width Δx. The following rules can be used to numerically compute $\int_a^b f(x)dx$

Rectangular Rule:

$$\int_a^b f(x)dx = \Delta x \sum_{i=0}^{n-1} f(x_{i+1/2})$$

Trapezoidal Rule:

$$\int_a^b f(x)dx = \Delta x \left(\frac{1}{2} f(x_0) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2} f(x_n) \right)$$

Simpson's Rule:

$$\int_a^b f(x)dx = \frac{\Delta x}{3} \left(f(x_0) + 4 \sum_{i=1}^{n/2} f(x_{2i-1}) + 2 \sum_{i=1}^{n/2-1} f(x_{2i}) + f(x_n) \right)$$

Solution of nonlinear equation – Newton Raphson method:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$