Tutorial: Integration

1. Integrate the following analytically (by hand):

   a. $\int_0^1 x^3 \, dx$

   b. $\int_{0.5}^5 2x^3 - 5x \, dx$

   c. $\int_2^4 x^5 + 2x^2 + 5x^4 \, dx$

   d. $\int_{-2}^1 205x^{12} \, dx$

   e. $\int_{-2}^{-2} 205x^{12} \, dx$ (explain your answer)

   f. $\int_{1.5}^3 e^{2x} \, dx$

2. Use the Trapezium rule with 4 strips (n=4) to work out by hand the area under the curve of
   $$f(x) = 200x^2 + 65x^3 + 20x^4 + 15x^5$$

   from a lower limit a=0.5 to an upper limit b=1.1 .

   Work out the analytical (exact answer) and then calculate the absolute true error for the numerical scheme.

3. Use Simpsons Rule with 6 strips (n=6) to calculate by hand

   $$f(x) = 10x^2 - 6x^3 - 90x^4 + 400x^5$$

   from a lower limit a=1 to an upper limit b=3 .

   Work out the analytical (exact answer) and then calculate the absolute relative true error for this numerical scheme.

4. Write a java program that will solve the following integral using either Simpson's rule, Trapezium rule and the Rectangular rule. Each numerical technique should be defined using a Java method and the user should be prompted to choose which method is to be used and how many strips should be used.
   $$\int_0^{25} (x^9 - x^7 + x^5 - x^3 + x - 100)dx$$

   Compute the answer using n=10, n=50, n=100. Verify which method is the most accurate by comparing with the exact solution.