

What Do We Mean by “Fitting Models to Data”?

Brady T. West

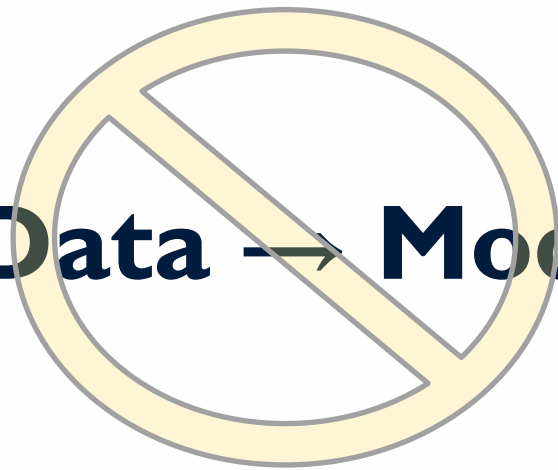
Fitting Models to Data

Goal: How to fit **statistical models** to **data**
to help answer research questions

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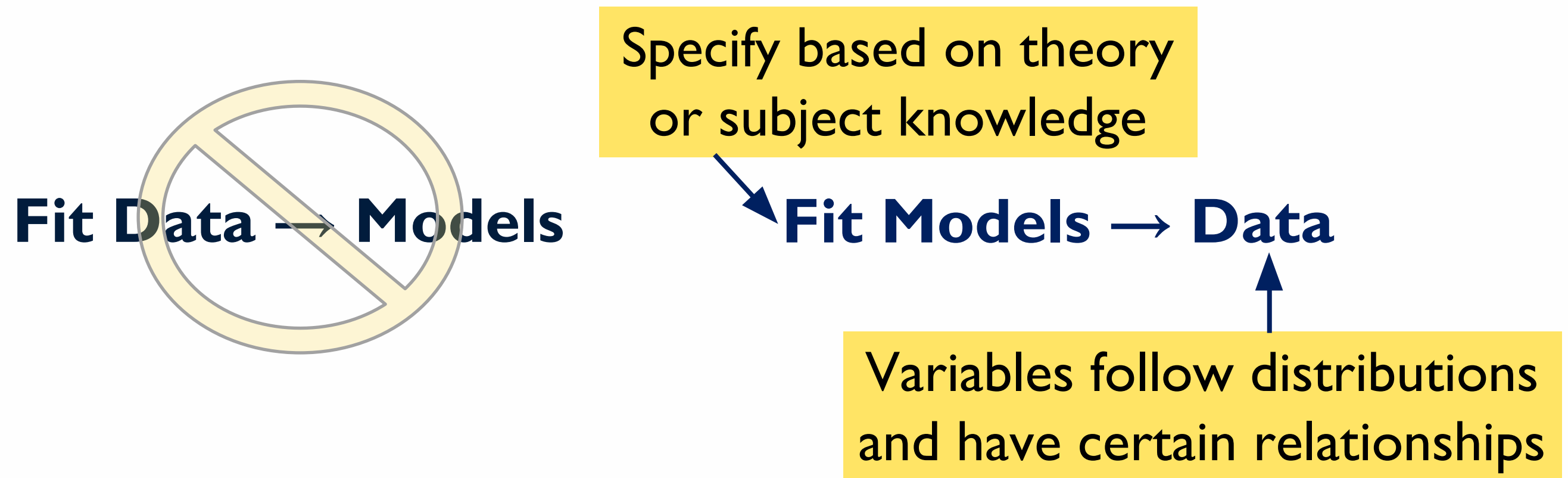
Fit Data → Models



Fit Models → Data

Fitting Models to Data

Goal: How to fit **statistical models** to **data** to help answer research questions



Fitting Models to Data

Why do we fit models to data?

- **Estimate** distributional properties of variables, potentially conditional on other variables
- Concisely **summarize relationships** between variables, and make inferential statements about those relationships
- **Predict** values of variables of interest conditional on values of other predictor variables, and characterize prediction uncertainty

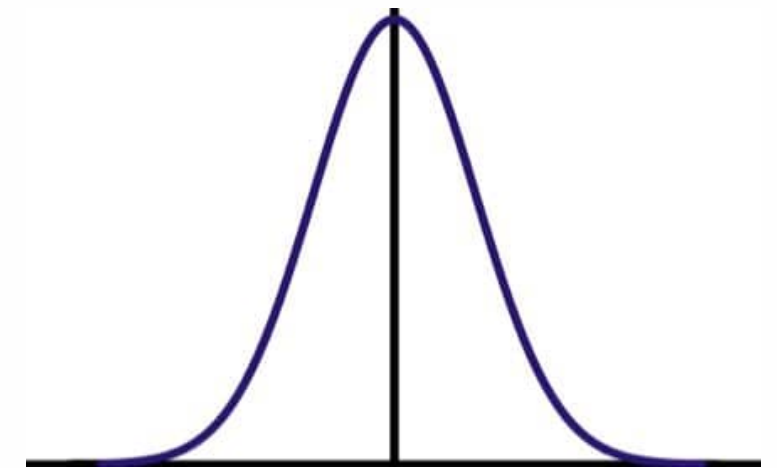
means
variances
quantiles

Fitting Models to Data

- Focus on **parametric models** → estimating **parameters** that describe the distributions of variables

Fitting Models to Data

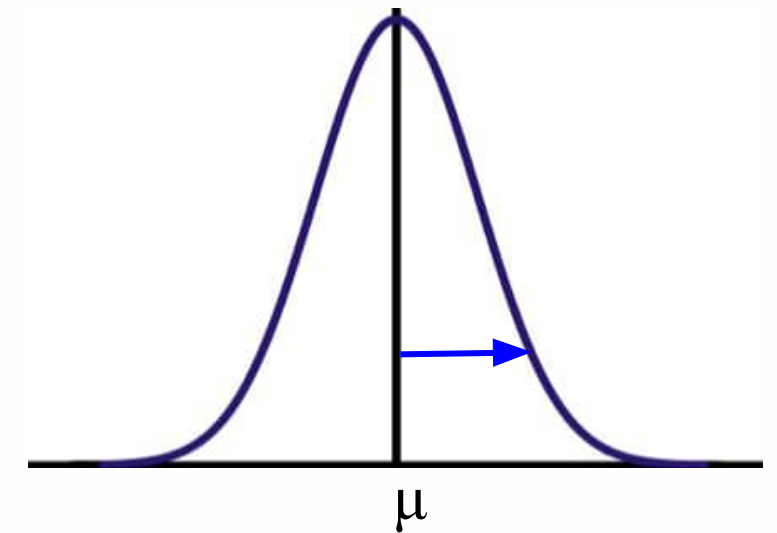
- Focus on **parametric models** → estimating **parameters** that describe the distributions of variables
- Given data, suggest **variable of interest** follows certain **probability model**



e.g. normal distribution

Fitting Models to Data

- Focus on **parametric models** → estimating **parameters** that describe the distributions of variables
- Given data, suggest **variable of interest** follows certain **probability model**
→ estimate parameters that define that model



e.g. normal distribution

e.g. mean and variance

Fitting Models to Data

Estimate model parameters + sampling variance
= **make inference about parameters**
by testing hypotheses or generating confidence intervals

Course 2!

Fitting Models to Data

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= **make inference about parameters**
by testing hypotheses or generating confidence intervals


Course 2!

Up next ...

Example of specifying probability model (given a research question)
and estimating the parameters of that model

Idea of **assessing model fit**: *Does model seem to fit observed data well?*

Example: Test Performance and Age



Variable of Interest
Test Performance
(0 – 8 points)

Example: Test Performance and Age


Possible Predictor
Age
(Standardized)



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Believe age has **curvilinear**
relationship with
performance

Variable of Interest
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moderate values of age → performance best
smaller or larger values of age → performance tends to be worse

Example: Test Performance and Age

Possible Predictor
Age
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Believe age has **curvilinear**
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Variable of Interest
Test Performance
(0 – 8 points)

moderate values of age → performance best
smaller or larger values of age → performance tends to be worse

- Goals:**
- 1) estimate **marginal mean** of performance across all ages
 - 2) estimate mean performance **conditional** on age

Example: Test Performance and Age

“mean-only” model
for performance

Performance follows **normal distribution** overall
defined by mean and variance (two parameters)

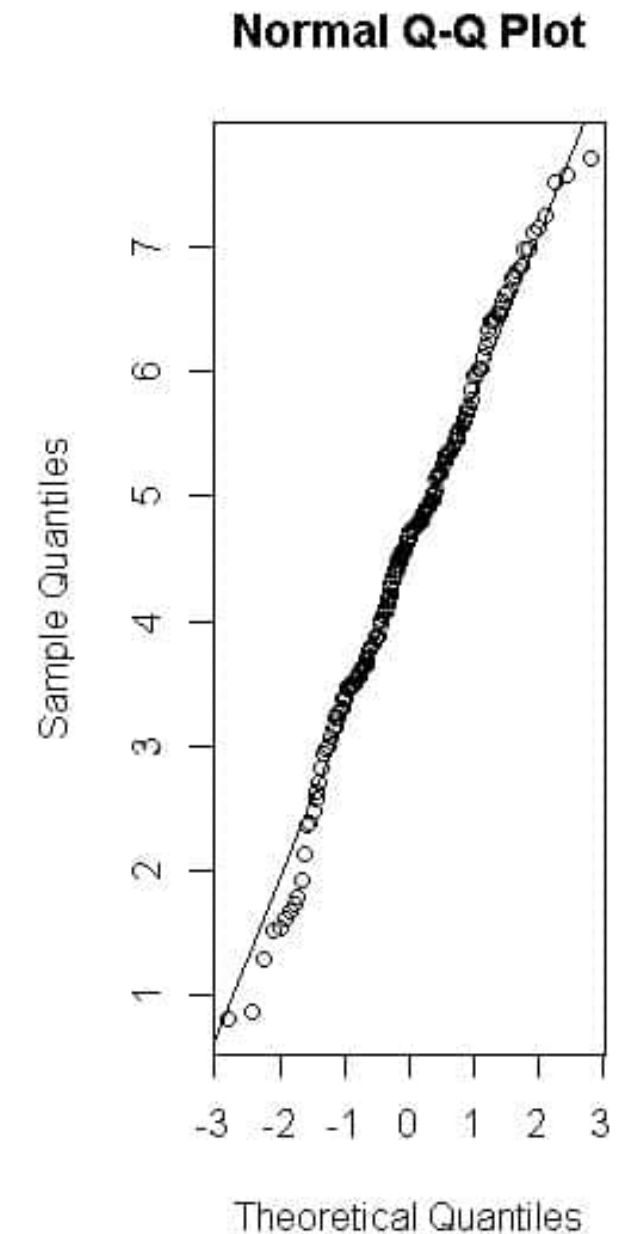
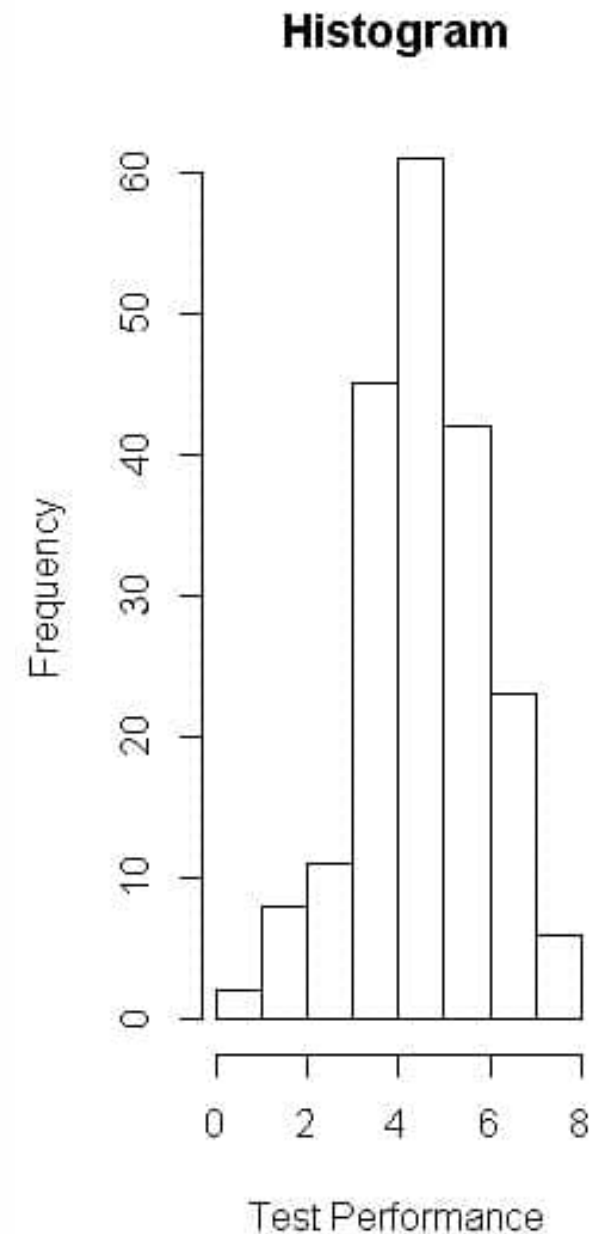
Conditional model
for performance

Conditional on age, performance follows **normal distribution**
mean defined by a **quadratic function** of age
 $a + b \cdot \text{age} + c \cdot \text{age}^2$ (three parameters: a , b , and c)
and variance σ^2 (one parameter)

The Data: Performance

**Examine marginal distribution
of performance ($n = 200$)
via Histogram and Normal Q-Q plot**

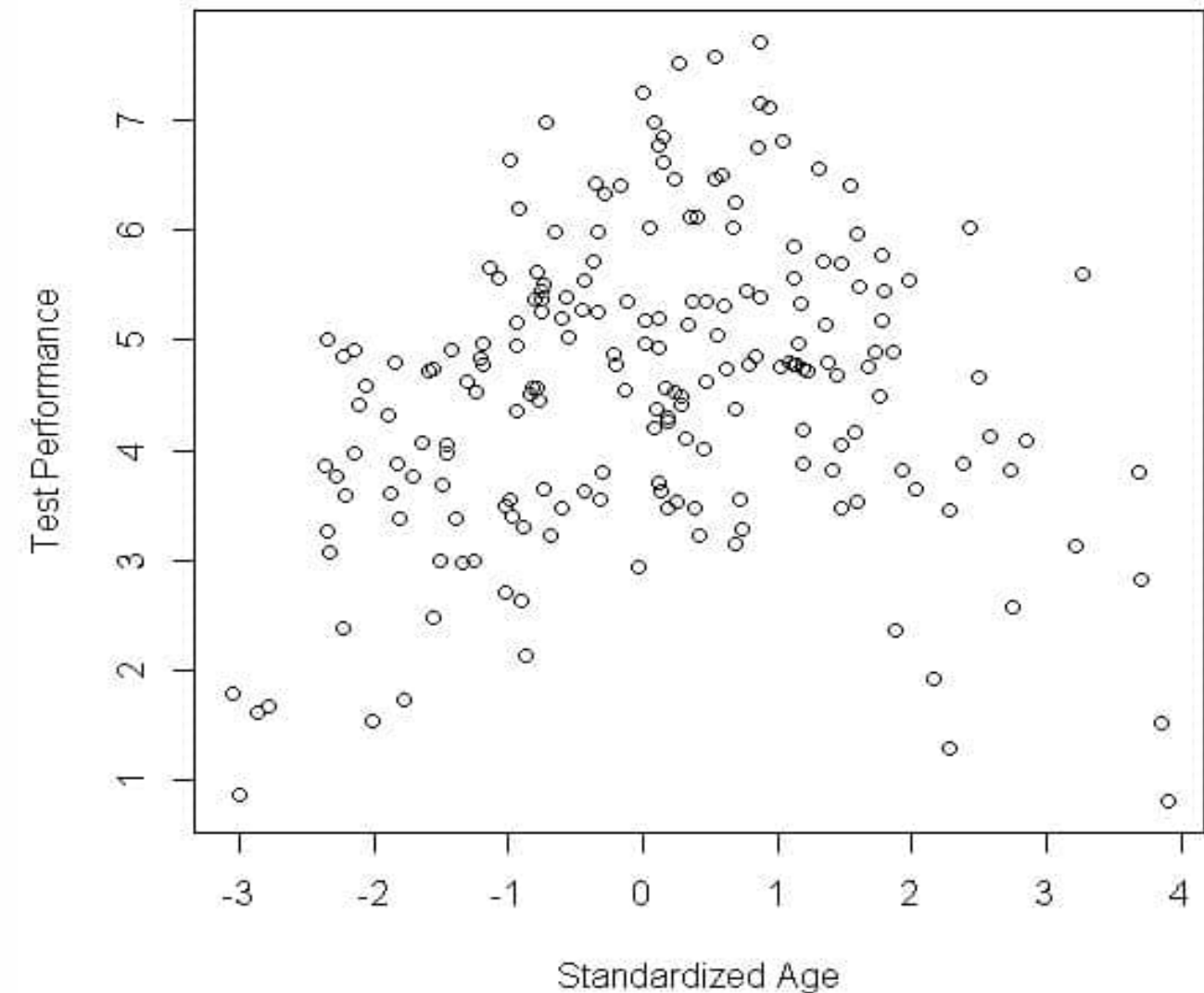
**Does the normal distribution
seem like a reasonable model
for performance?**



The Data: Performance and Age

Visualize relationship
between age and test performance
via scatter plot

Support for our theory regarding
curvilinear relationship?



Fit the “Mean-Only” Model

Fit **regression model** to performance data

$$\text{perf} = m + e$$

1st parameter
(*unknown constant*)

m = marginal mean

Fit the “Mean-Only” Model

Fit **regression model** to performance data

$$\text{perf} = m + e$$

1st parameter
(*unknown constant*)

m = marginal mean

e = random error defining each observation's deviation from the overall mean m,

Errors are **normally distributed** with mean 0 and variance σ^2

where $e \sim N(0, \sigma^2)$

2nd parameter

Fit the “Mean-Only” Model

Fit **regression model** to performance data

$$\text{perf} = m + e$$

where $e \sim N(0, \sigma^2)$

Parameter Estimates

Estimate of overall (marginal) mean $m = 4.57$ points

with $SE = 0.10$ points

→ support overall mean non-zero!

Estimate of $\sigma^2 = 1.82$ (points²)

Fit the “Mean-Only” Model

Fit **regression model** to performance data

$$\text{perf} = m + e$$

where $e \sim N(0, \sigma^2)$

Need to check this!

Parameter Estimates

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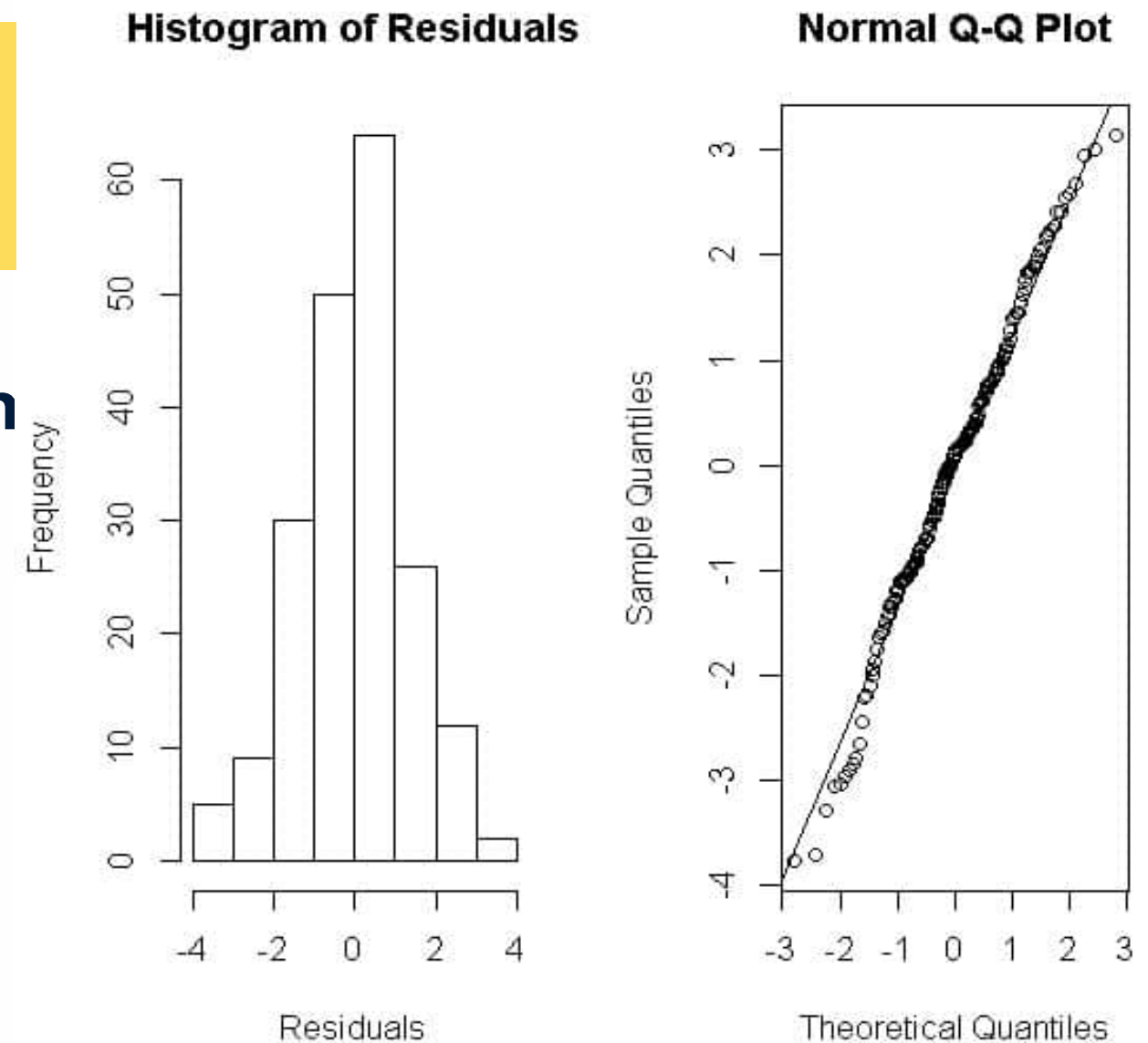
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Assess Fit of “Mean-Only” Model

Residuals = realized values of random errors
= observed performance – estimated mean m

- **Examine realized residuals via histogram and normal Q-Q plot**
to see if normal model is good fit for data
- **If normal model was not good fit**, would see large deviations from normality in realized residuals.



Fit the Conditional Model

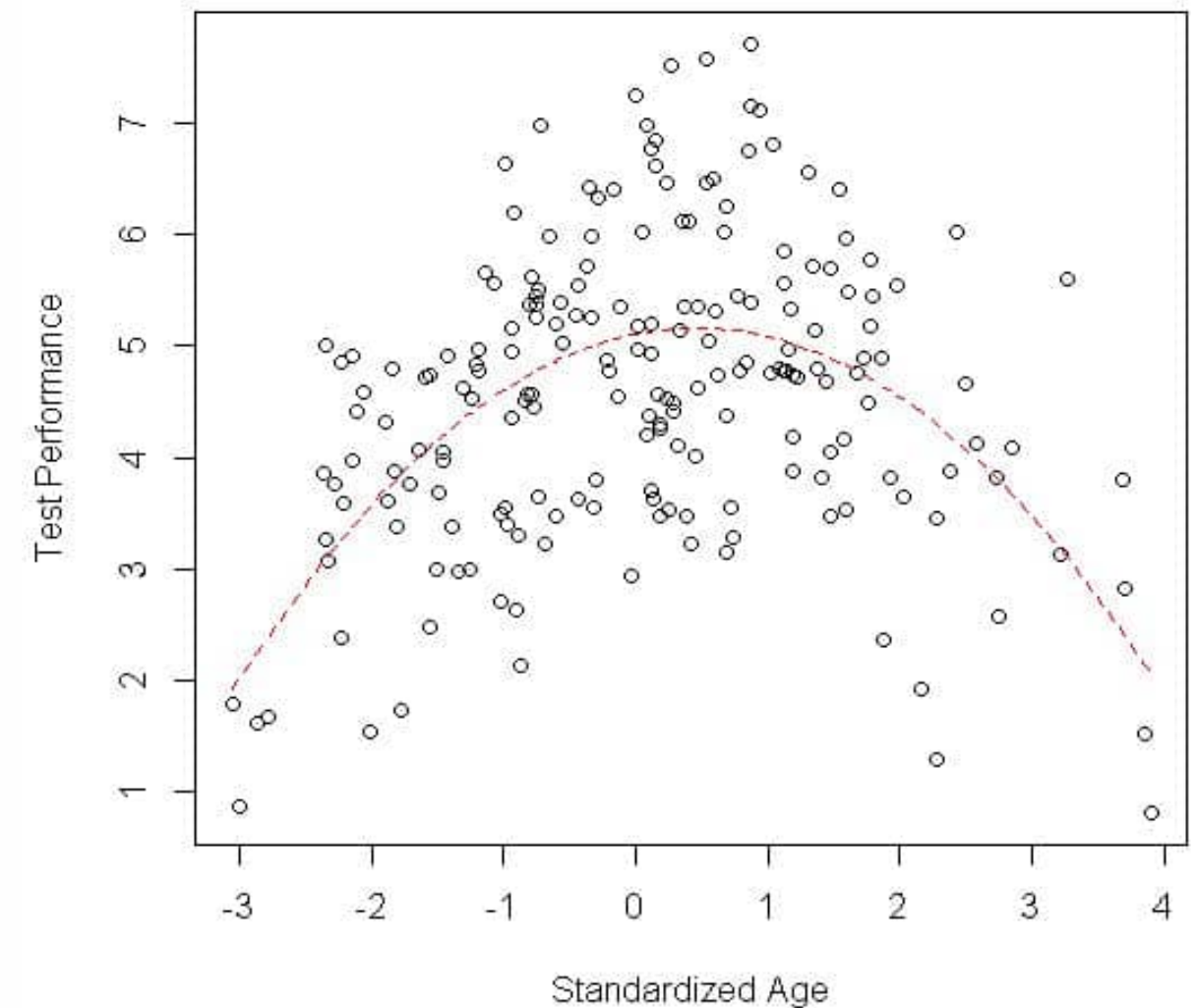
Fit **regression model**: regress performance on age and age²

$$\text{perf} = a + b \cdot \text{age} + c \cdot \text{age}^2 + e, \quad \text{where } e \sim \mathbf{N}(0, \sigma^2)$$

a, b, and c = three parameters

e = random error

Errors are normally distributed



Fit the Conditional Model

Fit **regression model**: regress performance on age and age²

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Parameter Estimates

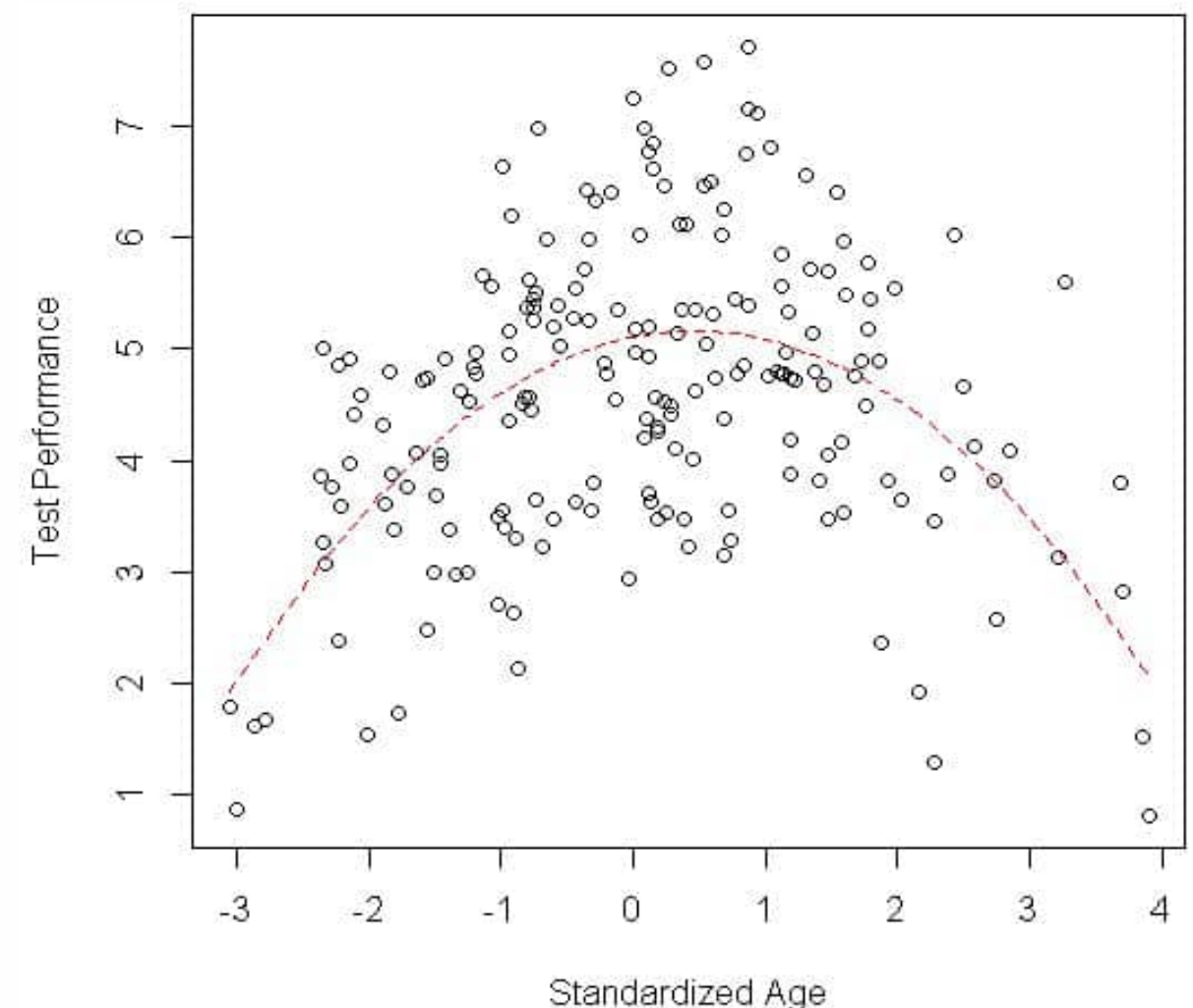
Estimate of $a = 5.11$ (SE = 0.10)

Estimate of $b = 0.24$ (SE = 0.06)

Estimate of $c = -0.26$ (SE = 0.03)

→ support a, b, c non-zero!

Estimate of $\sigma^2 = 1.29$ (points²)



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Parameter Estimates

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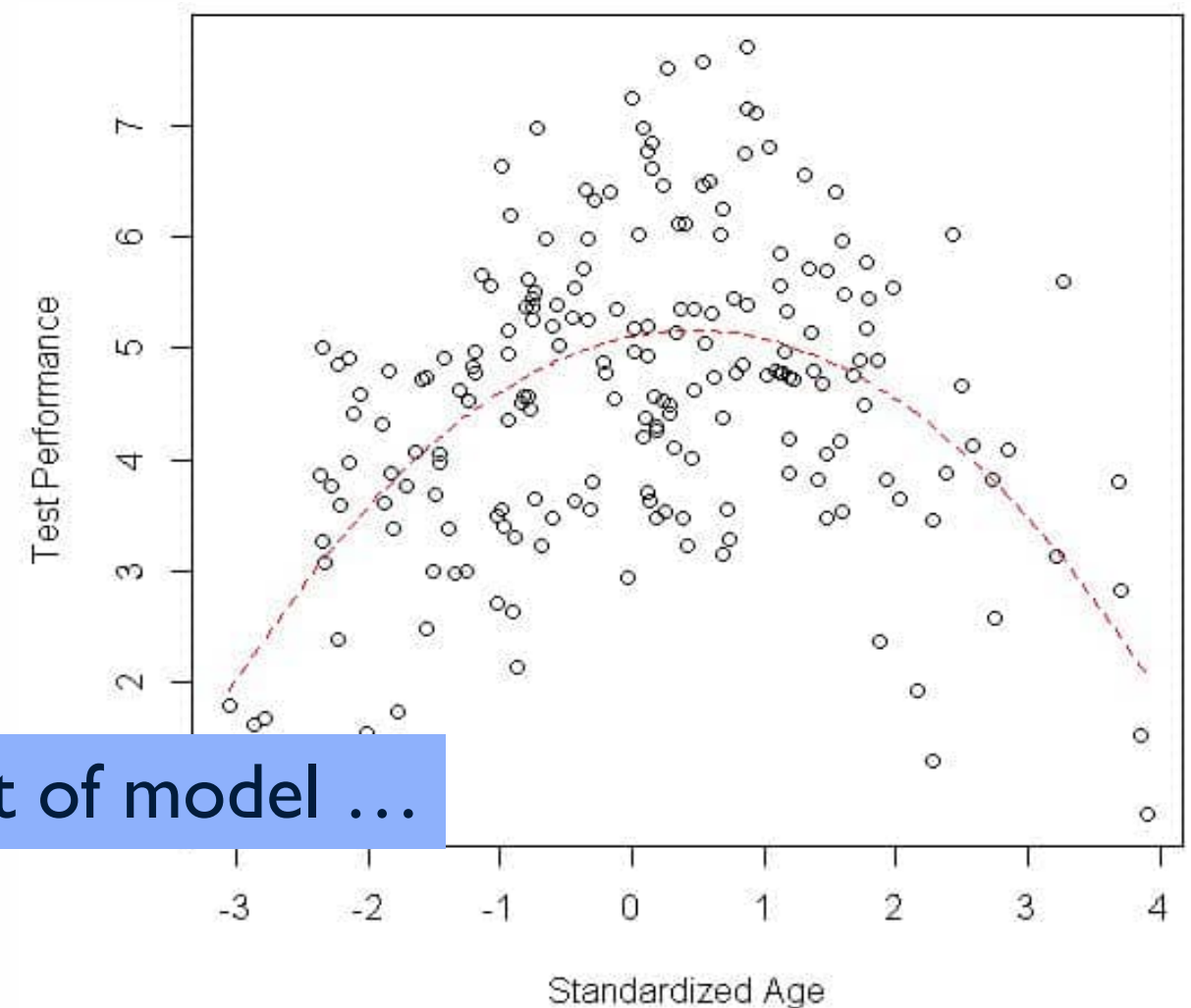
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Assess fit of model ...

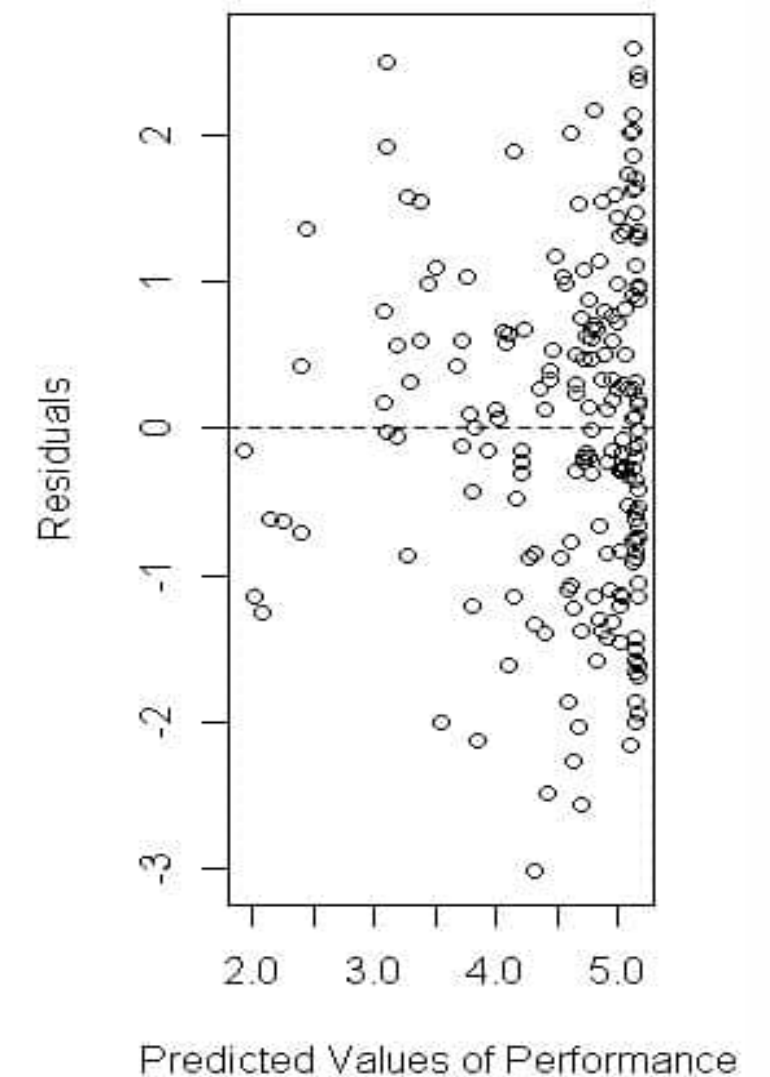
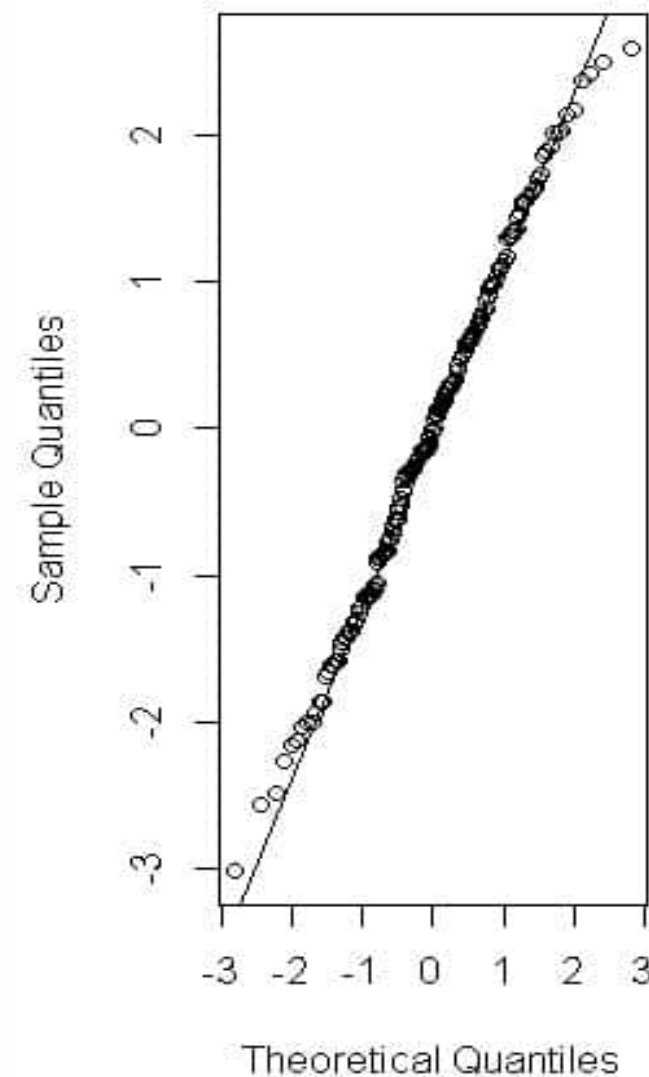


Assess Fit of Conditional Model

See if residuals (realized values of e):

- appear to be normally distributed

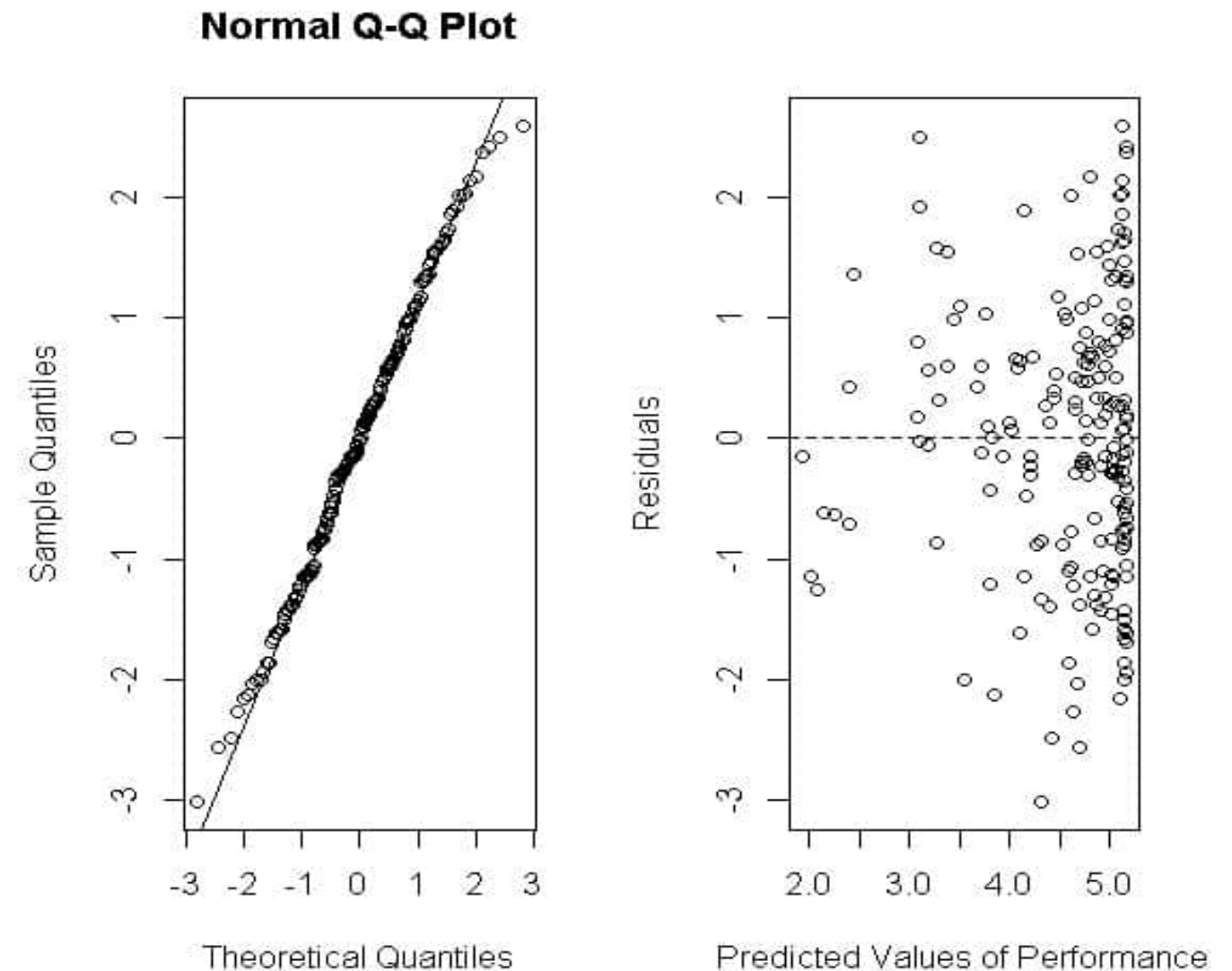
Normal Q-Q Plot



Assess Fit of Conditional Model

See if residuals (realized values of e):

- appear to be normally distributed
- are symmetrically distributed around zero with **constant variance**
(as function of **predicted values** of performance, given estimates of parameters a , b , and c)



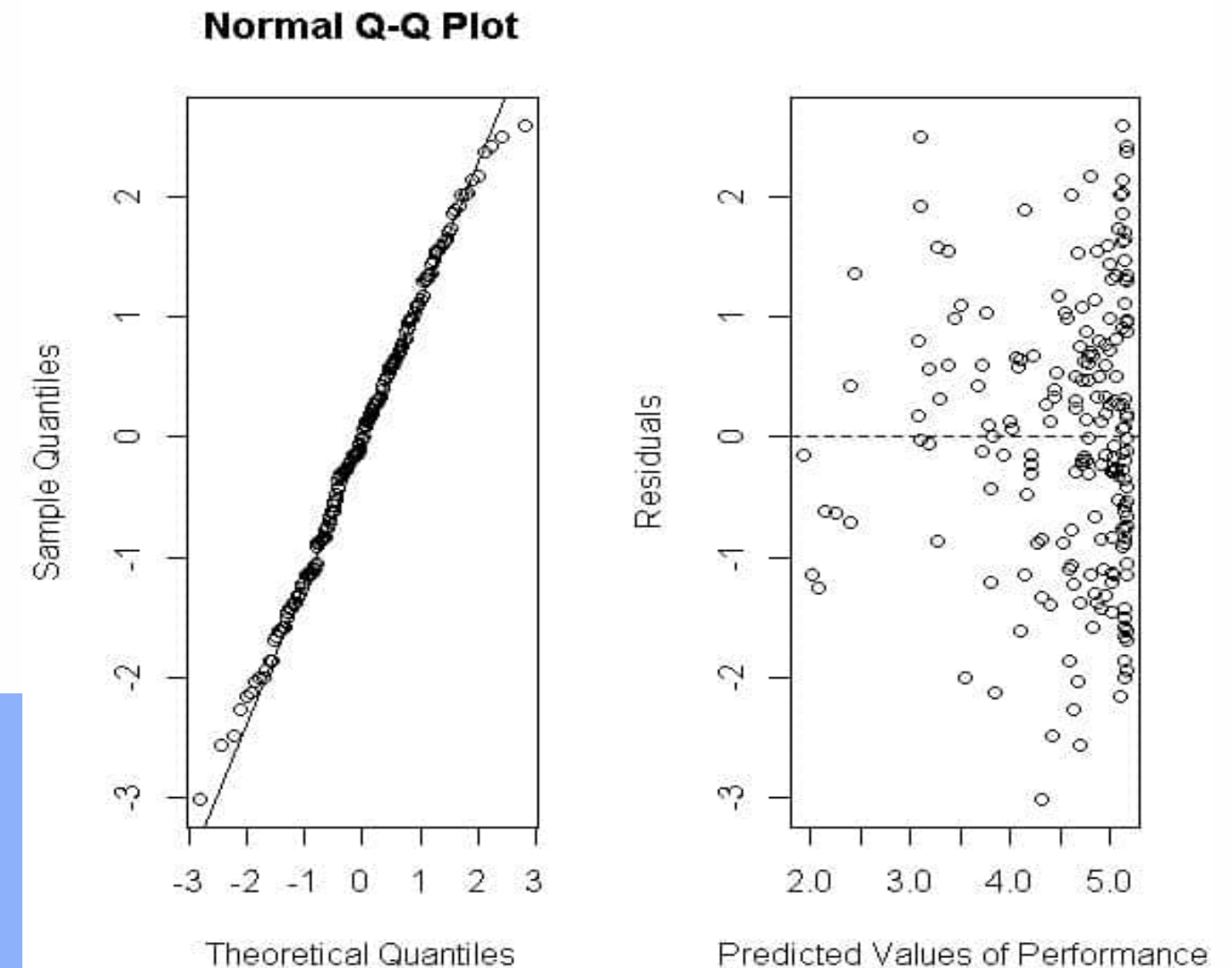
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- are symmetrically distributed around zero with **constant variance**
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Model fit looks good!

Could predict performance well,
given standardized age ... can we do better?



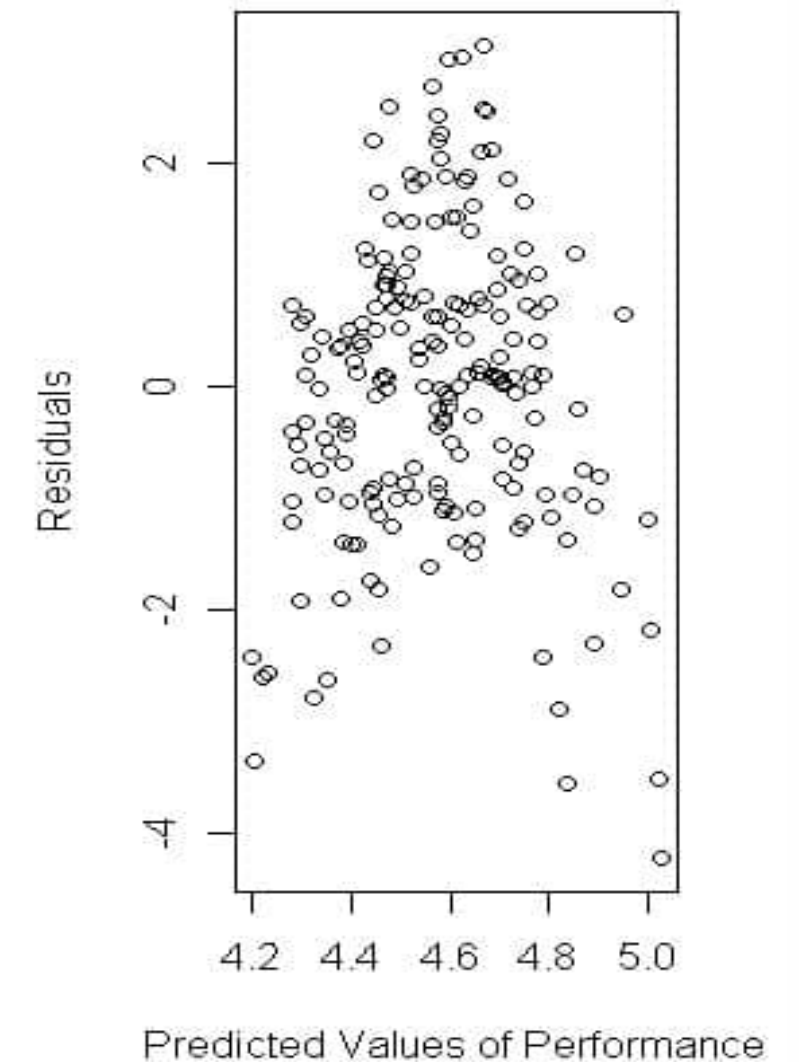
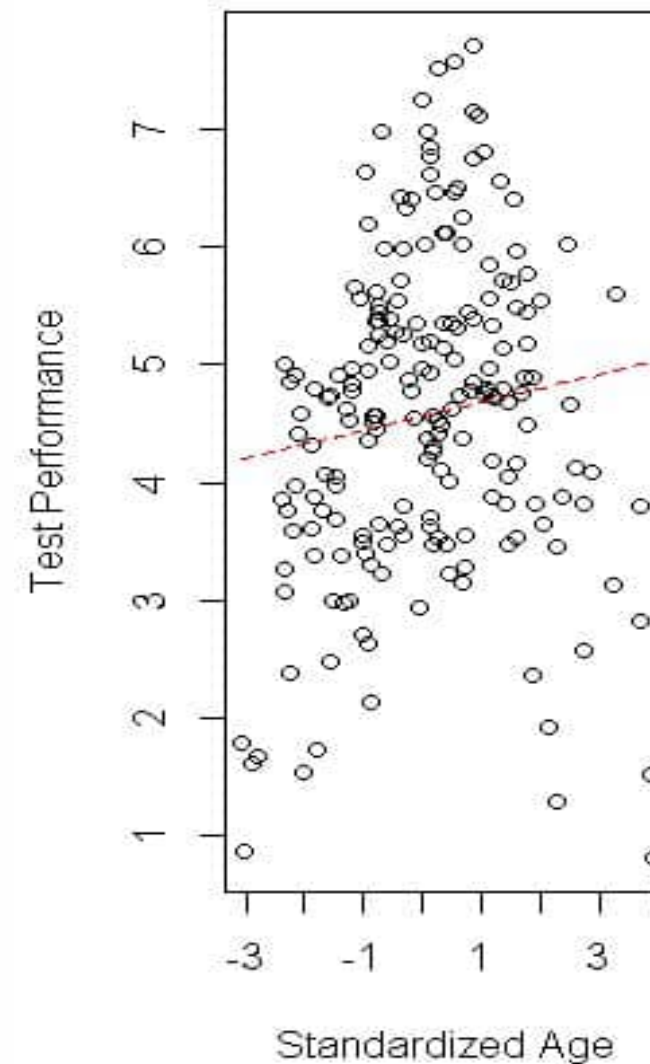
A model that does not fit well...

What if ...?

Fit **misspecified model** to data, assuming a **linear** relationship of performance and age?

Model fit looks poor ...

residuals are **not** symmetrically scattered around zero;
poor predictions at low and high values of age, higher σ^2



What did we see and what is next?

- **We have** ... introduced idea of fitting parametric models to data and assessing model fit
- **We will** ... talk about different types of variables interested in modeling, different types of data sets depending on study design (and implications for modeling), and different approaches to estimation and inference when fitting models
- **We will** ... discuss specific examples of modeling in detail!