

Multilevel Logistic Regression Models

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Model Specification

Multilevel model for binary dependent variable Y, measured on person i within cluster j

$$\ln\left[\frac{P(y_{ij}=1)}{1-P(y_{ij}=1)}\right] = \operatorname{logit}[P(y_{ij}=1)] = \beta_0 + \beta_1 x_{1ij} + u_{0j} + u_{1j} x_{1ij}$$
Fixed effects

Random effects



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$$\uparrow \qquad \uparrow \qquad \uparrow$$
Fixed effects

Random effects

Could use multilevel specification if desired!



Model Specification, cont'd

Same distributional assumptions about random cluster effects: normally distributed, mean vector 0, unique variances and covariances



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Recall: Multilevel model, because have explicit interest in estimating variance of random cluster effects!



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When we fit generalized linear regression models to non-normal outcomes and include random effects, estimation is more difficult mathematically ~ clear motivation is important!



Estimating the Model Parameters



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- I) approximate likelihood function
- 2) find parameter estimates that maximize approximate likelihood



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One approach = adaptive Gaussian quadrature
Simulation studies = works well in variety of scenarios
Deep dive: Reading by Kim et al. (2013)



Testing the Model Parameters

Compute confidence intervals or test hypotheses for model parameters

Test null hypotheses (e.g., fixed effect is zero, or variance component is zero – random effects don't vary!), can use **likelihood ratio testing**



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Reading this week: Provides specific details on how to perform these types of tests for parameters in multilevel models!



Revisiting NHANES Example

- Logistic regression to model probability of ever smoking 100 cigarettes as function of selected predictors.
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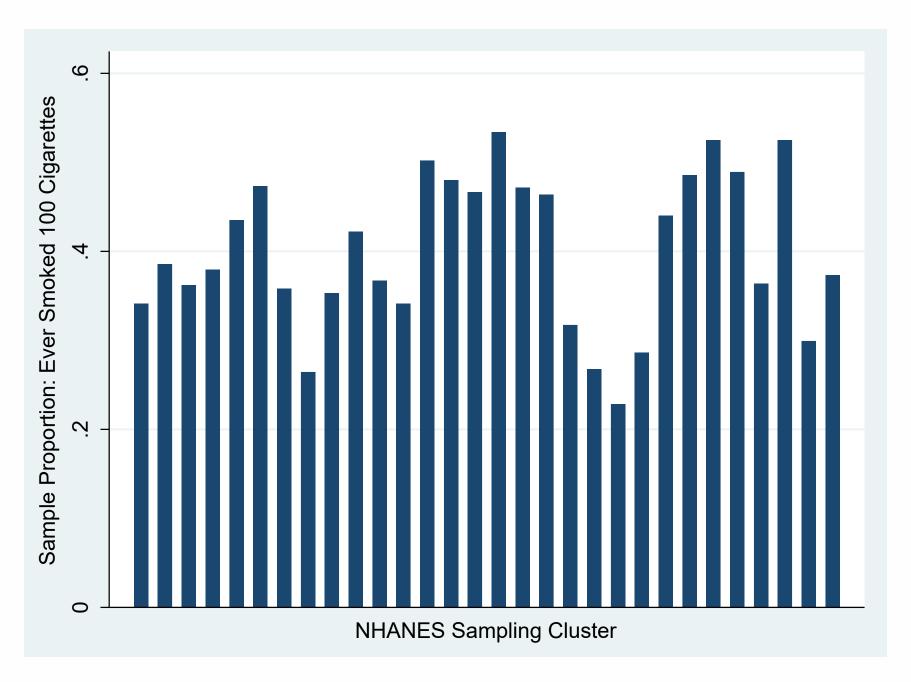


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- If smoking observations correlated within areas, standard errors in "naïve" logistic regression analysis <u>likely understated</u>.
- Plus **explicit interest** in estimating variance between sampling clusters in terms of probability of smoking!



Between-Cluster Variance in Smoking





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Logistic model including random effects of randomly sampled clusters (allows intercepts to randomly vary across clusters; no random slopes)



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- Same inferences regarding which predictors significant
- Slight changes in estimated fixed effects
- Standard errors of estimates are now larger!



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Even after adjusting for predictors, randomly sampled clusters still vary in terms of smoking prevalence!



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Including random cluster effects in logistic regression model improved fit



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Q: Look at distribution of **predicted** values of random interviewer effects, or EBLUPs ... **outliers**?

Remember: <u>no residuals</u> to worry about in simple logistic regression model!



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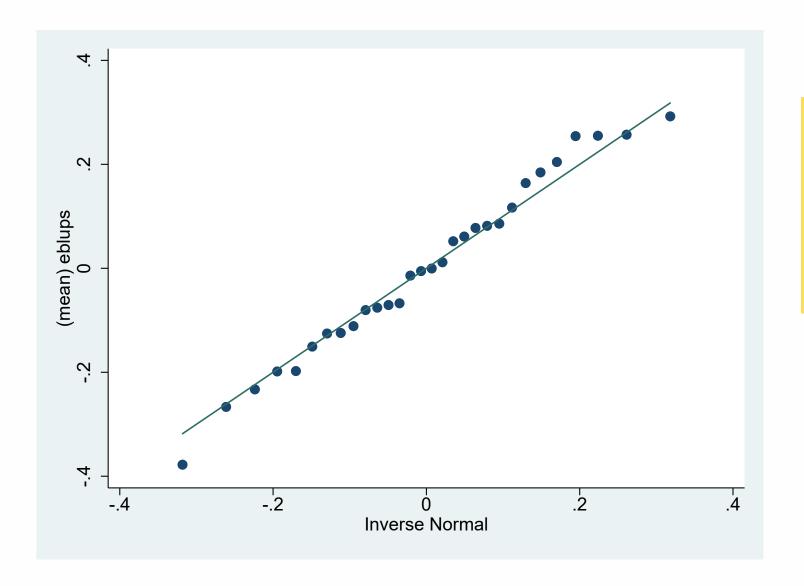
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Another Consideration:

Center continuous predictor variables so intercept is interpretable!



EBLUPs for Random Intercepts



QQ plot suggests
random effects on intercept
normally distributed
+ no outliers!



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- Same predictors of smoking still important!
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 explain variance by including fixed effects of cluster-level predictors
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- HOWEVER: When comparing variance components between multilevel models with different cluster-level fixed effects, both models must include same respondent-level fixed effects

Deeper Dive: Multilevel Analysis: Techniques and Applications, Hox et al, 3rd Edition, Section 6.5



What's Next?

- Full example: fitting multilevel models to longitudinal data with Python + making inference
- Marginal models for dependent data + alternatives for modeling clustered and longitudinal data sets