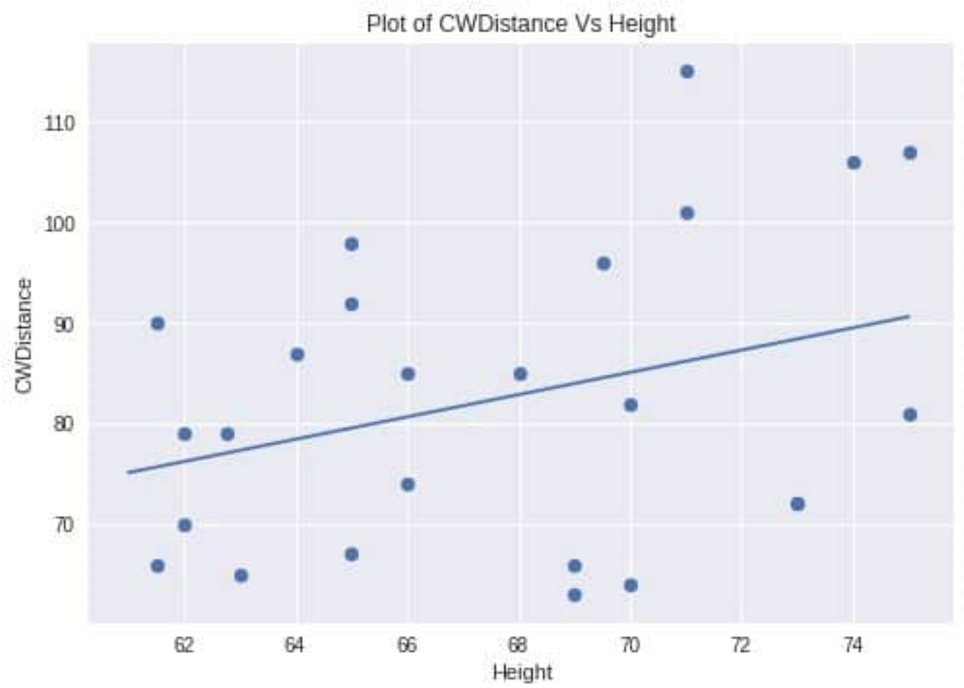
Linear Regression Inference

Brenda Gunderson

Regression of CWDistance on Height

Predicted CWDist = 7.5518 + 1.1076(height)



Regression Inference

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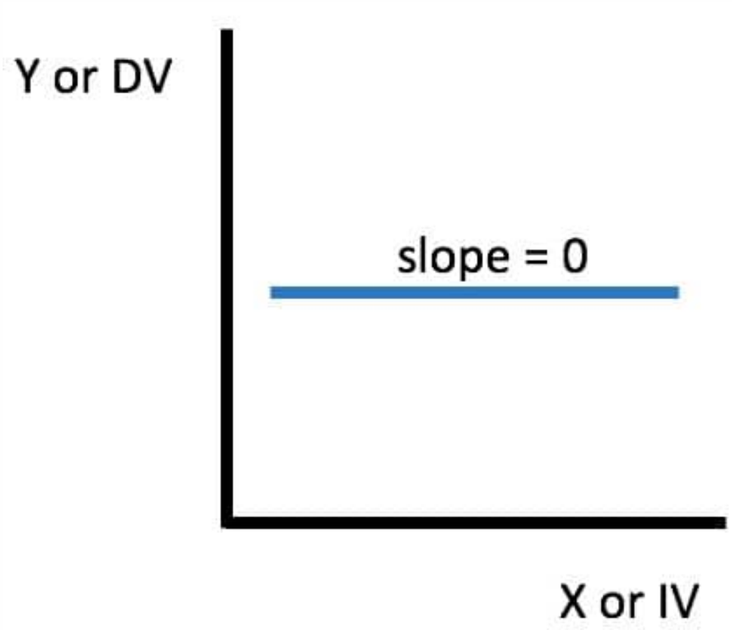
- **Think about it:**
  What would a slope = 0 imply?



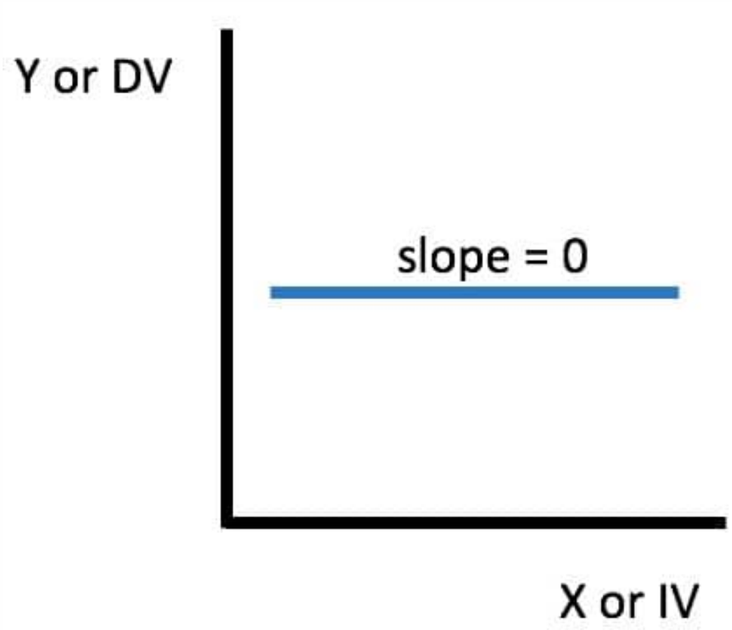
Regression Inference

Is there a significant (positive) linear relationship between CW Distance and Height?

- **Think about it:**
  What would a slope = 0 imply?

  - knowing x does not help to predict y

- Our slope b_1 = 1.1 ~ only an estimated slope



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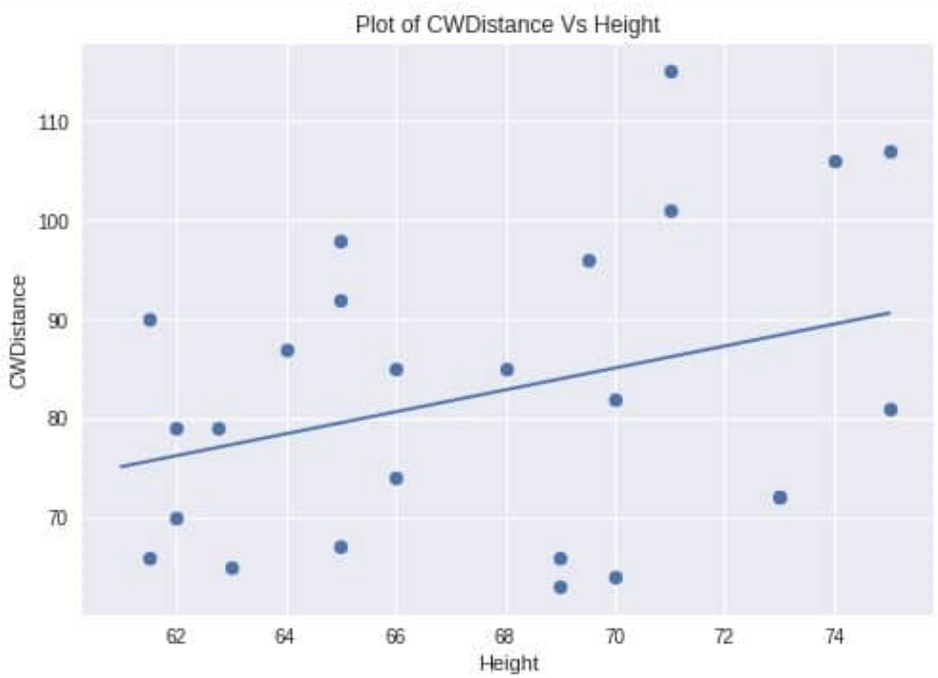
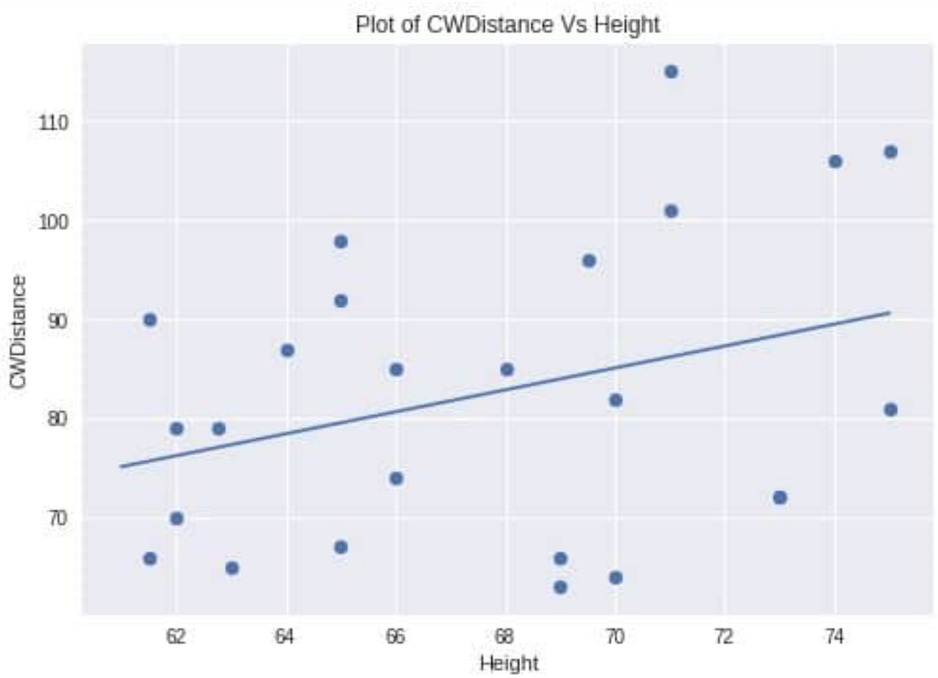
- **Imagine:** have population data on CW Distance and Height of all adults



Regression Inference

Is there a significant (positive) linear relationship between CW Distance and Height?

- **Imagine:** have population data on CW Distance and Height of all adults
- So there is an **underlying *true* slope b_1** want to assess if the true slope is 0 or not *(in our case is it positive > 0)*



Regression Inference

Test H_0: True slope (β_1) = 0

	coef	std err	t	P>|t|	[0.025	0.975]
const	7.5518	45.412	0.166	0.869	-86.391	101.494
Height	1.1076	0.670	1.653	0.112	-0.278	2.493

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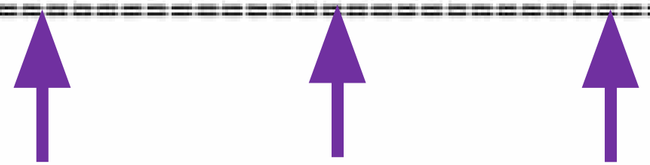

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Two-sided p-value of 0.112 is for testing H_a: True slope (β_1) $\neq$ 0
For significant positive association test H_a: True slope (β_1) > 0
p-value would be 0.112/2 = 0.056 *(marginally significant)*

Regression Inference

95% Confidence Interval for True slope (β_1)

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With 95% confidence, the population mean change in cartwheel distance for one inch increase in height is estimated to be anywhere from 0.2 inches shorter to 2.5 inches longer.

More Regression Inference

Used our regression line to **estimate mean cartwheel distance for all adults who are 64 inches tall** to be 78.4 inches

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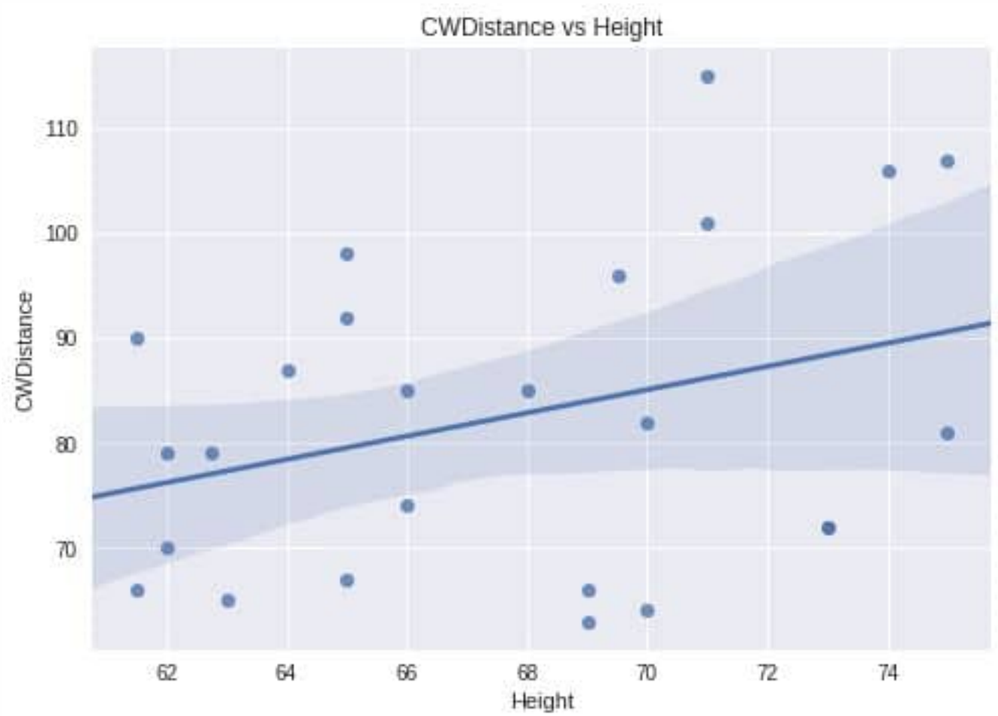
95% Confidence Interval Bands
for Mean CW Distance based on Height



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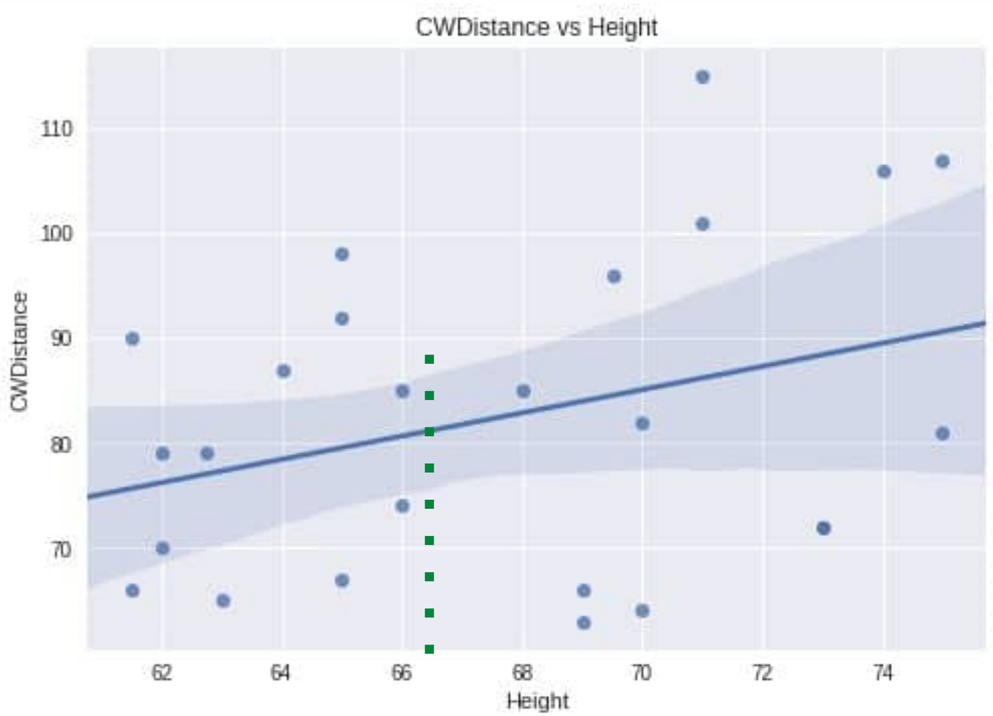

Notes:

1. **Intervals** are **narrower** for **values closer to sample mean** height of 67.6 inches

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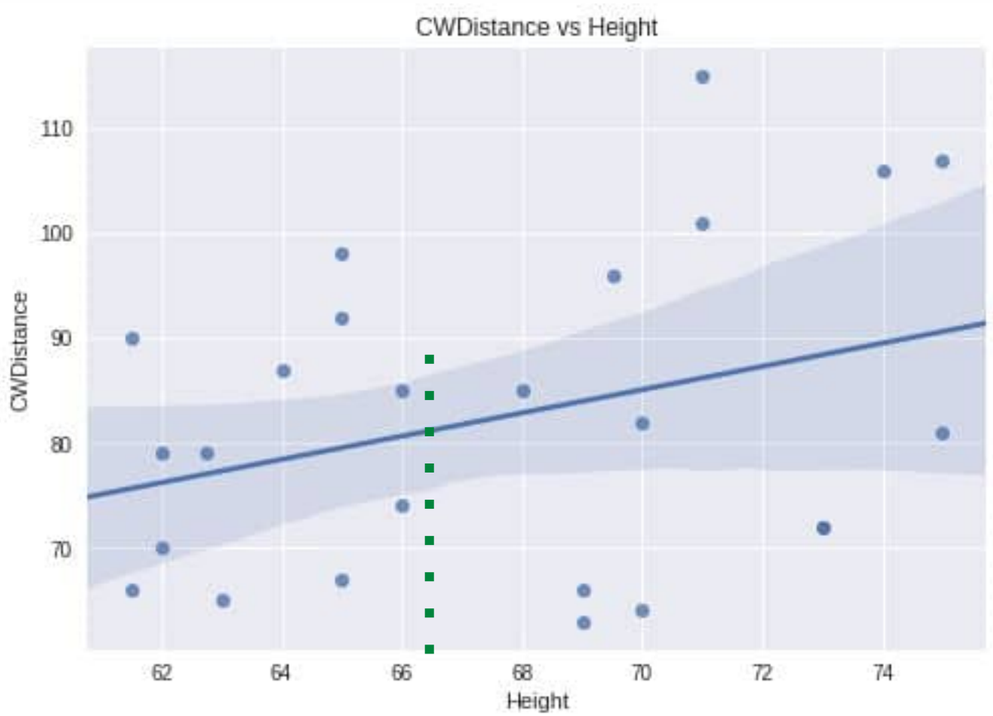


Notes:

1. **Intervals** are **narrower** for **values closer to sample mean** height of 67.6 inches
2. **Prediction Interval** for Individual Response (wider than corresponding CI for mean)

Underlying Assumptions

Fit (population) **regression model:** regressed cart wheel distance on height

$$\textbf{CWDist} = b_0 + b_1\,(\textbf{height}) + e, \quad \textbf{where } e \sim N(0, \sigma^2)$$

Need to check this!

b_0 and b_1 are two parameters

e = random error

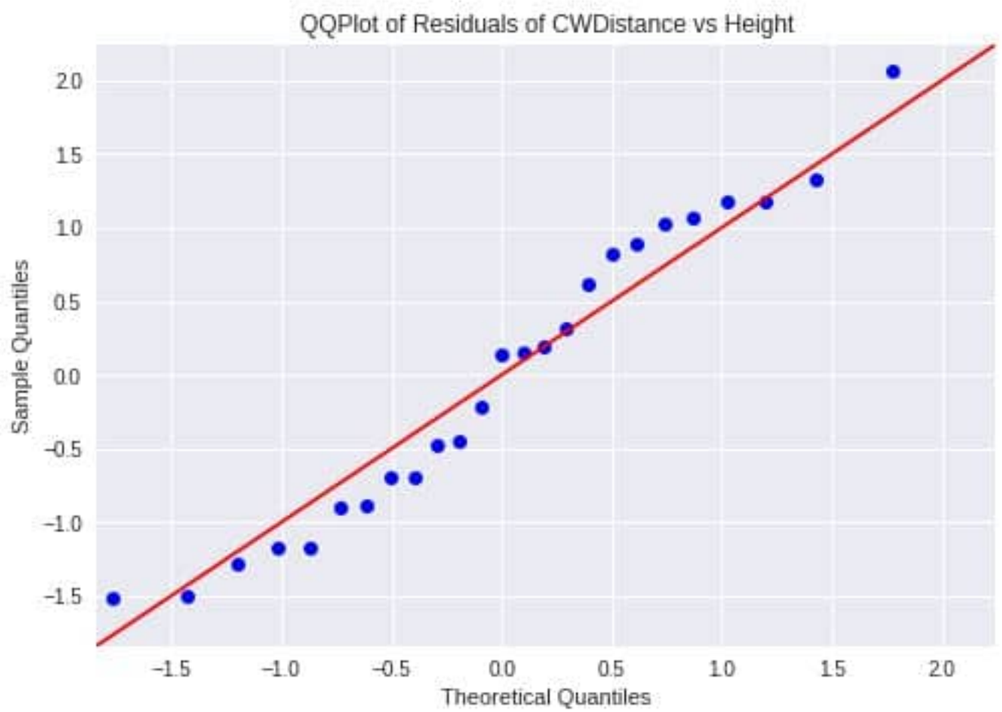
Errors are **normally distributed**

Checking Assumptions

True errors $e \sim N(0, \sigma^2)$

See if residuals (realized values of e):

- appear to be **normally distributed**



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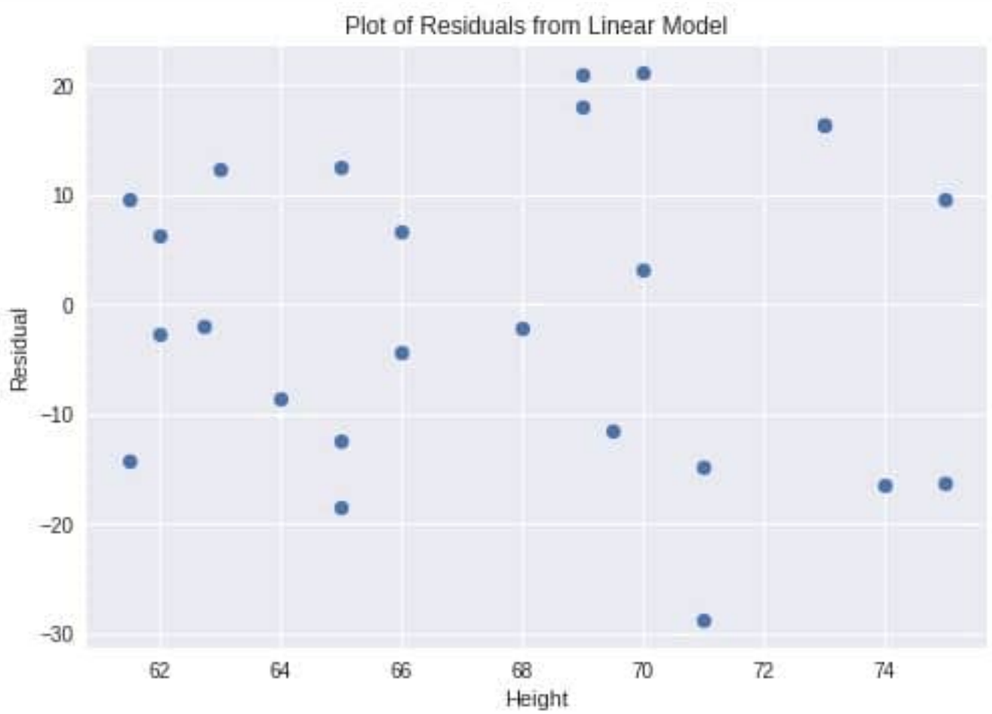
- Estimate of σ = 14.5 inches



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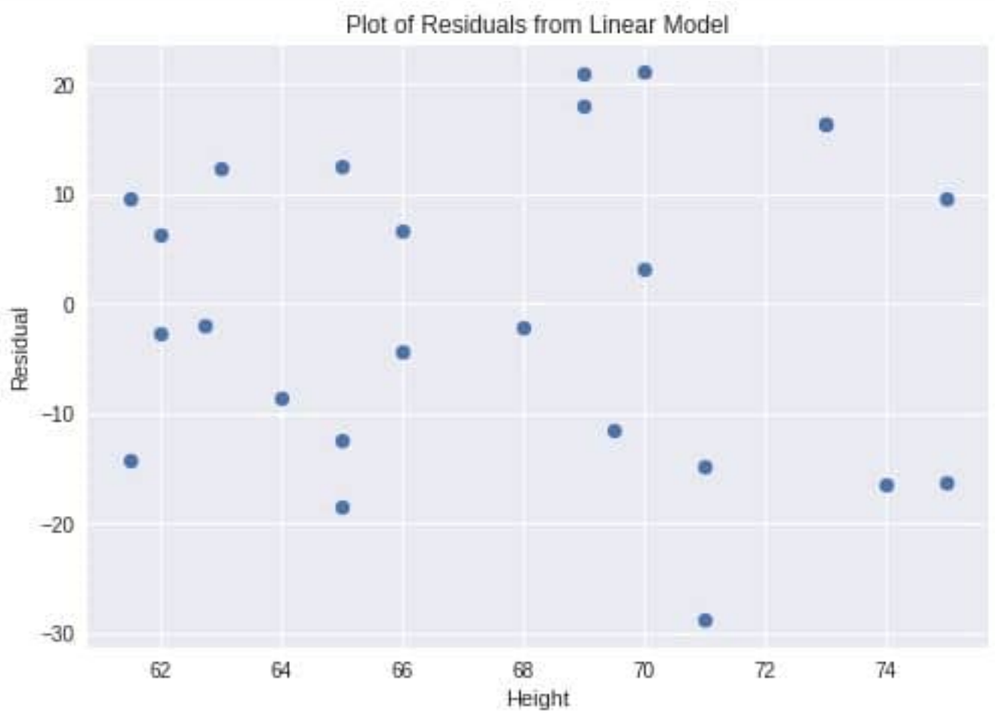

Model fit looks fine
... can we do better?

Adding a second variable

Does knowing if they actually *completed* the cartwheel make a difference in terms of cartwheel distance?

	ID	Age	Gender	GenderGroup	Glasses	GlassesGroup	Height	Wingspar	CWDistance	Complete	CompleteGroup	Score
0	1	56	F	1	Y	1	62.0	61.0	79	Y	1	7
1	2	26	F	1	Y	1	62.0	60.0	70	Y	1	8
2	3	33	F	1	Y	1	66.0	64.0	85	Y	1	7
3	4	39	F	1	N	0	64.0	63.0	87	Y	1	10
4	5	27	M	2	N	0	73.0	75.0	72	N	0	4

Regression Results

Predicted CWDist = -7.0457 + 1.2557(Height) + 6.0190(Complete)

OLS Regression Results

Dep. Variable:	CWDistance	R-squared:	0.135
Model:	OLS	Adj. R-squared:	0.056
Method:	Least Squares	F-statistic:	1.712
Date:	Mon, 26 Nov 2018	Prob (F-statistic):	0.204
Time:	05:06:55	Log-Likelihood:	-100.95
No. Observations:	25	AIC:	207.9
Df Residuals:	22	BIC:	211.6
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-7.0457	48.805	-0.144	0.887	-108.261	94.170
Height	1.2557	0.696	1.804	0.085	-0.188	2.699
Complete	6.0190	7.077	0.851	0.404	-8.657	20.695

Omnibus:	1.786	Durbin-Watson:	1.876
Prob(Omnibus):	0.409	Jarque-Bera (JB):	1.078
Skew:	0.137	Prob(JB):	0.583
Kurtosis:	2.020	Cond. No.	1.13e+03

Regression Results: Interpreting Coefficients

Predicted CWDist = -7.0457 + 1.2557(Height) + 6.0190(Complete)

Two adults with same completion status whose height differ by 1 inch tend to have cart wheel distances differing by 1.26 inches.

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Comparing adult who completed cartwheel with one of same height who did not: completer will on average have a CW Distance of 6 inches longer.

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Comparing adult who completed cartwheel with one of same height who did not: completer will on average have a CW Distance of 6 inches longer.

Height coefficient of 1.26 is only meaningful
when comparing two adults of the same completion status.
Complete coefficient of 6 is only meaningful
when comparing two adults of the same height.

Visualizing Regression Results

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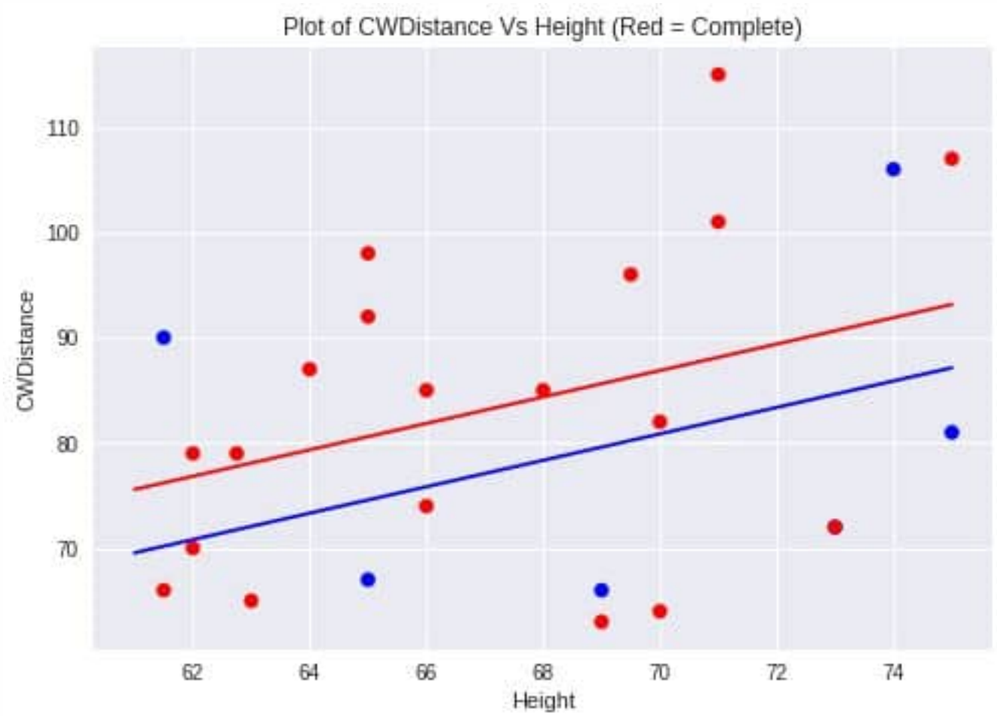

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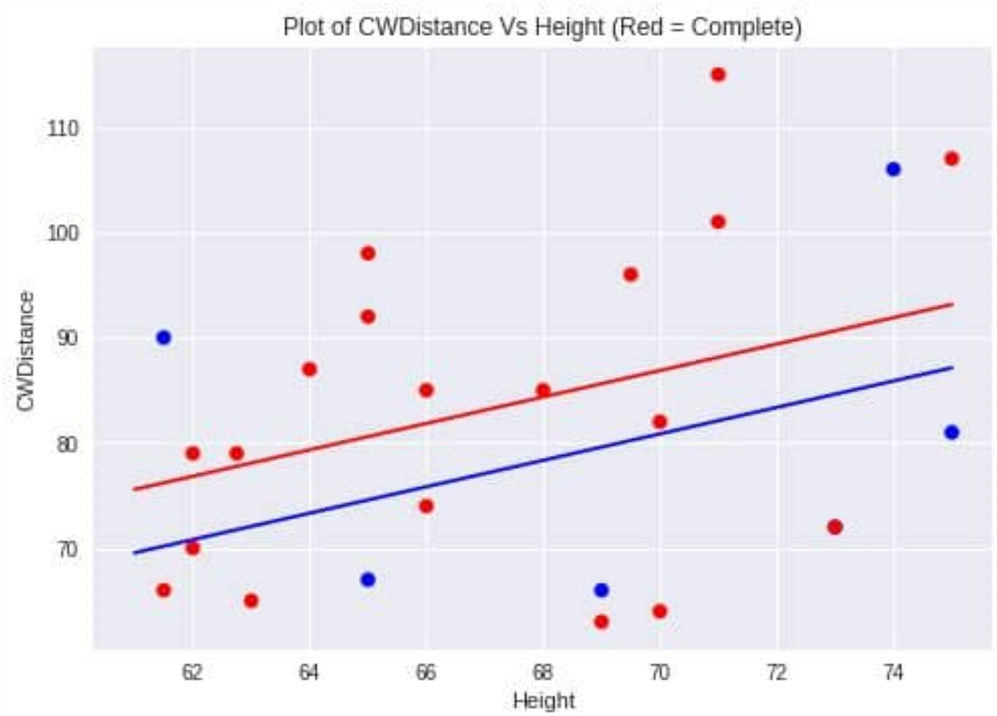

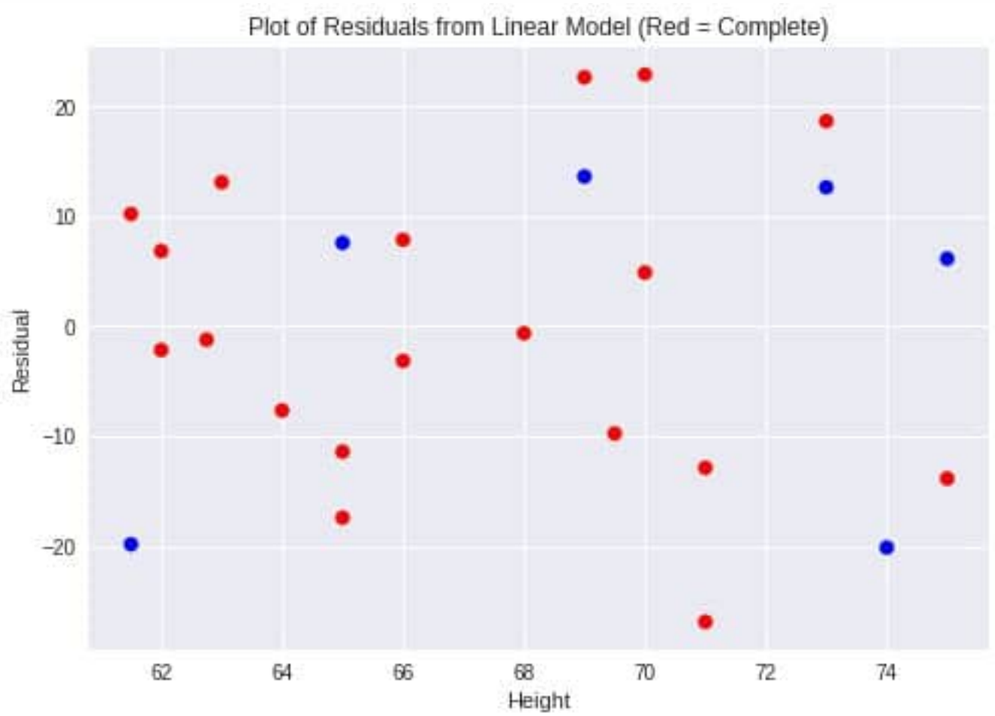

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	coef	std err	t	P>|t|
Intercept	-7.0457	48.805	-0.144	0.887
Height	1.2557	0.696	1.804	0.085
Complete	6.0190	7.077	0.851	0.404

After adjusting for completion status, does there appear to be a significant positive linear relationship between CW Distance and Height?

PAUSE HERE to provide time for IVQ

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After adjusting for completion status,

Estimate of Height coefficient = 1.26 (SE = 0.7)

p-value for assessing significant positive association = 0.085/2 = 0.0425

Estimate of σ = 14.6 inches

Summary

- **Regression** for predicting a quantitative response (DV) based on one or more explanatory variables (IV) (quantitative or categorical)
- **Inference** side: Confidence Intervals and Hypothesis Tests
- **Assumptions** for Inference
- ***Coming up next:***
  Regression models when the **response (DV) is binary** called Logistic Regression