

# Comparing Means for Two Independent Samples: An Example

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#### Example: Comparing Means in Two Groups

#### **Research Question:**

Considering African-American adults living in the U.S. in 2015-2016, did males and females have significantly different mean systolic blood pressure?

#### Inference Approaches:

- Form a confidence interval for the difference in the two means
- Perform a two-sample t-test for the difference in the two means
- Be sure to check assumptions!



#### Approach 1: Form a Confidence Interval

**Males:** Mean = 131.01, standard deviation = 20.59, n = 536**Females:** Mean = 125.79, standard deviation = 19.06, n = 599

- Best Point Estimate: Difference in sample means is 131.01 125.79 = 5.22 mmHg
- Interpretation: In 2015-2016, we estimate the mean systolic blood pressure for all male black adults was 5.22 mmHg higher than that for all female black adults.



#### Approach 1: Form a Confidence Interval

Males: Mean = 131.01, standard deviation = 20.59, n = 536

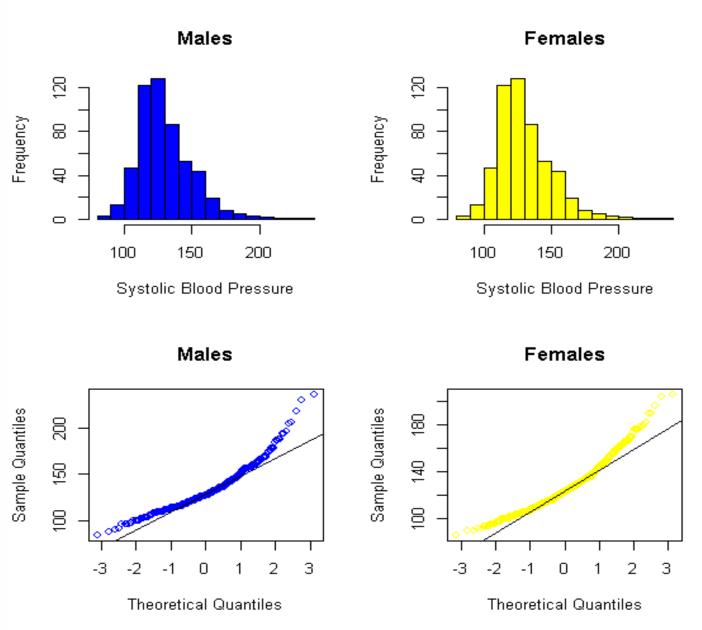
Females: Mean = 125.79, standard deviation = 19.06, n = 599

Note: sample standard deviations are similar.

Let's **check some assumptions** and decide which confidence interval approach is reasonable (pooled or unpooled?)



# Checking Assumptions



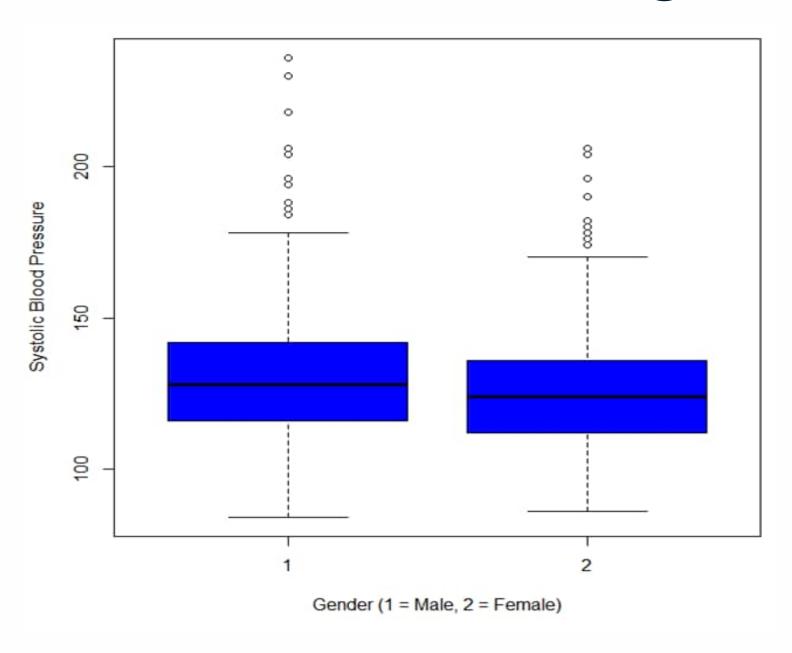
The distributions have a moderate right skew for both males and females...

The normality assumption is important for the two-sample t-test!

(large sample sizes, so can also rely on CLT)



# Checking Assumptions



Some evidence of slightly higher variance for males

→ consider robustness of results to different assumptions about variances in two groups!



#### Approach 1: Form a Confidence Interval

Assuming equal variance (pooling):

95% CI for difference in means = (2.91 mmHg, 7.53 mmHg)

Interval doesn't include  $0 \rightarrow Significant difference!$ 

Assuming unequal variance (no pooling):

95% CI for difference in means = (2.90 mmHg, 7.54 mmHg)

#### Same conclusion!

Result robust to possible violation of assumption.



# Approach 2: Two-Sample t-test

- Null: Males and Females have equal population means
- Alternative: Males and Females have different means

Alternative allows male mean to be either greater or less than female mean

→ two-tailed test need more evidence against null hypothesis to reject it!

Significance Level = 5%



# Approach 2: Two-Sample t-test

#### **Assumptions:**

- Normal distribution of blood pressure in each population
  - May not be reasonable, based on previous histograms and QQ plots for each sample
- Same standard deviation in each population
  - Somewhat reasonable and techniques robust,
     but we can examine both pooled and unpooled test results



# Approach 2: Two-Sample t-test

Assuming equal variance (pooling):

$$t = 4.436$$
,  $df = 1133$ , p-value < 0.001

We reject the null hypothesis; means are different!

Assuming unequal variance (no pooling):

$$t = 4.417$$
,  $df = 1094.3$ , p-value < 0.001

#### Same conclusion!

Result robust to possible violation of assumption.



# What if Normality Doesn't Hold?

- Not convinced the variable of interest follows a normal distribution in each population?
  - → non-parametric test that does not assume normality
- Non-parametric analog of two-sample t-test
  - = Mann-Whitney test
  - ~ compares locations of distributions using medians



### What if Normality Doesn't Hold?

- Mann-Whitney Test Result: p-value < 0.00 l</li>
  - We reject null that both distributions have identical "locations"
- Conclusion is robust to potential violations of normality!

Consistent evidence of robust difference in central tendencies of the two distributions, regardless of assumptions made and approach to inference used



#### What's Next?

How to compare two means based on paired data

#### **Examples:**

- blood pressure measurements
   from right and left arms of same subjects
- 2. measures of a continuous outcome before and after an intervention