

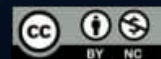


# Setting up a Test for a Population Proportion

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Reed Coots

Statistics Course Developer



# Hypothesis Testing

**Why do we do Hypothesis Tests?**

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Could the value of the parameter be \_\_\_\_\_?

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## Why do we do Hypothesis Tests?

Could the value of the parameter be \_\_\_\_\_?

Use data to help support that claim

# C.S. Mott Children's Hospital Poll

C.S. Mott Children's Hospital conducted a national poll on an issue in children's health, sleep habits. We will be looking at an example about lack of sleep in teens.

# Research Question

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**Parameter of Interest** -  $p$

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**Population** - Parents with a teenager (age 13-18)

**Parameter of Interest** -  $p$

**Test for a significant increase in the proportion of parents with a teenager who believe that electronics and social media is the cause for lack of sleep.**



# Hypotheses

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$$H_0 : p = 0.52$$

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$$H_a : p \neq 0.52$$

IVQ

What symbol should replace the ?

# Hypotheses

$$H_0 : p = 0.52$$

$$H_a : p > 0.52$$

“Significant Increase”

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$$\alpha = 0.05$$

# Survey Results

A random sample of **1018** parents with a teenager was taken and **56%** said they believe electronics and social media was the cause of their teenager's lack of sleep.



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We also need a **large enough sample size** to ensure our distribution of sample proportions is normal

That is:  $n \cdot p$  be at least 10  $\rightarrow n \cdot p_o$   
 $n \cdot (1-p)$  be at least 10  $\rightarrow n \cdot (1-p_o)$

# Checking Assumptions

Random Sample

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Random Sample ✓

In background  
information

# Checking Assumptions

Random Sample ✓

$$n \cdot p_o$$

$$n \cdot (1 - p_o)$$

$$p_o = 0.52$$

# Checking Assumptions

Random Sample ✓

$$n \cdot p_o = 1018 \cdot (0.52) = 529 \quad \checkmark$$

$$n \cdot (1-p_o) = 1018 \cdot (1-0.52) = 489 \quad \checkmark$$







# Testing a One Population Proportion

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# Hypotheses

$$H_0 : p = 0.52$$

$$H_a : p > 0.52$$

Best Estimate of  $p$  is  $\hat{p} = 0.56$

Where  $p$  is the population proportion of parents with a teenager who believe that electronics and social media is the cause of their teenager's lack of sleep

$$\alpha = 0.05$$

# Test Statistic

**Best estimate - Hypothesized estimate**

**Standard error of estimate**

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**Standard error of estimate**

$$\frac{\hat{p} - p_o}{s.e.}$$

$$s.e. (\hat{p}) = \sqrt{\frac{p \cdot (1-p)}{n}} \quad \longrightarrow \quad s.e. (\hat{p}) = \sqrt{\frac{p_o \cdot (1-p_o)}{n}}$$

# Test Statistic

$$\frac{\hat{p} - p_o}{s.e.}$$

$$\text{Null } s.e. (\hat{p}) = \sqrt{\frac{p_o \cdot (1 - p_o)}{n}}$$



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$$\frac{\hat{p} - p_o}{s.e.} \quad \text{Null } s.e. (\hat{p}) = \sqrt{\frac{p_o \cdot (1 - p_o)}{n}}$$

$$Z = \frac{0.56 - 0.52}{0.0157}$$

$$\mathbf{Z = 2.555}$$

# Test Statistic Interpretation

$$Z = 2.555$$

That means that our observed sample proportion is 2.555  
null standard errors above our hypothesized population  
proportion

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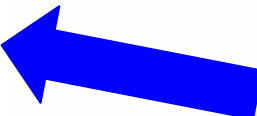
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$$\frac{\hat{p} - p_o}{s.e.(\hat{p})}$$

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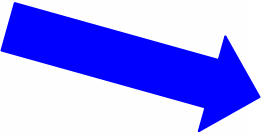
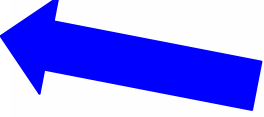
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 **Centers Data**

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**Scales Data**   $\frac{\hat{p} - p_o}{s.e.(\hat{p})}$   **Centers Data**



# The P-Value



# Conclusions

$$p\text{-value} = 0.0053 < \alpha = 0.05$$

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$p\text{-value} = 0.0053 < \alpha = 0.05$

Reject the null hypothesis ( $H_0: p = 0.52$ )

There is sufficient evidence to conclude that the population proportion of parents with a teenager who believe that electronics and social media is the cause for lack of sleep is greater than 52%.

# Summary

- 4 main steps to a hypothesis test
  - Stating hypothesis & select significance level ( $\alpha$ )
  - Checking assumptions
  - Calculating a test statistic and getting a p-value from the test statistic
  - Drawing a conclusions from the p-value

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- 4 main steps to a hypothesis test
  - Stating hypothesis & select significance level ( $\alpha$ )
  - Checking assumptions
  - Calculating a test statistic and getting a p-value from the test statistic
  - Drawing a conclusions from the p-value
- The Z test statistic distribution is  $N(0,1)$