Available online at www.sciencedirect.com
ScienceDirect
Nuclear Instruments and Methods in Physics Research 00 (2014) 1–3
Procedia Computer Science

High-accuracy Determination of the Absolute Efficiency Curve of a HPGe Clover Detector Over an Energy Range of 240-3300 keV

Usman Rizwan, Aaron Chester, Krzystof Starosta

Simon Fraser University

Abstract

Keywords:

1. Introduction

2. Relative Full energy Peak Efficiency Curve

The basic idea that motivates this work is that in gamma-ray spectroscopy spectroscopy it is important to know the absolute efficiency of a detector.

3. Analysis Technique

Absolute efficiency is defined as

$$\epsilon_{\text{abs.}}(E_i) = \frac{N}{I_\gamma \times br(E_1) \times t \times K}, \tag{1}$$

where $\epsilon_{\text{abs.}}(E_i)$ is the full-energy peak absolute efficiency, N is the number of counts in the full-energy peak, I_γ gamma-decay probability per decay of the parent, $br(E_i)$ is the branching ratio for the gamma-ray energy E_i and t is the total live-time and K is the factor correcting for dead time. The ratio of peak counts for any two gamma-ray lines from the same source is

$$\frac{N_1}{N_2} = \frac{\epsilon_{\text{abs.}}(E_1)}{\epsilon_{\text{abs.}}(E_1)} \times \frac{br(E_1)}{br(E_2)}, \tag{2}$$

Let $\epsilon_{\text{rel.}}$ = full-energy peak relative efficiency efficiency. By definition,

$$\epsilon_{\text{rel.}} = F \times \epsilon_{\text{abs.}}, \tag{3}$$

where F is a constant that scales $\epsilon_{\text{abs.}}$. This definition ensures that the ratio of relative efficiency is the same as the ratio of absolute efficiency,

$$\frac{\epsilon_{\text{rel.}}(E_1)}{\epsilon_{\text{rel.}}(E_2)} = \frac{\epsilon_{\text{abs.}}(E_1)}{\epsilon_{\text{abs.}}(E_2)}, \tag{4}$$

Absolute efficiency over a broad energy range of 240-3300 keV can be modelled using a quadratic equation in the log-log scale.

$$\log(\epsilon_{\text{abs.}}(E_i)) = a + b(\log(E_i)) + c(\log(E_i))^2 \tag{5}$$

or

$$\log(\epsilon_{\text{rel.}}(E_i)) = a + b(\log(E_i)) + c(\log(E_i))^2 + \log(F) \tag{6}$$

The basic idea that motivates this work is to demonstrate a simple method to calibrate a gamma-ray detector over a broad energy range using just one calibrated source which emits atleast two gamma-rays of different energy.

The data points were were fitted on a log-log scale including a normalising factor for each data set measured with an uncalibrated source. In the present case the activity of the ^{60}Co source was known with an accuracy of 1%. This allows determination of an absolute efficiency function and the correction factor for each uncalibrated source in a single step. Relative efficiency curve has the same fitting parameters as the absolute efficiency curve but is only shifted by an unknown factor which can be found.

This function was used in order to include the uncalibrated in the fitting procedure. Once calibrated source and three uncalibrated sources have been used. The normalizing factors are also fitted parameters.

$$\begin{aligned} \ln(\epsilon_{\text{rel.}}(E_i)) = a &+ b(\ln(E_i)) + c(\ln(E_i))^2 + \\ &a + b(\ln(E_i)) + c(\ln(E_i))^2 + d \\ &a + b(\ln(E_i)) + c(\ln(E_i))^2 + e \\ &a + b(\ln(E_i)) + c(\ln(E_i))^2 + f \end{aligned} \tag{7}$$

To find the values of the parameters, chi-squared method was used.

$$\begin{aligned} \chi^2 = &\sum_{i=1}^{N} \{\epsilon_{abs.}(E_i) - [a + b(\ln(E_i)) + c(\ln(E_i))^2]\}^2 \\ &\sum_{j=1}^{N} \{\epsilon_{rel.}(E_j) - [a + b(\ln(E_j)) + c(\ln(E_j))^2 + d]\}^2 \\ &\sum_{k=1}^{N} \{\epsilon_{rel.}(E_k) - [a + b(\ln(E_k)) + c(\ln(E_k))^2 + e]\}^2 \\ &\sum_{k=1}^{N} \{\epsilon_{rel.}(E_k) - [a + b(\ln(E_k)) + c(\ln(E_k))^2 + f]\}^2 \end{aligned} \tag{8}$$

Take a partial derivative of Eq. 8 with respect to a, b, c, d, e and f and set them equal to zero.

$$\frac{\partial\chi^2}{\partial a} = 0 \,;\; \frac{\partial\chi^2}{\partial b} = 0 \,;\; \frac{\partial\chi^2}{\partial c} = 0 \,;\; \frac{\partial\chi^2}{\partial d} = 0 \,;\; \frac{\partial\chi^2}{\partial e} = 0 \,;\; \frac{\partial\chi^2}{\partial f} = 0$$

$$MV = X \tag{9}$$

Let $\log(E_m)$=x_m, where m=i,j ,k or l.

$$M = \begin{pmatrix} \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}\frac{1}{\sigma_m^2}) & \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}\frac{x_m}{\sigma_m^2}) & \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}\frac{x_m^2}{\sigma_m^2}) & \sum\limits_{j=1}^{N_j}\frac{1}{\sigma_m^2} & \sum\limits_{k=1}^{N_k}\frac{1}{\sigma_m^2} & \sum\limits_{l=1}^{N_l}\frac{1}{\sigma_m^2} \\ \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}x_m) & \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}x_m^2) & \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}x_m^3) & \sum\limits_{j=1}^{N_j}x_j & \sum\limits_{k=1}^{N_k}x_k & \sum\limits_{l=1}^{N_l}x_l \\ \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}x_m^2) & \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}x_m^3) & \sum\limits_{m=i,j,k,l}(\sum\limits_{m}^{N_m}x_m^4) & \sum\limits_{j=1}^{N_j}x_j^2 & \sum\limits_{k=1}^{N_k}x_k^2 & \sum\limits_{l=1}^{N_l}x_l^2 \\ \sum\limits_{j=1}^{N_j}1 & \sum\limits_{j=1}^{N_j}x_j & \sum\limits_{j=1}^{N_j}x_j^2 & \sum\limits_{j=1}^{N_j}1 & 0 & 0 \\ \sum\limits_{k=1}^{N_k}1 & \sum\limits_{k=1}^{N_k}x_k & \sum\limits_{k=1}^{N_k}x_k^2 & 0 & \sum\limits_{k=1}^{N_k}1 & 0 \\ \sum\limits_{l=1}^{N_l}1 & \sum\limits_{l=1}^{N_l}x_l & \sum\limits_{l=1}^{N_l}x_l^2 & 0 & 0 & \sum\limits_{l=1}^{N_l}1 \end{pmatrix}$$

$$V = \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix}$$

$$X = \begin{pmatrix} \sum\limits_{m=i,j,k,l}[\sum\limits_{m}^{N_m}\log(\epsilon_{abs.}(E_m))] \\ \sum\limits_{m=i,j,k,l}[\sum\limits_{m}^{N_m}\log(\epsilon_{abs.}(E_m))x_m] \\ \sum\limits_{m=i,j,k,l}[\sum\limits_{m}^{N_m}\log(\epsilon_{abs.}(E_m))x_m^2] \\ \sum\limits_{j=1}^{N_j}\log(\epsilon_{rel.}(E_j)) \\ \sum\limits_{k=1}^{N_k}\log(\epsilon_{rel.}(E_k)) \\ \sum\limits_{l=1}^{N_l}\log(\epsilon_{rel.}(E_l)) \end{pmatrix}$$

where M is the so-called design matrix, V is the column vector of parameters that have to be determined and X is the column vector of logarithm of measured efficiencies. The square of the uncertainity for the parameters in vertor V is given by the diagonal elements of the vector Ω:

$$\Omega = \frac{\sum\limits_{m=i,j,k,l}[\sum\limits_{m}^{N_m}\chi^2]}{p-n}M^{-1} = \begin{pmatrix} \sigma^2(a) & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ \Omega_{21} & \sigma^2(b) & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} \\ \Omega_{31} & \Omega_{32} & \sigma^2(c) & \Omega_{34} & \Omega_{35} & \Omega_{36} \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & \sigma^2(d) & \Omega_{45} & \Omega_{46} \\ \Omega_{51} & \Omega_{52} & \Omega_{53} & \Omega_{54} & \sigma^2(e) & \Omega_{56} \\ \Omega_{61} & \Omega_{62} & \Omega_{63} & \Omega_{64} & \Omega_{65} & \sigma^2(f) \end{pmatrix} \tag{10}$$

where p are the number of fitting points and n are the number of fitting parameters. p-n is thus the degree of independence.