Unit No. 4

Factorization and Algebraic Manipulation

Exercise No. 4.1

Question No. 1

Factorize by identifying common factors.

(i). $6x + 12$

Solution:

$6x + 12$

The greatest common factor (GCF) of 6 and 12 is 6.

$= 6(x + 2)$

(ii). $15y^2+20y$

Solution:

$15y^2 + 20y$

- The GCF of 15 and 20 is 5.
- The lowest power of y common in both terms is y.

$= 5y (3y + 4)$

(iii). $-12x^2 - 3x$

Solution:

$-12x^2 - 3x$

- The GCF of 12 and 3 is 3.
- The lowest power of x is x.
- Since both terms are negative, we factor out $-3x$.

$= -3x(4x + 1)$

(iv). $4a^2b + 8ab^2$

Solution:

$4a^2b + 8ab^2$

- The GCF of 4 and 8 is 4.
- The lowest power of a is a.
- The lowest power of b is b.

$4a^2b + 8ab^2$

$= 4ab(a + 2b)$

(v). $xy - 3x^2 + 2x$

Solution:

$xy \quad - \quad 3x^2 + 2x$

- The GCF of all terms is x.

= x(y − 3x + 2)

(vi). $3a^2b - 9ab^2 + 15ab$

Solution:

$3a^2b - 9ab^2 + 15ab$

- The GCF of 3, 9, and 15 is 3.
- The lowest power of a is a.
- The lowest power of b is b.

= 3ab(a − 3b + 5)

Question No. 2

Factorize and represent pictorially:

(i). 5x + 15

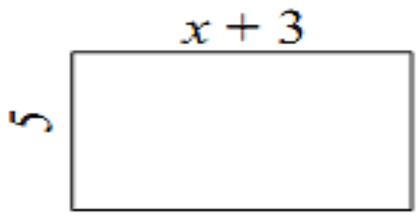
Solution:

5x + 15

- The greatest common factor (GCF) of 5 and 15 is 5.

= 5(x + 3)

Pictorial Form:



(ii). $x^2 + 4x + 3$

Solution:

$x^2 + 4x + 3$

- **We need two numbers whose product is 3 and sum is 4**
- **The numbers are 1 and 3 because:**

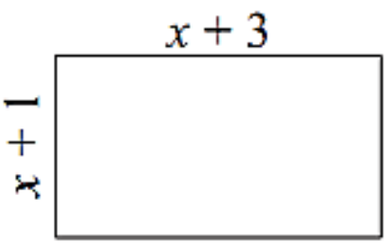
$1 \times 3 = 3$, $1 + 3 = 4$

$= x^2 + x + 3x + 3$

= x(x + 1) + 3(x + 1)

= (x + 1)(x + 3)

Pictorial Form:



(iii). $x^2 + 6x + 8$

Solution:

$x^2 + 6x + 8$

- We need two numbers whose product is 8 and sum is 6.
- The numbers are 2 and 4 because:

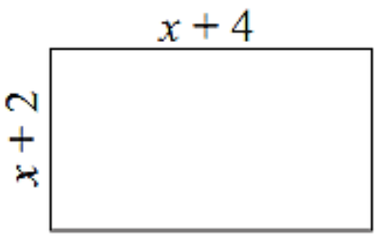
$2 \times 4 = 8, \qquad 2 + 4 = 6$

$= x^2 + 2x + 4x + 8$

$= x(x + 2) + 4(x + 2)$

$= (x + 2)\,(x + 4)$

Pictorial Form:



(iv). $x^2 + 4x + 4$

Solution:

$x^2 + 4x + 4$

- We need two numbers whose product is 4 and sum is 4.
- The numbers are 2 and 2 because:

$2 \times 2 = 4, \qquad 2 + 2 = 4$

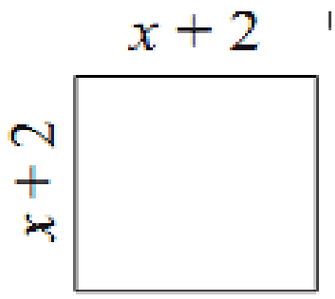
$= x^2 + 2x + 2x + 4$

$= x(x + 2) + 2(x + 2)$

$= (x + 2)(x + 2)$

$= (x + 2)^2$

Pictorial Form:



Question No. 3

Factorize:

(i). $x^2 + x - 12$

Solution:

$x^2 + x - 12$

- Find two numbers that multiply to -12 and add up to 1 (coefficient of x): (+4, -3)

$= x^2 + 4x - 3x - 12$

$= x(x + 4) - 3(x + 4)$

$= (x + 4)(x - 3)$

(ii). $x^2 + 7x + 10$

Solution:

$x^2 + 7x + 102$

- Find two numbers that multiply to 10 and add up to 7: (+5, +2)

$= x^2 + 5x + 2x + 10$

$= x(x + 5) + 2(x + 5)$

$= (x + 5)(x + 2)$

(iii). $x^2 - 6x + 8$

Solution:

$x^2 - 6x + 8$

- Find two numbers that multiply to 8 and add up to -6: (-4, -2)

$= x^2 - 4x - 2x + 8$

$= x(x - 4) - 2(x - 4)$

$= (x - 4)(x - 2)$

(iv). $x^2 - x - 56$

Solution:

$x^2 - x - 56$

- Find two numbers that multiply to -56 and add up to -1: (-8, +7)

$= x^2 - 8x + 7x - 56$

$= x(x - 8) + 7(x - 8)$

$= (x - 8)(x + 7)$

(v). $x^2 - 10x - 24$

Solution:

$x^2 - 10x - 24$

- Find two numbers that multiply to -24 and add up to -10: (-12, +2)

$= x^2 - 12x + 2x - 24$

$= x(x - 12) + 2(x - 12)$

$= (x - 12)(x + 2)$

(vi). $y^2 + 4y - 12$

Solution:

$y^2 + 4y - 12$

- Find two numbers that multiply to -12 and add up to 4: (+6, -2)

$= y^2 + 6y - 2y - 12$

$= y(y + 6) - 2(y + 6)$

$= (y + 6)(y - 2)$

(vii). $y^2 + 13y + 36$

Solution:

$y^2 + 13y + 36$

Find two numbers that multiply to 36 and add up to 13: (+9, +4)

$= y^2 + 9y + 4y + 36$

$= y(y + 9) + 4(y + 9)$

$= (y + 9)(y + 4)$

(viii). $x^2 - x - 2$

Solution:

$x^2 - x - 2$

- Find two numbers that multiply to -2 and add up to -1: (-2, +1)

$= x^2 - 2x + x - 2$

$= x(x - 2) + 1(x - 2)$

$= (x - 2)(x + 1)$

Question No. 4

Factorize:

(i). $2x^2 + 7x + 3$

Solution:

$2x^2 + 7x + 3$

- Multiply $2 \times 3 = 6$
- Find two numbers that multiply to 6 and add to 7 → (+6, +1)

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (2x + 1)(x + 3)$

(ii). $2x^2 + 11x + 15$

Solution:

$2x^2 + 11x + 15$

- Multiply $2 \times 15 = 30$
- Find two numbers that multiply to 30 and add to 11 → (+6, +5)

$= 2x^2 + 6x + 5x + 15$

$= 2x(x + 3) + 5(x + 3)$

$= (2x + 5)(x + 3)$

(iii). $4x^2 + 13x + 3$

Solution:

$4x^2 + 13x + 3$

- Multiply $4 \times 3 = 12$
- Find two numbers that multiply to 12 and add to 13 → (+12, +1)

$= 4x^2 + 12x + x + 3$

$= 4x(x + 3) + 1(x + 3)$

$= (4x + 1)(x + 3)$

(iv). $3x^2 + 5x + 2$

Solution:

$3x^2 + 5x + 2$

- Multiply $3 \times 2 = 6$
- Find two numbers that multiply to 6 and add to 5 → (+3, +2)

$= 3x^2 + 3x + 2x + 2$

$= 3x(x + 1) + 2(x + 1)$

$= (3x + 2)(x + 1)$

(v). $3y^2 - 11y + 6$

Solution:

$3y^2 - 11y + 6$

- Multiply $3 \times 6 = 18$
- Find two numbers that multiply to 18 and add to -11 → (-9, -2)

$= 3y^2 - 9y - 2y + 6$

$= 3y(y - 3) - 2(y - 3)$

$= (3y - 2)(y - 3)$

(vi). $2y^2 - 5y + 2$

Solution:

$2y^2 - 5y + 2$

- Multiply $2 \times 2 = 4$
- Find two numbers that multiply to 4 and add to -5 → (-4, -1)

$= 2y^2 - 4y - y + 2$

$= 2y(y - 2) - 1(y - 2)$

$= (2y - 1)(y - 2)$

(vii). $4z^2 - 11z + 6$

Solution:

$4z^2 - 11z + 6$

- Multiply $4 \times 6 = 24$
- Find two numbers that multiply to 24 and add to -11 → (-8, -3)

$= 4z^2 - 8z - 3z + 6$

$= 4z(z - 2) - 3(z - 2)$

$= (4z - 3)(z - 2)$

(viii). $6 + 7x - 3x^2$

Solution:

$6 + 7x - 3x^2$

- Multiply $6 \times (-3) = -18$
- Find two numbers that multiply to -18 and add to +7 → (+9, -2)

$= 6 - 2x + 9x - 3x^2$

$= 2(3 - x) + 3x(3 - x)$

$= (2 + 3x)(3 - x)$

$= (3x + 2)(3 - x)$