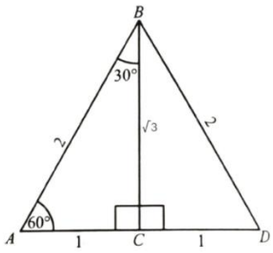
Unit No. 6

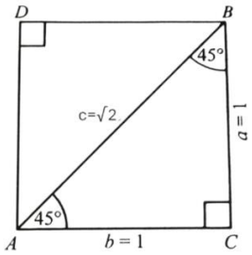
Trigonometry

Exercise No. 6.4

Question No. 1

Find the value of the following trigonometric ratios without using the calculator.





Θ	0	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

(i) sin 30°

Solution:

sin 30°

As sin θ = $\frac{Perpendicular}{Hypotenuse} = \frac{a}{c}$

sin 30° = $\frac{1}{2}$

(ii) cos 30°

Solution:

cos 30°

As cos θ = $\frac{Base}{Hypotenuse} = \frac{b}{c}$

cos 30° = $\frac{\sqrt{3}}{2}$

(iii) tan $\frac{\pi}{6}$

Solution:

tan $\frac{\pi}{6}$

1st convertint radian into degree:

= tan $\frac{\pi}{6} \times \frac{180}{\pi}$

= tan 30°

As $\tan\theta = \frac{Perpendicular}{Base} = \frac{a}{b}$

$\tan 30° = \frac{1}{\sqrt{3}}$

By rationalizing:

$\tan 30° = \frac{\sqrt{3}}{3}$

(iv) tan 60°

Solution:

= tan 60°

As $\tan\theta = \frac{Perpendicular}{Base} = \frac{a}{b}$

$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

(v) sec 60°

Solution:

sec 60°

As $\sec\theta = \frac{Hypotenuse}{Base} = \frac{c}{b}$

$\sec 60° = \frac{2}{1} = 2$

(vi) cos $\frac{\pi}{3}$

Solution:

$\cos\frac{\pi}{3}$

1st convertint radian into degree:

$= \cos\frac{\pi}{3} \times \frac{180}{\pi}$

= cos 60°

As $\cos\theta = \frac{Base}{Hypotenuse} = \frac{b}{c}$

$\cos 60° = \frac{1}{2}$

(vii) cot 60°

Solution:

cot 60°

As $\cot\theta = \frac{Base}{Perpendicular} = \frac{b}{a}$

$\cot 60° = \frac{1}{\sqrt{3}}$

By rationalizing:

$\cot 60° = \frac{\sqrt{3}}{3}$

(viii) sin 60°

Solution:

sin 60°

As sin θ = $\frac{Perpendicular}{Hypotenuse} = \frac{a}{c}$

sin 60° = $\frac{\sqrt{3}}{2}$

(ix) sec 30°

Solution:

sec 30°

As sec θ = $\frac{Hypotenuse}{Base} = \frac{c}{b}$

sec 30° = $\frac{2}{\sqrt{3}}$

By rationalizing:

sec 30° = $\frac{2\sqrt{3}}{3}$

(x) cosec 30°

Solution:

cosec 30°

As cosec θ = $\frac{Hypotenuse}{Perpendicular} = \frac{c}{a}$

cosec 30° = $\frac{2}{1} = 2$

(xi) sin 45°

Solution:

sin 45°

As sin θ = $\frac{Perpendicular}{Hypotenuse} = \frac{a}{c}$

sin 45° = $\frac{1}{\sqrt{2}}$

By rationalizing:

sin 45° = $\frac{\sqrt{2}}{2}$

(xii) cos $\frac{\pi}{4}$

Solution:

cos $\frac{\pi}{4}$

1st convertint radian into degree:

= cos $\frac{\pi}{4} \times \frac{180}{\pi}$

= cos 45°

As $\cos\theta = \frac{Perpendicular}{Hypotenuse} = \frac{a}{c}$

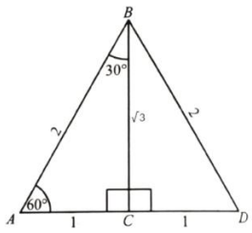
$\cos 45° = \frac{1}{\sqrt{2}}$

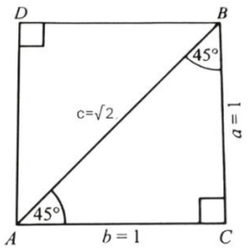
By rationalizing:

$\cos 45° = \frac{\sqrt{2}}{2}$

Question No. 2

Evaluate:





Θ	0	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

(i) 2 sin 60° cos 60°

Solution:

2 sin 60° cos 60°

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2}$

(ii) $2\cos\frac{\pi}{6}\sin\frac{\pi}{6}$

Solution:

$2\cos\frac{\pi}{6}\sin\frac{\pi}{6}$

1st convertint radian into degree:

$= \frac{\pi}{6} \times \frac{180}{\pi} = 30°$

$= 2\cos 30° \sin 30°$

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2}$

(iii) 2 sin 45° + 2 cos 45°

Solution:

2 sin 45° + 2 cos 45°

$= 2\left(\frac{1}{\sqrt{2}}\right) + 2\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}$

$= \frac{2+2}{\sqrt{2}}$

$= \frac{4}{\sqrt{2}}$

By rationalizing:

$= \frac{4\sqrt{2}}{2}$

$= 2\sqrt{2}$

(iv) sin 60° cos 30° + cos 60° sin 30°

Solution:

sin 60° cos 30° + cos 60° sin 30°

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{3+1}{4}$

$= \frac{4}{4}$

$= 1$

(v) cos 60° cos 30° - sin 60° sin 30°

Solution:

cos 60° cos 30° - sin 60° sin 30°

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= \frac{\sqrt{3}-\sqrt{3}}{4}$

$= 0$

(vi) sin 60° cos 30° - cos 60° sin 30°

Solution:

sin 60° cos 30° - cos 60° sin 30°

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

(vii) cos 60° cos 30° + sin 60° sin 30°

Solution:

cos 60° cos 30° + sin 60° sin 30°

$= (\frac{1}{2})(\frac{\sqrt{3}}{2}) + (\frac{\sqrt{3}}{2})(\frac{1}{2})$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{\sqrt{3}+\sqrt{3}}{4}$

$= \frac{2\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

(viii) tan $\frac{\pi}{6}$ cot $\frac{\pi}{6}$ + 1

Solution:

tan $\frac{\pi}{6}$ cot $\frac{\pi}{6}$ + 1

1st convertint radian into degree:

$= \frac{\pi}{6} \times \frac{180}{\pi} = 30°$

= tan 30° cot 30° + 1

$= (\frac{1}{\sqrt{3}})(\sqrt{3}) + 1$

= 1 + 1

= 2

Question No. 3

If sin $\frac{\pi}{4}$ and cos $\frac{\pi}{4}$ equal to $\frac{1}{\sqrt{2}}$ each, then find the value of the followings:

Data:

sin $\frac{\pi}{4}$ = 30° = $\frac{1}{\sqrt{2}}$

cos $\frac{\pi}{4}$ = 30° = $\frac{1}{\sqrt{2}}$

(i) 2 sin 45° - 2 cos 45°

Solution:

2 sin 45° - 2 cos 45°

$= 2\left(\frac{1}{\sqrt{2}}\right) - 2\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}}$

$= 0$

(ii) 3 cos 45° + 4 sin 45°

Solution:

3 cos 45° + 4 sin 45°

$= 3\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}}$

$= \frac{3+4}{\sqrt{2}}$

$= \frac{7}{\sqrt{2}}$

By rationalizing:

$= \frac{7\sqrt{2}}{2}$

(iii) 5 cos 45° - 3 sin 45°

Solution:

5 cos 45° - 3 sin 45°

$= 5\left(\frac{1}{\sqrt{2}}\right) - 3\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}}$

$= \frac{5-3}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

By rationalizing:

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$