Unit No. 6

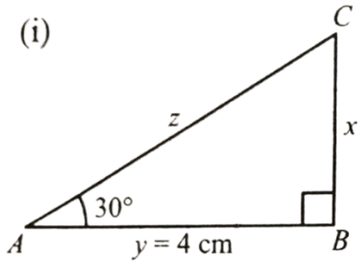
Trigonometry

Exercise No. 6.5

Question No. 1

Find the values of x, y and z from the following right-angled triangles.

(i)



Data:

Base = y = 4 cm

Angle = θ = 30°

To Find:

Perpendicular = x = ?

Hypotenuse = z = ?

Solution:

$$\text{Tan } \theta = \frac{Perpendicular}{Base}$$

$$\text{Tan } 30 = \frac{x}{4}$$

$$x = 4 \times Tan\ 30$$

$$x = 4 \times \frac{1}{\sqrt{3}}$$

$$x = \frac{4}{\sqrt{3}}\ cm$$

$$\text{Cos } \theta = \frac{Base}{Hypotenuse}$$

$$\text{Cos } 30 = \frac{4}{z}$$

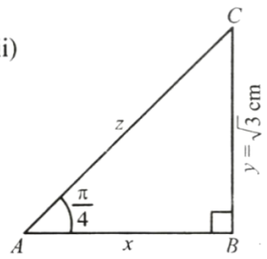
$$z = \frac{4}{\text{Cos } 30}$$

$$z = \frac{4}{\frac{\sqrt{3}}{2}}$$

$$z = \frac{4 \times 2}{\sqrt{3}}$$

$$z = \frac{8}{\sqrt{3}}\ cm$$

(ii)



Data:

Perpendicular = y = $\sqrt{3}$ cm

Angle = θ = $\frac{\pi}{4} = \frac{\pi}{4} \times \frac{180}{\pi}$ = 45°

To Find:

Base = x = ?

Hypotenuse = z = ?

Solution:

$$\text{Tan}\ \theta = \frac{Perpendicular}{Base}$$

$$\text{Tan}\ 45 = \frac{\sqrt{3}}{x}$$

$$x = \frac{\sqrt{3}}{\text{Tan}\ 45}$$

$$x = \frac{\sqrt{3}}{1}$$

$$x = \sqrt{3}\ cm$$

$$\text{Sin}\ \theta = \frac{Perpendicular}{Hypotenuse}$$

$$\text{Sin}\ 45 = \frac{\sqrt{3}}{z}$$

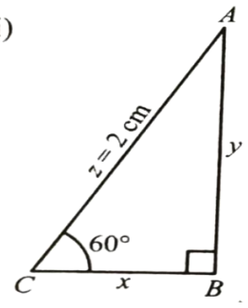
$$\text{z} = \frac{\sqrt{3}}{\text{Sin}\ 45}$$

$$\text{z} = \frac{\sqrt{3}}{\frac{1}{\sqrt{2}}}$$

$$\text{z} = \sqrt{3} \times \sqrt{2}$$

$$\text{z} = \sqrt{6}\ cm$$

(iii)



Data:

Hypotenuse = z = 2 cm

Angle = θ = 60°

To Find:

Base = x = ?

Perpendicular = y = ?

Solution:

$$\text{Sin } \theta = \frac{Perpendicular}{Hypotenuse}$$

$$\text{Sin } 60 = \frac{y}{2}$$

$$y = 2 \times \text{Sin } 60$$

$$y = 2 \times \frac{\sqrt{3}}{2}$$

$$y = \sqrt{3}\ cm$$

$$\text{Tan } \theta = \frac{Perpendicular}{Base}$$

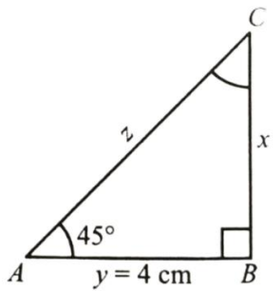
$$\text{Tan } 60 = \frac{\sqrt{3}}{x}$$

$$x = \frac{\sqrt{3}}{Tan\ 60}$$

$$x = \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = 1 \text{ cm}$$

(iv).



Data:

Base = y = 4 cm

Angle = θ = 45°

To Find:

Perpendicular = x = ?

Hypotenuse = z = ?

Solution:

$$\text{Tan } \theta = \frac{Perpendicular}{Base}$$

$$\text{Tan } 45 = \frac{x}{4}$$

$$1 \times 4 = x$$

$$x = 4\ cm$$

$$\text{Sin } \theta = \frac{Perpendicular}{Hypotenuse}$$

$$\text{Sin } 45 = \frac{4}{z}$$

$$z = \frac{4}{\text{Sin } 45}$$

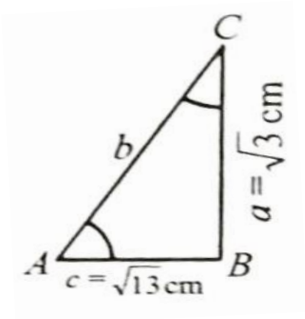
$$z = \frac{4}{\frac{1}{\sqrt{2}}}$$

$$z = 4\sqrt{2}\ cm$$

Question No. 2

Find the unknown side and angles of the following triangles.

(i).



Data:

Base = c = $\sqrt{13}$ cm

Perpendicular = a = $\sqrt{3}$ cm

To Find:

Hypotenuse = b = ?

m ∠A = ?

m ∠C = ?

Solution:

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$b^2 = (\sqrt{3})^2 + (\sqrt{13})^2$

$b^2 = 3 + 13$

$b^2 = 16$

$b = \sqrt{16}$

$b = 4$ cm

$m\angle A = \tan^{-1}\left[\frac{\sqrt{3}}{\sqrt{13}}\right]$

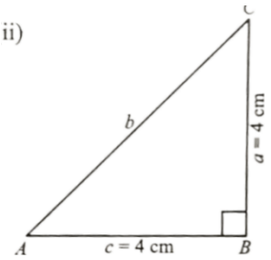
$m\angle A = \tan^{-1}\left[\frac{\sqrt{3}}{\sqrt{13}}\right]$

$m\angle A = 25.64°$

$m\angle C = 90° - 25.64°$

$m\angle C = 64.36°$

ii)



Data:

Base = c = 4 cm

Perpendicular = a = 4 cm

To Find:

Hypotenuse = b = ?

m ∠A = ?

m ∠C = ?

Solution:

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$b^2 = (4)^2 + (4)^2$

$b^2 = 16 + 16$

$b^2 = 32$

$b = \sqrt{32}$

$b = 4\sqrt{2}$ cm

$m\angle A = \tan^{-1}[\frac{4}{4}]$

$m\angle A = \tan^{-1}[1]$

$m\angle A = 45°$

$m\angle C = 90° - 45°$

$m\angle C = 45°$

Question No. 3

Each side of a square field is 60 m long. Find the lengths of the diagonals of the field.

Data:

Base = b = 60 m

Perpendicular = a = 60 m

To Find:

Hypotenuse = c = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$c^2 = (60)^2 + (60)^2$

$c^2 = 3600 + 3600$

$c^2 = 7200$

$c = \sqrt{7200}$

$c = 60\sqrt{2}$ m

Therefore, the length of each diagonal is 84.85 m.

Question No. 4

Solve the following triangles when

m∠B = 90°:

(i). m∠C = 60°, c = 3√3 cm

Data:

m∠C = 60°

c = 3√3 cm

To Find:

m∠A = ?

a = ?

b = ?

Solution:

$m\angle A = 90° - m\angle C$

$m\angle A = 90° - 60°$

$m\angle A = 30°$

$\text{Tan } \theta = \frac{Perpendicular}{Base}$

$\text{Tan } 30 = \frac{a}{3\sqrt{3}}$

$a = Tan\ 30 \times 3\sqrt{3}$

$a = \frac{1}{\sqrt{3}} \times 3\sqrt{3}$

$a = 3\ cm$

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$b^2 = (3)^2 + (3\sqrt{3})^2$

$b^2 = 9 + 9(3)$

$b^2 = 9 + 27$

$b^2 = 36$

$b = 6$ cm

(ii). m∠C = 45°, a = 8 cm

Data:

m∠C = 45°

a = 8 cm

To Find:

m∠A = ?

b = ?

c = ?

Solution:

$m\angle A = 90° - m\angle C$

$m\angle A = 90° - 45°$

$m\angle A = 45°$

$\text{Tan}\ \theta = \frac{Perpendicular}{Base}$

$\text{Tan}\ 45 = \frac{8}{c}$

$c = \frac{8}{\text{Tan}\ 45}$

$c = \frac{8}{1}$

$c = 8\ cm$

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$b^2 = (8)^2 + (8)^2$

$b^2 = 64 + 64$

$b^2 = 128$

$b = 8\sqrt{2}$ cm

(iii). a = 12 cm, c = 6 cm

Data:

a = 12 cm

c = 6 cm

To Find:

b = ?

m∠A = ?

m∠C = ?

Solution:

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$b^2 = (12)^2 + (6)^2$

$b^2 = 144 + 36$

$b^2 = 180$

$b = 6\sqrt{5}$ cm

$$\text{Tan } \theta = \frac{Perpendicular}{Base}$$

$$\text{Tan } \theta = \frac{12}{6}$$

$\text{Tan } \theta = 2$

$\theta = tan^{-1} 2$

m∠A = 63.4°

m∠C = 90° - m∠A

m∠C = 90° - 63.4°

m∠C = 26.6°

(iv). m∠A = 60°, c = 4 cm

Data:

m∠A = 60°

c = 4 cm

To Find:

m∠C = ?

a = ?

b = ?

Solution:

m∠C = 90° - m∠A

m∠C = 90° - 60°

m∠C = 30°

$$\text{Tan } \theta = \frac{Perpendicular}{Base}$$

$$\text{Tan } 60 = \frac{a}{4}$$

$a = 4 \times \tan 60$

$a = 4 \times \sqrt{3}$

$a = 4\sqrt{3}\ cm$

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$b^2 = (4\sqrt{3})^2 + (4)^2$

$b^2 = 16(3) + 16$

$b^2 = 48 + 16$

$b^2 = 64$

$b = 8$ cm

(v). m∠A = 30°, c = 4 cm

Data:

m∠ A = 30°

c = 4 cm

To Find:

m∠C = ?

a = ?

b = ?

Solution:

$m\angle C = 90° - m\angle A$

$m\angle C = 90° - 30°$

$m\angle C = 60°$

$\text{Tan}\ \theta = \frac{Perpendicular}{Base}$

$\text{Tan}\ 30 = \frac{a}{4}$

$a = 4 \times \tan 30$

$a = 4 \times \frac{1}{\sqrt{3}}$

$a = \frac{4}{\sqrt{3}}$

$a = \frac{4}{\sqrt{3}}\ cm$

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$b^2 = (\frac{4}{\sqrt{3}})^2 + (4)^2$

$b^2 = \frac{16}{3} + 16$

$b^2 = \frac{16+48}{3}$

$b^2 = \frac{64}{3}$

$b = \frac{8}{\sqrt{3}}$ cm

(vi). b = 10 cm, a = 6 cm

Data:

a = 6 cm

b = 10 cm

To Find:

c = ?

m∠A = ?

m∠C = ?

Solution:

Using Pythagoras Theorem:

$b^2 = a^2 + c^2$

$(10)^2 = (6)^2 + c^2$

$c^2 = 100 - 36$

$c^2 = 64$

$c = 8$ cm

$\text{Sin } \theta = \frac{Perpendicular}{Hypotenuse}$

$\text{Sin } m\angle A = \frac{6}{10}$

$\text{Sin } m\angle A = 0.6$

$m\angle A = sin^{-1}\ 0.6$

$m\angle A = 36.9°$

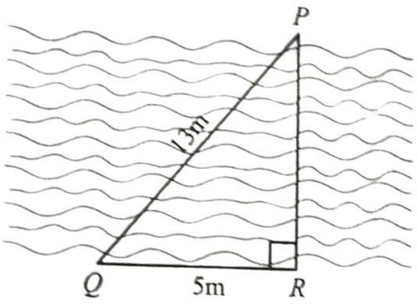
$m\angle C = 90° - m\angle A$

$m\angle C = 90° - 36.9°$

$m\angle C = 53.1°$

Question No. 5

Let Q and R be the two points on the same bank of a canal. The point P is placed on the other bank straight to point R. Find the width of the canal and the angle PQR in radians.



Data:

Base = b = 5 m

Hypotenuse = c = 13 m

To Find:

Width of canal = a = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$(13)^2 = a^2 + (5)^2$

$169 = a^2 + 25$

$a^2 = 169 - 25$

$a^2 = 144$

Therefore, the width of canal is 12 m and the angle PQR is 1.18 radians.

Finding direction:

$$\text{Tan } \theta = \frac{Perpendicular}{Base}$$

$$\text{Tan } \theta = \frac{12}{5}$$

$$\theta = tan^{-1}2.4$$

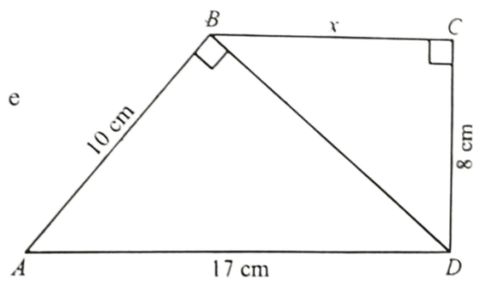
$\theta = 67.4°$

$$\theta = 67.4 \times \frac{\pi}{180}\, rad$$

$$\theta = 1.18\, rad$$

Question No. 6

Calculate the length x in the adjoining figure.



For ΔABD;

Data:

Base = b = 10 cm

Hypotenuse = c = 17 cm

To Find:

Perpendicular = a = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$(17)^2 = a^2 + (10)^2$

$289 = a^2 + 100$

$a^2 = 289 - 100$

$a^2 = 189$

$a = 3\sqrt{21}$ cm

For ΔBCD;

Data:

Perpendicular = a = 8 cm

Hypotenuse = c = $3\sqrt{21}$ cm

To Find:

Base = b = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$(3\sqrt{21})^2 = (8)^2 + b^2$

$9(21) = 64 + b^2$

$189 = 64 + b^2$

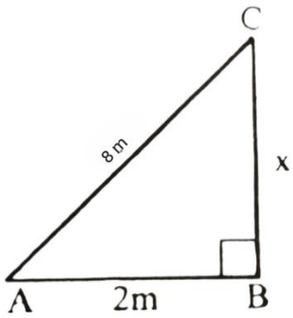
$b^2 = 189 - 64$

$b^2 = 125$

$b = 5\sqrt{5}$ cm

Question No. 7

If the ladder is placed along the wall such that the foot of the ladder is 2 m away from the wall. If the length of the ladder is 8 m, find the height of the wall.



Data:

Distance of ladder from wall = b = 2 m

Length of the ladder = c = 8 m

To Find:

Height of wall = a = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$(8)^2 = a^2 + (2)^2$

$64 = a^2 + 4$

$a^2 = 64 - 4$

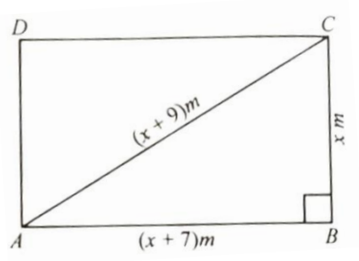
$a^2 = 60$

$a = 2\sqrt{15}$ m

Height of wall = a = $2\sqrt{15}$ m = 7.75 m

Question No. 8

The diagonal of a rectangular field ABCD is (x + 9)m and the sides are (x + 7)m and x m. Find the value of x.



Data:

Base = b = (x + 7) m

Perpendicular = a = x m

Hypotenuse = c = (x + 9) m

To Find:

x = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$(x + 9)^2 = x^2 + (x + 7)^2$

$x^2 + 2(x)(9) + 9^2 = x^2 + x^2 + 2(x)(7) + 7^2$

$x^2 + 18x + 81 = x^2 + x^2 + 14x + 49$

$x^2 + 18x + 81 - x^2 - x^2 - 14x - 49 = 0$

$- x^2 + 4x + 32 = 0$

$x^2 - 4x - 32 = 0$

By factorization:

$x^2 - 8x + 4x - 32 = 0$

$x(x – 8) + 4(x – 8) = 0$

$(x – 8)(x + 4) = 0$

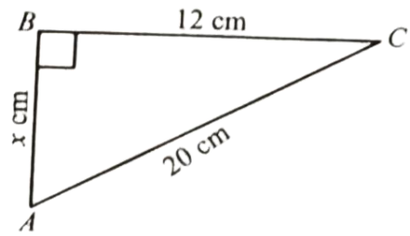
$(x – 8) = 0$; $(x + 4) = 0$

$x = 8m$; $x = -4$ m (Not required as -ve)

$x = 8m$

Question No. 9

Calculate the value of 'x' in each case.



Data:

Perpendicular = a = 12 cm

Hypotenuse = c = 20 cm

To Find:

Base = b = x = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

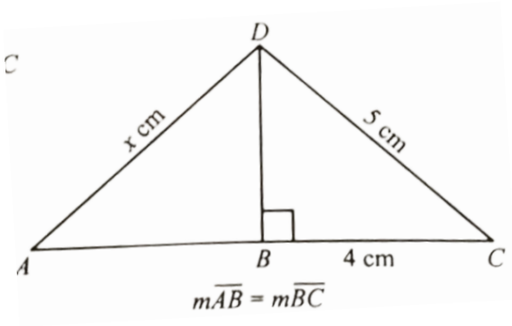
$(20)^2 = (12)^2 + x^2$

$400 = 144 + x^2$

$x^2 = 400 - 144$

$x^2 = 256$

$x = 16$ cm



For ΔBCD;

Data:

Base = b = 4 cm

Hypotenuse = c = 5 cm

To Find:

Perpendicular = a = ?

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$(5)^2 = a^2 + 4^2$

$25 = a^2 + 16$

$a^2 = 25 - 16$

$a^2 = 9$

$a = 3$ cm

For ΔABD;

Data:

Perpendicular = a = 3cm

As mAB = m BC = 4 cm, So;

Base = b = 4 cm

To Find:

Hypotenuse = c = x

Solution:

Using Pythagoras Theorem:

$c^2 = a^2 + b^2$

$x^2 = (3)^2 + (4)^2$

$x^2 = 9 + 16$

$x^2 = 25$

$x = 5$ cm