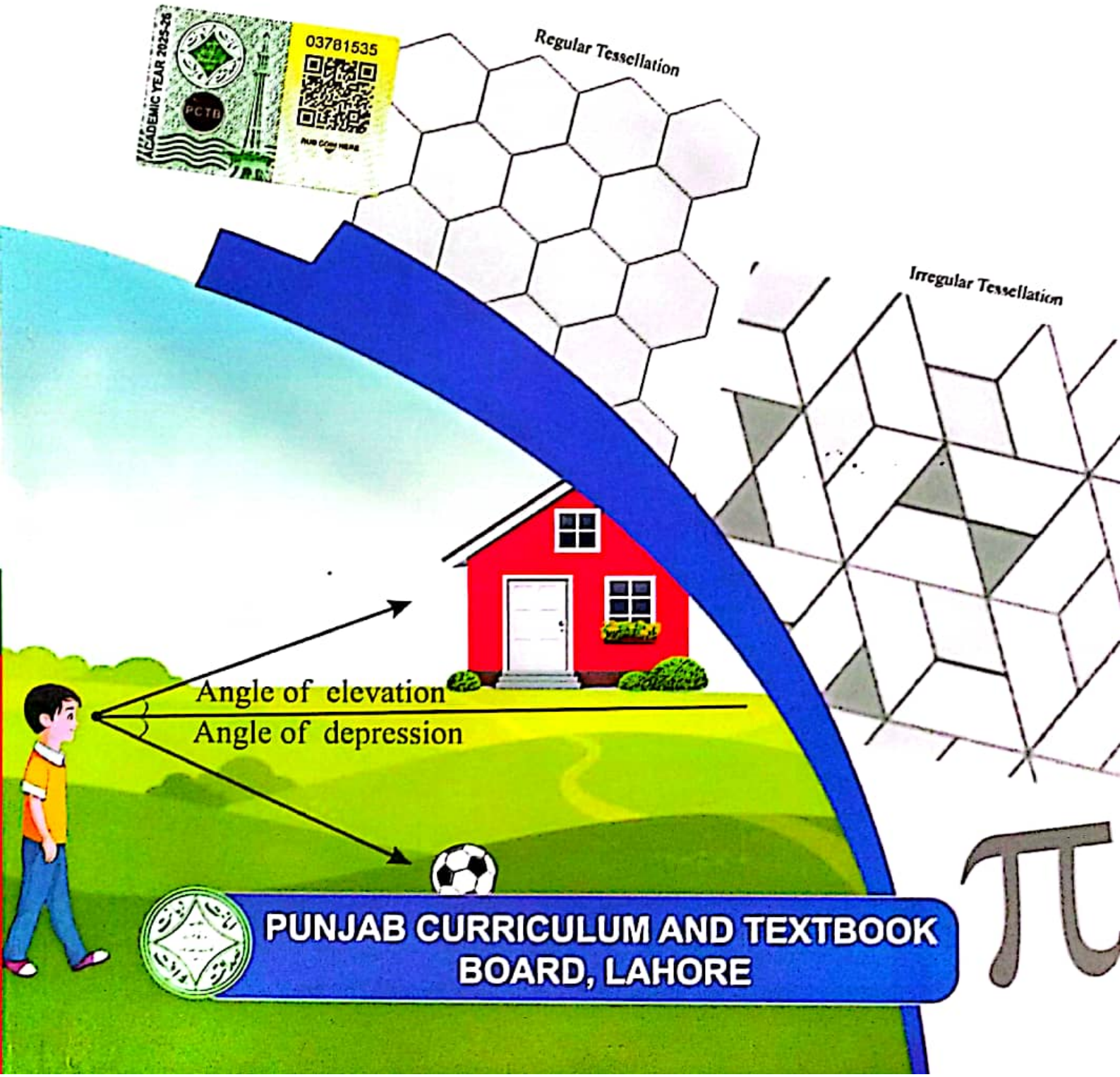
MATHEMATICS 9



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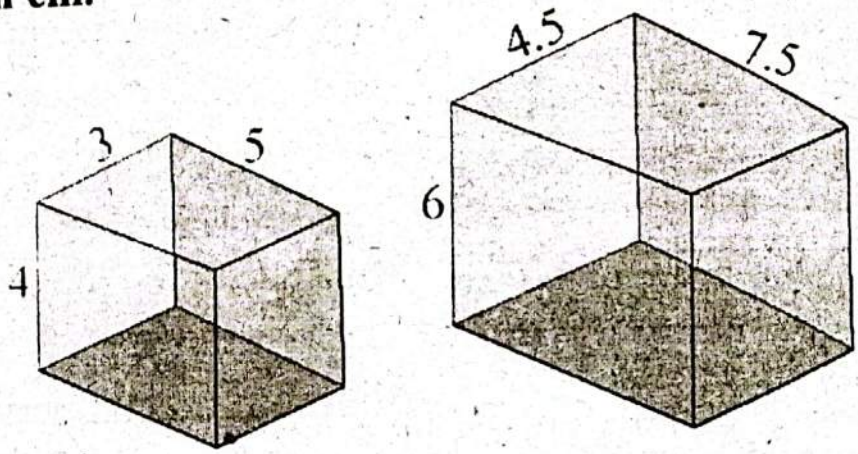
UNIT 9 SIMILAR FIGURES

EXERCISE 9.1

Q.1. Find whether the solids are similar. All lengths are in cm.



Sol. Dimension of smaller cuboid = 4 cm by 3 cm by 5 cm

Dimension of bigger cuboid = 6 cm by 4.5 cm by 7.5 cm

Let us find the ratios of sides of both cuboids.

$$\frac{4}{6} = \frac{3}{4.5} = \frac{5}{7.5}$$

$$0.6667 = 0.6667 = 0.6667$$

Since all ratios have same values. So both shapes are similar.

Q.2. In triangle ABC, the sides are given as $m\overline{AB} = 6cm$, $m\overline{BC} = 9cm$ and $m\overline{CA} = 12cm$. In triangle DEF, the sides are given as $m\overline{DE} = 10.5cm$, $m\overline{EF} = 15.75cm$, and $m\overline{FD} = 21cm$. Prove that the triangles are similar.

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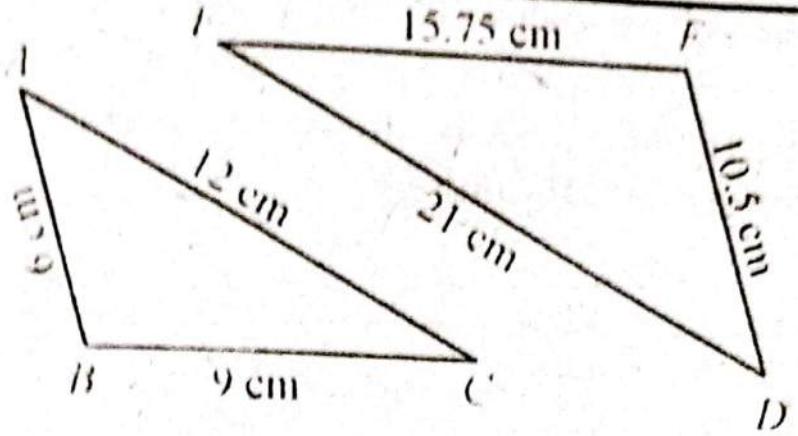
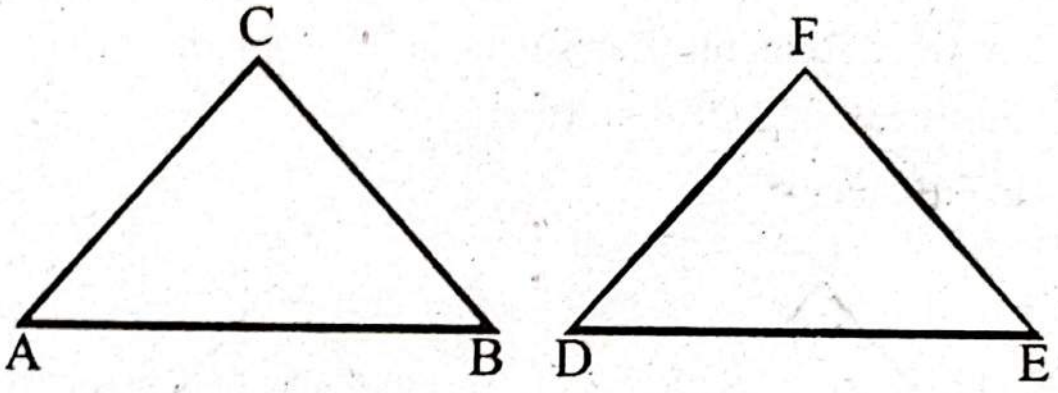


Sol. To prove: ΔABC is similar to ΔDEF

Given that $m\overline{AB} = 6cm$, $m\overline{BC} = 9cm$, $m\overline{CA} = 12cm$

$m\overline{DE} = 10.5cm$, $m\overline{EF} = 15.75cm$, $m\overline{FD} = 21cm$

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{CA}}{m\overline{FD}}$$



Let us find all these ratios to reach conclusion

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{6}{10.5} = 0.5714$$

$$\frac{m\overline{BC}}{m\overline{EF}} = \frac{9}{15.75} = 0.5714$$

$$\frac{m\overline{CA}}{m\overline{FD}} = \frac{12}{21} = 0.5714$$

Since all ratios are equal to 0.5714 corresponding sides are proportional.

So ΔABC and ΔDEF are similar.

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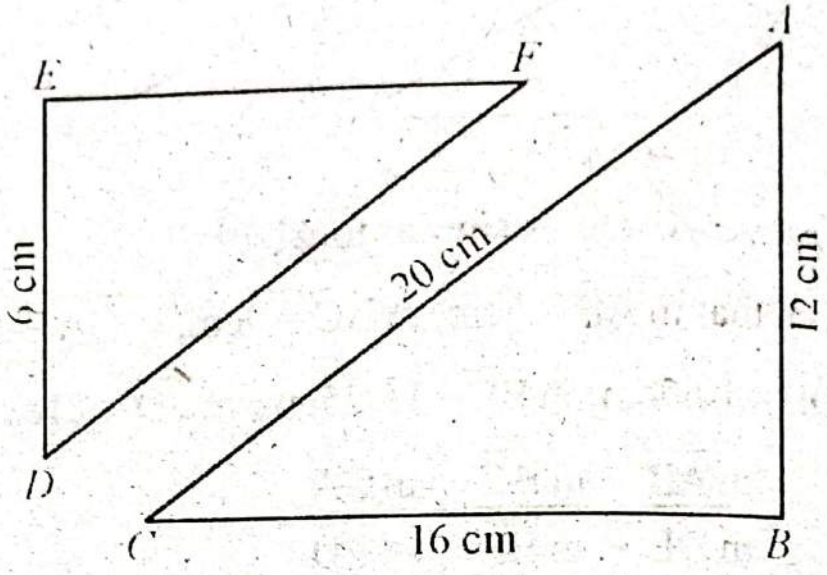
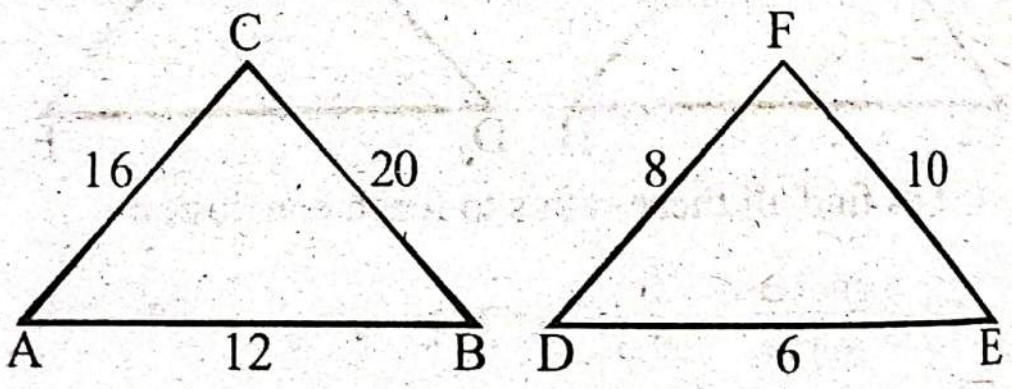
Q.3. **In the figure below, $\Delta ABC \sim \Delta DEF$. The length of AB is 12 cm, AC is 16 cm, and BC is 20 cm. The perimeter of ΔDEF is 30 cm and mDE = 6 cm. Find the lengths of DF and EF.**



Sol. $m\overline{AB} = 12cm$, $m\overline{AC} = 16cm$, $m\overline{BC} = 20cm$

Perimeter of $\Delta DEF = 30$ cm.

$m\overline{DE} = 6cm$



To find: length of $\overline{DF}$ and of $\overline{EF}$

$$\frac{\overline{AC}}{\overline{DF}} = \frac{\overline{AB}}{\overline{DE}}$$

$$\frac{16}{\overline{DF}} = \frac{12}{6}$$

$$12\,\overline{DF} = 16 \times 6$$

$$12\,\overline{DF} = 96$$

$$\overline{DF} = \frac{96}{12}$$

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$$\overline{DF} = 8 \text{ cm}$$

Now $$\frac{\overline{BC}}{\overline{EF}} = \frac{\overline{AC}}{\overline{DF}}$$

$$\frac{20}{\overline{EF}} = \frac{16}{8}$$

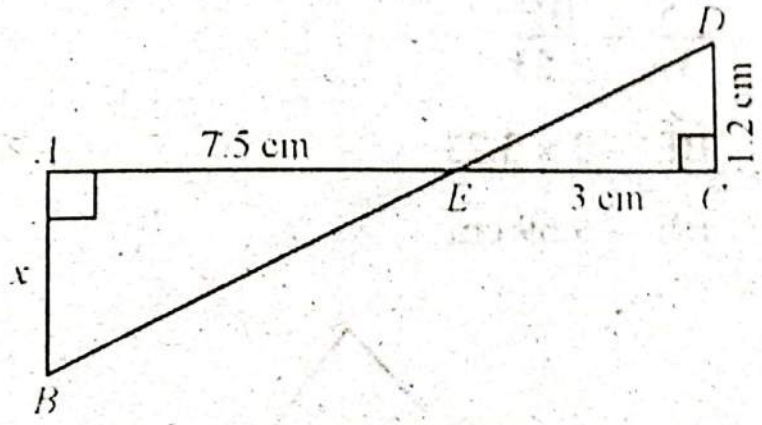
$$16\,\overline{EF} = 20 \times 8$$

$$\overline{EF} = \frac{160}{16}$$

$$\overline{EF} = 10 \text{ cm}$$

Q.4. Find the value of x in each of the following:

(i)



Sol. Since ΔABE and ΔDEC are similar.

$m\,\overline{AE} = 7.5$ cm, $m\,\overline{EC} = 3$ cm

$m\,\overline{AB} = ?$, $m\,\overline{DC} = 1.2$ cm

$$\frac{m\,\overline{AE}}{m\,\overline{EC}} = \frac{m\,\overline{AB}}{m\,\overline{DC}}$$

$$\frac{7.5}{3} = \frac{m\,\overline{AB}}{1.2}$$

$$\frac{m\,\overline{AB}}{1.2} = 2.5$$

$$m\,\overline{AB} = 1.2 \times 2.5$$

$$m\,\overline{AB} = 3 \text{ cm}$$

By Pythagoras theorem,

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$$|\overline{DE}|^2 = (1.2)^2 + 3^2$$
$$= 1.44 + 9$$
$$= 10.44$$
$$\overline{DE} = \sqrt{10.44} = 3.23 \text{ cm}$$

Now

$$\frac{\overline{AE}}{\overline{EC}} = \frac{\overline{BE}}{\overline{DE}}$$

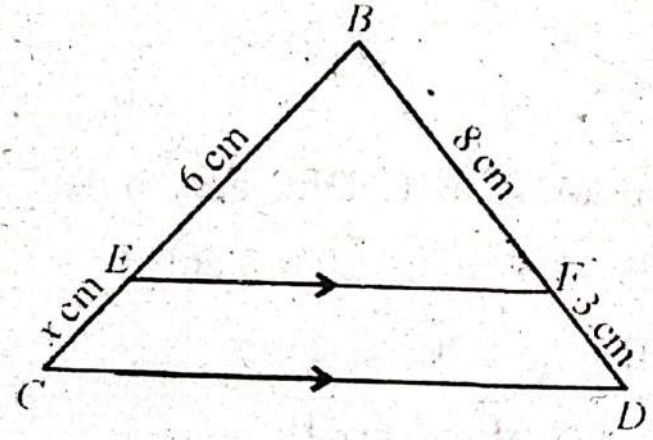
$$\frac{7.5}{\overline{EC}} = \frac{\overline{BE}}{3.23}$$

$$\frac{7.5}{3} = \frac{\overline{BE}}{3.23}$$

$$\overline{BE} = 3 \times 3.23$$

$$\overline{BE} = 9.69 \text{ cm}$$

(ii)



Sol.

$$\frac{6}{x+6} = \frac{8}{11}$$

$$8(x+6) = 11 \times 6$$

$$8x + 48 = 66$$

$$8x = 66 - 48$$

$$8x = 18$$

$$x = \frac{18}{8}$$

$$x = 4.5 \text{ cm}$$

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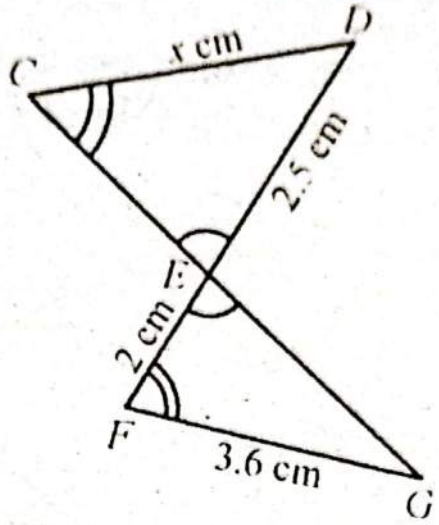
(iii)



Sol.

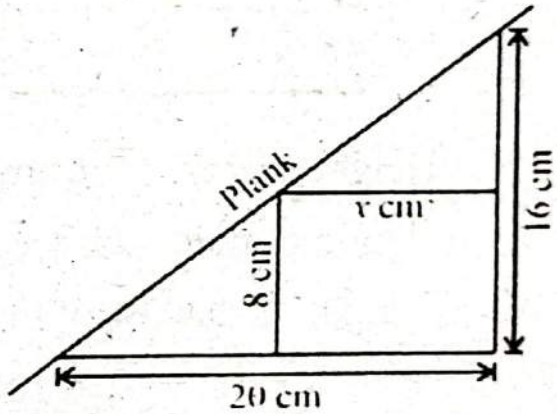
$$\frac{x}{3.6} = \frac{2.5}{2}$$

$$x = 3.6 \times \frac{2.5}{2}$$

$$= 1.8 \times 2.5$$

$$x = 4.5 \text{ cm}$$

Q.5. A plank is placed straight upstairs that 20 cm wide and 16 cm deep. A rectangular box of height 8 cm and width x cm is placed on a stair under the plank. Find the value of x.



Sol. Width of upstairs = 20 cm

Depth of upstairs = 16 cm

Height of box = 8 cm

Width of box = xcm = ?

Height of right angled triangle above the rectangular box = 16 – 8 = 8cm

$$\frac{16}{8} = \frac{20}{x}$$

$$16x = 8 \times 20$$

$$x = \frac{160}{16}$$

$$x = 10 \text{ cm}$$

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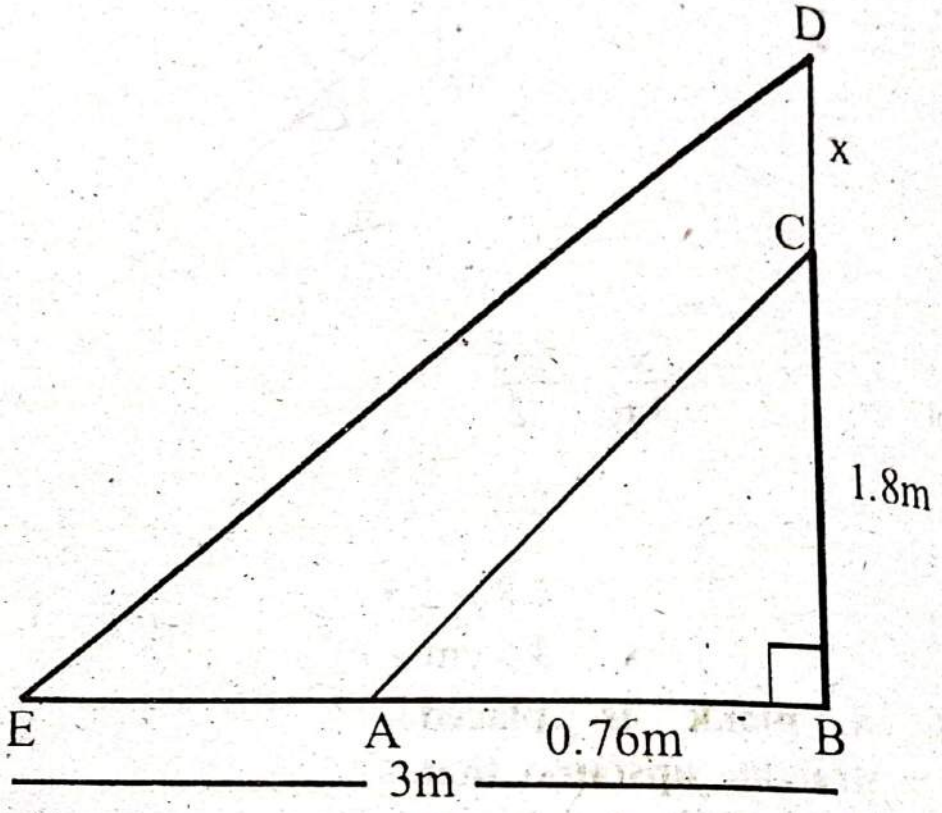
Q.6. **A man who is 1.8 m tall casts a shadow of a 0.76 m in length. If at the same time a telephone pole costs a 3 m shadow, find the height of the pole.**

Sol.



Height of man = 1.8 cm

Height of pole = (1.8 + x) m

Length of shadow of man = 0.76 m

$$\frac{1.8}{0.76} = \frac{1.8 + x}{3}$$

$$0.76(1.8 + x) = 3 \times 1.8$$

$$1.368 \times 0.76\,x = 5.4$$

$$0.76\,x = 5.4 - 1.368$$

$$0.76\,x = 4.032$$

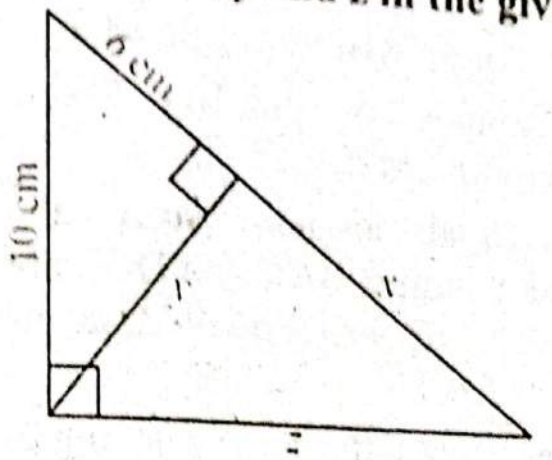
$$x = \frac{4.032}{0.76}$$

$$x = 5.3 \text{ m}$$

Height of pole = 5.3 + 1.8

= 7.1 m

Q.7. Find the values of x, y and z in the given figure.



Sol. Both triangles are similar.

Using Pythagoras theorem,

Perpendicular of $\Delta = \sqrt{10^2 - 6^2}$

$= \sqrt{100 - 36}$

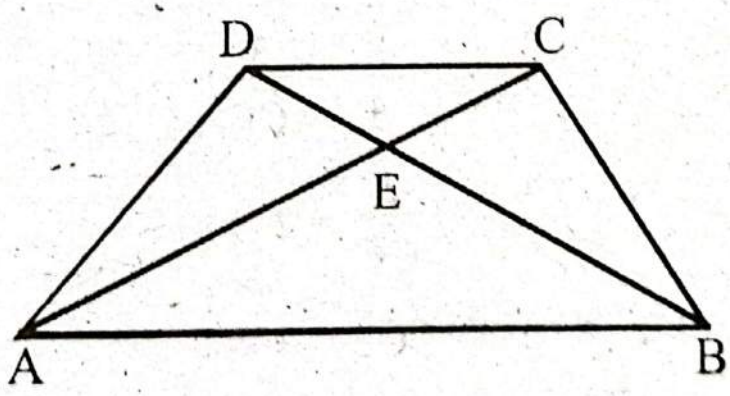
$y = \sqrt{64}$

$y = 8$ cm

Since both triangles are similar.

Q.8. Draw an isosceles trapezoid ABCD where $\overline{AB} \parallel \overline{CD}$ and $m\overline{AB} > m\overline{CD}$. Draw diagonal $\overline{AC}$ and $\overline{BD}$, intersecting at E. Prove that ΔABE is similar to ΔCDE. If $m\overline{AB} = 8$ cm, $m\overline{CD} = 4$cm, and $m\overline{AE} = 3$ cm, find the length of $\overline{CE}$.

Sol.



To prove that triangles ABE and CDE are similar in an isosceles trapezoid ABCD, we can use the fact that diagonals of a trapezoid are transversals, which means that the marked angles are equal.

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$$\angle BAE = \angle DCE$$

and Angle ABD = Angle CDE

Draw diagonals AC and BD. The diagonals intersect each other at point E.

Identify equal angles: Since the diagonals are transversals, angles BAE and DCE are equal and angles ABD are CDE are equal. Use the AA similarity criterion.

Two triangles are similar if one angle of one triangle is equal to one angle of the other triangle, and sides including these angles are proportional.

In an isosceles trapezoid, the diagonals are equal in length.

A trapezoid is isosceles if

- Each pair of base angles is equal.
- Each pair of opposite angles is supplementary.

Now

$$m\overline{AB} = 8 \text{ cm}, \; m\overline{CD} = 4 \text{ cm}$$

$$m\overline{AE} = 3 \text{ cm}, \text{ length of } \overline{CE} = ?$$

ΔAEB and ΔDEC are similar. So

$$\frac{m\overline{AB}}{m\overline{CD}} = \frac{m\overline{AE}}{m\overline{CE}}$$

$$\frac{8}{4} = \frac{3}{\overline{CE}}$$

$$\frac{3}{\overline{CE}} = 2$$

$$2\overline{CE} = 3$$

$$\overline{CE} = \frac{3}{2}$$

$$\overline{CE} = 1.5 \text{ cm}$$

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Q.9. A regular dodecagon has its side lengths decreased by a factor of $\frac{1}{\sqrt{2}}$. If the perimeter of the original dodecagon is 72 cm. What is the side length of scaled dodecagon?

Sol. Dodecagon is one of the types of polygon that has 12 sides. It is a regular polygon.

Given that its perimeter is 72 cm.

Let side a length of scaled dodecagon = x cm

Then $$x = \frac{72}{12} \times \frac{1}{\sqrt{2}}$$

$$= \frac{6}{\sqrt{2}}$$

$$= \frac{6 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2} \text{ cm}$$

EXERCISE 9.2

Q.1. Find the ratio of the area of similar figures if the ratio of their corresponding lengths are:

(i) 1 : 3

Sol. Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 : \ell_2 = 1 : 3$

So, $$\frac{A_1}{A_2} = \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{9}$$

$$A_1 : A_2 = 1 : 9$$

(ii) 3 : 4

Sol. Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

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Here $\ell_1 : \ell_2 = 3 : 4$

So, $$\frac{A_1}{A_2} = \left(\frac{3}{4}\right)^2$$

$$= \frac{9}{16}$$

$$A_1 : A_2 = 9 : 16$$

(iii) **2 : 7**

Sol. Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 : \ell_2 = 2 : 7$

So, $$\frac{A_1}{A_2} = \left(\frac{2}{7}\right)^2$$

$$= \frac{4}{49}$$

$$A_1 : A_2 = 4 : 49$$

(iv) **8 : 9**

Sol. Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 : \ell_2 = 8 : 9$

So, $$\frac{A_1}{A_2} = \left(\frac{8}{9}\right)^2$$

$$= \frac{64}{81}$$

$$A_1 : A_2 = 64 : 81$$

(v) **6 : 5**

Sol. Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 : \ell_2 = 6 : 5$

So, $$\frac{A_1}{A_2} = \left(\frac{6}{5}\right)^2$$

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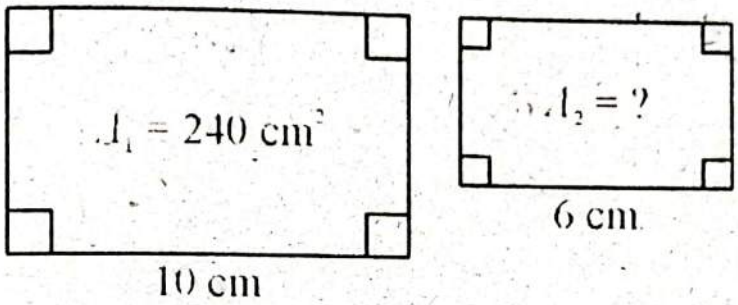
$$= \frac{36}{25}$$

$$A_1 : A_2 = 36 : 25$$

Q.2. Find the unknowns in the following figures:

(i)



Sol. Area of bigger shape $= A_1 = 240 \text{ cm}^2$

Length of bigger shape $= \ell_1 = 10$ cm

Length of smaller shape $= \ell_2 = 6$ cm

Area of smaller shape $= A_2 = ?$

Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{240}{A_2} = \left(\frac{10}{6}\right)^2$$

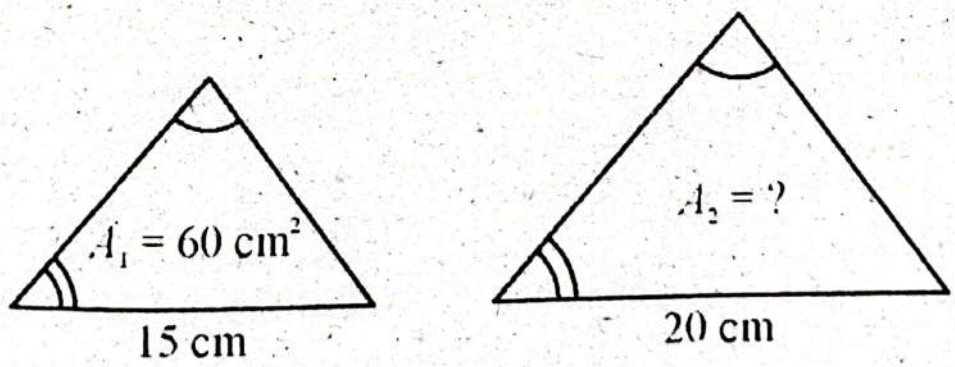
So, $$\frac{240}{A_2} = \frac{100}{36}$$

$$100\, A_2 = 240 \times 36$$

$$A_2 = \frac{240 \times 36}{100}$$

$$A_2 = 86.4 \text{ cm}^2$$

(ii)



Sol. Area of smaller triangle $= A_1 = 60 \text{ cm}^2$

Side of smaller triangle $= \ell_1 = 15$ cm

Side of bigger triangle $= \ell_2 = 20$ cm

Area of bigger triangle $= A_2 = ?$

Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{60}{A_2} = \left(\frac{15}{20}\right)^2$$

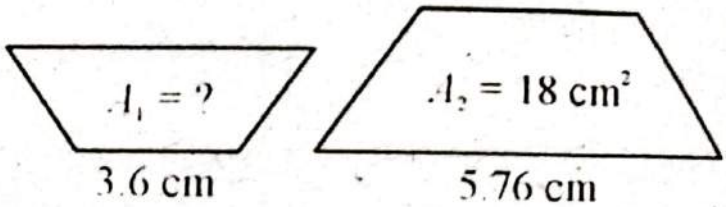
So, $$\frac{60}{A_2} = \frac{225}{400}$$

$$225\ A_2 = 60 \times 400$$

$$A_2 = \frac{2400}{225}$$

$$A_2 = 106.67\ cm^2$$

(iii)



Sol. Area of bigger trapezium $= A_2 = 18\ cm^2$

Base of bigger trapezium $= \ell_2 = 5.76$ cm

Area of smaller trapezium $= A_1 = ?$

Base of smaller trapezium $= \ell_1 = 3.6$ cm

Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{A_1}{18} = \left(\frac{3.6}{5.76}\right)^2$$

So, $$A_1 = (0.625)^2 \times 18$$

$$= 0.390625 \times 18$$

$$A_1 = 7.03125\ cm^2$$

So Area of smaller trapezium $= 7.03125\ cm^2$

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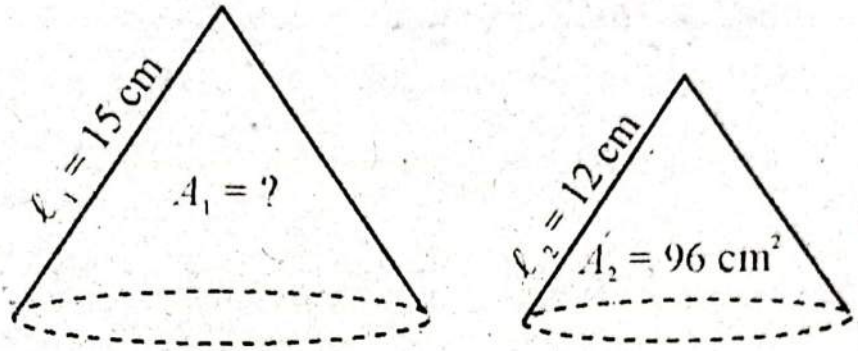
(iv)



Sol. Area of big cone $= A_1 = ?$

Slant height $= \ell_1 = 15$ cm

Area of smaller cone $= A_2 = 96$ cm^2

Slant height $= \ell_2 = 12$ cm

Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{A_1}{96} = \left(\frac{15}{12}\right)^2$$

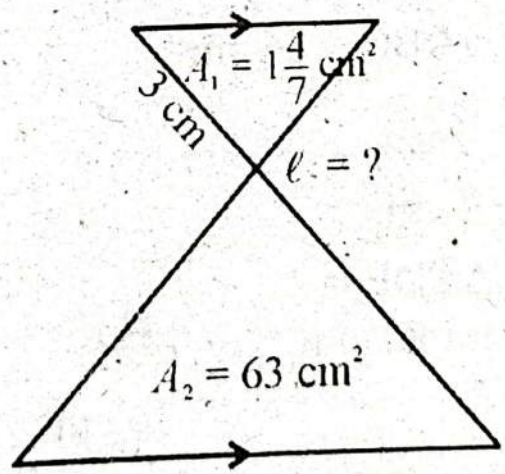
So, $$A_1 = \left(\frac{5}{4}\right)^2 \times 96$$

$$= \frac{25}{16} \times 96$$

$$= 25 \times 6$$

$$A_1 = 150 \text{ cm}^2$$

(v)



Sol. Area of smaller triangle $= A_1 = 1\frac{4}{7}$ cm^2 $= \frac{11}{7}$ cm^2

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Side of smaller triangle $= \ell_1 = 3$ cm

Area of bigger triangle $= A_2 = 63\ cm^2$

Side of bigger triangle $= \ell_2 = ?$

Using $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{\frac{11}{7}}{63} = \left(\frac{3}{\ell_2}\right)^2$$

So, $$\frac{11}{7 \times 63} = \frac{9}{\ell_2^2}$$

$$11\,\ell_2 = 9 \times 7 \times 63$$

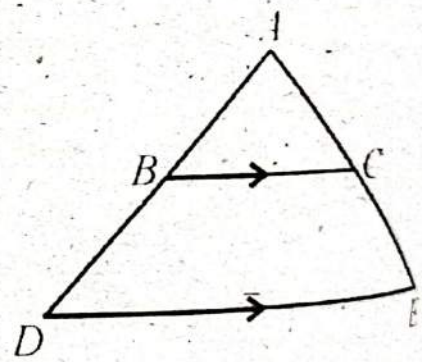
$$\ell_2 = \frac{63 \times 63}{11} = 360.812$$

$$\ell_2 = 19 \text{ cm}$$

Q.3 Given that area of $\Delta ABC = 36cm^2$ and, $m\overline{AB} = 6$ cm, $m\overline{BD} = 4$ cm. Find

(a) the area of ΔADE

(b) the area of trapezium BCED



Sol. Area of $\Delta ABC = 36\ cm^2$

$m\overline{AB} = 6$ cm

$m\overline{BC} = 4$ cm

(a) Area of $\Delta ADE = ?$

(b) Area of trapezium = BCED

Let $A_1 = 36\ cm^2$

$m\overline{AB} = \ell_1 = 6$ cm

Area of $\Delta ADE = A_2 = ?$

Using

Using $\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$

$$\frac{36}{A_2} = \left(\frac{6}{10}\right)^2$$

So, $\frac{36}{A_2} = \frac{36}{100}$

$$36\,A_2 = 36 \times 100$$

$$A_2 = \frac{36 \times 100}{36}$$

$$A_2 = 100 \text{ cm}^2$$

So area of $\Delta ADE = 100 \text{ cm}^2$

(b) Area of trapezium BCED $= \text{Area of } \Delta ADE - \Delta ABC$

$= 100 - 36 = 64 \text{ cm}^2$

Q.4. Given that ΔABC and ΔDEF are similar, with a scale factor of k = 3. If the area of ΔABC is 50 cm^2, find the area of triangle ΔDEF?

Sol. Given that $\Delta ABC \sim \Delta DEF$

Scale factor of k = 3

Area of $\Delta ABC = 50 \text{ cm}^2$

Area of $\Delta DEF = k^2\,\Delta ABC$

$= 3^2 \times 50$

$= 9 \times 50$

$= 450 \text{ cm}^2$

Q.5. Quadrilaterals ABCD and EFGH are similar, with a scale factor of $k = \frac{1}{4}$. If the area of quadrilateral ABCD is 64 cm^2, find the area of quadrilateral EFGH.

Sol. Given that quadrilateral

ABCD and EFGH are similar scale factor $k = \frac{1}{4}$

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Area of quadrilateral EFGH = k × area of quadrilateral ABCD

$$= \left(\frac{4}{1}\right)^2 \times 64$$

$$= 16 \times 64$$

Area of quadrilateral EFGH = 1024 cm^2

Q.6. The areas of two similar triangles are 16 cm^2 and 25 cm^2. What is the ratio of a pair of corresponding sides?

Sol. Let $A_1 = 6$

$A_2 = 25$

Now by formula

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

or $$\left(\frac{\ell_1}{\ell_2}\right)^2 = \frac{A_1}{A_2}$$

$$= \frac{16}{25}$$

$$\sqrt{\left(\frac{\ell_1}{\ell_2}\right)^2} = \sqrt{\frac{16}{25}}$$

$$\frac{\ell_1}{\ell_2} = \frac{4}{5}$$

Q.7. The area of two similar triangles are 144 cm^2 and 81 cm^2. If the base of the large triangle is 30 cm, find the corresponding base of the smaller triangle.

Sol. Let A_1 = Area of bigger triangle = 144 cm^2

Let A_2 = Area of smaller triangle = 81 cm^2

Base of large triangle $= \ell_1 = 30$

Base of smaller triangle $= \ell_2 = ?$

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Using $\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$

$$\frac{144}{81} = \left(\frac{30}{\ell_2}\right)^2$$

So, $\frac{144}{81} = \frac{900}{\ell_2^2}$

$$144\,\ell_2^2 = 81 \times 900$$

$$\ell_2 = \frac{81 \times 900}{144}$$

$$= \sqrt{\frac{81 \times 900}{144}}$$

$$= \frac{9 \times 30}{12} = \frac{270}{12}$$

$$\ell_2 = 22.67 \text{ cm}$$

Q.8. A regular heptagon is inscribed in a larger regular heptagon and each side of the larger heptagon is 1.7 times the side of the smaller heptagon. If the area of the smaller heptagon is known to be 100 cm^2, find the area of the larger heptagon.

Sol. Let A_2 be area of large heptagon $= 100 \text{ cm}^2$

Let A_1 is be area of smaller heptagon = ?

Here Scale factor = 1.7

Using $\frac{A_1}{A_2} = k^2$

$$A_1 = k^2 A_2$$

$$= (1.7)^2 \times 100$$

$$= 2.89 \times 100$$

Area of larger heptagon = 289 cm^2

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EXERCISE 9.3

Q.1. The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes?

Sol. Ratio between radii of two spheres = 3 : 4

Let V_1, V_2 be the volumes of two spheres. Then

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$= \left(\frac{3}{4}\right)^3$$

$$\frac{V_1}{V_2} = \frac{27}{64}$$

Q.2. Two regular tetrahedrons have volumes in the ratio 8 : 27. What is the ratio of their sides?

Sol. Ratio of volumes of regular tetrahedrons $= \frac{V_1}{V_2} = \frac{8}{27}$

Ratio of their sides $= \frac{\ell_1}{\ell_2}$

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{8}{21} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = \left(\frac{8}{27}\right)^{1/3}$$

$$\frac{\ell_1}{\ell_2} = \frac{2}{3}$$

Q.3. Two right cones have volumes in the ratio 64 : 125. What is the ratio of:

(a) their heights **(b) their base areas?**

Sol.(a) Ratio of volume of two right comes = 64 : 125

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{64}{125} = \left(\frac{h_1}{h_2}\right)^3$$

Where h_1 and h_2 are heights of cones.

$$\frac{h_1}{h_2} = \left(\frac{64}{125}\right)^{1/3}$$

$$= \left(\frac{4^3}{5^3}\right)^{1/3}$$

$$\frac{h_1}{h_2} = \frac{4}{5}$$

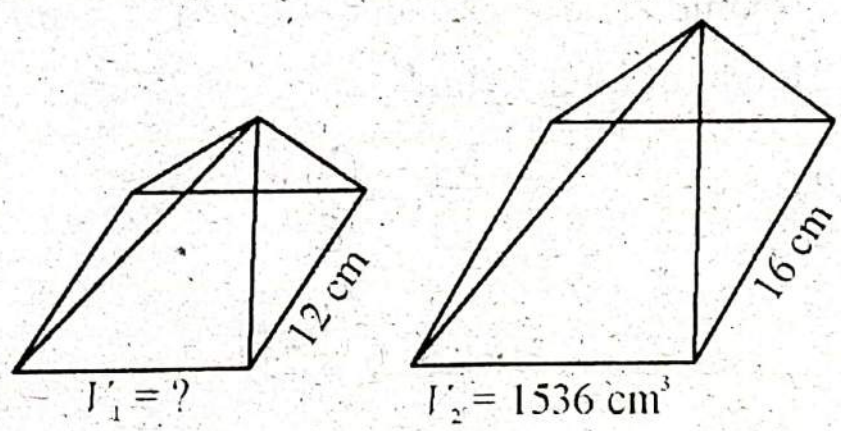
(b) Ratio of their base areas.

$$\frac{A_1}{A_2} = \left(\frac{4}{5}\right)^2$$

$$= \frac{16}{25}$$

Q.4. Find the missing value in the following similar solids.

(i)



Sol. V_1 = Volume of smaller shape = ?

V_2 = Volume of larger shape = 1536 cm^3

Side of smaller shape = ℓ_1 = 12 cm

Side of larger shape = ℓ_2 = 16 cm

Using $$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

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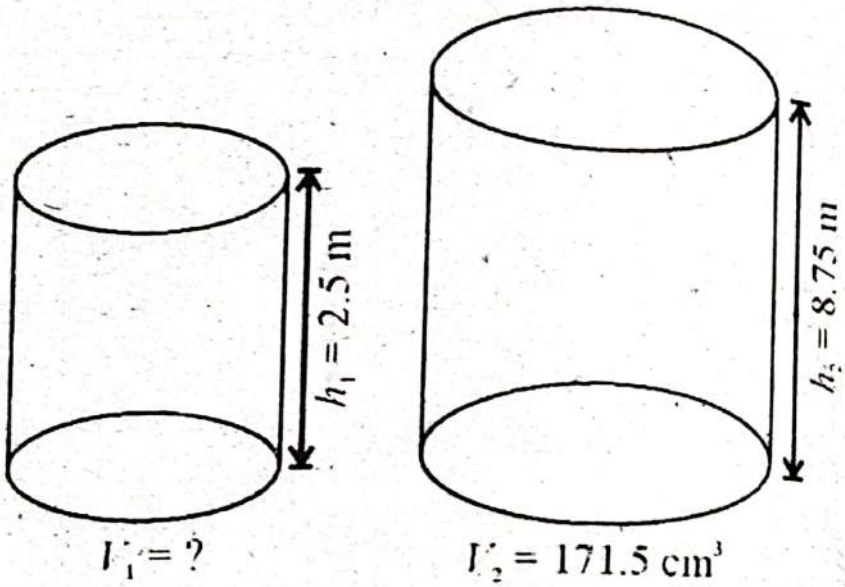
$$\frac{V_1}{1536} = \left(\frac{12}{16}\right)^3$$

$$V_1 = 1536 \times \frac{1728}{4096}$$

$$V_1 = 648 \text{ cm}^3$$

(ii)



Sol. V_1 = Volume of small cylinder = ?

Height of small cylinder = h_1 = 2.5 cm

Volume of large cylinder = V_2 = 171.5 cm^3

Height of large cylinder = h_2 = 8.75 cm

Using $$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{171.5} = \left(\frac{2.5}{8.75}\right)^3$$

$$V_1 = 171.5 \times \frac{(2.5)^3}{(8.75)^3}$$

$$= 171.5 \times \frac{15.625}{699.921875}$$

$$= \frac{2679.6875}{699.921875}$$

$$= 3.83 \text{ cm}^3$$

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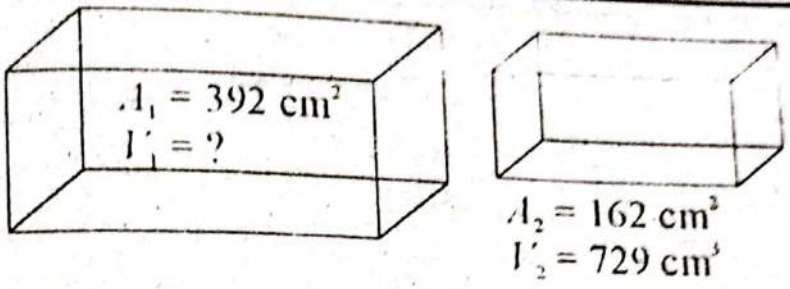
(iii)



Sol. $$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2 \Rightarrow \frac{392}{162} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\Rightarrow \quad \left(\frac{\ell_1}{\ell_2}\right)^2 = \frac{196}{81}$$

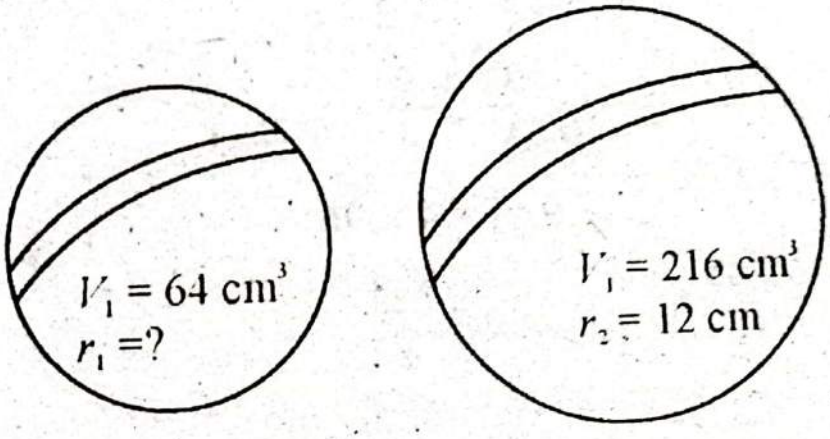
$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = \frac{14}{9}$$

Now $$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^2 \Rightarrow \frac{V_1}{729} = \left(\frac{14}{9}\right)^2$$

$$\Rightarrow \quad V_1 = 729 \times \frac{2744}{729}$$

$$V_1 = 2744 cm^3$$

(iv)



Sol. Here $V_1 = 64\ cm^3$, $V_2 = 216\ cm^3$

$r_1 = ?$, $r_2 = 12$ cm

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{64}{216} = \left(\frac{r_1}{12}\right)^3$$

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$$\frac{r_1^3}{1728} = \frac{64}{216}$$

$$r_1^3 = \frac{64}{216} \times 1728$$

$$= \frac{110592}{216}$$

$$r_1^3 = 512$$

$$r_1 = (512)^{1/3}$$

$$r_1 = 8 \text{ cm}$$

Q.5. The ratio of the corresponding lengths of two similar canonical cans is 3 : 2.

(i) The larger canonical can have surface area of 96 m^2. Find the surface area of the smaller canonical can.

Sol. Ratio of lengths of cans is 3 : 2

Larger can has surface area

$$A_1 = 96 \text{ m}^2$$

Smaller can has area = A_2 = ?

Using $$\frac{A_1}{A_2} = \left(\frac{3}{2}\right)^2$$

$$\frac{96}{A_2} = \frac{9}{4}$$

$$9A_2 = 4 \times 96$$

$$A_2 = \frac{384}{9}$$

$$A_2 = 42.67 \text{ m}^2$$

(ii) The smaller canonical can have a volume of 240 m^3. Find the volume of larger canonical can.

Sol.(ii) Now $V_2 = 240 \text{ m}^3$

V_1 = ?

Also ratio between lengths of cans = 3 : 2

Using $$\frac{V_1}{V_2} = \left(\frac{3}{2}\right)^3$$

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$$\frac{V_1}{240} = \frac{27}{8}$$

$$V_1 = \frac{27}{8} \times 240$$

$$V_1 = 27 \times 30$$

$$V_1 = 810 \text{ m}^3$$

Q.6. The ratio of the heights of two similar cylindrical water tanks is 5 : 3.

(i) If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank.

Sol. Surface area of larger tank = 250 m^2 = A_1

Surface area of smaller tank = A_2 = ?

Using $$\frac{A_1}{A_2} = \left(\frac{3}{2}\right)^2$$

$$\frac{250}{A_2} = \frac{25}{9}$$

$$25A_2 = 9 \times 250$$

$$A_2 = \frac{9 \times 250}{25}$$

$$= 9 \times 10$$

$$A_2 = 90 \text{ m}^2$$

(ii) If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank.

Sol. Volume of smaller tank = V_2 = 270 m^3

Volume of larger tank = V_1 = ?

Also

Ratio between heights of tank = 5 : 3

Using $$\frac{V_1}{V_2} = \left(\frac{5}{3}\right)^3$$

$$\frac{V_1}{270} = \frac{125}{27}$$

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$$V_1 = \frac{125}{27} \times 270$$

$$V_1 = 125 \times 10$$

$$V_1 = 1250 \text{ m}^3$$

EXERCISE 9.4

Q.1. (i) What is the sum of the interior angles of a decagon (10-sided polygon)?

Sol. The formula for sum of interior angles of n-sided polygon is $(n - 2) \times 180^\circ$.

Where n is the number of sides of a polygon. Here we have $n = 10$

Sum of interior angles of a decagon $= (10 - 2) \times 180^\circ$

$= 8 \times 180^\circ$

$= 1440^\circ$

(ii) Calculate the measure of each interior angle of a regular hexagon.

Sol. No. of sides of regular hexagon = 6

Sum of interior angles of a regular hexagon = 720°

So measure of each interior angle of regular hexagon $= \frac{720}{6}$

$= 120^\circ$

(iii) What is each exterior angle of a regular pentagon?

Sol. Regular pentagon has 5 sides

Sum of exterior angle of a polygon = 360°

Each exterior angle of regular pentagon $= \frac{360^\circ}{5}$

$= 72^\circ$.

(iv) If the sum of the interior angles of a polygon is 1440°, how many sides does the polygon have?

Sol. Sum of interior angles of polygon = 1440°

No. of sides of polygon = n = ?

Using

Sum of interior angles of polygon $= (n-2) \times 180^\circ$

So, $(n-2) \times 180^\circ = 1440^\circ$

$$= \frac{1440}{180}$$

$$n - 2 = 8$$

$$n = 2 + 8$$

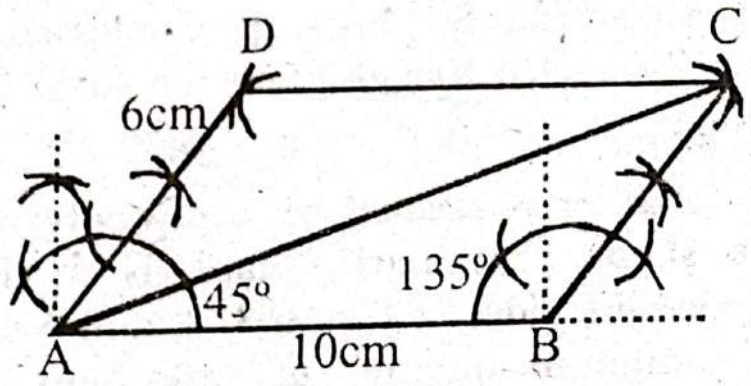
No. of sides $= n = 10$

So polygon is decagon.

Q.2. In a parallelogram ABCD, $m\overline{AB} = 10cm$, $m\overline{AD} = 6cm$ and $m\angle BAD = 45^\circ$. Calculate the length of diagonal $m\overline{AC}$.

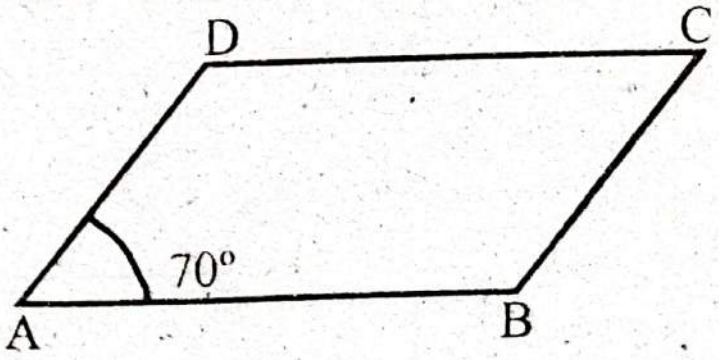
Sol. By actual construction of parallelogram with given measurements we find that

Length of diagonal $= \overline{AC} = 7.15cm$



Q.3. In a parallaelogram ABCD if $m\angle DAB = 70^\circ$, find the measures of all other angles in the parallelogram.

Sol. Given that ABCD is a parallelogram with $m\angle DAB = 70^\circ$



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Properties of Parallelogram

Opposite corner angles are equal.

Opposite sides are equal in length and parallel.

$m\angle DCB = m\angle DAB$

$m\angle DCB = 70^o$ $(\because m\angle DAB = 70^o)$

Now $m\angle ABC = 180^o - 70^o$

$= 110^o$

& $m\angle ABC = m\angle CDA$

or $m\angle CDA = m\angle ABC$

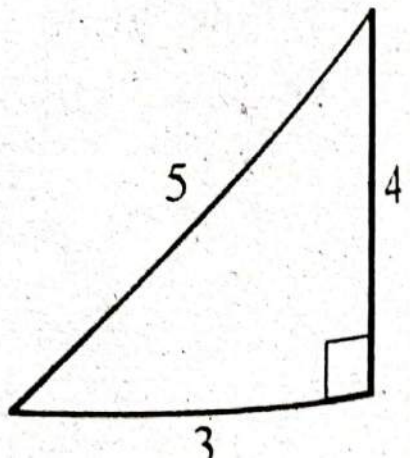
$m\angle CDA = 110^o$ $(\because m\angle ABC = 110^o)$

Q.4. A shape is created by cutting a square in half diagonally and then attaching a right-angled triangle to the hypotenuse of each half. Explain why this shape can tessellate and calculate the interior angle of the new shape.

Sol. A square is cut in half diagonally. Place right angled triangle to the hypotenuse of each half. Since both halves are similar. So, this shape can tessellate. Sum of both right angled triangles sum up to interior angles equal to 360°. $(\because 180^o + 180^o = 360^o)$

Q.5. A tessellation is created by repeatedly reflecting a basic shape. The basic shape is a right-angled triangle with sides of length 3, 4, and 5 units. Find: The minimum number of reflections needed to create a tessellation that covers a square with an area of 3600 square units.

Sol. Given dimension of right angled triangle are 3by 4 by5



Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3 \times 4$

$= 6$ sq. unit

Area conveyed by tessellation = 3600 sq. units

Minimum No. of reflections $= \frac{\text{Area covered by tessellation}}{\text{Area of triangle}}$

$= \frac{3600}{6}$

$= 600$

Q.6. A tessellation is created using regular hexagons. Each hexagon has a side length of 5 cm. Find the total area of the tessellation if it consists of 25 hexagons and total perimeter of the outer edge of the tessellation, assuming it's a perfect hexagon.

Sol. Length of side of regular hexagon = 5cm

Hexagon has 6 sides.

Area of regular hexagon $= \frac{3\sqrt{3}}{2} a^2$

Where 'a' is the side of hexagon

Area of hexagon $= \frac{3\sqrt{3}}{2} (5)^2$

$= \frac{3\sqrt{3}}{2} \times 25$

$= 64.95$ sq. unit

Total area of tessellation if it consists of 25 hexagons

$= 64.95 \times 25$

$= 1623.75$ sq. unit

Perimeter of outer edge of tessellation $= 24 \times 4$

$= 100$

Q.7. A rectangular floor is 12 m by 15 m. How many square tiles, each 1 m by 1 m, are needed to cover the floor?

Ans. Length of floor = 15 m

Width of floor = 12 m

Area of floor $= L \times W$

$= 15\text{ m} \times 12\text{ m}$

$= 180\text{ m}^2$

Dimensions of square tile = 1m by 1m

Area of each tile $= 1 \times 1$

$= 1\text{ m}^2$

No. of tiles to cover floor $= \frac{180}{1}$

= 180 tiles

Q.8. A rectangular wall is 10 m tall and 12 m wide. How many gallons of paint are needed to cover the wall, if one gallon covers 350 m^2?

Sol. Height of wall = 10m

Width of wall = 12 m

Area of wall $= 10 \times 12$

$= 120\text{ m}^2$

One gallon covers 350m^2.

or $350\text{m}^2 = 1$ gallon

$1\text{ m}^2 = \frac{1}{350}$ gallon

So, $120\text{ m}^2 = \frac{120}{350}$ gallon

= 0.34 gallons needed to paint 120m^2

Q.9. A rectangular wall has a length of 10 m and a width of 4 meters. If 1 litre of paint covers 2 m^2, how many liters of paint are needed to cover the wall?

Ans. Length of wall = 10 m

Width of wall = 4 m

Area of wall $= 10\text{ m} \times 4\text{m}$
$= 40\text{ m}^2$

Given that

1 litres of point covers 2m^2.

$$2\text{ m}^2 = 1\text{ litre}$$

$$1\text{ m}^2 = \frac{1}{2}\ell$$

So, $40\text{ m}^2 = 40 \times \frac{1}{2}\ell$

$= 20$ litres

So, 20 litres of paint is needed for the wall.

Q.10. A window has a trapezoidal shape with parallel sides of 3 m and 1.5 m and a height of 2 m. Find the area of the window.

Sol. Parallel sides of trapezium are 3 m & 1.5 m

Height of window = h = 2m

Let a = 3 m, b = 1.5 m, h = 2

Area of trapezoidal window

$$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (3 + 1.5) \times 2$$

$$= 4.5\text{ m}^2$$

REVIEW EXERCISE 9

Q.1. Four options are given against each statement. Encircle the correct one.

(i) If two polygons are similar, then:

(a) their corresponding angles are equal.

(b) their areas are equal.

(c) their volumes are equal.

(d) their corresponding sides are equal.

Sol. **(a)**

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(ii) The ratio of the areas of two similar polygons is:

(a) equal to the ratio of their perimeters.

(b) equal to the square of the ratio of their corresponding sides.

(c) equal to the cube to the ratio of their corresponding sides.

(d) equal to the sum of their corresponding sides.

Sol. **(b)**

(iii) If the volume of two similar solids is 125 cm^3 and 27 cm^3, the ratio of their corresponding heights is _____.

(a) 3 : 5 (b) 5 : 3

(c) 25 : 9 (d) 9 : 25

Sol. **(b)**

(iv) The exterior angle of regular pentagon is:

(a) 40° (b) 45°

(c) 60° (d) 72°

Sol. **(d)**

(v) A parallelogram has an area of 64 cm^2 and a similar parallelogram has an area of 144 cm^2. If a side of the smaller parallelogram is 8cm, the corresponding side of the larger parallelogram is:

(a) 10 cm (b) 12 cm

(c) 18 cm (d) 16 cm

Sol. **(c)**

(vi) The total number of diagonals in a polygon with 9 sides is:

(a) 18 (b) 21

(c) 25 (d) 27

Sol. **(d)**

(vii) Two spheres are similar, and their radii are in the ration 4 : 5. If the surface area of the larger sphere is 500π cm^2, what is the surface area of the smaller sphere?

(a) 256π cm^2 (b) 320π cm^2

(c) 400π cm^2 (d) 405π cm^2

Sol. **(b)**

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(viii) A regular polygon has an exterior angle of 30°. How many diagonals does the Polygon have?

(a) 54 (b) 90

(c) 72 (d) 108

Sol. The number of diagonals = 66 – 12 = 54

So option (a) is correct.

(ix) In a regular hexagon, the ratio of the length of a diagonal to the side length is:

(a) $\sqrt{3} : 1$ (b) 2 : 1

(c) 3 : 2 (d) 2 : 3

Sol. **(b)**

(x) A regular polygon has an interior angle of 165°. How many sides does it have?

(a) 15 (b) 16

(c) 20 (d) 24

Sol. The interior angle of polygon is 165°.

The exterior angle $= 180 - 165$

$= 15^\circ$

Sum of exterior angles of a polygon $= 360^\circ$

So, No. of sides a polygon have $= \frac{360}{15}$

$= 24$ sides

Sol. So, option (d) is correct.

Q.2. If the sum of the interior angles of a polygon is 1080°, how many sides does the polygon has?

Sol. Given that

Sum of interior angles of polygon $= 1080^\circ$

We know that

Sum of interior angles of polygon $= (n - 2) \times 180^\circ$

$1080 = (n - 2) \times 180^\circ$

$n - 2 = \frac{1080}{180}$

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$$n - 2 = 6$$
$$n = 6 + 2$$
$$n = 8 \text{ sides}$$

So polygon has 8 sides. i.e., (octagon).

Q.3. Two similar bottles are such that one is twice as high as the other. What is the ratio of their surface areas and their capacities?

Sol. Since it is given that

One bottle is twice as high as the other.

Let A_1, A_2 be the surface areas of two bottles. V_1, V_2 be the capacities.

Then $$\frac{A_1}{A_2} = \left(\frac{2}{1}\right)^2$$
$$= \frac{4}{1} = 4 : 1$$

Now $$\frac{V_1}{V_2} = \left(\frac{2}{1}\right)^2$$
$$\frac{V_1}{V_2} = \frac{8}{1}$$
$$\frac{V_1}{V_2} = 8 : 1$$

Q.4. Each dimension of a model car is $\frac{1}{10}$ of the corresponding car dimension. Find the ratio of:

(a) the areas of their windscreens

(b) the capacities of their boots

(c) the widths of the cars

(d) the number of wheels they have.

Sol. Given that

Dimension of model car is $\frac{1}{10}$ of the corresponding car dimension.

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(a) Ratio of areas of their wind screens = ?

As there is similarity between windscreen of model car and corresponding car.

So $$\frac{A_1}{A_2} = \left(\frac{1}{10}\right)^2$$

$$= \frac{1}{100}$$

$$A_1 : A_2 = 1 : 100$$

(b) Ratio of their boots $= \frac{V_1}{V_2} = \left(\frac{1}{10}\right)^3$

$$V_1 : V_2 = 1 : 1000$$

(c) Ratio between widths of cars.

Let W_1, W_2 be the widths of model car and corresponding car

Then $$\frac{W_1}{W_2} = \frac{1}{10}$$

$$W_1 : W_2 = 1 : 10$$

(d) Ratio of the number of wheels they have = ?

Clearly No. of wheels of model car = No. of wheels of corresponding car

So Required ratio = 4 : 4

= 1 : 1

Q.5. Three similar jugs have heights 8 cm, 12 cm and 16 cm. If the smallest jug holds $\frac{1}{2}$ litre, find the capacities of the other two.

Sol. Given that three similar jugs have height 8 cm, 12 cm, 16 cm.

Smallest jug holds $\frac{1}{2}$ litre.

Capacities of other two jugs are to be found.

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

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(d) **the area of the rear window of the actual car if the area of the rear window of the model is 3 cm^2.**

Sol. Area of rear window of actual car = ?

Area of rear window of model car is 3cm^2.

Given in statement,

1 cm^2 represents 2500 cm^2

So 3 cm^2 represents = 3 × 2500 cm^2

= 7500 cm^2

Q.8. **The ratio of the areas of two similar labels on two similar jars of coffee is 144 : 169. Find the ratio of**

(a) **the heights of the two jars**

(b) **their capacities.**

Sol. Given that

Ratio of area of two similar labels on two similar jars of coffee = 144 : 169

(a) Ratio of heights of two jars = ?

Using $$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$$

$$\frac{144}{169} = \left(\frac{h_1}{h_2}\right)^2$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{12}{13}$$

So, Required ratio = 12 : 13

(b) To find ratio of their capacities,

Using $$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$= \left(\frac{12}{13}\right)^3$$

$$= \frac{12^3}{13^3}$$

$$= 1728 : 2197$$

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