Unit No. 2

Logarithms

Review Exercise No. 2

Question No. 1

Choose the correct option.

(i). The standard form of 5.2×10^6 is:

(a) 52,000

(b) 520,000

(c) 5,200,000

(d) 52,000,000

(ii). Scientific notation of 0.00034 is:

(a) 3.4×10^3

(b) 3.4×10^{-4}

(c) 3.4×10^4

(d) 3.4×10^{-3}

(iii). The base of common logarithm is:

(a) 2

(b) 10

(c) 5

(d) e

(iv). $\log_2 2^3$ = _________

(a) 1

(b) 2

(c) 5

(d) 3

(v). log 100 = _________

(a) 2

(b) 3

(c) 10

(d) 1

(vi) If log 2 = 0.3010, then log 2000 is:

(a) 1.3010

(b) 0.6010

(c) 2.3010

(d) 2.6010

(vii) log (0) = ______

(a) positive

(b) negative

(c) zero

(d) undefined

(viii) log 10,000 =

(a) 2

(b) 3

(c) 4

(d) 5

(ix). log 5 + log 3 = _______

(a) log 0

(b) log 2

(c) log $[\frac{5}{3}]$

(d) log 15

(x). $3^4 = 81$ in logarithmic form is:

(a) $\log_3 4 = 81$

(b) $\log_4 3=81$

(c) $\log_3 81 = 4$

(d) $\log_4 81 = 3$

Question No. 2

Express the following numbers in scientific notation:

(i). 0.000567

Solution:

0.000567

$= \frac{567}{1000000}$

$= \frac{5.67}{1000000} \times 100$

$= \frac{567}{10000}$

$= \frac{567}{10^4}$

In scientific notation:

$= 5.67 \times 10^{-4}$

(ii). 734

Solution:

734

$= 7.34 \times 100$

In scientific notation:

$= 7.34 \times 10^2$

(iii) 0.33×10^3

Solution:

0.33×10^3

$= \frac{33}{100} \times 10^3$

$= 33 \times 10^{-2} \times 10^3$

$= 3.3 \times 10 \times 10^{-2} \times 10^3$

$= 3.3 \times 10^{1-2+3}$

In scientific notation:

$= 3.3 \times 10^2$

Question No. 3

Express the following numbers in ordinary notation:

(i). 2.6×10^3

Solution:

2.6×10^3

$= \frac{26}{10} \times 10^3$

$= 26 \times 10^{-1} \times 10^3$

$= 26 \times 10^{-1+3}$

$= 26 \times 10^2$

$= 26 \times 100$

In ordinary notation:

$= 2600$

(ii). 8.794×10^{-4}

Solution:

8.794×10^{-4}

$= \frac{8794}{1000} \times 10^{-4}$

$= 8794 \times 10^{-3} \times 10^{-4}$

$= 8794 \times 10^{-3-4}$

$= 8794 \times 10^{-7}$

$= \frac{8794}{10^7}$

$= \frac{8794}{10000000}$

In ordinary notation:

$= 0.0008794$

(iii). 6×10^{-6}

Solution:

6×10^{-6}

$= \frac{6}{10^6}$

$= \frac{6}{1000000}$

In ordinary notation:

$= 0.000006$

Question No. 4

Express each of the following in logarithmic form.

(i). $3^7=2187$

Solution:

Logarithmic form:

$\log_3 2187 = 7$

(ii). $a^b= c$

Solution:

$a^b= c$

In logarithmic form:

$\log_a c = b$

(iii). $(12)^2 = 144$

Solution:

$(12)^2 = 144$

In logarithmic form:

$\log_{12} 144 = 2$

Question No. 5

Express each of the following is exponential form:

(i). $\log_4 8= x$

Solution:

$\log_4 8= x$

In exponential form:

$4^x = 8$

(ii). $\log_9 729 = 3$

Solution:

$\log_9 729 = 3$

Exponential form is

$9^3 = 729$

(iii). $\log_4 1024 = 5$

Solution:

$\log_4 1024 = 5$

Exponential form:

$4^5 = 1024$

Question No. 6

Find the value of x in the following:

(i). $\log_9 x = 0.5$

Solution:

$\log_9 x = 0.5$

By writing in exponential form:

$9^{0.5} = x$

$x = 3$

(ii). $[\frac{1}{9}]^{3x} = 27$

Solution:

$[\frac{1}{9}]^{3x} = 27$

$[\frac{1}{3^2}]^{3x} = 3^3$

$[3^{-2}]^{3x} = 3^3$

$3^{-6x} = 3^3$

Base are same, so:

$-6x = 3$

$$x = -\frac{3}{6}$$

$$x = -\frac{1}{2}$$

(iii). $\left[\frac{1}{32}\right]^{2x} = 64$

Solution:

$\left[\frac{1}{32}\right]^{2x} = 64$

$\left[\frac{1}{2^5}\right]^{2x} = 2^6$

$[2^{-5}]^{2x} = 2^6$

$2^{-10x} = 2^6$

Base are same, So:

$-10x = 6$

$x = -\frac{6}{10}$

$x = -\frac{3}{5}$

Question No. 7

Write the following as a single logarithm:

(i). 7 log x – 3 log y^2

Solution:

7 log x – 3 log y^2

Applying power rule:

= log $(x)^7$ – log $(y^2)^3$

= log x^7 – log y^6

In a single logarithm:

= log x^7y^6

(ii). 3 log 4 – log 32

Solution:

3 log 4 – log 32

Applying power rule:

= log $(4)^3$ – log 32

In a single logarithm:

= log $\frac{4^3}{32}$

= log $\frac{64}{32}$

= log 2

(iii). $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

Solution:

$\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

Applying power rule:

$= (\log_5 8 + \log_5 27)^{\frac{1}{3}} - \log_5 3$

In a single logarithm:

$= \log_5 \frac{\sqrt[3]{8 \times 27}}{3}$

$= \log_5 \frac{\sqrt[3]{216}}{3}$

$= \log_5 \frac{\sqrt[3]{6^3}}{3}$

$= \log_5 \frac{6}{3}$

$= \log_5 2$

Question No. 8

Expand the following using laws of logarithm:

(i). log (xyz^6)

Solution:

$\log (xyz^6)$

Applying product rule:

$= \log x + \log y + \log z^6$

Applying power rule:

$= \log x + \log y + 6 \log z$

(ii). $\log_3 \sqrt[6]{m^5n^3}$

Solution:

$\log_3 \sqrt[6]{m^5n^3}$

Applying power rule:

$= \log_3 (m^5n^3)^{\frac{1}{6}}$

$= \frac{1}{6}[\log_3 (m^5n^3)]$

Applying product rule:

$= \frac{1}{6}[\log_3 m^5 + \log_3 n^3]$

Applying power rule:

$= \frac{1}{6}[5\log_3 m + 3\log_3 n]$

(iii). $\log\sqrt{8x^3}$

Solution:

$\log\sqrt{8x^3}$

Applying power rule:

$= log\ (8x^3)^{\frac{1}{2}}$

$= log\ (2^3)^{\frac{1}{2}}x^{\frac{3}{2}}$

$= log\ (2^{\frac{3}{2}})x^{\frac{3}{2}}$

$= log\ (2^{\frac{3}{2}})x^{\frac{3}{2}}$

Applying product rule:

$= log\ 2^{\frac{3}{2}} + logx^{\frac{3}{2}}$

Applying power rule:

$= \frac{3}{2}log\ 2 + \frac{3}{2}logx$

$= \frac{3}{2}[log\ 2 + logx]$

Question No. 9

Find the values of the following with the help of logarithm table:

(i). $\sqrt[3]{\mathbf{68.24}}$

Solution:

$\sqrt[3]{68.24}$

Let:

$x = \sqrt[3]{68.24}$

Taking log on both sides:

$logx = log\sqrt[3]{68.24}$

Applying power rule:

$logx = \log\ (68.24)^{\frac{1}{3}}$

$logx = \frac{1}{3}\log\ 68.24$

$logx = \frac{1}{3}\ (1.8340)$

$logx = 0.6113$

Taking antilog:

$x = antilog\ 0.6113$

$x = 4.0860$

(ii). 319.8 × 3.543

Solution:

$$319.8 \times 3.543$$

Let:

$$x = 319.8 \times 3.543$$

Taking log on both sides:

$$logx = \log(319.8 \times 3.543)$$

Applying product rule:

$$logx = \log 319.8 + \log 3.543$$

$$logx = 2.5049 + 0.5494$$

$$logx = 3.0543$$

Taking antilog:

$$x = antilog\ 3.0543$$

$$x = 1133.18$$

(iii). $\frac{\mathbf{36.12 \times 750.9}}{\mathbf{113.2 \times 9.98}}$

Solution:

$$\frac{36.12 \times 750.9}{113.2 \times 9.98}$$

Let:

$$x = \frac{36.12 \times 750.9}{113.2 \times 9.98}$$

Taking log on both sides:

$$logx = \log \frac{36.12 \times 750.9}{113.2 \times 9.98}$$

Applying product and quetient rules:

$$logx = \log 36.12 + \log 750.9 - [log113.2 + log9.98]$$

$$logx = \log 36.12 + \log 750.9 - log113.2 - log9.98$$

$$logx = 1.557 + 2.8756 - 2.0538 - 0.9991$$

$$logx = 1.3797$$

Taking antilog:

$$x = antilog\ 1.3797$$

$$x = 24.01$$

Question No. 10

In the year 2016, the population of a city wa 22 millions and was growing at a rate of 2.5% per year. The function

$p(t) = 22(1.025)^t$ gives the population in millions, t years after 2016. Use model to determine in which year the population will reach 35 millions. Round the answer to the nearest year.

Data:

$p(t) = 22(1.025)^t$

$p(t) = 35$ millions

Rate = 2.5%

To find:

Time = t = ?

Solution:

$p(t) = 22(1.025)^t$

By putting values:

$35 = 22(1.025)^t$

Taking log on both sides:

$\log 35 = \log 22(1.025)^t$

Applying product rule:

$\log 35 = \log 22 + \log (1.025)^t$

Applying power rule:

$\log 35 = \log 22 + t\log (1.025)$

$1.5441 = 1.3424 + t\,(0.01072)$

$0.2017 = t\,(0.01072)$

$$\frac{0.2017}{0.01072} = t$$

$t = 18.82\ years$

$t = 19\ years$

So, in (2016 + 19) **2035** the population will reach 35 millions.