Unit No. 4

Factorization and Algebraic Manipulation

Exercise No. 4.3

Question No. 1

Find HCF by factorization method.

(i). $21x^2y, 35xy^2$

Solution:

Factors of $21x^2y = 3 \times 7\, x.x.y$

Factors of $35xy^2 = 5 \times 7\, x.y.y$

Common factor = C.F = $7.x.y$

H.C.F = $7xy$

(ii). $4x^2 - 9y^2, 2x^2 - 3xy$

Solution:

Factors of $4x^2 - 9y^2 = (2x)^2 - (3y)^2$

$= (2x + 3y)(2x - 3y)$

Factors of $2x^2 - 3xy = 2x^2 - 3xy$

$= x\,(2x - 3y)$

Common factor = C.F = $(2x - 3y)$

H.C.F = $(2x - 3y)$

(iii). $x^3 - 1, x^2 + x + 1$

Solution:

Factors of $x^3 - 1 = x^3 - 1$

$= (x - 1)(x^2 + x.1 + 1^2)$

$= (x - 1)(x^2 + x + 1)$

Factors of $x^2 + x + 1 = x^2 + x + 1$

Common factor = C.F = $(x^2 + x + 1)$

H.C.F = $(x^2 + x + 1)$

(iv). $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

Solution:

Factors of $a^3 + 2a^2 - 3a = a(a^2 + 2a - 3)$

$= a(a^2 + 3a - a - 3)$

$= a[a(a + 3) - 1(a + 3)]$

$= a(a + 3)(a - 1)$

Factors of $2a^3 + 5a^2 - 3a = a(2a^2 + 5a - 3)$

$= a(2a^2 + 6a - a - 3)$

$= a[2a(a + 3) - 1(a + 3)]$

$= a(a + 3)(2a - 1)$

Common factor = C.F = $a(a + 3)$

H.C.F = $a(a + 3)$ or $(a^2 + 3a)$

(v). $\boldsymbol{t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1}$ (Wrong value in book)

Solution:

Factors of $t^2 - 3t - 4 = t^2 - 3t - 4$

$= t^2 + t - 4t - 4$

$= t(t + 1) - 4(t + 1)$

$= (t + 1)(t - 4)$

Factors of $t^2 + 5t + 4 = t^2 + 5t + 4$

$= t^2 + t + 4t + 4$

$= t(t + 1) + 4(t + 1)$

$= (t + 1)(t + 4)$

Factors of $t^2 - 1 = t^2 - 1^2$

$= (t + 1)(t - 1)$

Common factor = C.F = $(t + 1)$

H.C.F = $(t + 1)$

(vi). $\boldsymbol{x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x}$

Solution:

Factors of $x^2 + 15x + 56 = x^2 + 7x + 8x + 56$

$= x(x + 7) + 8(x + 7)$

$= (x + 7)(x + 8)$

Factors of $x^2 + 5x - 24 = x^2 + 8x - 3x - 24$

$= x(x + 8) - 3(x + 8)$

$= (x + 8)(x - 3)$

Factors of $x^2 + 8x = x^2 + 8x$

$= x(x + 8)$

Common factor = C.F = $(x + 8)$

H.C.F = $(x + 8)$

Question No. 2

Find HCF of the following expressions by using division method:

(i). $27x^3 + 9x^2 - 3x - 10,\ 3x - 2$ (Wrong value in book)

Solution:

$$\begin{array}{r|l|l} 3x-2 & 27x^3+9x^2-3x-10 & 9x^2+9x+5 \\ & \pm 27x^3 \mp 18x^2 & \\ \hline & 27x^2-3x-10 & \\ & \pm 27x^2 \mp 18x & \\ \hline & 15x-10 & \\ & \pm 15x \mp 10 & \\ \hline & 0 & \end{array}$$

H.C.F = 3x - 2

(ii). $x^3 - 9x^2 + 21x - 9,\ x^2 - 4x + 3$ (Wrong value in book)

Solution:

$$\begin{array}{r|l|l} x^2-4x+3 & x^3-9x^2+21x-9 & x-5 \\ & \pm x^3 \mp 4x^2 \pm 3x & \\ \hline & -5x^2+18x-9 & \\ & \mp 5x^2 \pm 20x \mp 15 & \\ \hline & -2x+6 & \end{array}$$

we may write it $-2(x-3)$

Now

$$\begin{array}{r|l|l} x-3 & x^2-4x+3 & x-1 \\ & \pm x^2 \mp 3x & \\ \hline & -x+3 & \\ & \mp x \pm 3 & \\ \hline & 0 & \end{array}$$

H.C.F = x - 3

(iii). $2x^3 + 2x^2 + 2x + 2,\ 6x^3 + 12x^2 + 6x + 12$

Solution:

$2x^3 + 2x^2 + 2x + 2,\ 6x^3 + 12x^2 + 6x + 12$

$= 2(x^3 + x^2 + x + 1),\ 6(x^3 + 2x^2 + x + 2)$

Ignore common number:

$$\begin{array}{r|l|l} x^3+x^2+x+1 & x^3+2x^2+x+2 & 1 \\ & \pm x^3 \pm x^2 \pm x \pm 1 & \\ \hline & x^2+1 & \end{array}$$

Now

$$\begin{array}{r|l|l} x^2+1 & x^3+x^2+x+1 & x+1 \\ & \pm x^3 \quad \pm x & \\ \hline & x^2+1 & \\ & \pm x^2 \pm 1 & \\ \hline & 0 & \end{array}$$

H.C.F = $x^2 + 1$ (Wrong answer in book)

(iv). $2x^3 - 4x^2 + 6x$, $x^3 - 2x$, $3x^2 - 6x$

Solution:

$x^3 - 2x$) $2x^3 - 4x^2 + 6x$ (2
$\pm 2x^3 \mp 4x$
$-4x^2 + 10x$ we may write it $-2(2x^2 - 5x)$

Now multiply $x^3 - 2x$ with 2 and simplify

$2x^2 - 5x$) $2x^3 - 4x$ (x
$\pm 2x^3 \mp 5x^2$
$5x^2 - 4x$

Now multiply $5x^2 - 4x$ with 2 and simplify

$2x^2 - 5x$) $10x^2 - 8x$ (5
$\pm 10x^2 \mp 25x$
$17x$

Now

x) $2x^2 - 5x$ ($2x - 5$
$\pm 2x^2 \mp 5x$
0

And

x) $3x^2 - 6x$ ($3x - 6$
$\pm 3x^2 \mp 6x$
0

HCF = x wrong answer in book

Question No. 3

Find L.C.M of the following expressions by using prime factorization method.

(i) $2a^2b$, $4ab^2$, $6ab$

Solution:

$2a^2b$, $4ab^2$

Factors of $2a^2b$ = 2.a.a.b

Factors of $4ab^2$ = 2.2.a.b.b

Factors of 6ab = 2.3.a.b

C.F = 2.a.b

N.C.F = 2.3.a.b

L.C.M = C.F × N.C.F

L.C.M = 2.a.b × 2.3.a.b

L.C.M = $12a^2b^2$

(ii) $x^2 + x, x^3 + x^2$

Solution:

$x^2 + x, x^3 + x^2$

Factors of $x^2 + x = x(x + 1)$

Factors of $x^3 + x^2 = x.x\ (x + 1)$

C.F = $x(x + 1)$

N.C.F = x

L.C.M = C.F × N.C.F

L.C.M = $x(x + 1) \times x$

L.C.M = $x^2(x + 1)$

(iii) $a^2 - 4a + 4, a^2 - 2a$

Solution:

$a^2 - 4a + 4, a^2 - 2a$

Factors of $a^2 - 4a + 4 = a^2 - 2a - 2a + 4$

$= a(a - 2) - 2(a - 2)$

$= (a - 2)(a - 2)$

Factors of $a^2 - 2a = a\ (a - 2)$

C.F = $(a - 2)$

N.C.F = $a\ (a - 2)$

L.C.M = C.F × N.C.F

L.C.M = $(a - 2) \times a.\ (a - 2)$

L.C.M = $a(a - 2)^2$

(iv) $x^4 - 16, x^3 - 4x$

Solution:

$x^4 - 16, x^3 - 4x$

Factors of $x^4 - 16 = (x^2)^2 - (4)^2$

$= (x^2 - 4)(x^2 + 4)$

Factors of $x^3 - 4x = x(x^2 - 4)$

C.F = $(x^2 - 4)$

N.C.F = $x.\ (x^2 + 4)$

L.C.M = C.F × N.C.F

L.C.M = $(x^2 - 4) \times x.\ (x^2 + 4)$

L.C.M = $x(x^2 - 4)(x^2 + 4)$
L.C.M = $x(x^4 - 16)$

(v) $16 - 4x^2, x^2 + x - 6, 4 - x^2$

Solution:

$16 - 4x^2, x^2 + x - 6, 4 - x^2$

Factors of $16 - 4x^2 = 4(4 - x^2)$

$= 2.2(2 - x)(2 + x)$

Factors of $x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= x(x + 3) - 2(x + 3)$

$= (x + 3)(x - 2)$

$= (x + 3)[-(-x + 2)]$

$= (-x - 3)(2 - x)$

Factors of $4 - x^2 = 2^2 - x^2$

$= (2 + x)(2 - x)$

C.F = $(2 - x)$

N.C.F = $2.2.(-x - 3)(x + 2)$

$= -4(x + 3)(x + 2)$

L.C.M = C.F × N.C.F

L.C.M = $(2 - x) \times [-4(x + 3)(x + 2)]$

L.C.M = $-4(4 - x^2)(x - 3)$

L.C.M = $4(-4 + x^2)(x - 3)$

L.C.M = $4(x^2 - 4)(x - 3)$ (Wrong answer in book)

Question No. 4

The HCF of two polynomials is $y - 7$ and their LCM is $y^3 - 10y^2 + 11y + 70$. If one of the polynomials is $y^2 - 5y - 14$, find the other.

Given:

H.C.F = $y - 7$

L.C.M = $y^3 - 10y^2 + 11y + 70$

$P(x) = y^2 - 5y - 14$

To Find:

$Q(x) = ?$

Solution:

Simplifying P(x):

$P(x) = y^2 - 5y - 14 = y^2 - 7y + 2y - 14$

$= y(y - 7) + 2(y - 7)$

$= (y - 7)(y + 2)$

Simplifying L.C.M:

$= y^3 - 10y^2 + 11y + 70$

Possible factors of constant term +70 are: (±1,±2,±5,±7,±10,±14,±35,±70)

We check for a root by substituting values:

Where possible factors are:

$(y+2)(y-5)(y-7)$, So;

$y^3 - 10y^2 + 11y + 70 = (y+2)(y-5)(y-7)$

$P(x) \times Q(x) = H.C.F \times L.C.M$

By putting values:

$(y - 7)(y + 2) \times Q(x) = (y - 7) \times (y+2)(y-5)(y-7)$

$Q(x) = \frac{(y - 7) \times (y+2)(y-5)(y-7)}{(y - 7)(y + 2)}$

$Q(x) = (y - 5)(y - 7)$

$Q(x) = y^2 - 5y - 7y + 35$

$Q(x) = y^2 - 12y + 35$

Question No. 5

The LCM and HCF of two polynomials p(x) and q(x) are $36x^3(x + a)(x^3 - a^3)$ and $x^2(x - a)$ respectively. If $p(x) = 4x^2 (x^2 - a^2)$, find q(x).

Given:

$L.C.M = 36x^3(x + a)(x^3 - a^3)$

$H.C.F = x^2 (x - a)$

$P(x) = 4x^2 (x^2 - a^2)$

To Find:

$Q(x) = ?$

Solution:

Simplifying $L.C.M = 36x^3(x + a)(x^3 - a^3)$

$= 2.2.3.3.x.x.x(x + a)(x - a)(x^2 - ax + a^2)$

Simplifying $H.C.F = x^2 (x - a)$

$= x.x (x - a)$

Simplifying $P(x) = 4x^2 (x^2 - a^2)$

$= 2.2.x.x (x + a) (x - a)$

$P(x) \times Q(x) = H.C.F \times L.C.M$

By putting values:

$2.2.x.x (x + a) (x - a) \times Q(x) = x.x (x - a) \times 2.2.3.3.x.x.x(x + a)(x - a)(x^2 - ax + a^2)$

$$Q(x) = \frac{x.x\,(x-a) \times 2.2.3.3.x.x.x(x+a)(x-a)(x^2-ax+a^2)}{2.2.x.x\,(x+a)\,(x-a)}$$

$$Q(x) = 3.3.x.x.x(x-a)(x^2-ax+a^2)$$

$$Q(x) = 9x^3(x^3-a^3)$$

Question No. 6

The HCF and LCM of two polynomials is $(x + a)$ and $12x^2\,(x + a)(x^2 - a^2)$ respectively. Find the product of the two polynomials.

Given:

$$H.C.F = (x+a)$$

$$L.C.M = 12x^2\,(x+a)(x^2-a^2)$$

Simplifying $L.C.M = 12x^2\,(x+a)(x^2-a^2)$

$$= 12x^2\,(x+a)(x+a)(x-a)$$

$$= 12x^2\,(x+a)^2(x-a)$$

To Find:

$$P(x) \times Q(x) = ?$$

Solution:

$$P(x) \times Q(x) = H.C.F \times L.C.M$$

By putting values:

$$P(x) \times Q(x) = (x+a) \times 12x^2\,(x+a)^2(x-a)$$

$$P(x) \times Q(x) = 12x^2\,(x-a)\,(x+a)^3$$