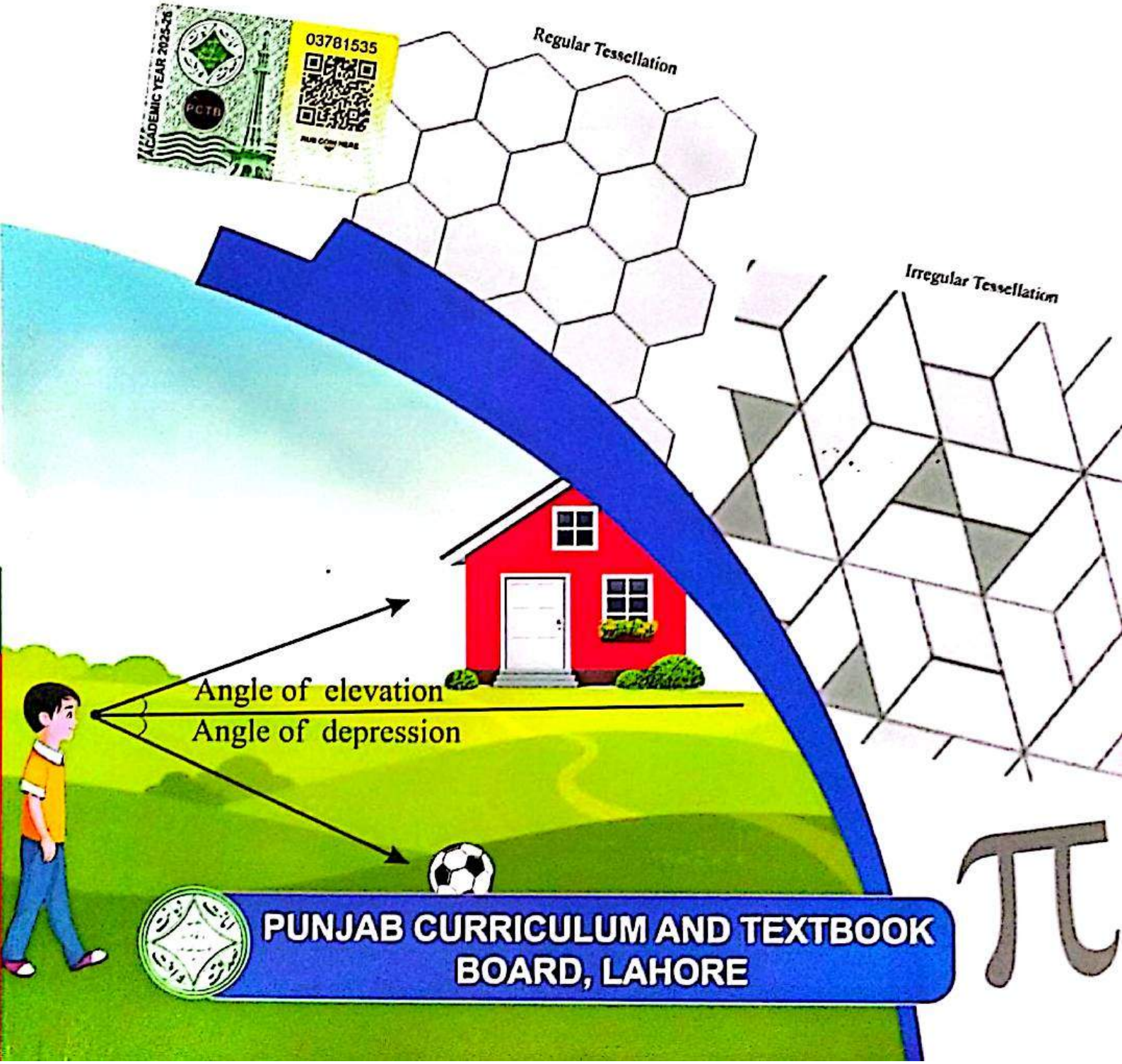
MATHEMATICS 9



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UNIT 3 SETS AND FUNCTIONS

EXERCISE 3.1

Q.1. Write the following sets in set builder notation:

(i) **{1, 4, 9, 16, 25, 36, …, 484}**

Sol. Set builder notation is

$\{x \mid x = n^2, n \in N \wedge 1 \leq n \leq 22\}$

(ii) **{2, 4, 8, 16, 32, …, 128}**

Sol. Set builder notation is

$\{x \mid x \in 2^n, n \in N \wedge 1 \leq n \leq 7\}$

(iii) **{0, ±1, ±2, …, ±1000}**

Sol. Builder notation is

$\{x \mid x \in Z \wedge -1000 \leq x \leq 1000\}$

(iv) **{6, 12, 18, …, 120}**

Sol. Set builder notation is

$\{x \mid x = 6n, n \in N \wedge 1 \leq n \leq 20\}$

(v) **{100, 102, 104, …, 400}**

Sol. Set builder notation is

$\{x \mid x \in E \wedge 100 \leq x \leq 400\}$

(vi) **{1, 3, 9, 27, 81, …}**

Sol. Set builder notation is

$\{x \mid x = 3^n, n \in W\}$

(vii) **{1, 2, 4, 5, 10, 20, 25, 50, 100}**

Sol. Set builder notation is

{x | x is a divisor of 100}

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(viii) $\{5, 10, 15, \ldots, 100\}$

Sol. Set builder notation is

$\{x \mid x = 5n, n \in N \wedge 1 \leq n \leq 20\}$

(ix) **The set of all integers between – 100 and 1000**

Sol. Set builder notation is

$\{x \mid x \in Z \wedge -100 \leq x \leq 1000\}$

Q.2. **Write each of the following sets in tabular forms:**

(i) $\{x \mid x \text{ is a multiple of } 3 \wedge x \leq 35\}$

Sol. Tabular form is

$\{3, 6, 9, \ldots, 33\}$

(ii) $\{x \mid x \in R \wedge 2x + 1 = 0\}$

Sol. Tabular form is

$\left\{\frac{-1}{2}\right\}$

(iii) $\{x \mid x \in P \wedge x < 12\}$

Sol. Tabular form is

$\{2, 3, 5, 7, 11\}$

(iv) $\{x \mid x \text{ is a divisor of } 128\}$

Sol. Tabular form is

$\{1, 2, 4, 8, 16, 32, 64, 128\}$

(v) $\{x \mid x = 2^n, n \in N \wedge n < 8\}$

Sol. Tabular form is

$\{2, 4, 8, 16, 32, 64, 128\}$

(vi) $\{x \mid x \in N \wedge x + 4 = 0\}$

Sol. Natural numbers are simply counting numbers.

Tabular form is $\{\ \}$.

(vii) $\{x \mid x \in N \wedge x = x\}$

Sol. Tabular form is

$\{1, 2\ 3, 4, 5, 6, \ldots\}$

(viii) $\{x \mid x \in Z \wedge 3x + 1 = 0\}$

Sol. Tabular form is

$\{\ \}$

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Q.3. Write two proper subsets of each of the following sets:

(i) **{ a, b, c}**

Sol. Two proper subsets are

ϕ, {a}

(ii) **{0, 1}**

Sol. Two proper subsets are

ϕ, {0}

(iii) **N**

Sol. Two proper subsets are

ϕ, {1, 2}

(iv) **Z**

Sol. Two proper subsets are

ϕ, {1, 2, 3}

(v) **Q**

Sol. Two proper subsets are

$\phi, \left\{\frac{-1}{2}\right\}$

(vi) **R**

Sol. Two proper subsets are

ϕ, {5, 6}

(vii) $\{x \mid x \in Q \wedge 0 < x \leq 2\}$

Sol. Two proper subsets are

$\phi, \left\{\frac{3}{2}\right\}$

Q.4. Is there any set which has no proper subset? If so, name that set.

Sol. Yes, Empty set has no proper subset.

Q.5. What is the difference between {a, b} and { {a, b} }?

Sol. {a, b} and {{a, b}}

{a, b} is set containing two elements a and b while {{a, b}} is set containing one element {a, b}

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Q.6. What is the number of elements of the power set of each of the following sets?

(i) { }

Sol. Empty set has no element,

Here n = 0, Formula $2^0 = 1$

No. of elements in power set of { } is 1.

(ii) **{0, 1}**

Sol. No. of elements in set are 2.

No. of elements in power set = $2^2 = 4$

(iii) **{1, 2, 3, 4, 5, 6, 7}**

Sol. Here n = 7

No. of elements in power set = $2^7 = 128$

(iv) **{0, 1, 2, 3, 4, 5, 6, 7}**

Sol. No. of elements = n = 8

No. of elements in power set = $2^8 = 256$

(v) **{a, {b, c}}**

Sol. No. of elements = n = 2

No. of elements in power set = $2^2 = 4$

(vi) **{{a, b}, {b, c}, {d, e}}**

Sol. No. of elements in set = n = 3

No. of elements of power set = $2^3 = 8$

Q.7. Write down the power set of each of the following sets:

(i) **{9, 11}**

Sol. Here n = 2

No. of elements of power set = $2^2 = 4$

Let A = {9, 11}

P(A) = {ϕ, {9}, {11}, {9, 11}}

(ii) **{+, −, ×, ÷}**

Sol. Here n = 4.

No.

$P(A) = \{\phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\},$
$\{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\},$
$\{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$

(iii) $\{\phi\}$

Sol. Let $A = \{\phi\}$

No. of elements $= n = 1$

No. of elements in power set $= 2^1 = 2$

$P(A) = \{\phi, \{\phi\}\}$

(iv) $\{a, \{b, c\}\}$

Sol. Let $A = \{a, \{b, c\}\}$

No. of elements in set $A = n = 2$

No. of elements in power set $= 2^2 = 4$

$P(A) = \{\phi, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

EXERCISE 3.2

Q.1. Consider the universal set $U = \{x : x$ is multiple of 2 and $0 < x \leq 30\}$.

$A = \{x : x$ is a multiple of 6$\}$ and

$B = \{x : x$ is a multiple of 8$\}$

Sol. $U = \{x : x$ is a multiple of 2 and $0 < x \leq 30\}$

$= \{2, 4, 6, 8, 10, \ldots, 30\}$

$A = \{x : x$ is a multiple of 6$\}$

$B = \{x : x$ is a multiple of 8$\}$

(i) List all elements of sets A and B in tabular form

Sol. $A = \{6, 12, 18, 24, 30\}$

$B = \{8, 16, 24\}$

(ii) Find $A \cap B$

Sol. $A \cap B = \{6, 12, 18, 24, 30\} \cap \{8, 16, 24\}$

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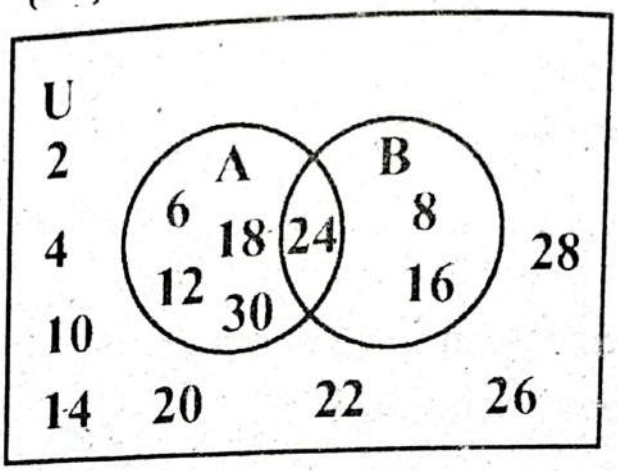
(iii) Draw a Venn diagram

Sol. $U = \{2, 4, 6, \ldots, 30\}$

$A = \{6, 12, 18, 24, 30\}$

$B = \{8, 16, 24\}$

$A \cap B = \{24\}$



Q.2. Let, U = {x : x is an integer and $0 < x \leq 150$},

G = {x : $x = 2^m$ for integer m and $0 \leq m \leq 12$} and

H = {x : x is a square}

Sol. $U = \{x : x \text{ is an integer and } 0 < x \leq 150\}$

$= \{1, 2, 3, \ldots, 150\}$

$G = \{x : x = 2^m \text{ for integer } m \text{ and } 0 < m \leq 12\}$

$H = \{x : x \text{ is a square}\}$

(i) List all elements of sets G and H in tabular form

Sol. $G = \{x : x = 2^m \text{ for integer } m \text{ and } 0 < m \leq 12\}$

$= \{1, 2, 4, 8, 16, 32, 64, 128\}$

$H = \{x : x \text{ is a square}\}$

$= \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$

(ii) Find $G \cup H$

Sol. $G \cup H = \{1, 2, 4, 8, 16, 32, 64, 128\} \cup \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$

$= \{1, 2, 4, 8, 9, 16, 25, 32, 36, 49, 64, 81, 100, 121, 128, 144\}$

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(iii) Find $G \cap H$

Sol. $G \cap H = \{1, 2, 4, 8, 16, 32, 64, 128\} \cup \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$

$= \{1, 4, 16, 64\}$

Q.3. Consider the sets $P = \{x : x$ is a prime number and $0 < x \leq 20\}$ and $Q = \{x : x$ is a divisor of 210 and $0 < x \leq 20\}$

Sol. $P = \{x : x$ is a prime number and $0 < x \leq 20\}$

$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$Q = \{x : x$ is a divisor of 210 and $0 < x \leq 20\}$

$Q = \{1, 2, 3, 5, 6, 7, 10, 14, 15\}$

(i) Find $P \cap Q$

Sol. $P \cap Q = \{2, 3, 5, 7, 11, 13, 17, 19\} \cap \{1, 2, 3, 5, 6, 7, 10, 14, 15\}$

$= \{2, 3, 5, 7\}$

(ii) Find $P \cup Q$

Sol. $P \cup Q = \{2, 3, 5, 7, 11, 13, 17, 19\} \cup \{1, 2, 3, 5, 6, 7, 10, 14, 15\}$

$= \{1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19\}$

Q.4. Verify the commutative properties of unions and intersection for the following pairs of sets:

Sol. Commutative property of union: $A \cup B = B \cup A$

Commutative property of intersection: $A \cap B = B \cap A$

(i) $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 6, 8, 10\}$

Sol. Commutative property of union:

$A \cup B = \{1, 2, 3, 4, 5\} \cup \{4, 6, 8, 10\}$

$= \{1, 2, 3, 4, 5, 6, 8, 10\}$

$B \cup A = \{4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5, 6, 8, 10\}$

$\therefore \quad A \cup B = B \cup A$

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Commutative property of intersection:

$A \cap B = \{1, 2, 3, 4, 5\} \cup \{4, 6, 8, 10\}$

$= \{4\}$

$B \cap A = \{4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5\}$

$= \{4\}$

$\therefore \quad A \cap B = B \cap A$

(ii) **N, Z**

Sol. $N = \{1, 2, 3, \ldots\}$, $Z = \{0, \pm1, \pm2, \pm3, \ldots\}$

Commutative property of union:

$A \cup B = B \cup A$

$A \cup B = \{1, 2, 3, \ldots\} \cup \{0, \pm1, \pm2, \pm3, \ldots\}$

$= \{0, \pm1, \pm2, \pm3, \ldots\}$

$= Z$

$B \cup A = \{0, \pm1, \pm2, \pm3, \ldots\} \cup \{1, 2, 3, \ldots\}$

$= \{0, \pm1, \pm2, \pm3, \ldots\}$

$= Z$

$\therefore \quad A \cup B = B \cup A$

Commutative property of intersection:

$A \cap B = B \cap A$

$A \cap B = \{1, 2, 3, \ldots\} \cap \{0, \pm1, \pm2, \pm3, \ldots\}$

$= \{0, 1, 2, 3, \ldots\}$

$= N$

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$B \cap A = \{0, \pm 1, \pm 2, \pm 3, \ldots\} \cap \{1, 2, 3, \ldots\}$

$= \{0, 1, 2, 3, \ldots\}$

$= N$

$\therefore \quad A \cap B = B \cap A$

(iii) $\mathbf{A = \{x \mid x \in R \wedge x \geq 0\}, B = R.}$

Sol. Commutative property of union:

$$A \cup B = B \cup A$$

$A \cup B = \{x \mid x \in R \wedge x \geq 0\} \cup R$

$= R$

$B \cup A = R \cup \{x \mid x \in R \wedge x \geq 0\}$

$= R$

$A \cup B = B \cup A$

Commutative property of intersection:

$$A \cap B = B \cap A$$

$A \cap B = \{x \mid x \in R \wedge x \geq 0\} \cap R$

$= \{x \mid x \in R \wedge x \geq 0\}$

$B \cap A = R \cap \{x \mid x \in R \wedge x \geq 0\}$

$= \{x \mid x \in R \wedge x \geq 0\}$

$A \cap B = B \cap A$

Q.5. **Let $U = \{a, b, c, d, e, f, g, h, i, j\}$**

$A = \{a, b, c, d, g, h\}, B = \{c, d, e, f, j\}$

Verify De Morgan's Laws for these sets. Draw Venn diagram

Sol. $U = \{a, b, c, d, e, f, g, h, i, j\}$

$A = \{a, b, c, d, g, h\}$

$B = \{c, d, e, f, j\}$

De Morgan's Laws

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

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$A' = U - A$

$= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$

$= \{e, f, i, j\}$

$B' = U - B$

$= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$

$= \{a, b, c, d, e, f, g, h, j\}$

$A \cap B = \{a, b, c, d, g, h\} \cap \{c, d, e, f, j\}$

$= \{c, d\}$

(i) $(A \cup B)' = A' \cap B'$

$(A \cup B)' = U - A \cup B$

$= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, e, f, g, h, j\}$

$= \{i\}$

$A' \cap B' = \{e, f, i, j\} \cap \{a, b, g, h, i\}$

$= \{i\}$

$\therefore \quad (A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

$(A \cap B)' = U - A \cap B$

$= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d\}$

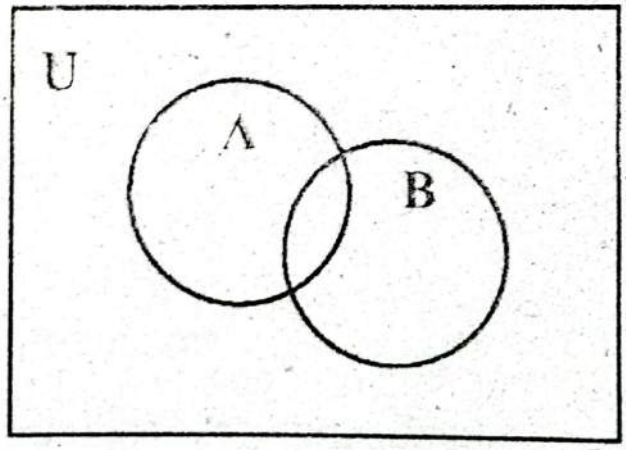
$= \{a, b, c, d, e, f, g, h, i, j\}$

$A' \cup B' = \{e, f, i, j\} \cup \{a, b, c, d, e, f, g, h, j\}$

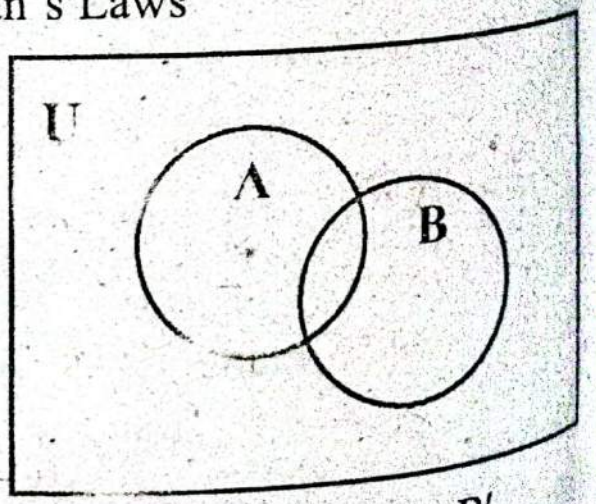
$= \{a, b, c, d, e, f, g, h, i, j\}$

$\therefore \quad (A \cap B)' = A' \cup B'$

Hence verified De Morgan's Laws



$(A \cup B)' = A' \cap B'$



$(A \cap B)' = A' \cup B'$

Q.6. If U = {1, 2, 3, ..., 20} and A = {1, 3, 5, ..., 19}, verify the following:

(i) $A \cup A' = U$

Sol. $A' = U - A$

$= \{1, 2, 3, \ldots, 20\} - \{1, 3, 5, \ldots, 19\}$

$= \{2, 4, 6, \ldots, 20\}$

$A \cup A' = \{1, 3, 5, \ldots, 19\} \cup \{2, 4, 6, \ldots, 20\}$

$= \{1, 2, 3, \ldots, 20\}$

$= U$

Hence proved that $A \cup A' = U$

(ii) $A \cap U = A$

Sol. $A \cap U = \{1, 3, 5, \ldots, 19\} \cap \{1, 2, 3, \ldots, 20\}$

$= \{1, 3, 5, \ldots, 19\}$

$= A$

Hence proved that $A \cap U = A$

(iii) $A \cap A' = \phi$

Sol. $A' = U - A$

$= \{1, 2, 3, \ldots, 20\} - \{1, 3, 5, \ldots, 19\}$

$= \{2, 4, 6, \ldots, 20\}$

$A \cup A' = \{1, 3, 5, \ldots, 19\} \cup \{2, 4, 6, \ldots, 20\}$

$= \{\ \} = \phi$

Hence proved that $A \cap A' = \phi$

Q.7. In a class of 55 studetns, 34 like to play cricket and 30 like to play hockey. Also each student likes to play at least one of the two games. How many students like to play both games?

Sol. U = {total students in a class}

A = {students who like to play cricket}

B = {students who like to play hockey}

From the statement we have $n(U) = n(A \cup B) = 55$

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$n(A) = 34$, $n(B) = 30$, we want to find the total number of students who like to play both games.

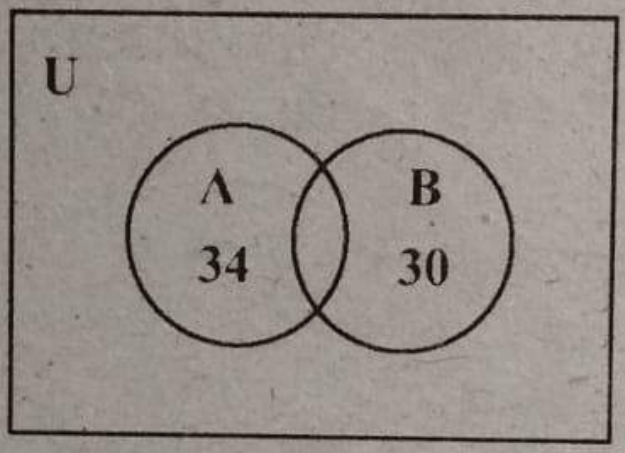
$$n(A \cap B) = ?$$

Using the principles of inclusion and exclusion for two sets.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\begin{aligned} n(A \cap B) &= n(A) + n(B) - n(A \cup B) \\ &= 34 + 50 - 55 \\ &= 64 - 55 \\ &= 9 \end{aligned}$$

Hence 9 students like to play both games.



Q.8. In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi, and 10 can speak Urdu and Punjabi. How many can speak all three languages?

Sol. Let U = {total number of employees} = 500

A = {employees who can speak Urdu} = 250

B = {employees who can speak English} = 150

C = {employees who can speak Punjabi} = 50

From the given statement, we have

$n(A \cup P \cup C) = 500$, $n(A) = 250$, $n(B) = 150$, $n(C) = 50$

$n(A \cap B) = 40$, $n(B \cap C) = 30$, $n(A \cap C) = 10$

We want to find the employees who can speak all three languages.

$$N(A \cap B \cap C) = ?$$

Using the principle of inclusion and exclusion of three sets

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

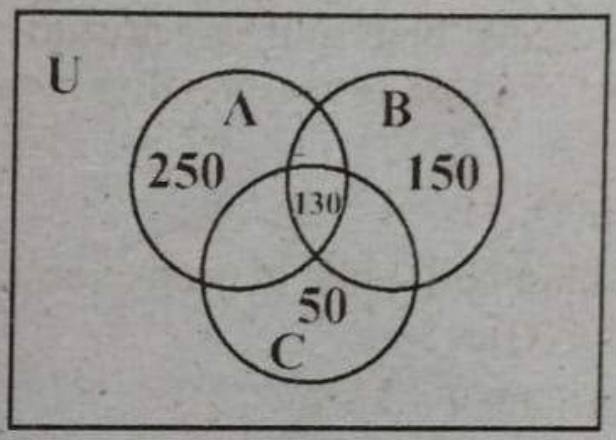
$$n(A \cap B \cap C) = n(A \cup B \cup C) - n(A) - n(B) - n(C) + n(A \cap B) + n(B \cap C) + n(A \cap C)$$

$$= 500 - 250 - 150 - 50 + 40 + 30 + 10$$

$$= 580 - 450$$

$$= 130$$

Therefore 130 employees can speak all three languages.



Q.9. In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps?

Sol. Let

U = {total number of people with either a blue or green shirt or cap}

B = {people wear blue shirts}

G = {people wear green shirt}

C = {people wear cap}

From the given statement

$n(B \cup G \cup C) = n(U) = 25$, $n(B) = 19$, $n(G) = 15$,

$n(C) = ?$, $n(B \cap G) = 3$, $n(B \cap C) = 4$, $n(G \cap C) = 2$

We want to find the people wearing caps, denoted as n(C)

Using the principle of inclusion and exclusion for three sets

$$n(U) = n(B) + n(G) + n(C) - n(B \cap G) - n(B \cap C) - n(G \cap C) + n(A \cap B \cap C)$$

$$25 = 19 + 15 + n(C) - 3 - 4 - 2 + n(A \cap B \cap C)$$

$$25 = 25 + n(C) - 9 + n(A \cap B \cap C)$$

$$n(C) + n(A \cap B \cap C) = 25 - 25 + 9$$

$$n(C) + n(A \cap B \cap C) = 9$$

$n(A \cap B \cap C)$ represents the number of people wearing all three items: blue shirt, green shirt and a cap. Since this specific value is not directly provided, let denote it as x.

Thus $n(C) = 9 - x$

Since x must be a subset of people contained in $n(B \cap G)$, and $n(B \cap G) = 3$

$\therefore \quad n(C) = 9 - 3 = 6$

Hence 6 people are wearing caps.

(Note: Answer in book is wrong)

Q.10. In a training session, 17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Sol. Let U = {total number of participants}

L = {participants have laptops}

T = {participants have tablets}

B = {participants have books}

From the given statements

$n(U) = n(L \cup T \cup B) = 35$, $n(L) = 17$, $n(T) = 11$

$n(L \cap T) = 9$, $n(L \cap B) = 6$, $n(T \cap B) = 4$,

$n(L \cap T \cap B) = 8$

We have to find the number of participants who have books, n(B) = ?

Using the principle of inclusion and exclusion for three sets

$$n(L \cup T \cup B) = n(L) + n(T) + n(B) - n(L \cap T) - n(L \cap B) - n(T \cap B) + n(L \cap T \cap B)$$

$$35 = 17 + 11 + n(B) - 9 - 6 - 4 + 8$$

$$35 = 36 - 19 + n(B)$$

$$35 = 17 + n(B)$$

$$n(B) = 35 - 17 = 18$$

Therefore 18 participants have books.

Q.11. A shopping mall has 150 employees labelled 1 to 150, representing the Universal set U. The employees fall into the following categories:

- **Set A: 40 employees with a salary range of 30k-45k, labelled from 50 to 89.**
- **Set B: 50 employees with a salary range of 50k-80k, labelled from 101 to 150.**
- **Set C: 60 employees with a salary range of 100k-150k, labelled from 1 to 49 and 90 to 100.**

Sol. Total employees = 150

Labelled from 1 to 150

U = {1, 2, 3, ..., 150}

Employees with a salary range of 30k to 45k, labelled from 50 to 89.

A = {50, 51, 52, ..., 89}

Employees with a salary range of 50k to 80k, labelled from 101 to 150

B = {101, 102, ..., 150}

Employees with a salary range of 100k to 150k, labelled from 1 to 49 and 90 to 100.

C = {1, 2, 3, ..., 49, 90, 91, ..., 100}

$\therefore$ n(U) = 150, n(A) = 40, n(B) = 50, n(C) = 60

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(a) **Find $(A' \cup B') \cap C$**

Sol. $A' = U - A$

$= \{1, 2, 3, \ldots, 150\} - \{50, 51, 52, \ldots, 89\}$

$= \{1, 2, 3, \ldots, 49, 90, 91, \ldots, 150\}$

$B' = U - B$

$= \{1, 2, 3, \ldots, 150\} - \{101, 102, \ldots, 150\}$

$= \{1, 2, 3, \ldots, 100\}$

$A' \cup B' = \{1, 2, 3, \ldots, 49, 90, 91, \ldots, 150\} \cup \{1, 2, 3, \ldots, 100\}$

$= \{1, 2, 3, \ldots, 150\}$

$(A' \cup B') \cap C = \{1, 2, 3, \ldots, 150\} \cap \{1, 2, \ldots, 49, 90, 91, \ldots, 100\}$

$= \{1, 2, 3, \ldots, 49, 90, 91, \ldots, 100\}$

(b) **Find $n\{A \cap (B^C \cap B^C)\}$**

Sol. $B^C = B' = \{1, 2, 3, \ldots, 100\}$

$C^C = U - C$

$= \{1, 2, 3, \ldots, 150\} - \{1, 2, 3, \ldots, 49, 90, 91, \ldots, 100\}$

$= \{50, 51, \ldots, 89\}$

$A \cap (B^C \cap C^C) = \{50, 51, \ldots, 89\} \cap \{50, 51, \ldots, 89\}$

$= \{50, 51, \ldots, 89\}$

$n\{A \cap (B^C \cap C^C)\} = 40$

Q.12. In a secondary school with 125 students participate in at least one of the following sports: cricket, football, or hockey.

- **60 students play cricket.**
- **70 students play football.**
- **40 students play hockey.**
- **25 students play both cricket and football.**
- **15 students play both football and hockey.**
- **10 students play both cricket and hockey.**

Sol. Let U = {total number of students who play cricket, football or hockey}

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C = {students who play cricket}

F = {students who play football}

H = {students who play hockey}

From the given statement

$n(U) = n(C \cup F \cup H) = 125$, $n(C) = 60$, $n(F) = 70$,

$n(H) = 40$, $n(C \cap F) = 25$, $n(F \cap H) = 15$, $n(C \cap H) = 10$

(a) How many students play all three sports?

Sol. We want to find that students play all three sports

i.e., $n(C \cap F \cap H) = ?$

Using the principle of inclusion and exclusion for three sets.

$$n(C \cup F \cup H) = n(C) + n(F) + n(H) - n(C \cap F) - (F \cap H) - n(C \cap H)$$

$$125 = 60 + 70 + 40 - 25 - 15 - 10 + n(C \cap F \cap H)$$

$$125 = 170 - 50 + n(C \cap F \cap H)$$

$$125 = 120 + n(C \cap F \cap H)$$

$$n(C \cap F \cap H) = 125 - 120$$

$$= 5$$

$\therefore$ 5 students play all three games.

(b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Sol. Let $x = n(C \cap F \cap H) = 5$

Now calculate the number of students in each overlapping region.

1. $n(C \cap F) - n(C \cap F \cap H) = 25 - 5 = 20$ (students who play cricket and football but not hockey)
2. $n(F \cap H) - n(C \cap F \cap H) = 15 - 5 = 10$ (students who play football and hockey but not cricket)
3. $n(C \cap H) - n(C \cap F \cap H) = 10 - 5 = 5$ (students who play cricket and hockey, but not football)

Now calculate the remaining students in each sport who do not play multiple sports.

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1. Students who play only cricket.

$$n(C) - [n(C \cap F) + n(C \cap H) - n(C \cap F \cap H)] = 60 - (25 + 10 - 5)$$
$$= 60 - 30$$
$$= 30$$

2. Students who play only football.

$$n(F) - [n(C \cap F) + n(F \cap H) - n(C \cap F \cap H)] = 70 - (25 + 15 - 5)$$
$$= 70 - 35$$
$$= 35$$

3. Students who play only hockey.

$$n(H) - [n(C \cap H) + n(F \cap H) - n(C \cap F \cap H)] = 40 - (10 + 15 - 5)$$
$$= 40 - 20$$
$$= 20$$

Now

Only cricket = 30

Only football = 35

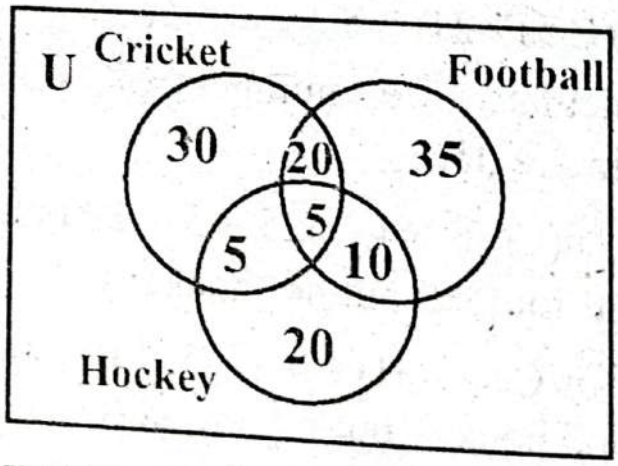
Only hockey = 20

Cricket and football only = 20

Football and hockey only = 10

Cricket and hockey only = 5

All three sports = 5



Q.13. A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

- **40 people said they liked nihari**

- **65 people said they liked biryani**
- **50 people said they liked korma**
- **20 people said they liked nihari and biryani**
- **35 people said they liked biryani and korma**
- **27 people said they liked nihari and korma**
- **12 people said they liked all three foods nihari, biryani, and korma.**

Sol. Let

U = {total number of people who were asked about their favourite foods}

N = {people who liked nihari}

B = {people who liked biryani}

K = {people who liked korma}

From the given statement

$n(N) = 40$, $n(B) = 65$, $n(K) = 60$, $n(N \cap B) = 20$, $n(B \cap K) = 27$

$n(N \cap B \cap K) = 12$

(a) At least how many people like nihari, biryani or korma?

Sol. Using the principle of inclusion and exclusion for three sets

$$n(N \cup B \cup K) = n(N) + n(B) + n(K) - n(N \cap B) - n(B \cap K) - n(N \cap K) + n(N \cap B \cap K)$$

$$= 40 + 65 + 50 - 20 - 35 - 27 + 12$$

$$= 167 - 82$$

$$= 85$$

$\therefore$ 85 people like at least one of Nihari, Baryani or Korma.

(b) How many people did not like nihari, biryani, or korma?

Sol. Total surveyed = 130

People who did not like any of the three

$$130 - n(N \cup B \cup K) = 130 - 85$$

$$= 45$$

So 45 people did not like any of these foods.

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(c) How many people like only one of the following foods: nihari, biryani, or korma?

Sol. To find people who like only one food.

1. People who like only nihari:

$= n(N) - n(N \cap B) - n(N \cap K) + n(N \cap B \cap K)$

$= 40 - 20 - 27 + 12$

$= 52 - 47$

$= 5$

2. People who like only biryani:

$= n(B) - n(N \cap B) - n(B \cap K) + n(N \cap B \cap K)$

$= 65 - 20 - 35 + 12$

$= 77 - 55$

$= 22$

3. People who like only korma:

$= n(K) - n(N \cap K) - n(B \cap K) + n(N \cap B \cap K)$

$= 50 - 27 - 35 + 12$

$= 62 - 62$

$= 0$

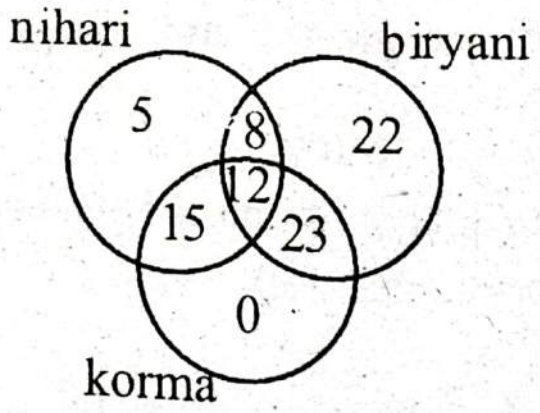
Total number of people who like only one food $= 5 + 22 + 0$

$= 27$

$\therefore$ 27 people like only one of the foods.

(d) Draw a Venn diagram.

Sol. Venn diagram of people who like nihari, biryani and korma.



EXERCISE 3.3

Q.1. For A = {1, 2, 3, 4}, find the following relations in A. State the domain and range of each relation.

Sol. $A = \{1, 2, 3, 4\}$

$A \times A = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

(i) $\{(x, y) \mid y = x\}$

Sol. $r_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Dom $r_1 = \{1, 2, 3, 4\}$

Range $r_1 = \{1, 2, 3, 4\}$

(ii) $\{(x, y) \mid y + x = 5\}$

Sol. $r_2 = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

Dom $r_2 = \{1, 2, 3, 4\}$

Range $r_2 = \{1, 2, 3, 4\}$

(iii) $\{(x, y) \mid x + y < 5\}$

Sol. $r_3 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

Dom $r_3 = \{1, 2, 3\}$

Range $r_3 = \{1, 2, 3\}$

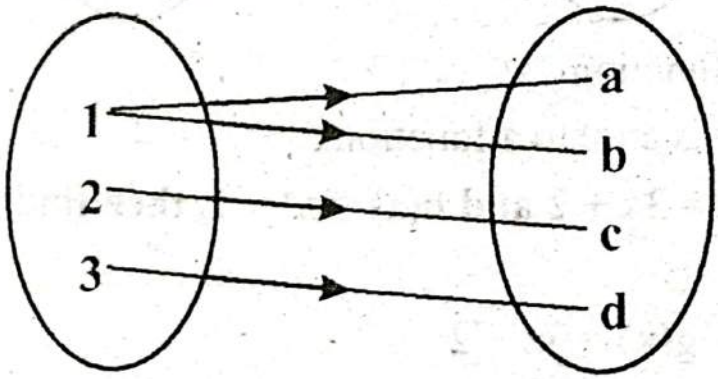
(iv) $\{(x, y) \mid x + y > 5\}$

Sol. $r_4 = \{(2, 4), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$

Dom $r_4 = \{2, 3, 4\}$

Range $r_4 = \{2, 3, 4\}$

Q.2. Which of the following diagrams represent functions and of which type?



It is not a function.

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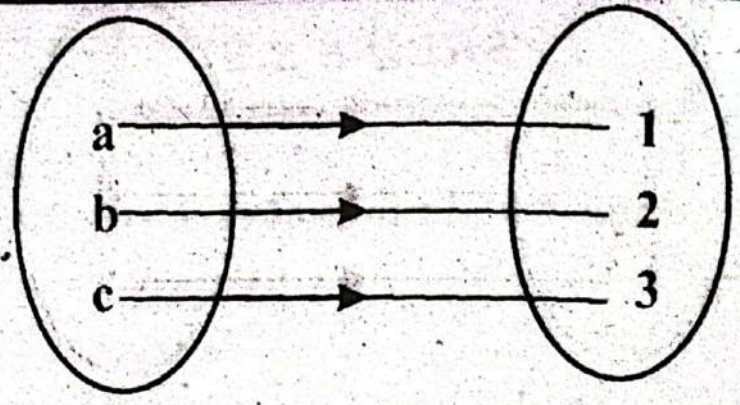
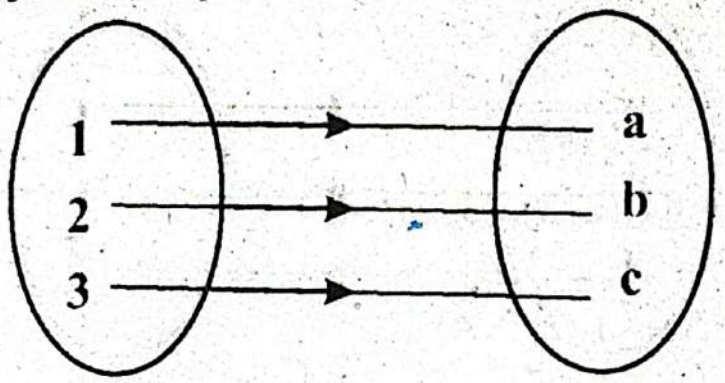
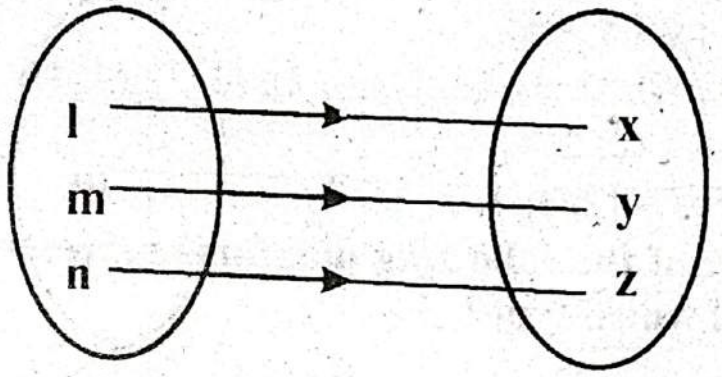


It is a function.
(one-one) and onto function
(or Bijective function)



It is a function.
(one-one) and onto function
(or Bijective function)



It is a function.
Which is an into a function.

Q.3. If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) $g(0)$

Sol.

$$g(x) = 3x + 2$$
$$g(0) = 3(0) + 2$$
$$g(0) = 2$$

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(ii) $g(-3)$

Sol.

$$g(x) = 3x + 2$$
$$g(-3) = 3(-3) + 2$$
$$g(-3) = -9 + 2$$
$$g(-3) = -7$$

(iii) $g\left(\frac{2}{3}\right)$

Sol.

$$g(x) = 3x + 2$$
$$g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2$$
$$= 2 + 2$$
$$= 4$$

(iv) $h(1)$

Sol.

$$h(x) = x^2 + 1$$
$$h(1) = (1)^2 + 1$$
$$h(1) = 1 + 1$$
$$h(1) = 2$$

(v) $h(-4)$

Sol.

$$h(x) = x^2 + 1$$
$$h(-4) = (-4)^2 + 1$$
$$= 16 + 1$$
$$= 17$$

(vi) $h\left(-\frac{1}{2}\right)$

Sol.

$$h(x) = x^2 + 1$$
$$h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1$$
$$= \frac{1}{4} + 1$$
$$= \frac{1+4}{4} = \frac{5}{4}$$
$$= 1\frac{1}{4}$$

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Q.4. Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b.

Sol.

$$f(x) = ax + b + 1$$

$$f(3) = 3a + b + 1$$

But $f(3) = 8$

$$\therefore \quad 3a + b + 1 = 8$$

$$3a + b = 8 - 1$$

$$3a + b = 7 \qquad \ldots(i)$$

$$f(6) = 6a + b + 1$$

But $f(6) = 14$

$$6a + b + 1 = 14$$

$$6a + b = 14 - 1$$

$$6a + b = 13 \qquad \ldots(ii)$$

Subtracting equation (i) from equation (ii)

$$\begin{array}{r} 6a + b = 13 \\ \underline{-3a \pm b = -7} \\ 3a \quad = 6 \end{array}$$

$$a = \frac{6}{3} = 2$$

Putting the value of a in equation (i)

$$3a + b = 7$$

$$3(2) + b = 7$$

$$6 + b = 7$$

$$b = 7 - 6$$

$$b = 1$$

$$\therefore \quad a = 2, b = 1$$

Q.5. Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.

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Sol.

$$g(x) = ax + b + 5$$

$$g(-1) = 0$$

$$g(-1) = a(-1) + b + 5$$

$$= -a + b + 5$$

$$-a + b + 5 = 0$$

$$a + b = -5 \quad \ldots(i)$$

$$g(2) = 10$$

$$g(2) = a(2) + b + 5$$

$$= 2a + b + 5$$

$$2a + b + 5 = 10$$

$$2a + b = 5 \quad \ldots(ii)$$

Subtracting equation (i) from equation (ii)

$$\begin{array}{rcl} 2a - b &=& 5 \\ \mp a \pm b &=& \mp 5 \\ \hline 3a &=& 10 \end{array}$$

$$a = \frac{10}{3}$$

Putting the value of a in equation (i)

$$-a + b = -5$$

$$\frac{-10}{3} + b = -5$$

$$b = -5 + \frac{10}{3}$$

$$b = \frac{-15 + 10}{3}$$

$$b = \frac{-5}{3}$$

$$\therefore \quad a = \frac{10}{3}, \; b = \frac{-5}{3}$$

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Q.6. Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value.

Sol.

$$f(x) = 5x + 1$$
$$f(x) = 32$$
$$\therefore \quad 5x + 1 = 32$$
$$5x = 32 - 1$$
$$5x = 31$$
$$x = \frac{31}{5}$$
$$x = 6\frac{1}{5}$$

(Note: Answer in book in wrong)

Q.7. Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

Sol.

$$f(x) = cx^2 + d$$
$$f(1) = 6$$
$$f(1) = c(1)^2 + d$$
$$= c + d$$
$$\therefore \quad c + d = 6 \qquad \ldots(i)$$
$$f(-2) = 10$$
$$f(-2) = c(-2)^2 + d$$
$$= c(4) + d$$
$$= 4c + d$$
$$\therefore \quad 4c + d = 10 \qquad \ldots(ii)$$

Subtracting equation (i) from equation (ii)

$$\begin{array}{rcl} 4c + d & = & 10 \\ \underline{\ \ c \pm d} & = & \underline{\ \ 6} \\ 3c & = & 4 \end{array}$$

$$c = \frac{4}{3}$$

Putting the value of c in equation (i)

$$c + d = 6$$

$$\frac{4}{3} + d = 6$$

$$d = 6 - \frac{4}{3}$$

$$d = \frac{18 - 4}{3}$$

$$d = \frac{14}{3}$$

$$\therefore \quad c = \frac{4}{3}, d = \frac{14}{3}$$

REVIEW EXERCISE 3

Q.1. Four options are given against each statement. Encircle the correct one.

(i) The set builder form of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$

(a) $\left\{x \mid x = \frac{1}{n}, n \in W\right\}$ (b) $\left\{x \mid x = \frac{1}{2n+1}, n \in W\right\}$

(c) $\left\{x \mid x = \frac{1}{n+1}, n \in W\right\}$ (d) $\{x \mid x = 2n + 1, n \in W\}$

Sol. Option (b) is correct.

(ii) If $A = \{\ \}$, then $P(A)$ is:

(a) $\{\ \}$ (b) $\{1\}$

(c) $\{\{\ \}\}$ (d) ϕ

Sol. Option (c) is correct.

(iii) If $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $U - (A \cap B)$ is:

(a) $\{1, 2, 4, 5\}$ (b) $\{2, 3\}$

(c) $\{1, 3, 4, 5\}$ (d) $\{1, 2, 3\}$

Sol. Option (a) is correct.

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(iv) If A and B are overlapping sets, then $n(A - B)$ is equal to:

(a) $n(A)$ (b) $n(B)$

(c) $A \cap B$ (d) $n(A) - n(A \cap B)$

Sol. Option (d) is correct.

(v) If $A \subseteq B$ and $B - A \neq \phi$, then $n(B - A)$ is equal to:

(a) 0 (b) $n(B)$

(c) $n(A)$ (d) $n(B) - n(A)$

Sol. Option (d) is correct.

(vi) If $n(A \cup B) = 50$, $n(A) = 30$ and $n(B) = 35$, then $n(A \cap B) =$:

(a) 23 (b) 15

(c) 9 (d) 40

Sol. Option (b) is correct.

(vii) If $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$, then Cartesian product of A and B contains exactly _____ elements.

(a) 13 (b) 12

(c) 10 (d) 6

Sol. Option (b) is correct.

(viii) If $f(x) = x^2 - 3x + 2$, then the value of $f(a + 1)$ is equal to:

(a) $a + 1$ (b) $a^2 + 1$

(c) $a^2 + 2a + 1$ (d) $a^2 - a$

Sol. Option (d) is correct.

(ix) Given that $f(x) = 3x + 1$, if $f(x) = 28$, then the value of x is:

(a) 9 (b) 27

(c) 3 (d) 18

Sol. Option (a) is correct.

(x) Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$ two non-empty sets and $f : A \rightarrow B$ be a function defined as $f = \{(1, a), (2, b), (3, b)\}$, then which of the following statement is true?

(a) f is injective (b) f is surjective

(c) f is bijective (d) f is into only

Sol. Option (b) is correct.

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Q.2. Write each of the following sets in tabular forms:

(i) $\{x \mid x = 2n, n \in N\}$

Sol. $\{2, 4, 6, 8, \ldots\ldots\}$

(ii) $\{x \mid x = 2m + 1, m \in N\}$

Sol. $\{3, 5, 7, 9, \ldots\ldots\}$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$

Sol. $\{0, 11, 22, 33, 44, 55, 66, 77, 88, 99, 110\}$

(iv) $\{x \mid x \in E \wedge 4 < x < 6\}$

Sol. $\{\ \}$

(v) $\{x \mid x \in O \wedge 5 \leq x < 7\}$

Sol. $\{5\}$

(vi) $\{x \mid x \in Q \wedge x^2 = 2\}$

Sol. ϕ

(vii) $\{x \mid x \in Q \wedge x = -x\}$

Sol. $\{0\}$

(viii) $\{x \mid x\ R \wedge x \notin Q'\}$

Sol. Q

Q.3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{1, 3, 5, 7, 9\}$

List the members of each of the following sets:

(i) A'

Sol. $A' = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

(ii) B'

Sol. $B' = U - B$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4, 5\}$

$= \{6, 7, 8, 9, 10\}$

(iii) $A \cup B$

Sol. $A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5, 6, 8, 10\}$

(iv) $A - B$

Sol. $A - B = \{2, 4, 6, 8, 10\} - \{1, 2, 3, 4, 5\}$

$= \{6, 8, 10\}$

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(v) $A \cap C$

Sol. $A \cap C = \{2, 4, 6, 8, 10\} - \{1, 3, 5, 7, 9\}$

$= \phi$

(vi) $A' \cup C'$

Sol. $A' = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$C' = U - C$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $A' \cup C$

Sol. $A' = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$A' \cup C = \{1, 3, 5, 7, 9\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 3, 5, 7, 9\}$

(viii) U'

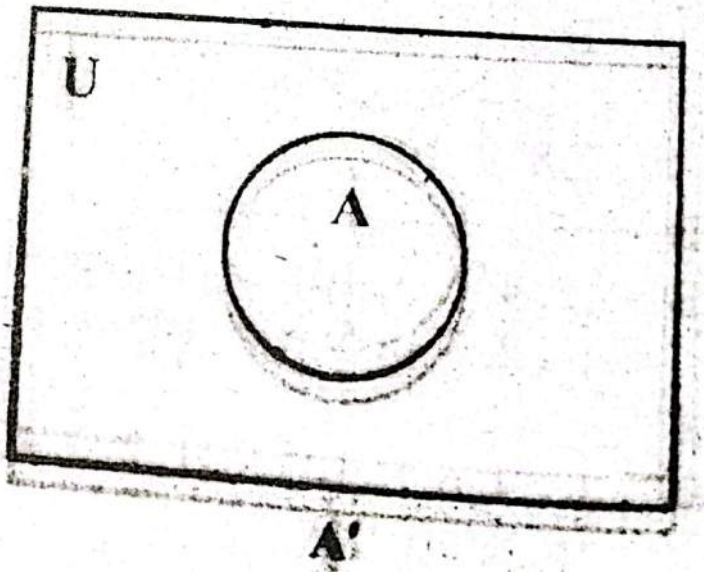
Sol. $U' = U - U$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

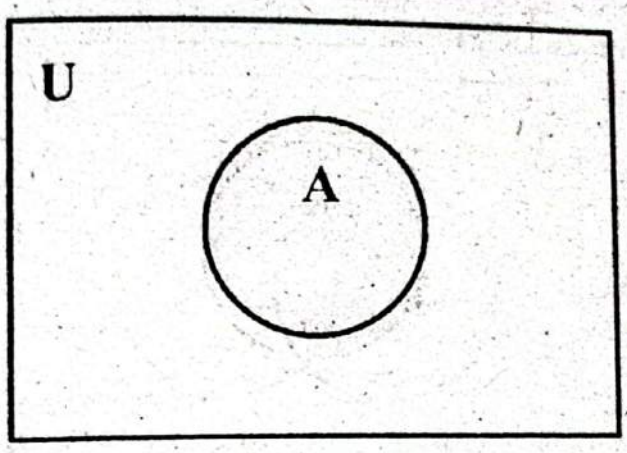
$= \phi$

Q.4. Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i) A'

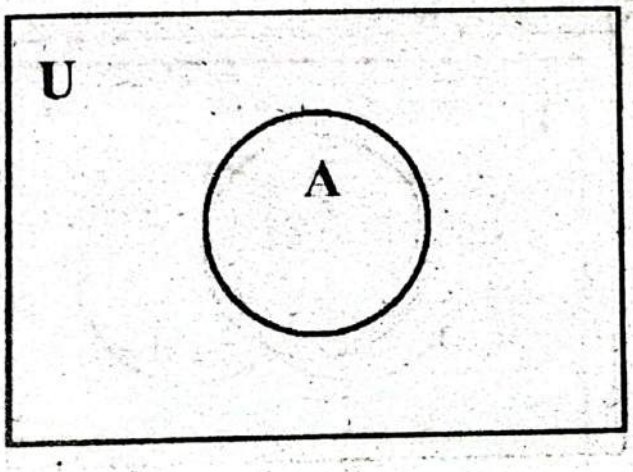


(ii) $A \cap U$



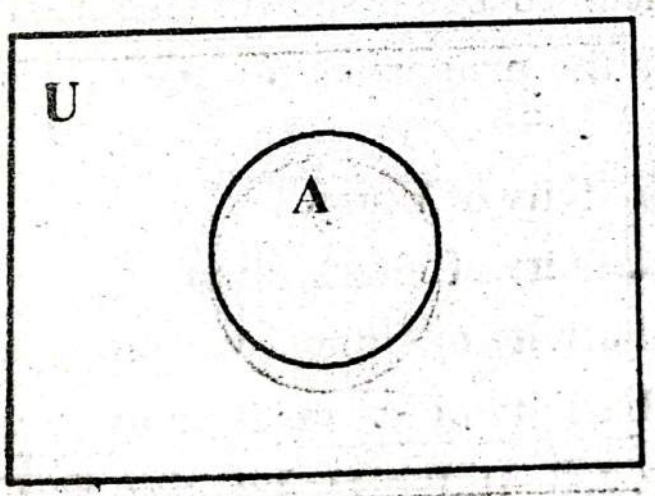
$A \cap U$

(iii) $A \cup U$



$A \cup U$

(iv) $A \cup \phi$



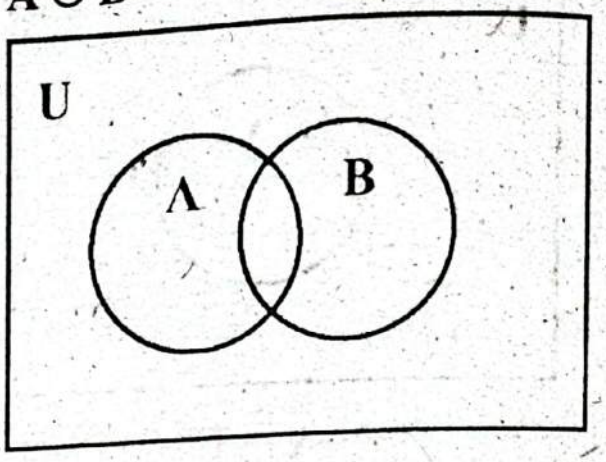
$A \cup \phi$

(v) $\phi \cap \phi$

Sol. $\phi \cap \phi = \phi$

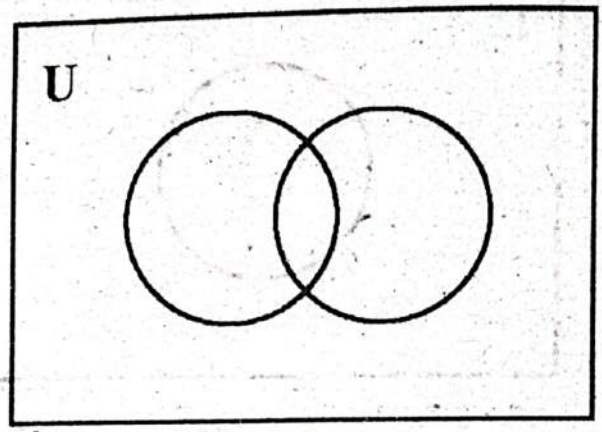
Q.5. Use Venn diagrams to verify the following:

(i) $A - B = A \cup B'$



$A - B = A \cup B'$

(ii) $(A - B)' \cap B = B$



$(A - B)' \cap B = B$

Double lined area is showing the result which is set B.

Q.6. Verify the properties for the sets A, B and C given below:

(i) Associativity of Union

(ii) Associativity of intersection

(iii) Distributivity of Union over intersection.

(iv) Distributivity of intersection over union.

(a) $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$

(b) $A = \phi$, $B = \{0\}$, $C = \{0, 1, 2\}$

(c) $A = N$, $B = Z$, $C = Q$

Sol.(i) Associativity of Union

(a) $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6, 7, 8\}$

$C = \{5, 6, 7, 9, 10\}$

Sol. $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S $= A \cup (B \cup C)$

$= \{1, 2, 3, 4\} \cup [\{3, 4, 5, 6, 7, 8\} \cup \{5, 6, 7, 9, 10\}]$

$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

R.H.S $= (A \cup B) \cup C$

$= [\{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}] \cup \{5, 6, 7, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{5, 6, 7, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\therefore$ L.H.S = R.H.S

Hence $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A = \phi$

$B = \{0\}$

$C = \{0, 1, 2\}$

Sol. $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S $= A \cup (B \cup C)$

$= \phi \cup [\{0\} \cup \{0, 1, 2\}]$

$= \phi \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$

R.H.S $= (A \cup B) \cup C$

$= [\phi \cup \{0\}] \cup \{0, 1, 2\}$

$= \{0\} \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$

$\therefore$ L.H.S = R.H.S

$A \cup (B \cup C) = (A \cup B) \cup C$

(c) $A = N$

$B = Z$

$C = Q$

Sol. $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S $= A \cup (B \cup C)$

$= N \cup (Z \cup Q)$

$= N \cup Q$

$= Q$

R.H.S $= (A \cup B) \cup C$

$= Z \cup Q$

$= Q$

$\therefore$ L.H.S = R.H.S

Hence $A \cup (B \cup C) = (A \cup B) \cup C$

(ii) Associativity of intersection

(a) $\mathbf{A = \{1, 2, 3, 4\}}$

$\mathbf{B = \{3, 4, 5, 6, 7, 8\}}$

$\mathbf{C = \{5, 6, 7, 9, 10\}}$

Sol. $A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S $= A \cap (B \cap C)$

$= \{1, 2, 3, 4\} \cap [\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 7, 9, 10\}]$

$= \{1, 2, 3, 4\} \cap \{5, 6, 7\}$

$= \phi$

R.H.S $= (A \cap B) \cap C$

$= [\{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}] \cap \{5, 6, 7, 9, 10\}$

$= \{3, 4\} \cap \{5, 6, 7, 9, 10\}$

$= \phi$

$\therefore$ L.H.S = R.H.S

Hence $A \cap (B \cap C) = (A \cap B) \cap C$

(b) $A = \phi$

$B = \{0\}$

$C = \{0, 1, 2\}$

Sol. $A \cap (B \cap C) = (A \cap B) \cap C$

$$\text{L.H.S} = A \cap (B \cap C)$$
$$= \phi \cap [\{0\} \cap \{0, 1, 2\}]$$
$$= \phi \cap \{0\}$$
$$= \phi$$
$$\text{R.H.S} = (A \cap B) \cap C$$
$$= [\phi \cap \{0\}] \cap \{0, 1, 2\}$$
$$= \phi \cap \{0, 1, 2\}$$
$$= \phi$$
$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

Hence $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $A = N$

$B = Z$

$C = Q$

Sol. $A \cap (B \cap C) = (A \cap B) \cap C$

$$\text{L.H.S} = A \cap (B \cap C)$$
$$= N \cap (Z \cap Q)$$
$$= N \cap Z$$
$$= N$$
$$\text{R.H.S} = (A \cap B) \cap C$$
$$= (N \cap Z) \cap Q$$
$$= N \cap Q$$
$$= N$$
$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

Hence $A \cap (B \cap C) = (A \cap B) \cap C$

(iii) Distributivity of Union over intersection.

(a) $\mathbf{A = \{1, 2, 3, 4\}}$

$\mathbf{B = \{3, 4, 5, 6, 7, 8\}}$

$\mathbf{C = \{5, 6, 7, 9, 10\}}$

Sol. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$= \{1, 2, 3, 4\} \cup [\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 7, 9, 10\}]$

$= \{1, 2, 3, 4\} \cup \{5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

R.H.S $= (A \cup B) \cap (A \cup C)$

$= [\{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}] \cap [\{1, 2, 3, 4\} \cup \{5, 6, 7, 9, 10\}]$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{1, 2, 3, 4, 5, 6, 7, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$\therefore$ L.H.S = R.H.S

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A = \phi$

$B = \{0\}$

$C = \{0, 1, 2\}$

Sol. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$= \phi \cup [\{0\} \cap \{0, 1, 2\}]$

$= \phi \cup \{0\}$

$= \{0\}$

R.H.S $= (A \cup B) \cap (A \cup C)$

$= [\phi \cup \{0\}] \cap [\phi \cup \{0, 1, 2\}$

$= \{0\} \cap \{0, 1, 2\}$

$= \{0\}$

$\therefore$ L.H.S = R.H.S

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(c) $A = N$

$B = Z$

$C = Q$

Sol. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

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$$\text{L.H.S} = A \cup (B \cap C)$$
$$= N \cup (Z \cap Q)$$
$$= N \cup Z$$
$$= Z$$
$$\text{R.H.S} = (A \cup B) \cap (A \cup C)$$
$$= (N \cup Z) \cap (N \cup Q)$$
$$= Z \cap Q$$
$$= Z$$
$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) Distributivity of intersection over union.

(a) $\mathbf{A = \{1, 2, 3, 4\}}$

$\mathbf{B = \{3, 4, 5, 6, 7, 8\}}$

$\mathbf{C = \{5, 6, 7, 9, 10\}}$

Sol. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$\text{L.H.S} = A \cap (B \cup C)$$
$$= \{1, 2, 3, 4\} \cap [\{3, 4, 5, 6, 7, 8\} \cup \{5, 6, 7, 9, 10\}]$$
$$= \{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8, 9, 10\}$$
$$= \{3, 4\}$$
$$\text{R.H.S} = (A \cap B) \cup (A \cap C)$$
$$= [\{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}] \cup [\{1, 2, 3, 4\} \cap \{5, 6, 7, 9, 10\}]$$
$$= \{3, 4\} \cup \phi$$
$$= \{3, 4\}$$
$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $A = \phi$

$B = \{0\}$

$C = \{0, 1, 2\}$

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Sol. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= \phi \cap [\{0\} \cup \{0, 1, 2\}]$

$= \phi \cap \{0, 1, 2\}$

$= \phi$

R.H.S $= (A \cap B) \cup (A \cap C)$

$= [\phi \cap \{0\}] \cup [\phi \cap \{0, 1, 2\}$

$= \phi \cup \phi$

$= \phi$

$\therefore$ L.H.S = R.H.S

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $A = N$

$B = Z$

$C = Q$

Sol. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= N \cap (Z \cup Q)$

$= N \cap Q$

$= N$

R.H.S $= (A \cap B) \cup (A \cap C)$

$= (N \cap Z) \cup (N \cap Q)$

$= N \cup N$

$= N$

$\therefore$ L.H.S = R.H.S

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Q.7. Verify De Morgan's Laws for the following sets:

$U = \{1, 2, 3, \ldots, 20\}$, $A = \{2, 4, 6, \ldots, 20\}$ and

$B = \{1, 3, 5, \ldots, 19\}$

Sol. De Morgan's Law

$(A \cup B)^C = A^C \cap B^C$

L.H.S $= (A \cup B)^C$

$= U - (A \cup B)$

$= U - [\{2, 4, 6, \ldots, 20\} \cup \{1, 3, 5, \ldots, 19\}]$

$= \{1, 2, 3, \ldots, 20\} - \{1, 2, 3, 4, \ldots, 20\}$

$= \phi$

R.H.S $= A^C \cap B^C$

$= (U - A) \cap (U - B)$

$= [(1, 2, 3, \ldots, 20\} - \{2, 4, 6, \ldots, 20\}] \cap$

$[\{1, 2, \ldots, 20\} - \{1, 3, 5, \ldots, 19\}]$

$= \{1, 3, 5, 7, \ldots, 19\} \cap \{2, 4, 6, \ldots, 20\}$

$= \phi$

L.H.S = R.H.S

Hence $(A \cup B)^C = A^C \cap B^C$

2. $(A \cap B)^C = A^C \cup B^C$

L.H.S $= (A \cap B)^C$

$= U - (A \cap B)$

$= U - [\{2, 4, 6, \ldots, 20\} \cap \{1, 3, 5, \ldots, 19\}]$

$= \{1, 2, 3, \ldots, 20\} - \phi$

$= \{1, 2, 3, \ldots, 20\}$

R.H.S $= A^C \cup B^C$

$= (U - A) \cup (U - B)$

$= [(1, 2, 3, \ldots, 20\} - \{2, 4, 6, \ldots, 20\}] \cup$

$[\{1, 2, \ldots, 20\} - \{1, 3, 5, \ldots, 19\}]$

$= \{1, 3, 5, 7, \ldots, 19\} \cup \{2, 4, 6, \ldots, 20\}$

$= \{1, 2, 3, \ldots, 20\}$

L.H.S = R.H.S

Hence $(A \cap B)^C = A^C \cup B^C$

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Q.8. Consider the set $P = \{x \mid x = 5m, m \in N\}$ and $Q = \{x \mid x = 2m, m \in N\}$. Find $P \cap Q$

Sol. $P = \{5, 10, 15, 20, \ldots\ldots\}$

$Q = \{2, 4, 6, 8, 10, \ldots\ldots\}$

$P \cap Q = \{5, 10, 15, 20, \ldots\ldots\} \cap \{2, 4, 6, 8, 10, \ldots\ldots\}$

$= \{10, 20, 30, \ldots\ldots\}$

Q.9. From suitable properties of union and intersection, deduce the following results:

(i) $\mathbf{A \cap (A \cup B) = A \cup (A \cap B)}$

(ii) $\mathbf{A \cup (A \cap B) = A \cap (A \cup B)}$

Sol. To deduce the given results we will use basic properties of union (U) and intersection ($\cap$) of sets.

1. Idempotent Law. $A \cup A = A$, $A \cap A = A$
2. Commutative Law: $A \cup B = B \cup A$, $A \cap B = B \cap A$
3. Associative Law: $A \cup (B \cup C) = (A \cup B) \cup C$

   $A \cap (B \cap C) = (A \cap C) \cup C$
4. Distributive Law: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   $A \cup (B \cup C) = (A \cup C) \cap (A \cup C)$

(i) $\mathbf{A \cap (A \cup B) = A \cup (A \cap B)}$

Sol. Proof

1. Expand $A \cap (A \cup B)$ using the distributive law:

   $$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$$
2. By the idempotent law $A \cap A = A$

   $$(A \cap A) \cup (A \cap B) = A \cup (A \cap B)$$

   Thus we have

   $$A \cap (A \cup B) = A \cup (A \cap B)$$

(ii) $\mathbf{A \cup (A \cap B) = A \cap (A \cup B)}$

Sol. Proof

1. From the result in (1), we know that

   $$A \cap (A \cup B) = A \cup (A \cap B)$$

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2. Using the commutative law, we can interchange the sides:

$$A \cup (A \cap B) = A \cap (A \cup B)$$

Hence it proved.

Q.10. If $g(x) = 7x - 2$ and $s(x) = 8x^2 - 3$ find:

Sol. $g(x) = 7x - 2$

$s(x) = 8x^2 - 3$

(i) g(0)

Sol.

$$g(x) = 7x - 2$$
$$g(0) = 7(0) - 2$$
$$= 2$$

(ii) g(–1)

Sol.

$$g(x) = 7x - 2$$
$$g(-1) = 7(-1) - 2$$
$$= -7 - 2$$
$$= -9$$

(iii) $g\left(-\frac{5}{3}\right)$

Sol.

$$g(x) = 7x - 2$$
$$g\left(-\frac{5}{3}\right) = 7\left(-\frac{5}{3}\right) - 2$$
$$= \frac{-35}{3} - 2$$
$$= \frac{-35 - 6}{3}$$
$$= \frac{-41}{3} = -13\frac{2}{3}$$

(iv) s(1)

Sol.

$$s(x) = 8x^2 - 3$$
$$s(1) = 8(1)^2 - 3$$
$$= 8 - 3$$
$$= 5$$

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(v) $s(-9)$

Sol.

$$s(x) = 8x^2 - 3$$
$$s(-9) = 8(-9)^2 - 3$$
$$= 8(81) - 3$$
$$= 648 - 3 = 645$$

(vi) $s\left(\frac{7}{2}\right)$

Sol.

$$s(x) = 8x^2 - 3$$
$$s\left(\frac{7}{2}\right) = 8\left(\frac{7}{2}\right)^2 - 3$$
$$= 8\left(\frac{49}{4}\right) - 3$$
$$= 98 - 3 = 95$$

Q.11. Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.

Sol.

$$f(x) = ax + b$$

If $f(-2) = 3$

Then $f(-2) = a(-2) + b = 3$

$\therefore$ $-2a + b = 3$ ...(i)

If $f(4) = 10$

Then $f(4) = a(4) + b = 10$

$\therefore$ $4a + b = 10$ ...(ii)

Subtract equation (i) from equation (ii)

$$\begin{array}{rl} 4a + b &= 10 \\ \mp 2a \pm b &= \underline{\ 3} \\ \hline 6a &= 7 \end{array}$$

$$a = \frac{7}{6}$$

Putting the value of a in equation (ii)

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$$4\left(\frac{7}{6}\right)+6=10$$

$$\frac{14}{3}+6=10$$

$$b=10-\frac{14}{3}$$

$$=\frac{30-14}{3}=\frac{16}{3}$$

$$\therefore \quad a=\frac{7}{6},\ b=\frac{16}{3}$$

Q.12. Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x.

Sol. $k(x) = 7x - 5$

If $k(x) = 100$

Then $7x - 5 = 100$

$7x = 100 + 5$

$7x = 105$

$x = \frac{105}{7}$

$x = 15$

Q.13. Consider the function $g(x) = mx^2 + n$, where m and n are constant numbers. If $g(4) = 20$ and $g(0) = 5$, find the values of m and n.

Sol. $g(x) = mx^2 + n$

If $g(4) = 20$

$g(4) = m(4)^2 + n = 20$

$\therefore$ $16m + n = 20$ ...(i)

If $g(0) = 5$

$g(0) = m(0) + n = 5$

$n = 5$

Putting the value of $n = 5$ in equation (i)

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$$16m + 5 = 20$$
$$16m = 20 - 5$$
$$16m = 15$$
$$m = \frac{15}{16}$$

Q.14. A shopping mall has 100 products from various categories labelled 1 to 100, representing the universal set U. The products are categorized as follows:

- **Set A: Electronics, consisting of 30 product labelled from 1 to 30.**
- **Set B: Clothing comprises 25 products labelled from 31 to 55.**
- **Set C: Beauty Products, comprising 25 products labelled from 76 to 100.**

Write each set in tabular form, and find the union of all three sets.

Sol. Sets in tabular form

$A = \{\text{Electronics}\}$

Products labelled 1 to 30

$A = \{1, 2, 3, \ldots, 30\}$

$B = \{\text{Clothing}\}$

Products labelled 31 to 55

$B = \{31, 32, 33, \ldots, 55\}$

$C = \{\text{Beauty products}\}$

Products labelled 76 to 100

$C = \{76, 77, \ldots, 100\}$

$$A \cup B \cup C = \{1, 2, 3, \ldots, 30\} \cup \{31, 32, \ldots, 55\} \cup \{76, 77, \ldots, 100\}$$
$$= \{1, 2, 3, \ldots, 55, 76, 77, \ldots, 100\}$$

$$\therefore \quad n(A \cup B \cup C) = n(A) + n(B) + n(C)$$
$$= 30 + 25 + 25$$
$$= 80$$

Since the sets are disjoint (no overlap).

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Q.15. Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.

Sol. U = {students who appeared in the annual examination}

M = {students who passed in math test}

S = {students who passed in science test}

By the given statement

$n(U) = 180, n(M) = 120, n(S) = 90, n(M \cap S) = 60$

(a) How many passed either the math or science test?

Sol. The number of students who passed either the math or science test $n(M \cup S)$ is given by the formula

$$n(M \cup S) = n(M) + n(S) - n(M \cap S)$$
$$= 120 + 90 - 60$$
$$= 210 - 60$$
$$= 150$$

Hence 150 students passed either the math or science test.

(b) How many did not pass either of the two tests?

Sol. The number of students who did not pass either test is the total students minus those who passed at least one test.

Not passed either = total students $- n(M \cup S)$

$$= 180 - 150 = 30$$

$\therefore$ 30 students did not pass either test.

(c) How many passed the science test but not the math test?

Sol. The number of students who passed the science test but not the math test $n(S - M)$ is given by

$$n(S - M) = n(S) - n(M \cap S)$$
$$= 90 - 60 = 30$$

$\therefore$ 30 students passed the science test but not the math test.

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(d) How many failed the science test?

Sol. The number of students who failed the science test is the total students minus those who passed the science test.

$$\text{Failed in science} = n(U) - n(S)$$
$$= 180 - 90$$
$$= 90$$

$\therefore$ 90 students failed the science test.

Q.16. In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

- **150 developers like Python.**
- **130 developers like Java.**
- **120 developers like PHP.**
- **70 developers like both Python and Java.**
- **60 developers like both Python and PHP.**
- **50 developers like both Java and PHP.**
- **40 developers like all three languages: Python, Java and PHP.**

Sol. Let U = {total number of software developers}

P = {developers who like python}

J = {developers who like Java}

H = {developers who like PHP}

By the given statement

$n(U) = 300$, $n(P) = 150$, $n(J) = 130$, $n(H) = 120$

$n(P \cap J) = 70$, $n(P \cap H) = 60$, $n(J \cap P) = 50$, $n(P \cap J \cap H) = 40$

(a) How many developers use at least one of these languages?

Sol. The formula for the union of three sets is

$$n(P \cup J \cup H) = n(P) + n(J) + n(H) - n(P \cap J) - n(P \cap H) - n(J \cap P) + n(P \cap J \cap H)$$

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(c) How many developers do not use any of these languages?

Sol. The total number of developers is 300. There who like at least one language in $n(P \cup J \cup H) = 260$

$\therefore$ Not using any language $= 300 - 260$

$= 40$

(d) How many developers use only PHP?

Sol. Developers who like only PHP

$n(H)$ only $= n(H) - n(P \cap H) - n(J \cap H) + n(P \cap J \cap H)$

$= 120 - 60 - 50 + 40$

$= 160 - 110$

$= 50$

$\therefore$ The number of developers who use only PHP is 50.

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