Unit No. 5

Linear Equations and Inequalities

Exercise No. 5.2

Question No. 1

Maximize f(x, y) = 2x + 5y; subject to the constraints:

$2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$

Solution:

$2y - x \leq 8$ … eq. (i)

$x - y \leq 4$ … eq. (ii)

Associated equation of (i).

$2y - x = 8$

- x-intercept: Set $y = 0$:

$2(0) - x = 8$
$0 - x = 8$

$x = -8$

So, the point is (-8, 0).

- y-intercept: Set $x = 0$:

$2y - 0 = 8$
$2y = 8$

$y = 8/26$

$y = 4$

So, the point is (0, 4).

Origin Test for eq. (i):

Put $x = y = 0$ in $2y - x \leq 8$:

$2(0) - 0 \leq 8$

$0 - 0 \leq 8$

$0 \leq 8$ True

Shading Region:

Shading lies towards the origin side.

Associated equation of (ii).

$x - y = 4$

- x-intercept: Set $y = 0$:

$x - 0 \leq 4$
$x = 4$

So, the point is (4, 0).

- y-intercept: Set x = 0:

  $0 - y = 4$
  $- y = 4$

  $y = -4$

So, the point is (0, -4).

Origin Test for eq. (ii):

Put $x = y = 0$ in $x - y \leq 4$:

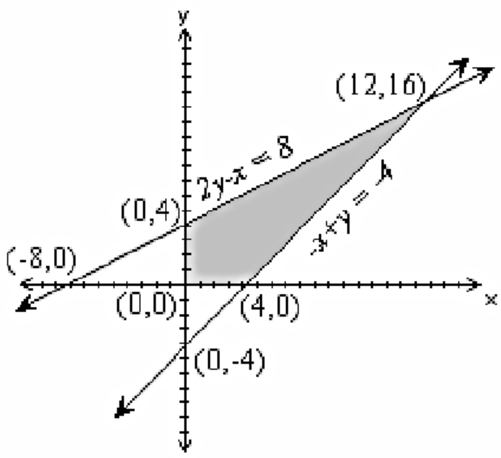
  $0 - 0 \leq 4$

  $0 \leq 4$ True

Shading Region:

  Shading lies towards the origin side.

Graphical Representation:



Feasible Solution Region:

  OABC is feasible solution region.

O(0, 0), A(4, 0), C(0, 4), B(x, y)

To find B(x, y), solve both equations for corner points.

Adding eq. (i) & eq. (ii):

$(2y - x) + (x - y) = (8) + (4)$

$2y - x + x - y = 8 + 4$

$y = 12$

Put y = 12 in eq. (i):

$x – 12 = 4$

$x = 4 + 12$

$x = 16$

Thus B(x, y) = (16, 12)

Now;

$f(x, y) = 2x + 5y$ … eq. (iii)

Put O(0, 0) in eq. (iii);

$f(0, 0) = 2(0) + 5(0) = 0 + 0 = 0$

Put A(4, 0) in eq. (iii);

$f(4, 0) = 2(4) + 5(0) = 8 + 0 = 8$

Put C(0, 4) in eq. (iii);

$f(0, 4) = 2(0) + 5(4) = 0 + 20 = 20$

Put B(16, 12) in eq. (iii);

$f(0, 4) = 2(16) + 5(12) = 32 + 60 = 92$

Maximum Value of Given Function:

The maximum value of $f(x, y)$ is 92 at corner point B(16, 12).

Question No. 2

Maximize f(x, y) = x + 3y; subject to the constraints:

$2x + 5y \leq 30$; $5x + 4y \leq 20$; $x \geq 0$; $y \geq 0$

Solution:

$2x + 5y \leq 30$ … eq. (i)

$5x + 4y \leq 20$ … eq. (ii)

Associated equation of (i).

$2x + 5y = 30$

- x-intercept: Set $y = 0$:

$2x + 5(0) = 30$
$2x + 0 = 30$

$x = 30/2$

$x = 15$

So, the point is (15, 0).

- y-intercept: Set $x = 0$:

$2(0) + 5y = 30$
$0 + 5y = 30$

$y = 30/5$

$y = 6$

So, the point is (0, 6).

Origin Test for eq. (i):

Put $x = y = 0$ in $2x + 5y \leq 30$:

$2(0) + 5(0) \leq 30$

$0 + 0 \leq 30$

$0 \leq 30$ True

Shading Region:

Shading lies towards the origin side.

Associated equation of (ii).

$5x + 4y = 20$

- x-intercept: Set y = 0:

$5x + 4(0) = 20$
$5x + 0 = 20$
$x = 20/5$
$x = 4$

So, the point is (4, 0).

- y-intercept: Set x = 0:

$5(0) + 4y = 20$
$0 + 4y = 20$

$y = 20/4$

$y = 5$

So, the point is (0, 5).

Origin Test for eq. (ii):

Put $x = y = 0$ in $5x + 4y \leq 20$:

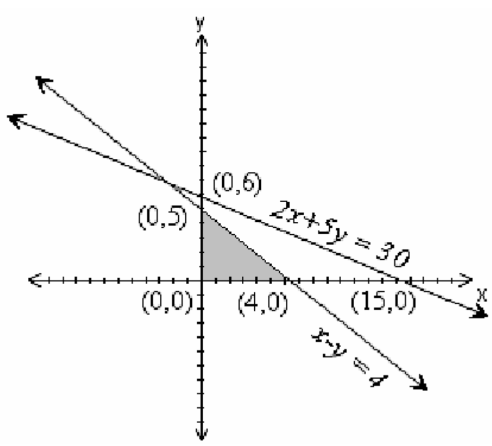
$5(0) + 4(0) \leq 20$

$0 \leq 20$ True

Shading Region:

Shading lies towards the origin side.

Graphical Representation:



Feasible Solution Region:

OAB is feasible solution region.

O(0, 0), A(4, 0), B(0, 5)

Now;

$f(x, y) = x + 3y$ … eq. (iii)

Put O(0, 0) in eq. (iii);

$f(0, 0) = 0 + 3(0) = 0 + 0 = 0$

Put A(4, 0) in eq. (iii);

$f(4, 0) = 4 + 3(0) = 4 + 0 = 4$

Put B(0, 5) in eq. (iii);

$f(0, 5) = 0 + 3(5) = 0 + 15 = 15$

Maximum Value of Given Function:

The maximum value of $f(x, y)$ is 15 at corner point B(0, 5).

Question No. 3

Maximize $z = 2x + 3y$; subject to the constraints:

$2x + y \leq 4$; $4x - y \leq 4$; $x \geq 0$; $y \geq 0$

Solution:

$2x + y \leq 4$ … eq. (i)

$4x - y \leq 4$ … eq. (ii)

Associated equation of (i).

$2x + y = 4$

- x-intercept: Set $y = 0$:

$2x + 0 = 4$
$2x = 4$

$x = 4/2 = 2$

So, the point is (2, 0).

- y-intercept: Set $x = 0$:

$2(0) + y = 4$

$y = 4$

So, the point is (0, 4).

Origin Test for eq. (i):

Put $x = y = 0$ in $2x + y \leq 4$:

$2(0) + 0 \leq 4$

$0 \leq 4$ True

Shading Region:

Shading lies towards the origin side.

Associated equation of (ii).

$4x - y = 4$

- x-intercept: Set $y = 0$:

$4x - 0 = 4$
$4x = 4$
$x = 4/4$
$x = 1$

So, the point is (1, 0).

- y-intercept: Set x = 0:

$4(0) - y = 4$
$- y = 4$

$y = -4$

So, the point is (0, -4).

Origin Test for eq. (ii):

Put $x = y = 0$ in $4x - y \leq 4$:

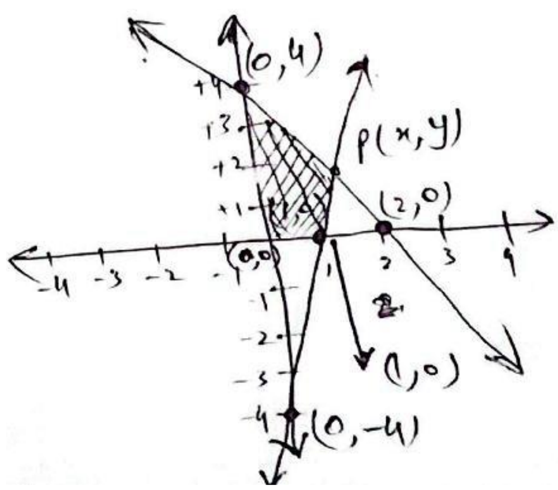
$4(0) - 0 \leq 4$

$0 \leq 4$ True

Shading Region:

Shading lies towards the origin side.

Graphical Representation:



Feasible Solution Region:

OABC is feasible solution region.

O(0, 0), A(1, 0), B(0, 4), C(x, y) = ?

To find C(x, y), solve both equations for corner points.

Adding eq. (i) & eq. (ii):

$(2x + y) + (4x - y) = (4) + (4)$

$2x + y + 4x - y = 4 + 4$

$6x = 8$

$x = \frac{8}{6} = \frac{4}{3}$

Put $y = \frac{4}{3}$ in eq. (i):

$2(\frac{4}{3}) + Y = 4$

$\frac{8}{3} + Y = 4$

$Y = 4 - \frac{8}{3}$

$Y = \frac{12-8}{3}$

$Y = \frac{4}{3}$

Thus $C(x, y) = (\frac{4}{3}, \frac{4}{3})$

Now;

$f(x, y) = 2x + 3y$ … eq. (iii)

Put O(0, 0) in eq. (iii);

$f(0, 0) = 2(0) + 3(0) = 0 + 0 = 0$

Put A(1, 0) in eq. (iii);

$f(1, 0) = 2(1) + 3(0) = 2 + 0 = 2$

Put B(0, 4) in eq. (iii);

$f(0, 4) = 2(0) + 3(4) = 0 + 12 = 12$

Put $C(\frac{4}{3}, \frac{4}{3})$ in eq. (iii);

$f(\frac{4}{3}, \frac{4}{3}) = 2(\frac{4}{3}) + 3(\frac{4}{3}) = \frac{8}{3} + \frac{12}{3} = \frac{20}{3} \approx 7$

Maximum Value of Given Function:

The maximum value of $f(x, y)$ is 12 at corner point B(0, 4).

Question No. 4

Minimize z = 2x + y; subject to the constraints:

$x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$

Solution:

$x + y \geq 3$ … eq. (i)

$7x + 5y \leq 35$ … eq. (ii)

Associated equation of (i).

$x + y = 3$

- x-intercept: Set y = 0:

$x + 0 = 3$
$x = 3$

So, the point is (3 0).

- y-intercept: Set x = 0:

$0 + y = 3$
$y = 3$

So, the point is (0, 3).

Origin Test for eq. (i):

Put $x = y = 0$ in $x + y \geq 3$:

$0 + 0 \geq 3$

$0 \geq 3$ False

Shading Region:

Shading lies away from the origin side.

Associated equation of (ii).

$7x + 5y = 35$

- x-intercept: Set $y = 0$:

$7x + 5(0) = 35$
$7x + 0 = 35$
$7x = 35$
$x = 35/7$
$x = 5$

So, the point is (5, 0).

- y-intercept: Set $x = 0$:

$7(0) + 5y = 35$
$5y = 35$

$y = 35/7 = 5$

So, the point is (0, 5).

Origin Test for eq. (ii):

Put $x = y = 0$ in $7x + 5y \leq 35$:

$7(0) + 5(0) \leq 35$

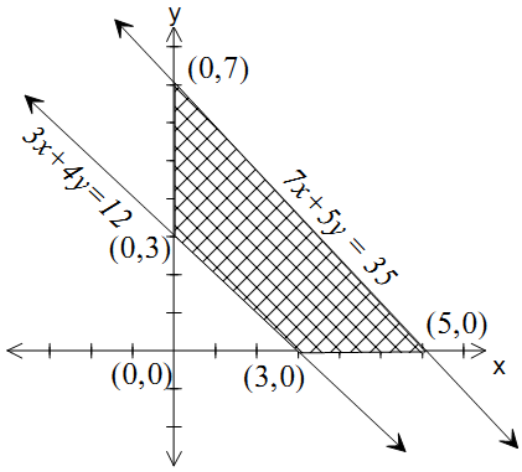
$0 + 0 \leq 35$

$0 \leq 35$ True

Shading Region:

Shading lies towards the origin side.

Graphical Representation:



Feasible Solution Region:

ABCD is feasible solution region.

A(3, 0), B(5, 0), C(0, 7), D(0, 3)

Now;

$f(x, y) = 2x + y$ … eq. (iii)

Put A(3, 0) in eq. (iii);

$f(3, 0) = 2(3) + (0) = 6 + 0 = 6$

Put B(5, 0) in eq. (iii);

$f(5, 0) = 2(5) + (0) = 10 + 0 = 10$

Put C(0, 7) in eq. (iii);

$f(0, 7) = 2(0) + (7) = 0 + 7 = 7$

Put D(0, 3) in eq. (iii);

$f(0, 3) = 2(0) + (3) = 0 + 3 = 3$

Minimum Value of Given Function:

The minimum value of $f(x, y)$ is 3 at corner point D(0, 3).

Question No. 5

Maximize the function defined as; $f(x, y) = 2x + 3y$ subject to the constraints:

$2x + y \leq 8$; $x + 2y \leq 14$; $x \geq 0$; $y \geq 0$

Solution:

$2x + y \leq 8$ … eq. (i)

$x + 2y \leq 14$ … eq. (ii)

Associated equation of (i).

$2x + y = 8$

- x-intercept: Set $y = 0$:

$2x + 0 = 8$
$x = 8/2$

$x = 4$

So, the point is (4, 0).

- y-intercept: Set $x = 0$:

$2(0) + y = 8$
$0 + y = 8$
$y = 8$

So, the point is (0, 8).

Origin Test for eq. (i):

Put $x = y = 0$ in $2x + y \leq 8$:

$2(0) + 0 \leq 8$

$0 + 0 \leq 8$

$0 \leq 8$ True

Shading Region:

Shading lies towards the origin side.

Associated equation of (ii).

$x + 2y = 14$

- x-intercept: Set $y = 0$:

$x + 2(0) = 14$
$x = 14$

So, the point is (14, 0).

- y-intercept: Set $x = 0$:

$0 + 2y = 14$
$y = 14/2$

$y = 7$

So, the point is (0, 7).

Origin Test for eq. (ii):

Put $x = y = 0$ in $x + 2y \leq 14$:

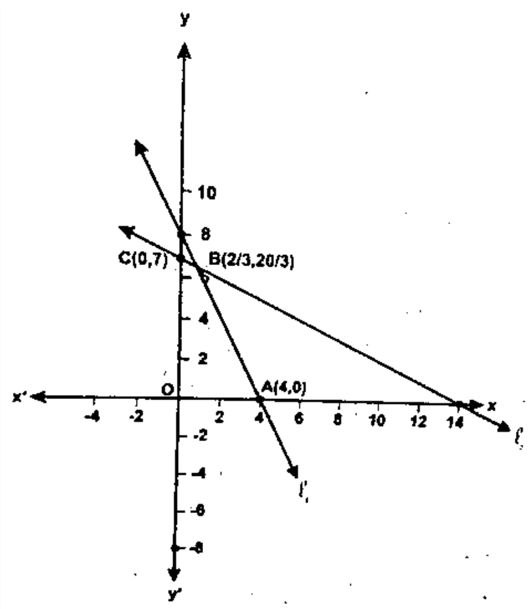
$0 + 2(0) \leq 14$

$0 \leq 14$ True

Shading Region:

Shading lies towards the origin side.

Graphical Representation:



Feasible Solution Region:

OABC are feasible solution region.

O(0, 0), A(4, 0), C(0, 7), B(x, y)

To find B(x, y), solve both equations for corner points.

Multiply eq. (i) by 2 & subtract eq. (i) from eq. (ii):

(x + 2y) - (4x + 2y) = (14) - (16)

x + 2y - 4x - 2y = 14 - 16

-3x = -2

$x = \frac{2}{3}$

Put $x = \frac{2}{3}$ in eq. (i):

$2(\frac{2}{3}) + y = 8$

$\frac{4}{3} + y = 8$

$y = 8 - \frac{4}{3}$

$y = \frac{24-4}{3}$

$y = \frac{20}{3}$

Thus $B(x, y) = (\frac{2}{3}, \frac{20}{3})$

Now;

$f(x, y) = 2x + 3y$ … eq. (iii)

Put O(0, 0) in eq. (iii);

$f(0, 0) = 2(0) + 3(0) = 0 + 0 = 0$

Put A(4, 0) in eq. (iii);

$f(4, 0) = 2(4) + 3(0) = 8 + 0 = 8$

Put B$(\frac{2}{3}, \frac{20}{3})$ in eq. (iii);

$f(\frac{2}{3}, \frac{20}{3}) = 2(\frac{2}{3}) + 3(\frac{20}{3}) = \frac{4}{3} + \frac{60}{3} = \frac{64}{3} = 21.33$

Put C(0, 7) in eq. (iii);

$f(0, 7) = 2(0) + 3(7) = 0 + 21 = 21$

Maximum Value of Given Function:

The maximum value of $f(x, y)$ is $\frac{64}{3}$ at corner point B$(\frac{2}{3}, \frac{20}{3})$.

Question No. 6

Find minimum and maximum values of z = 3x + y; subject to the constraints:

$3x + 5y \geq 15$; $x + 6y \geq 9$; $x \geq 0$; $y \geq 0$

Solution:

$3x + 5y \geq 15$ … eq. (i)

$x + 6y \geq 9$ … eq. (ii)

Associated equation of (i).

$3x + 5y = 15$

- x-intercept: Set y = 0:

$3x + 5(0) = 15$
$3x + 0 = 15$

$x = 15/3 = 5$

So, the point is (5, 0).

- y-intercept: Set x = 0:

$3(0) + 5y = 15$
$5y = 15$

$y = 15/5$

$y = 3$

So, the point is (0, 3).

Origin Test for eq. (i):

Put x = y = 0 in $3x + 5y \geq 15$:

$3(0) + 5(0) \geq 15$

$0 + 0 \geq 15$

$0 \geq 15$ False

Shading Region:

Shading lies away from the origin side.

Associated equation of (ii).

$x + 6y = 9$

- x-intercept: Set y = 0:

  $x + 6(0) = 9$
  $x = 9$

So, the point is (9, 0).

- y-intercept: Set x = 0:

  $0 + 6y = 9$
  $y = \frac{9}{6}$

  $y = \frac{3}{2}$

So, the point is $(0, \frac{3}{2})$.

Origin Test for eq. (ii):

Put $x = y = 0$ in $x + 6y \geq 9$:

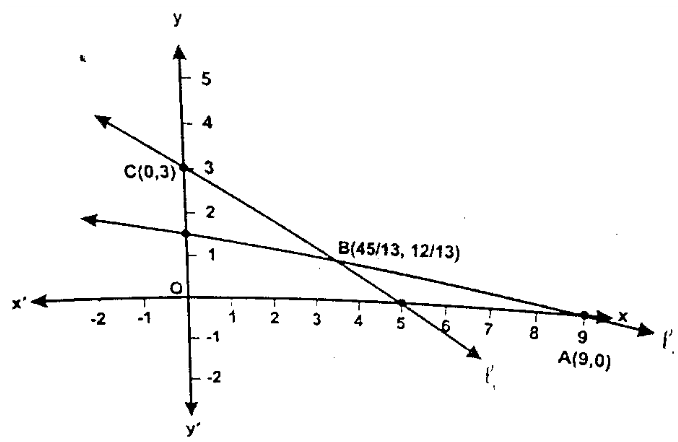
$0 + 6(0) \geq 9$

$0 \geq 9$ False

Shading Region:

Shading lies away from the origin side.

Graphical Representation:



Feasible Solution Region:

ABC are feasible solution region.

A(0, 3), B(9, 0), C(x, y)

To find C(x, y), solve both equations for corner points.

Multiply eq. (i) by 3 & Subtract both equations:

$3(x + 6y) - (3x + 5y) = 3(9) - (15)$

$3x + 18y - 3x - 5y = 27 - 15$

$13y = 12$

$y = \frac{12}{13}$

Put $y = \frac{12}{13}$ in eq. (i):

$x + 6(\frac{12}{13}) = 9$

$x + \frac{72}{13} = 9$

$x = 9 - \frac{72}{13}$

$x = \frac{117-72}{13}$

$x = \frac{45}{13}$

Thus $C(x, y) = (\frac{45}{13}, \frac{12}{13})$

Now;

$f(x, y) = 3x + y$ … eq. (iii)

Put A(0, 3) in eq. (iii);

$f(0, 3) = 3(0) + (3) = 0 + 3 = 3$

Put B(9, 0) in eq. (iii);

$f(9, 0) = 3(9) + 0 = 27 + 0 = 27$

Put $C(\frac{45}{13}, \frac{12}{13})$ in eq. (iii);

$f(\frac{45}{13}, \frac{12}{13}) = 3(\frac{45}{13}) + (\frac{12}{13}) = \frac{135}{13} + \frac{12}{13} = \frac{147}{13} \approx 11$

Maximum and Minimum Values of Given Function:

The maximum value of $f(x, y)$ is 27 at corner point B(9, 0) and the minimum value of $f(x, y)$ is 3at corner point C(0, 3).