

Unit No. 7

Coordinate Geometry

Exercise No. 7.3

Question No. 1

If the houses of two friends are represented by coordinates (2, 6) and (9, 12) on a grid. Find the straight line distance between their houses if the grid units represent kilometres?

Data:

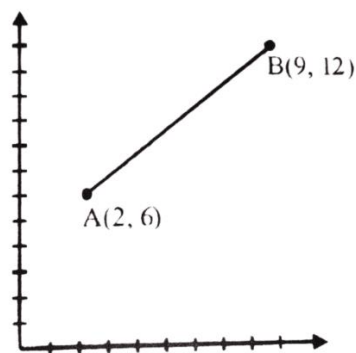
$$A = (2, 6)$$

$$B = (9, 12)$$

To Find:

Distance between houses = ?

Pictorial Form:



Solution:

Using Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(9 - 2)^2 + (12 - 6)^2}$$

$$|AB| = \sqrt{(7)^2 + (6)^2}$$

$$|AB| = \sqrt{49 + 36}$$

$$|AB| = \sqrt{85}$$

$$|AB| = 9.22 \text{ km}$$

So, the Distance between houses is 9.22 km.

Question No. 2

Consider a straight trail (represented by coordinate plane) that starts at point (5,7) and ends at point (15,3). What are the coordinates of the midpoint?

Data:

$$A = (5, 7)$$

$$B = (15, 3)$$

To Find:

Coordinates of the midpoint = ?

Solution:

Using Mid-Point Formula:

$$\text{Mid - Point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Mid - Point}(AB) = \left(\frac{5 + 15}{2}, \frac{7 + 3}{2} \right)$$

$$\text{Mid - Point}(AB) = \left(\frac{20}{2}, \frac{10}{2} \right)$$

$$\text{Mid - Point}(AB) = (10, 5)$$

So, coordinates of the midpoint are (10, 5).

Question No. 3

An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assume the coordinates are in metres.

Data:

$$A = (10, 8)$$

$$B = (4, 3)$$

To Find:

Distance between the buildings = ?

Pictorial Form:

Solution:

Using Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(4 - 10)^2 + (3 - 8)^2}$$

$$|AB| = \sqrt{(-6)^2 + (-5)^2}$$

$$|AB| = \sqrt{36 + 25}$$

$$|AB| = \sqrt{61}$$

$$|AB| = 7.81 \text{ m}$$

So, Distance between the buildings is 7.81 m.

Question No. 4

A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other is at (12, 10) on the city grid map, where each unit represents kilometres. What is the distance between the two locations?

Data:

$$A = (7, 2)$$

$$B = (12, 10)$$

To Find:

Distance between delivery locations = ?

Pictorial Form:

Solution:

Using Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(12 - 7)^2 + (10 - 2)^2}$$

$$|AB| = \sqrt{(5)^2 + (8)^2}$$

$$|AB| = \sqrt{25 + 64}$$

$$|AB| = \sqrt{89}$$

$$|AB| = 9.43 \text{ km}$$

So, Distance between delivery locations is 9.43 km.

Question No. 5

The start and end points of a race track are given by coordinates (3,9) and (9,13). What is the midpoint of the track?

Data:

$$A = (3,9)$$

$$B = (9,13)$$

To Find:

Midpoint of the track = ?

Pictorial Form:

Solution:

Using Mid-Point Formula:

$$\text{Mid - Point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Mid - Point}(AB) = \left(\frac{3 + 9}{2}, \frac{9 + 13}{2} \right)$$

$$\text{Mid - Point}(AB) = \left(\frac{12}{2}, \frac{22}{2} \right)$$

$$\text{Mid} - \text{Point}(AB) = (6, 11)$$

Midpoint of the track is (6, 11) equidistant from water stations.

Question No. 6

The coordinates of two points on a road are A (3, 4) and B (7, 10). Find the midpoint of the road.

Data:

$$A = (3, 4)$$

$$B = (7, 10)$$

To Find:

Midpoint of the road = ?

Pictorial Form:

Solution:

$$\text{Mid} - \text{Point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Mid} - \text{Point}(AB) = \left(\frac{3 + 7}{2}, \frac{4 + 10}{2} \right)$$

$$\text{Mid} - \text{Point}(AB) = \left(\frac{10}{2}, \frac{14}{2} \right)$$

$$\text{Mid} - \text{Point}(AB) = (5, 7)$$

So, midpoint of the road is (5, 7).

Question No. 7

A ship is navigating from port A located at (12°N, 65°W) to port B at (20°N, 45°W). If the ship travels along the shortest path on the surface of the Earth, calculate the straight line distance between the points.

Data:

$$A = (12^\circ\text{N}, 65^\circ\text{W})$$

$$B = (20^\circ\text{N}, 45^\circ\text{W}).$$

To Find:

Distance between the points = ?

Pictorial Form:

Solution:

$$\text{Mid} - \text{Point Latitude} = \frac{x_1 + x_2}{2}$$

$$\text{Mid} - \text{Point Latitude} = \frac{12+20}{2} = \frac{32}{2} = 16^\circ\text{N}$$

$$\text{Mid} - \text{Point Longitude} = \frac{y_1 + y_2}{2}$$

$$\text{Mid - Point Longitude} = \frac{65+45}{2} = \frac{110}{2} = 55^\circ\text{W}$$

To Find Straight Line Distance:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(20 - 12)^2 + (45 - 65)^2}$$

$$|AB| = \sqrt{(8)^2 + (-20)^2}$$

$$|AB| = \sqrt{64 + 400}$$

$$|AB| = \sqrt{464} = 21.5 \text{ units}$$

So, distance between the points is 21.5 units.

Question No. 8

Farah is fencing around a rectangular field with corners at (0, 0), (0, 5), (8, 5) and (8, 0). How much fencing material will she need to cover the entire perimeter of the field?

Data:

$$A = (0,0)$$

$$B = (0,5)$$

$$C = (8,5)$$

$$D = (8,0)$$

To Find:

Perimeter = ?

Length of Fence = ?

Pictorial Form:

Solution:

1. Finding distance between AB:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(0 - 0)^2 + (5 - 0)^2}$$

$$|AB| = \sqrt{(0)^2 + (5)^2} = \sqrt{(5)^2} = 5$$

2. Finding distance between BC:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|BC| = \sqrt{(8 - 0)^2 + (5 - 5)^2}$$

$$|BC| = \sqrt{(8)^2 + (0)^2} = \sqrt{(8)^2} = 8$$

3. Finding distance between CD:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|CD| = \sqrt{(8 - 8)^2 + (0 - 5)^2}$$

$$|CD| = \sqrt{(0)^2 + (-5)^2} = \sqrt{(5)^2} = 5$$

4. Finding distance between DA:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|DA| = \sqrt{(0 - 8)^2 + (0 - 0)^2}$$

$$|DA| = \sqrt{(-8)^2} = \sqrt{(8)^2} = 8$$

Conclusion:

$$|AB| = 5 \text{ and } |CD| = 5$$

$$|BC| = 8 \text{ and } |DA| = 8$$

Since opposite sides are equal ($|AB| = |CD|$ and $|BC| = |DA|$), the given points do form a rectangle.

$$\text{Perimeter of Rectangle} = 2(l + b)$$

$$= 2(8 + 5)$$

$$= 2(13)$$

$$= 26 \text{ units}$$

$$\text{Length of Fencing material} = 26 \text{ units}$$

Question No. 9

An airplane is flying from city X at (40° N, 100° W) to city Y at (50° N, 80° W). Use coordinate geometry, calculate the shortest distance between these two cities.

Data:

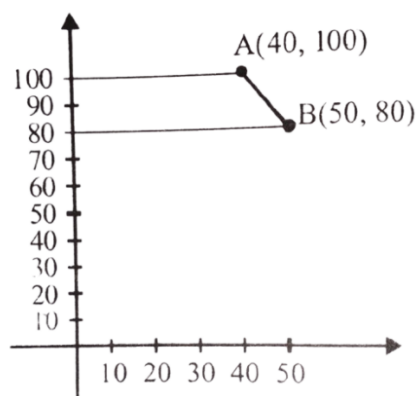
$$X = (40^\circ \text{ N}, 100^\circ \text{ W})$$

$$Y = (50^\circ \text{ N}, 80^\circ \text{ W})$$

To Find:

$$\text{Distance between these two cities} = ?$$

Pictorial Form:



Solution:

$$\text{Mid - Point Latitude} = \frac{x_1 + x_2}{2}$$

$$\text{Mid - Point Latitude} = \frac{40+50}{2} = \frac{90}{2} = 45^\circ\text{N}$$

$$\text{Mid - Point Longitude} = \frac{y_1 + y_2}{2}$$

$$\text{Mid - Point Longitude} = \frac{100+80}{2} = \frac{180}{2} = 90^\circ\text{W}$$

To Find Straight Line Distance:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|XY| = \sqrt{(50 - 40)^2 + (80 - 100)^2}$$

$$|XY| = \sqrt{(10)^2 + (-20)^2}$$

$$|XY| = \sqrt{100 + 400}$$

$$|XY| = \sqrt{500} = 22.4 \text{ units}$$

So, distance between two cities is 22.4 units.

Question No. 10

A land surveyor is marking out a rectangular plot of land with corners at (3, 1), (3, 6), (8, 6), and (8, 1). Calculate the perimeter.

Data:

$$\mathbf{A = (3, 1)}$$

$$\mathbf{B = (3, 6)}$$

$$\mathbf{C = (8, 6)}$$

$$\mathbf{D = (8, 1)}$$

To Find:

$$\text{Perimeter} = ?$$

Pictorial Form:

Solution:

1. Finding distance between AB:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(3 - 3)^2 + (6 - 1)^2}$$

$$|AB| = \sqrt{(0)^2 + (5)^2} = \sqrt{(5)^2} = 5$$

2. Finding distance between BC:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|BC| = \sqrt{(8 - 3)^2 + (6 - 6)^2}$$

$$|BC| = \sqrt{(5)^2 + (0)^2} = \sqrt{(5)^2} = 5$$

3. Finding distance between CD:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|CD| = \sqrt{(8 - 8)^2 + (1 - 6)^2}$$

$$|CD| = \sqrt{(0)^2 + (-5)^2} = \sqrt{(5)^2} = 5$$

4. Finding distance between DA:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|DA| = \sqrt{(8 - 3)^2 + (1 - 1)^2}$$

$$|DA| = \sqrt{(5)^2} = \sqrt{(5)^2} = 5$$

$$\text{Perimeter} = 5 + 5 + 5 + 5 = 20 \text{ units}$$

Thus, required perimeter is 20 units.

Question No. 11

A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the coordinates: A (0, 0), B (5, 0), C (5, 3), and D (0, 3). How much fencing is required?

Data:

$$\mathbf{A = (0, 0)}$$

$$\mathbf{B = (5, 0)}$$

$$\mathbf{C = (5, 3)}$$

$$\mathbf{D = (0, 3)}$$

To Find:

$$\text{Length of Fence} = ?$$

Pictorial Form:

Solution:

1. Finding distance between AB:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(5 - 0)^2 + (0 - 0)^2}$$

$$|AB| = \sqrt{(5)^2 + (0)^2} = \sqrt{(5)^2} = 5$$

2. Finding distance between BC:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|BC| = \sqrt{(5 - 5)^2 + (3 - 0)^2}$$

$$|BC| = \sqrt{(0)^2 + (3)^2} = \sqrt{(3)^2} = 3$$

3. Finding distance between CD:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|CD| = \sqrt{(0 - 5)^2 + (3 - 3)^2}$$

$$|CD| = \sqrt{(-5)^2 + (0)^2} = \sqrt{(5)^2} = 5$$

4. Finding distance between DA:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|DA| = \sqrt{(0 - 0)^2 + (3 - 0)^2}$$

$$|DA| = \sqrt{(3)^2} = 3$$

$$\text{Perimeter} = 5 + 3 + 5 + 3 = 16 \text{ units}$$

Thus, required length of fencing is 16 units.