Unit No. 3

Set and Functions

Exercise No. 3.3

Question No. 1

For A = {1, 2, 3, 4}, find the following relations in A. State the domain and range of each relation.

(i) {(x, y) | y = x}

(ii) {(x, y) | y + x = 5}

(iii) {(x, y) | x + y < 5}

(iv) {(x, y) | x + y > 5}

Understanding Relations:

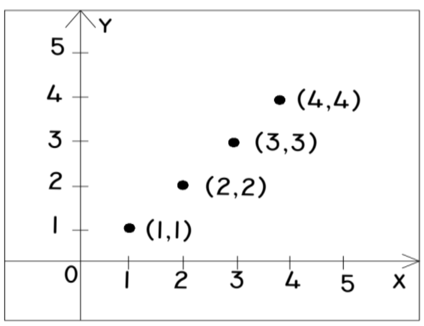
A relation on A is a set of ordered pairs (x,y) where x,y∈A, and the relation follows a given rule.

(i) {(x, y) | y = x}

Solution:

R_1 = {(1,1),(2,2),(3,3),(4,4)}

- Domain: {1,2,3,4} (all x-values)
- Range: {1,2,3,4} (all y-values)

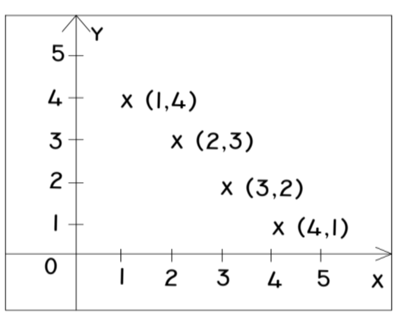


(ii) {(x, y) | y + x = 5}

Solution:

R_2={(1,4),(2,3),(3,2),(4,1)}

- **Domain:** {1,2,3,4} (all x-values) **Range:** {1,2,3,4} (all y-values)

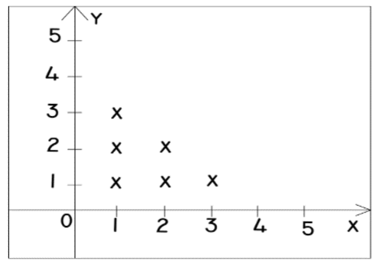


(iii) {(x, y) | x + y < 5}

Solution:

R_3={(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)}

- Domain: {1,2,3} (all x-values)
- Range: {1,2,3} (all y-values)

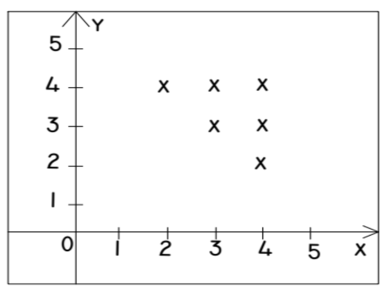


(iv) {(x, y) | x + y > 5}

Solution:

R_4 = {(2,4),(3,3),(3,4),(4,2),(4,3),(4,4)}

- Domain: {2,3,4} (all x-values)
- Range: {2,3,4} (all y-values)



Question No. 2

Which of the following diagrams represent functions and of which type?

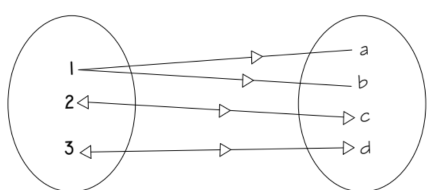


Fig (1)

Answer:

It does not represent a function.

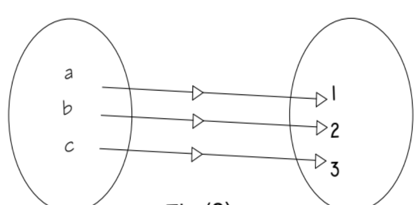


Fig (2)

Answer:

It represents a function, which is a bijective function.

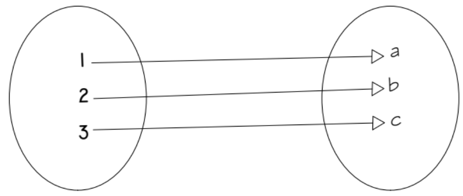


Fig (3)

Answer:

It represents a function, which is a bijective function.

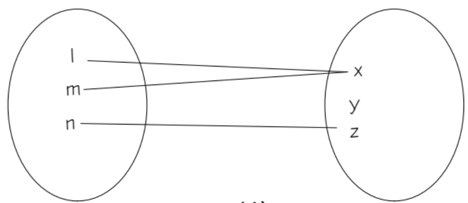


Fig (4)

Answer:

It represents a function, which is an into function.

Question No. 3

If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) $g(0)$

Solution:

$g(0)$

Substituting $x = 0$ into $g(x)$:

$g(0) = 3(0) + 2 = 0 + 2 = 2$

(ii) $g(-3)$

Solution:

$g(-3)$

Substituting $x = -3x$ into $g(x)$:

$g(-3) = 3(-3) + 2 = -9 + 2 = -7$

(iii) $g(\frac{2}{3})$

Solution:

$g(\frac{2}{3})$

Substituting $x = \frac{2}{3}$ into $g(x)$:

$g(\frac{2}{3}) = (3 \times \frac{2}{3}) + 2 = 2 + 2 = 4$

(iv) h(1)

Solution:

$h(1)$

Substituting x=1 into h(x):

$h(1) = (1)2 + 1 = 1 + 1 = 2$

(v) h(-4)

Solution:

$h(-4)$

Substituting $x = -4$ into h(x):

$h(-4) = (-4)^2 + 1 = 16 + 1 = 17$

(vi) h($-\frac{1}{2}$)

Solution:

$h(-\frac{1}{2})$

Substituting $x = -\frac{1}{2}$ into h(x):

$h(-\frac{1}{2}) = (-\frac{1}{2})^2 + 1 = \frac{1}{4} + 1 = \frac{1+4}{4} = \frac{5}{4}$

Question No. 4

Given that f(x) = ax + b + 1, where a and b are constant numbers. If f(3) = 8 and f(6) = 14, then find the values of a and b.

Given function:

$\mathbf{f(x) = ax + b + 1}$

$\mathbf{f(3) = 8}$

$\mathbf{f(6) = 14}$

To Find:

$\mathbf{a = ?}$

$\mathbf{b = ?}$

Solution:

Steps:

1. **Substitute f(3) = 8 into f(x):**

$f(3) = a(3) + b + 1 = 8$

$3a + b + 1 = 8$

$3a + b = 8 - 1$

$3a + b = 7 \quad \ldots(\text{Eq. i})$

2. **Substitute f(6) = 14 into f(x):**

    f(6) = a(6) + b + 1 = 14

    6a + b = 14 – 1

    6a + b = 13 …(Eq. ii)

3. **Solve the system of equations (i) and (ii) for a and b:**

    Subtract Equation (i) from Equation (ii): (6a + b) − (3a + b) = 13 − 7

    6a + b − 3a - b = 6

    3a = 6

    $$a = \frac{6}{3}$$

    a = 2

4. **Substitute the value of "a" back into either Equation 1 or 2 to find b:**

    Let's use Equation i:

    3(2) + b = 7

    6 + b = 7

    b = 7 – 6

    b = 1

    a = 2 , b = 1

Question No. 5

Given that g(x) = ax + b + 5, where a and b are constant numbers. If g(-1) = 0 and g(2) = 10, find the values of a and b.

Given function:

g(x) = ax + b + 5

g(-1) = 0

g(2) = 10

To Find:

a = ?

b = ?

Solution:

Steps:

1. **Substitute g(-1) = 0 into g(x):**

    g(−1) = a(−1) + b + 5 = 0

    −a + b = −5 …(Equation 1)

2. **Substitute g(2) = 10 into g(x):**

    g(2) = a(2) + b + 5 = 10

$2a + b = 10 - 5$

$2a + b = 5$ …(Equation 2)

3. **Solve the system of equations (1) and (2) for a and b:**

   Subtract Equation 1 from Equation 2:

   $(2a + b) - (-a + b) = 5 - (-5)$

   $2a + b + a - b = 5 + 5$

   $3a = 10$

   $a = \frac{10}{3}$

4. **Substitute the value of "a" back into either Equation 1 or 2 to find b:**

   Let's use Equation 1:

   $-(\frac{10}{3}) + b = -5$

   $b = -5 + \frac{10}{3}$

   $b = \frac{-15+10}{3}$

   $b = \frac{-15+10}{3}$

   $b = \frac{-5}{3}$

   $a = \frac{10}{3}$, $b = \frac{-5}{3}$

Question No. 6

Consider the function defined by f(x)=5x+1. If f(x) = 32, find the x value.

Given function:

$f(x) = 5x + 1$

$f(x) = 32$

To Find:

$x = ?$

Solution:

$f(x) = 5x + 1$

$32 = 5x + 1$

$32 - 1 = 5x$

$31 = 5x$

$x = \frac{31}{5}$ … (Either wrong value in book or wrong answer)

- There are two possibilities either change 32 by 31 or 1 by 2 to solve according to book`s answer.

Question No. 7

Consider the function $f(x) = cx^2+d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2)= 10$, then find the values of c and d.

Given Function:

$f(x) = cx^2 + d$

$f(1) = 6$

$f(-2) = 10$

To Find:

$c = ?$

$d = ?$

Solution:

$f(x) = cx^2 + d$

$f(1) = c(1)^2 + d = 6$

$c + d = 6 \quad \ldots \text{(Equation 1)}$

$f(x) = cx^2 + d$

$f(-2) = c(-2)^2 + d = 10$

$4c + d = 10 \quad \ldots \text{(Equation 2)}$

Subtract Equation 1 from Equation 2:

$(4c + d) - (c + d) = 10 - 6$

$3c = 4$

$c = \frac{4}{3}$

Let's use Equation 1:

$\frac{4}{3} + d = 6$

$d = 6 - \frac{4}{3}$

$d = \frac{18-4}{3}$

$d = \frac{14}{3}$

$c = \frac{4}{3} \quad , \quad d = \frac{14}{3}$