nit No. 10

Graphs of Functions

Exercise 10.2

Question No. 1

Plot the graph of $y = 2x^2 - 4x + 3$ from −1 to 3. Draw tangent at (2,3) and find the gradient.

Solution:

$y = 2x^2 - 4x + 3$

$x = -1, 0, 1, 2, 3$

$x = -1;\ y = 2(-1)^2 - 4(-1) + 3 = 2 + 4 + 3 = 9$

$x = 0;\ y = 2(0)^2 - 4(0) + 3 = 0 - 0 + 3 = 3$

$x = 1;\ y = 2(1)^2 - 4(1) + 3 = 2 - 4 + 3 = 1$

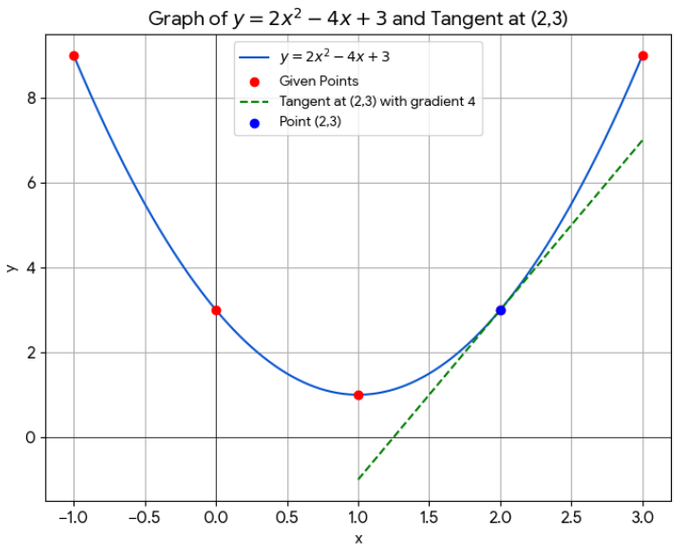
$x = 2;\ y = 2(2)^2 - 4(2) + 3 = 8 - 8 + 3 = 3$

$x = 3;\ y = 2(3)^2 - 4(3) + 3 = 18 - 12 + 3 = 9$

Table:

	A	B	C	D	E
x-axis	-1	0	1	2	3
y-axis	9	3	1	3	9

Graphical Representation:



Finding point on the tangent:

Let's find another point on the tangent line.

From graph drawn above two points are (2, 3) and (1.5, 1)

Slope of tangent line = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - 1.5} = \frac{2}{0.5} = \frac{2 \times 10}{5} = 4$

So, required gradient is 4.

Question No. 2

Plot the graph of $y = 3x^2 + x + 1$ and draw tangent at (1,5). Also find gradient of the tangent line at this point.

Solution:

$y = 3x^2 + x + 1$

$x = -2, -1, 0, 1, 2, 3$

$x = -2; y = 3(-2)^2 + (-2) + 1 = 12 - 2 + 1 = 11$

$x = -1; y = 3(-1)^2 + (-1) + 1 = 3 - 1 + 1 = 3$

$x = 0; y = 3(0)^2 + (0) + 1 = 0 - 0 + 1 = 1$

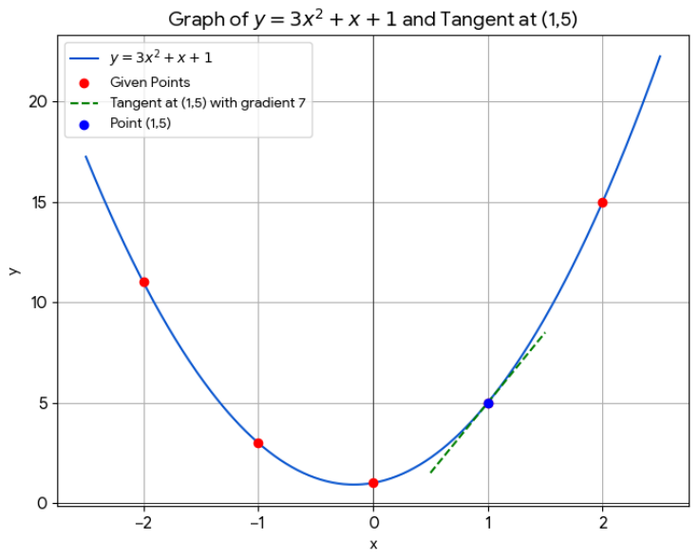
$x = 1; y = 3(1)^2 + (1) + 1 = 3 + 1 + 1 = 5$

$x = 2; y = 3(2)^2 + (2) + 1 = 12 + 2 + 1 = 1$

Table:

	A	B	C	D	E
x-axis	-2	-1	0	1	2
y-axis	11	3	1	5	15

Graphical Representation:



Find gradient of the tangent line:

We find another point on tangent line. Other point on tangent line is (2, 12),

slope of tangent line through (1, 5) and (2, 12) is,

Gradient = m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 5}{2 - 1} = \frac{7}{1} = 7$

Gradient = 7

Question No. 3

The strength of students in a school was 1000 in 2016. If the strength decay according to the equation $S=1000\ e^{-t}$, where S is the number of students at time t. (a) Graph the given equation for t = 0 (in 2016) to t=9 (in 2025).

(b) From the graph, estimate the student's strength in 2019 and in 2023.

Solution:

$S=1000\ e^{-t}$

In 2016 when t = 0; $S=1000\ e^{-0} = 1000\ (1) = 1000$

In 2017 when t = 1; S=1000 e−1 = 1000 (0.367879) = 367.879

In 2018 when t = 2; S=1000 e−2 = 1000 (0.131762) = 131.762

In 2019 when t = 3; S=1000 e−3 = 1000 (0.048237) = 48.237

In 2020 when t = 4; S=1000 e−4 = 1000 (0.017753) = 17.753

In 2021 when t = 5; S=1000 e−5 = 1000 (0.006547) = 6.547

In 2022 when t = 6; S=1000 e−6 = 1000 (0.002415) = 2.415

In 2023 when t = 7; S=1000 e−7 = 1000 (0.000891) = 0.891

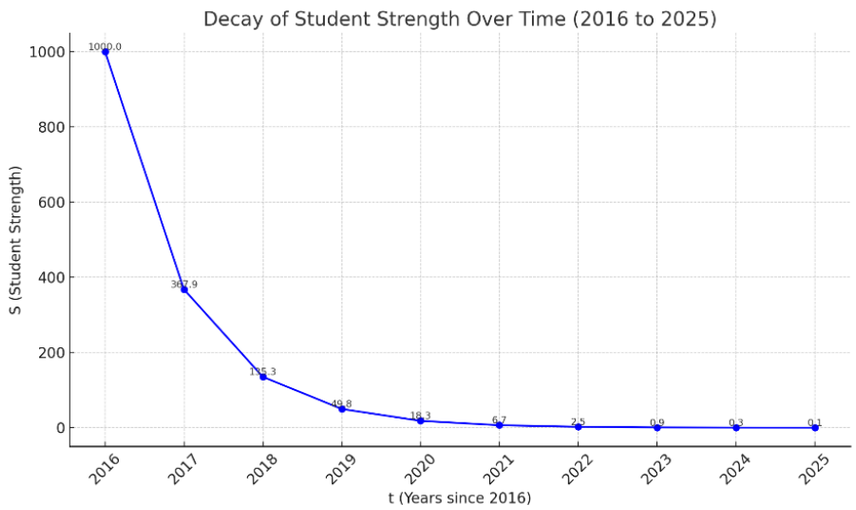
In 2024 when t = 8; S=1000 e−8 = 1000 (0.000329) = 0.329

In 2025 when t = 9; S=1000 e−9 = 1000 (0.000121) = 0.121

Table:

	x-axis (t)	y-axis (S)
2016	0	1000
2017	1	367.879
2018	2	131.762
2019	3	48.237
2020	4	17.753
2021	5	6.547
2022	6	2.415
2023	7	0.891
2024	8	0.329
2025	9	0.121

Graphical Representation:



Answer to (b):

From the graph:

In 2019 (t = 3): Estimated student strength = 49.8 ≈ 50 students

In 2023 (t = 7): Estimated student strength = 0.9 ≈ 1 student

Question No. 4

The demand and supply functions for a product are given by the equations

$P_d = 400 - 5Q$, $P_s = 3Q + 24$.

Plot the graph of each function over the interval Q = 0 to Q = 300.

Solution:

Given that; Q = 0 to Q = 300

For solving take Q = 0, 40, 80, 100

Put in Pd first:

Q = 0, $P_d = 400 - 5(0) = 400 - 0 = 400$

Q = 40, $P_d = 400 - 5(40) = 400 - 200 = 200$

Q = 80, $P_d = 400 - 5(80) = 400 - 400 = 0$

Q = 100, $P_d = 400 - 5(100) = 400 - 500 = -100$

Now, put in Ps;

Q = 0, $P_s = 3(0) + 24 = 0 + 24 = 24$

Q = 40, $P_s = 3(40) + 24 = 120 + 24 = 144$

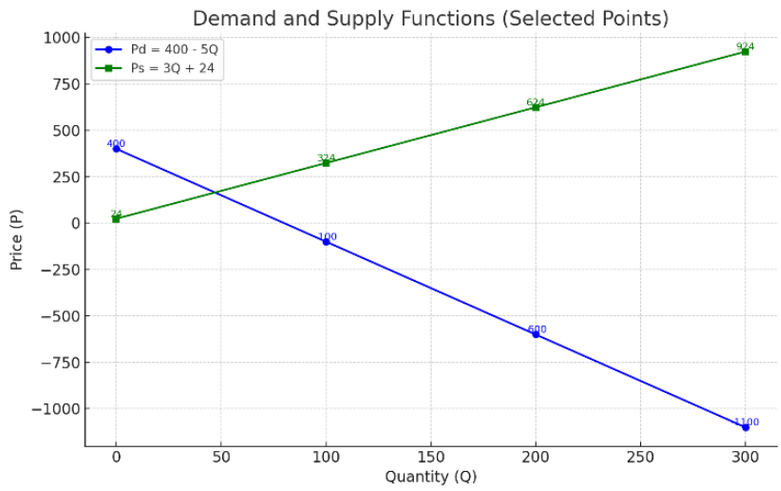
Q = 80, $P_s = 3(80) + 24 = 240 + 24 = 264$

Q = 100, $P_s = 3(100) + 24 = 300 + 24 = 324$

Table:

	A	B	C	D
Q	0	40	80	100
P_d	400	200	0	-100
P_s	24	144	264	324

Graphical Representation:



Question No. 5

Shahid's salary S(x) in rupees is based on the following formula:

S(x) = 45000 + 4500x,

where x is the number of years he has been with the company. Sketch and interpret the graph of salary function for 0 ≤ x ≤5.

Solution:

S(x) = 45000 + 4500x

0 ≤ x ≤5; x = 0, 1, 2, 3, 4, 5

x = 0; S(x) = 45000 + 4500(0) = 45000 + 0 = 45000

x = 1; S(x) = 45000 + 4500(1) = 45000 + 4500 = 49500

x = 2; S(x) = 45000 + 4500(2) = 45000 + 9000= 54000

x = 3; S(x) = 45000 + 4500(3) = 45000 + 13500= 58500

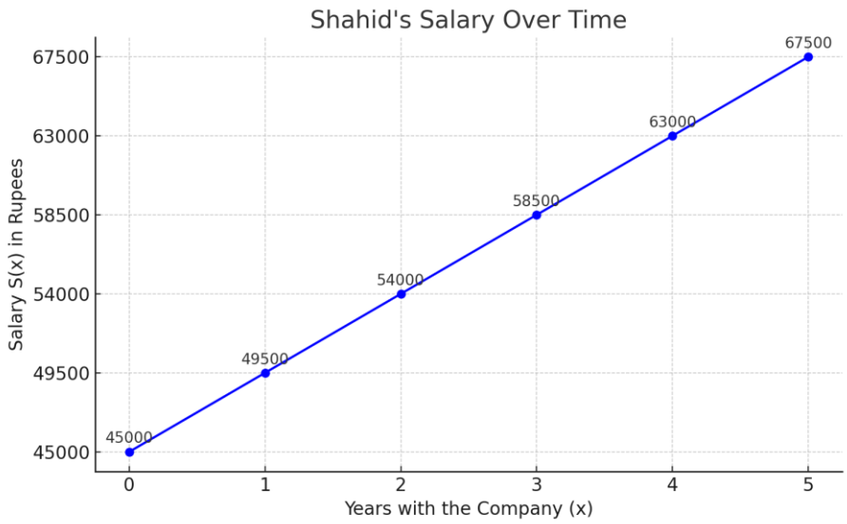
x = 4; S(x) = 45000 + 4500(4) = 45000 + 18000= 63000

x = 5; S(x) = 45000 + 4500(5) = 45000 + 22500= 67500

Table:

		A	B	C	D	E	F
x-axis	x	0	1	2	3	4	5
y-axis	S(x)	45000	49500	54000	58500	63000	67500

Graphical Representation:



Shahid's salary increases line only with years of service and rises Rs. 4500 for every year.

Question No. 6

A company manufactures school bags. The cost function of producing x bags is C(x) = 1200 + 20x and the revenue from selling x bags is R(x) = 50x.

(a) Find the break-even point.

(b) Determine the profit or loss when 250 bags are sold.

(c) Plot the graphs of both the functions and identify the break-even point.

Solution:

C(x) = 1200 + 20x

R(x) = 50x

Let; x = 0, 20, 40, 60, 250

Put these values in C(x) turn by turn:

x = 0; C(x) = 1200 + 20(0) = 1200 + 0 = Rs. 1200

x = 20; C(x) = 1200 + 20(20) = 1200 + 400 = Rs. 1600

x = 40; C(x) = 1200 + 20(40) = 1200 + 800 = Rs. 2000

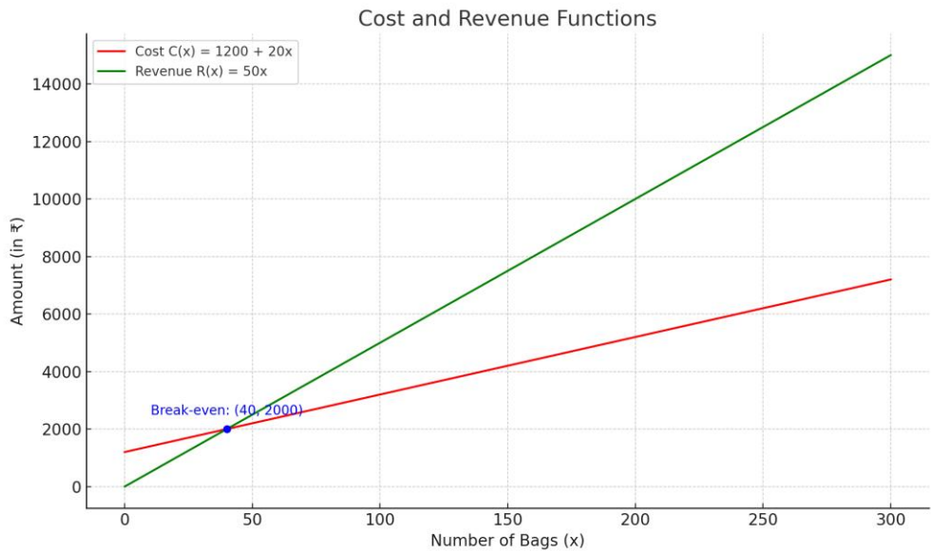
x = 60; C(x) = 1200 + 20(60) = 1200 + 1200 = Rs. 2400

x = 250; C(x) = 1200 + 20(250) = 1200 + 5000 = Rs. 6200

Table:

No. of bags	0	20	40	60	250
Cost (Rs.)	1200	1600	2000	2400	6200

Graphical Representation:



From graph:

Break-Even point (40, 2000)

i.e. At x = 40 bags

C.P = 1200 + 20 × 250

C.P = 1200 + 5000

C.P = Rs. 6200 (for x = 250)

Sale price (S.P)= 50 × 250

S.P = Rs. 12500

Since S.P > C.P so

Profit is realized

Profit = Rs. 12500 − Rs. 6200

Profit = Rs. 6300

Question No. 7

A newspaper agency fixed cost of Rs. 70 per edition and marginal printing and distribution costs of Rs. 40 per copy. Profit function is p(x) = 10 x − 70, where x is the number of newspapers. Plot the graph and find profit for 500 newspapers.

Solution:

P(x) = 10x − 70

Let; x = 7, 14, 21

x = 7; P (7) = 10 × 7 – 70 = 70 – 70 = 0

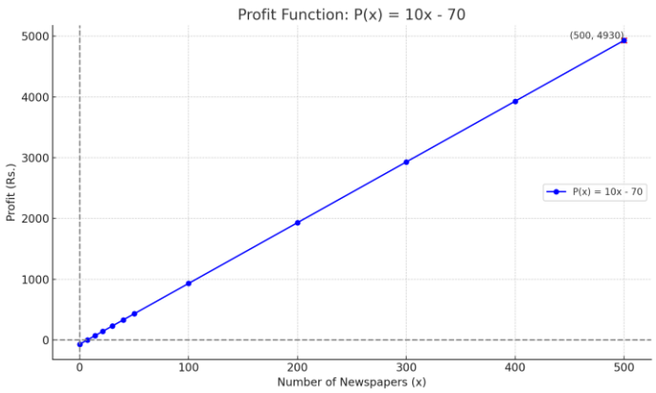
x = 14; P (14) = 10 × 14 – 70 = 140 – 70 = 70

x = 21; P (21) = 10 × 21 – 70 = 210 – 70 = 140

Table:

x	7	14	21
P	0	70	140

Graphical Representation:



Finding Profit for 500 Newspapers:

$P(x) = 10x - 70$

$P(500) = 10 \times 500 - 70$

$P(500) = 5000 - 70$

$P(500) = Rs.\ 4930$

The profit for **500** newspapers is **Rs. 4930**.

Question No. 8

Ali manufactures expensive shirts for sale to a school. Its cost (in rupees) for x shirts is $C(x) = 1500 + 10x + 0.2x^2$, $0 \leq x \leq 150$. Plot the graph and find the cost of 200 shirts.

Solution:

$C(x) = 1500 + 10x + 0.2x^2$

$0 \leq x \leq 150$; x = 0, 1, 2, 3, ….., 150

For x = 0; $C(0) = 1500 + 10(0) + 0.2(0)^2 = 1500 + 0 + 0 = Rs.\ 1500$

For x = 50; $C(50) = 1500 + 10(50) + 0.2(50)^2 = 1500 + 500 + 5000 = Rs.\ 2500$

For x = 100; $C(100) = 1500 + 10(100) + 0.2(100)^2 = 1500 + 1000 + 0.2(10000)$

$C(100) = 1500 + 1000 + 2000 = Rs.\ 4500$

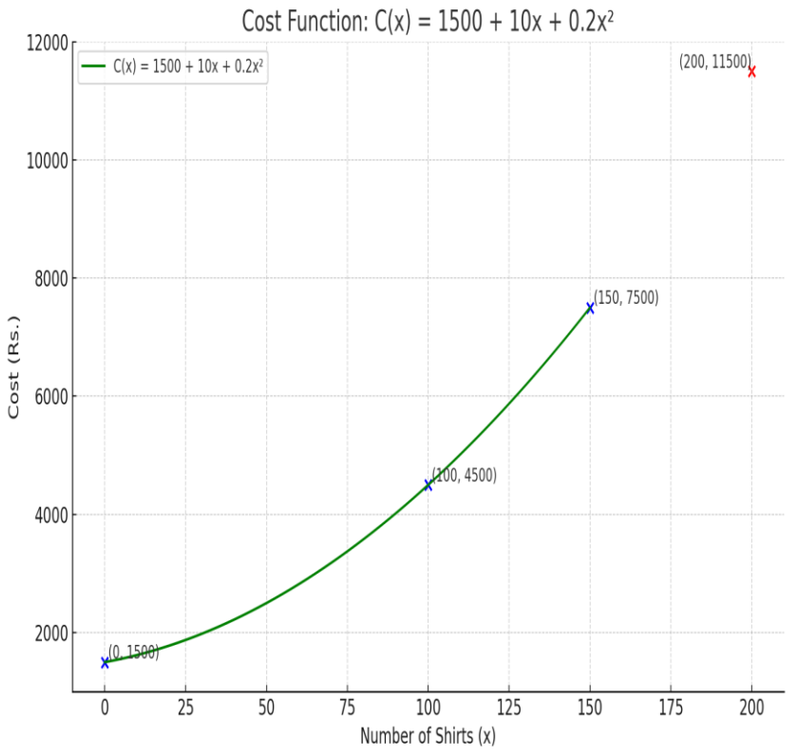
For x = 150; $C(150) = 1500 + 10(150) + 0.2(150)^2 = 1500 + 1500 + 0.2(22500)$

$C(150) = 1500 + 1500 + 4500 = Rs.7500$

Table:

x (shirts)	0	50	100	150
C(x) (Rs.)	1500	2500	4500	7500

Graphical Representation:



Finding cost of 200 shirts:

For x = 200;

C (200) = 1500 + 10(200) + 0.2(200)^2

C (200) = 1500 + 2000 + 8000

C (200) = Rs. 11500

The cost of **200** shirts will be **Rs**. **11500**.