Unit No. 2

Logarithms

Basic Concepts

Scientific Notation:

Scientific notation is a method to express very large or very small numbers in a more manageable form.

Usage: It's commonly used in science, engineering, and mathematics to simplify complex calculations.

Form: A number in scientific notation is written as $a \times 10^n$, where $1 \leq a < 10$ and $n \in Z$ (meaning n is an integer).

Coefficient: The term "a" is called the coefficient or base number.

Rule for the exponent:

- If the number is greater than 1, then n is positive.
- If the number is less than 1, then n is negative.

Remember!

- If the number is greater than 1 then *n* is positive.
- If the number is less than 1 then *n* is negative.

Remember!

Positive exponent: If the exponent (n) is positive, then the decimal point in the coefficient (a) will move to the right when converting back to the standard form of the number.

Negative exponent: If the exponent (n) is negative, then the decimal point in the coefficient (a) will move to the left when converting back to the standard form of the number.

Logarithm!

- ✓ The word "logarithm" is based on two Greek words: "logos" (ratio) and "arithmos" (proportion).
- ✓ John Napier, a Scottish mathematician, introduced the word "logarithm."
- ✓ Logarithms are a way to simplify complex calculations, especially those involving multiplication and division of large numbers.
- ✓ Logarithms remain fundamental in mathematics and have applications in science, finance, and technology.

Logarithm of a Real Number!

The general form of a logarithm is:

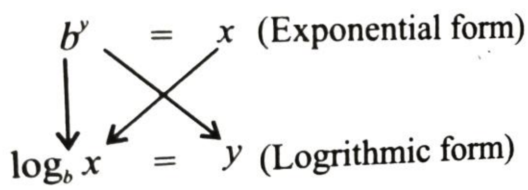
$$\log_b(x) = y$$

Where:

- b is the **base**,
- x is the **result** or the number whose logarithm is being taken,
- y is the **exponent** or the logarithm of x to the base b.

This means that: $b^y = x$

In words, "**the logarithm of x to the base b is y,** means that when b is raised to the power y, it equals x.



The relationship between logarithmic form and exponential form is given: $\log_b(x) = y \Leftrightarrow b^y = x$

where $b > 0$, $x > 0$ and $b \neq 1$

Common Logarithm!

The **common logarithm** is the logarithm with a base of 10. It is written as log10 or simply as log (when no base is mentioned, it is usually assumed to be base 10).

For example:

$10^1 = 10 \Leftrightarrow \log_{10} 10 = 1$

$10^2 = 100 \Leftrightarrow \log_{10} 100 = 2$

$10^{-1} = 10 \Leftrightarrow \log_{10} 0.1 = -1$

$10^{-2} = 100 \Leftrightarrow \log_{10} 0.01 = -2$

History:

English mathematician Henry Briggs extended Napier's work and developed the common logarithm. He also introduced logarithmic tables.

Characteristic and Mantissa of Logarithms:

The logarithm of a number consists of two parts: the **characteristic** and the **mantissa**.

Characteristic:

The characteristic is the integral part of the logarithm. It tells us how big or small the number is.

Remember!

When the characteristic is negative, we write it with a bar ($\overline{\text{char}}$).

Rules for Finding the Characteristic:

(i) For a number greater than 1:

Characteristic = number of digits to the left of the decimal point - 1

For example, in log567

the characteristic = 3 – 1 = 2

(ii) For a number less than 1:

Characteristic = - (number of zeros between the decimal point and the first non-zero digit + 1)

For example, in log0.0123

the characteristic = $-(1 + 1) = -2$ or $\bar{2}$

Mantissa:

The mantissa is the decimal part of the logarithm. It represents the "fractional" component and is always positive.

For example, in log5000=3.698 the mantissa is 0.698.

Remember!

log(Number) = Characteristic + Mantissa

Do you know?

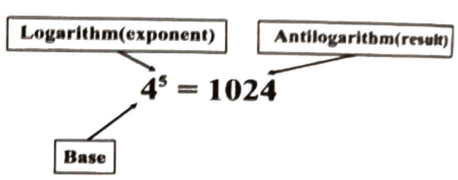
log(0) = undefined

log(1) = 0

$\log_a(a) = 1$

Antilogarithm:

An **antilogarithm** is the inverse operation of a logarithm. An antilogarithm helps to find a number whose logarithmic value is given. If $\log_b x = y \Leftrightarrow b^y = x$ then the process of finding x is called the antilogarithm of y.



Remember!

The word antilogarithm is another word for the number or result. For example, in

$4^3 = 64$, the result 64 is the antilogarithm.

Natural Logarithm:

The **natural logarithm** is the logarithm with base e, where *e* is a mathematical constant approximately equal to 2.71828. It is denoted as ***ln***.

Usage: The natural logarithm is commonly used in mathematics, particularly in calculus, to describe exponential growth, decay, and many other natural phenomena.

For example, ***ln*** $e^2 = 2$, i.e., the logarithm of e^2 to the base e is 2.

History:

Swiss mathematician and physicist Leonhard Euler introduced 'e' for the base of the natural logarithm.

Reference Position!

The place between the first non-zero digit from the left and its next digit is called the **reference position**.

For example, in 1332, the reference position is between 1 and 3 (1∧332). It is represented by mark ∧.

Do you know?

ln (0) = undefined

ln (1) = 0, ***ln*** (e) = 1

Difference between Common Logarithm and Natural Logarithm:

#	Common Logarithm	Natural Logarithm
i	The base of a common logarithm is 10.	The base of a natural logarithm is e.
ii	It is written as $\log_{10}(x)$ or simply log(x) when no base is specified.	It is written as ln(x).
iii	Common logarithms are widely used in everyday calculations, especially in scientific and engineering applications.	Natural logarithms are commonly used in higher-level mathematics, particularly calculus and applications involving growth/decay processes.

Laws of logarithms:

1. Product Law:

The logarithm of the product is the sum of the logarithms of the factors.

$\log_b (xy) = \log_b x + \log_b y$

Proof:

Let

$m = \log_b x$ ... (i) and $n = \log_b y$ ... (ii)

Express (i) and (ii) in exponential form:

$x = b^m$ and $= b^n$

Multiply x *and* y we get:

$$x.y = b^m.b^n$$

$$x.y = b^{m+n}$$

By writing in log form:

$\log_b (xy) = m + n$

Recall m & n:

$\log_b (xy) = \log_b x + \log_b y$

2. Quotient Law:

The logarithm of a quotient is the difference between the logarithms of the numerator and the denominator.

$\mathbf{\log_b (\frac{x}{y}) = \log_b x - \log_b y}$

Proof:

Let

$m = \log_b x$ ... (i) and $n = \log_b y$ ... (ii)

Express (i) and (ii) in exponential form:

$x = b^m$ and $y = b^n$

Divide x & y we get:

$$\frac{x}{y} = \frac{b^m}{b^n}$$

$$\frac{x}{y} = b^{m-n}$$

By writing in log form:

$\log_b (\frac{x}{y}) = m - n$

Recall m & n:

$\log_b (\frac{x}{y}) = \log_b x - \log_b y$

3. Power Law:

The logarithm of a number raised to a power is the product of the power and the logarithm of the base number.

$\mathbf{\log_b(x^n) = n \log_b x}$

Proof:

Let

$m = \log_b x$ ... (i)

Its exponential form is:

$$x = b^m$$

Raise both sides to the power n:

$x^n = (b^m)^n$

$x^n = b^{nm}$

Its logarithmic form is:

$\log_b (x^n) = nm$

Recall m:

$\log_b (x^n) = n \cdot \log_b x$

4. Change of Base Law:

This law allows changing the base of a logarithm from "b" to any other base "a."

$\boldsymbol{\log_b x = \frac{\log_b x}{\log_a b}}$

Proof:

Let

$m = \log_b x$ ... (i)

Its exponential form is:

$b^m = x$

Taking $\log_a$ on both sides, we get:

$\log_a (b^m) = \log_a x$

$m \log_a (b) = \log_a x$

$m = \frac{\log_a x}{\log_a b}$

Recall m:

$\log_b x = \frac{\log_a x}{\log_a b}$