Unit No. 9

Similar Figures

Review Exercise No. 9

Question No.1

Four options are given against each statement. Encircle the correct one.

(i) If two polygons are similar, then:

(a) their corresponding angles are equal.

(b) their areas are equal.

(c) their volumes are equal.

(d) their corresponding sides are equal.

(ii) The ratio of the areas of two similar polygons is:

(a) equal to the ratio of their perimeters.

(b) equal to the square of the ratio of their corresponding sides.

(c) equal to the cube of the ratio of their corresponding sides.

(d) equal to the sum of their corresponding sides.

(iii) If the volume of two similar solids is 125 cm^3 and 27 cm^3, the ratio of their corresponding heights is ----------.

(a) 3:5

(b) 5:3

(c) 25:9

(d) 9:25

(iv) The exterior angle of regular pentagon is: (a) 40°

(b) 45°

(c) 60°

(d) 72°

(v) A parallelogram has an area of 64 cm^2 and a similar parallelogram has an area of 144 cm^2. If a side of the smaller parallelogram is 8 cm, the corresponding side of the larger parallelogram is:

(a) 10 cm

(b) 12 cm

(c) 18 cm

(d) 16 cm

(vi) The total number of diagonals in a polygon with 9 sides is:

(a) 18

(b) 21

(c) 25

(d) 27

(vii) Two spheres are similar, and their radii are in the ratio 4:5. If the surface area of the larger sphere is 500 πcm^2, what is the surface area of the smaller sphere?

(a) 256 πcm^2

(b) 320 πcm^2

(c) 400 πcm^2

(d) 405 πcm^2

(viii) A regular polygon has an exterior angle of 30°. How many diagonals does the Polygon have?

(a) 54

(b) 72

(c) 108

(d) 90

(ix) In a regular hexagon, the ratio of the length of a diagonal to the side length is:

(a) 3:1

(b) 2 : 1

(c) 3 : 2

(d) 2 : 3

(x) A regular polygon has an interior angle of 165°. How many sides does it have?

(a) 15

(b) 16

(c) 20

(d) 24

Question No.2

If the sum of the interior angles of a polygon is 1080°, how many sides does the polygon has?

Given:

Sum of interior angles of polygon = 1080°

To Find:

No. of sides of polygon = ?

Formula:

Sum of interior angles of polygon = (n − 2) × 180°

Solution:

Sum of interior angles of polygon = (n − 2) × 180°

By putting value:

1080° = (n − 2) × 180°

(n − 2) = 1080 / 180

(n − 2) = 6

n = 6 + 2

n = 8 sides

So, polygon has **8 sides**; i.e. **Octagon**.

Question No.3

Two similar bottles are such that one is twice as high as the other. What is the ratio of their surface areas and their capacities?

Given:

One bottle is twice as high as the other.

If one bottle is twice as high as other, the ratio of their heights be $h_1 / h_2 = 2 / 1$

To Find:

Ratio of their surface areas = ?

Ratio of their capacities = ?

Let;

A_1 and A_2 be the surface areas of two bottles.

V_1 and V_2 be the capacities of two bottles.

Solution:

Finding Ratio of their surface areas:

$$A_1 / A_2 = \left(\frac{2}{1}\right)^2 = 4 / 1$$

$A_1 : A_2 = 4 : 1$

Finding Ratio of their capacities:

$$V_1 / V_2 = \left(\frac{2}{1}\right)^3 = 8 / 1$$

$V_1 : V_2 = 8 : 1$

Question No.4

Each dimension of a model car is 1 / 10 of the corresponding car dimensions. Find the ratio of:

(a) the areas of their windscreens

(b) the capacities of their boots

(c) the widths of the cars

(d) the number of wheels they have.

Given:

Dimension of model car is 1 / 10 of the corresponding car dimension.

This means the ratio of a linear dimension of the model car (*lm*) to the actual car (*la*) is; *lm* / *la* =1 / 10

To Find:

(a) the areas of their windscreens = ?

(b) the capacities of their boots = ?

(c) the widths of the cars = ?

(d) the number of wheels they have = ?

Solution:

(a) The areas of their windscreens:

For similar figures, the ratio of their areas is the square of the ratio of their corresponding linear dimensions.

Let A_m be the area of the model car's windscreen and A_a be the area of the actual car's windscreen. $A_m / A_a=(1 / 10)^2$

$= 1 / 100$

So, the ratio of the areas of their windscreens is 1 : 100

(b) The capacities of their boots:

Capacities refer to volumes. For similar figures, the ratio of their volumes is the cube of the ratio of their corresponding linear dimensions. Let V_m be the capacity of the model car's boot and V_a be the capacity of the actual car's boot.

$V_m / V_a = (1 / 10)^3$

$= 1 / 1000$

So, the ratio of the capacities of their boots is 1 : 1000

(c) The widths of the cars:

Width is a linear dimension. Let Wm be the width of the model car and W_a be the width of the actual car. The ratio of their widths is given directly by the scale factor:

$W_m / W_a =1 / 10$

So, the ratio between widths of cars is 1 : 10

(d) The number of wheels they have:

The number of wheels is a count, not a dimension that scales with similarity. Both a model car and an actual car (typically) have 4 wheels.

So, the ratio of the number of wheels they have = 4 : 4

So, the ratio of the number of wheels is 1 : 1

Question No.5

Three similar jugs have heights 8 cm, 12 cm and 16 cm. If the smallest jug holds 1 / 2 litre, find the capacities of the other two.

Given:

Heights of three similar jugs are $h_1 = 8$ cm, $h_2 = 12$ cm, and $h_3 = 16$ cm

Smallest jug (8 cm height) holds $V_1 = 1 / 2$ litre

To Find:

Capacities of the other two jugs = ?

Solution:

Capacity of the second jug (12 cm height):

Let V_2 be the capacity of the jug with height 12 cm.

The ratio of volumes of similar objects is the cube of the ratio of their corresponding linear dimensions (heights).

V1 / V2 = (H1 / H2)3

1 / 2V2 = (8 / 12)3

$1 / 2V_2 = (2 / 3)^3$

$1 / 2V_2 = 8 / 27$

$1 \times 27 = 8 \times 2\ V_2$

$V_2 = 27 / 16$

$V_2 = 1.6875\ l$

Capacity of the third jug (16 cm height):

$V_2 / V_3 = (H_2 / H_3)^3$

$1.6875 / V_3 = (3 / 4)^3$

$1.6875 / V_3 = 27 / 64$

$1.6875 \times 64 = 27 \times V_3$

$V_3 = 108 / 27$

$V_3 = 4\ l$

The capacities of the all three jugs : 1/2 *l* : 1.6875 *l* : 4 *l*

Question No.6

Three similar drinking glasses have heights 7.5 cm, 9 cm and 10.5 cm. If the tallest glass holds 343 millilitres, find the capacities of the other two.

Given:

Heights of three similar glasses are $h_1 = 7.5$ cm, $h_2 = 9$ cm, $h_3 = 10.5$ cm

Tallest glass ($h_3 = 10.5$ cm) holds $V_3 = 343$ millilitres

To Find:

The capacities of the other two glasses = ?

Solution:

Capacity of the first glass (7.5 cm height):

Let V_1 be the capacity of the glass with height 7.5 cm

$V_1 / V_3 = (H_1 / H_3)^3$

$V_1 / 343 = (7.5 / 10.5)^3$

$V_1 / 343 = (421.875 / 1157.625)$

$V_1 = (421.875 / 1157.625) \times 343$

$V_1 = 125\ ml$

Capacity of the second glass (9 cm height):

Let V_2 be the capacity of the glass with height 9 cm.

$V_1 / V_2 = (H_1 / H_2)^3$

$125 / V_2 = (7.5 / 9)^3$

$125 / V_2 = 421.875 / 729$

$125 \times 729 = 421.875 \times V_2$

$V_2 = 91125 / 421.875$

$V_2 = 216\ ml$

Question No.7

A toy manufacturer produces model cars which are similar in every way to the actual cars. If the ratio of the door area of the model to the door area of the car is 1 cm to 2500 cm, find:

(a) the ratio of their lengths

(b) the ratio of the capacities of their petrol tanks

(c) the width of the model, if the actual car is 150 cm wide

(d) the area of the rear window of the actual car if the area of the rear window of the model is 3 cm^2.

Given:

Ratio of door area of model car (A_m) to door area of actual car (A_a) is

$A_m : A_a = 1 : 2500$

Solution:

(a) The ratio of their lengths:

The ratio of areas is the square of the ratio of corresponding lengths (l).

$A_m / A_a = (l_m / l_a)^2$

$1 / 2500 = (l_m / l_a)^2$

$(l_m / l_a)^2 = (1 / 50)^2$

Taking the square root of both sides:

$l_m / l_a = 1 / 50$

So, the ratio of their lengths is 1 : 50.

(b) The ratio of the capacities of their petrol tanks:

Capacities refer to volumes. The ratio of volumes (Va and Vm) is the cube of the ratio of corresponding lengths.

$V_m / V_a = (l_m / l_a)^3$

$V_m / V_a = (1 / 50)^3$

$V_m / V_a = 1 / 125000$

So, the ratio of the capacities of their petrol tanks is 1: 125000.

(c) The width of the model, if the actual car is 150 cm wide:

Let W_m be the width of the model car and W_a be the width of the actual car.

Given; $W_a = 150$ cm

$W_m = 1 / 50 \times W_a = 1 / 50 \times 150$ cm $= 3$ cm

The width of the model car is 3 cm.

(d) The area of rear window of actual car:

Given:

The area of the rear window of the model car = 3 cm^2

According statement of the question;

1 cm^2 represents 2500 cm^2

So, 3 cm^2 represents 3×2500 $cm^2 = 7500$ cm^2

Question No.8

The ratio of the areas of two similar labels on two similar jars of coffee is 144 : 169. Find the ratio of:

(a) the heights of the two jars

(b) their capacities.

Given:

Ratio of area of two similar labels on two similar jars of coffee;

$A_1 : A_2 = 144 : 169$

Solution:

(a) The ratio of heights of the two jars:

$A_1 / A_2 = (h_1 / h_2)^2$

$144 / 169 = (h_1 / h_2)^2$

$(h_1 / h_2)^2 = (12 / 13)^2$

$h_1 / h_2 = 12 / 13$

So, required ration: 12 : 13

(b) The ratio of their capacities:

$V_1 / V_2 = (h_1 / h_2)^3$

$V_1 / V_2 = (12 / 13)^3$

$V_1 / V_2 = 1728 / 2197$

So, the ratio of their capacities is 1728 : 2197

Question No.9

A tessellation of tiles on a floor has been made using a repeating pattern of a regular hexagon, six squares and six equilateral triangles. Find the total area of a single pattern with side length 21 metre of each polygon.

Given:

The tessellation has a repeating pattern of:

- **1 regular hexagon**
- **6 squares**
- **6 equilateral triangles**

Side length of each polygon = 1/2 metre

To Find:

Total Area of single pattern = ?

Solution:

1. Area of Hexagon:

Formula:

$Area_{hex} = (3\sqrt{3}) / 2 \times side^2$

By putting value:

$Area_{hex} = (3\sqrt{3}) / 2 \times (1/2)^2$

$Area_{hex} = (3\sqrt{3}) / 2 \times 1/4$

$Area_{hex} = (3\sqrt{3}) / 8\ m^2$

2. Area of Equilateral Triangle:

To find height by using Pythagoras theorem:

$$\text{Height} = \sqrt{(\frac{1}{2})^2 - (\frac{1}{4})^2} = \sqrt{\frac{1}{4} - \frac{1}{16}} = \sqrt{\frac{4-1}{16}} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}\ \text{m}$$

$Area_{triangle} = \frac{1}{2} \times base \times height$

$\text{Area}_{\text{triangle}} = \frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{4}$

$\text{Area}_{\text{triangle}} = \frac{\sqrt{3}}{16}\ \text{m}^2$

$\text{Area of 6 triangles} = 6 \times \frac{\sqrt{3}}{16}\ \text{m}^2$

$\text{Area of 6 triangles} = \frac{3\sqrt{3}}{8}\ \text{m}^2$

3. Area of Squares:

$\text{Area}_{\text{square}} = l^2$

$\text{Area}_{\text{square}} = (\frac{1}{2})^2 = \frac{1}{4}\ \text{m}^2$

$\text{Area of 6 squares} = 6 \times \frac{1}{4}\ \text{m}^2$

$\text{Area of 6 squares} = \frac{3}{2}\ \text{m}^2$

$\text{Total Area} = \text{Area}_{\text{hex}} + \text{Area of 6 triangles} + \text{Area of 6 squares}$

$\text{Total Area} = \frac{3\sqrt{3}}{8}\ \text{m}^2 + \frac{3\sqrt{3}}{8}\ \text{m}^2 + \frac{3}{2}\ \text{m}^2$

$\text{Total Area} = 0.6495\ \text{m}^2 + 0.6495\ \text{m}^2 + 1.5\ \text{m}^2$

$\text{Total Area} = 2.80\ \text{m}^2$