Unit No. 7

Coordinate Geometry

Exercise No. 7.2

Question No. 1

Find the slope and inclination of the line joining the points:

(i) (-2, 4); (5, 11)

Data:

P = (-2, 4)

Q = (5, 11)

Solution:

Let; $x_1 = -2$; $y_1 = 4$

$x_2 = 5$; $y_2 = 11$

Formula of Slope:

Slope of line PQ = m = $\frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{11-4}{5-2} = \frac{7}{7} = 1$$

Inclination:

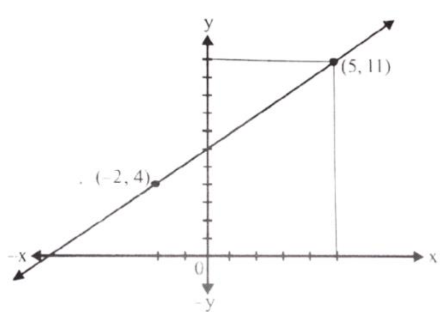
$\tan(\alpha) = m$

$\tan(\alpha) = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45°$

Pictorial Form:



(ii) (3, -2); (2, 7)

P = (3, -2)

Q = (2, 7)

Solution:

Let; $x_1 = 3$; $y_1 = -2$

$x_2 = 2$; $y_2 = 7$

Formula of Slope:

Slope of line PQ = m = $\frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{7--2}{2-3} = \frac{9}{-1} = -9$$

Inclination:

$\tan(\alpha) = m$

$\tan(\alpha) = -9$

$\alpha = \tan^{-1}(-9)$

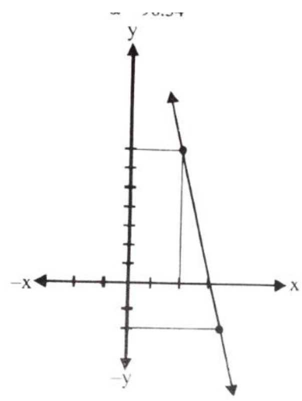
$\alpha = 180° - 83.67°$

$\alpha = 96.33°$

$\alpha = 96°(0.33 \times 60)'$

$\alpha = 96°20'$

Pictorial Form:



(iii) (4, 6); (4, 8)

P = (4, 6)

Q = (4, 8)

Solution:

Let; $x_1 = 4$; $y_1 = 6$

$x_2 = 4$; $y_2 = 8$

Formula of Slope:

Slope of line PQ = m = $\frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{8-6}{4-4} = \frac{2}{0} = \infty$$

Inclination:

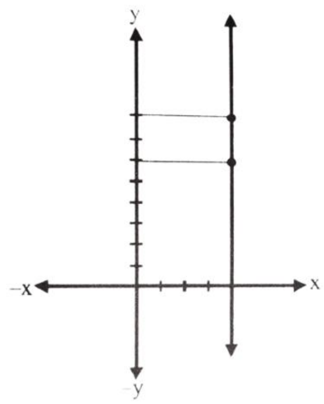
$\tan(\alpha) = m$

$\tan(\alpha) = -9$

$\alpha = \tan^{-1}(\infty)$

$\alpha = 90°$

Pictorial Form:



Question No. 2

By means of slopes, show that the following points lie on the same line:

(i) A (-1, -3); B (1, 5); C (2, 9)

Data:

A = (-1, -3)

B = (1, 5)

C = (2, 9)

Solution:

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

- Slope of AB (mAB):

$$mAB = \frac{5+3}{1+1} = \frac{8}{2} = 4$$

- Slope of BC (mBC):

$$mBC = \frac{9-5}{2-1} = \frac{4}{1} = 4$$

Since slope of mAB = mBC = 4,

So, the points A, B, and C lie on the same line.

(ii) P (4, -5); Q (7, 5); R (10, 15)

Data:

P = (4, -5)

Q = (7, 5)

R = (10, 15)

Solution:

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

- Slope of PQ (mPQ):

$$mPQ = \frac{5+5}{7-4} = \frac{10}{3} = 3.33$$

- Slope of QR (mQR):

$$mQR = \frac{15-5}{10-7} = \frac{10}{3} = 3.33$$

Since slope of mPQ = mQR = 3.33,

So, the points P, Q, and R lie on the same line.

(iii) L (-4, 6); M (3, 8); N (10, 10)

Data:

L = (-4, 6)

M = (3, 8)

N = (10, 10)

Solution:

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

- Slope of LM (mLM):

$$mLM = \frac{8-6}{3--4} = \frac{2}{7} = 0.29$$

- Slope of MN (mMN):

$$mMN = \frac{10-8}{10-3} = \frac{2}{7} = 0.29$$

Since slope of mLM = mMN = 0.29,

So, the points L, M, and N lie on the same line.

(iv) X (a, 2b); Y (c, a+b); Z (2c-a, 2a)

Data:

X = (a, 2b)

Y = (c, a+b)

Z = (2c-a, 2a)

Solution:

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

- Slope of XY (mXY):

$$mXY = \frac{a+b-2b}{c-a} = \frac{a-b}{c-a}$$

- Slope of YZ (mYZ):

$$mYZ = \frac{2a-(a+b)}{2c-a-c} = \frac{2a-a-b}{c-a} = \frac{a-b}{c-a}$$

Since slope of mXY = mYZ = $\frac{a-b}{c-a}$, So, the points X, Y, and Z lie on the same line.

Question No. 3

Find k so that the line joining A(7, 3); B(k, -6) and the line joining C(-4, 5); D(-6, 4) are:

(i) parallel

(ii) perpendicular

Data:

A = (7, 3)

B = (k, -6)

C = (-4, 5)

D = (-6, 4)

Solution:

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

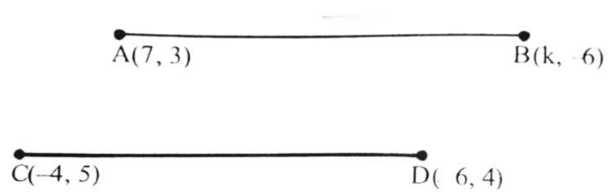
- Slope of AB (m_1):

$$m_1 = \frac{-6-3}{k-7} = \frac{-9}{k-7}$$

- Slope of CD (m_2):

$$m_2 = \frac{4-5}{-6+4} = \frac{-1}{-2} = \frac{1}{2}$$

Pictorial Representation:



First Condition:

As AB || CD, so that are equal i.e;

$$\frac{-9}{k-7} = \frac{1}{2}$$

$$-9(2) = 1(k-7)$$

$$-18 = k - 7$$

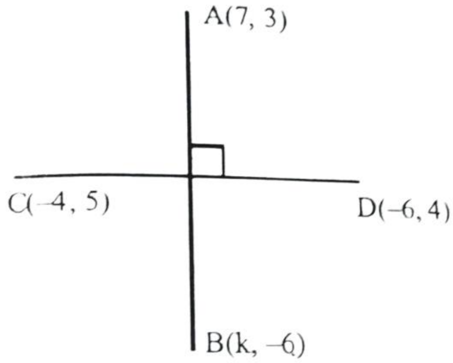
$$-18 + 7 = k$$

$$k = -11$$

Second Condition:

Since AB ⊥ CD, so product of these slopes is equal to -1;

Pictorial Representation:



$$\frac{-9}{k-7} \times \frac{1}{2} = -1$$

$$\frac{-9}{2k-14} = -1$$

$$-9 = -1(2k-14)$$

$$-9 = -2k + 14$$

$$2k = 9 + 14$$

$$k = \frac{23}{2}$$

Question No. 4

Using slopes, show that the triangle with its vertices A(6, 1) B(2, 7) and C(-6, -7) is a right triangle.

Data:

A = (6, 1)

B = (2, 7)

C = (-6, -7)

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

Solution:

- Slope of AB (m_1):

  $m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$

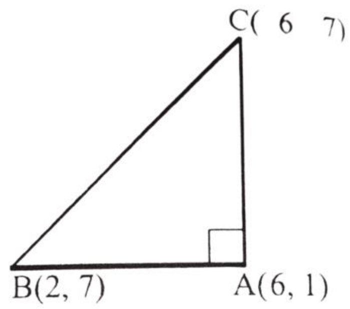
- Slope of CD (m_2):

  $m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$

- Slope of AC (m_3):

  $m_3 = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$

Pictorial Representation:



Given Condition:

$$(m_1) \times (m_3) = (\frac{-3}{2}) \times (\frac{2}{3})$$

$$(m_1) \times (m_3) = -1$$

So, side AB ⊥ AC, ΔABC is Right Triangle.

Question No. 5

Two pairs of points are given. Find whether the two lines determined by these points are:

(i) parallel

(ii) perpendicular

(iii) none

(a) (1, 2), (2, 4) and (4, 1), (-8, 2)

Data:

A = (1, 2)

B = (2, 4)

C = (4, 1)

D = (-8, 2)

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

Solution:

- Slope of AB (m_1):

$$m_1 = \frac{4--2}{2-1} = \frac{6}{1} = 6$$

- Slope of CD (m_2):

$$m_2 = \frac{2-1}{-8-4} = \frac{1}{-12}$$

(i). As $m_1 \neq m_2$, so AB is not parallel to CD.

(ii). Since $m_1 \times m_2 = (6)(\frac{1}{-12}) = \frac{1}{-2} \neq -1$,

So AB is not perpendicular to CD.

(iii). AB and CD are neither parallel nor perpendicular.

(b) (-3, 4), (6, 2) and (4, 5), (-2, -7)

Data:

A = (-3, 4)

B = (6, 2)

C = (4, 5)

D = (-2, -7)

Formula of Slope:

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

Solution:

- Slope of AB (m_1):

$$m_1 = \frac{2-4}{6--3} = \frac{-2}{9}$$

- Slope of CD (m_2):

$$m_2 = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$$

(i). As $m_1 \neq m_2$, so AB is not parallel to CD.

(ii). Since $m_1 \times m_2 = (\frac{-2}{9})(2) = \frac{-4}{9} \neq -1$,

So AB is not perpendicular to CD.

(iii). AB and CD are neither parallel nor perpendicular.

Question No. 6

Find an equation of:

(a) the horizontal line through (7, -9)

Solution:

The horizontal line through (7, -9)

Here $(x_1, y_1) = (7, -9)$

As line is parallel to x-axis;

So slope m = 0

Equation of line is;

$$y - y_1 = m(x - x_1)$$

$$y - (-9) = 0(x - 7)$$

$$y + 9 = 0$$

(b) the vertical line through (-5, 3)

Solution:

The vertical line through (-5, 3)

Here $(x_1, y_1) = (-5, 3)$

As line is vertical i.e parallel to y-axis, so

$m = \infty$ or $\quad m = \frac{1}{0}$

Equation of line is;

$$y - y_1 = m(x - x_1)$$

$$y - (3) = \frac{1}{0}(x - -5)$$

$$0(y - 3) = x + 5$$

$$x + 5 = 0$$

(c) through A(-6, 5) having slope 7

Solution:

Here $(x_1, y_1) = (-6, 5)$

$m = 7$

Equation of line is;

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 7\,(x - -6)$$

$$y - 5 = 7x + 42$$

$$y = 7x + 42 + 5$$

$$y = 7x + 47$$

$$7x - y + 47 = 0$$

(d) through (8, -3) having slope 0

Solution:

Here $(x_1, y_1) = (8, -3)$

$m = 0$

Equation of line is;

$$y - y_1 = m(x - x_1)$$

$$y - -3 = 0\,(x - 8)$$

$$y + 3 = 0$$

(e) through (-8, 5) having slope undefined

Solution:

Here $(x_1, y_1) = (-8, 5)$

$m = \infty = \frac{1}{0}$

Equation of line is;

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{0}(x - -8)$$

$0(y - 5) = 1(x - -8)$

$0 = x + 8$

$x + 8 = 0$

(f) through (-5, -3) and (9, -1)

Solution:

Here $(x_1, y_1) = (-5, -3)$

$(x_2, y_2) = (9, -1)$

Slope of line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-1+3}{9--5} = \frac{2}{14} = \frac{1}{7}$

Equation of line is;

$y - y_1 = m(x - x_1)$

$y - -3 = \frac{1}{7}(x - -5)$

$7(y + 3) = 1\ (x + 5)$

$7y + 21 = x + 5$

$0 = x + 5 - 7y - 21$

$x - 7y - 16 = 0$

(g) y-intercept: -7 and slope: -5

Solution:

Slope intercept form:

$y = mx + c$

Here $m = -5$, $c = -7$

Equation of line is;

$y = -5x - 7$

$5x + y + 7 = 0$

(h) x-intercept: -3 and y-intercept: 4

Solution:

Here $a = -3$ and $b = 4$

Two intercepts form:

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-3} + \frac{y}{4} = 1$

$\frac{4x-3y}{-12} = 1$

$-4x + 3y = 1(12)$

$-4x + 3y = 12$

$-4x + 3y - 12 = 0$

$4x - 3y + 12 = 0$

(i) x-intercept: -9 and slope: -4

Solution:

Line intercept x-axis at x = -9

So, $(x_1, y_1) = (-9, 0)$, $m = -4$

Equation of line is;

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x - -9)$$

$$y = -4x - 36$$

$$y + 4x + 36 = 0$$

$$4x + y + 36 = 0$$

Question No. 7

Find an equation of the perpendicular bisector of the segment joining the points A(3, 5) and B(9, 8).

Solution:

Formula of Slope:

Slope of line AB = $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_1 = \frac{8-5}{9-3} = \frac{3}{6} = \frac{1}{2}$$

Slope of required line = $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-1}{m_1} = \frac{-1}{\frac{1}{2}} = -2$$

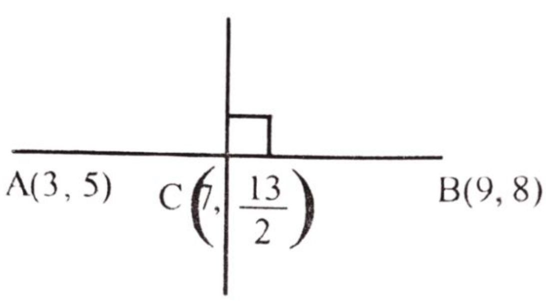
(Since lines are perpendicular),

Using Mid-Point Formula:

$$Mid - Point(AB) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{3+9}{2}, \frac{5+8}{2}\right) = = \left(\frac{12}{2}, \frac{13}{2}\right) = \left(6, \frac{13}{2}\right)$$

Pictorial Representation:



Equation of line passing through $\left(6, \frac{13}{2}\right)$ and slope -2 is ;

$y - y_1 = m(x - x_1)$

$y - \frac{13}{2} = -2(x - 6)$

$\frac{2y - 13}{2} = -2x + 12$

$2y - 13 = 2(-2x + 12)$

$2y - 13 = -4x + 24$

$2y - 13 + 4x - 24 = 0$

$4x + 2y - 37 = 0$

Question No. 8

Find an equation of the line through (-4, -6) and perpendicular to a line having slope $-\frac{3}{2}$.

Solution:

Slope of given line = $-\frac{3}{2}$

Slope of required line = m = $\frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-1}{m_1} = \frac{-1}{\frac{-3}{2}} = \frac{2}{3}$$

Equation of the line passing through (-4, -6) having slope $-\frac{3}{2}$;

$y - y_1 = m(x - x_1)$

$y + 6 = \frac{2}{3}(x + 4)$

$3(y + 6) = 2(x + 4)$

$3y + 18 = 2x + 8$

$3y + 18 - 2x - 8 = 0$

$-2x + 3y + 10 = 0$

$2x - 3y - 10 = 0$

Question No. 9

Find an equation of the line through (11, -5) and parallel to a line with slope -24.

Solution:

Slope of given line = -24

Slope of required line = m = -24

(As lines are parallel);

Equation of the line passing through (11, -5) having slope -24;

$y - y_1 = m(x - x_1)$

$y + 5 = -24(x - 11)$

$y + 5 = -24x + 264$

24x - y – 264 + 5 = 0

24x - y – 259 = 0

Question No. 10

Convert each of the following equations into:

(i) slope-intercept form

(ii) two-intercept form

(iii) normal form

(a) 2x - 4y + 11 = 0

Solution:

(i) slope-intercept form: (y = mx + c)

2x - 4y + 11 = 0

2x + 11 = 4y

4y = 2x + 11

y = $\frac{2x}{4} + \frac{11}{4}$

y = $\frac{x}{2} + \frac{11}{4}$

which is slope intercep form with:

m = $\frac{1}{2}$, c = $\frac{11}{4}$

(ii) two-intercept form: ($\frac{x}{a} + \frac{y}{b} = 1$)

Solution:

2x - 4y + 11 = 0

2x - 4y = -11

Divide both sides by -11:

$$\frac{2x}{-11} - \frac{4y}{-11} = \frac{-11}{-11}$$

$$\frac{-2x}{11} + \frac{4y}{11} = 1$$

$$\frac{x}{\frac{-11}{2}} + \frac{y}{\frac{11}{4}} = 1$$

Which is two intercept form with:

$a = \frac{-11}{2}$; $b = \frac{11}{2}$

(iii) normal form: (x cos α + y sin α = p)

Solution:

2x - 4y + 11 = 0

-2x + 4y = 11

Divide both sides by $\sqrt{(-2)^2 + (4)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20} = \mathbf{2\sqrt{5}}$

$$\frac{-2}{2\sqrt{5}}x + \frac{4}{2\sqrt{5}}y = \frac{11}{2\sqrt{5}}$$

$$\frac{-1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = \frac{11}{2\sqrt{5}}$$

Which is normal form with:

$\cos\alpha = \frac{-1}{\sqrt{5}}$; $p = \frac{11}{2\sqrt{5}}$

$\alpha = cos^{-1}\frac{-1}{\sqrt{5}}$ = 116.57

x cos α + y sin α = p

x cos (116.57) + y sin (116.57) = $\frac{11}{2\sqrt{5}}$

Length of perpendicular from (0, 0) to line 2x - 4y + 11 = 0 is $\frac{11}{2\sqrt{5}}$.

(b) 4x + 7y - 2 = 0

(i) slope-intercept form: (y = mx + c)

Solution:

4x + 7y - 2 = 0

7y = -4x + 2

y = $\frac{-4x}{7} + \frac{2}{7}$

which is slope intercep form with:

m = $\frac{-4}{7}$, c = $\frac{2}{7}$

(ii) two-intercept form: ($\frac{x}{a} + \frac{y}{b} = 1$)

Solution:

4x + 7y - 2 = 0

4x + 7y = 2

Divide both sides by 2:

$$\frac{4x}{2} + \frac{7y}{2} = \frac{2}{2}$$

$$2x + \frac{7y}{2} = 1$$

$$\frac{x}{\frac{1}{2}} + \frac{y}{\frac{2}{7}} = 1$$

Which is two intercept form with:

$$a = \frac{1}{2} \qquad ; \qquad b = \frac{2}{7}$$

(iii) normal form: (x cos α + y sin α = p)

Solution:

4x + 7y - 2 = 0

4x + 7y = 2

Divide both sides by $\sqrt{(4)^2 + (7)^2}$

$$= \sqrt{16 + 49}$$

$$= \sqrt{65}$$

$$\frac{4}{\sqrt{65}}x + \frac{7}{\sqrt{65}}y = \frac{2}{\sqrt{65}}$$

Which is normal form with:

$\cos\alpha = \frac{4}{\sqrt{65}}$; $p = \frac{2}{\sqrt{65}}$

$\alpha = cos^{-1}\frac{4}{\sqrt{65}} = 60.26$

x cos α + y sin α = p

x cos (60.26) + y sin (60.26) = $\frac{2}{\sqrt{65}}$

Length of perpendicular from (0, 0) to line 4x + 7y -2 = 0 is $\frac{2}{\sqrt{65}}$.

(c) 15y - 8x + 3 = 0

(i) slope-intercept form: (y = mx + c)

Solution:

15y - 8x + 3 = 0

15y = 8x - 3

$y = \frac{8x}{15} - \frac{3}{15}$

$y = \frac{8x}{15} - \frac{1}{5}$

which is slope intercep form with:

$m = \frac{8}{15} c = \frac{-1}{5}$

(ii) two-intercept form: ($\frac{x}{a} + \frac{y}{b} = 1$)

Solution:

15y - 8x + 3 = 0

-8x + 15y = -3

Divide both sides by -3:

$$\frac{-8x}{-3} + \frac{15y}{-3} = \frac{-3}{-3}$$

$$\frac{8x}{3} + \frac{5y}{-1} = 1$$

$$\frac{x}{\frac{3}{8}} + \frac{y}{\frac{-1}{5}} = 1$$

Which is two intercept form with:

$a = \frac{3}{8}$; $b = \frac{-1}{5}$

(iii) normal form: (x cos α + y sin α = p)

Solution:

15y - 8x + 3 = 0

-8x + 15y = -3

8x - 15y = 3

Divide both sides by $\sqrt{(8)^2 + (-15)^2}$

= $\sqrt{64 + 225}$

= $\sqrt{289} = \mathbf{17}$

$$\frac{8}{17}x + \frac{-15}{17}y = \frac{3}{17}$$

Which is normal form with:

$\cos \alpha = \frac{8}{17}$; $p = \frac{3}{17}$

$\alpha = cos^{-1}\frac{8}{17}$ = 61.93

As α lies in quadrent IV, So;

α = 360 – 61.93 = 298.0 7 °

x cos α + y sin α = p

x cos (298.0 7) + y sin (298.0 7) = $\frac{3}{17}$

Length of perpendicular from (0, 0) to line 15y - 8x + 3 = 0 is $\frac{3}{17}$.

Question No. 11

In each of the following, check whether the two lines are:

(i) parallel

(ii) perpendicular

(iii) neither parallel nor perpendicular

(a) 2x + y - 3 = 0 ; 4x + 2y + 5 = 0

Solution:

l_1 : 2x + y - 3 = 0

l_2 : 4x + 2y + 5 = 0

Slope of 1st line $l_1 = m_1 = \frac{-a}{b}$

$$m_1 = \frac{-2}{1} = -2$$

Slope of 1st line $l_2 = m_2 = \frac{-a}{b}$

$$m_2 = \frac{-4}{2} = -2$$

Since $m_1 = m_2 = -2$

So, given lines are parallel.

(b) 3y = 2x + 5 ; 3x + 2y - 8 = 0

Solution:

l_1 : 3y = 2x + 5 = 2x -3y + 5 = 0

l_2 : 3x + 2y - 8 = 0

Slope of 1st line $l_1 = m_1 = \frac{-a}{b}$

$m_1 = \frac{-2}{-3} = \frac{2}{3}$

Slope of 1st line $l_2 = m_2 = \frac{-a}{b}$

$m_2 = \frac{-3}{2}$

Now;

$$m_1 \times m_2 = \frac{2}{3} \times \frac{-3}{2} = -1$$

So, given lines are perpendiculr.

(c) 4y + 2x - 1 = 0 ; x - 2y - 7 = 0

Solution:

l_1 : 4y + 2x - 1 = 0;   2x + 4y - 1 = 0

l_2 : x - 2y - 7 = 0

Slope of 1st line $l_1 = m_1 = \frac{-a}{b}$

$$m_1 = \frac{-2}{4} = \frac{-1}{2}$$

Slope of 1st line $l_2 = m_2 = \frac{-a}{b}$

$$m_2 = \frac{-1}{-2} = \frac{1}{2}$$

Now;

$$m_1 = \frac{-1}{2} \neq m_2 = \frac{1}{2}$$

And;

$$m_1 \times m_2 = -1$$

$\frac{-1}{2} \times \frac{1}{2} \neq -1$; $\frac{-1}{4} \neq -1$

Since $m_1 \neq m_2$ and $m_1 \times m_2 \neq -1$

So, given lines are neither parallel nor perpendicular.

Question No. 12

Find an equation of the line through (-4, 7) and parallel to the line 2x - 7y + 4 = 0.

Solution:

2x - 7y + 4 = 0

Slope of line = m = $-\frac{2}{-7} = \frac{2}{7}$

As required line is parallel to the given line:

So slope of required line = $\frac{2}{7}$

Equation of line passing through (-4, 7) and having slope $\frac{2}{7}$;

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{2}{7}(x - (-4))$

7(y – 7) = 2(x + 4)

7y – 49 = 2x + 8

– 49 - 8 = 2x – 7y

2x – 7y = -57

2x – 7y + 57 = 0

Question No. 13

Find an equation of the line through (5, -8) and perpendicular to the join of A (-15, -8), B (10, 7).

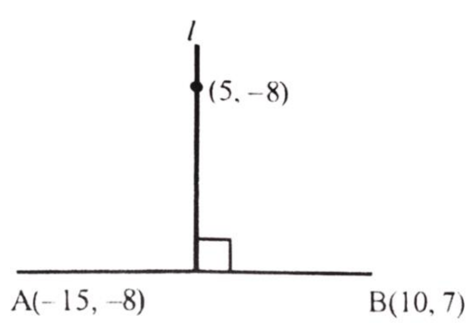
Solution:

Slope of line AB = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7+8}{10+15}$

$= \frac{15}{25} = \frac{3}{5}$

Pictorial Form:



As required line is perpendicular to the given line AB.

So,

Slope of required line = m' = $\frac{-1}{\frac{3}{5}} = \frac{-5}{3}$

Equation of line passing through (5, -8) and having slope $\frac{-5}{3}$ is;

$y - y_1 = m(x - x_1)$

$y - (-8) = \frac{-5}{3}(x - 5)$

$3(y + 8) = -5(x - 5)$

$3y + 24 = -5x + 25$

$0 = -5x + 25 - 3y - 24$

$-5x - 3y + 1 = 0$

$5x + 3y - 1 = 0$