Unit No. 5

Linear Equations and Inequalities

Basic Concepts

Linear Equation:

An equation of the form $ax + b = 0$ where 'a' and 'b' are constants, $a \neq 0$ and 'x' is a variable, is called a linear equation in one variable. In a linear equation, the highest power of the variable is always 1.

Remember!

$ax + b = 0$ and $a \neq 0$ is also called the general form of a linear equation in one variable.

Remember!

A linear equation in one variable has only one solution.

Remember!

We check the solution after solving a linear equation to ensure the accuracy of our work.

Linear Inequalities

Inequalities are expressed by the following four symbols:

$>$ (greater than), $<$ (less than), $\geq$ (greater than or equal to), $\leq$ (less than or equal to).

For example,

(i) $a x < b$

(ii) $a x + b \geq c$

(iii) $a x + b y > c$

(iv) $a x + b y \leq c$

are inequalities. Inequalities (i) and (ii) are in one variable while inequalities (iii) and (iv) are in two variables. The following operations will not affect the order of inequality while changing it to a simpler equivalent form:

(i) Adding or subtracting a constant to each side of it.

(ii) Multiplying or dividing each side by a positive constant.

Do you know?

The order of an inequality is changed by multiplying or dividing each side by a negative constant.

Solution of a Linear Inequality:

A solution of a linear inequality in x and y is an ordered pair of numbers which satisfies the inequality.

For example, the ordered pair (1,1) is a solution of the inequality $x + 2y < 6$ because

$1 + 2(1) = 3 < 6$, which is true.

Do you know?

A test point is a point selected to determine which side of the boundary line represents the solution region for an inequality. Usually, we take origin (0,0) as a test point.

- If the inequality holds true with the test point, the region containing this point is part of the solution.
- If the inequality is false, the opposite region is the solution region.

Solution of Two Linear Inequalities in Two Variables:

The graph of a system of linear inequalities consists of the set of all ordered pairs (x, y) in the xy-plane which simultaneously satisfies all the inequalities in the system.

To find the graph of such a system, we draw the graph of each inequality in the system on the same coordinate axes and then take the intersection of all the graphs. The common region so obtained is called the solution region for the system of inequalities.

Feasible Solution:

While tackling a certain problem from everyday life each linear inequality concerning the problem is named as problem constraint. The system of linear inequalities involved in the problem concerned is called problem constraints. The variables used in the system of linear inequalities relating to the problems of everyday life are non-negative and are called non-negative constraints. These non-negative constraints play an important role for taking decision. So, these variables are called decision variables. A region which is restricted to the first quadrant is referred to as a feasible region for the set of given constraints. Each point of the feasible region is called a feasible solution of the system of linear inequalities (or for the set of a given constraints).

Remember!

A point of a solution region where two of its boundary lines intersect, is called a corner point or vertex of the solution region.

Maximum and Minimum Values of a Function in the Feasible Solution:

A function which is to be maximized or minimized is called an objective function.

Note that there are infinitely many feasible solutions in the feasible region. The feasible solution which maximizes or minimizes the objective function is called the optimal solution.

Procedure for determining optimal solution:

(i) Graph the solution set of linear inequality constraints to determine feasible region.

(ii) Find the corner points of the feasible region.

(iii) Evaluate the objective function at each corner point to find the optimal solution.