Unit No. 6

Trigonometry

Exercise No. 6.6

Question No. 1

The angle of elevation of the top of a flag post from a point on the ground level 40 m away from the flag post is 60°. Find the height of the post.

Data:

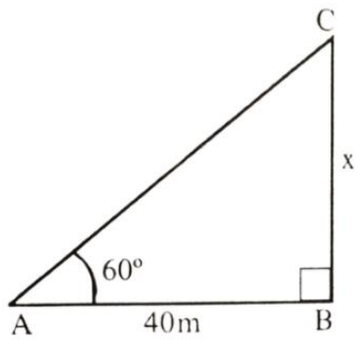
Base (AB) = 40 m

Angle of elevation (θ) = 60°

To Find:

Height of post (BC) = H = ?

Pictorial Form:



Solution:

$\text{Tan } \theta = \frac{Perpendicular}{Base} = \frac{BC}{AB}$

$\text{Tan } 60° = \frac{H}{40}$

$H = 40 \times \text{Tan } 60$

$H = 40 \times \sqrt{3}$

$H = 40\sqrt{3}$

$H = 69.28\ m$

The height of the flag post is approximately 69.28 meters.

Question No. 2

An isosceles triangle has a vertical angle of 120° and a base 10 cm long. Find the length of its altitude.

Data:

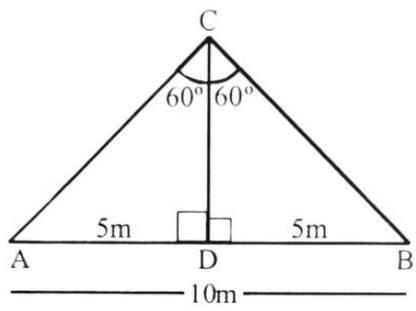
Angle ACB = 120°

Distance from A to B = AB = 10 cm

To Find:

Altitude CD = H = ?

Pictorial Form:



Explanation:

In an isosceles triangle, the altitude bisects the base and the vertical angle.

Therefore, angle BCD = $\frac{120}{2}$ = 60° and

AD = BD = $\frac{AB}{2} = \frac{10}{2}$ = 5 cm,

Solution:

$\text{Tan } \theta = \frac{Perpendicular}{Base} = \frac{BD}{CD}$

$\text{Tan } 60° = \frac{5}{H}$

$H = \frac{5}{\text{Tan } 60°}$

$H = \frac{5}{\sqrt{3}}$

$H = 2.89\ m$

The length of the altitude is approximately 2.89 m.

Question No. 3

A tree is 72 m high. Find the angle of elevation of its top from a point 100 m away on the ground level.

Data:

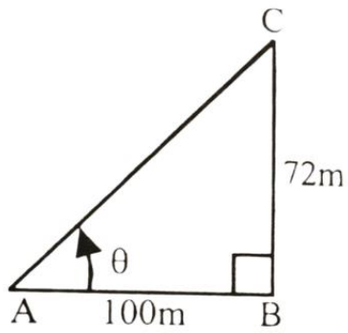
Height of tree (BC) = 72 m

Distance from point to tree (AB) = 100 m

To Find:

Angle of elevation (θ) = ?

Pictorial Form:



Solution:

Tan θ = $\frac{Perpendicular}{Base} = \frac{BC}{AB}$

Tan θ = $\frac{72}{100}$

θ = tan^{-1} 0.72

θ = 35.7°

The angle of elevation of the top of the tree from the point on the ground is approximately 35.77°.

Question No. 4

A ladder makes an angle of 60° with the ground and reaches a height of 10m along the wall. Find the length of the ladder.

Data:

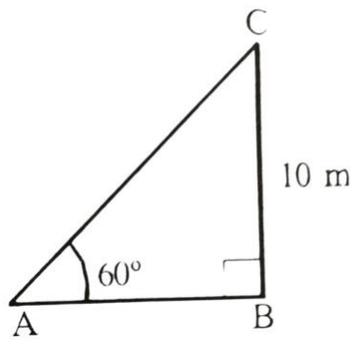
Height reached on the wall (BC) = 10 m

Angle with the ground = θ = 60°

To Find:

Length of the ladder (AC) = L = ?

Pictorial Form:



Solution:

$Sin\ \theta = \frac{Perpendicular}{Base} = \frac{BC}{AC}$

$Sin\ 60 = \frac{10}{L}$

$L = \frac{10}{Sin\ 60}$

$$L = \frac{10}{\frac{\sqrt{3}}{2}}$$

$$L = \frac{10}{\sqrt{3}} \times 2$$

$$L = \frac{20}{\sqrt{3}}$$

$$L = 11.55\ m$$

The length of the ladder is approximately 11.55 meters.

Question No. 5

A light house tower is 150 m high from the sea level. The angle of depression from the top of the tower to a ship is 60°. Find the distance between the ship and the tower.

Data:

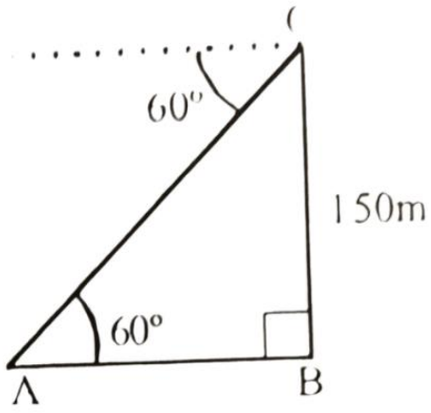
Height of the lighthouse (BC) = 150 m

Angle of depression = θ = 60°

To Find:

Distance between ship and tower (BC) = x = ?

Pictorial Form:



Solution:

$$\text{Tan } \theta = \frac{Perpendicular}{Base} = \frac{BC}{AB}$$

$$\text{Tan } 60 = \frac{150}{x}$$

$$x = \frac{150}{\tan 60}$$

$$x = \frac{150}{\sqrt{3}}$$

$$x = 86.60\ m$$

The distance between the ship and the tower is approximately 86.60 meters.

Question No. 6

Measure of an angle of elevation of the top of a pole is 15° from a point on the ground, in walking 100 m towards the pole the measure of angle is found to be 30°. Find the height of the pole.

Data:

Angle of elevation from A = 15°

Angle of elevation from B = 30°

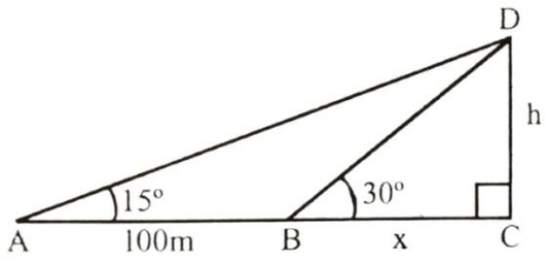
Distance AB = 100 m

To Find:

Distance between pole and point B = x = ?

Height of the pole (CD) = H = ?

Pictorial Form:



Solution:

In triangle BCD;

$\text{Tan } \theta = \frac{Perpendicular}{Base} = \frac{CD}{BC}$

$\text{Tan } 30 = \frac{H}{x}$

$\frac{1}{\sqrt{3}} = \frac{H}{x}$

$x = \sqrt{3}\text{H}$

In triangle ACD;

Let:

Distance AC = AB + BC = 100m + x

$\text{Tan } \theta = \frac{Perpendicular}{Base} = \frac{CD}{AC}$

$\text{Tan } 15 = \frac{H}{100 + x}$

$0.2679 = \frac{H}{100 + \sqrt{3}\text{H}}$

$0.2679\left(100 + \sqrt{3}\text{H}\right) = \text{H}$

$\text{H} = 26.79 + 0.4640\text{H}$

$\text{H} - 0.4640\text{H} = 26.79$

$0.5360H = 26.79$

$H = \frac{26.79}{0.5360}$

$H = 50\,m$

The height of the pole is 50 meters.

Question No. 7

Find the measure of an angle of elevation of the Sun, if a tower 300 m high casts a shadow 450 m long.

Solution:

Data:

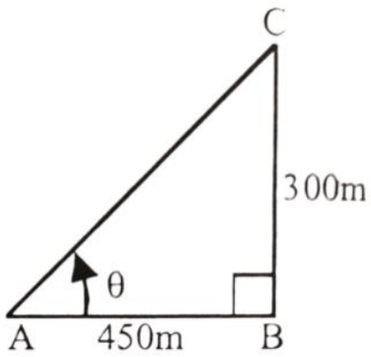
Height of the tower (BC) = 300 m

Length of the shadow (AB) = 450 m

To Find:

Angle of elevation of the Sun (θ) = ?

Pictorial Form:



Solution:

$\text{Tan}\,\theta = \frac{Perpendicular}{Base} = \frac{BC}{AB}$

$\text{Tan}\,\theta = \frac{300}{450}$

$\theta = tan^{-1}\,\frac{2}{3}$

$\theta = tan^{-1}\,0.6667$

$\theta = 33.69$

The measure of the angle of elevation of the Sun is approximately 33.69°.

Question No. 8

Measure of angle of elevation of the top of a cliff is 25°, on walking 100 metres towards the cliff, measure of angle of elevation of the top is 45°. Find the height of the cliff.

Data:

Angle of elevation from A = 25°

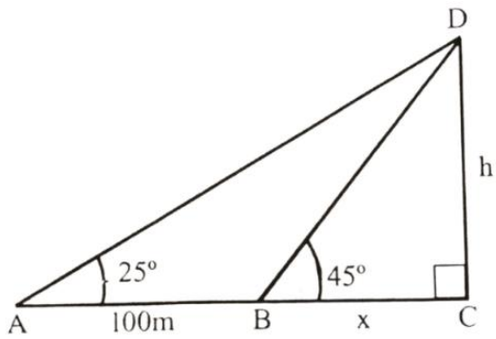
Angle of elevation from B = 45°

Distance AB = 100 m

To Find:

Height of the cliff (AB) = H = ?

Pictorial Form:



Solution:

Let BC = x and CD = H

In triangle BCD;

Tan θ = $\frac{Perpendicular}{Base} = \frac{CD}{BC}$

Tan 45 = $\frac{H}{x}$

1 = $\frac{H}{x}$

$H = x$

In triangle ACD;

AC = AB + BC = 100 m + x

Tan 25 = $\frac{CD}{AC}$

$0.4663 = \frac{H}{100 + x}$

$0.4663\ (100 + x) =$ H

$H = 46.63 + 0.4663H$ as H = x

$H - 0.4663H = 46.63$

$0.5337H = 46.63$

$H = \frac{46.63}{0.5337}$

$H = 87.37$ m

The height of the cliff is approximately 87.37 meters.

Question No. 9

From the top of a hill 300 m high, the measure of the angle of depression of a point on the nearer shore of the river is 70° and measure of the angle of depression of a point,

directly across the river is 50°. Find the width of the river. How far is the river from the foot of the hill?

Data:

Height of the hill (CD) = 300 m

Angle of depression of point B = 70°

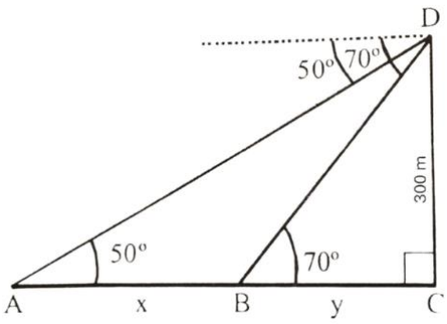
Angle of depression of point A = 50°

To Find:

Width of the river (BC) = y = ?

Distance of the nearer shore from the foot of the hill (AB) = x = ?

Pictorial Form:



Solution:

In triangle BCD:

Tan θ = $\frac{Perpendicular}{Base} = \frac{CD}{BC}$

Tan 70 = $\frac{300}{y}$

2.7475 = $\frac{300}{y}$

2.7475 y = 300

$y = \frac{300}{2.7475}$

$y = 109.2\ m$

In triangle ACD:

Let: AB + BC = x + y

Tan θ = $\frac{Perpendicular}{Base} = \frac{CD}{BC}$

Tan 50 = $\frac{300}{x+y}$

1.1918 = $\frac{300}{x + 109.19}$

1.1918 (x + 109.19) = 300

1.1918x + 130.1326 = 300

1.1918x = 300 - 130.1326

1.1918x = 169.8674

$x = \frac{169.8674}{1.1918}$

$x = 142.53\, m$

The width of the river is approximately 142.53 meters.

The river is approximately 109.2 meters from the foot of the hill.

Question No. 10

A kite has 120 m of string attached to it when at an angle of elevation of 50°. How far is it above the hand holding it? (Assume that the string is stretched.)

Data:

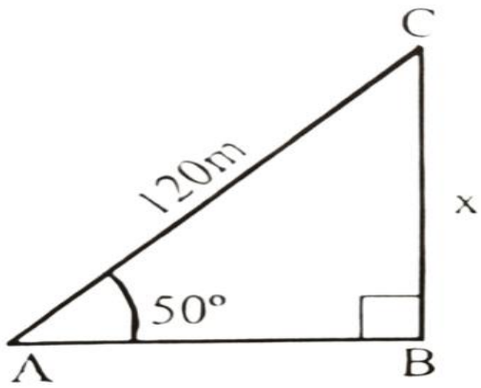
Length of the string (AC) = 120 m

Angle of elevation (θ) = 50°

To Find:

Height of the kite (AB) = H = ?

Pictorial Form:



Solution:

Sin θ = $\frac{Perpendicular}{Hypotenuse} = \frac{BC}{AC}$

Sin 50 = $\frac{H}{120}$

0.7660 = $\frac{H}{120}$

H = 120 × 0.7660

H = 91.92 m

The kite is approximately 91.92 meters above the hand holding it.