Unit No. 10

Graphs of Functions

Exercise No. 10.1

Question No. 1

To sketch a linear function y = m x + c:

c is the y-intercept (where the line crosses the y-axis, i.e., when x = 0).

m is the slope (rise over run).Sketch the graph of the following linear functions:

(i) y = 3 x − 5

Solution:

y = 3 x – 5 ….. (i)

P). Let; x = -2

Put in given equation (i):

y = 3 (- 2) – 5

y = - 6 − 5

y = - 11

Ordered Pair P: (- 2, - 11)

Q). Let; x = -1

Put in given equation (i):

y = 3 (- 1) – 5

y = - 3 − 5

y = - 8

Ordered Pair Q: (- 1, - 8)

R). Let; x = 0

Put in given equation (i):

y = 3 (0) – 5

y = 0 − 5

y = - 5

Ordered Pair R: (0, - 5)

S). Let; x = 1

Put in given equation (i):

y = 3 (1) – 5

y = 3 − 5

y = - 2

Ordered Pair S: (1, - 2)

T). Let; x = 2

Put in given equation (i):

y = 3 (2) – 5

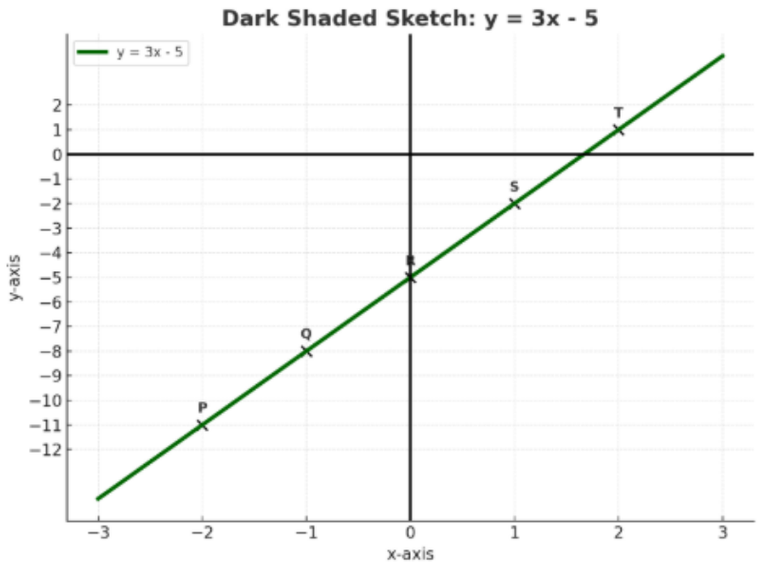
y = 6 − 5

y = 1

Ordered Pair P: (2, 1)

	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	-11	-8	-5	-2	1

Graphical Representation:



(ii) y = − 2 x + 8

Solution:

y = − 2 x + 8 …... (i)

P). Let; x = -2

Put in given equation (i):

y = − 2 (- 2) + 8

y = 4 + 8

y = 12

Ordered Pair P: (- 2, 12)

Q). Let; x = -1

Put in given equation (i):

y = − 2 (- 1) + 8

y = 2 + 8

y = 10

Ordered Pair Q: (- 1, 10)

R). Let; x = 0

Put in given equation (i):

$y = -2(0) + 8$

$y = 0 + 8$

$y = 8$

Ordered Pair R: (0, 8)

S). Let; x = 1

Put in given equation (i):

$y = -2(1) + 8$

$y = -2 + 8$

$y = 6$

Ordered Pair S: (1, 6)

T). Let; x = 2

Put in given equation (i):

$y = -2(2) + 8$

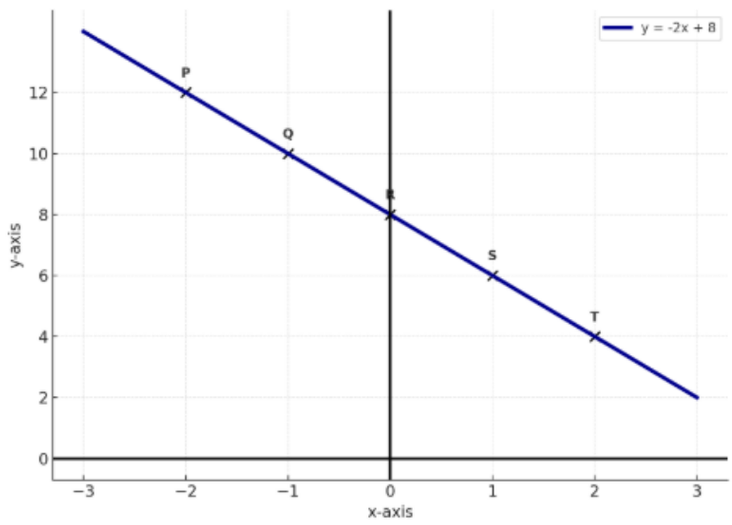
$y = -4 + 8$

$y = 4$

Ordered Pair P: (2, 4)

	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	12	10	8	6	4

Graphical Representation:



(iii) $y = 0.5x - 1$

Solution:

$y = 0.5x - 1$ ….. (i)

P). Let; x = -2

Put in given equation (i):

$y = 0.5 (- 2) - 1$

$y = - 1 - 1$

$y = - 2$

Ordered Pair P: (- 2, - 2)

Q). Let; x = -1

Put in given equation (i):

$y = 0.5 (- 1) - 1$

$y = - 0.5 - 1$

$y = - 1.5$

Ordered Pair Q: (- 1, - 1.5)

R). Let; x = 0

Put in given equation (i):

$y = 0.5 (0) - 1$

$y = 0 - 1$

$y = - 1$

Ordered Pair R: (0, - 1)

S). Let; x = 1

Put in given equation (i):

$y = 0.5 (1) - 1$

$y = 0.5 - 1$

$y = - 0.5$

Ordered Pair S: (1, - 0.5)

T). Let; x = 2

Put in given equation (i):

$y = 0.5 (2) - 1$

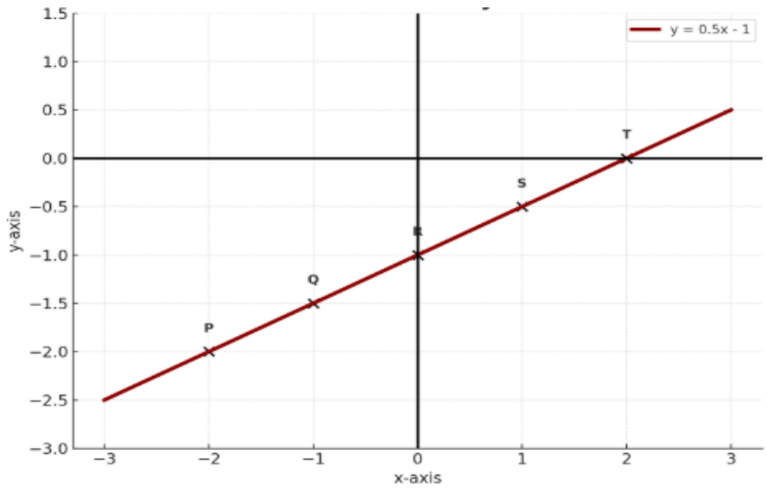
$y = 1 - 1$

$y = 0$

Ordered Pair P: (2, 0)

	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	-2	-1.5	-1	-0.5	0

Graphical Representation:



Question No. 2

Plot the graph of the following quadratic and cubic functions:

(i) $y = x^3 + 2x^2 - 5x - 6; -3.5 \leq x \leq 2.5$

Solution:

Type: Cubic function.

If $x = -3.5$,

$y = (-3.5)^3 + 2(-3.5)^2 - 5(-3.5) - 6 = -42.875 + 24.5 + 17.5 - 6 = -6.875$

If $x = -3$, $y = (-3)^3 + 2(-3)^2 - 5(-3) - 6 = -27 + 18 + 15 - 6 = 0$

If $x = -1$, $y = (-1)^3 + 2(-1)^2 - 5(-1) - 6 = -1 + 2 + 5 - 6 = 0$

If $x = 0$, $y = 0^3 + 2(0)^2 - 5(0) - 6 = -6$

If $x = 1$, $y = (1)^3 + 2(1)^2 - 5(1) - 6 = 1 + 2 - 5 - 6 = -8$

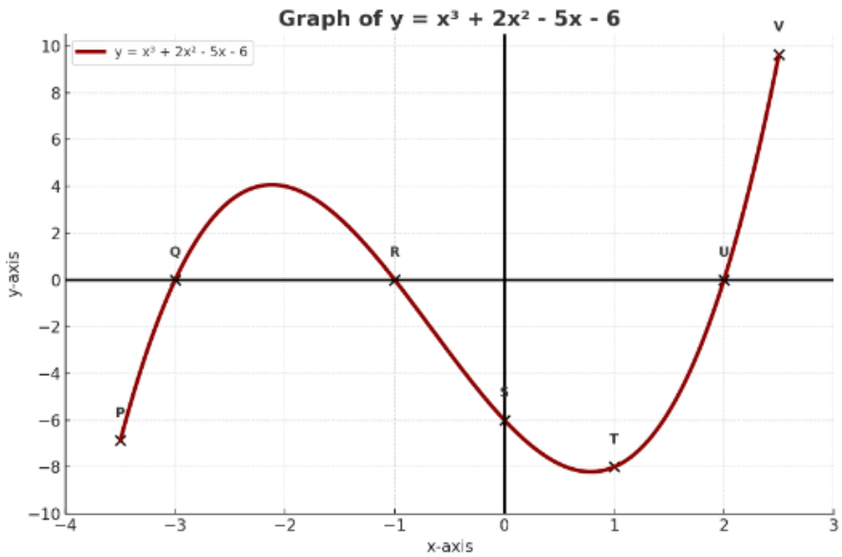
If $x = 2$, $y = 2^3 + 2(2)^2 - 5(2) - 6 = 8 + 8 - 10 - 6 = 0$

If $x = 2.5$, $y = 2.5^3 + 2(2.5)^2 - 5(2.5) - 6 = 15.625 + 12.5 - 12.5 - 6 = 9.625$

Table:

Points	P	Q	R	S	T	U	V
x-axis	-3.5	-3	-1	0	1	2	2.5
y-axis	-6.875	0	0	-6	-8	0	9.625

Graphical Representation:



(ii) $y = x^2 + x - 2$

Solution:

Type: Quadratic function (parabola).

If $x = -2$, $y = (-2)^2 + (-2) - 2 = 4 - 2 - 2 = 0$

If $x = -1$, $y = (-1)^2 + (-1) - 2 = 1 - 1 - 2 = -2$

If $x = 0$, $y = 0^2 + 0 - 2 = -2$

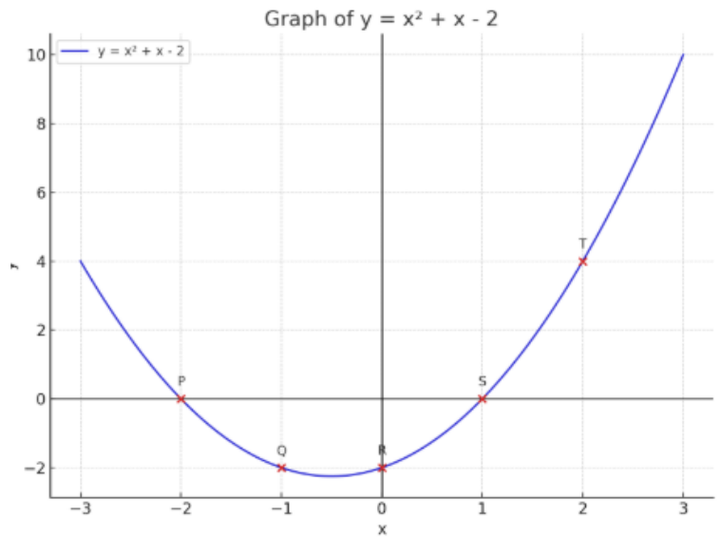
If $x = 1$, $y = 1^2 + 1 - 2 = 0$

If $x = 2$, $y = (2)^2 + (2) - 2 = 4 + 2 - 2 = 4$

Table:

Points	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	0	-2	-2	0	4

Graphical Representation:



(iii) $y = x^3 + 3x^2 + 2x$; $-2.5 \le x \le 0.5$

Solution:

Type: Cubic function.

For example:

If x = − 2, $y = (-2)^3 + 3(-2)^2 + 2(-2) = -8 + 12 - 4 = 0$

If x = − 1, $y = (-1)^3 + 3(-1)^2 + 2(-1) = -1 + 3 - 2 = 0$

If x = 0, $y = (0)^3 + 3(0)^2 + 2(0) = 0 + 0 + 0 = 0$

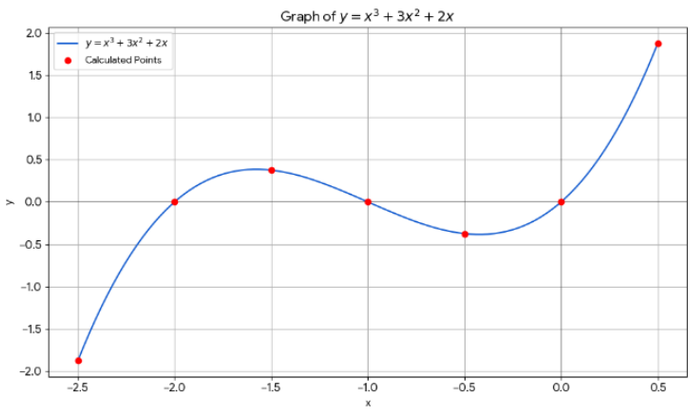
If x = 1, $y = (1)^3 + 3(1)^2 + 2(1) = 1 + 3 + 2 = 6$

If x = 2, $y = (2)^3 + 3(2)^2 + 2(2) = 8 + 12 + 4 = 24$

Table:

Points	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	0	0	0	6	24

Graphical Representation:



(iv) $y = 5x^2 - 2x - 3$

Solution:

Type: Quadratic function (parabola).

If x = -2, $y = 5(-2)^2 - 2(-2) - 3 = 20 + 4 - 3 = 21$

If x = -1, $y = 5(-1)^2 - 2(-1) - 3 = 5 + 2 - 3 = 4$

If x = 0, $y = 5(0)^2 - 2(0) - 3 = 0 - 0 - 3 = -3$

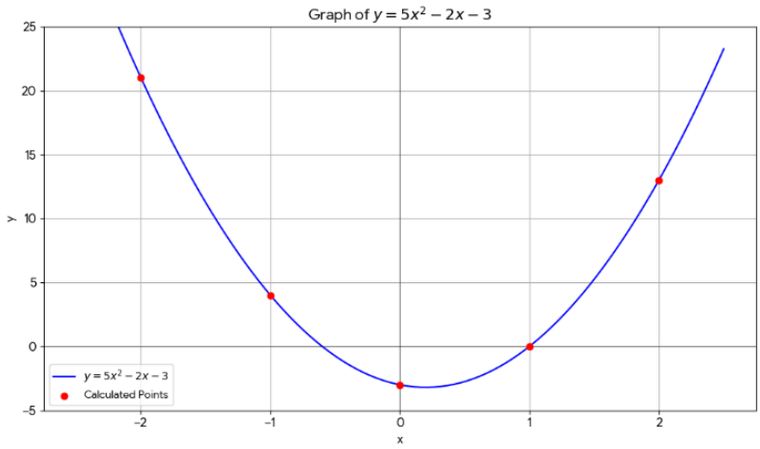
If x = 1, $y = 5(1)^2 - 2(1) - 3 = 5 - 2 - 3 = 0$

If x = 2, $y = 5(2)^2 - 2(2) - 3 = 20 - 4 - 3 = 13$

Table:

Points	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	21	4	-3	0	13

Graphical Representation:



Question No. 3

Plot the graph of the following functions:

(i) $y = 4^x$

Solution:

Type: Exponential function.

If $x = -2$, $y = 4^{-2} = 1/16$

If $x = -1$, $y = 4^{-1} = 1/4$

If $x = 0$, $y = 4^0 = 1$ (y-intercept)

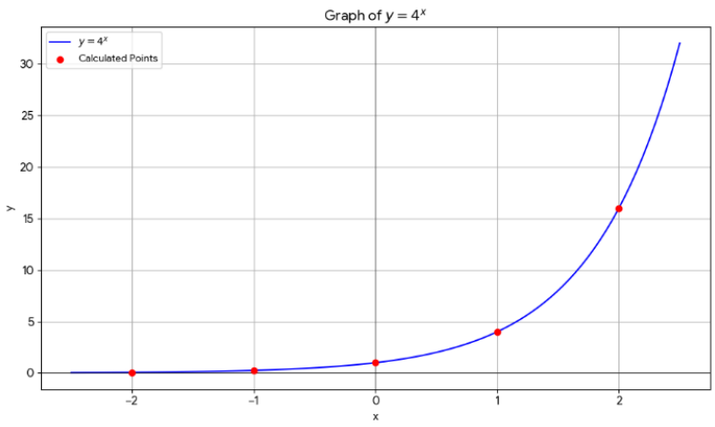
If $x = 1$, $y = 4^1 = 4$

If $x = 2$, $y = 4^2 = 16$

Table:

Points	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	1/16	1/4	1	4	16

Graphical Representation:



(ii) $y = 5^{-x}$

Solution:

Type: Exponential function.

If $x = -2$, $y = 5^{-(-2)} = 5^2 = 25$

If x = − 1, $y = 5^{-(-1)} = 5^1 = 5$

If x = 0, $y = 5^0 = 1$ (y-intercept)

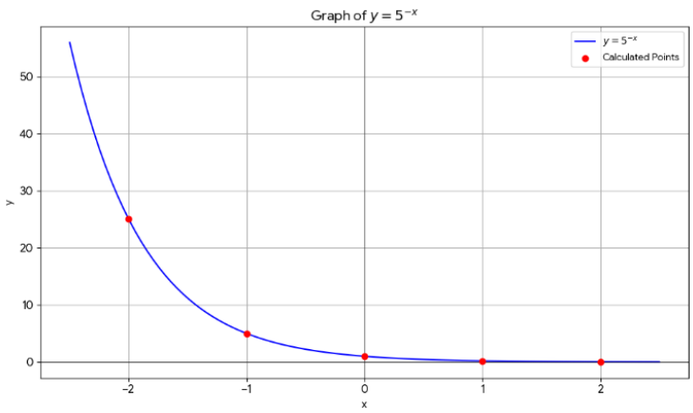
If x = 1, $y = 5^{-1} = 1/5$

If x = 2, $y = 5^{-2} = 1/25$

Table:

Points	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	25	5	1	1/5	1/25

Graphical Representation:



(iii) $\mathbf{y = \frac{1}{x-3}}$**, x ≠ 3**

Solution:

Type: Rational function.

If x = -2, y = 1/(- 2 − 3) = 1/-5 = - 1/5

If x = -1, y = 1/(- 1 − 3) = 1/-4 = - 1/4

If x = 0, y = 1/(0 − 3) = 1/-3 = - 1/3

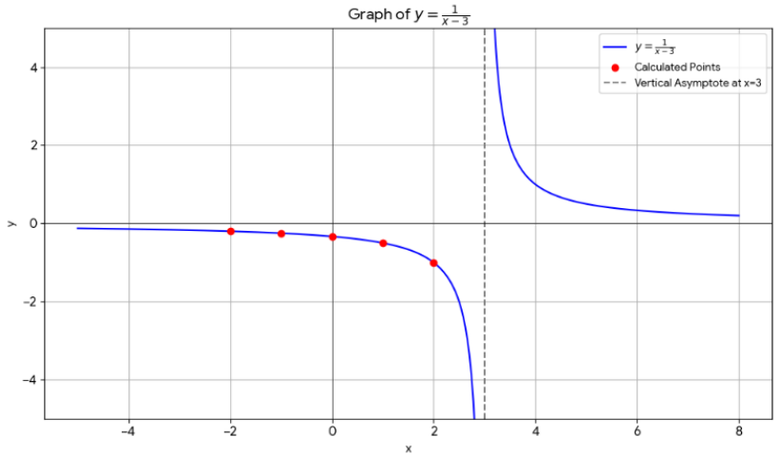
If x = 1, y = 1/(1 − 3) = 1/-2 = - 1/2

If x = 2, y = 1/(2 − 3) = 1/-1 = - 1

Table:

Points	P	Q	R	S	T
x-axis	-2	-1	0	1	2
y-axis	-1/5	-1/4	-1/3	-1/2	-1

Graphical Representation:



(iv) $y = \frac{2}{x} + 3$, $x \neq 0$

Solution:

Type: Rational function (reciprocal function).

If $x = -2$, $y = \frac{2}{-2} + 3 = -1 + 3 = 2$

If $x = -1$, $y = \frac{2}{-1} + 3 = -2 + 3 = 1$

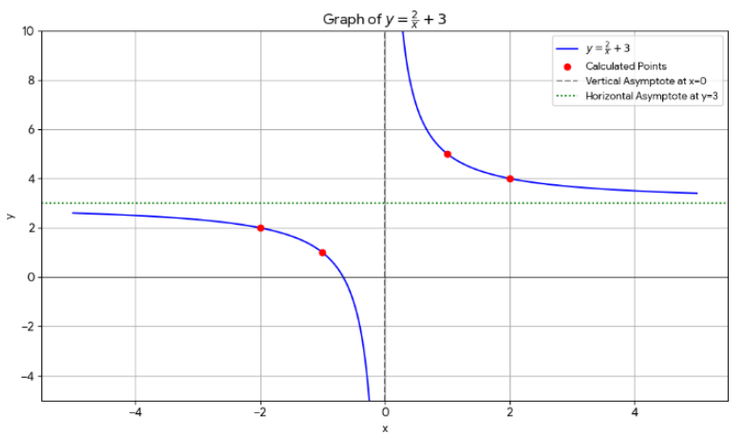
If $x = 1$, $y = \frac{2}{1} + 3 = 2 + 3 = 5$

If $x = 2$, $y = \frac{2}{2} + 3 = 1 + 3 = 4$

Table:

Points	P	Q	R	S
x-axis	-2	-1	1	2
y-axis	2	1	5	4

Graphical Representation:



(v) $y = x^{\frac{1}{2}}$

Solution:

Type: Square root function.

If x = 0, y = $(0)^{\frac{1}{2}} = 0$

If x = 1, y = $(1)^{\frac{1}{2}} = 1$

If x = 4, y = $(4)^{\frac{1}{2}} = 2$

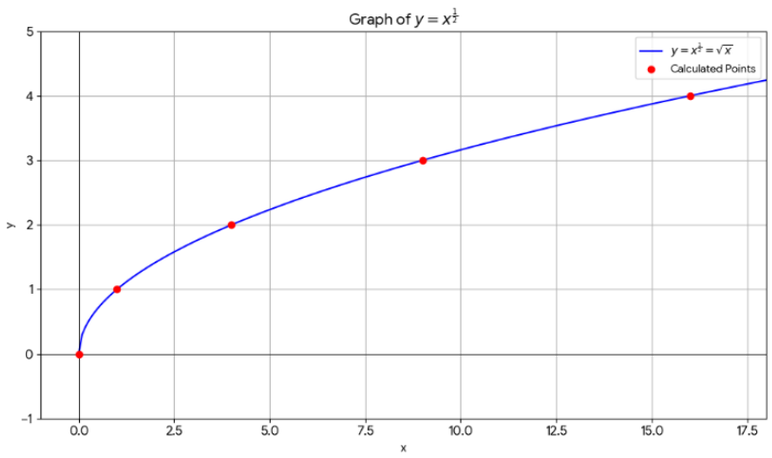
If x = 9, y = $(9)^{\frac{1}{2}} = 3$

If x = 16, y = $(16)^{\frac{1}{2}} = 4$

Table:

Points	P	Q	R	S	T
x-axis	0	1	4	9	16
y-axis	0	1	2	3	4

Graphical Representation:



(vi) $y = 3x^{\frac{1}{3}}$

Solution:

Type: Cube root function.

If x = - 1, y = $3(-1)^{\frac{1}{3}} = -3$

If x = 0, y = $3(0)^{\frac{1}{3}} = 0$

If x = 1, y = $3(1)^{\frac{1}{3}} = 3$

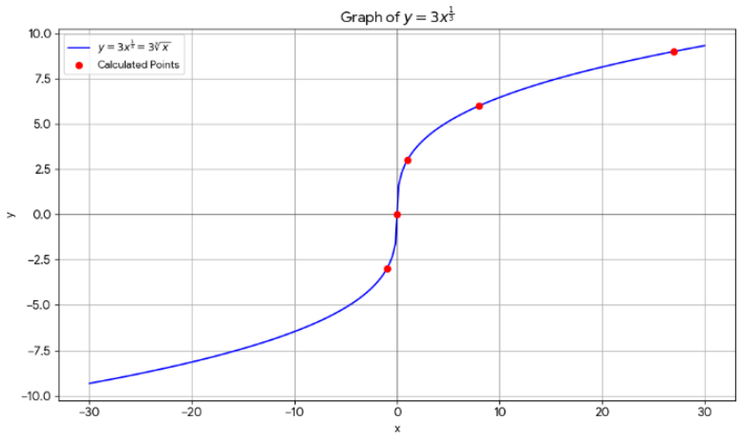
If x = 8, y = $3(8)^{\frac{1}{3}} = 6$

If x = 27, y = $3(27)^{\frac{1}{3}} = 9$

Table:

Points	P	Q	R	S	T
x-axis	-1	0	1	8	27
y-axis	-3	0	3	6	9

Graphical Representation:



(vii) y = $2x^{-2}$

Solution:

Type: Rational function.

If x = - 2, y = $2(-2)^{-2}$ = 1/2

If x = - 1, y = $2(-1)^{-2}$ = 2

If x = 1, y = $2(1)^{-2}$ = 2

If x = 2, y = $2(2)^{-2}$ = 1/2

If x = 3, y = $2(3)^{-2}$ = 2/9

Table:

Points	P	Q	R	S	T
x-axis	-2	-1	1	2	3
y-axis	1/2	2	2	1/2	2/9

Graphical Representation: