Unit No. 10

Graphs of Functions

Basic Concepts

Graph of Linear Functions:

A linear function is a mathematical expression that represents a straight-line relationship between two variables. Its general form is $f(x) = mx + c$, where "m" is the slope or gradient of the line, indicating how steep it is and "c" is the y-intercept (the point where the line crosses the y-axis). It can also be written as $y = mx + c$.

Graph of Quadratic Functions:

A quadratic function is a type of polynomial function that involves x^2 term. Its general form is: $y = ax^2 + bx + c$

Where a, b, c are constants and $a \neq 0$.

Keep in mind!

The graph of a quadratic function is always a **parabola**.

- If $a > 0$, then the parabola opens upward.
- If $a < 0$, then the parabola opens downward.

Graph of Cubic Functions:

A cubic function is a type of polynomial function of degree 3. Its standard form is:

$y = ax^3 + bx^2 + cx + d$

Where a, b, c, d are constants and $a \neq 0$.

Remember!

- The graph of a cubic function is a curve that can have at most two turning points.
- It has a general "S-shaped" appearance and depending on the coefficients, the shape may vary.
- Such functions are much more complicated and show more varied behaviour than linear and quadratic ones.

Graph of Reciprocal Functions:

A reciprocal function is a function of the form: $y = \frac{a}{x}$

Where a is any real number and $x \neq 0$.

Remember!

An asymptote is a line that a graph approaches but never touches.

Graph of Exponential Functions:

($y = ka^x$ where x is real number, $a > 1$)

An exponential function is a mathematical function of the form:

$y = ka^x$

Where a, k are constants, x is variable and $a > 1$.

Graphs of $y = ax^n$ where n is +ve integer, -ve integer or rational number for

$x > 0$ and a is any real number):

The graph of the function $y = ax^n$, where n is a positive integer, negative integer or rational number for $x > 0$ and a is any real number, exhibits distinct behaviours depending on the value of n.

Exponential Growth/Decay Applications:

Exponential growth and decay are widely observed in real-world phenomenon and their graphical representations offer critical insights into these processes. In exponential growth, such as population expansion, compound interest in finance or the spread of infectious diseases, the graph starts slowly but accelerates rapidly as time progresses. The curve increases steeply, showcasing how growth becomes more pronounced with time due to constant proportional changes. Conversely, in exponential decay, observed in cooling of objects or depreciation of assets, the graph starts high and decreases sharply before levelling off, indicating a gradual reduction over time. These graphs are essential for interpreting trends, making predictions and informing decision-making in diverse fields.

Gradients of Curves by Drawing Tangents:

The gradient or slope of a graph at any point is equal to the gradient of the tangent to the curve at that point. Remember that a tangent is a line that just touches a curve only at one point (and doesn't cross it).

The gradient between two points is defined as:

$$\text{Gradient} = \frac{\text{Change in y}}{\text{Change in x}} = \frac{\Delta y}{\Delta x} = \frac{y2-y1}{x2-x1}$$

Applications of Graphs in Real-Life:

- Tax Optimization:
  Identifying optimal income levels, tax brackets, and tax liability.
- Income & Salary Analysis:
  Visualizing compensation packages and income growth; revealing patterns or anomalies in salary versus experience.
- Business Cost-Profit Analysis:
  Visualizing cost-profit relationships, determining break-even points, and optimizing production levels.