# MATHEMATICS 9

PUNJAB CURRICULUM AND TEXTBOOK BOARD, LAHORE

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# UNIT 7 COORDINATE GEOMETRY

## EXERCISE 7.1

**Q.1. Describe the location in the plane of the point P(x, y), for which**

**(i)** $x > 0$

**Sol.** Right half plane

**(ii)** $x > 0$ **and** $y > 0$

**Sol.** $1^{st}$ quadrant

**(iii)** $x = 0$

**Sol.** $x = 0$ is the equation of y-axis

**(iv)** $y = 0$

**Sol.** $y = 0$ is the equation of x-axis

**(v)** $x \geq 0$ **and** $y < 0$

**Sol.** $4^{th}$ quadrant and negative y-axis

**(vi)** $y = 0, x = 0$

**Sol.** Origin

**(vii)** $|x| = |y|$

**Sol.** It is a line bisecting $1^{st}$ and $3^{rd}$ quadrant.

**(viii)** $x \geq 3$

**Sol.** The set of points lying on and right side of the line $x = 3$

**(ix)** $y > 2$

**Sol.** The set of points lying above x-axis.

**(x)** **x and y have opposite signs.**

**Sol.** The set of points in $2^{nd}$ and $4^{th}$ quadrants.

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**Q.2. Find in distance between the points.**

**(a) A(6, 7), B(0, –2)**

**Sol.** Distance between A and B is

$$d(A, B) = |\overline{AB}| = \sqrt{(0-6)^2 + (-2-7)^2}$$
$$= \sqrt{36 + 81}$$
$$= \sqrt{117} = \sqrt{9 \times 13}$$
$$= 3\sqrt{3}$$

**(ii) C(–5, –2), D(3, 2)**

**Sol.** Distance between points C and D

$$d(C, D) = |\overline{CD}| = \sqrt{(3-(-5))^2 + (2-(-2))^2}$$
$$= \sqrt{8^2 + 4^2}$$
$$= \sqrt{64 + 16}$$
$$= \sqrt{80}$$
$$= \sqrt{16 \times 5} = 4\sqrt{5}$$

(iii) L(0, 3), M(–2, 4)

Sol. $d(L, M) = |\overline{LM}| = \sqrt{(-2-0)^2 + (-4-3)^2}$

$$= \sqrt{4 + 49} = \sqrt{53}$$

(iv) P(–8, –7), Q(0, 0)

Sol. $d(P, Q) = |\overline{PQ}| = \sqrt{(-8-0)^2 + (-7-0)^2}$

$$= \sqrt{64 + 49} = \sqrt{113}$$

**Q.3. Find in each of the following:**

**(i) The distance between the two given points**

**(ii) Midpoint of the line segment joining the two points.**

**(a) A(3, 1), B(–2, –4)** **(b) A(–8, 3), B(2, –1)**

**(c)** $A\left(-\sqrt{5}, -\frac{1}{3}\right), B(-3\sqrt{5}, 5)$

**Sol.**(i)(a) $|\overline{AB}| = \sqrt{(-2-3)^2 + (-4-1)^2}$

$$= \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

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(b) $|\overline{AB}| = \sqrt{(2-(-8))^2 + (-1-3)^2}$
$= \sqrt{100+16} = \sqrt{116} = 2\sqrt{29}$

(c) $|\overline{AB}| = \sqrt{(-3\sqrt{5}+\sqrt{5})^2 + \left(5+\frac{1}{3}\right)^2}$

$= \sqrt{(-2\sqrt{5})^2 + \left(\frac{16}{3}\right)^2} = \sqrt{20+\frac{256}{9}}$

$= \sqrt{\frac{436}{9}} = \frac{2\sqrt{109}}{3}$

(ii)(a) Midpoint of $\overline{AB} = \left(\frac{3-2}{2}, \frac{1-4}{2}\right) = \left(\frac{-1}{2}, \frac{-3}{2}\right)$

(b) Midpoint of $\overline{AB} = \left(\frac{-8+2}{2}, \frac{3-1}{2}\right) = \left(\frac{-6}{2}, \frac{2}{2}\right) = (-3, 1)$

(c) Midpoint of $\overline{AB} = \left(\frac{-\sqrt{5}-3\sqrt{5}}{2}, \frac{-\frac{1}{3}+5}{2}\right)$

$= \left(\frac{-4\sqrt{5}}{2}, \frac{-1+15}{2\times 3}\right) = \left(-2\sqrt{5}, \frac{7}{3}\right)$

**Q.4. Which of the following points are at a distance of 15 units from the origin?**

**(i) $(\sqrt{176}, 7)$ (ii) (10, –10) (iii) (1, 15)**

**Sol.**(i) $= \sqrt{(176-0)^2 + (7-0)^2} = \sqrt{176+49} = \sqrt{225} = 15$

(ii) $= \sqrt{(10-0)^2 + (-10-0)^2} = \sqrt{100+100}$

$= \sqrt{200} = \sqrt{100+2} = 10\sqrt{2}$

(iii) $= \sqrt{(15-0)^2 + (1-0)^2} = \sqrt{225+1} = \sqrt{226}$

Only in part (i) we see that point $(\sqrt{176}, 1)$ is at a distance of 15 units from origin.

**Q.5. Show that:**

**(i) the points A(0, 2), B($\sqrt{3}$, 1) and C(0, –2) are vertices of a right triangle.**

**Sol.** A(0, 2), B($\sqrt{3}$, 1), C(0, –2)

Using distance formula to find $\overline{AB}, \overline{BC}, \overline{CA}$

$|\overline{AB}| = \sqrt{(\sqrt{3}-0)^2 + (1-2)^2}$

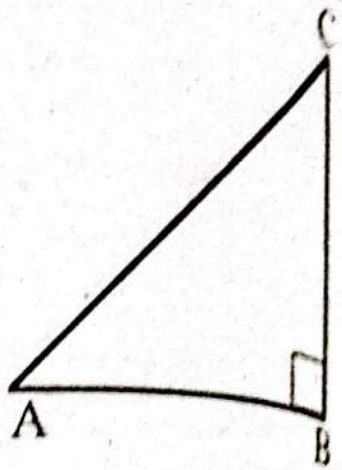
$= \sqrt{3+1} = \sqrt{4} = 2$

$|\overline{AB}| = (2)^2$

$|\overline{AB}| = 4 \quad \ldots(1)$

$|\overline{BC}| = \sqrt{(0-\sqrt{3})^2 + (-2-1)^2}$

$= \sqrt{3+9} = \sqrt{12}$




$|\overline{BC}| = (\sqrt{12})^2$

$|\overline{BC}| = 12 \quad \ldots(2)$

$|\overline{CA}| = \sqrt{(0-0)^2 + (-2-2)^2}$

$= \sqrt{0+(-4)^2} = \sqrt{4^2}$

$|\overline{CA}| = 4$

$|\overline{CA}|^2 = 4^2$

$|\overline{CA}|^2 = 16 \quad \ldots(3)$

From (1), (2), (3)

$|\overline{AB}|^2 + |\overline{BC}|^2 = |\overline{AC}|^2$

So given vertices are related to right angled triangle.

**(ii) the points A(3, 1), B(–2, –3) and C(2, 2) are vertices of an isosceles triangle.**

**Sol.** $\overline{AB} = \sqrt{(-2-3)^2 + (-3-1)^2}$

$= \sqrt{(-5)^2 + (-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$

$|\overline{BC}| = \sqrt{(2-(-2))^2 + (2-(-3))^2}$

$= \sqrt{(2+2)^2 + (2+3)^2} = \sqrt{4^2+5^2}$

$= \sqrt{16+25}$

$|\overline{BC}| = \sqrt{41}$

$|\overline{AC}| = \sqrt{(2-3)^2 + (2-1)^2}$

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$$= \sqrt{(-1)^2 + 1^2} = \sqrt{1+1}$$

$$|\overline{AC}| = \sqrt{2}$$

Since

$$|\overline{AB}|^2 = |\overline{BC}|^2 = \sqrt{41}$$

So given vertices are related to an isosceles triangle.

(iii) **the points A(5, 2), B(–2, 3), C(–3, –4) and D(4, –5) are vertices of a parallelogram. Is the parallelogram a square?**

**Sol.** $|\overline{AB}| = \sqrt{(-2-5)^2 + (3-2)^2}$

$$= \sqrt{(-7)^2 + 1^2}$$

$$= \sqrt{47+1} = \sqrt{50}$$

$$|\overline{BC}| = \sqrt{(-3-(-2))^2 + (-4-3)^2}$$

$$= \sqrt{(-3+2)^2 + (-7)^2} = \sqrt{(-1)^2 + 49}$$

$$= \sqrt{50}$$

$$|\overline{CD}| = \sqrt{(-3-4)^2 + (-4+5)^2}$$

$$= \sqrt{(-7)^2 + 1^2} = \sqrt{49+1}$$

$$= \sqrt{50}$$

$$|\overline{DA}| = \sqrt{(4-5)^2 + (-5-2)^2}$$

$$= \sqrt{(-1)^2 + (-7)^2} = \sqrt{1+49}$$

$$= \sqrt{50}$$

Since

$$|\overline{AB}| = |\overline{BC}| = |\overline{CD}| = |\overline{DA}| = \sqrt{50}$$

Now

$$|\overline{AC}| = \sqrt{(-3-5)^2 + (-4-2)^2}$$

$$= \sqrt{(-8)^2 + (-6)^2} = \sqrt{64+36}$$

$$|\overline{AC}| = \sqrt{100}$$

$$|\overline{AC}|^2 = 10^2$$

$$|\overline{AC}|^2 = 100 \qquad \ldots(1)$$

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$$|\overline{AB}|^2 = (\sqrt{50})^2 = 50$$

$$|\overline{BC}|^2 = (\sqrt{50})^2 = 50$$

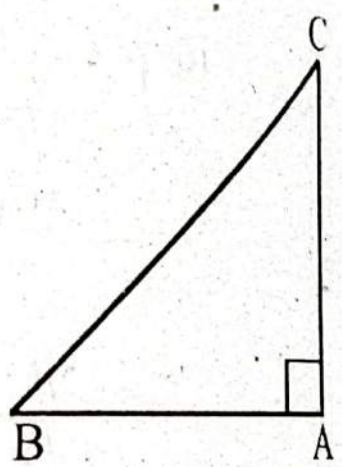
Adding

$$|\overline{AB}|^2 + |\overline{BC}|^2 = 50 + 50$$
$$= 100$$
$$= |\overline{AC}|^2$$

Since Pythagoras theorem is satisfied. So ABCD is a square.

**Q.6. Find h such that the points $A(\sqrt{3}, 1)$, $B(0, 2)$ and $C(h, -2)$ are vertices of a right triangle with right angle at the vertex A.**

**Sol.**




$$|\overline{AB}| = \sqrt{(0-\sqrt{3})^2 + (2-1)^2}$$
$$= \sqrt{3+1}$$
$$= \sqrt{4}$$
$$|\overline{AB}|^2 = (\sqrt{4})^2$$
$$|\overline{AB}|^2 = 4 \qquad \ldots(1)$$
$$|\overline{AC}| = \sqrt{(h-\sqrt{3})^2 + (-2-1)^2}$$
$$= \sqrt{h^2 - 2\sqrt{3}h + 3 + 9}$$
$$|\overline{AC}|^2 = \sqrt{h^2 - 2\sqrt{3}h + 12}$$
$$|\overline{AC}|^2 = h^2 - 2\sqrt{3}h + 12 \qquad \ldots(2)$$
$$|\overline{BC}|^2 = \sqrt{(h-2)^2 + (-2-2)^2}$$
$$= \sqrt{h^2 - 4h + 4 + 16}$$
$$= \sqrt{h^2 - 4h + 20}$$
$$|\overline{BC}|^2 = h^2 - 4h + 20 \qquad \ldots(3)$$

Now $\quad |\overline{AB}|^2 + |\overline{AC}|^2 = |\overline{BC}|^2$

$$4 + h^2 - 2\sqrt{3}h + 12 = h^2 - 4h + 20$$

$$-2\sqrt{3}h + 16 = -4h + 20$$
$$-2\sqrt{3}h + 4h = 20 - 16$$
$$(4 - 3.64)h = 4$$
$$0.36h = 4$$
$$h = \frac{4}{0.36}$$
$$h = 11.1$$

**Q.7. Find h such that A(–1, h), B(3, 2) and C(7, 3) are collinear.**

**Sol.** As points A, B, C are collinear. So, using the condition

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

$$-1\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} - h\begin{vmatrix} 3 & 1 \\ 7 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 7 & 3 \end{vmatrix} = 0$$

$$-(2 - 3) - h(3 - 7) + 1(9 - 14) = 0$$
$$-(-1) - h(-4) + 1(-5) = 0$$
$$4h + 1 - 5 = 0$$
$$4h = 4$$
$$h = \frac{4}{4}$$
$$h = 1$$

**Q.8. The points A(–5, –2) and B(5, –4) are ends of a diameter of a circle. Find the centre and radius of the circle.**

**Sol.** A(–5, –2), B(5, –4) let C(h, k) be the centre of the circle having radius r. Since C(h, k) is midpoint of $\overline{AB}$. So coordinates of C are

$$h = \left(\frac{-5+5}{2}\right), \quad k = \left(\frac{-2+(-4)}{2}\right)$$

$$h = 0, \qquad k = \left(\frac{-2-4}{2}\right)$$

$$k = \frac{-6}{2}$$

$$k = -3$$

So C(0, –3)

$$r = \overline{AC}$$

$$= \sqrt{(0+5)^2 + (-3+2)^2}$$

$$= \sqrt{25+1}$$

$$= \sqrt{26}$$

So radius of the circle is $\sqrt{26}$.

**Q.9. Find h such that the points A(h, 1), B(2, 7) and C(–6, –7) are vertices of a right triangle with right angle at the vertex A.**

**Sol.** $|\overline{AB}|^2 = (2-h)^2 + (7-1)^2$

$= 4 + h^2 - 4h + 36$

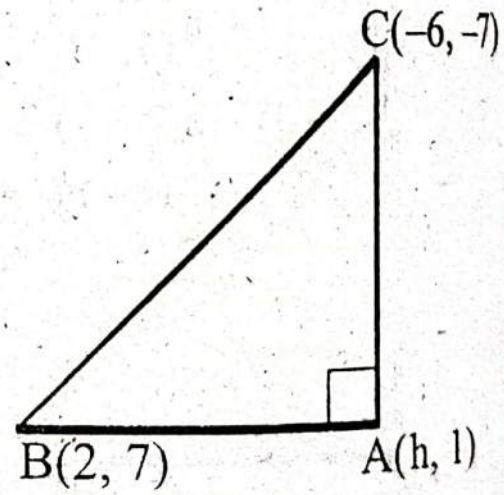
$= h^2 - 4h + 40$

$|\overline{BC}|^2 = (-6-2)^2 + (-7-7)^2$

$= (-8)^2 + (-14)^2$

$= 64 + 196$

$|\overline{BC}|^2 = 260$




$|\overline{AC}|^2 = (-6-h)^2 + (-7-1)^2$

$= 36 + 12h + h^2 + 64$

$|\overline{AC}|^2 = h^2 + 12h + 100$

Since ABC is a right angle triangle with right angle at vertex A. By Pythagoras theorem

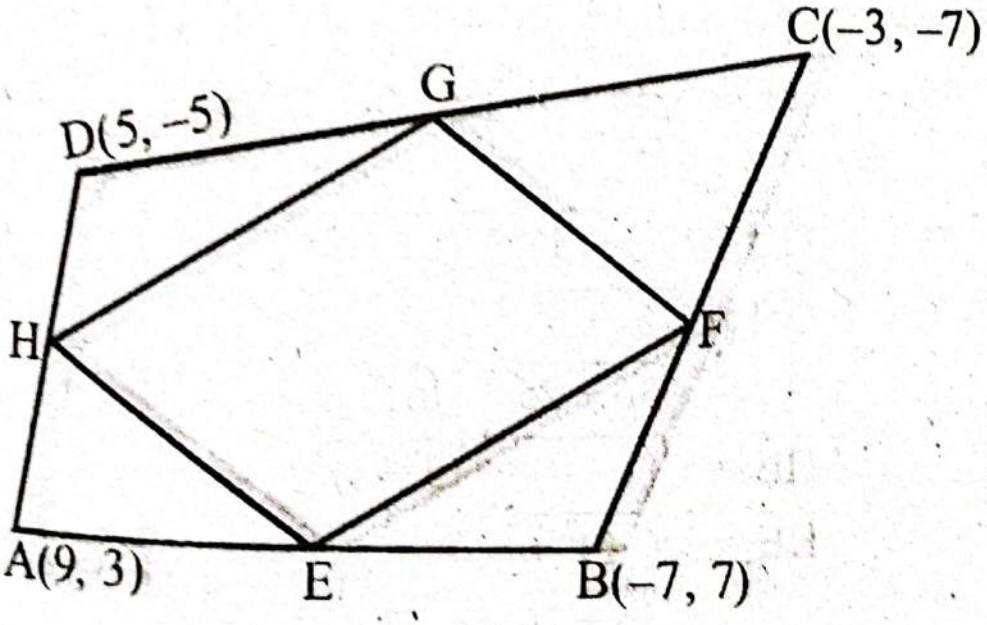
$|\overline{AB}|^2 = |\overline{BC}|^2 + |\overline{AC}|^2$

$$260 = h^2 - 4h + 40 + h^2 + 12h + 100$$
$$260 = 2h^2 + 8h - 140$$
$$2h^2 + 8h + 140 - 260 = 0$$
$$2h^2 + 8h - 120 = 0$$
$$2(h^2 + 4h - 60) = 0$$
$$h^2 + 4h - 60 = 0$$
$$h^2 + 10h - 6h - 60 = 0$$
$$h(h + 10) - 6(h + 10) = 0$$
$$(h + 10)(h - 6) = 0$$
$$h + 10 = 0 \quad ; \quad h - 6 = 0$$
$$h = -10 \quad ; \quad h = 6$$

So $h = -10$ or $h = 6$

**Q.10. A quadrilateral has the points A(9, 3), B(–7, 7), C(–3, –7) and D(5, –5) as its vertices.**

**Sol.** Let E, F, G, H be the midpoints of the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ respectively.




Midpoint E of $\overline{AB}$ $= \left(\frac{9-7}{2}, \frac{3+7}{2}\right)$

$$= \left(\frac{2}{2}, \frac{10}{2}\right)$$

$$= E(1, 5)$$

Midpoint F of $\overline{BC}$ $= \left(\frac{-3-7}{2}, \frac{-7+7}{2}\right)$

$= \left(\frac{-10}{2}, \frac{0}{2}\right)$

$= F(-5, 0),$

Midpoint G of $\overline{CD}$ $= \left(\frac{5-3}{2}, \frac{-5-7}{2}\right)$

$= \left(\frac{2}{2}, \frac{-12}{2}\right)$

$= G(1, -6)$

Midpoint of $\overline{AD}$ $= H\left(\frac{5+9}{2}, \frac{-5+3}{2}\right)$

$= H(7, -1)$

$\overline{EF} = \sqrt{(-5-1)^2 + (0-5)^2}$

$\overline{EF} = \sqrt{(-6)^2 + (-5)^2}$

$= \sqrt{36+25}$

$= \sqrt{61}$

$\overline{FG} = \sqrt{(1+5)^2 + (-6-0)^2}$

$\overline{FG} = \sqrt{36+36}$

$= \sqrt{72}$

$\overline{GH} = \sqrt{(7-1)^2 + (-1+6)^2}$

$\overline{GH} = \sqrt{6^2 + 5^2}$

$= \sqrt{36+25} = \sqrt{61}$

$\overline{HE} = \sqrt{(1-7)^2 + (5+1)^2}$

$\overline{HE} = \sqrt{(-6)^2 + 6^2}$

$= \sqrt{36+36} = \sqrt{72}$

$|\overline{EF}| = |\overline{GH}| = \sqrt{61}$

& $|\overline{FG}| = |\overline{HE}| = \sqrt{72}$

Show that the figure obtained by joining the midpoints consecutively is a parallelogram.

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# EXERCISE 7.2

**Q.1. Find the slope and inclination of the line joining the points:**

(i) $(-2, 4)$ ; $(5, 11)$

(ii) $(3, -2)$ ; $(2, 7)$

(iii) $(4, 6)$ ; $(4, 8)$

**Sketch each line in the plane.**

(i) $(-2, 4)$ ; $(5, 11)$

**Sol.** Let $P(x_1, y_1) = (-2, 4)$

$Q(x_2, y_2) = (5, 11)$

Slope of line $\overline{PQ} = m = \dfrac{y_2 - y_1}{x_2 - x_1}$

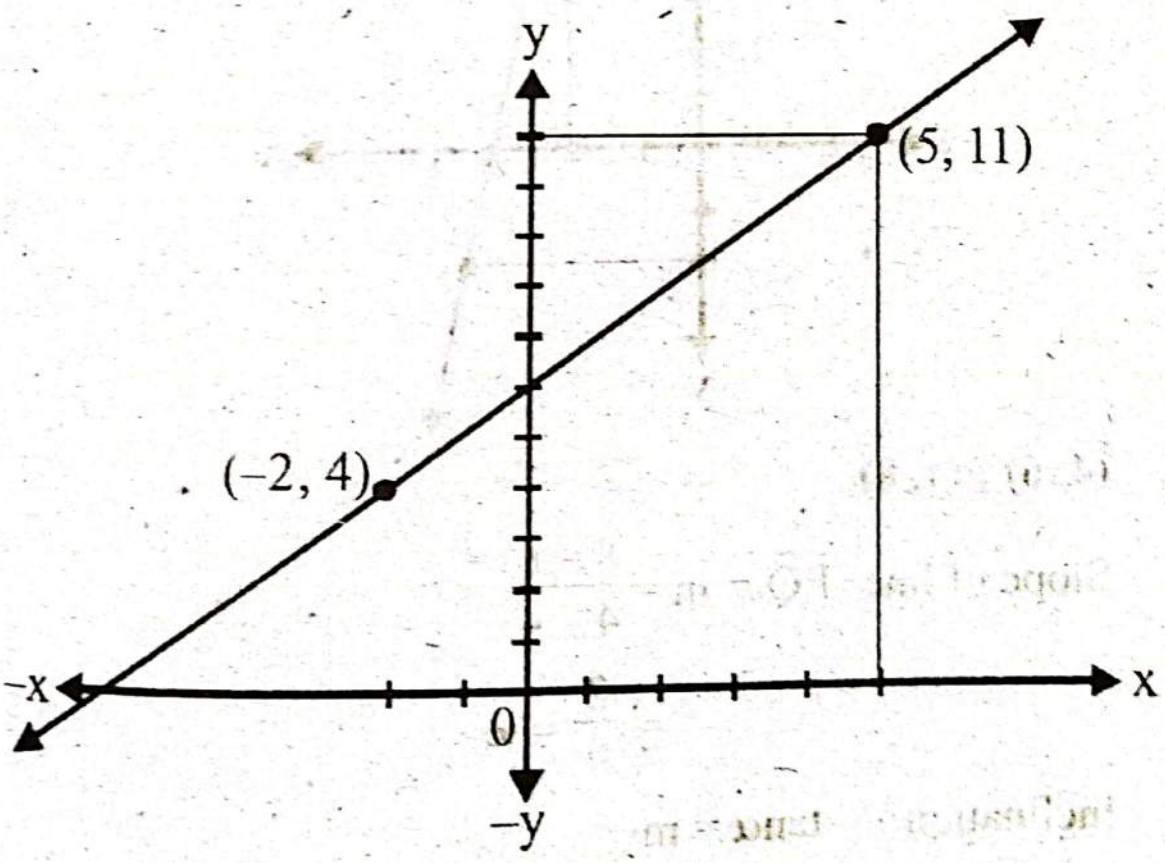
$$= \frac{11 - 4}{5 - (-2)} = \frac{7}{7} = 1$$

Inclination $\tan\alpha = m$

$\Rightarrow$ $\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45^\circ$



**(ii)** **(3, –2) ; (2, 7)**

**Sol.** Slope of line $\overline{PQ} = m = \dfrac{7-(-2)}{2-3}$

$$= \frac{7+2}{-1} = -9$$

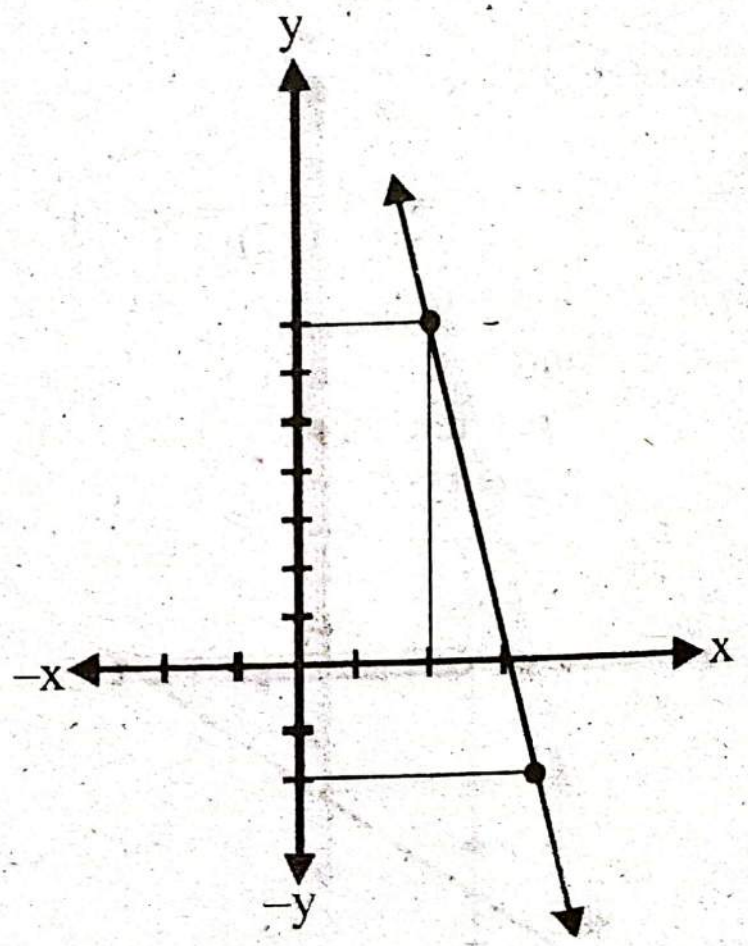
Inclination $\tan\alpha = m$

$\Rightarrow$ $\tan\alpha = -9$

$\alpha = \tan^{-1}(-9)$

$\alpha = 180^\circ - 83.66^\circ$

$\alpha = 96.34^\circ$




**(iii)** **(4, 6) ; (4, 8)**

**Sol.** Slope of line $\overline{PQ} = m = \dfrac{8-6}{4-4}$

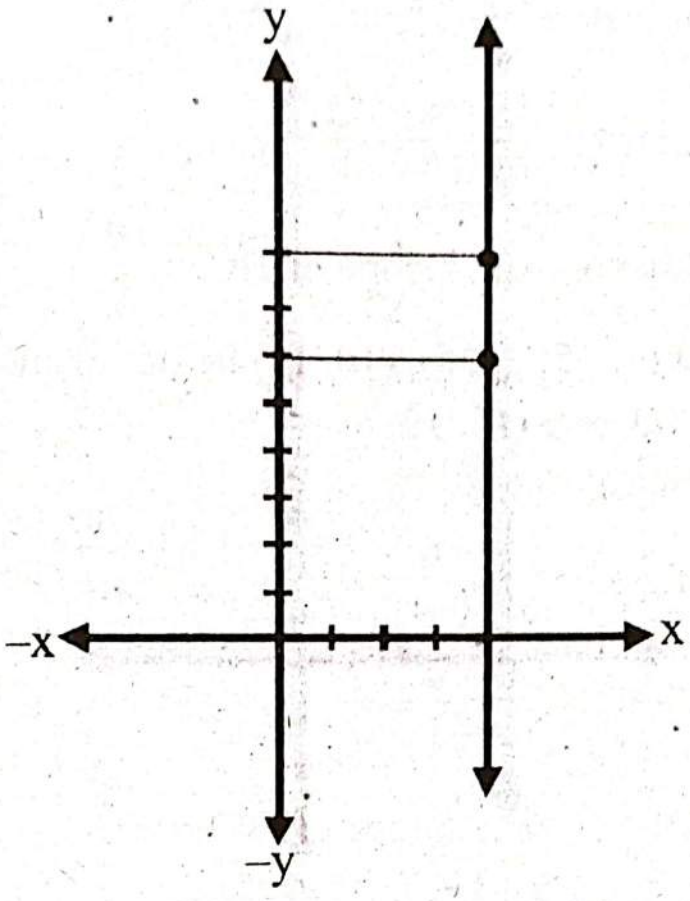
$$= \frac{2}{0} = \infty$$

Inclination $\tan\alpha = m$

$$\Rightarrow \quad \tan\alpha = \infty$$

$$\alpha = \tan^{-1}(\infty)$$

$$\alpha = 90^\circ$$




**Q.2. By means of slopes, show that the following points lie on the same line:**

**(i) (–1, –3) ; (1, 5) ; (2, 9)**

**Sol.** Let A(–1, –3), B(1, 5), C(2, 9)

If the said points lie on a same line, then

Slope of $\overline{AB}$ = slope of $\overline{BC}$

$$\text{Slope of } \overline{AB} = \frac{5+3}{1+1} = \frac{8}{2} = 4$$

$$\text{Slope of } \overline{BC} = \frac{9-5}{2-1} = \frac{4}{1} = 4$$

Since slope of $\overline{AB}$ = slope of $\overline{BC}$ = 4

So point, (–1, –3), (1, 5), (2, 9) lie on the same line.

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**(ii)** **(4, –5) ; (7, 5) ; (10, 15)**

**Sol.** Let A(4, –5), B(7, 5), C(10, 15)

Slope of $\overline{AB} = \frac{5-(-5)}{7-4} = \frac{5+5}{3} = \frac{10}{3}$

Slope of $\overline{BC} = \frac{15-5}{10-7} = \frac{10}{3}$

Since slope of $\overline{AB}$ = slope of $\overline{BC} = \frac{10}{3}$

So point (4, –5), (7, 5), (10, 15) lie on a same line.

**(iii)** **(–4, 6) ; (3, 8) ; (10, 10)**

**Sol.** Let A(–4, 6), B(3, 8), C(10, 10)

Slope of $\overline{AB} = \frac{8-6}{3-(-4)} = \frac{2}{7}$

Slope of $\overline{BC} = \frac{10-8}{10-3} = \frac{2}{7}$

Since slope of $\overline{AB}$ = slope of $\overline{BC} = \frac{2}{7}$

So point A(–4, 6), B(3, 8), C(10, 10) lie on a same line.

**(iv)** **(a, 2b) ; (c, a + b) ; (2c – a, 2a)**

**Sol.** Let A(a, 2b), B(c, a + b), C(2c – a, 2a)

Slope of $\overline{AB} = \frac{a+b-2b}{c-a} = \frac{a-b}{c-a}$

Slope of $\overline{BC} = \frac{2a-(a+b)}{2c-a-c} = \frac{2a-a-b}{c-a} = \frac{a-b}{c-a}$

Since slope of $\overline{AB}$ = slope of $\overline{BC} = \frac{a-b}{c-a}$

**Q.3.** **Find k so that the line joining A(7, 3) ; B(k, –6) and the line joining C(–4, 5) ; D(–6, 4) are:**

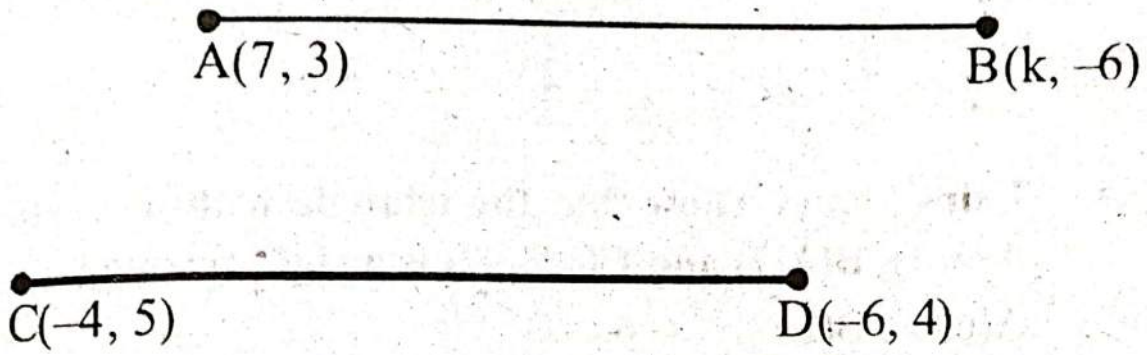
**(i)** **parallel** **(ii)** **perpendicular**

**Sol.**(i) $m_1$ = slope of $\overline{AB} = \frac{-6-3}{k-7}$

$$= \frac{-9}{k-7}$$

$$\text{slope of } \overline{CD} = \frac{4-5}{-6+4}$$

$$= \frac{-1}{-2} = \frac{1}{2}$$




As $\overline{AB} \parallel \overline{CD}$, so that are equal. i.e.

$$\frac{-9}{k-7} = \frac{1}{2}$$

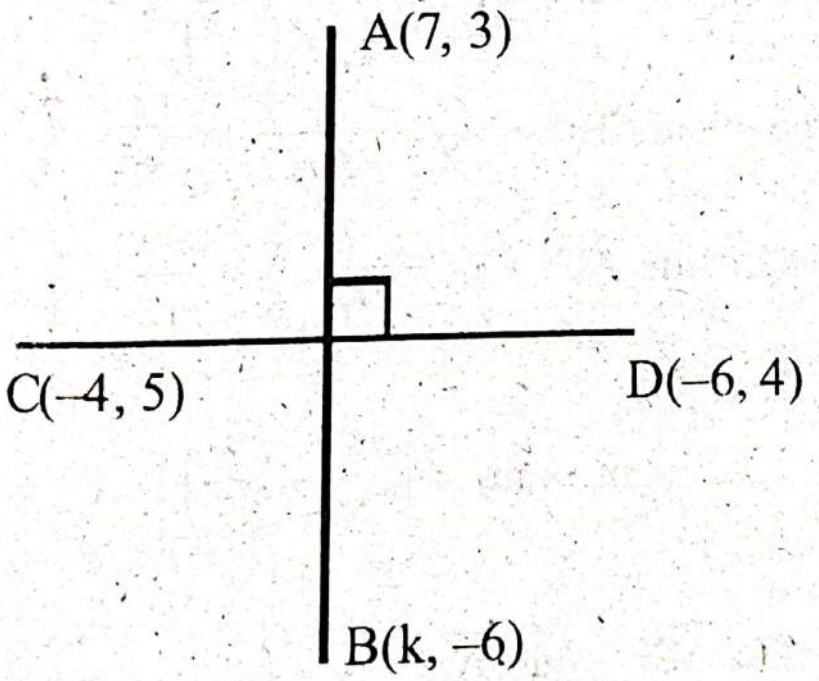
$$k - 7 = 2(-9)$$

$$k - 7 = -18$$

$$k = -18 + 7$$

$$k = -11$$

(ii) Since $\overline{AB} \perp \overline{CD}$, so product of these slopes is equal to $-1$.

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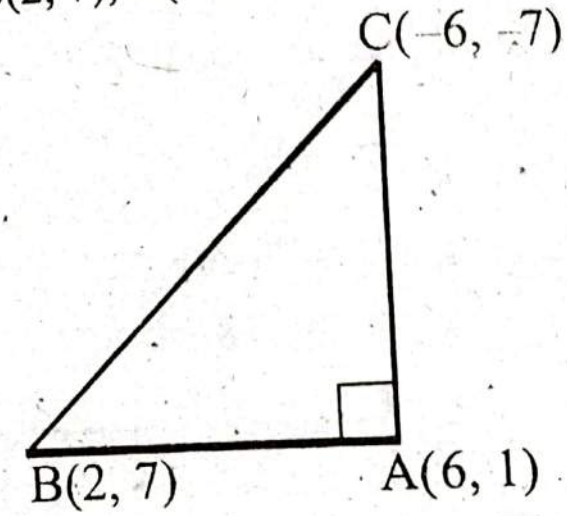
i.e. $\left(\frac{-9}{k-7}\right)\left(\frac{1}{2}\right) = -1$

$$-9 = -2(k-7)$$
$$-9 = -2k + 14$$
$$-2k = 14 + 9$$
$$-2k = 23$$
$$k = \frac{23}{2}$$

**Q.4. Using slopes, show that the triangle with its vertices A(6, 1), B(2, 7) and C(–6, –7) is a right triangle.**

**Sol.** A(6, 1), B(2, 7), C(–6, –7)




Slope of side $\overline{AB} = m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$

Slope of side $\overline{BC} = m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$

slope of side $\overline{AC} = m_3 = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$

Carefully see that

$$m_1 \times m_3 = \left(\frac{-3}{2}\right) \times \left(\frac{2}{3}\right)$$
$$= -1$$

So, side $\overline{AB} \perp$ side $\overline{AC}$

So, $\Delta ABC$ is a right triangle.

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**Q.5. Two pairs of points are given. Find whether the two lines determined by these points are:**

**(i) parallel (ii) perpendicular**

**(iii) none**

**(a) (1, –2), (2, 4) and (4, 1), (–8, 2)**

**Sol.**(a) Let A(1, –2), B(2, 4) and C(4, 1), D(–8, 2)

$$\text{Slope of } \overline{AB} = \frac{4-(-2)}{2-1} = \frac{4+2}{1} = \frac{6}{1} = 6 = m_1$$

$$\text{Slope of } \overline{CD} = \frac{2-1}{-8-4} = \frac{1}{-12} = m_2$$

(i) As $m_1 \neq m_2$, so $\overline{AB}$ is not parallel to $\overline{CD}$.

(ii) Since $m_1 \times m_2 = (6)\left(\frac{-1}{12}\right) = \frac{-1}{2} \neq -1$

So $\overline{AB}$ is not perpendicular to $\overline{CD}$.

(iii) So $\overline{AB}$ and $\overline{CD}$ are neither parallel nor perpendicular.

**(b) (–3, 4), (6, 2) and (4, 5), (–2, –7)**

**Sol.** Let A(–3, 4), B(6, 2), C(4, 5), D(–2, –7)

$$\text{Slope of } \overline{AB} = \frac{2-4}{6-(-3)} = \frac{-2}{6+3} = \frac{-2}{9} = m_1$$

$$\text{Slope of } \overline{CD} = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2 = m_2$$

(i) As $m_1 \neq m_2$, so $\overline{AB}$ and $\overline{CD}$ are not parallel.

(ii) Since $m_1 \times m_2 = \frac{-2}{9} \times 2 = \frac{-4}{9} \neq -1$

So $\overline{AB}$ and $\overline{CD}$ are not perpendicular.

**Q.6. Find an equation of:**

**(a) the horizontal line through (7, –9)**

**Sol.** The horizontal line through (7, –9)

Here $(x_1, y_1) = (7, -9)$

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As line is parallel to x-axis.

So slope $m = 0$

Equation of line is

$$y - y_1 = m(x - x_1)$$
$$y - (-9) = 0(x - 7)$$
$$y + 9 = 0$$

**(b) the vertical line through (–5, 3)**

**Sol.** The vertical line through (–5, 3)

Here $(x_1, y_1) = (-5, 3)$

As line is vertical i.e. parallel to y-axis. So $m = \infty$.

or $m = \frac{1}{0}$

Equation of line is

$$y - y = m(x - x_1)$$
$$y - 3 = \frac{1}{0}(x - (-5))$$
$$0(y - 3) = x + 5$$
$$x + 5 = 0$$

**(c) through A(–6, 5) having slope 7**

**Sol.** Here $(x_1, y_1) = (-6, 5)$, $m = 7$

Equation of line is

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 7(x - (-6))$$
$$y - 5 = 7x + 42$$
$$7x - y + 5 + 42 = 0$$
$$7x - y + 47 = 0$$

**(d) through (8, –3) having slope 0**

**Sol.** Here $(x_1, y_1) = (8, -3)$, slope $= m = 0$

Equation of line is

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 0(x - 8)$$
$$y + 3 = 0$$

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**(e) through (–8, 5) having slope undefined**

**Sol.** $m = \infty = \frac{1}{0}$

Equation of line is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{0}(x - (-8))$$

$$0(y - 5) = 1(x + 8)$$

$$x + 8 = 0$$

**(f) through (–5, –3) and (9, –1)**

**Sol.** $(x_1, y_1) = (-5, -3)$

$(x_2, y_2) = (9, -1)$

Slope of line $= m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-1 + 3}{9 - (-5)} = \frac{2}{14} = \frac{1}{7}$$

Equation of line is

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{1}{7}(x - (-5))$$

$$y + 3 = \frac{1}{7}(x + 5)$$

$$7(y + 3) = x + 5$$

$$7y + 21 = x + 5$$

$$x - 7y - 21 + 5 = 0$$

$$x - 7y - 16 = 0$$

**(g) y-intercept: –7 and slope: –5**

**Sol. Slope intercept form:**

Equation of straight line is

$$y = mx + c$$

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Here $m = -5, c = -7$

$$y = -5x - 7$$

or $5x + y + 7 = 0$

**(h) x-intercept: –3 and y-intercept: 4**

**Sol.** Here $a = -3, b = 4$

Two intercepts form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-3} + \frac{y}{4} = 1$$

$$\frac{-x}{3} + \frac{y}{4} = 1$$

$$\frac{-4x + 3y}{12} = 1$$

$$-4x + 3y = 12$$

or $4x - 3y + 12 = 0$

**(i) x-intercept: –9 and slope: –4**

**Sol.** Line intercept x-axis at $x = -9$

So $(x_1, y_1) = (-9, 0), m = -4$

Equation of line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x - (-9))$$

$$y = -4(x + 9)$$

$$y = -4x - 36$$

or $4x + y + 36 = 0$

**Q.7. Find an equation of the perpendicular bisector of the segment joining the points A(3, 5) and B(9, 8).**

**Sol.** Slope of the line through A and B

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 5}{9 - 3} = \frac{3}{6} = \frac{1}{2}$$

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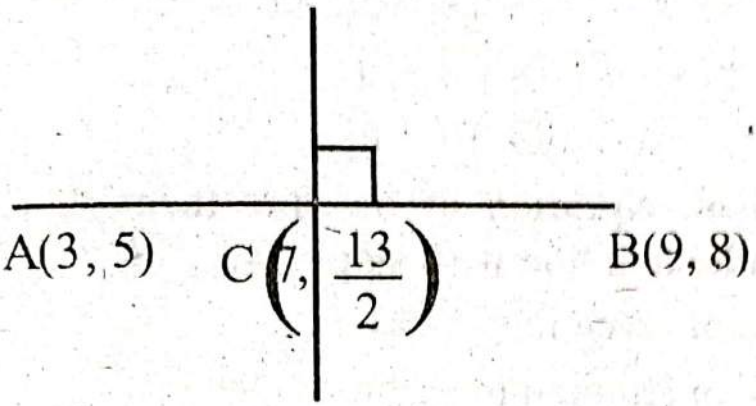
Slope of required line $= m = \frac{-1}{m_1}$

$$= \frac{-1}{\frac{1}{2}} = -2$$

(Since lines are perpendicular)

Midpoint of A and B $= \left(\frac{3+9}{2}, \frac{5+8}{2}\right)$

$$= \left(\frac{12}{2}, \frac{13}{2}\right) = C\left(6, \frac{13}{2}\right)$$




Equation of line passing through $\left(6, \frac{13}{2}\right)$ and slope $-2$ is

$$y - y_1 = m(x - x_1)$$

$$y - \frac{13}{2} = -2(x - 6)$$

$$\frac{2y - 13}{2} = -2x + 12$$

$$2y - 13 = 2(-2x + 12)$$

$$2y - 13 = -4x + 24$$

$$4x + 2y - 13 - 24 = 0$$

$$4x + 2y - 37 = 0$$

**Q.8. Find an equation of the line through (–4, –6) and perpendicular to a line having slope $\frac{-3}{2}$.**

**Sol.** Slope of given line $= \frac{-3}{2}$

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Slope of required line $= m = \dfrac{-1}{\frac{-3}{2}} = \dfrac{2}{3}$

Equation of the line passing through $(-4, -6)$ having slope $\dfrac{2}{3}$

$$y - y_1 = m(x - x_1)$$
$$y + 6 = \frac{2}{3}(x + 4)$$
$$3(y - 6) = 2(x + 4)$$
$$3y + 18 = 2x + 8$$
$$2x - 3y - 18 + 8 = 0$$
$$2x - 3y - 10 = 0$$

**Q.9. Find an equation of the line through (11, –5) and parallel to a line with slope-24.**

**Sol.** Slope of given line $= -24$

Slope of required line $= m = -24$

(As lines are parallel)

Equation of a line passing through $(11, -5)$ and having slope $-24$.

$$y - y_1 = m(x - x_1)$$
$$y + 5 = -24(x - 11)$$
$$y + 5 = -24x + 264$$
$$24x + y + 5 - 264 = 0$$
$$24x + y - 259 = 0$$

**Q.10. Convert each of the following equation into:**

**(i) Slope intercept form (ii) two intercept form**

**(iii) normal form**

**(a)** $2x - 4y + 11 = 0$ **(b)** $4x + 7y - 2 = 0$

**(c)** $-8x + 15y + 3 = 0$

**Sol.(a)** $2x - 4y + 11 = 0$

**(i) Slope intercept form:** $(y = mx + c)$

$$2x - 4y + 11 = 0$$

$$4y = 2x + 11$$

$$y = \frac{2}{4}x + \frac{11}{4}$$

$$y = \frac{1}{2}x + \frac{11}{4}$$

which is slope intercept form with $m = \frac{1}{2}$, $c = \frac{11}{4}$

**(ii) Two intercept form:** $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

$$2x - 4y = -11$$

Divide both sides by $-11$

$$\frac{2x}{-11} - \frac{4y}{-11} = \frac{-11}{-11}$$

$$\frac{x}{\frac{-11}{2}} + \frac{y}{\frac{11}{4}} = 1$$

Which is two intercept form with

$$a = \frac{-11}{2}$$

$$b = \frac{11}{4}$$

**(iii) Normal form:** $(x\cos\alpha + y\sin\alpha = p)$

$$2x - 4y = -11$$

$$\Rightarrow \quad -2x + 4y = 11 \qquad \ldots(1)$$

Divide both sides by $\sqrt{(-2)^2 + 4^2}$

$$= \sqrt{4 + 16} = \sqrt{20}$$

$$= \sqrt{4 \times 5} = 2\sqrt{5}$$

Equation (i) becomes

$$\frac{-2}{2\sqrt{5}}x + \frac{4}{2\sqrt{5}}y = \frac{11}{2\sqrt{5}}$$

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$$\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = \frac{11}{2\sqrt{5}}$$

Which is normal form with

$$\cos\alpha = \frac{-1}{\sqrt{5}}, \sin\alpha = \frac{2}{\sqrt{5}}$$

$$y = \frac{11}{2\sqrt{5}}$$

Length of perpendicular from (0, 0) to the line $2x - 4y + 11 = 0$ is $\frac{11}{2\sqrt{5}}$

**(b)** $\mathbf{4x + 7y - 2 = 0}$

**(i)** **Slope intercept form:** $(y = mx + c)$

$$7y = -4x + 2$$

$$y = \frac{-4}{7}x + \frac{2}{7}$$

which is slope intercept form with $m = \frac{-4}{7}$, $c = \frac{2}{7}$

**(ii)** **Two intercept form:** $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

$$4x + 7y = 2$$

$$\frac{4x}{2} + \frac{7y}{2} = \frac{2}{2}$$

$$2x + \frac{7y}{2} = 1$$

$$\frac{x}{\frac{1}{2}} + \frac{y}{\frac{2}{7}} = 1$$

Which is two intercept form with $a = \frac{1}{2}$, $b = \frac{2}{7}$

**(iii)** **Normal form:** $(x\cos\alpha + y\sin\alpha = p)$

$$4x + 7y = 2 \qquad \ldots(1)$$

Dividing both sides by

$$\sqrt{4^2+7^2} = \sqrt{16+49} = \sqrt{65}$$

So equation (1) becomes

$$\frac{4x}{\sqrt{65}} + \frac{7y}{\sqrt{65}} = \frac{2}{\sqrt{65}}$$

Which is normal form with

$$\cos\alpha = \frac{4}{\sqrt{65}}$$

$$\sin\alpha = \frac{7}{\sqrt{65}}, \; p = \frac{2}{\sqrt{65}}$$

Length of perpendicular from (0, 0) to line $4x + 7y - 2 = 0$ is $\frac{2}{\sqrt{65}}$

(c) $-8x + 15y + 3 = 0$

(i) **Slope intercept form:** $(y = mx + c)$

$$15y = 8x - 3$$

$$y = \frac{8}{15}x - \frac{3}{15}$$

$$= \frac{8}{15}x - \frac{1}{5}$$

Which is slope intercept form with $m = \frac{8}{15}$, $c = \frac{-1}{5}$

(ii) **Two intercept form:** $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

$$-8x + 15y = -3$$

Divide both sides by $-3$

$$\frac{-8x}{-3} + \frac{15y}{-3} = \frac{-3}{-3}$$

$$\frac{8x}{3} - 5y = 1$$

$$\frac{8x}{3} + (-5y) = 1$$

$$\frac{x}{\frac{3}{8}} + \frac{y}{\frac{-1}{5}} = 1$$

Which is two intercept form with $a = \frac{3}{8}$, $b = \frac{-1}{5}$

**(iii)** **Normal form:** $(x \cos\alpha + y \sin\alpha = p)$

$$-8x + 15y = -3$$

$$\Rightarrow \quad 8x - 15y = 3 \qquad \ldots(1)$$

Dividing both sides by

$$\sqrt{8^2 + (-15)^2} = \sqrt{64 + 225}$$

$$= \sqrt{289} = 17$$

So equation (1) becomes

$$\frac{8x}{17} + \frac{15y}{17} = \frac{3}{17}$$

Which is normal form with

$$\cos\alpha = \frac{8}{17}$$

$$\sin\alpha = \frac{-15}{17}, \; p = \frac{3}{17}$$

**Q.11. In each of the following check whether the two lines are**

**(i) parallel** **(ii) perpendicular**

**(iii) neither parallel nor perpendicular**

**(a)** $2x + y - 3 = 0$ ; $4x + 2y + 5 = 0$

**(b)** $3y = 2x + 5$ ; $3x + 2y - 8 = 0$

**(c)** $4y + 2x - 1 = 0$ ; $x - 2y - 7 = 0$

**Sol.(a)** $\ell_1 : 2x + y - 3 = 0$ ; $\ell_2 = 4x + 2y + 5 = 0$

Slope of $\ell_1 = m_1 = \frac{-9}{10}$

$$= \frac{-2}{1} = -2$$

Slope of $\ell_2 = m_2 = \frac{-a}{b}$

$$= \frac{-4}{2} = -2$$

Since $m_1 = m_2 = -2$

So given lines are parallel.

**(b)** $\ell_1 : 3y = 2x + 5$ ; $\ell_2 = 3x + 2y - 8 = 0$

**Sol.** Slope of $\ell_1 = m_1 = \frac{-a}{b}$

$$= \frac{-2}{3} = \frac{2}{3}$$

Slope of $\ell_2 = m_2 = \frac{-a}{b} = \frac{-3}{2}$

Now $m_1 \times m_2 = \frac{2}{3} \times \frac{-3}{2}$

$$m_1 m_2 = -1$$

So given lines are perpendicular.

**(c)** $2x + 4y - 1 = 0$ ; $x - 2y - 7 = 0$

**Sol.** $\ell_1: 2x + 4y - 1 = 0$

Slope of $\ell_1 = m_1 = \frac{-a}{b} = -\frac{-2}{3} = \frac{2}{3}$

$\ell_2: x - 2y - 7 = 0$

Slope of $\ell_2 = m_2 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$

As $m_1 \neq m_2$ and $m_1 \times m_2 \neq -1$

So given lines are neither parallel nor perpendicular.

**Q.12. Find an equation of the line through (–4, 7) and parallel to the line $2x - 7y + 4 = 0$.**

**Sol.** $2x - 7y + 4 = 0$

Slope $= m = -\frac{2}{-7} = \frac{2}{7}$

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As required line is parallel to the given line.

So slope of required line is $\frac{2}{7}$.

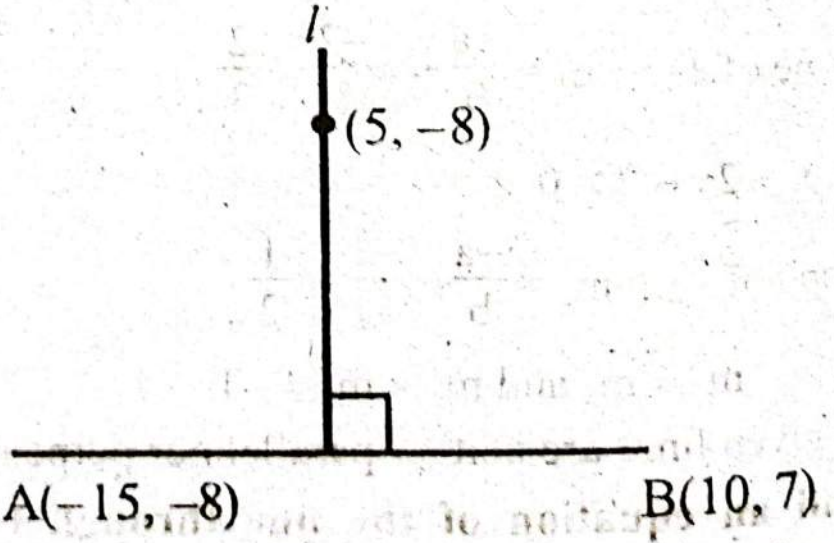
Equation of line passing through (–4, 7) and having slope $\frac{2}{7}$

$$y - y_1 = m(x - x_1)$$
$$y - 7 = \frac{2}{7}(x + 4)$$
$$7(y - 7) = 2(x + 4)$$
$$7y - 49 = 2x + 8$$
$$2x - 7y + 49 + 8 = 0$$
$$2x - 7y + 57 = 0$$

**Q.13. Find an equation of the line through (5, –8) and perpendicular to the join of A(–15, –8), B(10, 7).**

**Sol.** Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{7 + 8}{10 + 15} = \frac{15}{25}$$
$$m = \frac{3}{5}$$




As required line is perpendicular to the given line AB.

So slope of required line $= m' = \frac{-1}{\frac{3}{5}}$

$$m' = \frac{-5}{3}$$

Equation of line passing through (5, –8) and having slope $\frac{-5}{3}$ is

$$y - y_1 = m'(x - x_1)$$

$$y - (-8) = \frac{-5}{3}(x - 5)$$

$$y + 8 = \frac{-5}{3}(x - 5)$$

$$3(y + 8) = -5(x - 5)$$

$$3y + 24 = -5x + 25$$

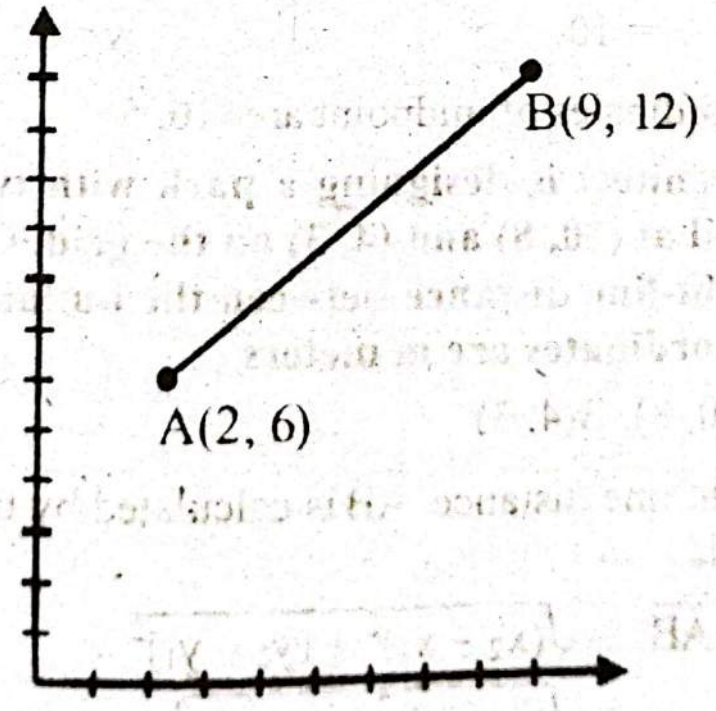
$$5x + 3y + 24 - 25 = 0$$

$$5x + 3y - 1 = 0$$

## EXERCISE 7.3

**Q.1. If the houses of two friends is represented by coordinates (2, 6) and (9, 12) on a grid. Find the straight line distance between their houses if the grid units represent kilometres?**

**Sol.** A(2, 6) for House 1, B(9, 12) for House 2

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We can find straight line distance between two houses by using distance formula.

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(9-2)^2 + (12-6)^2}$$
$$= \sqrt{7^2 + 6^2}$$
$$= \sqrt{49 + 36}$$
$$= \sqrt{85}$$
$$= 9.22 \text{ kilometres}$$

**Q.2. Consider a straight trail (represented by coordinate plane) that starts at point (5, 7) and ends at point (15, 3). What is the coordinate of the midpoint?**

**Sol.** Let A(5, 7), B(15, 3)

Let C(x, y) be the midpoint of straight trail.

Then

| | |
|---|---|
| $x = \frac{x_1 + x_2}{2}$ | $y = \frac{y_1 + y_2}{2}$ |
| $x = \frac{5 + 15}{2}$ | $y = \frac{7 + 3}{2}$ |
| $x = \frac{20}{2}$ | $y = \frac{10}{2}$ |
| $x = 10$ | $y = 5$ |

So coordinates of midpoint are (10, 5)

**Q.3. An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assume the coordinates are in meters.**

**Sol.** Let (10, 8), B(4, 3)

Straight line distance $\overline{AB}$ is calculated by using distance formula.

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-10)^2 + (3-8)^2}$$

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$$= \sqrt{(-6)^2 + (-5)^2}$$
$$= \sqrt{36 + 25}$$
$$= \sqrt{61}$$
$$= 7.81 \text{ metres}$$

**Q.4. A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other at (12, 10) on the city grid map, where each unit represents kilometers. What is the distance between the two locations?.**

**Sol.** Let A(7, 2), B(12, 10) be the two delivery locations on the city grid map.

Distance between two locations is calculated by using distance formula.

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(12 - 7)^2 + (10 - 2)^2}$$
$$= \sqrt{5^2 + 8^2}$$
$$= \sqrt{25 + 64}$$
$$= \sqrt{89}$$
$$= 9.43 \text{ km}$$

**Q.5. The start and end points of a race track are given by coordinates (3, 9) and (9, 13). What is the midpoint of the track, and how might it be used for planning water stations?**

**Sol.** Let A(3, 9), B(9, 13)

Midpoint C(x, y) = ?

Midpoint of race track

$$= \left(\frac{3+9}{2}, \frac{9+13}{2}\right)$$
$$= \left(\frac{12}{2}, \frac{22}{2}\right)$$
$$= (6, 11)$$

This midpoint will be equidistant (at equal distance) from water stations.

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**Q.6. The coordinates of two points on a road are A(3, 4) and B(7, 10). Find the midpoint of the road.**

**Sol.** Let A(3, 4), B(7, 10) be two points on the road.
Then let C(x, y) the midpoint of road. So

$$C(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{3+7}{2}, \frac{4+10}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{14}{2}\right)$$

$$= (5, 7)$$

So midpoint of the road is (5, 7).

**Q.7. A ship is navigating from Port A located at (12°N, 65°W) to Port B at (20°N, 45°W). If the ship travels along the shortest path on the surface of the Earth, describe the process to calculate the straight line distance between the points.**

**Sol.** Point A's location (12°N, 65°W)
Point B's location (20°N, 45°W)

$$\text{Midpoint latitude} = \frac{12^\circ + 20^\circ}{2} = \frac{32}{2} = 16^\circ\text{N}$$

$$\text{Midpoint longitude} = \frac{65^\circ + 45^\circ}{2} = \frac{110^\circ}{2} = 55^\circ\text{W}$$

$$\text{Straight line distance} = \sqrt{(20-12)^2 + (45-65)^2}$$

$$= \sqrt{8^2 + (-20)^2}$$

$$= \sqrt{64 + 400} = \sqrt{464} = 21.5$$

**Q.8. Farah is fencing around a rectangular field with corners at (0, 0), (0, 5), (8, 5) and (8, 0). How much fencing material will you need to cover the entire perimeter of the field?**

**Sol.** Let A(0, 0), B(0, 5), C(8, 5), D(8, 0)

$$|\overline{AB}| = \sqrt{(0-0)^2 + (5-0)^2}$$

$$= \sqrt{5^2} = 5$$

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$$|\overline{BC}| = \sqrt{(8-0)^2 + (5-5)^2}$$
$$= \sqrt{8^2 + 0^2}$$
$$= \sqrt{8^2} = 8$$
$$|\overline{CD}| = \sqrt{(8-8)^2 + (0-5)^2}$$
$$= \sqrt{0^2 + (-5)^2}$$
$$= \sqrt{25} = 5$$
$$|\overline{DA}| = \sqrt{(0-8)^2 + (0-5)^2}$$
$$= \sqrt{(-8)^2 + (-5)^2}$$
$$= \sqrt{64 + 25}$$
$$= \sqrt{89}$$
$$|\overline{DA}| = 9.43$$

Since $|\overline{BC}| \neq |\overline{DA}|$.

So given points are not related to rectangle.

We just add all these lengths to find perimeter of field. (fencing material)

Fencing material needed = 5 + 8 + 5 + 9.43

= 27.43 unit

**Q.9. An airplane is flying from city X at (40°N, 100°W) to city Y at (50°N, 80°W). Use coordinate geometry, calculate the shortest distance between these two cities.**

**Sol.** City X's location (40°N, 100°W)

City Y's location (50°N, 80°W)

$$\text{Midpoint latitude} = \frac{40+50}{2} = \frac{90}{2} = 45^\circ\text{N}$$

$$\text{Midpoint longitude} = \frac{100+80}{2} = \frac{180}{2} = 90^\circ\text{W}$$

Let d be the shortest distance between to cities. Then

$$d = \sqrt{(50-40)^2 + (80-100)^2}$$

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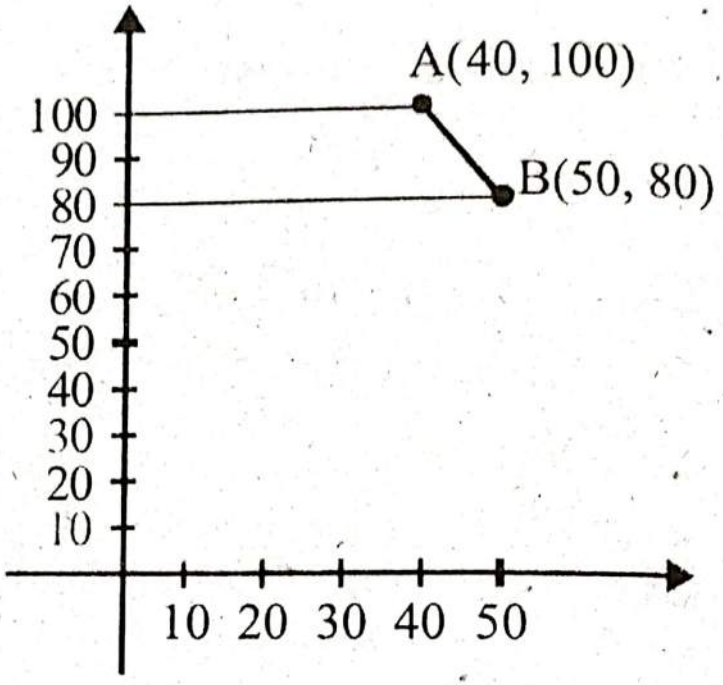
$= \sqrt{10^2 + (-20)^2}$

$= \sqrt{100 + 400}$

$= \sqrt{500} = 22.4$




**Q.10. A land surveyor is marking out a rectangular plot of land with corners at (3, 1), (3, 6), (8, 6), and (8, 1). Calculate the perimeter.**

**Sol.** Let A(3, 1), B(3, 6), C(8, 6), D(8, 1)

$$|\overline{AB}| = \sqrt{(3-3)^2 + (6-1)^2}$$
$$= \sqrt{0^2 + 5^2}$$
$$= \sqrt{25} = 5$$

$$|\overline{BC}| = \sqrt{(8-3)^2 + (6-6)^2}$$
$$= \sqrt{5^2 + 0^2}$$
$$= \sqrt{25} = 5$$

$$|\overline{CD}| = \sqrt{(8-8)^2 + (1-6)^2}$$
$$= \sqrt{0^2 + (-5)^2}$$
$$= \sqrt{25} = 5$$

$$|\overline{DA}| = \sqrt{(8-3)^2 + (1-1)^2}$$
$$= \sqrt{5^2 + 0^2}$$
$$= \sqrt{25} = 5$$

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Perimeter of pond $= 5 + 5 + 5 + 5$

$= 20$ units

**Q.11. A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the coordinates: A(0, 0), B(5, 0), C(5, 3), and D(0, 3). How much fencing is required?**

**Sol.** A(0, 0), B(5, 0), C(5, 3), D(0, 5)

$$|\overline{AB}| = \sqrt{(5-0)^2 + (0-0)^2}$$
$$= \sqrt{25} = 5$$
$$|\overline{BC}| = \sqrt{(5-5)^2 + (3-0)^2}$$
$$= \sqrt{0+9} = 3$$
$$|\overline{CA}| = \sqrt{(0-5)^2 + (3-3)^2}$$
$$= \sqrt{25+0} = 5$$
$$|\overline{DA}| = \sqrt{(0-0)^2 + (3-0)^2}$$
$$= \sqrt{9} = 3$$

Fencing required $= 5 + 3 + 5 + 3 = 16$

## REVIEW EXERCISE 7

**Q.1. Four options are given against each statement. Encircle the correct option.**

(i) The equation of a straight line in the slope-intercept form is written as:

(a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$

(c) $y = c + mx$ (d) $(ax + by + c = 0)$

**Sol.** So option (c) is correct.

(ii) The gradient of two parallel lines is:

(a) Equal (b) Zero

(c) Negative reciprocals of each other

(d) Always undefined

**Sol.** So option (a) is correct.

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(iii) If the product of the gradients of two lines is $-1$, then the lines are:

(a) parallel (b) perpendicular

(c) collinear (d) coincident

**Sol.** So option (b) is correct.

(iv) Distance between two points P(1, 2) and Q(4, 6) is:

(a) 5 (b) 6

(c) $\sqrt{13}$ (d) 4

**Sol.** P(1, 2), Q(4, 6)

Distance between P and Q

$$= \sqrt{(4-1)^2 + (6-2)^2}$$
$$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$$
$$= \sqrt{25} = 5$$

So option (a) is correct.

(v) The midpoint of a line segment with endpoint $(-2, 4)$ and $(6, -2)$ is:

(a) (4, 2) (b) (2, 1)

(c) (1, 1) (d) (0, 0)

**Sol.** $(-2, 4), (6, -2)$

$$\text{Midpoint} = \left(\frac{-2+6}{2}, \frac{4+(-2)}{2}\right)$$
$$= \left(\frac{4}{2}, \frac{2}{2}\right)$$
$$= (2, 1)$$

So option (b) is correct.

(vi) A line passing through points (1, 2) and (4, 5) is:

(a) $y = x + 1$ (b) $y = 2x + 3$

(c) $y = 3x - 2$ (d) $y = x + 2$

**Sol.** A(1, 2), B(4, 5)

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

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$$= \frac{5-2}{4-1} = \frac{3}{3} = 1$$

Equation of line is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = x - 1$$

or $$y = x - 1 + 2$$

$$y = x + 1$$

Option (a) is correct.

(vii) The equation of a line in point-slope form is:

(a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$

(c) $y = c + mx$ (d) $(ax + by + c = 0)$

**Sol.** $y - y_1 = m(x - x_1)$ is point-slope form of equation of line.

So option (b) is correct.

(viii) $2x + 3y - 6 = 0$ in the slope-intercept form is.

(a) $y = \frac{-2}{3}x + 2$ (b) $y = \frac{2}{3}x - 2$

(c) $y = \frac{2}{3}x + 1$ (d) $y = \frac{-2}{3}x - 2$

**Sol.**

$$2x + 3y - 6 = 0$$

$$3y = -2x + 6$$

$$y = \frac{-2}{3}x + \frac{6}{3}$$

$$y = \frac{-2}{3}x + 2$$

So option (a) is correct.

(ix) The equation of a line in symmetric form is:

(a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $\frac{x - x_1}{l} + \frac{y - y_1}{m} = \frac{z - z_1}{l}$

(b) $ax + by + c = 0$ (d) $y - y_1 = m(x - x_1)$

**Sol.** $\frac{x}{a} + \frac{y}{b} = 1$ is symmetric form.

So option (a) is correct.

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(x) The equation of a line in normal form is:

(a) $y = mx + c$ (b) $\frac{x}{a} + \frac{y}{b} = 1$

(c) $\frac{x - x_1}{\cos\alpha}$ (d) $x\cos\alpha + y\sin\alpha = p$

**Sol.** So option (d) is correct.

**Q.2. Find the distance between two points A(2, 3) and B(7, 8) on a coordinate plane.**

**Sol.**

$$|\overline{AB}| = \sqrt{(7-2)^2 + (8-3)^2}$$
$$= \sqrt{5^2 + 5^2}$$
$$= \sqrt{25 + 25} = \sqrt{50}$$
$$= 7.07 \text{ units}$$

**Q.3. Find the midpoint of the line segment joining the points (4, –2) and (–6, 3).**

**Sol.** Midpoint $= \left(\frac{4 + (-6)}{2}, \frac{-2 + 3}{2}\right)$

$$= \left(\frac{4 - 6}{2}, \frac{1}{2}\right)$$
$$= \left(\frac{-2}{2}, \frac{1}{2}\right) = \left(-1, \frac{1}{2}\right)$$

**Q.4. Calculate the gradient (slope) of the line passing through the points (1, 2) and (4, 6)**

**Sol.** Let A(1, 2), B(4, 6)

Gradient (slope) of the line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{6 - 2}{4 - 1} = \frac{4}{3}$$

**Q.5. Find the equation of the line in the form $y = mx + c$ that passes through the points (3, 7) and (5, 11).**

**Sol.** Let A(3, 7), B(5, 11)

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Slope of the line = m $= \frac{11-7}{5-3}$

$= \frac{4}{2} = 2$

Equation of line is

$$y - y_1 = m(x - x_1)$$
$$y - 7 = 2(x - 3)$$
$$y - 7 = 2x - 6$$
$$y = 2x - 6 + 7$$
$$y = 2x + 1$$

**Q.6. If two lines are parallel, and one line has a gradient of $\frac{2}{3}$, what is the gradient of the other line? What about if the lines were perpendicular?**

**Sol.** Since given that two lines are parallel. If one line has gradient (slope) $= \frac{2}{3}$, then other line will also have the same gradient (slope) i.e. gradient of other line $\frac{2}{3}$.

If lines were perpendicular, then product of their gradient would be $-1$.

**Q.7. An airplane needs to fly from city A at coordinates (12, 5) to city B at coordinates (8, –4). Calculate the straight-line distance between these two cities.**

**Sol.** City A has coordinates (12, 5)

City B has coordinates (8, –4)

Straight line distance $= \sqrt{(8-12)^2 + (-4-5)^2}$

$= \sqrt{(-4)^2 + (-9)^2}$

$= \sqrt{16 + 81}$

$= \sqrt{97} = 9.85$ units

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**Q.8. In a landscaping project, the path starts at (2, 3) and ends at (10, 7). Find the midpoint.**

**Sol.** Let A(2, 3), B(10, 7)

$$\text{Midpoint of garden path} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2 + 10}{2}, \frac{3 + 7}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{10}{2}\right)$$

$$= (6, 5)$$

**Q.9. A drone is flying from point (2, 3) to point (10, 15) on the grid. Calculate the gradient of the line along which the drone is flying and the total distance travelled.**

**Sol.** Let A(2, 3), B(10, 15)

$$\text{Gradient of line} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{15 - 3}{10 - 2} = \frac{12}{8} = \frac{3}{2}$$

Total distance travelled is calculated by using distance formula.

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(10 - 2)^2 + (15 - 3)^2}$$

$$= \sqrt{8^2 + 12^2} = \sqrt{64 + 144}$$

$$= \sqrt{208} = 14.42 \text{ units}$$

**9. For a line with a gradient of (–3) and a y-intercept of (2), write the equation of the line in:**

**(a) Slope-intercept form**

**(b) Point-slope form (using the point (1, 2))**

**(c) Two-point form (using the points (1, 2) and (4, –7))**

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(d) **Intercept form**

(e) **Symmetric form**

(f) **Normal form**

**Sol.**(a) Gradient = m = –3, y-intercept = c = 2

Slope intercept form is $y = mx + c$

$$y = -3x + 2$$

(b) Slope = m = –3, point $(x_1, y_1) = (1, 2)$

Point slope form is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -3(x - 1)$$

$$y - 2 = -3x + 3$$

$$3x + y - 2 - 3 = 0$$

$$3x + y - 5 = 0$$

(c) Two point form is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - 1}{x - 2} = \frac{-7 - 2}{4 - 1}$$

or $$\frac{y - 1}{-7 - 2} = \frac{x - 2}{4 - 1}$$

(d) Intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

We consider first part as

$$y = -3x + 2$$

$$\frac{3}{2}x + \frac{y}{2} = \frac{2}{2}$$

$$\frac{x}{\frac{2}{3}} + \frac{y}{2} = 1$$

(e) Symmetric form consider

$$y = -3x + 2$$

$$3x + y = 2$$

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Divide both sides by $\sqrt{3^2 + 1^2} = \sqrt{10}$

$$\frac{3}{\sqrt{10}}x + \frac{1}{\sqrt{10}}y = \frac{2}{\sqrt{10}}$$

or $$\frac{y}{\sqrt{10}} + \frac{3x}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$

(f) Normal form

Normal form is $x \cos\alpha + y \sin\alpha = p$

Consider

$$y = -3x + 2$$

$$3x + y = 2$$

Divide both sides by $\sqrt{3^2 + 1^2} = \sqrt{10}$

$$\frac{3}{\sqrt{10}}x + \frac{1}{\sqrt{10}}y = \frac{2}{\sqrt{10}}$$

Here $\cos\alpha = \frac{3}{\sqrt{10}}$, $\sin\alpha = \frac{1}{\sqrt{10}}$, $p = \frac{2}{\sqrt{10}}$

$$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$$

$$\alpha = 18.43^\circ$$

Normal form is

$$x\cos(18.43^\circ) + y\sin(18.43^\circ) = \frac{2}{\sqrt{10}}$$

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