Unit No. 6

Trigonometry

Exercise No. 6.1

Question No. 1

Find in which quadrant the following angles lie. Write a co-terminal angle for each:

(i) 65°

Solution:

65°

Quadrant: 1st quadrant (0° to 90°)

Co-terminal = 65° + 360° = 425°

(ii) 135°

Solution:

135°

Quadrant: 2nd quadrant (90° to 180°)

Co-terminal = 135° − 360° = −225°

(iii) –40°

Solution:

–40°

Negative angle: Rotate clockwise from 0°

Standard angle = −40° + 360° = 320°

Quadrant: 4th quadrant (270° to 360°)

Co-terminal angle: 320°

(iv) 210°

Solution:

210°

Quadrant: 3rd quadrant (180° to 270°)

Co-terminal = 210° − 360° = −150°

(v) –150°

Solution:

–150°

Standard angle: –150° + 360° = 210°

Quadrant: 3rd quadrant (180° to 270°)

Co-terminal angle: 210°

Question No. 2

Convert the following into degrees, minutes, and seconds:

(i) 123.456°

Solution:

123.456°

Degrees = 123°

Decimal = 0.456 × 60 = 27.36′

Minutes = 27′

Seconds = 0.36 × 60 = 21.6″

123° 27′ 21.6″

(ii) 58.7891°

Solution:

58.7891°

Degrees = 58°

Decimal = 0.7891 × 60 = 47.346′

Minutes = 47′

Seconds = 0.346 × 60 = 20.76″

58° 47′ 20.76″

(iii) 90.5678°

Solution:

90.5678°

Degrees = 90°

Decimal = 0.5678 × 60 = 34.068′

Minutes = 34′

Seconds = 0.068 × 60 = 4.08″

90° 34′ 4.08″

Question No. 3

Convert the following into decimal degrees:

(i) 65° 32′ 15″

Solution:

Decimal Degrees= $D + \frac{M}{60} + \frac{S}{3600}$

65° 32′ 15″

$= 65 + \frac{32}{60} + \frac{15}{3600}$

$= 65 + 0.5333 + 0.0042$

$= 65.5375°$

(ii) 42° 18′ 45″

Solution:

Decimal Degrees= $D + \frac{M}{60} + \frac{S}{3600}$

$42° 18′ 45″$

$= 42 + \frac{18}{60} + \frac{45}{3600}$

$= 42 + 0.3 + 0.0125$

$= 42.3125°$

(iii) 78° 45′ 36″

Solution:

Decimal Degrees= $D + \frac{M}{60} + \frac{S}{3600}$

$78° 45′ 36″$

$= 78 + \frac{45}{60} + \frac{36}{3600}$

$= 78 + 0.75 + 0.01$

$= 78.76°$

Question No. 4

Convert the following into radians:

(i) 36°

Solution:

$36°$

Radians = Degrees × $\frac{\pi}{180}$

Radians = $36 \times \frac{\pi}{180}$

Radians = $\frac{\pi}{5}\ rad$

(ii) 22.5°

Solution:

$22.5°$

Radians = Degrees × $\frac{\pi}{180}$

Radians = $22.5 \times \frac{\pi}{180}$

Radians = $\frac{\pi}{8}\ rad$

(iii) 67.5°

Solution:

67.5°

Radians = Degrees × $\frac{\pi}{180}$

Radians = 67.5 × $\frac{\pi}{180}$

Radians = $\frac{3\pi}{8}\ rad$

Question No. 5

Convert the following into degrees:

(i) $\frac{\pi}{16}\ rad$

Solution:

$\frac{\pi}{16}\ rad$

Degrees = Radians × $\frac{180}{\pi}$

Degrees = $\frac{\pi}{16}$ × $\frac{180}{\pi}$

Degrees = 11.25°

(ii) $\frac{11\pi}{5}\ rad$

Solution:

$\frac{11\pi}{5}\ rad$

Degrees = Radians × $\frac{180}{\pi}$

Degrees = $\frac{11\pi}{5}$ × $\frac{180}{\pi}$

Degrees = 396°

(iii) $\frac{7\pi}{6}\ rad$

Solution:

$\frac{7\pi}{6}\ rad$

Degrees = Radians × $\frac{180}{\pi}$

Degrees = $\frac{7\pi}{6}$ × $\frac{180}{\pi}$

Degrees = 210°

Question No. 6

Find the arc length and area of a sector if:

(i) r = 6 cm and central angle θ = π/3 radians.

Data:

$r = 6\ cm$

$central\ angle\ \theta = \frac{\pi}{3}\ rad$

To Find:

$l = ?$

$A = ?$

Solution:

$l = r\ \theta$

$l = 6 \times \frac{\pi}{3}$

$l = 2\pi$

$l = 6.28\ cm$

$A = \frac{1}{2}\ r^2\ \theta$

$A = \frac{1}{2}\ (6)^2\ (\frac{\pi}{3})$

$A = (36)\ (\frac{\pi}{6})$

$A = 6\pi\ cm^2$

$A = 18.84\ cm^2$

(ii) r = 4.8/π cm and central angle θ = 5π/6 radians.

Data:

$\boldsymbol{r = \frac{4.8}{\pi}\ cm}$

$\boldsymbol{central\ angle\ \theta = \frac{5\pi}{6}\ rad}$

To Find:

$\boldsymbol{l} = ?$

$\boldsymbol{A} = ?$

Solution:

$l = r\ \theta$

$l = \frac{4.8}{\pi} \times \frac{5\pi}{6}$

$$l = 4\ cm$$

$$A = \frac{1}{2}\ r^2\ \theta$$

$$A = \frac{1}{2}\ (\frac{4.8}{\pi})^2\ (\frac{5\pi}{6})$$

$$A = (\frac{23.04}{\pi})\ (\frac{5}{12})$$

$$A = \frac{9.6}{\pi}$$

$$A = 3.06\ cm^2$$

Question No. 7

If the central angle of a sector is 60° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it represents.

Data:

$$\boldsymbol{r = 12\ cm}$$

$$\boldsymbol{central\ angle\ \theta = 60°}$$

$$\boldsymbol{central\ angle\ \theta = 60 \times \frac{\pi}{180}\ rad}$$

$$\boldsymbol{central\ angle\ \theta = \frac{\pi}{3}\ rad}$$

To Find:

A = ?

% age = ?

Solution:

$$Area\ of\ Sector = \frac{1}{2}\ r^2\ \theta$$

$$Area\ of\ Sector = \frac{1}{2}\ (12)^2\ (\frac{\pi}{3})$$

$$Area\ of\ Sector = (144)\ (\frac{\pi}{6})$$

$$Area\ of\ Sector = 24\pi$$

$$Area\ of\ Sector = 75.36\ cm^2$$

$$Area\ of\ Circle = \pi r^2$$

$$Area\ of\ Circle = \pi(12)^2$$

$$Area\ of\ Circle = \pi\ (144)$$

$$Area\ of\ Circle = 452.16\ cm^2$$

$$Required\ \%age = \frac{Area\ of\ Sector}{Area\ of\ Circle} \times 100\%$$

$$Required\ \%age\ = \frac{75.36}{452.16}\ \times 100\%$$

$$Required\ \%age\ =\ 16.67\ \%$$

Question No. 8

Find the percentage of the area of sector subtending an angle π/8 radians.

Data:

$$\boldsymbol{central\ angle\ \theta = \frac{\pi}{8}\ rad}$$

To Find:

% age = ?

Solution:

$$Area\ of\ Sector = \frac{1}{2}\ r^2\ \theta$$

$$Area\ of\ Sector = \frac{1}{2}\ r^2\ (\frac{\pi}{8})$$

$$Area\ of\ Sector = \frac{\pi\ r^2}{16}$$

$$Area\ of\ Circle\ =\ \pi r^2$$

$$Required\ \%age\ = \frac{Area\ of\ Sector}{Area\ of\ Circle}\ \times 100\%$$

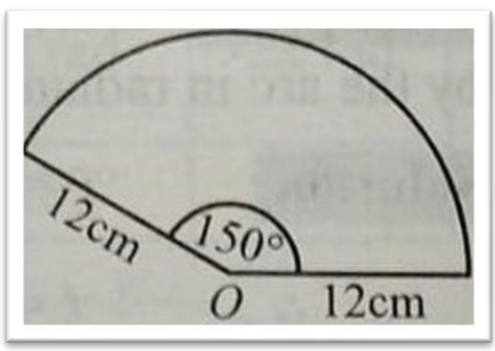
$$Required\ \%age\ = \frac{\pi r^2}{16\pi r^2}\ \times 100\%$$

$$Required\ \%age\ = \frac{100}{16}\ \%$$

$$Required\ \%age\ =\ 6.25\ \%$$

Question No. 9

A circular sector of radius r = 12 cm has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?



Hint: Arc length of sector = circumference of cone.

Data:

$$r\ =\ 12\ cm$$

$central\ angle\ \theta = 150°$

$$central\ angle\ \theta = 150\ \times \frac{\pi}{180}\ rad$$

$$central\ angle\ \theta = \frac{5\pi}{6}\ rad$$

To Find:

$l = ?$

Solution:

Arc length of the sector becomes the cone's base circumference:

$$Arc\ length = r\ .\ \theta$$

$$Arc\ length = 12\ .\ (\frac{5\pi}{6})$$

$$Arc\ length = 10\pi\ cm$$

Cone base circumference:

$$R = \frac{10\ \pi}{2\ \pi}$$

$$R = 5\ cm$$

Slant height of cone:

$l = 12\ cm$ (same as radius of sector)