Unit No. 3

Set and Functions

Exercise No. 3.2

Question No. 1

Consider the universal set ,

U={x : x is a multiple of 2 and 0 < x ≤ 30},

A= {x : x is a multiple of 6} and

B = {x : x is a multiple of 8}

(i) List all elements of sets A and B in tabular form.

(ii) Find A∩B

(iii) Draw a Venn diagram.

Step 1:

U = {x : x is a multiple of 2 and 0 < x ≤ 30}

Solution:

U = {2,4,6,8,10,12,14,16,18,20,22,24,26,28,30}

A = {x : x is a multiple of 6}

Solution:

A = {6,12,18,24,30}

B = {x : x is a multiple of 8}

Solution:

B = {8,16,24}

Step 2:

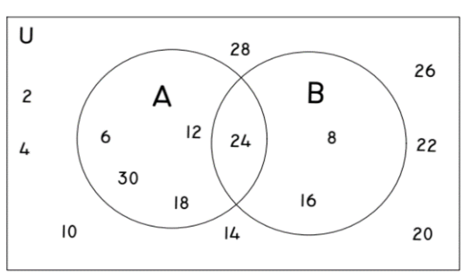
A∩B = {6,12,18,24,30} ∩ {8,16,24}

Intersection means the common elements in both sets:

A∩B = {24}

Step 3:

Draw a Venn Diagram:



Question No. 2

Let, U = {x : x is aninteger and 0 ≤ x ≤ 150}

G = {x : x = 2^m integer m and 0 < x ≤ 150}

H = {x : x is a square}

(i) List all elements of sets G and H in tabular form.

(ii) Find G∪H

(iii) Find G ∩ H

Given:

U = {x : x is aninteger and 0 ≤ x ≤ 150}

G = {x : x = 2^m integer m and 0 < x ≤ 150}

H = {x : x is a square}

Step 1:

List elements of G and H:

G = {x:x = 2^m integer m and 0 < x ≤ 150}

G = {1, 2, 4, 8, 16, 32, 64, 128}

H = {x : x is a square}

H = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}

Step 2:

Find G ∪ H

Union includes all unique elements from both sets:

G ∪ H = {1, 2, 4, 8, 16, 32, 64, 128} ∪ {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}

G∪H={1, 2, 4, 8, 9, 16, 25, 32, 36, 49, 64, 81, 100, 121, 128, 144}

Step 3:

Find G ∩ H

Intersection includes common elements in both sets:

G ∩ H = {1, 2, 4, 8, 16, 32, 64, 128} ∩ {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}

G ∩ H = {1, 4, 16, 64}

Question No. 3

Consider the sets

P = {x : x is a prime number and 0 < x ≤ 20}
Q = {x : x is a divisor of 210 and 0 < x ≤ 20}

(i) Find P ∩ Q
(ii) Find P ∪ Q

Given:

P = {x : x is a prime number and 0 < x ≤ 20}

Q = {x : x is a divisor of 210 and 0 < x ≤ 20}

Step 1:

List elements of P and Q

Elements of P (prime numbers up to 20):

P = {2, 3, 5, 7, 11, 13, 17, 19}

Elements of Q (divisors of 210 up to 20):

Q = {1, 2, 3, 5, 6, 7, 10, 14, 15}

Step 2:

Find P ∩ Q

Common elements in both sets:

P ∩ Q = {2, 3, 5, 7, 11, 13, 17, 19} ∩ {1, 2, 3, 5, 6, 7, 10, 14, 15}

P ∩ Q = {2, 3, 5, 7}

Step 3:

Find P ∪ Q

All unique elements from both sets:

P ∪ Q = {2, 3, 5, 7, 11, 13, 17, 19} ∪ {1, 2, 3, 5, 6, 7, 10, 14, 15}

P ∪ Q = {1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19}

Question No. 4

Verify the Commutative Properties of Union and Intersection for the following pairs of sets:

(i). A = {1, 2, 3, 4, 5},　　B = {4, 6, 8, 10}

(ii). N, Z

(iii). A = {x|x ∈ R ^ x ≥ 0},　　B = R

The commutative property states that for any two sets A and B:

- **A ∪ B = B ∪ A**
- **A ∩ B = B ∩ A**

Step 1:

(i). Given sets:

A = {1, 2, 3, 4, 5}

B = {4, 6, 8, 10}

- **Union:**

A ∪ B = {1, 2, 3, 4, 5} ∪ {4, 6, 8, 10}

A ∪ B = {1, 2, 3, 4, 5, 6, 8, 10}

B ∪ A = {4, 6, 8, 10} ∪ {1, 2, 3, 4, 5}

B ∪ A = {1, 2, 3, 4, 5, 6, 8, 10}

So proved that $A \cup B = B \cup A$

- **Intersection:**

$A \cap B = \{1, 2, 3, 4, 5\} \cap \{4, 6, 8, 10\}$
$A \cap B = \{4\}$
$B \cap A = \{4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5\}$
$B \cap A = \{4\}$

So proved that $A \cap B = B \cap A$

(ii). Given sets:

$A = N = \{1, 2,3, 4, 5,\ldots\}$

$B = Z = \{0, \pm1, \pm2, \pm3, \pm4, \pm5\ldots\}$

- **Union:**

$A \cup B = \{1, 2, 3, 4, 5,\ldots\} \cup \{0, \pm1, \pm2, \pm3, \pm4, \pm5\ldots\}$

$A \cup B = \{0, \pm1, \pm2, \pm3, \pm4, \pm5\ldots\}$

$B \cup A = \{0, \pm1, \pm2, \pm3, \pm4, \pm5 \ldots\} \cup \{1, 2, 3, 4, 5,\ldots\}$

$B \cup A = \{0, \pm1, \pm2, \pm3, \pm4, \pm5\ldots\}$

So proved that $A \cup B = B \cup A$

- **Intersection:**

$A \cap B = \{1, 2, 3, 4, 5,\ldots\} \cap \{0, \pm1, \pm2, \pm3, \pm4, \pm5\ldots\}$
$A \cap B = \{1, 2, 3, 4, 5,\ldots\}$
$B \cap A = \{0, \pm1, \pm2, \pm3, \pm4, \pm5\ldots\} \cap \{1, 2, 3, 4, 5,\ldots\}$
$B \cap A = \{1, 2, 3, 4, 5,\ldots\}$

So proved that $A \cap B = B \cap A$

2nd Method:

$A = N$

$B = Z$

- **Union:**

$A \cup B = N \cup Z$

$A \cup B = Z$

$B \cup A = Z \cup N$

$B \cup A = Z$

So proved that $A \cup B = B \cup A$

- **Intersection:**

$A \cap B = N \cap Z$
$A \cap B = N$
$B \cap A = Z \cap N$
$B \cap A = N$

So proved that $A \cap B = B \cap A$

(iii) Given sets:

$A = \{x|x \in R \wedge x \geq 0\}$

$B = R$

- **Union:**

  $A \cup B = \{x|x \in R \wedge x \geq 0\} \cup R$

  $A \cup B = R$

  $B \cup A = R \cup \{x|x \in R \wedge x \geq 0\}$

  $B \cup A = R$

  So proved that $A \cup B = B \cup A$

- **Intersection:**

  $A \cap B = \{x|x \in R \wedge x \geq 0\} \cap R$
  $A \cap B = \{x|x \in R \wedge x \geq 0\}$
  $B \cap A = R \cap \{x|x \in R \wedge x \geq 0\}$
  $B \cap A = \{x|x \in R \wedge x \geq 0\}$

  So proved that $A \cap B = B \cap A$

Question No. 5

Let, U = {a,b,c,d,e,f,g,h,i,j}

A = {a,b,c,d,g,h}

B = { c,d,e,f,j }

Verify De Morgan`s Laws for these sets.

Given sets:

U = {a,b,c,d,e,f,g,h,i,j}

A = {a,b,c,d,g,h}
B = {c,d,e,f,j}

Draw Venn diagram.

1. $(A \cup B)^C = A^C \cap B^C$

2. $(A \cap B)^C = A^C \cup B^C$

- **Left side of first law:**
  $A \cup B = \{a,b,c,d,g,h\} \cup \{c,d,e,f,j\}$

  $A \cup B = \{a,b,c,d,e,f,g,h,j\}$
  $(A \cup B)^C = \{a,b,c,d,e,f,g,h,i,j\} - \{a,b,c,d,e,f,g,h,j\}$

  $(A \cup B)^C = \{i\}$

- **Right side of first law:**
  $A^C = \{a,b,c,d,e,f,g,h,i,j\} - \{a,b,c,d,g,h\}$

  $A^C = \{e,f,i,j\}$
  $B^C = \{a,b,c,d,e,f,g,h,i,j\} - \{c,d,e,f,j\}$

$B^C = \{a,b,g,h,i\}$
$A^C \cap B^C = \{e,f,i,j\} \cap \{a,b,g,h,i\}$

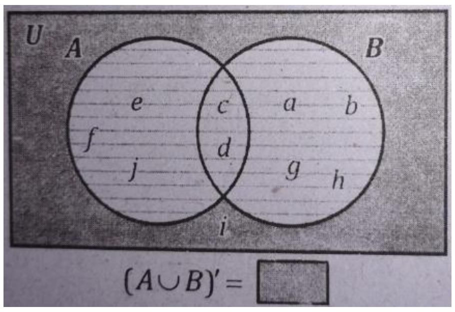
$(A \cap B)^C = \{i\}$

Thus, first De Morgan's law proved.

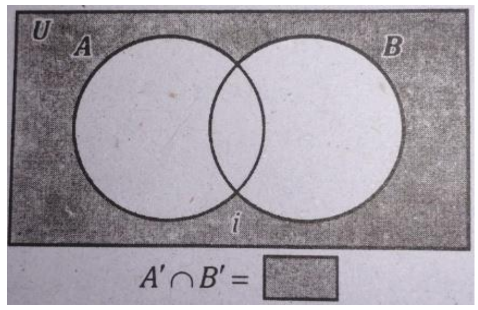
$(A \cup B)^C = A^C \cap B^C$

- **De Morgan`s Laws:**

Left side of first law:



Right side of first law:



- **Left side of second law:**

$A \cap B = \{a,b,c,d,g,h\} \cap \{c,d,e,f,j\}$
$A \cap B = \{c,d\}$

$(A \cap B)^C = \{a,b,c,d,e,f,g,h,i,j\} - \{c,d\}$

$(A \cap B)^C = \{a,b,e,f,g,h,i,j\}$

- **Right side of second law:**

$A^C = \{a,b,c,d,e,f,g,h,i,j\} - \{a,b,c,d,g,h\}$

$A^C = \{e,f,i,j\}$
$B^C = \{a,b,c,d,e,f,g,h,i,j\} - \{c,d,e,f,j\}$

$B^C = \{a,b,g,h,i\}$
$A^C \cup B^C = \{e,f,i,j\} \cup \{a,b,g,h,i\}$

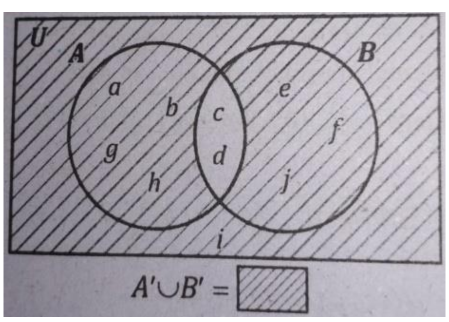
$(A \cup B)^C = \{a,b,e,f,g,h,i,j\}$

Thus, second De Morgan's law proved.

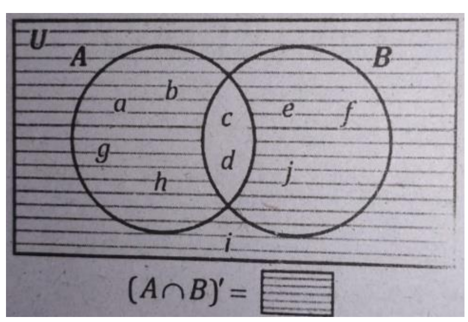
$(A \cap B)^C = A^C \cup B^C$

- **De Morgan`s Laws:**

Left side of second law:



Right side of second law:



Question No. 6

If U = {1,2,3,…,20} and A = {1,3,5,…,19}, verify the following:

(i). A U A' = U

(ii). A ∩ U = A

(iii). A ∩ A' = Φ

Given:

U = {1,2,3,...,20}
A = {1,3,5,...,19}

(i) A U A' = U

Solving Left Hand Side:

A' = U – A = {1,2,3,...,20} - {1,3,5,...,19}

A' = {2,4,6,...,20}

A U A' = {1,3,5,...,19} U {2,4,6,...,20}

A U A' = {1,2,3,…,20} = U
Verified.

(ii). A ∩ U = A

Solving Left Hand Side:

A ∩ U = {1,3,5,...,19} ∩ {1,2,3,...,20}

A ∩ U = {1,3,5,...,19} = A

Verified.

(iii). A ∩ A' = Φ

Solving Left Hand Side:

A' = U – A = {1,2,3,...,20} - {1,3,5,...,19}

A' = {2,4,6,...,20}

A ∩ A' = {1,3,5,...,19} ∩ {2,4,6,...,20}

A ∩ A' = Φ

Verified.

Question No. 7

In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each student likes to play at least one of the two games. How many students like to play both games?

Given:

Total students = n(U) = 55

Cricket players = n(C) = 34

Hockey players = n(H) = 30

To Find:

Both Games = n(C ∩ H) = ?

Solution:

Using the Inclusion-Exclusion Principle

$n(U) = n(C) + n(H) - n(C \cap H)$

$55 = 34 + 30 - n(C \cap H)$

$n(C \cap H) = 64 - 55$

$n(C \cap H) = 9$

Thus, 9 students play both games.

Question No. 8

In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi, and 10 can speak Urdu and Punjabi. How many can speak all three languages?

Given data:

Total employees = n(U) = 500

Urdu speaking = n(Ur) = 250

English speaking = n(E) = 150

Punjabi speaking = n(P) = 50

Urdu and English speaking = n(Ur∩E) = 40

English and Punjabi speaking = n(E∩P) = 30

Urdu and Punjabi speaking = n(Ur∩P) = 10

To Find:

Speak all three languages = n(Ur∩P∩E) = ?

Solution:

Using the Inclusion-Exclusion Principle

$n(U) = n(Ur) + n(E) + n(P) - n(Ur \cap E) - n(E \cap P) - n(Ur \cap P) + n(Ur \cap P \cap E)$

Substituting the given values:

$500 = 250 + 150 + 50 - 40 - 30 - 10 + n(Ur \cap P \cap E)$

$500 = 250 + 150 + 50 - 40 - 30 - 10 + n(Ur \cap P \cap E)$

$500 = 450 - 80 + n(Ur \cap P \cap E)$

$500 = 370 + n(Ur \cap P \cap E)$

$500 - 370 = n(Ur \cap P \cap E)$

$n(Ur \cap P \cap E) = 130$

130 employees can speak all three languages.

Question No. 9

In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts.
The total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps?

Data:

Total people = n(U) = 25

Wearing blue shirts = n(B) = 19

Wearing green shirts = n(G) = 15

Wearing blue and green shirts = n(B∩G) = 3

Wearing caps and blue shirts = n(C∩B) = 4

Wearing caps and green shirts = n(C∩G) = 2

To Find:

Wearing caps = n(C) = ?

Solution:

Use the Inclusion-Exclusion Formula

$n(U) = n(B) + n(G) + n(C) - n(B\cap G) - n(B\cap C) - n(C\cap G) + n(B\cap G\cap C)$

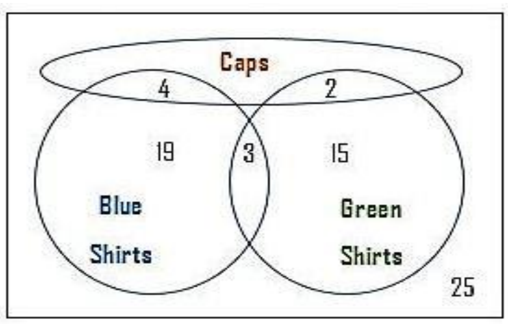
$25 = 19 + 15 + n(C) - 3 - 4 - 2 + n(B\cap G\cap C)$

$25 = 34 - 9 + n(C) + n(B\cap G\cap C)$

$25 = 25 + n(C) + n(B\cap G\cap C)$

$n(C) = 25 - 25$

$n(C) + n(B\cap G\cap C) = 0$ (wrong answer in book)



Question No. 10

In a training session, 17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Data:

Total Participants = n(U) = 35

Participants have laptop = n(L) = 17

Participants have tablet = n(T) = 11

Participants have laptop and tablet = n(L∩T) = 9

Participants have laptop and book = n(L∩B) = 6

Participants have tablet and book= n(T∩B) = 4

Participants have laptop, book or tablet = n(L∩B ∩T) = 8

To Find:

Participants have book = n(B) = ?

Solution:

Using the Inclusion-Exclusion Principle

$n(U) = n(L) + n(T) + n(B) – n(L∩T) – n(L∩B) – n(T∩B) + n(L∩T∩B)$

$35 = 17 + 11 + n(B) - 9 – 6 – 4 + 8$

$35 = 36 + n(B) - 19$

$35 – 36 + 19 = n(B)$

$n(B) = 18$

Question No. 11

A shopping mall has 150 employees labelled 1 to 150, representing the Universal set U. The employees fall into the following categories:

• Set A: 40 employees with a salary range of 30k-45k, labelled from 50 to 89.

• Set B: 50 employees with a salary range of 50k-80k, labelled from 101 to 150.

• Set C: 60 employees with a salary range of 100k-150k, labelled from 1 to 49 and 90 to 100.

Data:

U = {1, 2, 3, …, 150}

A = {50, 51, 52, …, 89}

B = {101, 102, 103, …, 150}

C = {1, 2, 3, …, 49, 90, 91, 92, …, 100}

(a). **Find (A'UB') ∩ C**

(b). **Find n {A ∩ (B' ∩ C')}**

Solution (Part a):

$U – A = \{1, 2, 3, …, 150\} - \{50, 51, 52, …, 89\}$

$A' = \{1, 2, 3, …, 49, 90, 91, …, 150\}$

$U – B = \{1, 2, 3, …, 150\} - \{101, 102, 103, …, 150\}$

$B' = \{1, 2, 3, …, 100\}$

$(A'UB') = \{1, 2, 3, …, 49, 90, 91, …, 150\}\ U\ \{1, 2, 3, …, 100\}$

$(A'UB') = \{1, 2, 3, …, 150\}$

$(A'UB') \cap C = \{1, 2, 3, \ldots, 150\} \cap \{1, 2, 3, \ldots, 49, 90, 91, 92, \ldots, 100\}$

$(A'UB') \cap C = \{1, 2, 3, \ldots, 49, 90, 91, \ldots, 100\}$

Solution (Part b):

$U - B = \{1, 2, 3, \ldots, 150\} - \{101, 102, 103, \ldots, 150\}$

$B' = \{1, 2, 3, \ldots, 100\}$

$U - C = \{1, 2, 3, \ldots, 150\} - \{1, 2, 3, \ldots, 49, 90, 91, \ldots, 100\}$

$C' = \{50, 51, 52, \ldots, 89\}$

$B' \cap C' = \{1, 2, 3, \ldots, 100\} \cap \{50, 51, 52, \ldots, 89\}$

$B' \cap C' = \{50, 51, 52, \ldots, 89\}$

$A \cap (B' \cap C') = \{50, 51, 52, \ldots, 89\} \cap \{50, 51, 52, \ldots, 89\}$

$A \cap (B' \cap C') = \{50, 51, 52, \ldots, 89\}$

$n\{A \cap (B' \cap C')\} = 40$ Elements

Question No. 12

In a secondary school with 125 students participate in at least one of the following sports: cricket, football, or hockey.

60 students play cricket.

70 students play football.

40 students play hockey.

25 students play both cricket and football.

15 students play both football and hockey.

10 students play both cricket and hockey.

(a) How many students play all three sports?

(b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Data:

Total No. of students = $n(U) = 125$

No. of students playing cricket = $n(C) = 60$

No. of students playing football = $n(F) = 70$

No. of students playing hockey = $n(H) = 40$

No. of students playing cricket and football = $n(C \cap F) = 25$

No. of students playing football and hockey = $n(F \cap H) = 15$

No. of students playing cricket and hockey = $n(C \cap H) = 10$

To Find:

Students play all three sports = $n(C \cap F \cap H) = ?$

Solution:

Using the Inclusion-Exclusion Principle

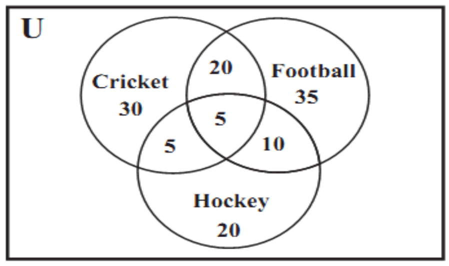
$n(U) = n(C) + n(F) + n(H) - n(C\cap F) - n(F\cap H) - n(C\cap H) + n(C\cap F\cap H)$

$125 = 60 + 70 + 40 – 25 – 15 – 10 + n(C\cap F\cap H)$

$125 - 60 - 70 - 40 + 25 + 15 + 10 = n(C\cap F\cap H)$

$n(C\cap F\cap H) = 5$

Venn diagram:



Question No. 13

A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

40 people said they liked nihari

65 people said they liked biryani

50 people said they liked korma

20 people said they liked nihari and biryani

35 people said they liked biryani and korma

27 people said they liked nihari and korma

12 people said they liked all three foods nihari, biryani, and korma

(a) At least how many people like nihari, biryani or korma?

(b) How many people did not like nihari, biryani, or korma?

(c) How many people like only one of the following foods: nihari, biryani, or korma?

Data:

Total people = n(U) = 130

No. of people liked nihari = n(N) = 40

No. of people liked biryani = n(B) = 65

No. of people liked korma = n(K) = 50

No. of people liked nihari and biryani = $n(N\cap B) = 20$

No. of people liked biryani and korma = $n(B\cap K) = 35$

No. of people liked nihari and korma = $n(N\cap K) = 27$

No. of people liked all three foods = $n(N\cap B \cap K) = 12$

To Find:

a). People like nihari, biryani or korma = n(NUBUK) = ?

b). People did not like nihari, biryani, or korma = ?

c). People like nihari, biryani, or korma = ?

Solution (Part a):

Using the Inclusion-Exclusion Principle

$n(U) = n(N) + n(B) + n(K) - n(N∩B) - n(B∩K) - n(N∩K) + n(N∩B∩K)$

$n(NUBUK) = 40 + 65 + 50 - 20 - 35 - 27 + 12$

$n(NUBUK) = 167 – 82$

$n(NUBUK) = 85$

85 people like at least one of nihari, biryani or korma.

Solution (Part b):

People did not like nihari, biryani, or korma = ?

Total surveyed = 130

People who did not like nihari, biryani, or korma = 130 - n(NUBUK)

People who did not like nihari, biryani, or korma = 130 – 85

People who did not like nihari, biryani, or korma = 45

45 people did not like any of these foods.

Solution (Part c):

People like nihari, biryani, or korma = ?

1. People who like only nihari:

$= n(N) - n(N∩B) - n(N∩K) + n(N∩B∩K)$

$= 40 - 20 - 27 + 12$

$= 52 – 47$

$= 5$

2. People who like only biryani:

$= n(B) - n(N∩B) - n(B∩K) + n(N∩B∩K)$

$= 65 - 20 - 35 + 12$

$= 77 – 55$

$= 22$

3. People who like only korma:

$= n(K) - n(N∩K) - n(B∩K) + n(N∩B∩K)$

$= 50 - 27 - 35 + 12$

$= 62 – 62$

$= 0$

Total number of people who like only one food = 5 + 22 + 0

= 27

27 people like only one of the foods.

(d) Draw a Venn diagram.

(d)