Unit No. 6

Trigonometry

Exercise No. 6.2

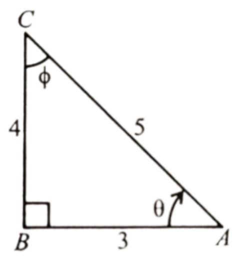
Question No. 1

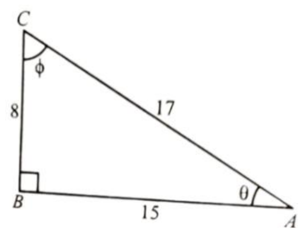
For each of the following right-angled triangles, find the trigonometric ratios:

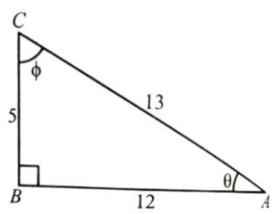
(i) sin θ **(ii) cos θ** **(iii) tan θ (iv) sec θ**

(v) cosec θ **(vi) cot φ**

(vii) tan φ **(viii) cosec φ** **(ix) sec φ (x) cos φ**

(a). 

(b). 

(c). 

Solution (fig. a):

Here a = 4, b = 3, c = 5 when θ is given:

(i) $\sin\theta = \frac{a}{c} = \frac{4}{5}$

(ii) $\cos\theta = \frac{b}{c} = \frac{3}{5}$

(iii) $\tan\theta = \frac{a}{b} = \frac{4}{3}$

(iv) $\sec\theta = \frac{c}{b} = \frac{5}{3}$

(v) $\operatorname{cosec}\theta = \frac{c}{a} = \frac{5}{4}$

Here a = 3, b = 4, c = 5 when φ is given:

(vi) $\cot\varphi = \frac{b}{a} = \frac{4}{3}$

(vii) $\tan\varphi = \frac{a}{b} = \frac{3}{4}$

(viii) $\operatorname{cosec}\varphi = \frac{c}{a} = \frac{5}{3}$

(ix) $\sec\varphi = \frac{c}{b} = \frac{5}{4}$

(x) $\cos\varphi = \frac{b}{c} = \frac{4}{5}$

Solution (fig. b):

Here a = 8, b = 15, c = 17 when θ is given:

(i) $\sin\theta = \frac{a}{c} = \frac{8}{17}$

(ii) $\cos\theta = \frac{b}{c} = \frac{15}{17}$

(iii) $\tan\theta = \frac{a}{b} = \frac{8}{15}$

(iv) $\sec\theta = \frac{c}{b} = \frac{17}{15}$

(v) $\operatorname{cosec}\theta = \frac{c}{a} = \frac{17}{8}$

Here a = 15, b = 8, c = 17 when φ is given:

(vi) $\cot\varphi = \frac{b}{a} = \frac{8}{15}$

(vii) $\tan\varphi = \frac{a}{b} = \frac{15}{8}$

(viii) $\operatorname{cosec}\varphi = \frac{c}{a} = \frac{17}{15}$

(ix) $\sec\varphi = \frac{c}{b} = \frac{17}{8}$

(x) $\cos\varphi = \frac{b}{c} = \frac{8}{17}$

Solution (fig. c):

Here a = 5, b = 12, c = 13 when θ is given:

(i) $\sin\theta = \frac{a}{c} = \frac{5}{13}$

(ii) $\cos\theta = \frac{b}{c} = \frac{12}{13}$

(iii) $\tan\theta = \frac{a}{b} = \frac{5}{12}$

(iv) $\sec\theta = \frac{c}{b} = \frac{13}{12}$

(v) $\operatorname{cosec}\theta = \frac{c}{a} = \frac{13}{5}$

Here a = 12, b = 5, c = 13 when φ is given:

(vi) $\cot\varphi = \frac{b}{a} = \frac{5}{12}$

(vii) $\tan\varphi = \frac{a}{b} = \frac{12}{5}$

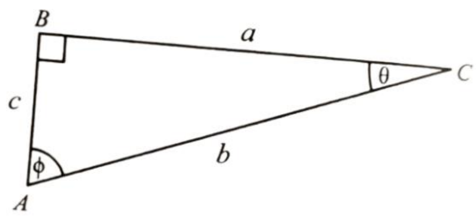
(viii) $\operatorname{cosec}\varphi = \frac{c}{a} = \frac{13}{12}$

(ix) $\sec\varphi = \frac{c}{b} = \frac{13}{5}$

(x) $\cos\varphi = \frac{b}{c} = \frac{5}{13}$

Question No. 2

For the following right-angled triangle ABC find the trigonometric ratios for which $m\angle A = \varphi$ and $m\angle C = \theta$:



(i) sin θ	**(ii) cos θ**	**(iii) tan θ**
(iv) sin φ	**(v) cos φ**	**(vi) tan φ**

Solution:

Here a = c, b = a, c = b when θ is given:

(i) $\sin\theta = \frac{a}{c} = \frac{c}{b}$

(ii) $\cos\theta = \frac{b}{c} = \frac{a}{b}$

(iii) $\tan\theta = \frac{a}{b} = \frac{c}{a}$

Here a = a, b = c, c = b when φ is given:

(iv) $\sin\varphi = \frac{a}{c} = \frac{a}{b}$

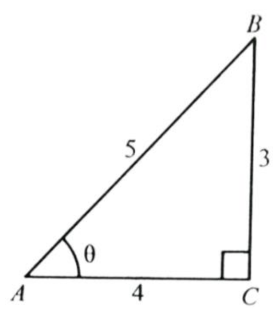
(v) $\cos\varphi = \frac{b}{c} = \frac{c}{b}$

(vi) $\tan\varphi = \frac{a}{b} = \frac{a}{c}$

Question No. 3

Considering the adjoining triangle ABC, verify that:

$a = 3, b = 4$, $c = 5$



(i) sin θ cosec θ = 1

Solution:

$\sin\theta \operatorname{cosec}\theta = 1$

We know that:

$\sin\theta = \frac{a}{c} = \frac{3}{5}$ and $\operatorname{cosec}\theta = \frac{c}{a} = \frac{5}{3}$

By putting these values in given equation:

$(\frac{3}{5})(\frac{5}{3}) = 1$

$1 = 1$

So, proved that:

$\sin\theta \operatorname{cosec}\theta = 1$

(ii) cos θ sec θ = 1

Solution:

$\cos\theta \sec\theta = 1$

We know that:

$\cos\theta = \frac{b}{c} = \frac{4}{5}$ and $\sec\theta = \frac{c}{b} = \frac{5}{4}$

By putting these values in given equation:

$(\frac{4}{5})(\frac{5}{4}) = 1$

$1 = 1$

So, proved that:

$\cos\theta \sec\theta = 1$

(iii) tan θ cot θ = 1

Solution:

$\tan\theta \cot\theta = 1$

We know that:

$\tan\theta = \frac{a}{b} = \frac{3}{4}$ and $\cot\theta = \frac{b}{a} = \frac{4}{3}$

By putting these values in given equation:

$(\frac{3}{4})(\frac{4}{3}) = 1$

$1 = 1$

So, proved that:

$\tan\theta \cot\theta = 1$

Question No. 4

Fill in the blanks.

(i) sin 30° = sin (90° − 60°) = ***cos60°***

(ii) cos 30° = cos (90° − 60°) = ***sin60°***

(iii) tan 30° = tan (90° − 60°) = ***cot60°***

(iv) tan 60° = tan (90° − 30°) = ***cot30°***

(v) sin 60° = sin (90° − 30°) = ***cos30°***

(vi) cos 60° = cos (90° − 30°) = ***sin30°***

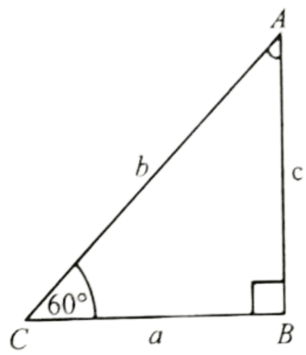
(vii) sin 45° = sin (90° − 45°) = ***cos45°***

(viii) tan 45° = tan (90° − 45°) = ***cot45°***

(ix) cos 45° = cos (90° − 45°) = ***sin45°***

Question No. 5

In a right-angled triangle ABC, m∠B = 90° and C is an acute angle of 60°. Also sin m∠A = a/b, then find the following trigonometric ratios:



From fig. $mAB = \mathbf{c}$ **,** $mBC = a,$ $mAC = \mathbf{b}$

(i) $\frac{mBC}{mAB}$

Solution:

$$\frac{mBC}{mAB}$$

$$= \frac{a}{c}$$

(ii) cos 60°

Solution:

cos 60°

$\cos 60° = \frac{mBC}{mAC}$

$$= \frac{a}{b}$$

(iii) tan 60°

Solution:

tan 60°

$\tan 60° = \frac{mAB}{mBC}$

$$= \frac{c}{a}$$

(iv) cosec $(\frac{\pi}{3})$

Solution:

Converting radian into degree:

$= \frac{\pi}{3} \times \frac{180}{\pi}$

= 60°

cosec 60°

$\text{cosec } 60° = \frac{mAC}{mAB}$

$= \frac{b}{c}$

(v) cot 60°

Solution:

cot 60°

$\cot 60° = \frac{mBC}{mAB}$

$= \frac{a}{c}$

(vi) sin 30°

Solution:

sin 30°

$\sin 30° = \frac{mBC}{mAC}$

$= \frac{a}{b}$

(vii) cos 30°

Solution:

cos 30°

$\cos 30° = \frac{mAB}{mBC}$

$= \frac{c}{b}$

(viii) tan $(\frac{\pi}{6})$

Solution:

Converting radian into degree:

$= \frac{\pi}{60} \times \frac{180}{\pi}$

$= 30°$

tan 30°

$\tan 30° = \frac{mAB}{mBC}$

$= \frac{a}{c}$

(ix) sec 30°

Solution:

sec 30°

$\sec 30^\circ = \frac{mAC}{mAB}$

$= \frac{b}{c}$

(x) cot 30°

Solution:

$\cot 30^\circ$

$\cot 30^\circ = \frac{mAB}{mBC}$

$= \frac{c}{a}$