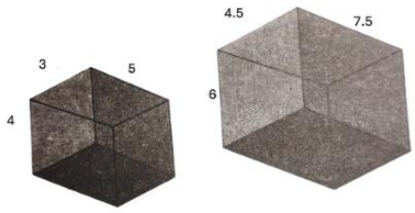
Unit No. 9

Similar Figures

Exercise No. 9.1

Question No. 1

Find whether the solids are similar. All lengths are in cm.



Solution:

We know that;

corresponding two polygons are similar if their corresponding angles are equal and the corresponding sides are proportional.

The given solids are cubes so the corresponding angles are equal.

The ratio of corresponding sides is:

Side i). 4 / 6 = 2 / 3

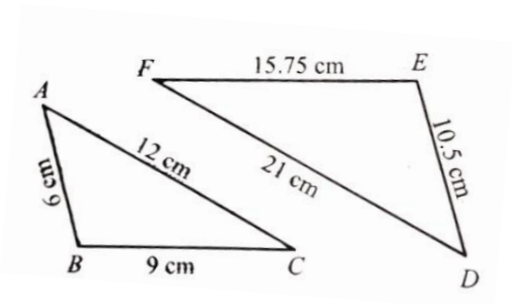
Side ii). 3 / 4.5 = 30 / 45 = 2 / 3

Side iii). 5 / 7.5 = 50 / 75 = 2 / 3

Since ratio of all given sides is equal so given solids are similar.

Question No. 2

In triangle ABC, the sides are given as m AB = 6 cm, m BC = 9 cm and m CA = 12 cm. In triangle DEF, the sides are given as m DE = 10.5 cm, m EF = 15.75 cm, and m FD = 21 cm.



Prove that the triangles are similar.

Solution:

We know that;

If ratio of all corresponding sides is equal, then the triangles are similar. The ratio of corresponding sides is:

m AB / m DE = m AC / m DF = m BC / m EF

6 / 10.5 = 12 / 21 = 9 / 15.75

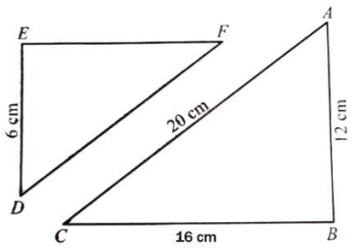
60 / 105 = 12 / 21 = 900 / 1575

4 / 7 = 4 / 7 = 4 / 7

Since, the ratios are equal, so the given triangles are similar.

Question No. 3

In the given figure, △ABC~△DEF, m AB = 12 cm, m AC = 20 cm and m BC = 16 cm. In △DEF, m DE = 6 cm. Find m DF and m EF.



Given:

△ABC ~ △DEF

Solution:

Since, two triangles are similar, the ratio of their corresponding sides must be equal.

m DF / m AC = m DE / m AB = m EF / m BC

Let; m DF = x and m EF = y

x / 20 = 6 / 12 = y / 16

x / 20 = 6 / 12 ➔ x = 6 / 12 × 20 ➔ x = 10 cm

y / 16 = 6 / 12 ➔ y = 6 / 12 × 16 ➔ y = 8 cm

So;

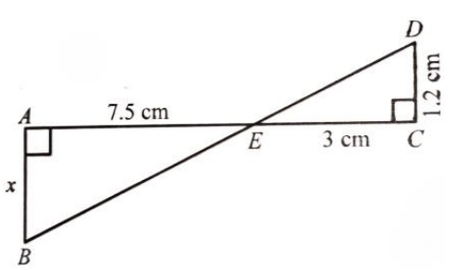
m DF = x = 10 cm

m EF = y = 8 cm

Question No. 4

Find the value of x in each of the following:

(i).



Given:

From figure of △ABE

m AE = 7.5 cm

From figure of △CED

m EC = 3 cm

m CD = 1.2 cm

To Find:

m AB = x = ?

Solution:

Since AB is parallel to CD so the triangles are similar and ratio of their corresponding sides must be equal.

m AB / m CD = m AE / m CE

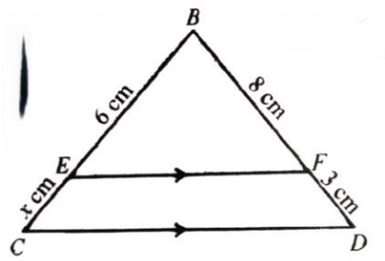
x / 1.2 = 7.5 / 3

x = 2.5 × 1.2

x = 3 cm

So, m AB = x = 3 cm

(ii).



Given:

From figure of △BEF

m BE = 6 cm

m BF = 8 cm

From figure of △ BCD

m BC = (x + 6) cm

m BD = 8 + 3 = 11 cm

To Find:

x = ?

Solution:

Since, EF is parallel to CD, so the triangles are similar.

m BE / m BC = m BF / m BD

6 / (x + 6) = 8 / 11

Cross-Multiplication:

$6 \times 11 = 8 \times (x + 6)$

$66 = 8x + 48$

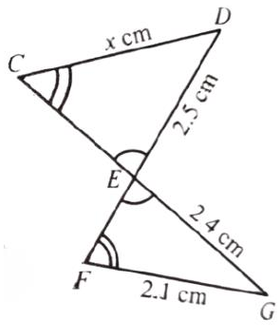
$66 - 48 = 8x$

$18 / 8 = x$

$x = 9 / 4$

x = 2.25 cm

(iii).



Given:

From figure of △CDE

m DE = 2.5 cm

From figure of △ EFG

m FG = 2.1 cm

m EG = 2.4 cm

m ∠C = m ∠F

m ∠E = m ∠E

To Find:

m CD = x = ?

Solution:

Side EF is parallel to ED; So, the triangles are similar.

m CD / m FG = m DE / m EG

$x / 2.1 = 2.5 / 2.4$

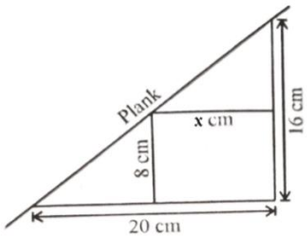
$x = 2.5 / 2.4 \times 2.1$

$x = 2.19$

So, m CD = x = 2.19 cm

Question No. 5

A plank is placed straight upstairs that 20 cm wide and 16 cm deep. A rectangular box of height 8 cm and width x cm is placed on a stair under the plank. Find the value of x.



Given:

Consider two triangles △AOB and △BCD

Since CD is parallel to side AO so the two triangles are similar and ratio of their corresponding sides are same.

To Find:

m CD = x = ?

Solution:

m OB / m ED = m OA / m CD

16 / 8 = 20 / x

Cross-Multiplication:

16 x = 20 × 8

x = 160 / 16

x = 10 cm

So, the width of rectangular box is 10 cm

Question No. 6

A man who is 1.8 m tall casts a shadow of a 0.76 m in length. If at the same time a telephone pole casts a 3 m shadow, find the height of the pole.

Given:

Height of man = m AB = 1.8 m

Length of shadow of man = m BE = 0.76 m

Length of shadow of pole = m ED = 3 m

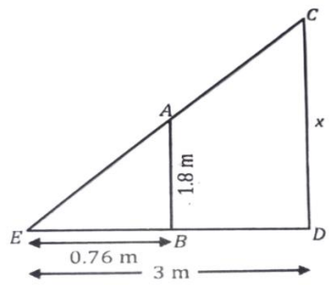
To Find:

Height of pole = x = ?

Solution:

Consider the two triangles △ABE and △CED.

Since side AB is parallel to CD so both triangles are similar.



Thus, the ratio of their corresponding sides is equal.

m CD / m AB = m ED / m EB

x / 1.8 = 3 / 0.76

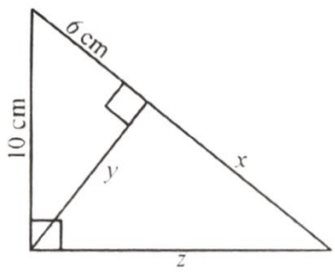
x = 3 / 0.76 × 1.8

x = 7.11 m

Height of pole = x = 7.11 m

Question No. 7

Find the values of x, y and z in the given figure.



Given:

In the given figure,

m ∠B = 90°

and

m BD ⊥ m AC

So, △ABC ~ △ADB ~ △BDC

In △ABC:

Hypotenuse = 10 cm

Legs = 6 cm

In △ADB:

Hypotenuse = 6 cm

To Find:

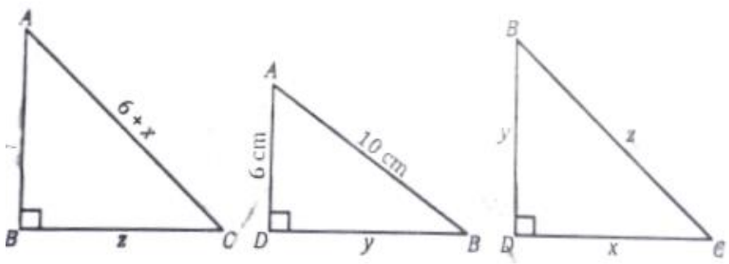
x = ?

y = ?

z = ?

Solution:

To find x, y and z, we first separate the triangles:



We can find y by using Pythagoras theorem for △ADB:

$(m\,AB)^2 = (m\,AD)^2 + (m\,BD)^2$

$(10)^2 = (6)^2 + (y)^2$

$100 = 36 + y^2$

$y^2 = 100 - 36$

$y^2 = 64$

Taking square root on both sides:

y = 8 cm

Since △ABC ~ △ADB, their corresponding sides are proportional.

$(6 + x) / 10 = 10 / 6$

Cross-Multiplication:

$6\,(6 + x) = 10\,(10)$

$36 + 6x = 100$

$6x = 100 - 36$

$x = 64 / 6$

$x = 32 / 3$

x = 10.67 cm

Now, consider triangles △ABD and △BDC,

Since △ABD ~ △BDC;

Their corresponding sides are proportional;

$10 / 6 = z / y$

Put y = 8 cm in above equation, we get:

$10 / 6 = z / 8$

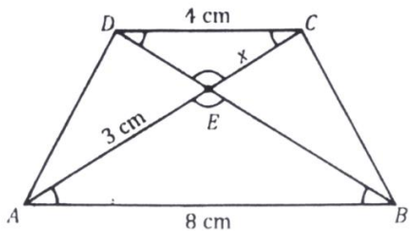
$10 \times 8 / 6 = z$

$z = 80 / 6$

z = 13. 33 cm

Question No. 8

Draw an isosceles trapezoid ABCD where AB∥CD and m AB > m CD. Draw diagonals AC and BD, intersecting at E. Prove that △ABE is similar to △CDE. If m AB = 8 cm, m CD =4 cm, and m AE = 3 cm, find the length of CE.



Given:

m AB = 8 cm

m AE = 3 cm

m CD = 4 cm

To Find:

m CE = x = ?

△ABE ~ △CDE

Solution:

In the isosceles trapezoid ABCD, the diagonals AC and BD cut each other at E and form two pairs of triangles △ABE and △CDE.

Since, the bases of both triangles AB and CD are parallel.

i.e.,

- AB || CD (Given)
- ∠A = ∠C (Alternate angles)
- ∠B = ∠D (Alternate angles)

So, by ∠A = ∠B = ∠C= ∠D (Opposite angles)

Since all three pairs of angles are the same, therefore △ABE and △ECD are similar for the AAA criteria. Hence, the corresponding sides of both triangles will be proportional.

m AB / m CD = m AE / m EC = m AB / m BC

8 / 4 = 3 / x

x = 3 × 4 / 8

x = 12 / 8

x = 3 / 2

m CE = x = 1.5 cm

Question No. 9

A regular dodecagon has its side lengths decreased by a factor of $\frac{1}{\sqrt{2}}$. If the perimeter of the original dodecagon is 72 cm. What is the side length of scaled dodecagon?

Given:

Perimeter of first dodecagon = 72 cm

To Find:

Length of scaled dodecagon = ?

Solution:

Side length of first regular dodecagon = 72 / 12 = 6 cm

(∴ dodecagon has 12 sides)

Side length of scaled dodecagon = $6 \times (\frac{1}{\sqrt{2}})$

By rationalizing:

$= 6 \times (\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}})$

$= 6 \times \frac{\sqrt{2}}{2}$

$= 3 \times \sqrt{2}$

Side length of scaled dodecagon = $3\sqrt{2}$ cm

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