

Unit No. 7

Coordinate Geometry

Basic Concepts

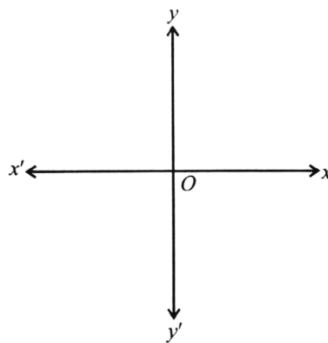
Geometry:

Geometry is one of the most ancient branches of mathematics. The Greeks systematically studied it about four centuries B.C. Most of the geometry taught in schools is due to Euclid who expounded thirteen books on the subject (300 B.C.). A French philosopher and mathematician Rene Descartes (1596–1650 A.D.) introduced algebraic methods in geometry which gave birth to analytic geometry (or coordinate geometry). Our aim is to present fundamentals of the subject in this book.

Coordinate Plane:

Draw in a plane two mutually perpendicular number lines $x'x$ and $y'y$, one horizontal and the other vertical. Let O be their point of intersection, called **origin** and the real number 0 of both the lines is represented by O . The two lines are called **coordinate axes**. The horizontal line $x'Ox$ is called the **x-axis** and the vertical line $y'Oy$ is called the **y-axis**.

Pictorial Representation:



Important information:

The Cartesian coordinate system or the rectangular coordinate system was invented by French mathematician René Descartes, when he tried to describe the path of a fly crawling along criss-cross beams on the ceiling while he lay on his bed. The Cartesian coordinate system created a link between algebra and geometry. Geometric shapes could now be described algebraically using the coordinates of the points that make up the shapes.

X - Coordinate or Abscissa:

The first component of the ordered pair (x, y) is called x-coordinate or abscissa of P .

Y - Coordinate or Ordinate:

The second component of the ordered pair (x, y) is called y-coordinate or ordinate of P .

Two dimensional rectangular (or Cartesian) coordinate system:

The method of pairing off in a one-to-one fashion the points in a plane with ordered pairs of real numbers is called the two dimensional rectangular (or Cartesian) coordinate system.

Quadrants:

The coordinate axes divide the plane into four equal parts called quadrants. They are defined as follows:

Quadrant I:

All points (x, y) with $x > 0, y > 0$

Quadrant II:

All points (x, y) with $x < 0, y > 0$

Quadrant III:

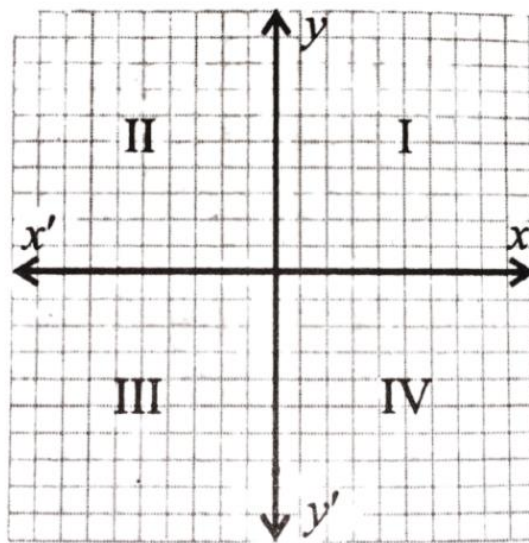
All points (x, y) with $x < 0, y < 0$

Quadrant IV:

All points (x, y) with $x > 0, y < 0$

Graph:

The point P in the plane that corresponds to an ordered pair (x, y) is called the graph.

Pictorial Representation:**The Distance Formula:**

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in the plane. To find the distance $d = |AB|$, we draw a horizontal line from A to a point C that lies directly below B, forming a right triangle ABC.

So that

$$|AC| = |x_2 - x_1| \text{ and } |BC| = |y_2 - y_1|$$

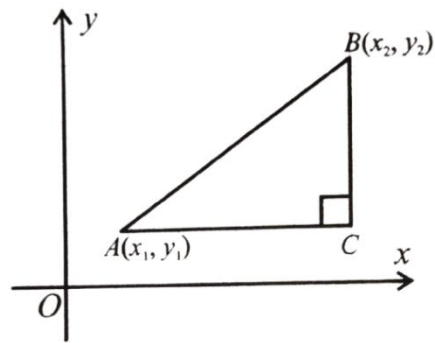
By using Pythagoras Theorem, we have:

$$d^2 = |AB|^2 = |AC|^2 + |BC|^2$$

$$|AB|^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Pictorial Representation:

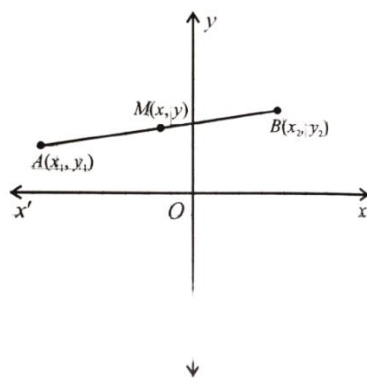


Note: The distance is always taken to be non-negative.

Mid Point Formula:

$$\text{Mid - Point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Pictorial Representation:

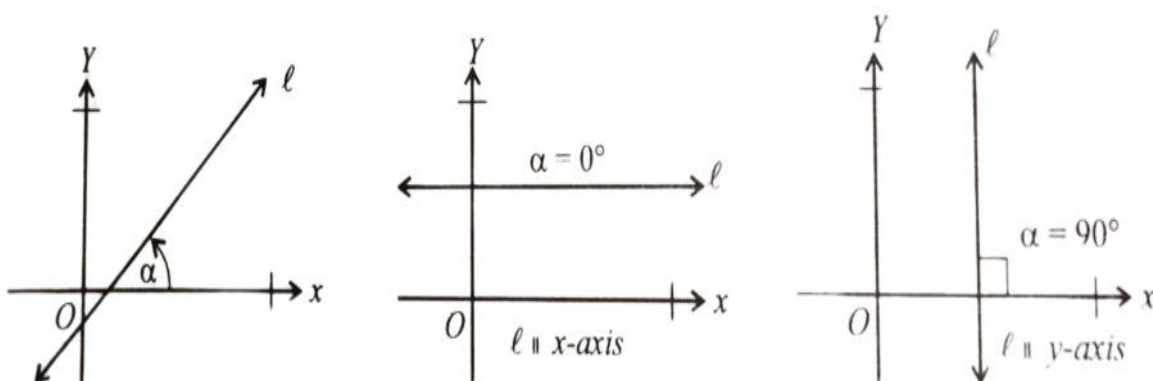


Equations of Straight Lines

Inclination of a Line:

The angle α ($0^\circ < \alpha < 180^\circ$) measured counterclockwise from the positive x-axis to a non-horizontal straight line ℓ is called the inclination of ℓ .

Pictorial Representation:



Note:

- (i) If ℓ is parallel to x-axis, then $\alpha = 0^\circ$
- (ii) If ℓ is parallel to y-axis, then $\alpha = 90^\circ$

Slope or Gradient of a Line:

The measure of steepness (ratio of rise to the run) is termed as slope or gradient of the inclined path and is denoted by m .

$$m = \frac{\text{rise}}{\text{run}} = \frac{y}{x} = \tan a$$

Why are slopes important?

The concept of slope is widely used in engineering, architecture and even in sports like skiing, where understanding the steepness of a hill or ramp is essential.

Note:

If $a > 0$, then the line ℓ is above the x-axis.

If $a < 0$, then the line ℓ is below the x-axis.

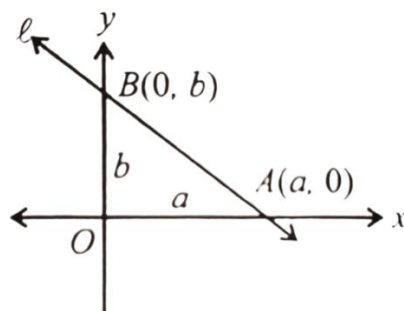
If $a = 0$, then the line ℓ becomes the x-axis.

Standard Forms of Equation of Straight Line:

Intercepts of a line:

- If a line intersects the x-axis at $(a, 0)$, then a is called the **x-intercept** of the line.
- If a line intersects the y-axis at $(0, b)$, then b is called the **y-intercept** of the line.

Pictorial Representation:



Point-Slope Form of Equation of a Straight Line:

$$m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = (y - y_1)$$

Symmetric Form of Equation of a Straight Line:

$$m = \frac{y - y_1}{x - x_1}$$

$$\frac{y - y_1}{x - x_1} = \frac{\sin a}{\cos a}$$

$$\frac{x - x_1}{\cos a} + \frac{y - y_1}{\sin a} = r$$

This is called symmetric form of equation of the line.

Intercept Form of Equation of a Straight Line:

$$\frac{x}{a} + \frac{y}{b} = r$$

A Linear Equation in two Variables Represents a Straight Line:

The linear equation $ax+by+c=0$ in two variables x and y represents a straight line. A linear equation in two variables x and y is:

$$ax + by + c = 0$$

Remember: The equation (i) represents a straight line and is called the general equation of a straight line.

Do you know?

Aviation is the operation and flight of aircraft, including airplanes, helicopters and drones.

Navigation is the process of determining and controlling the route of a vehicle, such as an aircraft, from one place to another.

Latitude:

Latitude measures how far a location is from the equator. It ranges from 0° at the equator to 90° north (at the North Pole) or 90° south (at the South Pole).

Longitude:

Longitude measures how far a location is from the Prime Meridian (which runs through Greenwich, London). It ranges from 0° at the Prime Meridian to 180° east and 180° west.