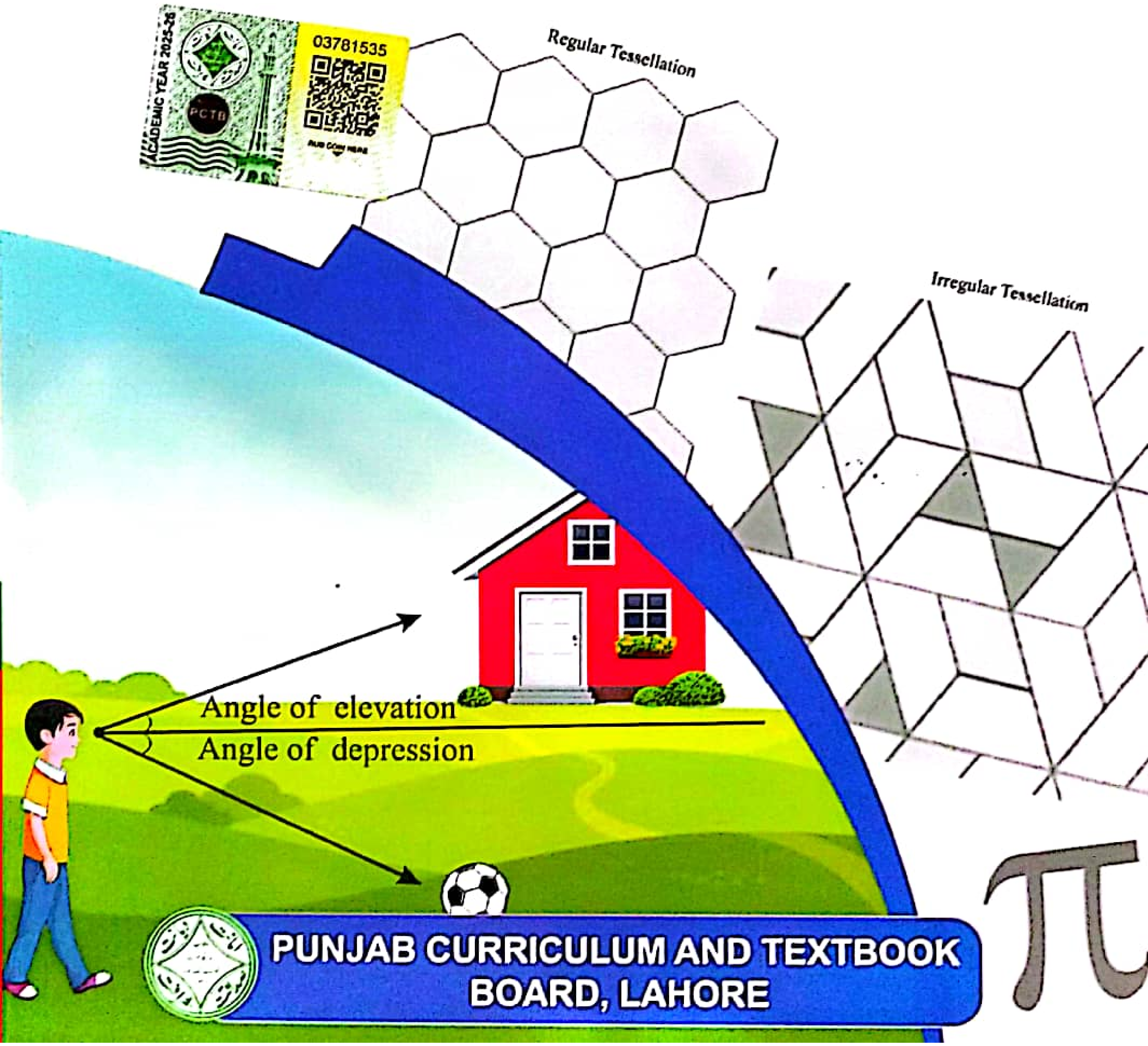
MATHEMATICS 9



PUNJAB CURRICULUM AND TEXTBOOK
BOARD, LAHORE

UNIT 4

FACTORIZATION AND ALGEBRAIC MANIPULATION

EXERCISE 4.1

Q.1. Factorize by identifying common factors.

(i) $\mathbf{6x + 12}$

Sol. 6 is common factor

$= 6(x + 2)$

(ii) $\mathbf{15y^2 + 20y}$

Sol. 5y is common factor

$= 5y(3y + 4)$

(iii) $\mathbf{-12x^2 - 3x}$

Sol. $-3x$ is common factor

$= -3x(4x + 1)$

(iv) $\mathbf{4a^2b + 8ab^2}$

Sol. 4ab is common factor

$= 4ab(a + 2b)$

(v) $\mathbf{xy - 3x^2 + 2x}$

Sol. x is common factor

$= x(y - 3x + 2)$

(vi) $\mathbf{3a^2b - 9ab^2 + 15ab}$

Sol. 3ab is common factor

$= 3ab(a - 3b + 5)$

Q.2. Factorize and represent pictorially:

(i) $\mathbf{5x + 15}$

Sol. 5 is common factor

$= 5x + 15 = 5(x + 3)$

(ii) $\mathbf{x^2 + 4x + 3}$

Sol. $= x^2 + 3x + x + 3$

$= x(x + 3) + 1(x + 3)$

$= (x + 3)(x + 1)$

(iii) $\mathbf{x^2 + 6x + 8}$

Sol. $= x^2 + 4x + 2x + 8$

$= x(x + 4) + 2(x + 4)$

$= (x + 4)(x + 2)$

(iv) $\mathbf{x^2 + 4x + 4}$

Sol. $= x^2 + 2x + 2x + 4$

$= x(x + 2) + 2(x + 2)$

$= (x + 2)(x + 2)$

Q.3. Factorize:

(i) $\mathbf{x^2 + x - 12}$

Sol. $= x^2 + 4x - 3x - 12$

$= x(x + 4) - 3(x + 4)$

$= (x + 4)(x - 3)$

(ii) $\mathbf{x^2 + 7x + 10}$

Sol. $= x^2 + 5x + 2x + 10$

$= x(x + 5) + 2(x + 5)$

$= (x + 5)(x + 2)$

(iii) $\mathbf{x^2 - 6x + 8}$

Sol. $= x^2 - 4x - 2x + 8$

$= x(x - 4) - 2(x - 4)$

$= (x - 4)(x - 2)$

(iv) $\mathbf{x^2 - x - 56}$

Sol. $= x^2 - 8x + 7x - 56$

$= x(x - 8) + 7(x - 8)$

$= (x - 8)(x + 7)$

(v) $\mathbf{x^2 - 10x - 24}$

Sol. $= x^2 - 12x + 2x - 24$

$= x(x - 12) + 2(x - 12)$

$= (x - 12)(x + 2)$

(vi) $\mathbf{y^2 + 4y - 12}$

Sol. $= y^2 + 6x - 2y - 12$

$= y(y + 6) - 2(y + 6)$

$= (y + 6)(y - 2)$

(vii) $\mathbf{y^2 + 13y + 36}$

Sol. $= y2 + 4y + 9y + 36$

$= y(y + 4) + 9(y + 4)$

$= (y + 4)(y + 9)$

(viii) $\mathbf{x^2 - x - 2}$

Sol. $= x^2 - 2x + x - 2$

$= x(x - 2) + 1(x - 2)$

$= (x - 2)(x + 1)$

Q.4. Factorize:

(i) $\mathbf{2x^2 + 7x + 3}$

Sol. $= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(ii) $\mathbf{2x^2 + 11x + 15}$

Sol. $= 2x^2 + 6x + 5x + 15$

$= 2x(x + 3) + 5(x + 3)$

$= (x + 3)(2x + 5)$

(iii) $\mathbf{4x^2 + 13x + 3}$

Sol. $= 4x^2 + 12x + x + 3$

$= 4x(x + 3) + 1(x + 3)$

$= (x + 3)(4x + 1)$

(iv) $3x^2 + 5x + 2$

Sol. $= 3x^2 + 3x + 2x + 2$

$= 3x(x + 1) + 2(x + 1)$

$= (x + 1)(3x + 2)$

(v) $3y^2 - 11y + 6$

Sol. $= 3y^2 - 9y - 2y + 6$

$= 3y(y - 3) - 2(y - 3)$

$= (y - 3)(3y - 2)$

(vi) $2y^2 - 5y + 2$

Sol. $= 2y^2 - 4y - y + 2$

$= 2y(y - 2) - 1(y - 2)$

$= (y - 2)(2y - 1)$

(vii) $4z^2 - 11z + 6$

Sol. $= 4z^2 - 8z - 3z + 6$

$= 4z(z - 2) - 3(z - 2)$

$= (z - 2)(4z - 3)$

(viii) $6 + 7x - 3x^2$

Sol. $= 6 + 9x - 2x - 3x^2$

$= 3(2 + 3x) - x(2 + 3x)$

$= (2 + 3x)(3 - x)$

EXERCISE 4.2

Q.1. Factorize each of the following expressions:

(i) $4x^4 + 81y^4$

Sol. $= (2x^2)^2 + (9y^2)^2 + 2(2x^2)(9y^2) - 2(2x^2)(9y^2)$

$= (2x^2 + 9y^2)^2 - 36x^2y^2$

$= (2x^2 + 9y^2)^2 - (6xy)^2$

$= (2x^2 + 9y^2 + 6xy)(2x^2 + 9y^2 - 6xy)$

(ii) $a^4 + 64b^4$

Sol. $= (a^2)^2 + (8b^2)^2 + 2(a^2)(8b^2) - 2(a^2)(8b^2)$

$= (a^2 + 8b^2)^2 - 16a^2b^2$

$= (a^2 + 8b^2) - (4ab)^2$

$= (a^2 + 8b^2 + 4ab)\ (a^2 + 8b^2 - 4ab)$

(iii) $x^4 + 4x^2 + 16$

Sol. $= x^4 + 16 + 2x^2$

$= (x^2)^2 + 4^2 + 2(x^2)(4) - 2(x^2)(4) + 2x^2$

$= (x^2 + 4)^2 - 8x^2 + 2x^2$

$= (x^2 + 4)^2 - 6x^2$

$= (x^2 + 4)^2 - (\sqrt{6}x)^2$

$= (x^2 + 4 + \sqrt{6}x)\ (x^2 + 4 - \sqrt{6}x)$

$= (x^2 + \sqrt{6}x + 4)\ (x^2 - \sqrt{6}x + 4)$

(iv) $x^4 - 14x^2 + 1$

Sol. $= x^4 + 1 - 14x^2$

$= (x^2)^2 + 1^2 + 2(x^2)(1) - 2(x^2)(1) - 14x^2$

$= (x^2 + 1)^2 - 2x^2 - 14x^2$

$= (x^2 + 1)^2 - (4x)^2$

$= (x^2 + 1 + 4x)(x^2 + 1 - 4x)$

$= (x^2 + 4x + 1)(x^2 - 4x + 1)$

(v) $x^4 - 30x^2y^2 + 9y^4$

Sol. $= x^4 + 9y^4 - 30x^2y^2$

$= (x^2)^2 + (3y^2)^2 + 6x^2y^2 - 36x^2y^2$

$= (x^2 + 3y^2)^2 - (6xy)^2$

$= (x^2 + 3y^2 + 6xy)\ (x^2 + xy^2 - 6xy)$

(vi) $x^4 + 11x^2y^2 + y^4$

Sol. $= x^4 + y^4 - 7x^2y^2$

Here correct question is $x^4 + y^2 - 7x^2y^2$

$= x^4 + y^4 + 2x^2y^2 - 2x^2y^2 - 7x^2y^2$

$= (x^2 + y^2)^2 - 9x^2y^2$

$= (x^2 + y^2 + 3xy)(x^2 + y^2 - 3xy)$

$= (x^2 + 3xy + y^2)(x^2 - 3xy + y^2)$

Q.2. Factorize each of the following expressions:

(i) $(x + 1)(x + 2)(x + 3)(x + 4) + 1$

Sol. Rearranging the terms

$= (x + 1)(x + 4)(x + 2)(x + 3) + 1$

$= (x^2 + 5x + 4)(x^2 + 5x + 6) + 1$

Let $x^2 + 5x = y$

$= (y + 4)(y + 6) + 1$

$= y^2 + 4y + 6y + 24 + 1$

$= y^2 + 10y + 25$

$= y^2 + 5y + 5y + 25$

$= y(y + 5) + 5(y + 5)$

$= (y + 5)(y + 5)$

$= (x^2 + 5x + 5)(x^2 + 5x + 5)$

(ii) $(x + 2)(x - 7)(x - 4)(x - 1) + 17$

Sol. Terms are already arranged

$= (x^2 - 7x + 2x - 14)(x^2 - 4x - x + 4) + 17$

$= (x^2 - 5x - 14)(x^2 - 5x + 4) + 17$

Let $x^2 - 5x = y$

$= (y - 14)(y + 4) + 17$

$= y^2 + 4y - 14y - 56 + 17$

$= y^2 - 10y - 39$

$= y^2 - 13y + 3y - 39$

$= y(y - 13) + 3(y - 13)$

$= (y - 13)(y + 3)$

$= (x^2 - 5x - 13)(x^2 - 5x + 3)$

(iii) $(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$

Sol. Let $2x^2 + 7x = y$

$= (y + 3)(y + 5) + 1$

$= y^2 + 3y + 5y + 15 + 1$
$= y^2 + 8y + 16$
$= y^2 + 4y + 4y + 16$
$= y(y + 4) + 4(y + 4)$
$= (y + 4)(y + 4)$
$= (2x^2 + 7x + 4)(2x^2 + 7x + 4)$

(iv) $\mathbf{(3x^2 + 5x + 3)(3x^2 + 5x + 5) + 8}$

Sol. Let $3x^2 + 5x = y$
$= (y + 3)(y + 5) - 3$
$= y^2 + 3y + 5y + 15 - 3$
$= y^2 + 8y + 12$
$= y^2 + 6y + 2y + 12$
$= y(y + 6) + 2(y + 6)$
$= (3x^2 + 5x + 6)(3x^2 + 5x + 2)$
$= (3x^2 + 5x + 6)(3x^2 + 3x + 2x + 2)$
$= (3x^2 + 5x + 6)(3x(x + 1) + 2(x + 1))$
$= (3x^2 + 5x + 6)(3x + 2)(x + 1)$

(v) $\mathbf{(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2}$

Sol. Re-arranging terms
$= (x + 1)(x + 6)(x + 2)(x + 3) - 3x^2$
$= (x^2 + x + 6x + 6)(x^2 + 2x + 3x + 6) - 3x^2$
$= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$
Let $x^2 + 6 = y$
$= (y + 7x)(y + 5x) - 3x^2$
$= y^2 + 5xy + 7xy + 35x^2 - 3x^2$
$= y^2 + 12xy + 32x^2$
$= y^2 + 8xy + 4xy + 32x^2$
$= y(y + 8x) + 4x(y + 8x)$
$= (y + 8x)(y + 4x)$
$= (x^2 + 6 + 8x)(x^2 + 6 + 4x)$
$= (x^2 + 8x + 6)(x^2 + 4x + 6)$

(vi) $(x+1)(x-1)(x+2)(x-2)+13x^2$

Sol. Rearranging terms

$= (x+1)(x+2)(x-1)(x-2)+13x^2$

$= (x^2+x+2x+2)(x^2-x-2x+2)+13x^2$

$= (x^2+3x+2)(x^2-3x+2)+13x^2$

$= (x^2+2+3x)(x^2+2-3x)+13x^2$

Let $x^2+2=y$

$= (y+3x)(y-3x)+13x^2$

$= y^2-9x^2+13x^2$

$= y^2+4x^2$

$= (x^2+2)^2+4x^2$

$= x^4+4x^2+4+4x^2$

$= x^4+4+8x^2$

$= x^4+4x^2+4-4x^2+8x^2$

$= (x^2-2)^2+4x^2$

Q.2. Factorize

(i) $\mathbf{8x^3+12x^2+6x+1}$

Sol. $= 8x^3+1+12x^2+6x$

$= 8x^3+13+6x(2x+1)$

$= (2x)^3+1^3+3(2x)(1)(2x+1)$

$[\because \ a^3+b^3+3ab(a+b)=(a+b)^3]$

$= (2x+1)^3$

(ii) $\mathbf{27a^3+108a^2b+144ab^2+64b^3}$

Sol. $= 27a^3+64b^3+108a^2b+144ab^2$

$= (3a)^3+(4b)^3+36ab(3a+4b)$

$= (3a)^3+(4b)^3+3(3a)(4b)(3a+4b)$

$= (3a+4b)^3$

(iii) $\mathbf{x^3+48x^2y+108xy^2+216y^3}$

Sol. $= x^3+216y^3+48x^2y+108xy^2$

$= (x)^3+(6y)^3+12xy(4x+9y)$

Which cannot be factorized.

(iv) $\mathbf{8x^3 - 125y^3 + 150xy^2 - 60x^2y}$

Sol. $= (2x)^3 - (5y)^3 - 60x^2y + 150xy^2$

$= (2x)^3 - (5y)^3 - 30xy(x - 5y)$

$= (2x)^3 - (5y)^3 - 3(2x)(5y)(x - 5y)$

$\because \quad a^3 - b^3 - 3ab(a - b) = (a - b)^3$

Q.3. Factorize:

(i) $\mathbf{125a^3 - 1}$

Sol. $= (5a)^3 - 1^3$

$[\because \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= (5a - 1)((5a)^2 + (5a)(1) + 1^2)$

$= (5a - 1)(25a^2 + 5a + 1)$

(ii) $\mathbf{64x^3 + 125}$

Sol. $= (4x)^3 + (5)^3$

$[\because \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= (4x + 5)((4x)^2 - (4x)(5) + 5^2)$

$= (4x + 5)(16x^2 - 20x + 25)$

(iii) $\mathbf{x^6 - 27}$

Sol. $= (x^2)^3 - (3)^3$

$= (x^2 - 3)((x^2)^2 + (x^2)(3) + 3^2)$

$= (x^2 - 3)(x^4 + 3x^2 + 9)$

(iv) $\mathbf{1000a^3 + 1}$

Sol. $= (10x)^3 + 1^3$

$= (10x + 1)((10x)^2 - (10x)(1) + 1^2)$

$= (10x + 1)(100x^2 - 10x + 1)$

(v) $\mathbf{343x^3 + 216}$

Sol. $= (7x)^3 + (6)^3$

$= (7x + 6)((7x)^2 - (7x)(6) + 6^2)$

$= (7x + 6)(49x^2 - 42x + 36)$

(vi) $\mathbf{27 - 512y^3}$

Sol. $= (3)^3 - (8y)^3$

$= (3 - 8y)(3^2 + 3(8y) + (8y)^2)$

$= (3 - 8y)(9 + 24y + 64y^3)$

EXERCISE 4.3

Q.1. Find HCF by factorization method.

(i) $21x^2y, 35xy^2$

Sol. $21x^2y = 3 \times 7 \times x \times x \times y$

$35xy^2 = 5 \times 7 \times x \times y \times y$

Common factorization

$= 7 \times x \times y$

HCF $= 7xy$

(ii) $4x^2 - 9y^2, 2x^2 - 3xy$

Sol. $4x^2 - 9y^2 = (2x)^2 - (3y)^2$

$= (2x + 3y)(2x - 3y)$

$2x^2 - 3xy = y(2x - 3y)$

Common factorization $= 2x - 3y$

HCF $= 2x - 3y$

(iii) $x^3 - 1, x^2 + x + 1$

Sol. $x^3 - 1 = x^3 - 1^3$

$= (x - 1)(x^2 + x + 1)$

Common factorization $= x^2 + x + 1$

HCF $= x^2 + x + 1$

(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

Sol.

$a^3 + 2a^2 - 3a$	$2a^3 + 5a^2 - 3a$
$= a(a^2 + 2a - 3)$	$= a(2a^2 + 5a - 3)$
$= a(a^2 + 3a - a - 3)$	$= a(2a^2 + 6a - a - 3)$
$= a(a(a + 3) - 1(a + 3))$	$= a(2a(a + 3) - 1(a +$
$= a(a + 3)(a - 1)$	$= a(a + 3)(2a - 1)$

Common factor $= a(a + 3)$

HCF $= a(a + 3)$

(v) $t^2 + 3t - 4, t^2 + 5t + 4, t^2 - 1$

Sol. $t^2 + 3t - 4 = t^2 + 4t - t - 4$

$= t(t + 4) - 1(t + 4)$

$= (t + 4)(t - 1)$

$t^2 + 5t + 4 = t^2 + 4t + t + 4$

$= t(t + 4) + 1(t + 4)$

$= (t + 4)(t + 1)$

$t^2 - 1 = t^2 - 1^2$

$= (t + 1)(t - 1)$

Common factor = 1

HCF = 1

(vi) $\mathbf{x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x}$

Sol. $x^2 + 15x + 56 = x^2 + 7x + 8x + 56$

$= x(x + 7) + 8(x + 7)$

$= (x + 7)(x + 8)$

$x^2 + 5x - 24 = x^2 + 8x - 3x - 24$

$= x(x + 8) - 3(x + 8)$

$= (x + 8)(x - 3)$

$x^2 + 8x = x(x + 8)$

Common factor = x + 8

HCF = x + 8

Q.2. Find HCF of the following expressions by using division method.

(i) $\mathbf{27x^3 + 9x^2 - 3x - 9, 3x - 2}$

Sol.

$$
\begin{array}{r|l}
 & 9x^2 + 9x + 5 \\
3x - 2 & 27x^3 + 9x^2 - 3x - 9 \\
 & \underline{{}_{-}27x^3 \mp 18x^2} \\
 & \quad 27x^2 - 3x - 9 \\
 & \quad \underline{{}_{-}27x^2 \mp 18x} \\
 & \qquad 15x - 9 \\
 & \qquad \underline{{}_{-}15x \mp 10} \\
 & \qquad\qquad 1
\end{array}
$$

Since remainder is not zero.

So HCF = 1

(ii) $\mathbf{x^3 - 9x^2 + 21x - 15,\ x^2 - 4x + 3}$

Sol.

$$
\begin{array}{r|l}
 & x - 5 \\
\hline
x^2 - 4x + 3 & x^3 - 9x^2 + 21x - 15 \\
 & \underline{\underset{-}{x^3} \mp 4x^2 \pm 3x} \\
 & -5x^2 + 18x - 15 \\
 & \underline{\mp 5x^2 \pm 20x \mp 15} \\
 & -2x
\end{array}
$$

HCF = 1

(iii) $\mathbf{2x^3 + 2x^2 + 2x + 2,\ 6x^3 + 12x^2 + 6x + 12}$

Sol.

$$
\begin{array}{r|l}
 & 3 \\
\hline
2x^3 + 2x^2 + 2x + 2 & 6x^3 + 12x^2 + 6x + 12 \\
 & \underline{\underset{-}{6x^3} \pm 6x^2 \pm 6x \pm 6} \\
 & 6x^2 + 6
\end{array}
$$

$$
\begin{array}{r|l}
 & 2x \\
\hline
6(x^2 + 1) & 2x^3 + 2x^2 + 2x + 2 \\
 & \underline{\underset{-}{2x^3} \quad \pm 2x} \\
 & 2x^2 + 2
\end{array}
$$

$$
\begin{array}{r|l}
2(x^2 + 1) & x^2 + 1 \\
 & \underline{\underset{-}{x^2} \pm 1} \\
 & \times
\end{array}
$$

HCF = $x^2 + 1$

(iv) $\mathbf{2x^3 - 4x^2 + 6x,\ x^3 - 2x,\ 3x^2 - 6x}$

Sol. $2x(x^2 - 2x + 3),\ x(x^2 - 2),\ 3x(x - 2)$

As x is common in all factorization, so x is HCF

Q.3. Find LCM of the following expressions by using prime factorization method.

(i) $2a^2b, 4ab^2, 6ab$

Sol. $2a^2b = 2 \times a \times a \times b$

$4ab^2 = 2 \times 2 \times a \times b \times b$

$6ab = 2 \times 3 \times a \times b$

Common factors $= 2 \times a \times b = 2ab$

Non-common factors $= 2 \times a \times b \times 3 = 6ab$

$$\begin{aligned} \text{LCM} &= 2ab \times 6ab \\ &= 12a^2b^2 \end{aligned}$$

(ii) $x^2 + x, x^3 + x^2$

Sol. $x^3 + x^2 = x^2(x + 1)$

Common factors $= x(x + 1)$

Non-common factors $= x$

So, $$\begin{aligned} \text{LCM} &= x \times x(x + 1) \\ &= x^2(x + 1) \end{aligned}$$

(iii) $a^2 - 4a + 4, a^2 - 2a$

Sol. $a^2 - 4a + 4 = (a - 2)^2$

$a^2 - 2a = a(a - 2)$

Common factors $= a - 2$

Non-common factors $= a(a - 2)$

$$\begin{aligned} \text{LCM} &= (a - 2) \times a(a - 2) \\ &= a(a - 2)^2 \end{aligned}$$

(iv) $x^4 - 16, x^3 - 4x$

Sol. $$\begin{aligned} x^4 - 16 &= (x^2)^2 - 4^2 \\ &= (x^2 + 4)(x^2 - 4) \\ &= (x^2 + 4)(x + 2)(x - 2) \end{aligned}$$

$x^3 - 4x = x(x^2 - 4) = x(x + 2)(x - 2)$

Common factors $= (x + 2)(x - 2)$

$$\begin{aligned} \text{LCM} &= x(x^2 + 4)(x^2 - 4) \\ &= x(x^4 - 16) \end{aligned}$$

(v) $16 - 4x^2, x^2 + x - 6, 4 - x^2$

Sol. $16 - 4x^2 = 4(4 - x^2)$

$= 4(2 + x)(2 - x)$

$x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= x(x + 3) - 2(x + 3)$

$= (x + 3)(x - 2)$

$= (-x - 3)(2 - x)$

$4 - x^2 \quad = 2^2 - x^2$

$= (2 + x)(2 - x)$

Common factorization $= 2 - x$

Non-common factorization $= 4(2 + x)(-x - 3)$

LCM $= 4(2 + x)(-x - 3)(2 - x)$

$= 4(2 + x)(x + 3)(x - 2)$

$= 4(x + 2)(x - 2)(x + 3)$

$= 4(x^2 - 4)(x + 3)$

4. **The HCF of two polynomials is $y - 7$ and their LCM is $y^3 - 10y^2 + 11y + 70$. If one of the polynomials is $y^2 - 5y - 14$, find the other.**

Sol. HCF $= y - 7$

LCM $= y^3 - 10y^2 + 11y + 70$

One polynomial $= y^2 - 5y - 14$

Other polynomial $= ?$

One polynomial × other polynomial = HCF × LCM

Other polynomial $= \dfrac{\text{HCF} \times \text{LCM}}{\text{one polynomial}}$

$= \dfrac{(y - 7)(y^3 - 10y^2 + 11y + 70)}{y^2 - 5y - 14}$

$$
\begin{array}{r|l}
 & y - 5 \\
\hline
x^2 - 4x + 3 & y^3 - 10y^2 + 11y + 70 \\
 & \underline{{}_{-}\,y^3 \mp 5y^2 \mp 14y} \\
 & -5y^2 + 25y + 70 \\
 & \underline{\mp 5y^2 \pm 25y \pm 70} \\
 & \times
\end{array}
$$

$$
\begin{aligned}
\text{Other polynomial} &= (y - 7)(y - 5) \\
&= y^2 - 7y - 5y + 35 \\
&= y^2 - 12y + 35
\end{aligned}
$$

5. The LCM and HCF of two polynomial p(x) and q(x) are $36x^3(x + a)(x^3 - a^3)$ and $x^2(x - a)$ respectively. If $p(x) = 4x^2(x^2 - a^2)$, find q(x).

Sol. $\text{LCM} = 36x^3(x + a)(x^3 - a^3)$

$\text{HCF} = x^2(x - a)$

$p(x) = 4x^2(x^2 - a^2)$

$q(x) = ?$

We know that

$$p(x) \times q(x) = \text{HCF} \times \text{LCM}$$

$$q(x) = \frac{\text{HCF} \times \text{LCM}}{p(x)}$$

$$q(x) = \frac{x^2(x - a)(36x^3)(x + a)(x^3 - a^3)}{4x^2(x^2 - a^2)}$$

$$= \frac{(x - a)(x + a)9x^3(x^3 - a^3)}{(x^2 - a^2)}$$

$$= \frac{9x^3(x^2 - a^2)(x^3 - a^3)}{(x^2 - a^2)}$$

$$q(x) = 9x^3(x^3 - a^3)$$

6. The HCF and LCM of two polynomials is $(x + a)$ and $12x^2(x + a)(x^2 - a^2)$ respectively. Find the product of the two polynomials.

Sol. HCF $= x + a$

LCM $= 12x^2(x + a)(x^2 - a^2)$

Let the polynomials be $p(x)$ and $q(x)$

Then $p(x) \times q(x) = \text{LCM} \times \text{HCF}$

$$= 12x^2(x + a)(x^2 - a^2) \times (x + a)$$
$$= 12x^2(x + a)(x + a)(x - a)(x + a)$$
$$= 12x^2(x + a)^3(x - a)$$

EXERCISE 4.4

Q.1. Find the square root of the following polynomials by factorization method:

(i) $x^2 - 8x + 16$

Sol. $= x^2 - 4x - 4x + 16$

$= x(x - 4) - 4(x - 4)$

$= (x - 4)(x - 4)$

$= (x - 4)^2$

Taking square root $= \pm\sqrt{(x - 4)^2}$

$= \pm(x - 4)$

(ii) $9x^2 + 12x + 4$

Sol. $= 9x^2 + 6x + 6x + 4$

$= 3x(3x + 2) + 2(3x + 2)$

$= (3x + 2)(3x + 2)$

$= (3x + 2)^2$

Taking square root $= \pm\sqrt{(3x + 2)^2}$

$= \pm(3x + 2)$

(iii) $36a^2 + 84a + 49$

Sol. $= (6a)^2 + (7)^2 + 2(6a)(7)$

$= (6a + 7)^2 \quad [\because\ a^2 + b^2 + 2ab = (a + b)^2]$

Taking square root $= \pm\sqrt{(6a + 7)^2}$

$= \pm(6a + 7)$

(iv) $\mathbf{64y^2 - 32y + 4}$

Sol. $= 4(16y^2 - 8y + 1)$

$= 4(16y^2 - 4y - 4y + 1)$

$= 4(4y(4y - 1) - 1(4y - 1))$

$= 4(4y - 1)(4y - 1)$

$= 4(4y - 1)^2$

Taking square root $= \pm\sqrt{4(4y - 1)^2}$

$= \pm 2(4y - 1)$

(v) $\mathbf{200t^2 - 120t + 18}$

Sol. $= 2(100t^2 - 60t + 9)$

$= 2((10t)^2 + (3)^2 - 2(10t)(3))$

$= 2(10t - 3)^2$

Taking square root $= \pm\sqrt{2}\,(10t - 3)$

(vi) $\mathbf{40x^2 + 120x + 90}$

Sol. $= 10(4x2 + 12x + 9)$

$= 10((2x)^2 + (3)^2 + 2(2x)(3))$

$= 10(2x + 3)^2$

Taking square root $= +\sqrt{10}(2x + 3)$

Q.2. Find the square root of the following polynomials by division method:

(i) $\mathbf{4x^4 - 28x^3 + 37x^2 + 42x + 9}$

Sol.

$$
\begin{array}{r|l}
 & \quad 2x^2 - 7x - 3 \\
\hline
2x^2 & 4x^4 - 28x^3 + 37x^2 + 42x + 9 \\
 & -4x^4 \\
\hline
4x^2 - 7x & \quad -28x^3 + 37x^2 + 42x + 9 \\
 & \quad \mp 28x^3 \pm 49x^2 \\
\hline
4x^2 - 14x - 3 & \qquad -12x^2 + 42x + 9 \\
 & \qquad \mp 12x^2 \pm 42x \pm 9 \\
\hline
 & \qquad \times
\end{array}
$$

So, Square root is $= \pm(2x^2 - 7x - 3)$

(ii) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

Sol.

$$
\begin{array}{r|l}
 & \quad 11x^2 - 9x - 12 \\ \hline
11x^2 & 121x^4 - 198x^3 - 183x^2 + 216x + 144 \\
 & -121x^4 \\ \hline
22x^2 - 9x & -198x^3 - 183x^2 + 216x + 144 \\
 & \mp 198x^3 \pm 81x^2 \\ \hline
22x^2 - 18x - 12 & -264x^2 + 216x + 144 \\
 & \mp 264x^2 \pm 216x \pm 144 \\ \hline
 & \times
\end{array}
$$

So, Square root is $= \pm(11x^2 - 9x - 12)$

(iii) $x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

Sol.

$$
\begin{array}{r|l}
 & \quad x^2 - 5xy + y^2 \\ \hline
x^2 & x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4 \\
 & \underline{\ } x^4 \mp 10x^2y \pm 25x^2y^2 \\ \hline
2x^2 - 5y & 2x^2y^2 - 10xy^3 + y^4 \\
 & \underline{\ } 2x^2y^2 \mp 10xy^3 \pm y^4 \\ \hline
 & \times
\end{array}
$$

So, Square root $= \pm(x^2 - 5xy + y^2)$

(iv) $4x^4 - 12x^3 + 37x^2 - 42x + 49$

Sol.

$$
\begin{array}{r|l}
 & \quad 2x^2 - 3x + 7 \\ \hline
11x^2 & 4x^4 - 12x^3 + 37x^2 - 42x + 49 \\
 & -4x^4 \\ \hline
22x^2 - 9x & -12x^3 + 37x^2 - 42x + 49 \\
 & \mp 12x^3 \pm 9x^2 \\ \hline
4x^2 - 6x + 7 & 28x^2 - 42x + 49 \\
 & \underline{\ } 28x^2 \mp 42x \pm 49 \\ \hline
 & \times
\end{array}
$$

So, Square root is $= \pm(2x^2 - 3x + 7)$

Q.3. An investor's return R(x) in rupees after investing x thousand rupees is given by quadratic expression.

$$R(x) = -x^2 + 6x - 8$$

Factor the expression and find the investment levels that result in zero return.

Sol.
$$\begin{aligned} R(x) &= -x^2 + 6x - 8 \\ &= -x^2 + 4x + 2x - 8 \\ &= -x(x-4) + 2(x-4) \\ &= (-x+2)(x-4) \\ &= (2-x)(x-4) \end{aligned}$$

Put $R(x) = 0$

$\Rightarrow \quad (2-x)(x-4) = 0$

$\Rightarrow \quad 2 - x = 0, x - 4 = 0$

$x = 2, x = 4$

Q.4. A company's profit P(x) in rupees from selling x units of a product is modeled by the cubic expression:

$$P(x) = x^3 - 15x^2 + 75x - 125$$

Find the break-even point(s), where the profit is zero.

Soil. $p(x) = x^3 - 15x^2 + 75x - 125$

Put $x = +5$ above

$$\begin{aligned} p(-5) &= (+5)^3 - 15(+5)^2 + 75(+5) - 125 \\ &= +125 - 15 \times 25 + 375 - 125 \\ &= -375 + 375 = 0 \end{aligned}$$

So $(x - 5)$ is a factor of $p(x)$.

$$\begin{array}{r|l} & x^2 - 10x + 25 \\ \hline x-5 & x^3 - 15x^2 + 75y - 125 \\ & \underline{-x^3 \mp 5x^2} \\ & \quad -10x^2 + 75x - 125 \\ & \quad \underline{\mp 10x^2 \pm 50x} \\ & \qquad 25x - 125 \\ & \qquad \underline{-25x \mp 125} \\ & \qquad \times \end{array}$$

$$= x^2 - 10x + 25$$
$$= (x - 5)^2$$

So

$$p(x) = x^3 - 15x^2 + 75x - 125$$
$$= (x - 5)(x - 5)(x - 5)$$

At $x = 5$

Profit is zero.

Q.5. The potential energy V(x) in an electric field varies as a cubic function of distance x, given by:

$$\mathbf{V(x) = 2x^3 - 6x^2 + 4x}$$

Determine where the potential energy is zero.

Sol. $V(x) = 2x^3 - 6x^2 + 4x$

$$= 2x(x^2 - 3x + 2)$$
$$= 2x(x^2 - 2x - x + 2)$$
$$= 2x(x(x - 2) - 1(x - 2))$$
$$= 2x(x - 2)(x - 1)$$

Potential energy is zero

i.e. $2x(x - 1)(x - 2) = 0$

$\Rightarrow$ $x = 0, 1, 2$

So potential energy is zero at $x = 0$ and $x = 1$ and $x = 2$.

Q.6. In structural engineering, the deflection y(x) of a beam is given by:

$$\mathbf{Y(x) = 2x^2 - 8x + 6}$$

This equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection.

Sol. $Y(x) = 2x^2 - 8x + 6$

$$= 2(x^2 - 4x + 3)$$
$$= 2(x^2 - 3x - x + 3)$$
$$= 2(x(x - 3) - 1(x - 3))$$
$$= 2(x - 1)(x - 3)$$

Points of zero deflection

$$x - 1 = 0, x - 3 = 0$$

$$x = 1, \quad x = 3$$

REVIEW EXERCISE 4

Q.1. Choose the correct option.

(i) The factorization of $12x + 36$ is:

(a) $12(x + 3)$ (b) $12(3x)$

(c) $12(3x + 1)$ (d) $x(12 + 36x)$

Sol. $12x + 36 = 12(x + 3)$

Option (a) is correct.

(ii) The factor of $4x^2 - 12x + 9$ are:

(a) $(2x + 3)^2$ (b) $(2x - 3)^2$

(c) $(2x - 3)(2x + 3)$ (d) $(2x + 3x)(2 - 3x)^2$

Sol. $4x^2 - 12x + 9$

$= 4x^2 - 6x - 6x + 9$

$= 2x(2x - 3) - 3(2x - 3)$

$= (2x - 3)(2x - 3)$

$= (2x - 3)^2$

Option (b) is correct.

(iii) The HCF of a^3b^3 and ab^2 is:

(a) a^3b^3 (b) ab^2

(c) a^4b^5 (d) a^2b

Sol. HCF of a^3b^3 & ab^2 is ab^2

Option (b) is correct.

(iv) The LCM of $16x^2$, $4x$ and $30xy$ is:

(a) $480x^3y$ (b) $240xy$

(c) $240x^2y$ (d) $120x^4y$

Sol. $16x^2 = 2 \times 2 \times 2 \times 2 \times x \times x$

$4x = 2 \times 2 \times x$

$30xy = 2 \times 3 \times 5 \times x \times y$

$LCM = 2 \times 2 \times 3 \times 2 \times 2 \times 5 \times x \times x \times y$

$= 240x^2y$

Option (c) is correct.

(v) Product of LCM and HCF = ________ of two polynomials.

(a) sum (b) difference

(c) product (d) quotient

Sol. Product of LCM and HCF = Product of two polynomials.

Option (c) is correct.

(vi) The square root of $x^2 - 6x + 9$ is:

(a) $\pm(x - 3)$ (b) $\pm(x + 3)$

(c) $x - 3$ (d) $x + 3$

Sol. $x^2 - 6x + 9 = (x - 3)^2$

Square root $= \pm(x - 3)$

Option (a) is correct.

(vii) The LCM of $(a - b)^2$ and $(a - b)^4$ is:

(a) $(a - b)^2$ (b) $(a - b)^3$

(c) $(a - b)^4$ (d) $(a - b)^4$

Sol. LCM of $(a - b)^2$ & $(a - b)^4 = (a - b)^4$

So option (c) is correct.

(viii) Factorization of $x^3 + 3x^2 + 3x + 1$ is:

(a) $(x + 1)^3$ (b) $(x - 1)^3$

(c) $(x + 1)(x^2 + x + 1)$ (d) $(x - 1)(x^2 - x + 1)$

Sol. $x^3 + 3x^2 + 3x + 1$

$= x^3 + 1 + 3x^2 + 3x$

$= x^3 + 1 + 3x(x + 1)$

$= (x + 1)^3$

Option (a) is correct.

(ix) Cubic polynomial has degree:

(a) 1 (b) 2

(c) 3 (d) 4

Sol. Cubic polynomial has degree 3.

Option (c) is correct.

(x) One of the factors of $x^3 - 27$ is:

(a) $x - 3$ (b) $x + 3$

(c) $x^2 - 3x + 9$ (d) Both a and c

Sol. $x^3 - 27 = x^3 - 27$

$= x^3 - 3^3$

$= (x - 3)(x^2 + 3x + 9)$

Option (a) is correct.

Q.2. Factorize the following expression.

(i) $\mathbf{4x^3 + 18x^2 - 12x}$

Sol. $= 2x(2x^2 + 9x - 6)$

(ii) $\mathbf{x^3 + 64y^3}$

Sol. $= (x)^3 + (4y)^3$

$= (x + 4y)(x^2 - (x)(4y) + (4y)^2)$

$= (x + 4y)(x^2 - 4xy + 16y^2)$

(iii) $\mathbf{x^3y^3 - 8}$

Sol. $= (xy)^3 - 2^3$

$= (xy - 2)((xy)^2 + (xy)(2) + 2^2)$

$= (xy - 2)(x^2y^2 + 2xy + 4)$

(iv) $\mathbf{-x^2 - 23x - 60}$

Sol. $= -(x^2 + 23x + 60)$

$= -(x^2 + 20x + 3x + 60)$

$= -(x(x + 20) + 3(x + 20))$

$= -(x + 20)(x + 3)$

(v) $\mathbf{2x^2 + 7x + 3}$

Sol. $= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(vi) $\mathbf{x^4 + 64}$

Sol. $= (x^2)^2 + 8^2 + 2(x^2)(8) - 2(x^2)(8)$

$= (x^2 + 8) - 16x^2$

$= (x^2 + 8) - (4x)^2$

$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$

(vii) $\mathbf{x^4 + 2x^2 + 9}$

Sol. $= (x^2)^2 + 3^2 + 2(x^2)(3) - 6x^2 + 2x^2$

$= (x^2 + 3)^2 - 4x^2$

$= (x^2 + 3)^2 - (2x)^2$

$= (x^2 + 3 + 2x)(x^2 + 3 - 2x)$

$= (x^2 + 2x + 3)(x^2 - 2x + 3)$

(viii) $\mathbf{(x + 3)(x + 4)(x + 5)(x + 6) - 360}$

Sol. $= (x + 3)(x + 60(x + 4)(x + 5) - 360$

$= (x^2 + 9x + 18)(x^2 + 9x + 20) - 360$

Let $x^2 + 9x = y$

$= (y + 18)(y + 20) - 360$

$= y^2 + 18y + 20y + 360 - 360$

$= y^2 + 38y$

$= y(y + 38)$

$= (x^2 + 9x)(x^2 + 9x + 38)$

$= x(x + 9)(x^2 + 9x + 38)$

(ix) $\mathbf{(x^2 + 6x + 3)(x^2 + 6x - 9) + 36}$

Sol. Let $x^2 + 6x = y$

$= (y + 3)(y - 9) + 36$

$= y^2 + 3y - 9y - 27 + 36$

$= y^2 - 6y + 9$

$= (y - 3)^2$

$= (x^2 + 6x - 3)^2$

Q.3. Find LCM and HCF by prime factorization method:

(i) $\mathbf{4x^3 + 12x^2, 8x^2 + 16x}$

Sol. $4x^3 + 12x^2 = 4x^2(x + 3)$

$8x^2 + 16x = 8x(x + 2)$

$\text{HCF} = 4x$

$\text{LCM} = 4x \times 2x \times (x + 3)(x + 2)$

$= 8x^2(x + 2)(x + 3)$

(ii) $\mathbf{x^3 + 3x^2 - 4x, x^2 - x - 6}$

Sol. $x^3 + 3x^2 - 4x = x(x^2 + 3x - 4)$

$= x(x^2 + 4x - x - 4)$

$= x(x(x + 4) - 1(x + 4))$

$= x(x - 1)(x + 4)$

$x^2 - x - 6 = x^2 - 3x + 2x - 6$

$= x(x - 3) + 2(x - 3)$

$= (x - 3)(x + 2)$

$\text{HCF} = 1$

$\text{LCM} = x(x - 1)(x + 4)(x + 2)(x - 3)$

$= x(x - 1)(x - 3)(x + 2)(x + 4)$

(iii) $\mathbf{x^2 + 8x + 16, x^2 - 16}$

Sol. $x^2 + 8x + 16 = x^2 + 2(x)(4) + (4)^2$

$= (x + 4)^2$

$x^2 - 16 = x^2 - 4^2$

$= (x + 4)(x - 4)$

$\text{HCF} = x + 4$

$\text{LCM} = (x + 4)^2(x - 4)$

$= (x + 4)(x + 4)(x - 4)$

$= (x + 4)(x^2 - 16)$

(iv) $\mathbf{x^3 - 9x, x^2 - 4x + 3}$

Sol.

$x^3 - 9x = x(x^2 - 9)$

$= x(x^2 - 3^2)$

$= x(x + 3)(x - 3)$

$$x^2 - 4x + 3 = x(x-3) - 1(x-3)$$
$$= (x-3)(x-1)$$

HCF $= x - 3$

LCM $= x(x+3)(x-1)(x-3)$

LCM $= x(x-1)(x^2-9)$

Q.4. Find square root by factorization and division method of the expression $16x^4 + 8x^2 + 1$.

Sol. $16x^4 + 8x^2 + 1 = (4x^2)^2 + 2(4x^2)(1) + 1^2$
$$= (4x^2+1)^2$$

Square root $= \pm\sqrt{4x^2+1}$

By division method

$$\begin{array}{r|l} & \quad 4x^2 + 1 \\ \hline 4x^2 & 16x^4 + 8x^2 + 1 \\ & -16x^4 \\ \hline 8x^2 + 1 & \quad -8x^2 + 1 \\ & \quad \underline{-}\,8x^2 \underline{\pm}\, 1 \\ \hline & \qquad \times \end{array}$$

Square root $= \pm\sqrt{4x^2+1}$

Q.5. Huraira is analyzing the total cost of a loan is modeled by the expression $C(x) = x^2 - 8x + 15$, where x is the number of years. Find the optimal repayment period for Huraira's loan?

Sol. $C(x) = x^2 - 8x + 15$
$$= x^2 - 5x - 3x + 15$$
$$= x(x-5) - 3(x-5)$$
$$= (x-5)(x-3)$$

To find optimal repayment period,

$$C(x) = 0$$

$x - 5 = 0$; $x - 3 = 0$

$x = 5$; $x = 3$