Unit No. 7

Coordinate Geometry

Review Exercise No. 7

Question No. 1

Four options are given against each statement.

Encircle the correct option.

(i) The equation of a straight line in the slope-intercept form is written as:

(a) $y = m(x + c)$

(b) $y - y1 = m(x - x1)$

(c) $y = c + mx$

(d) $ax + by + c=0$

(ii) The gradients of two parallel lines are:

(a) equal

(b) zero

(c) negative reciprocals of each other

(d) always undefined

(iii) If the product of the gradients of two lines is −1, then the lines are:

(a) Parallel

(b) perpendicular

(c) Collinear

(d) coincident

(iv) Distance between two points P (1, 2) and Q (4,6) is:

(a) 5

(b) 6

(c) $\sqrt{13}$

(d) 4

(v) The midpoint of a line segment with endpoints (−2, 4) and (6, −2) is:

(a) (4, 2)

(b) (2, 1)

(c) (1, 1)

(d) (0, 0)

(vi) A line passing through points (1, 2) and (4, 5) is:

(a) $y = x + 1$

(b) $y = 2x + 3$

(c) $y = 3x - 2$

(d) $y = x + 2$

(vii) The equation of a line in point-slope form is:

(a) $y = m\,(x + c)$

(b) $y - y_1 = m\,(x - x_1)$

(c) $y = c + mx$

(d) $ax + by + c = 0$

(viii) $2x + 3y - 6 = 0$ in the slope-intercept form is:

(a) $y = \frac{-2}{3}x + 2$

(b) $y = \frac{2}{3}x - 2$

(c) $y = \frac{2}{3}x + 1$

(d) $y = \frac{-2}{3}x - 2$

(ix) The equation of a line in symmetric form is:

(a) $\frac{x}{a} + \frac{y}{b} = 1$

(b) $\frac{x - x_1}{1} + \frac{y - y_1}{m} = \frac{z - z_1}{1}$

(c) $ax + by + c = 0$

(d) $y - y_1 = m\,(x - x_1)$

(x) The equation of a line in normal form is: (a) $y = mx + c$

(b) $\frac{x}{a} + \frac{y}{b} = 1$

(c) $\frac{x-x_1}{cos\alpha} = \frac{y-y_1}{sin\alpha}$

(d) $x cos\alpha + y sin\alpha = p$

Question No. 2

Find the distance between two points

A (2, 3) and B (7, 8) on a coordinate plane.

Data:

A = (2, 3)

B = (7, 8)

$\boldsymbol{x_1 = 2}$; $\boldsymbol{x_2 = 7}$

$\boldsymbol{y_1 = 3}$; $\boldsymbol{y_2 = 8}$

To Find:

|AB| = ?

Solution:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(7 - 2)^2 + (8 - 3)^2}$

$|AB| = \sqrt{(5)^2 + (5)^2}$

$|AB| = \sqrt{25 + 25}$

$|AB| = \sqrt{50}$

$|AB| = 5\sqrt{2}$

Question No. 3

Find the midpoint of the line segment joining the points (4, −2) and (−6, 3).

Data:

A = (4, −2)

B = (−6, 3)

$\boldsymbol{x_1 = 4}$; $\boldsymbol{x_2 = -6}$

$\boldsymbol{y_1 = -2}$; $\boldsymbol{y_2 = 3}$

To Find:

Mid-Point(AB) = ?

Solution:

$$\text{Mid} - \text{Point(AB)} = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

$$\text{Mid} - \text{Point(AB)} = (\frac{4 - 6}{2}, \frac{-2 + 3}{2})$$

$$\text{Mid} - \text{Point(AB)} = (\frac{-2}{2}, \frac{1}{2})$$

$$\text{Mid} - \text{Point(AB)} = (-1, \frac{1}{2})$$

Question No. 4

Calculate the gradient (slope) of the line passing through the points (1, 2) and (4, 6).

Data:

A = (1, 2)

B = (4, 6)

$\boldsymbol{x_1 = 1}$; $\boldsymbol{x_2 = 4}$

$\boldsymbol{y_1 = 2}$; $\boldsymbol{y_2 = 6}$

To Find:

Slope of the line AB = m = ?

Solution:

$$\text{Slope of the line AB} = \text{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{m} = \frac{6 - 2}{4 - 1} = \frac{4}{3}$$

Question No. 5

Find the equation of the line in the form y = mx + c that passes through the points (3, 7) and (5, 11).

Data:

A = (3, 7)

B = (5, 11)

$\boldsymbol{x_1 = 3}$; $\boldsymbol{x_2 = 5}$

$y_1 = 7 \quad ; \; y_2 = 11$

To Find:

Equation of the line AB = ?

Solution:

$$\text{Slope of the line AB} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11 - 7}{5 - 3} = \frac{4}{2} = 2$$

Equation of line is;

$$y - y_1 = m(x - x_1)$$
$$y - 7 = 2(x - 3)$$
$$y - 7 = 2x - 6$$
$$0 = 2x - 6 + 7 - y$$
$$y = 2x + 1$$

Question No. 6

If two lines are parallel and one line has a gradient of $\frac{2}{3}$, what is the gradient of the other line?

Solution:

If two lines are parallel, they have the same gradient (slope). Therefore, if one line has a gradient of $\frac{2}{3}$, the gradient of the other parallel line is also $\frac{2}{3}$.

If two lines are perpendicular, then product of their gradient would be -1.

Question No. 7

An airplane needs to fly from city A to coordinates (12, 5) to city B at coordinates (8, −4). Calculate the straight-line distance between these two cities.

Data:

A = (12, 5)
B = (8, −4)

To Find:

|AB| = ?

Solution:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(8 - 12)^2 + (-4 - 5)^2}$$

$$|AB| = \sqrt{(-4)^2 + (-9)^2}$$

$$|AB| = \sqrt{16 + 81}$$

$|AB| = \sqrt{97}$

$|AB| = 9.85$ Units

Question No. 8

In a landscaping project, the path starts at (2, 3) and ends at (10, 7). Find the midpoint.

Data:

A = (2, 3)

B = (10, 7)

$x_1 = 2 \quad ; \quad x_2 = 10$

$y_1 = 3 \quad ; \quad y_2 = 7$

To Find:

Mid-Point(AB) = ?

Solution:

$$\text{Mid} - \text{Point(AB)} = (\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2})$$

$$\text{Mid} - \text{Point(AB)} = (\frac{2 + 10}{2} , \frac{3 + 7}{2})$$

$$\text{Mid} - \text{Point(AB)} = (\frac{12}{2} , \frac{10}{2})$$

$$\text{Mid} - \text{Point(AB)} = (6 , 5)$$

Question No. 9

A drone is flying from point (2, 3) to point (10, 15) on the grid. Calculate the gradient of the line along which the drone is flying and the total distance travelled.

Data:

A = (2, 3)

B = (10, 15)

$x_1 = 2 \quad ; \quad x_2 = 10$

$y_1 = 3 \quad ; \quad y_2 = 15$

To Find:

Slope of line AB = m = ?

Distance between AB =|AB| = ?

Solution:

Slope of the line AB $= m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{15 - 3}{10 - 2} = \frac{12}{8} = \frac{3}{2}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(10 - 2)^2 + (15 - 3)^2}$

$|AB| = \sqrt{(8)^2 + (12)^2}$

$|AB| = \sqrt{64 + 144}$

$|AB| = \sqrt{208} = \sqrt{4 \times 4 \times 13}$

$|AB| = 4\sqrt{13}$

$|AB| = 14.42$ Units

Question No. 10

For a line with a gradient of −3 and a y-intercept of 2, write the equation of the line in:
(a) Slope-intercept form
(b) Point-slope form using the point (1, 2)
(c) Two-point form using the points (1, 2) and (4, −7)
(d) Intercepts form
(e) Symmetric form
(f) Normal form

Data:

Gradient = m = −3

y-intercept = c = 2

Solution Part (a):

Gradient = m = −3 ; y-intercept = c = 2

Slope of line form is;

$y = mx + c$

$y = -3x + 2$

Solution Part (b):

Gradient = m = −3 ; y-intercept = c = 2

Point = (1, 2)

Point slope of line form is;

$y - y_1 = m(x - x_1)$

$y - 2 = -3(x - 1)$

y – 2 = -3x + 3
y – 2 + 3x – 3 = 0
3x + y – 5 = 0

Solution Part (c):

Two-point form using the points (1, 2) and (4, −7)

$x_1 = 1 \quad ; \quad x_2 = 4$

$y_1 = 2 \quad ; \quad y_2 = -7$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{-7 - 2} = \frac{x - 1}{4 - 1}$$

Solution Part (d):

Intercept form:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Consider 1st part as;
y = -3x + 2

$$-\frac{3}{2}x + \frac{y}{2} = \frac{2}{2}$$

$$-\frac{x}{\frac{2}{3}} + \frac{y}{2} = \frac{2}{2}$$

Solution Part (e):

Symmetric form;
y = -3x + 2
3x + y = 2
Divide both sides by $\sqrt{3^2 + 1^2} = \sqrt{10}$;

$\frac{3x}{\sqrt{10}} + \frac{y}{\sqrt{10}} = \frac{2}{\sqrt{10}}$

$\frac{y}{\sqrt{10}} + \frac{3x}{\sqrt{10}} = \frac{2}{\sqrt{10}}$

Solution Part (f):

Normal Form;

$x \cos\alpha + y \sin\alpha = p$

Consider 1st part;
y = -3x + 2
3x + y = 2
Divide both sides by $\sqrt{3^2 + 1^2} = \sqrt{10}$;

$\frac{3x}{\sqrt{10}} + \frac{y}{\sqrt{10}} = \frac{2}{\sqrt{10}}$

Here $\cos\alpha = \frac{3}{\sqrt{10}} \quad ; \quad \sin\alpha = \frac{1}{\sqrt{10}} \quad ; p = \frac{2}{\sqrt{10}}$

$$\cos\alpha = \frac{3}{\sqrt{10}}$$

$$\alpha = \cos^{-1}\frac{3}{\sqrt{10}}$$

$$\alpha = 18.43°$$

Normal form is;

$$x\cos(18.43° - 90°) + y\ \sin(18.43° - 90°) = \frac{2}{\sqrt{10}}$$

$$x\cos(-71.56°) + y\ \sin(-71.56°) = \frac{2}{\sqrt{10}}$$