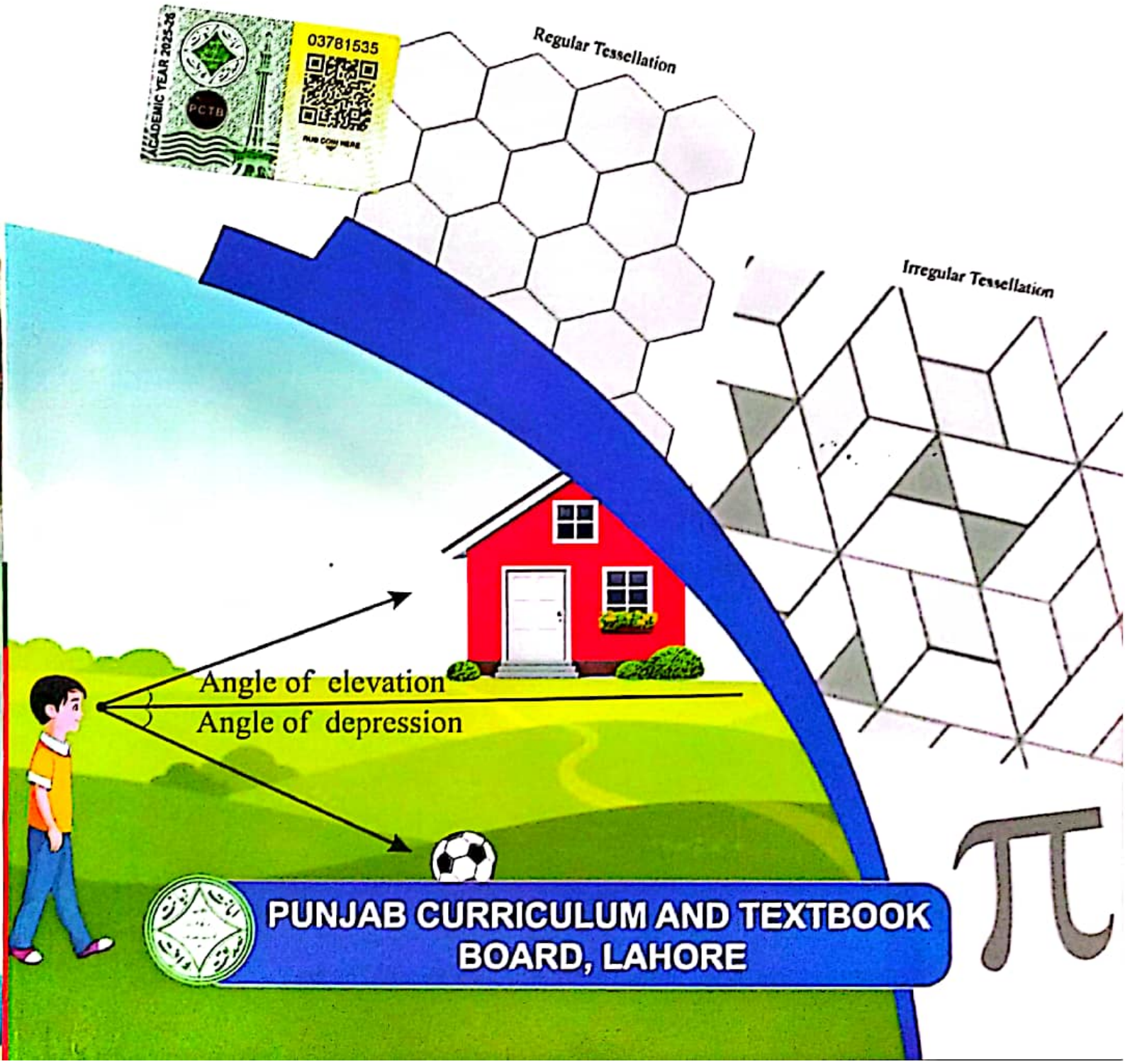
MATHEMATICS 9



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UNIT 12 INFORMATION HANDLING

EXERCISE 12.1

Q.1. The following distribution represents the scores achieved by a group of chemistry students in the chemistry laboratory.

Scores	24-28	29-33	34-38	39-43	44-48	49-53	Total
No. of students	3	6	12	23	15	6	65

Answer the following questions.

(i) What is the upper limit of the last class?

Sol. Last class is 49 – 53

Upper limit is 53

(ii) What is the lower limit of the class 39 – 34?

Sol. Lower limit of 39 – 43 is 39.

(iii) What is the midpoint of the class (34 – 38)?

Sol. Midpoint or class mark of the class 34 – 38 is

$$= \frac{34 + 38}{2}$$

$$= \frac{72}{2} = 36$$

(iv) What are the class frequencies of the classes 29 – 33 and 44 – 48?

Sol. From table, class frequency of the class 29 – 33 is 6 and that of the class 44 – 48 is 15.

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(v) What is the size of the class limits in the above frequency distribution?

Sol. Size of class limits $= \dfrac{\text{Greatest value - Smallest value}}{\text{No. of classes}}$

$$= \frac{53-24}{6} = \frac{29}{6} = 4.833$$

$$\approx 5$$

(vi) In which class or group does minimum number of students fall?

Sol. Carefully seeing at table, the minimum number of students fall in group 24 – 28.

(vii) What is the lower limit of the class having 15 as its class frequency?

Sol. Lower limit of class having 15 as class frequency is 44.

(viii) What is the number of students having scores between 24 and 43?

Sol. No. of students having scores between 24 and 43

$$= 3 + 6 + 12 + 33$$

$$= 44$$

Q.2. For a school staff, the following expenditures (rupees in hundred are required for the repair of chairs.

145, 152, 153, 156, 158, 160, 146, 152, 155, 159, 161, 163, 165, 147, 148, 151, 154, 156, 158, 160, 144, 167, 151, 150, 152, 149, 145, 153, 152, 155

Prepare a frequency distribution by tally bar method using 3 as the size of class limits and also write down what are the frequencies of the last three classes?

Sol. First we arrange the data in ascending order as

144, 145, 145, 146, 147, 148, 149, 150, 151, 151, 152, 152, 152, 152, 153, 153, 154, 155, 155, 156, 156, 158, 158, 159, 160, 160, 161, 163, 165, 167

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Frequency Distribution is

Class Limits	Tally Marks	f
144 – 146	IIII	4
147 – 149	III	3
150 – 152	~~IIII~~ II	7
153 – 155	~~IIII~~	5
156 – 158	IIII	4
159 – 161	IIII	4
162 – 164	I	1
165 – 167	II	2
Total		$\Sigma f = 30$

Frequencies of last 3 classes are 4, 1, –2 respectively.

Given below are the weights in kg of 30 students of a high school.

33, 24, 21, 15, 39, 37, 44, 42, 33,
28, 29, 32, 31, 28, 26, 32, 34, 35,
36, 41, 30, 35, 41, 23, 26, 18, 34

Taking 5 as the size of the class limit, prepare a frequency table and construct a frequency polygon.

Class Limits	Tally Marks	C.B	f
15 – 19	II	14.5 – 19.5	2
20 – 24	III	19.5 – 24.5	3
25 – 29	~~IIII~~	24.5 – 29.5	5
30 – 34	~~IIII~~ ~~IIII~~	29.5 – 34.5	10
35 – 39	~~IIII~~ I	34.5 – 39.5	6
40 – 44	IIII	39.5 – 44.5	4
Total			$\Sigma f = 30$

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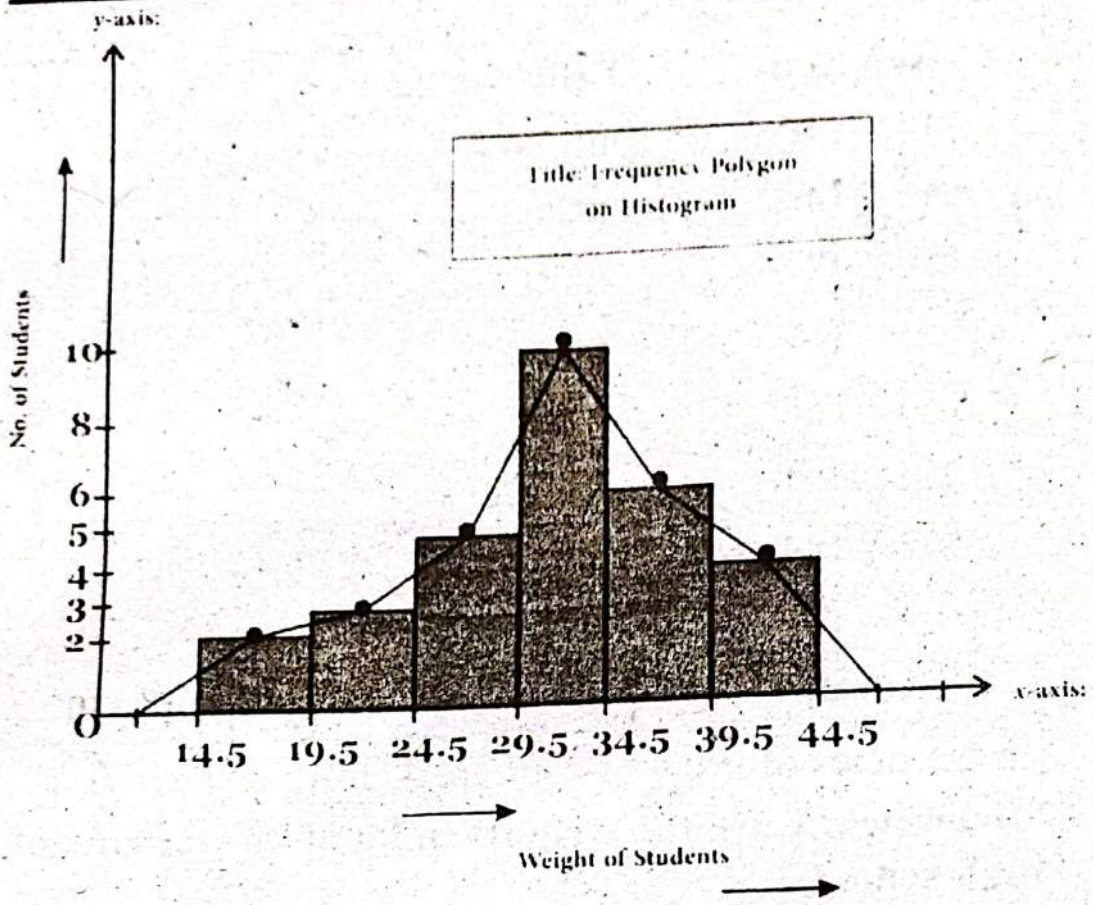

Q.4. **A group of Grade-10 students obtained the following marks out of 100 marks in English test.**

58, 59, 58, 33, 40, 58, 45, 46, 43, 45, 45,
50, 52, 49, 50, 57, 52, 55, 49, 50, 62, 49,
48, 44, 42, 47, 46, 47, 46, 53, 40, 44

Classify the data into a frequency distribution by (direct method) taking 6 as the size of class limit. Also find the class limit with least class frequency and construct histogram for the data.

Sol.

Class Limits	Tally Marks	C.B	f
33 – 38	I	32.5 – 38.5	1
39 – 44	~~IIII~~ I	38.5 – 44.5	6
45 – 50	~~IIII~~ ~~IIII~~ ~~IIII~~	44.5 – 50.5	15
51 – 56	IIII	50.5 – 56.5	4
57 – 62	~~IIII~~ I	56.5 – 62.5	6
Total			$\Sigma f = 32$

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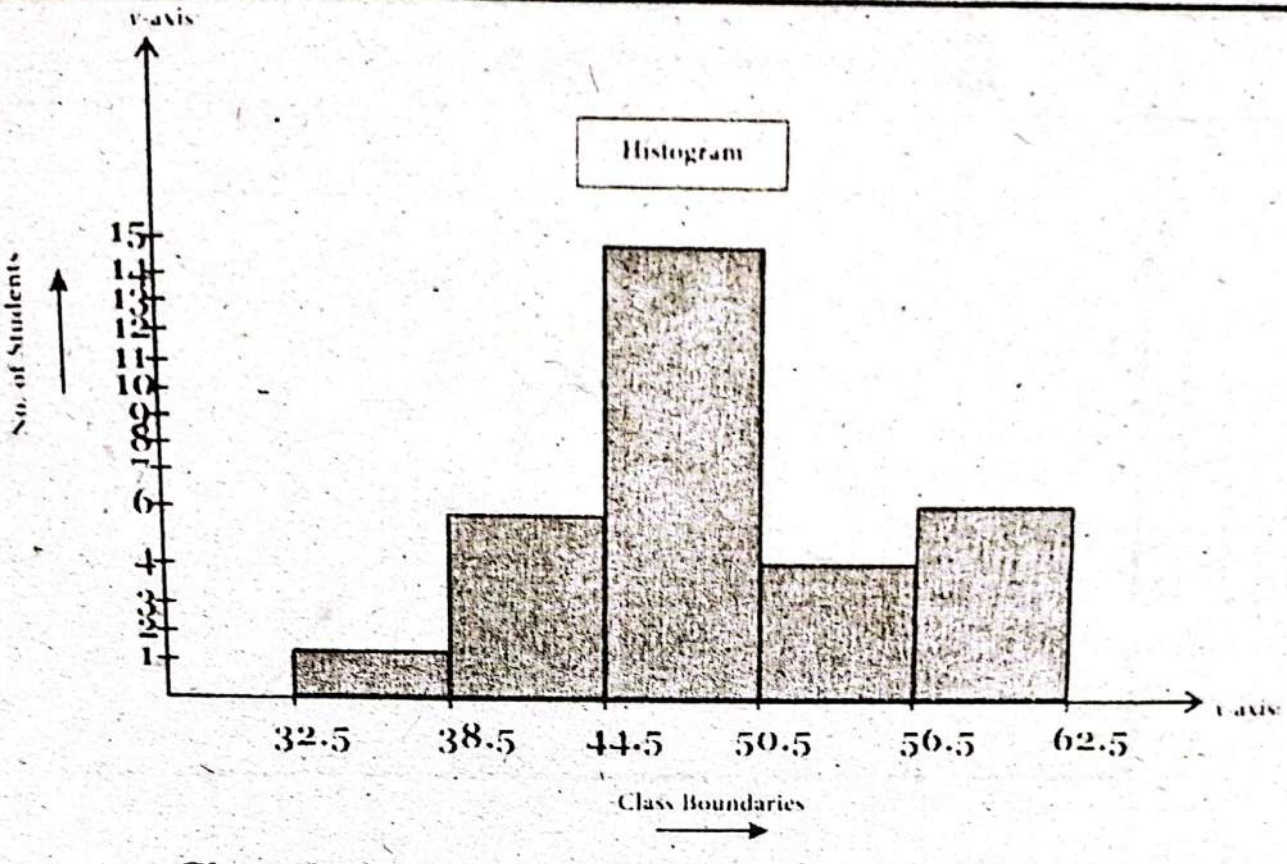

Class limit with least class frequency is 33 - 38

Q.5. From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (*f*)	06	17	23	30	22	13

Sol.

Weight (kg)	Class Boundaries	Frequency (*f*)
10 – 14	9.5 – 14.5	6
15 – 19	14.5 – 19.5	17
20 – 24	19.5 – 24.5	23
25 – 29	24.5 – 29.5	30
30 – 34	29.5 – 34.5	22
35 – 39	34.5 – 39.5	13

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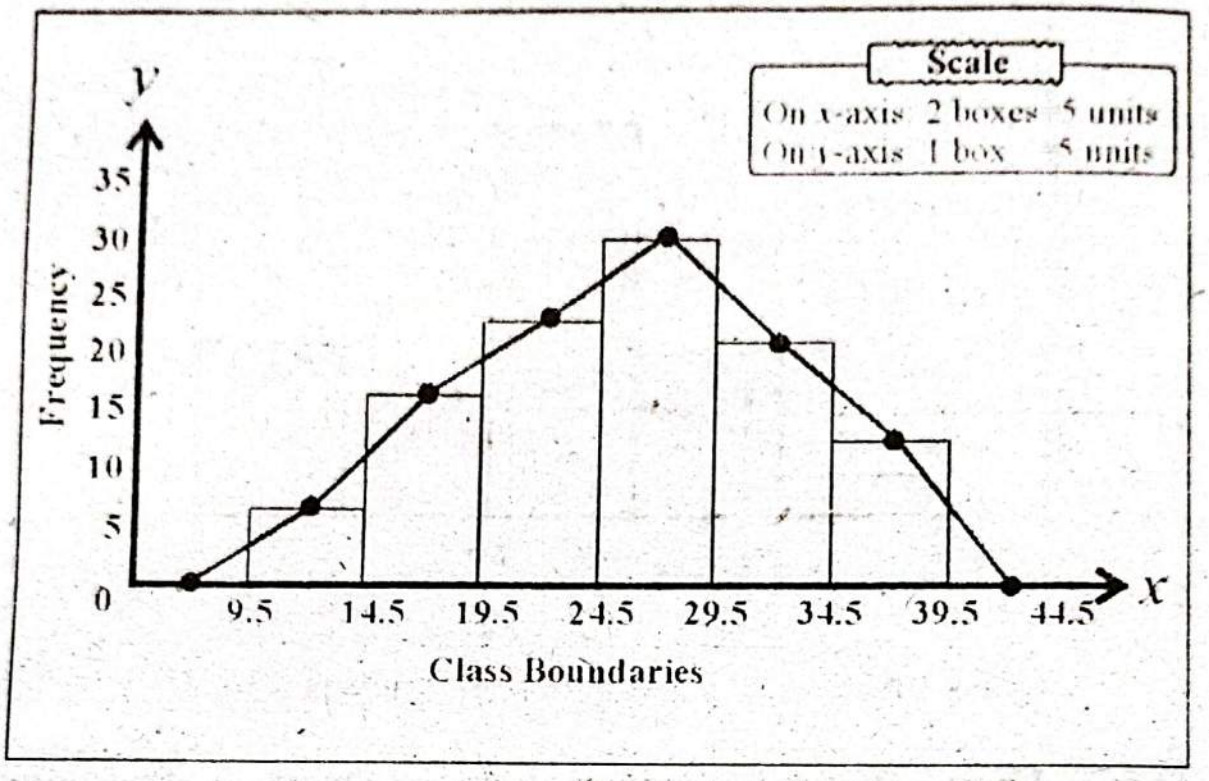
Title Frequency polygon on histogram



Weight (kg) ⟶

Q.6. The following data shows the number of heads in an experiment of 50 sets of tossing a coin 5 times. Make a discrete frequency distribution from the information.

3, 3, 4, 0, 5, 4, 3, 3, 1, 2, 4, 5, 0, 3, 2, 4, 4, 0, 0, 0, 5, 5, 3, 2, 1
4, 3, 2, 5, 3, 2, 1, 3, 5, 4, 3, 2, 1, 3, 2, 1, 3, 1, 3, 1, 4, 3, 2, 2, 4

Sol.

No. of heads	Tally Marks	f
0	~~IIII~~	5
1	~~IIII~~ II	7
2	~~IIII~~ IIII	9
3	~~IIII~~ ~~IIII~~ IIII	14
4	~~IIII~~ IIII	9
5	~~IIII~~ I	6
Total		$\Sigma f = 50$

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Q.7. **The marks obtained by the students of Grade-10 in mathematics test were grouped into the following frequency distribution.**

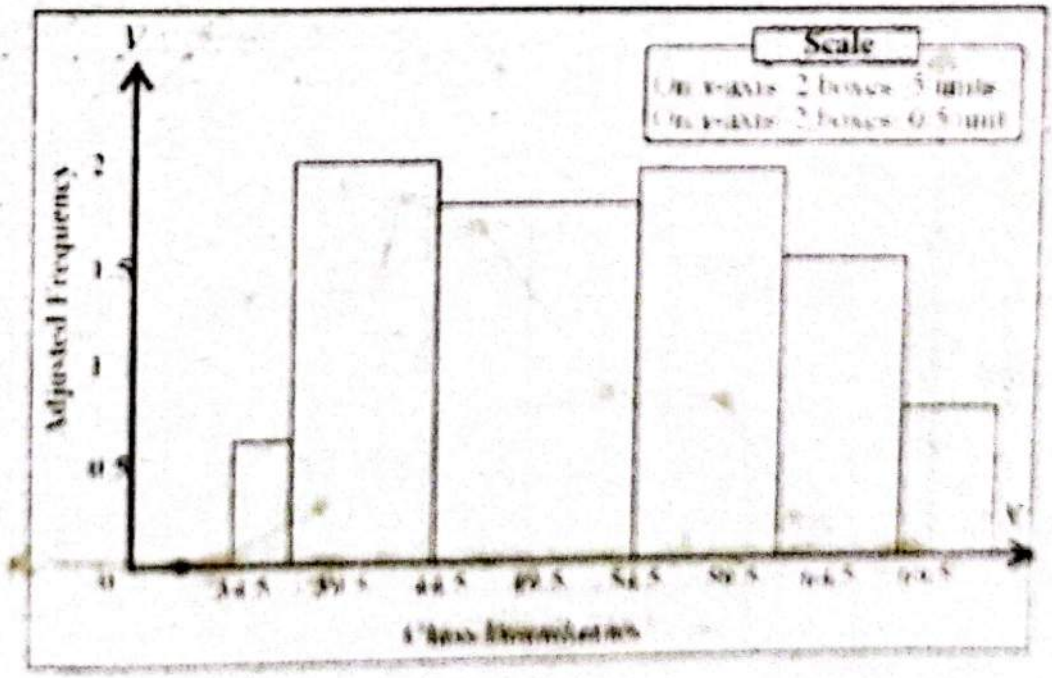
Marks	35-37	38-44	45-54	55-61	62-67	68-72
Frequency	2	12	16	13	9	3

Draw a histogram for the above distribution.

Sol.

Class Limits	Width of class	Adjusted frequency	f
35 – 37	37 – 35 = 2	$\frac{2}{2} = 1$	2
38 – 44	44 – 38 = 6	$\frac{12}{6} = 2$	12
45 – 54	54 – 45 = 9	$\frac{16}{9} = 1.8$	16
55 – 61	61 – 55 = 6	$\frac{13}{6} = 2.17$	13
62 – 67	67 – 62 = 5	$\frac{9}{5} = 1.8$	9
68 – 72	72 – 68 = 4	$\frac{3}{4} = 0.75$	3

Title **Histogram of marks obtained by students**



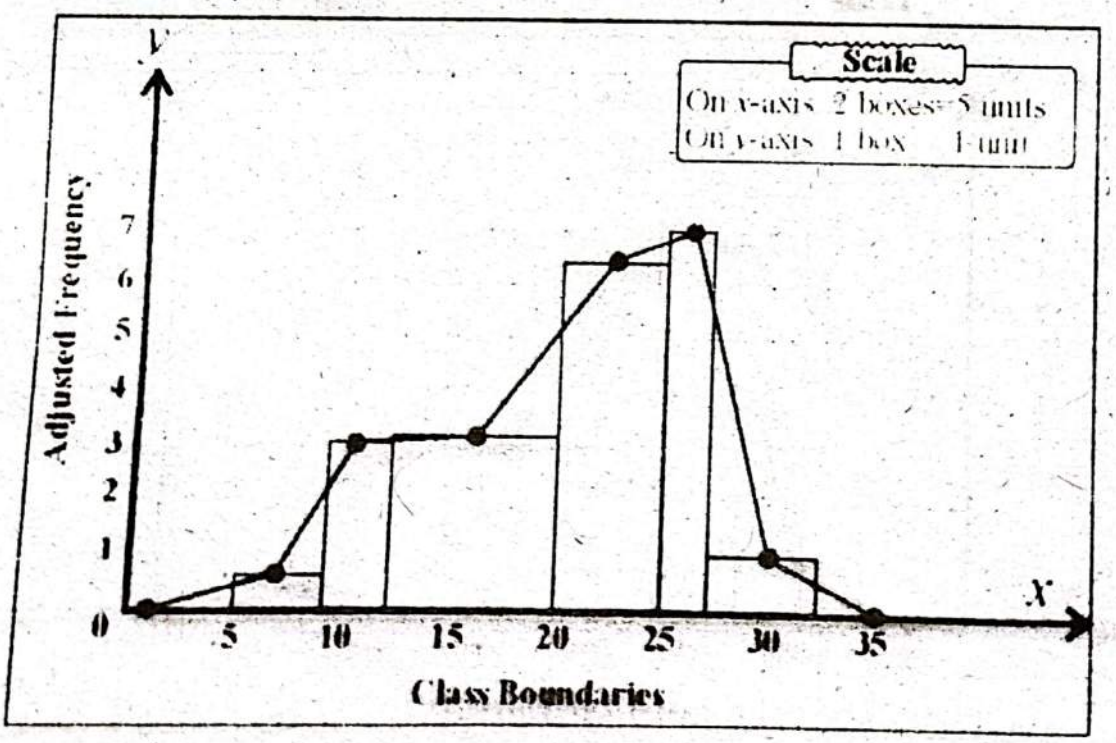
Q.8. Make a frequency polygon on histogram for the following grouped data:

Class limits	5-8	8-12	12-20	20-25	25-27	27-32
Frequency (f)	2	12	25	32	14	5

Sol.

Class Limits	Width of class	f	Adjusted frequency
5 – 8	8 – 5 = 3	2	$\frac{2}{3} = 0.7$
8 – 12	12 – 8 = 4	12	$\frac{12}{4} = 3$
12 – 20	20 – 12 = 8	25	$\frac{25}{8} = 3.125$
20 – 25	25 – 20 = 5	32	$\frac{32}{5} = 6.4$
25 – 27	27 – 25 = 2	14	$\frac{14}{2} = 7$
27 – 32	32 – 27 = 5	5	$\frac{5}{5} = 1$

Title Frequency polygon on histogram



EXERCISE 12.2

Q.1. Find the arithmetic mean in each of the following:

(i) 4, 6, 10, 12, 15, 20, 25, 28, 30.

Sol. $\Sigma x = 4 + 6 + 10 + 12 + 15 + 20 + 25 + 28 + 30$

$= 150$

No. of values = 9

Arithmetic mean $= \bar{x} = \dfrac{\Sigma x}{n}$

$= \dfrac{150}{9} = 16.67$

(ii) 12, 18, 19, 0, –19, –18, –12

Sol. $\Sigma x = 12 + 18 + 19 + 0 - 19 - 18 - 12$

$= 0$

No. of values = 7

Arithmetic mean $= \bar{x} = \dfrac{\Sigma x}{n}$

$= \dfrac{0}{7} = 0$

(iii) 6.5, 11, 12.3, 9, 8.1, 16, 18, 20.5, 25

Sol. $\Sigma x = 6.5 + 11 + 12.3 + 9 + 8.1 + 16 + 18 + 20.5 + 25$

$= 126.4$

No. of values = 9

Arithmetic mean $= \bar{x} = \dfrac{\Sigma x}{n}$

$\bar{x} = \dfrac{126.4}{9} = 14.04$

(iv) 8, 10, 12, 14, 16, 20, 22

Sol. $\Sigma x = 8 + 10 + 12 + 14 + 16 + 20 + 22$

$= 102$

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No. of values = 7

Arithmetic mean = $\bar{x} = \frac{\Sigma x}{n}$

$$\bar{x} = \frac{102}{7} = 14.57$$

Q.2. Following are the heights in (inches) of 12 students. Find the median height.

55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63.

Sol. 55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63

Arranging the data in ascending order first.

51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63

Since number of observations is even, i.e. n = 12

$$\text{Median } (\tilde{x}) = \frac{1}{2}\left[\frac{n}{2}\text{th observation} + \frac{n+2}{2}\text{th observation}\right]$$

$$= \frac{1}{2}\left[\frac{12}{2}\text{th observation} + \frac{12+2}{2}\text{th observation}\right]$$

$$= \frac{1}{2}[6^{th}\text{ observation} + 7^{th}\text{ observation}]$$

$$= \frac{1}{2}[56 + 57]$$

$$= \frac{1}{2}[113]$$

$$= 56.5 \text{ inches}$$

Q.3. Following are the earnings (in Rs.) of ten workers: 88, 70, 72, 125, 115, 95, 81, 90, 95, 90. Calculate

(i) Arithmetic Mean (ii) Median

(iii) Mode

Sol.(i) Arithmetic Mean

$$\Sigma x = 88 + 70 + 72 + 125 + 115 + 95 + 81 + 90 + 95 + 90$$

$$= 921$$

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No. of values = 10

Arithmetic mean = $\bar{x} = \frac{\Sigma x}{n}$

$$\bar{x} = \frac{921}{10} = 92.10$$

(ii) Median = ($\tilde{x}$) = ?

To find median, we first arrange the data in ascending order.

70, 72, 81, 88, 90, 90, 95, 95, 115, 125

Median = ($\tilde{x}$) = $\frac{1}{2}(90 + 90)$

$$= \frac{1}{2}(180) = 90$$

(iii) Mode = ?

Since 90 and 95 appear twice each. So mode of data is 90 and 95.

Q.4. The Marks obtained by the students in the subject of English are given below.

Marks obtained	**15-19**	**20-24**	**25-29**	**30-34**	**35-39**
Frequency	**9**	**18**	**35**	**17**	**5**

Find: (i) Arithmetic mean of their marks by direct and short formula.

Sol.

Marks obtained	**Midpoint (x)**	***f***	**f.x**
15 – 19	17	9	153
20 – 24	22	18	396
25 – 29	27	35	945
30 – 34	32	17	544
35 – 39	37	5	185
		$\Sigma f = 84$	$\Sigma fx = 2223$

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$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{2223}{84} = 26.46$$

Short Formula:

Marks	Midpoint	f	$D = x - A$
15 - 19	17	9	$17 - 27 = -10$
20 - 24	22	18	$22 - 27 = -5$
25 - 29	27	35	$27 - 27 = 0$
30 - 34	32	17	$32 - 27 = 5$
35 - 39	37	5	$37 - 27 = 10$

Now $\Sigma D = -10 - 5 + 0 + 5 + 10 = 0$

$$\bar{x} = A + \frac{\Sigma D}{n}$$

$$= 27 + \frac{0}{5}$$

$$= 27$$

(ii) Median of their marks.

Marks	Class Boundaries	f	c.f
15 - 19	14.5 – 19.5	9	9
20 - 24	19.5 – 24.5	18	27(c)
25 - 29	(ℓ)24.5 – 29.5	35(f)	62
30 - 34	29.5 – 34.5	17	79
35 - 39	34.5 – 39.5	5	84

Here $n = 84$

$$\frac{n}{2} = \frac{84}{2} = 42$$

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42nd item lies in the class boundaries

$$24.5 - 29.5$$

$$\ell = 24., h = 5$$

$$f = 35, c = 27$$

Using

$$\text{Median} = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 24.5 + \frac{5}{35}\left(\frac{84}{2} - 27\right)$$

$$= 24.5 + \frac{1}{7}(42 - 27)$$

$$= 24.5 + \frac{15}{7}$$

$$24.5 + 2.143$$

$$\tilde{x} = 26.64$$

Q.5. Given below is a frequency distribution.

Class Interval	5-9	10-14	15-19	20-24	25-29
Frequency	1	8	18	11	2

Find the mode of the frequency distribution.

Sol.

Class interval	Class boundaries	f
5 - 9	4.5 - 9.5	1
10 - 14	9.5 - 14.5	$8(f_1)$
15 - 19	14.5 - 19.5	$18\ (f_m)$
20 - 24	19.5 - 24.5	$11\ (f_2)$
25 - 29	24.5 - 29.5	2

Here $f_1 = 9, f_2 = 11, f_m = 18, h = 5, \ell = 14.5$

Using Mode $= \ell + \dfrac{(f_m - f_1) \times h}{f_m - f_1 + f_m - f_2}$

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$$\text{Mode} = 14.5 + \frac{(18-8)\times 5}{18-8+18-11}$$

$$= 14.5 + \frac{10\times 5}{17}$$

$$= 14.5 + \frac{50}{17}$$

$$= 14.5 + 2.94$$

$$\text{Mode} = 17.44$$

Q.6. Ten boys work on a petrol pump station. They get weekly wages as follows: Wages (in Rs.) 4250, 4350, 4400, 4250, 4350, 4410, 4500, 4300, 4500, 4390. Find the arithmetic mean by short formula, median and mode of their wages.

Sol. Arithmetic Mean

4250, 4350, 4400, 4250, 4350, 4410, 4500, 4300, 4500, 4390

Arranging the date first.

4250, 4250, 4300, 4350, 4350, 4390, 4400, 4410, 4500, 4500

Weekly wages (Rs)	$D = x - A$, $\boxed{A = 4350}$
4250	$D = 4250 - 4350 = -100$
4250	$= 4250 - 4350 = -100$
4300	$= 4300 - 4350 = -50$
4350	$= 4350 - 4350 = 0$
4350	$= 4350 - 4350 = 0$
4390	$= 4390 - 4350 = 40$
4400	$= 4400 - 4350 = 50$
4410	$= 4410 - 4350 = 60$
4500	$= 4500 - 4350 = 150$
4500	$= 4500 - 4350 = 150$

$\Sigma D = 200$

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Using $\bar{x} = A + \frac{\Sigma D}{n}$

$= 4350 + \frac{200}{10}$

$= 4350 + 20$

$= \text{Rs.}4370$

Median:

See the arranged data to find median.

$$\text{Median} = \tilde{x} = \frac{4350 + 4390}{2}$$

$$= \frac{8740}{2}$$

$$= \text{Rs.}4370$$

Mode:

Since 4250, 4350 and 4500 appear twice each.

So, Mode = $\hat{x}$ = Rs.4250, Rs.4350, Rs.4500

Q.7. The arithmetic mean of 45 numbers is 80. Find their sum.

Sol. Given that

$$\bar{x} = \frac{\Sigma x}{n}$$

or $\Sigma x = n \times \bar{x}$

$= 45 \times 80$

$\Sigma x = 3600$

So sum of 45 numbers whose arithmetic mean (80) is 3600.

Q.8. Five numbers are 1, 4, 0, 7, 9. Find their mean, median and mode.

Sol. Arithmetic Mean:

1, 4, 0, 7, 9

Arranged data is

<u>0, 1,</u> 4, <u>7, 9</u>

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$$\text{Mean} = \frac{\Sigma x}{n}$$

$$= \frac{0 + 14 + 7 + 9}{5}$$

$$= \frac{21}{5}$$

$$\bar{x} = 4.20$$

Median:

Median = $\tilde{x}$ = Most middle value is 4.

So median = 4

Modes:

There is no value which is repeated or most occurring. So data have no mode.

Q.9. A set of data contains the values as 148, 145, 160, 157, 156, 160.

Show that Mode > Median > Mean.

Sol. 148, 145, 160, 157, 156, 160

Arranging the data.

<u>145, 148</u>, 156, 157, <u>160, 160</u>

Mode: 160 repeats twice. So mode of data is 160.

$$\text{Median} = \frac{156 + 157}{2} = \tilde{x}$$

$$= \frac{313}{2}$$

$$\hat{x} = 156.5$$

$$\text{Mean} = \bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{145 + 148 + 156 + 157 + 160 + 160}{6}$$

$$= \frac{926}{6}$$

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$\bar{x} = 154.33$

Clearly $160 > 156.5 > 154.33$

i.e. Mode > Median > Mean

Q.10. The monthly attendance of 10 students for their lunch in the hostel is recorded as: 21, 15, 16, 18, 14, 17, 15, 12, 13, 11.

Find the median and mode of the attendance. Also find the mean if D = X – 20.

Sol. 21, 15, 16, 18, 14, 17, 15, 12, 13, 11

Arranging data in ascending order.

11, 12, 13, 14, 15, 15, 16, 17, 18, 21

$$\text{Median} = \tilde{x} = \frac{15+15}{2} = \frac{30}{2}$$

$$\tilde{x} = 15.$$

Mode: Most often item appearing in data is 15 which is two times.

So mode $= \hat{x} = 15$

Arithmetic Mean:

X	D = X – 20 $\boxed{A = 20}$	
11	11 – 20 = –9	
12	12 – 20 = –8	
13	13 – 20 = –7	
14	14 – 20 = –6	
15	15 – 20 = –5	
15	15 – 20 = –5	
16	16 – 20 = –4	
17	17 – 20 = –3	
18	18 – 20 = –2	
21	21 – 20 = 1	ΣD = –48

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Using $\bar{x} = A + \frac{\Sigma D}{n}$

$$= 20 + \frac{-48}{10}$$

$$= 20 - 4.8$$

$$\bar{x} = 15.2$$

Q.11. On a prize distribution day, 50 students brought pocket money as under:

Rupees	5-10	10-15	15-20	20-25	25-30
Frequency (f)	12	9	18	6	4

(i) Find the median and mode of the above data.

(ii) Find the arithmetic mean of the data given above using coding method.

Sol.

Rupees	f	c.f
5 – 10	12	12
10 – 15	9	12 + 9 = 21
15 – 20	18	21 + 18 = 39
20 – 25	7	39 + 7 = 46
25 – 30	4	46 + 4 = 50

Here, n = 50, so $\frac{50}{2} = 25$ lies in 15 – 20. So $\ell = 15$, h = 5, $f = 18$, c = 21

$$\text{Median } (\tilde{x}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 15 + \frac{5}{18}(25 - 21)$$

$$= 15 + \frac{5}{18} \times 4$$

$$= 15 + \frac{20}{18}$$

$$= 15 + 1.11$$

$$= 16.11$$

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Finding Arithmetic Mean by Using Coding Method

Rupees	f	x	$U = \frac{x - A}{h}$	fU
5 - 10	12	7.5	$\frac{7.5 - 18}{5} = -2.1$	$12x - 2.1 = -25.2$
10 - 15	9	12.5	$\frac{12.5 - 18}{5} = -1.1$	$9 \times (-1.1) = -9.9$
15 - 20	(18)A	17.5	$\frac{18 - 18}{5} = \frac{0}{5} = 0$	$18 \times 0 = 0$
20 - 25	7	22.5	$\frac{22.5 - 18}{5} = 0.9$	$7 \times 0.9 = 6.3$
25 - 30	4	27.5	$\frac{27.5 - 18}{5} = 1.9$	$4 \times 1.9 = 7.6$
	$\Sigma f = 50$			**$\Sigma fU = -21.2$**

Using formula $\bar{x} = A + \frac{\Sigma fu}{\Sigma f} \times h$

$$= 18 + \frac{-21.2}{50} \times 5$$

$$= 18 - \frac{106}{50}$$

$$\bar{x} = 18 - 2.12$$

$$\bar{x} = 15.88$$

Mode:

Rupees	f
5 - 10	12
10 - 15	9 (f_1)
(ℓ)15 - 20	18 (f_m)
20 - 25	7 (f_2)
25 - 30	4

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Here $\ell = 15$, $f_m = 18$, $f_1 = 9$, $f_2 = 7$, $h = 5$

Using Mode $= \ell + \dfrac{f_m - f_1}{f_m - f_1 + f_m - f_2} \times h$

$$= 15 + \frac{18 - 9}{18 - 9 + 18 - 7} \times 5$$

$$= 15 + \frac{9 \times 5}{20}$$

$$= 15 \times \frac{45}{20}$$

$$= 15 + 2.25$$

$$\hat{x} = 17.25$$

Q.12. The arithmetic mean of the ages of 20 boys is 13 years, 4 months and 5 days. Find the sum of their ages. If one of the boys is of age exactly 15 years. What is the average age of the remaining boys?

Sol. Arithmetic Mean of 20 boys $= \bar{x} = 13$ years, 4 months, 5 days

We convert in years as 13 year 4 months 5 days

$$= 13 + \frac{4}{12} + \frac{5}{365}$$

$$= 13 + 0.333 + 0.0136$$

$$\bar{x} = 13.3466 \text{ years}$$

No of Boys $= n = 20$

Using $\bar{x} = \dfrac{\Sigma x}{n}$

or $\Sigma x = n \times \bar{x}$

$$= 20 \times 13.3466$$

$$= 266.932 \text{ years}$$

$$= 266 \text{ years} + 0.932 \times 12 \text{ months}$$

$$= 266 \text{ years } 11.184 \text{ months}$$

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$= 266$ years 11 months 0.184×30

$= 266$ years 11 months 6 days

Given that one boys has age exactly 15 years.

Remaining sum = 266 years 11 month 6 days – 15

= 251 years 11 months 6 days

= 251.932 years

Now remaining boys = 20 – 1 = 19

Now $n = 19, \Sigma x = 251.932$ years

Average age of remaining boys $= \dfrac{251.932}{19}$

= 13.259 years

= 13 years + 0.259×12 months

= 13 years 3.108 months

= 13 years + 3 months + 0.108×30 days

= 13 years 3 months 4 days (approximately)

Q.13. Calculate the arithmetic means from the following information:

(i) If $D = X - 140$, $\Sigma D = 500$ and $n = 10$

Sol. $D = X - 140, \Sigma D = 500, n = 10, \bar{x} = ?$

As $D = X - A$, so $A = 140$

Using $\bar{x} = A + \dfrac{\Sigma D}{n} \times h$

$= 140 + \dfrac{500}{10}$

$= 140 + 50$

$\bar{x} = 190$

(ii) If $U = \dfrac{x - 130}{6}$, $\Sigma U = -150$ and $n = 15$

Sol. $U = \dfrac{x - 130}{6}, \Sigma u = -150, n = 15$

Here $U = \dfrac{x - A}{h}$

So $A = 130, h = 6$

Using $\bar{x} = A + \frac{\Sigma u}{n} \times h$

$$= 130 + \frac{-150}{15} \times 6$$

$$= 130 - 10 \times 6$$

$$= 130 - 60$$

$$\bar{x} = 70$$

(iii) **If $D = x - 25$, $\Sigma f D = 300$ and $\Sigma f = 20$**

Sol. Comparing $D = x - 25$ with $D = x - A$

We get $A = 25$

Using the formula.

$$\bar{x} = A + \frac{\Sigma fD}{\Sigma f}$$

$$= 25 + \frac{300}{20}$$

$$= 25 + 15$$

$$\bar{x} = 40$$

(iv) **If $U = \frac{x - 120}{5}$, $\Sigma f U = 60$ and $\Sigma f = 100$**

Sol. Comparing $U = \frac{x - 120}{5}$ with $U = \frac{x - A}{h}$

So, $A = 120, h = 5$

Using $\bar{x} = A + \frac{\Sigma fu}{\Sigma f} \times h$

$$= 120 + \frac{60}{100} \times 5$$

$$= 120 + \frac{300}{100}$$

$$= 120 + 3$$

$$\bar{x} = 123$$

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Q.14. The three children Haris, Maham and Minal made the following scores in a game conducted by a group of teachers in the school.

Haris scores	50	55	70	85	90
Maham scores	75	60	60	45	53
Minal scores	80	77	66	42	48

It is decided that the candidate who gets the highest average score will be awarded rupees 1000. Who will get the awarded amount?

Sol. Total scores Haris made = 50 + 55 + 70 + 85 + 90

= 350

Average score $= \frac{350}{5}$

$\bar{x} = 70$

Total score Maham made = 75 + 60 + 60 + 45 + 53

= 293

Average score $= \frac{293}{5}$

= 58.6

Total scores Minal made = 80 + 77 + 66 + 42 + 48

= 313

Average score $= \frac{313}{5}$

= 62.5

Since Minal's average is highest.

So, I decided to award Rs.1000 to Minal.

Q.15. Given below is a frequency distribution derived by making a substitution as D = X – 20. Calculate the arithmetic mean.

D	**–6**	**–4**	**–2**	**0**	**2**	**4**	**6**
f	**1**	**3**	**6**	**20**	**26**	**12**	**2**

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Sol. $D = x - 20$, $D = x - A \Rightarrow \boxed{A = 20}$

D	f	fD
−6	1	−6
−4	3	−12
−2	6	−12
0	20	0
2	26	52
4	12	48
6	2	12
	$\Sigma f = 70$	$\Sigma fD = 82$

Using $\bar{x} = A + \frac{\Sigma fD}{\Sigma f}$

$= 20 + \frac{82}{70}$

$= 20 + 1.17$

$\bar{x} = 21.17$

Q.16. Being partners Hafsa and Fatima took part in a quiz programme. They made the following number of points 45, 51, 58, 61, 74, 48, 46 and 50. Compute the average number of points using deviation $D = x - 58$.

Sol.

No. of Points	$D = x - 58 \Rightarrow \boxed{A = 58}$
45	$45 - 58 = -13$
51	$51 - 58 = -7$
58	$58 - 58 = 0$
61	$61 - 58 = 3$
74	$74 - 58 = 16$
48	$48 - 58 = -10$
46	$46 - 58 = -12$
50	$50 - 58 = -8$
	$\Sigma D = -31$

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Using the formula

$$\bar{x} = A + \frac{\Sigma D}{n}$$

$$= 58 + \frac{-31}{8}$$

$$= 58 - 3.875$$

$$\bar{x} = 54.125 = 54.13$$

Q.17. A person purchased the following food items:

Food item	Quantity (in kg)	Cost per Kg (in Rs.
Rice	10	96
Flour	12	48
Ghee	4	190
Sugar	3	49
Mutton	2	650

What is the weighted mean of cost of food items per kg?

Sol. Weighted mean of cost of food items per kg.

$$= \frac{10 \times 96 + 12 \times 48 + 4 \times 190 + 3 \times 49 + 2 \times 650}{10 + 12 + 4 + 3 + 2}$$

$$= \frac{960 + 576 + 760 + 147 + 1300}{31}$$

$$= \frac{3743}{31} = 120.74$$

Weighted mean = Rs.120.74 per kg

Q.18. For the following data, find the weighted mean.

Item	Quantity	Cost of item (in thousands)
Washing Machines	5	35
Heater	3	5
Stove	2	13
Dispensér	6	18

Sol. Formula

$$\bar{x} = \frac{\Sigma Wx}{W}$$

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$$\Sigma Wx = 5 \times 35 + 3 \times 5 + 2 \times 13 + 6 \times 18$$
$$= 175 + 15 + 26 + 108$$
$$\Sigma Wx = 324$$
$$\Sigma W = 5 + 3 + 2 + 6 = 16$$
$$\bar{x} = \frac{\Sigma Wx}{W}$$
$$= \frac{324}{16} = 20.25$$

Q.19. A company is planning its next year marketing budget across five years: yearly budgets (in million) are: 5, 7, 8, 6, 7. Find the average budget for the next year.

Sol. Let x = 5, 7, 8, 6, 7

$\Sigma x = 5 + 7 + 8 + 6 + 7$

$= 33$

No. of years = n = 5

Average budget for next year $= \frac{\Sigma x}{n}$

$= \frac{33}{5} = 6.6$ millions

Q.20. Ahmad obtained the following marks in a certain examination. Find the weighted mean if weights 5, 4, 2, 3, 2, 4 respectively are allotted to the subjects.

Urdu	English	Science	Math	Islamiyat	Computer
78	65	80	90	85	72

Sol.

Marks (x)	Weight (W)
78 (U)	5
65 (E)	4
80 (S)	2
90 (M)	3
85 (Isl)	2
72 (C)	4

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$\Sigma Wx = 5 \times 78 + 4 \times 65 + 2 \times 80 + 3 \times 90 + 2 \times 85 + 4 \times 72$

$= 390 + 260 + 160 + 270 + 170 + 288$

$= 1538$

$\Sigma W = 5 + 4 + 2 + 3 + 2 + 4$

$= 20$

Weighted mean $\bar{x} = \dfrac{\Sigma Wx}{W}$

$= \dfrac{1538}{20}$

$\bar{x} = 76.9$ Marks

REVIEW EXERCISE 12

Q.1. Four options are given against each statement. Encircle the correct option.

(i) Which data takes only some specific values?

(a) continuous data (b) discrete data

(c) grouped data (d) ungrouped data

Sol. **(b)**

(ii) The number of times a value occurs in a data is called:

(a) frequency (b) relative frequency

(c) class limit (d) class boundaries

Sol. **(a)**

(iii) Midpoint is also known as:

(a) mean (b) median

(c) class limit (d) class mark

Sol. **(d)**

(iv) Frequency polygon is also drawn/constructed by using:

(a) histogram (b) bar graph

(c) class boundaries (d) class limit

Sol. **(a)**

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(v) The difference between the greatest value and the smallest value is called:

(a) class limits (b) midpoint

(c) relative frequency (d) range

Sol. **(d)**

(vi) Measure of central tendency is used to find out the _____ of a data set.

(a) class boundaries (b) cumulative frequency

(c) middle or centre value (d) frequency

Sol. **(c)**

(vii) If the mean of 5, 7, 8, 9 and x is 7.5, what will be the value of x?

(a) 10 (b) 8

(c) 8.5 (d) 5.8

Sol. $\frac{5+7+8+9+x}{5} = 7.5$

$29 + x = 5 \times 7.5$

$x + 29 = 37.5$

$x = 37.5 - 29$

$x = 8.5$

So option (c) is correct.

(viii) Find the mode of the given data: 2, 5, 8, 9, 0, 1, 3, 7 and 10:

(a) 5 (b) 7

(c) 0 (d) no mode

Sol. **(d)**

(ix) In a data the values (observations) which appears or occurs most often is called:

(a) mean (b) mode

(c) median (d) weighted mean

Sol. **(b)**

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(x) Find the median of the given data: 110, 125, 122, 130, 124, 127 and 120:

(a) 124 (b) 120

(c) 125 (d) 127

Sol. Arrange the data first,

110, 120, 122, 124, 125, 127, 130

Most middle value = 124

So median = 124

So option (a) is correct.

Q.2. Define the following:

(i) frequency distribution

Ans. A distribution or table that represents classes or groups along with their respective class frequency is called frequency distribution.

(ii) histogram (unequal class limits)

Ans. Histogram (Unequal class limits)

This is a group of adjacent rectangles on xy-plane. A histogram is similar to bar graph but it is constructed for a frequency distribution.

In histogram, the values of the data are represented along the horizontal axis and frequencies are shown by bars perpendicular to horizontal axis. Bars of unequal width are used to represent individual classes of frequency table.

Class boundaries are marked on x-axis and a rectangle is made against each group with its width proportional to the size of class limits and height proportional to the class frequencies.

This can be achieved by adjusting the heights of rectangle. The height of each rectangle is obtained by dividing each class frequency on its class limit size.

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(iii) mean

Ans. It is defined as a value of variable which is obtained by dividing the sum of all the values (observations) by their number of observations.

Let $\bar{x}$ be the mean or Arithmetic mean, Σx denotes sum of observations and n, the no. of values, then

$$\bar{x} = \frac{\Sigma x}{n}$$

(iv) median

Ans. Median is the middle most value is an arranged data set. Median is the value which divides the data into two equal parts i.e. 50% data is before the median and 50% data after it. Median is denoted by $\tilde{x}$.

$$\text{Median } (\tilde{x}) = \left(\frac{n+1}{2}\right)^{th} \text{ observation.} \qquad \text{(when n is odd number)}$$

$$\text{Median } (\tilde{x}) = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{ observation} + \left(\frac{n+2}{2}\right)^{th} \text{ observation}\right]$$

(When n is an even number)

Q.3. Following are the weights of 40 students recorded to the nearest (lbs).

138, 164, 150, 132, 144, 125, 149, 157, 146, 158, 140, 147 136, 148, 152, 144, 168, 126, 138, 176, 163, 119, 154,165, 146, 173, 142, 147, 135, 153, 140, 135, 161, 145, 135, 142,150, 156, 145, 128, make a frequency table taking size of class limits as 10. Also draw histogram and frequency polygon of the given data.

Sol.(a)

Class limits	C.B	Tally Marks	f
119 – 128	118.5 – 128.5	IIII	4
129 – 138	128.5 – 138.5	~~IIII~~ II	7
139 – 148	138.5 – 148.5	~~IIII~~ ~~IIII~~ III	13
149 – 158	148.5 – 158.5	~~IIII~~ IIII	9
159 – 168	158.5 – 168.5	~~IIII~~	5
169 – 178	168.5 – 178.5	II	2

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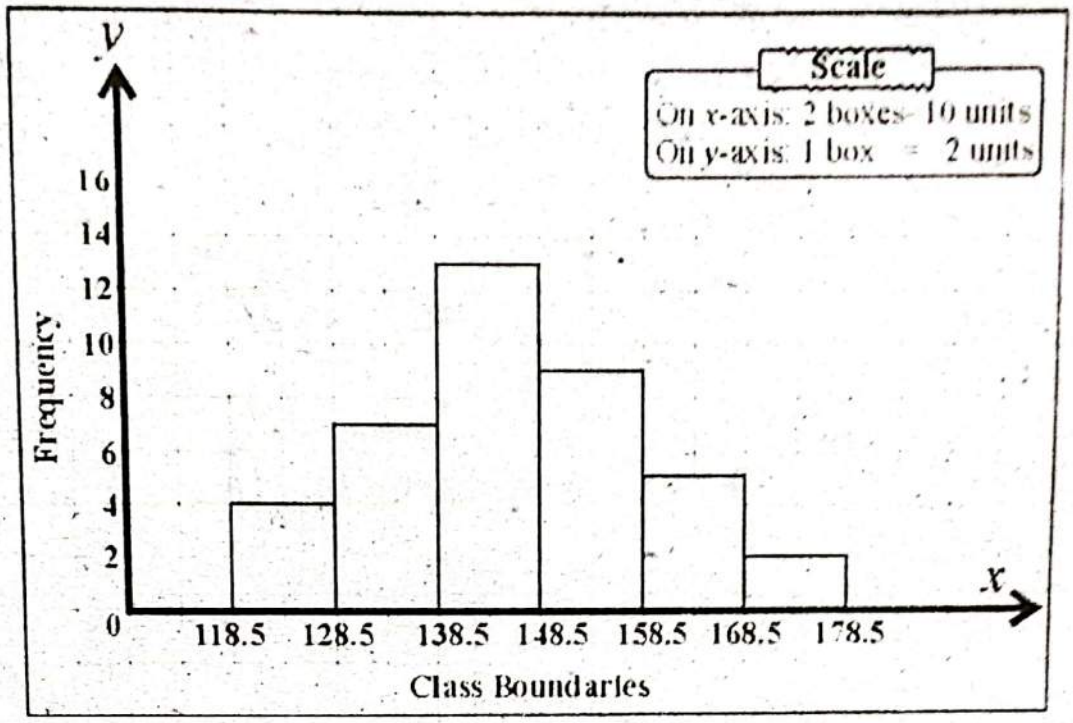
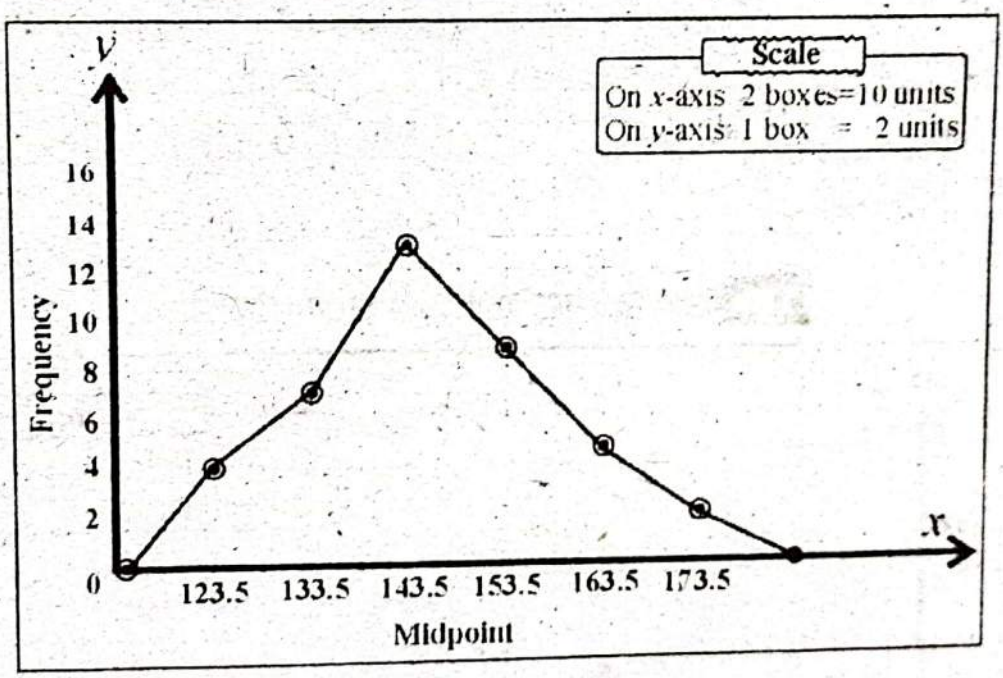
(b) **Title** Histogram of weight of 40 students



(c) **Title** Frequency polygon of weight of 40 students



Q.4. From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

Weight (kg)	50-56	57-59	60-64	65-72	7375	76-80
Frequency (f)	25	32	40	30	15	8

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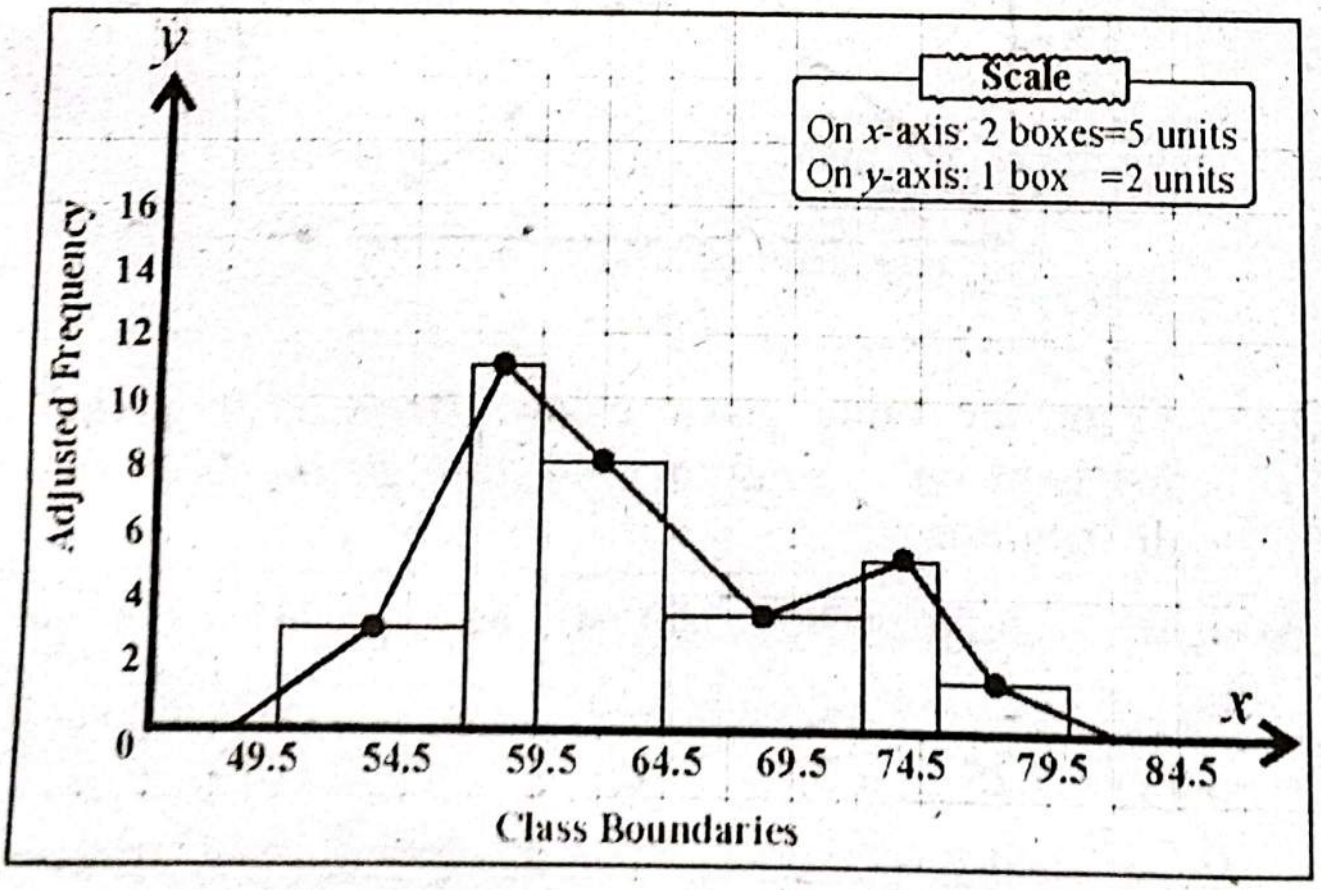
Weight (kg)	Frequency (f)	Width of Class	Height of rectangle
50 – 56	25	56 – 50 = 6	$\frac{25}{6} = 4.17$
57 – 59	32	59 – 57 = 2	$\frac{32}{2} = 16$
60 – 64	40	64 – 60 = 4	$\frac{40}{4} = 10$
65 – 72	30	72 – 65 = 7	$\frac{30}{7} = 4.29$
73 – 75	15	75 – 73 = 2	$\frac{15}{2} = 7.5$
76 – 80	8	80 – 76 = 4	$\frac{8}{4} = 2$

Title **Frequency polygon on histogram**



Q.5. Given below are marks obtained by 45 students in the monthly test of Biology:

Marks	20-24	25-29	30-34	35-39	40-44	45-49
No. of students	05	08	12	15	03	02

With reference to the above table find the following:

(i) upper class boundary of the 5th class.

(ii) lower class boundaries of all the classes.

(iii) midpoint of all the classes.

(iv) the class interval with the least frequency.

Sol.

Marks	Class Boundaries	Mid points
20 – 24	19.5 – 24.5	22
25 – 29	24.5 – 29.5	27
30 – 34	29.5 – 34.5	32
35 – 39	34.5 – 395	37
40 – 44	39.5 – 44.5	42
45 – 49	44.5 – 49.5	47

(i) Upper class boundary of 5th class = 44.5

(ii) Lower class boundaries of all the classes are as follows.
19.5, 24.5, 29.5, 34.5, 39.5, 44.5

(iii) Mid points of all the classes are
22, 27, 32, 37, 42, 47

(iv) The least frequency is 5 which falls in the class interval 20 – 24.

Q.6. Given below is frequency distribution.
Draw frequency polygon and histogram for the distribution.

Class limits	5-9	10-14	15-19	20-24	25-29	30-34
Frequency	1	8	18	11	2	5

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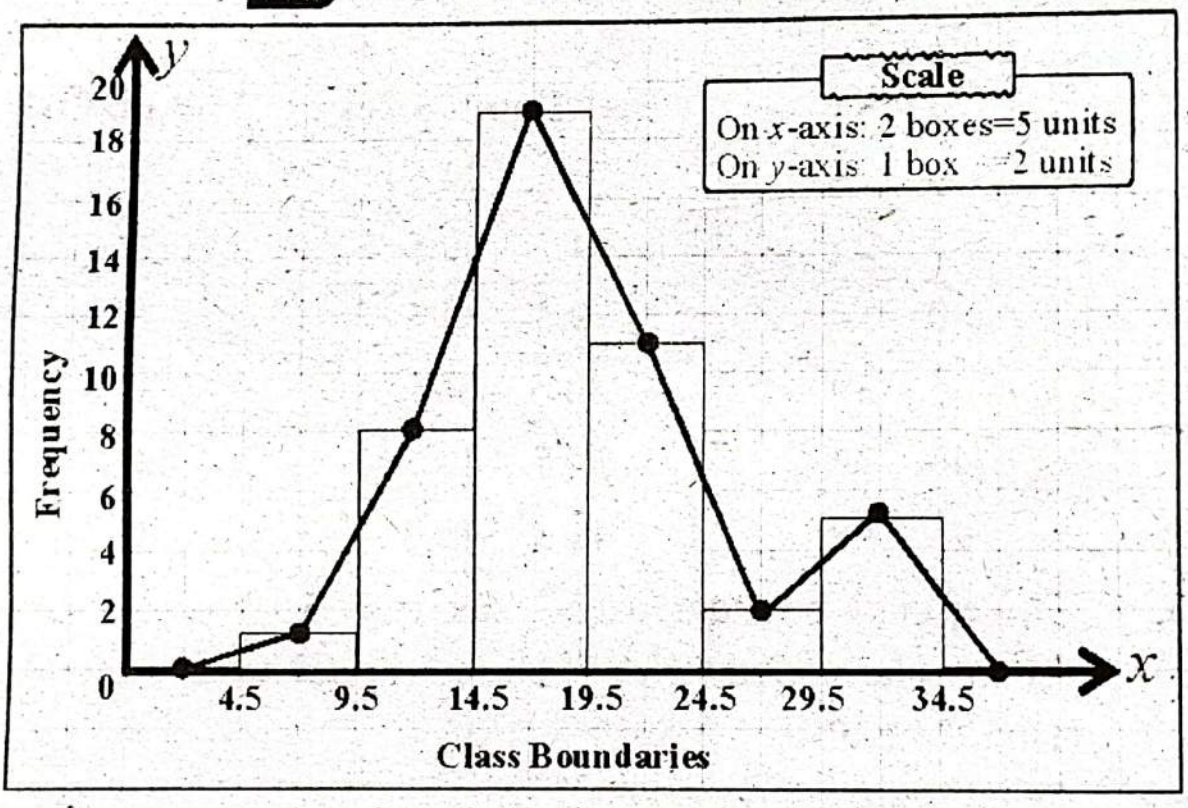
Sol.

Class limits	Class boundaries	Mid points	f
5 – 9	4.5 – 9.5	7	1
10 – 14	9.5 – 14.5	12	8
15 – 19	14.5 – 19.5	17	18
20 – 24	19.5 – 24.5	22	11
25 – 29	24.5 – 29.5	27	2
30 – 34	29.5 – 34.5	32	5

Title **Frequency polygon on histogram**



Q.7. For the following data, find the weighted mean.

Item	Quantity	Cost of item (Rs.)
Chair	20	500
Table	20	400
Black board	10	750
Tube light	25	230
Cupboard	09	950

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Sol.

Item	Quantity	Cost of item (Rs.)	Wx
Chair	20	500	10000
Table	20	400	8000
Black Board	10	750	7500
Tube light	25	230	5750
Cup board	09	950	8550
	$\Sigma w = 84$		ΣWx = Rs.39800

$$\text{Weighted Mean} = \frac{\Sigma Wx}{\Sigma W}$$

$$= \frac{39800}{84} = \text{Rs.}473.81$$

Q.8. A principal of a school allocates funds of Rs.50,000 to five different sectors:

(i) chairs: Rs.15000 (ii) table: Rs.12,000

(iii) black boards: Rs.6,000

(iv) room renovation: Rs.10,000

(v) gardening: Rs.7,000

Find the average of funds allocation in each sector of the school.

Sol. Chairs: Rs.15,000

Tables: Rs.12,000

Black boards: Rs.6,000

Room Renovation: Rs.10,000

Gardening: Rs.7,000

Average of funds allocation in each sector of school

$$= \frac{15000 + 12000 + 6000 + 10000 + 7000}{5}$$

$$= \frac{50000}{5} = \text{Rs.}10,000$$

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Q.9. The marks of a student Saad in six tests were 84, 91, 72, 68, 87, 78. Find the arithmetic mean of his marks.

Sol. Sum of tests marks = 84 + 91 + 72 + 68 + 87 + 78

$$\Sigma x = 480$$

No. of tests = n = 6

Arithmetic mean of his marks $= \frac{\Sigma x}{n}$

$$= \frac{480}{6} = 80 \text{ marks}$$

Q.10. Adjoining distribution showed maximum load (in kg) supported by certain ropes. Find the mean load using short method.

Max-Load kg	93-97	98-102	103-107	108-112	113-117	118-122
No. of ropes	2	5	8	12	6	2

Sol.

Max-load (kg)	No of ropes (f)	Midpoint (x)	D = x – A	fD
93 – 97	2	95	95 – 105 = –10	–20
98 – 102	5	100	100 – 105 = –5	–25
103 – 107	8	(105) A	105 – 105 = 0	0
108 – 112	12	110	110 – 105 = 5	60
113 – 117	6	115	115 – 105 = 10	60
118 – 122	2	120	120 – 105 = 15	30

$\Sigma fD = 105$

$\Sigma f = 35$, A = 105

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$$\text{Arithmetic mean} = \bar{x} = A + \frac{\Sigma fD}{\Sigma f}$$

$$= 105 + \frac{105}{35}$$

$$= 105 + 3$$

$$= 108 \text{ kg}$$

Q.11. Usman rolled a fair dice eight times. Each time thei sum was recorded as 8, 5, 6, 6, 9, 4, 3, 11. Find th median and mode of the sum.

Sol. 8, 5, 6, 6, 9, 4, 3, 11

Arranging the data.

3, 4, 5, 6, 6, 8, 9, 11

Here no. of values is even

i.e. $n = 8$

$$\text{Median } (\tilde{x}) = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+2}{2}\right)^{th} \text{observation}\right]$$

$$= \frac{1}{2}\left[\left(\frac{8}{2}\right)^{th} \text{observation} + \left(\frac{8+2}{2}\right)^{th} \text{observation}\right]$$

$$= \frac{1}{2}\left[4^{th} \text{ observation} + 5^{th} \text{ observation}\right]$$

$$= \frac{1}{2}(6+6)$$

$$\tilde{x} = \frac{1}{2}(12) = 6$$

Since value 6 is repeated most times so mode of sum = $\hat{x} = 6$

Q.12. Two partners Mr.Aslam and Mrs. Kalsoom run a company. In the following data the weekly wages (in Rs.) of employees who work in the company are given:

Wages (Rs.)	600-700	700-800	800-900	900-1000	1000-1100
Employees	3	5	7	21	11

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Sol.

Wages (Rs.)	Frequency (f)	Midpoint (x)	fx
600 – 700	3	650	1950
700 – 800	5	750	3750
800 – 900	7	850	5950
900 – 1000	21	950	19950
1000 – 1100	11	1050	11550
	$\Sigma f = 47$		$\Sigma fx = 43,150$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{43150}{47}$$

$$\bar{x} = 918.09$$

Median

Wages (Rs.)	Employees (f)	c.f
600 – 700	3	3
700 – 800	5	8
800 – 900	7	15
900 – 1000	21	36
1000 – 1100	11	47

Here n = 47

So $\frac{n}{2} = \frac{47}{2} = 23.5$

24th item lies in 900 – 1000.

$\ell = 900$, $h = 100$, $f = 21$

$c = 15$

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$$\text{Median} = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 900 + \frac{100}{47}(23.5 - 15)$$

$$= 900 + 2.1277\,(8.5)$$

$$= 900 + 18.08$$

$$\tilde{x} = 918.09$$

Mode

Wages (Rs.)	f
600 – 700	3
700 – 800	5
800 – 900	$\boxed{7}$ f_1
900 – 1000	$\boxed{21}$ f_m
1000 – 1100	$\boxed{11}$ f_2

Here $\ell = 900$

$f_m = 21,\ f_1 = 7,\ f_2 = 11$

$h = 100$

$$\text{Mode} = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)}$$

$$= 900 + \frac{(21 - 7) \times 100}{(21 - 7) + (21 - 11)}$$

$$= 900 + \frac{14 \times 100}{14 + 10}$$

$$= 900 + \frac{1400}{24}$$

$$= 900 + 58.33$$

$$\text{Mode} = \hat{x} = 958.33$$

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