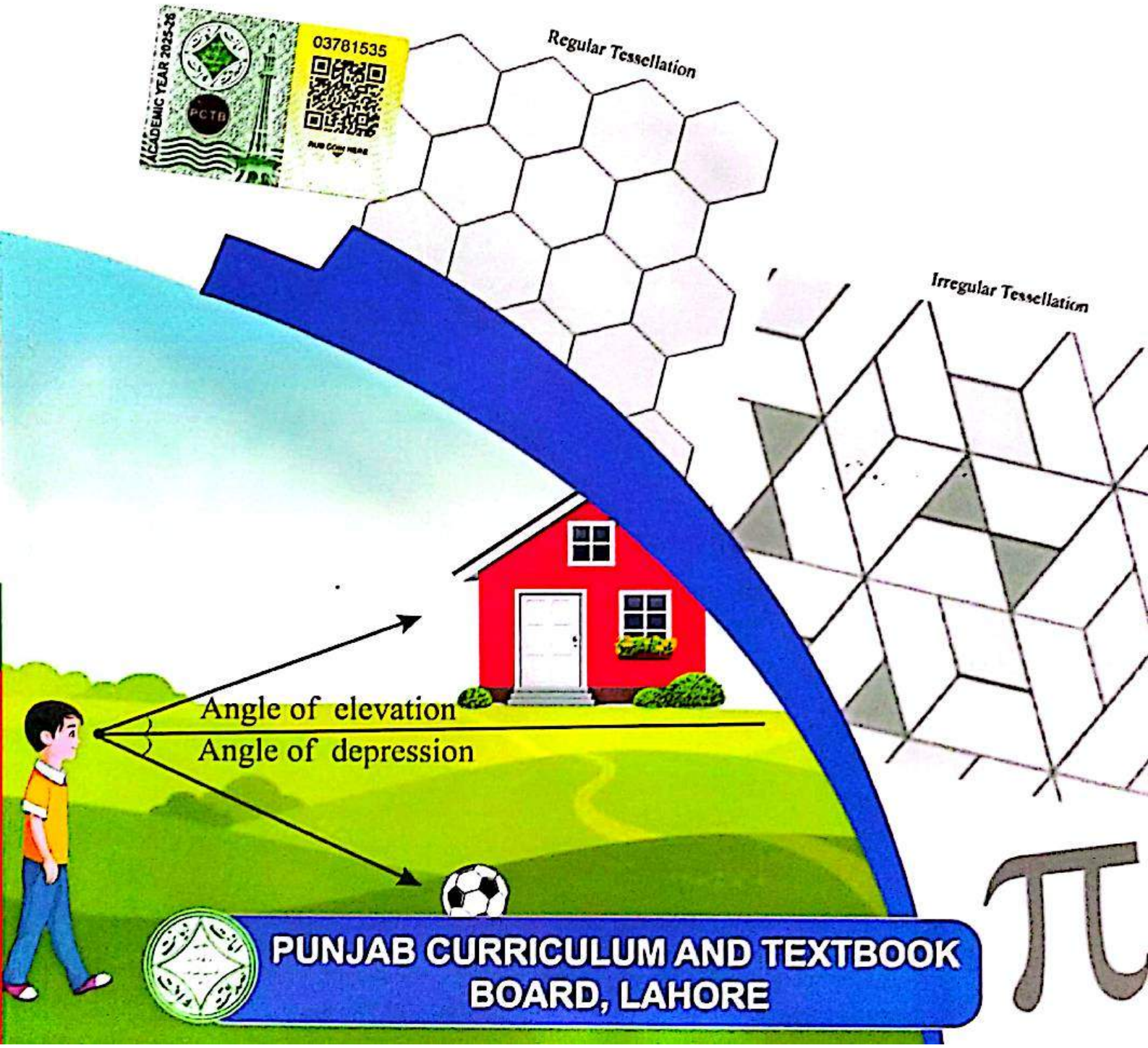
MATHEMATICS 9



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UNIT 5 LINEAR EQUATIONS AND INEQUALITIES

EXERCISE 5.1

Q.1. Solve and represent the solution set on a real line.

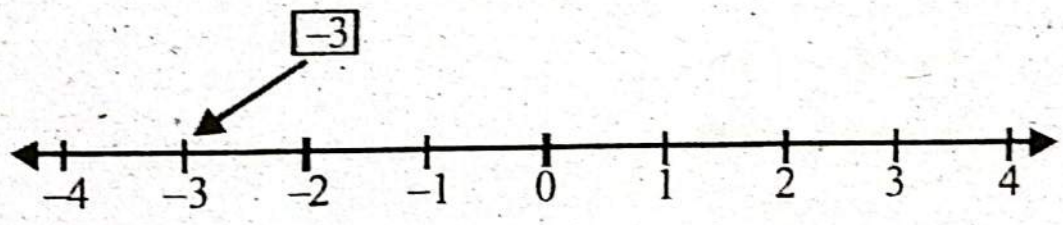
(i) $12x + 30 = -6$

Sol. $12x = -6 - 30$

$12x = -36$

$x = \frac{-36}{12}$

$x = 3$



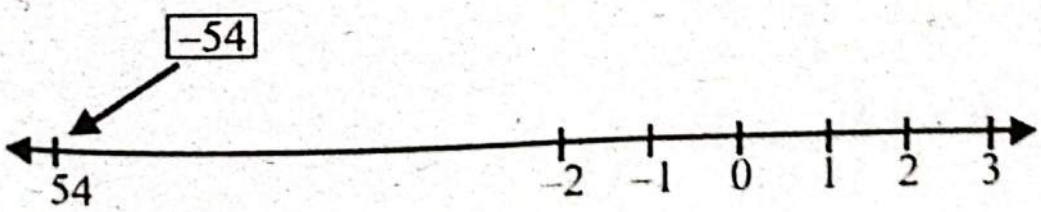
(ii) $\frac{x}{3} + 6 = -12$

Sol. $\frac{x}{3} = -6 - 12$

$\frac{x}{3} = -18$

$x = 3 \times (-18)$

$x = -54$



(iii) $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

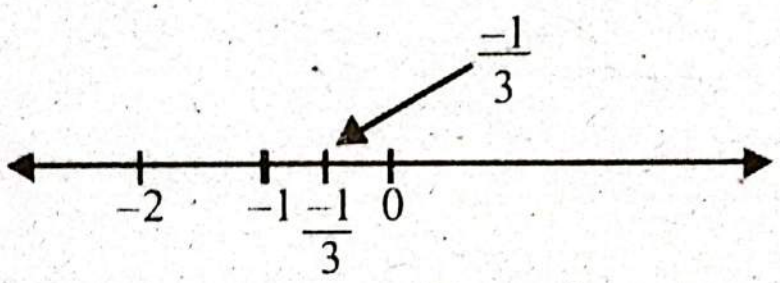
Sol. $\frac{2x - 3x}{4} = \frac{1}{12}$

$$\frac{-x}{4} = \frac{1}{12}$$

$$12x = -4$$

$$x = \frac{-4}{12}$$

$$x = \frac{-1}{3}$$



(iv) $2 = 7(2x + 4) + 12x$

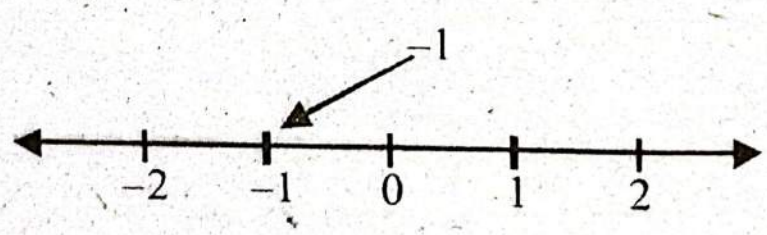
Sol. $2 = 14x + 28 + 12x$

$$2 - 28 = 26x$$

$$26x = -26$$

$$x = \frac{-26}{26}$$

$$x = -1$$



(v) $\frac{2x - 1}{3} - \frac{3x}{4} = \frac{5}{6}$

Sol. $\frac{4(2x - 1) - 3(3x)}{12} = \frac{5}{6}$

$$\frac{8x - 4 - 9x}{12} = \frac{5}{6}$$

$$\frac{-x - 4}{12} = \frac{5}{6}$$

By cross multiplication,

$$6(-x - 4) = 12 \times 5$$

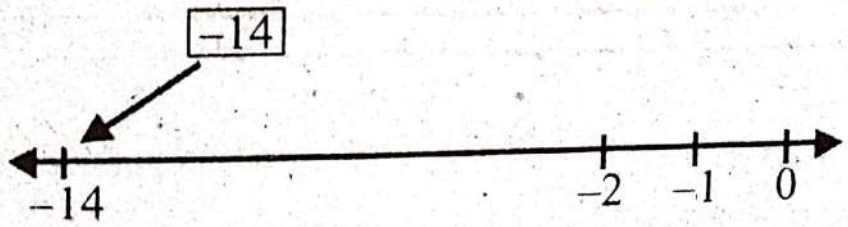
$$-6x - 24 = 60$$

$$-6x = 60 + 24$$

$$-6x = 84$$

$$x = \frac{84}{-6}$$

$$x = -14$$



(vi) $\mathbf{\frac{-5x}{10} = 9 - \frac{10}{5}x}$

Sol. $\frac{-5x}{10} + \frac{10}{5}x = 9$

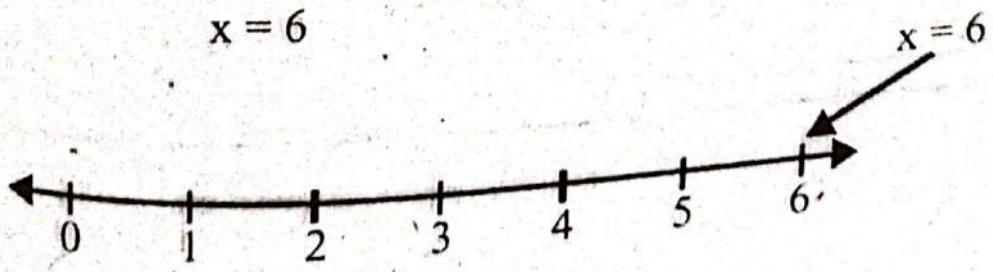
$$\frac{-5x + 20x}{10} = 9$$

$$\frac{15x}{10} = 9$$

$$15x = 90$$

$$x = \frac{90}{15}$$

$$x = 6$$

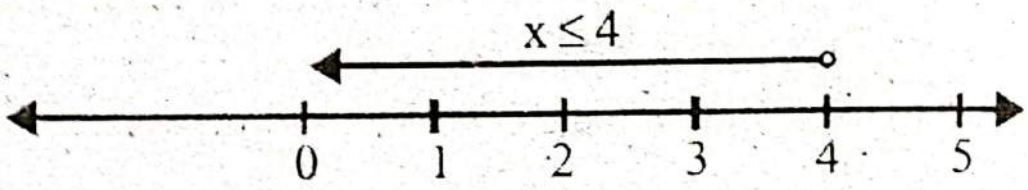


Q.2. Solve each inequality and represent the solution on a real line.

(i) $x - 6 \leq -2$

Sol. $x - 6 + 6 \leq -2 + 6$

$x \leq 4$

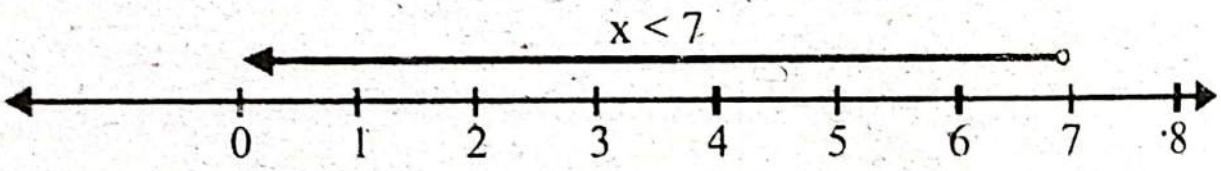


(ii) $-9 > -16 + x$

Sol. $-9 + 16 > -16 + 16 + x$

$7 > x$

or $x < 7$

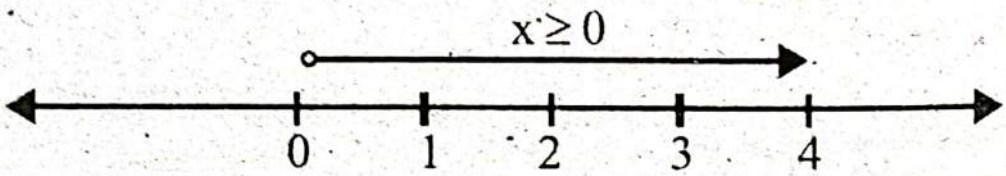


(iii) $3 + 2x \geq 3$

Sol. $3 - 3 + 2x \geq 3 - 3$

$2x \geq 0$

$x \geq 0$



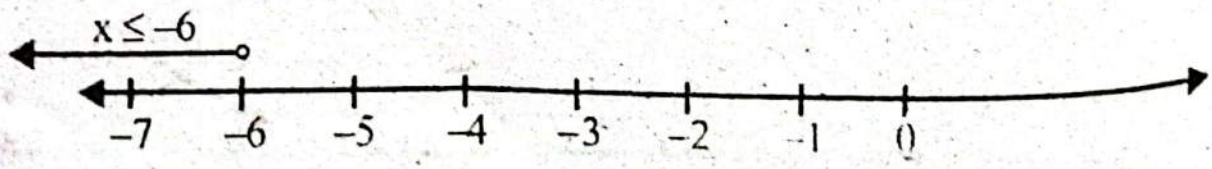
(iv) $6(x + 10) \leq 0$

Sol. $6x + 60 \leq 0$

$6x \leq -60$

$x \leq \frac{-60}{6}$

$x \leq -6$



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(v) $\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$

Sol. $\frac{5}{3}x - \frac{3}{4} + \frac{3}{4} < \frac{-1}{12} + \frac{3}{4}$

$$\frac{5}{3}x < \frac{-1+9}{12}$$

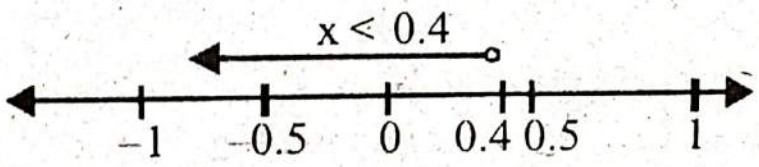
$$\frac{5}{3}x < \frac{8}{12}$$

$$\frac{5}{3}x < \frac{2}{3}$$

$$\frac{3}{5} \times \frac{5}{3}x < \frac{3}{5} \times \frac{2}{3}$$

$$x < \frac{2}{5}$$

$$x < 0.4$$



(vi) $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

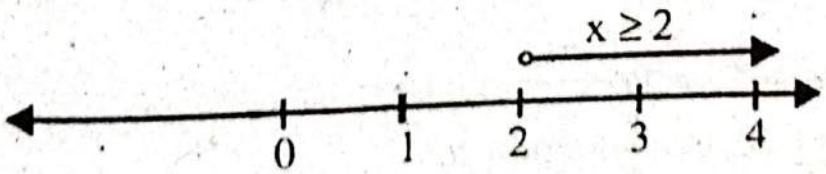
Sol. $-\frac{1}{2} + 1 \leq \frac{1}{2} \times \frac{1}{2}x$

$$\frac{1}{2} \leq \frac{x}{4}$$

or $\frac{x}{4} \geq \frac{1}{2}$

$$4 \times \frac{x}{4} \geq 4 \times \frac{1}{2}$$

$$x \geq 2$$



Q.3. Shade the solution region for the following linear inequalities in xy-plane.

(i) $2x + y < 6$

Sol. Associated equation is

$$2x + y = 6$$

For x-intercept, put $y = 0$

$$2x + 0 = 6$$

$$2x = 6$$

$$x = \frac{6}{2}$$

$$x = 3$$

1st ordered pair is (3, 0)

For y-intercept, put $x = 0$

$$2(0) + y = 6$$

$$y = 6$$

2nd ordered pair is (0, 6)

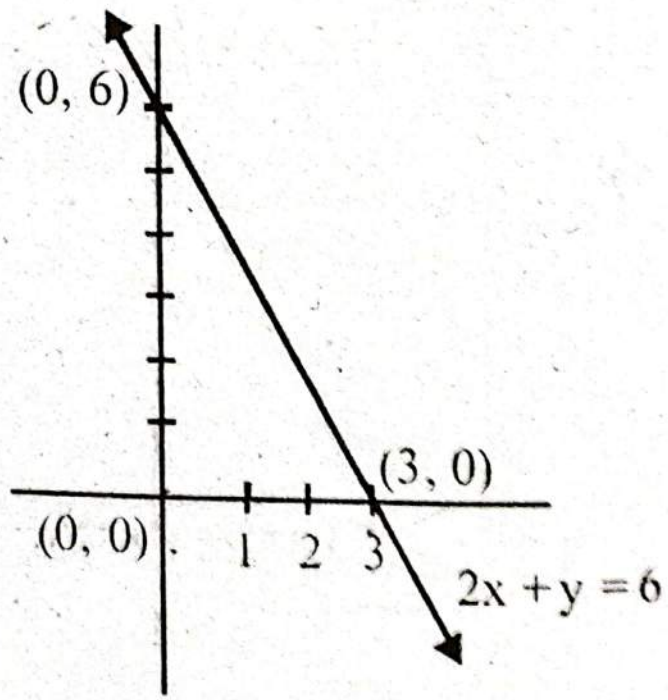
Now put $x = y = 0$ (origin test) in

$$2x + y < 6$$

$$2(0) + 0 < 6$$

$$0 < 6$$

Which is true, so shading and will lie towards (0, 0) i.e. origin. Now we draw the line joining the points (3, 0) and (0, 6).



(ii) $3x + 7y \geq 21$

Sol. Associated equation is

$$3x + 7y = 21$$

For x-intercept, put $y = 0$

$$3x + 7(0) = 21$$
$$3x = 21$$
$$x = \frac{21}{3} = 7$$

1st ordered pair = (7, 0)

For y-intercept, put $x = 0$

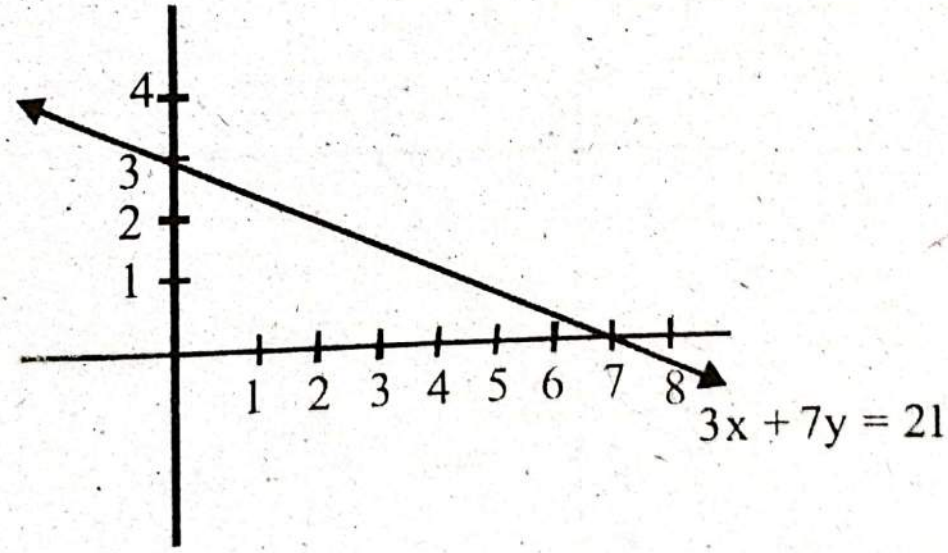
$$3(0) + 7y = 21$$
$$7y = 21$$
$$y = \frac{21}{7}$$
$$y = 3$$

2nd ordered pair is (0, 3)

Now put $x = y = 0$ (origin test) in

$$3x - 2y > 6$$
$$3(0) - 2(0) > 6$$
$$0 > 6$$

Which is false so shading lies away from origin (0, 0).



(iii) $\mathbf{3x - 2y \geq 6}$

Sol. Associate equation is

$$3x - 2y = 6$$

For x-intercept, put $y = 0$

$$3x - 2(0) = 6$$
$$3x = 6$$
$$x = \frac{6}{3} = 2$$

1st ordered pair = (2, 0)

For y-intercept, put x = 0

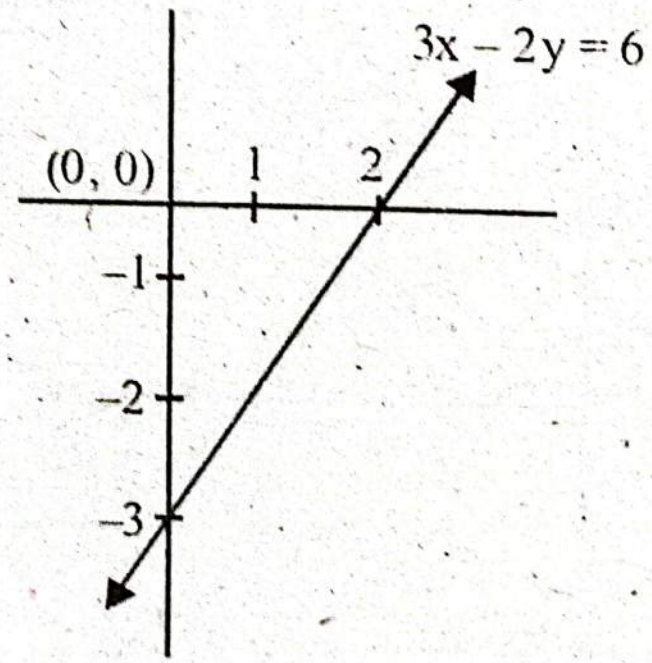
$$3(0) - 2y = 6$$
$$-2y = 6$$
$$y = \frac{6}{-2}$$
$$y = -3$$

2nd ordered pair is (0, –3)

Now put x = y = 0 (origin test) in

$$3x + 7y > 21$$
$$3(0) + 7(0) > 21$$
$$0 > 21$$

Which is false so shading lies away from origin (0, 0)



(iv) $\mathbf{5x - 4y \leq 20}$

Sol. Associate equation is

$$5x - 4y = 20$$

For x-intercept, put $y = 0$

$$5x - 4(0) = 20$$

$$5x = 20$$

$$x = \frac{20}{5} = 4$$

1st ordered pair = (4, 0)

For y-intercept, put $x = 0$

$$5(0) - 4y = 20$$

$$-4y = 20$$

$$y = \frac{20}{-4}$$

$$y = -5$$

2nd ordered pair is (0, –5)

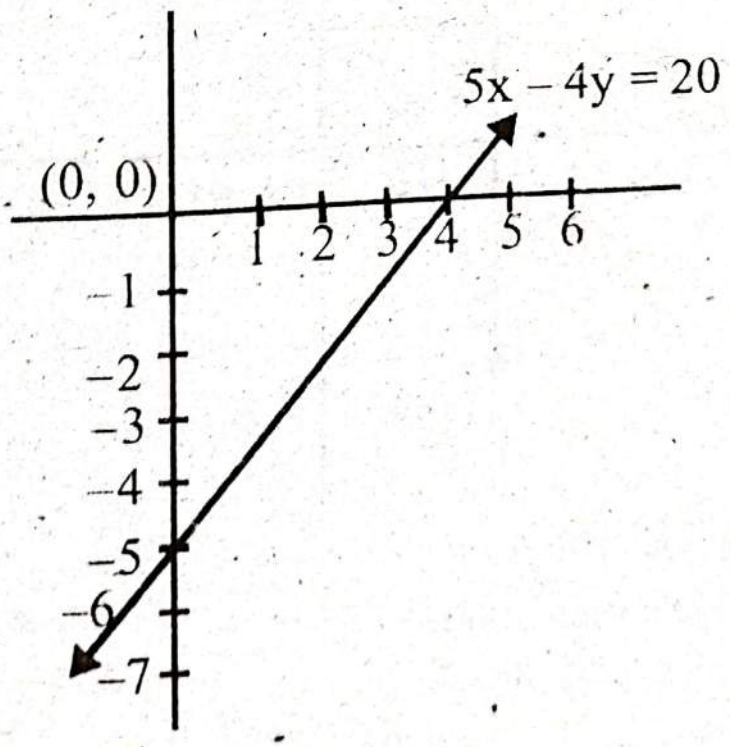
Now put $x = y = 0$ (origin test) in

$$5x - 4y < 20$$

$$5(0) - 4(0) < 20$$

$$0 < 20$$

Which is true so shading lies towards origin (0, 0)



(v) $\mathbf{2x + 1 > 0}$

Sol. Associate equation is

$$2x + 1 = 0$$

$$2x = -1$$

$$x = \frac{-1}{2}$$

The inequality in xy-plane is considered as $2x + ay > -1$ and its solution set consists of all points (x, y) such that $x, y \in R$ and $x > \frac{-1}{2}$

$$x = \frac{-1}{2} \qquad \ldots(i)$$

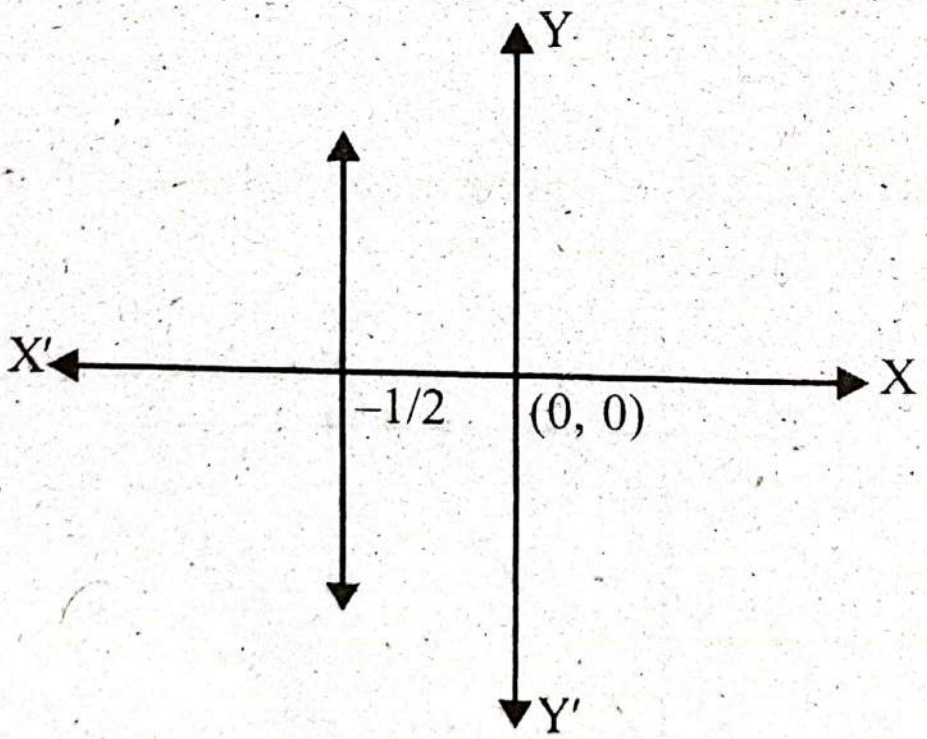
is vertical line parallel to y-axis.

Put $x = 0$ in $\quad 2x + 1 > 0$

$$2(0) + 1 > 0$$

$$1 > 0$$

Which is true. So shading lies towards origin (0, 0).



(vi) $\mathbf{3y - 4 \leq 0}$

Sol. Associated equation is

$$3y - 4 = 0$$

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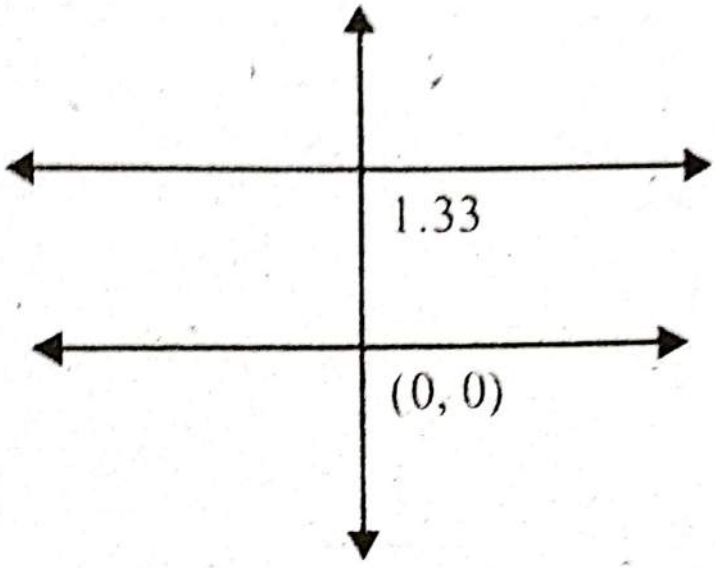
$$3y = 4$$

$$y = \frac{4}{3}$$

$$y = 1.33$$



Put $y = 0$ in

$$3y - 4 < 0$$

$$3(0) - 4 < 0$$

$$-4 < 0$$

Which is true. Shading lies towards origin.

Q.4. Indicate the solution region of the following linear inequalities by shading:

(i) $2x - 3y \leq 6$; $2x + 3y \leq 12$

Sol. Associated equation is

$$2x - 3y = 0$$

For x-intercept, put $y = 0$

$$2x - 3(0) = 6$$

$$2x = 6$$

$$x = \frac{6}{2} = 3$$

1st ordered pair = (3, 0)

For y-intercept, put $x = 0$

$$2(0) - 3y = 6$$

$$-3y = 6$$

$$y = \frac{6}{-3}$$

$$y = -2$$

other ordered pair is (0, –2)

Now put $x = y = 0$ (origin test) in

$$2x - 3y < 6$$

$$2(0) - 3(0) < 6$$

$$0 < 6$$

Which is true so shading lies towards origin.

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Now consider second inequality.

$$2x + 3y \leq 12$$

Associated equation is

$$2x + 3y = 12$$

For x-intercept, put $y = 0$

$$2x + 3(0) = 12$$

$$2x = 12$$

$$x = \frac{12}{2} = 6$$

1st ordered pair is (6, 0)

For y-intercept, put $x = 0$

$$2(0) + 3y = 12$$

$$3y = 12$$

$$y = \frac{12}{3}$$

$$y = 4$$

other ordered pair is (0, 4)

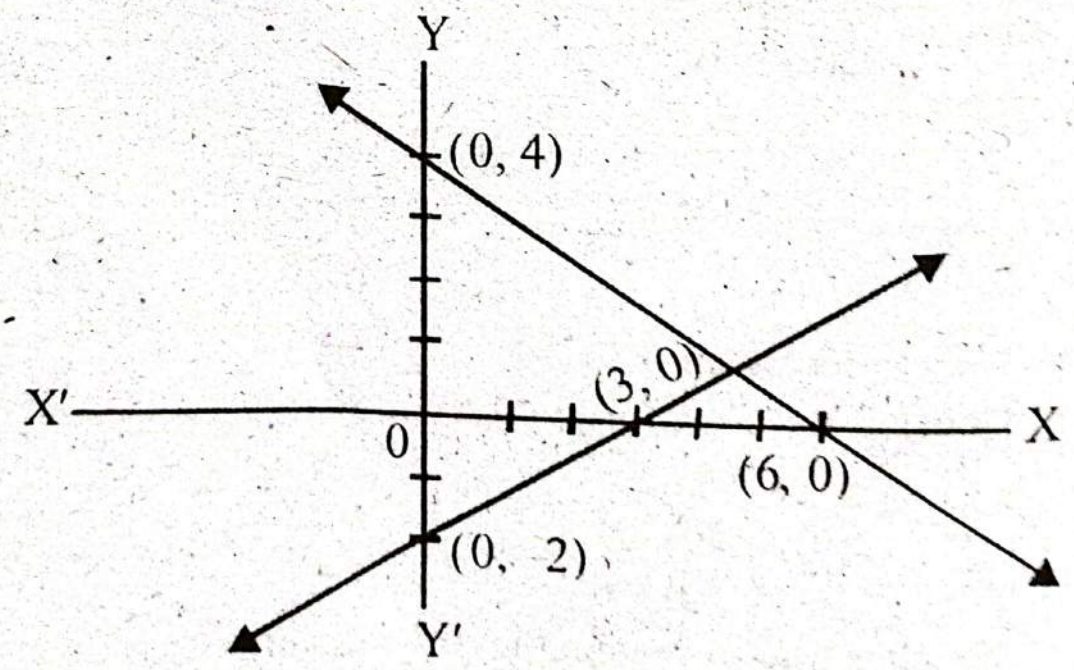
Now put $x = y = 0$ (origin test) in

$$2x + 3y < 12$$

$$2(0) + 3(0) < 12$$

$$0 < 12$$

Which is true so shading lies towards origin side. Now we draw the lines.



(ii) $x + y \geq 5$; $-y + x \leq 1$

Sol. Associated equation is

$$x + y = 5$$

For x-intercept, put $y = 0$

$$x + 0 = 5$$

$$x = 5$$

1st ordered pair = (5, 0)

For y-intercept, put $x = 0$

$$0 + y = 5$$

$$y = 5$$

2nd ordered pair = (0, 5)

Take $x = y = 0$ (origin test) in

$$0 + 0 < 5$$

$$0 < 5$$

Which is false so shading lies away from origin side.

Now consider

$$-y + x \leq 1$$

Associated equation is

$$-y + x = 1$$

For x-intercept, put $y = 0$

$$0 + x = 1$$

$$x = 1$$

1st ordered pair = (1, 0)

For y-intercept, put $x = 0$

$$-y + 0 = 1$$

$$y = -1$$

other ordered pair is $(0, -1)$

Now take $x = y = 0$ (origin test) in

$$0 + 0 < 1$$

$$0 < 1$$

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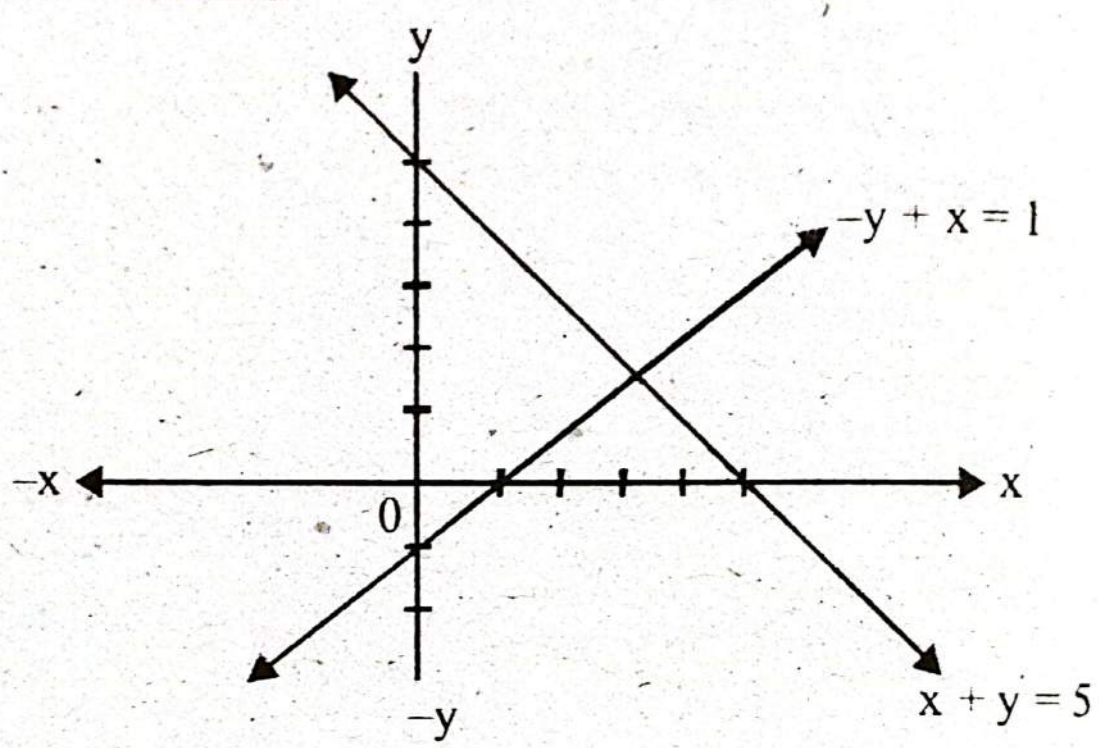
Which is true so shading lies towards origin side. Now we draw the lines.



(iii) $\mathbf{3x + 7y \geq 21}$; $\mathbf{x - y \leq 2}$

Sol. Associated equation is

$$3x + 7y = 21$$

For x-intercept, put y = 0

$$3x + 7(0) = 21$$

$$3x = 21$$

$$x = \frac{21}{3} = 7$$

1st ordered pair = (7, 0)

For y-intercept, put x = 0

$$3(0) + 7y = 21$$

$$7y = 21$$

$$y = \frac{21}{7}$$

$$y = 3$$

2nd ordered pair = (0, 3)

Now take x = y = 0 (origin test) in

$$3(0) + 7(0) > 21$$

$$0 > 21$$

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Which is false so shading lies away from origin side.

Now consider $x - y \leq 2$

Associated equation is

$$x - y = 2$$

For x-intercept, put $y = 0$

$$x - 0 = 2$$

$$x = 2$$

1st ordered pair is (2, 0)

For y-intercept, put $x = 0$

$$0 - y = 2$$

$$-y = 2$$

$$y = -2$$

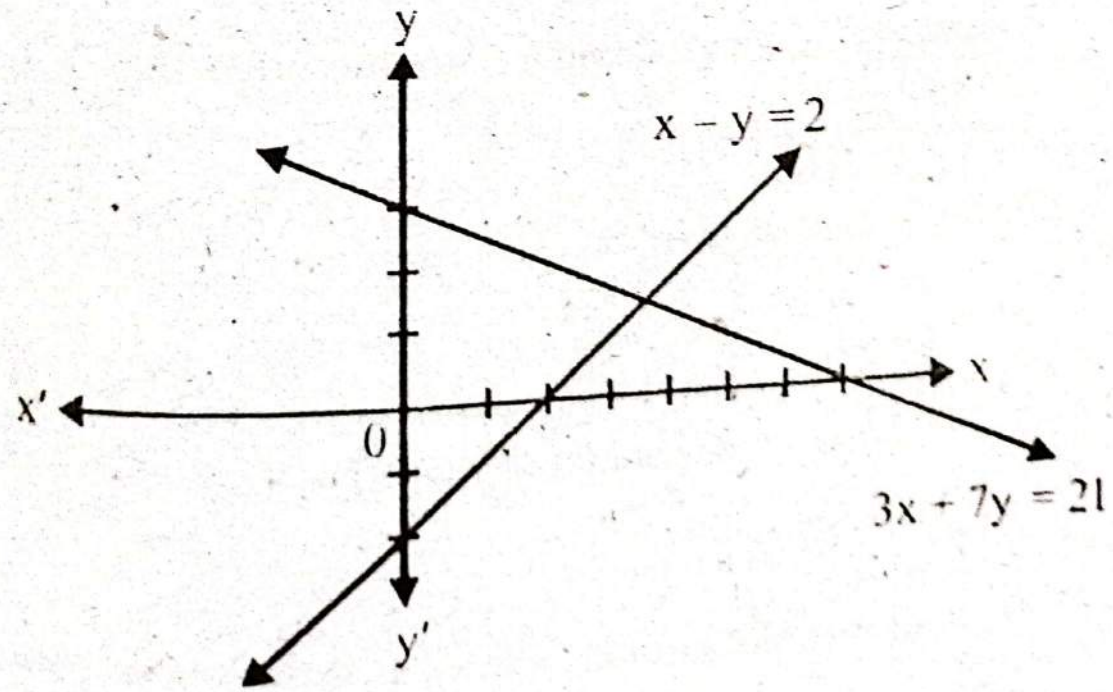
other ordered pair = (0, –2).

Now put $x = y = 0$ in $x - y < 2$ (origin test) in

$$0 - 0 < 2$$

$$0 < 2$$

Which is true so shading lies towards origin side. Now we draw the lines.



(iv) $4x - 3y \leq 12$; $x \geq -\frac{3}{2}$

Sol. Associated equation is

$$4x - 3y = 12$$

For x-intercept, put $y = 0$

$$4x - 3(0) = 12$$

$$4x = 12$$

$$x = \frac{12}{4} = 3$$

1st ordered pair $= (3, 0)$

For y-intercept, put $x = 0$

$$4(0) - 3y = 12$$

$$-3y = 12$$

$$y = \frac{12}{-3}$$

$$y = -4$$

other ordered pair is $(0, -4)$

Now put $x = y = 0$ (origin test) in

$$4(0) - 3(0) < 12$$

$$0 < 12$$

Which is true so shading lies towards origin side.

Now consider $x \geq -\frac{3}{2}$.

Associated equation is

$$x = -\frac{3}{2}$$

$x = -\frac{3}{2}$ is a line parallel to y-axis put $x = 0$ in $x > -\frac{3}{2}$

$\Rightarrow 0 > -\frac{3}{2}$ which is true.

So shading lies towards origin side. Now we draw lines.

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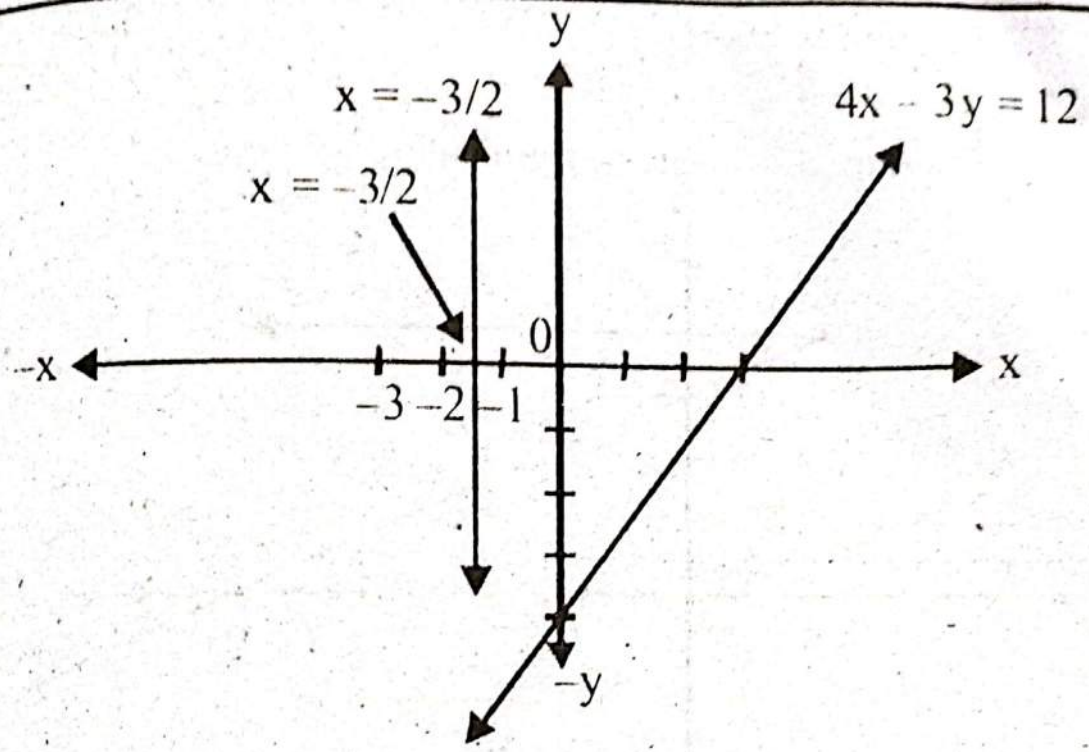


(v) $\mathbf{3x + 7y \geq 21}$; $\mathbf{y \leq 4}$

Sol. Associated equation is

$$3x + 7y \geq 21$$

For x-intercept

Put $y = 0$

$$3x = 21$$

$$x = 7$$

1st ordered pair is $(7, 0)$

For y-intercept, put $x = 0$

$$7y = 21$$

$$y = 3$$

2nd ordered pair is $(0, 3)$

Origin test, put $x = y = 0$ in

$$3x + 7y > 21$$

$$3(0) + 7(0) > 21$$

$$0 > 21 \text{ (False)}$$

So shading region away from origin side.

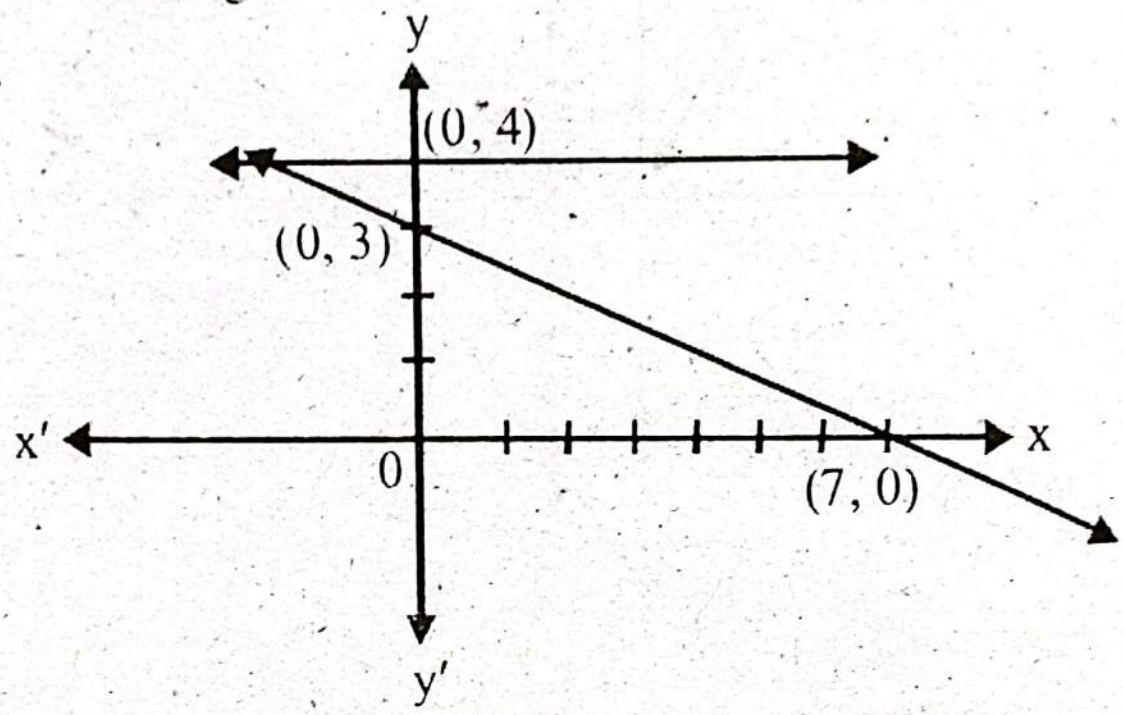
Now $y \leq 4$

$$y = 4$$

So point is (0, 4)

$$y = 0 \Rightarrow 0 < 4 \text{ (true)}$$

Shading lies towards origin side.



(vi) $\mathbf{5x + 7y \leq 35}$; $\mathbf{x - 2y \leq 2}$

Sol. Associated equation is

$$5x + 7y = 35$$

For x-intercept, put y = 0

$$5x + 7(0) = 35$$

$$5x = 35$$

$$x = \frac{35}{5} = 7$$

1st ordered pair = (7, 0)

For y-intercept, put x = 0

$$5(0) + 7y = 35$$

$$7y = 35$$

$$y = \frac{35}{7}$$

$$y = 5$$

2nd ordered pair = (0, 5)

Now put x = y = 0 (origin test) in

$$5(0) + 7(0) < 35$$

$$0 < 35$$

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Which is true so shading lies away origin side.

Now we consider $x - 2y \leq 2$

Associated equation is

$$x - 2y = 2$$

For x-intercept, put $y = 0$

$$x - 2(0) = 2$$

$$x = 2$$

1st ordered pair is $(2, 0)$

For y-intercept, put $x = 0$

$$0 - 2y = 2$$

$$-2y = 2$$

$$y = \frac{2}{-2}$$

$$y = -1$$

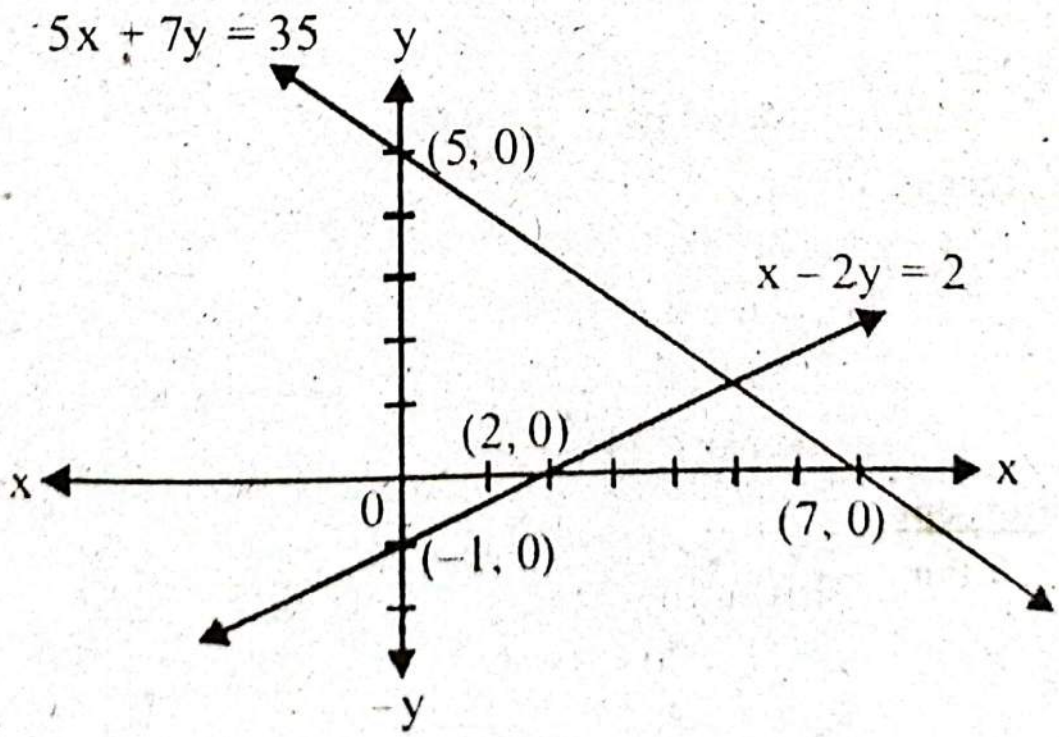
2nd ordered pair is $(0, -1)$

Take $x = y = 0$ (origin test) in

$$0 - 2(0) < 2$$

$$0 < 2$$

Which is true so shading lies towards origin side.



EXERCISE 5.2

Q.1. **Maximize $f(x, y) = 2x + 5y$; subject to the constraints,**

$2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$

Sol. Associated equations are

$$2y - x = 8 \quad \ldots(1)$$

$$x - y = 4 \quad \ldots(2)$$

x-intercept

Put $y = 0$ in (1)

$$2(0) - x = 8$$

$$-x = 8$$

$$x = -8$$

1st ordered pair = $(-8, 0)$

y-intercept

Put $x = 0$ in (1)

$$2y - 0 = 8$$

$$2y = 8$$

$$y = \frac{8}{2}$$

$$y = 4$$

Second ordered pair = $(0, 4)$

x-intercept

Put $y = 0$ in (2)

$$x - 0 = 4$$

$$x = 4$$

1st ordered pair = $(4, 0)$

y-intercept

Put $x = 0$ in (2)

$$0 - y = 4$$

$$-y = 4$$

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$y = -4$

2nd ordered pair = (0, –4)

Origin Test

Put $x = y = 0$ in $2y - x < 8$

$2(0) - 0 < 8$

$0 < 8$

Which is true, so shading lies towards the origin side.

Next put $x = 0$, $y = 0$ in

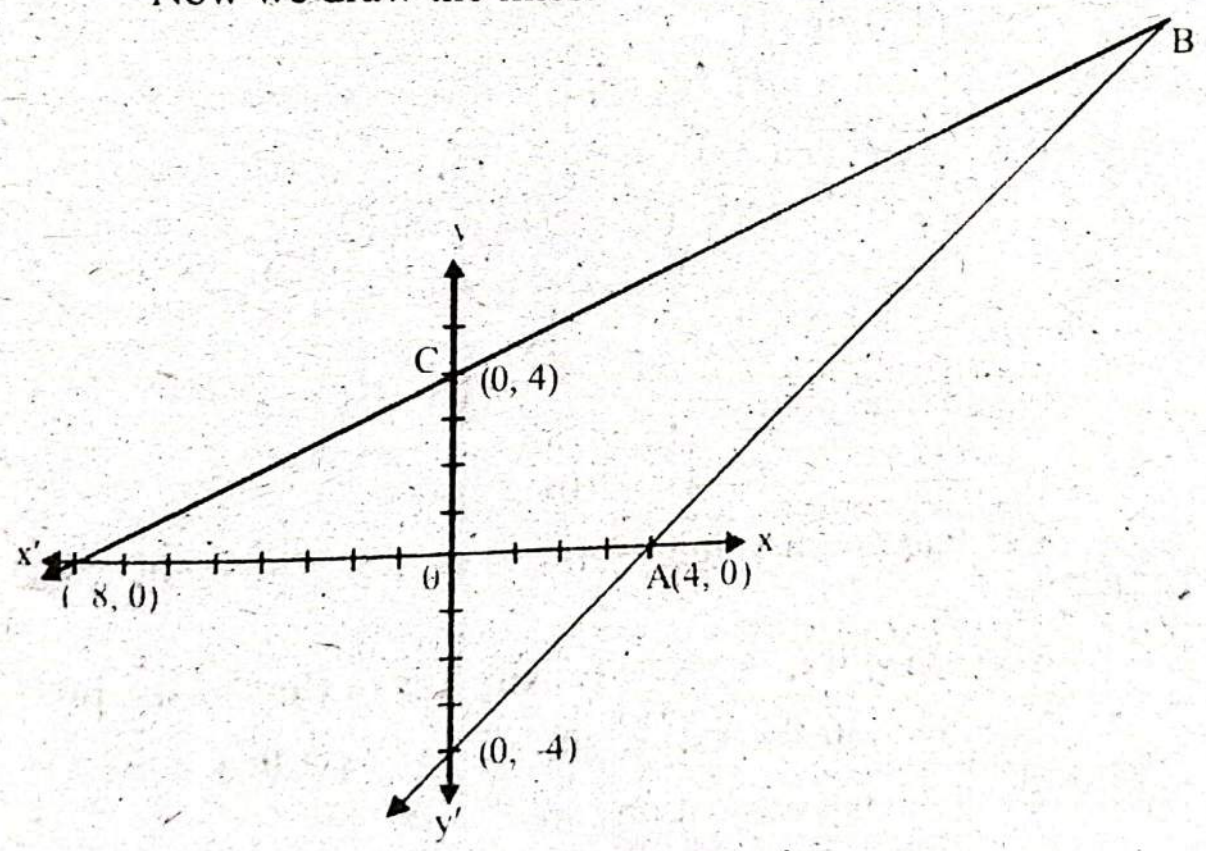
$x - y < 4$

$0 - 0 < 4$

$0 < 4$

Which is true, so shading lies towards origin side.

Now we draw the lines.



OABC is the feasible solution region.

So corner points are

O(0, 0), A(4, 0), C(0, 4) and B(x, y)

To find B(x, y), so solve both equations (1) & (2) for corner point.

Adding (1) and (2)

$$2y - x = 8$$
$$\underline{x - y = 4}$$
$$\underline{y = 12}$$

Put y = 2 in (2)

$$x - 12 = 4$$
$$x = 12 + 4$$
$$x = 16$$

So B(16, 12) corner point.

Now

$$f(x, y) = 2x + 5y \qquad \ldots(2)$$

Put O(0, 0) in (3)

$$f(0, 0) = 2(0) + 5(0) = 0$$

Put A(4, 0) in (3)

$$f(4, 0) = 2(4) + 5(0) = 8$$

Put B(16, 12) in (3)

$$f(16, 12) = 2(16) + 5(12)$$
$$= 32 + 60$$
$$= 92$$

Put C(0, 4) in (3)

$$f(0, 4) = 2(0) + 5(4) = 20$$

The maximum value of $f(x, y)$ is 92 at the corner point B(16, 12).

Q.2. Maximize $f(x, y) = x + 3y$; subject to the constraints $2x + 5y \leq 30$; $5x + 4y \leq 20$; $x \geq 0$; $y \geq 0$

Sol. The associated equations are

$$2x + 5y = 30 \qquad \ldots(1)$$

and $$5x + 4y = 20 \qquad \ldots(2)$$

<u>x-intercept</u>

Put y = 0 in (1)

$$2x + 5(0) = 30$$
$$2x = 30$$
$$x = 15$$

1st ordered pair = (15, 0)

y-intercept

Put x = 0 in (1)

$$2(0) + 5y = 30$$
$$5y = 30$$
$$y = \frac{30}{5}$$
$$y = 6$$

Second ordered pair = (0, 6)

Put x = y = 0 in $2x + 5y < 30$ (origin test)

$$2(0) + 5(0) < 30$$
$$0 < 30$$

Which is true. so shading lies towards origin test.

Now consider,

$$5x + 4y = 20$$

x-intercept

Put y = 0

$$5x + 4(0) = 20$$
$$5x = 20$$
$$x = \frac{20}{5} = 4$$

1st ordered pair is (4, 0)

y-intercept

Put x = 0

$$5(0) + 4y = 20$$
$$4y = 20$$
$$y = 5$$

2nd ordered pair is (0, 5)

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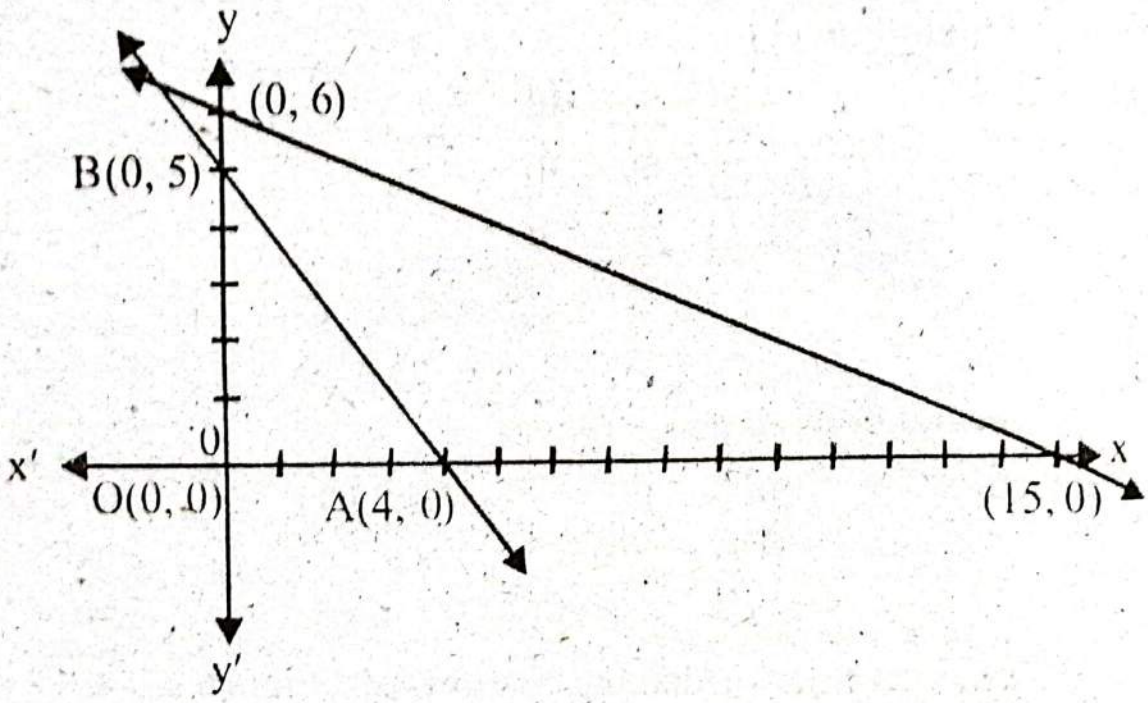
Put $x = y = 0$ in $5x + 4y < 20$

$$5(0) + 4(0) < 20$$

$$0 < 20$$

Which is true, so shading lies towards origin side.

Now we draw the lines.



OAB is the feasible region.

Corner points are

O(0, 0), A(4, 0), B(0, 5)

Now

$f(x, y) = x + 3y$

O(0, 0)

$$f(0, 0) = 0 + 3(0) = 0$$

A(4, 0)

$$f(4, 0) = 4 + 3(0) = 4$$

B(0, 5)

$$f(0, 5) = 0 + 3(5)$$

$$= 15$$

Clearly maximum value of $f(x, y)$ is 15 at corner point (0, 5).

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Q.3. **Maximize $z = 2x + 3y$; subject to the constraints**

$2x + y \leq 4$; $4x - y \leq 4$; **$x \geq 0$; $y \geq 0$**

Sol. The associated equations are

$$2x + y = 4 \quad \ldots(1)$$

and $$5x - y = 4 \quad \ldots(2)$$

x-intercept

Put $y = 0$ in (1)

$$2x = 4$$
$$x = 2$$

1st ordered pair = (2, 0)

y-intercept

Put $x = 0$

$$2(0) + y = 4$$
$$y = 4$$

2nd ordered pair = (0, 4)

Put $x = y = 0$ in $2x + y < 4$ (origin test)

$$2(0) + 0 < 4$$
$$0 < 4$$

Which is true, so shading lies towards origin test.

Now consider $4x - y = 4$

x-intercept

Put $y = 0$ in (2)

$$4x - 0 = 20$$
$$4x = 4$$
$$x = 1$$

1st ordered pair is (1, 0)

y-intercept

Put $x = 0$ in (2)

$$4(0) - y = 4$$
$$-y = 4$$
$$y = -3$$

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2nd ordered pair = (0, –3)

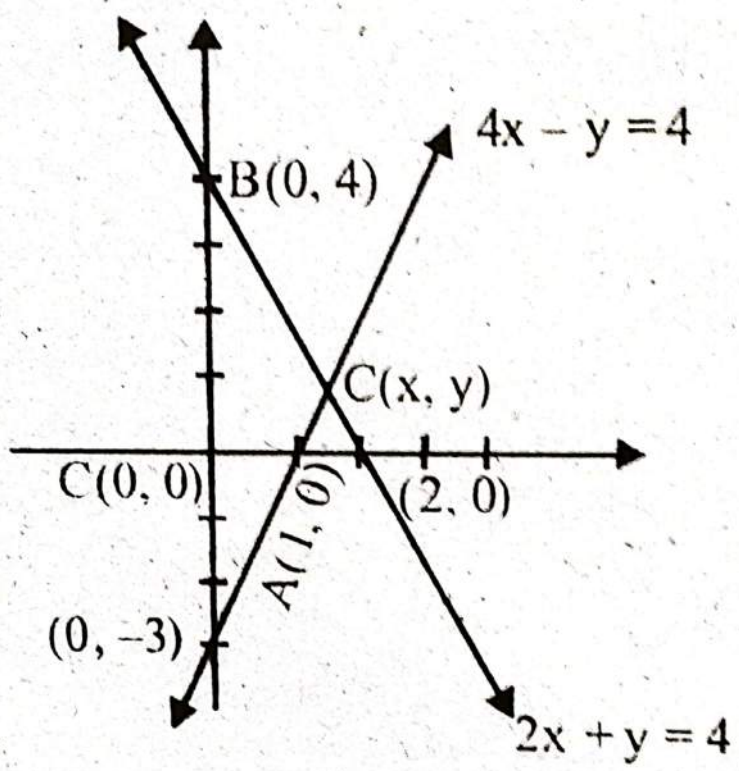
Now put $x = y = 0$ in $4x - y < 4$

$$4(0) - 0 < 4$$

$$0 < 4$$

Which is true, so shading lies towards origin side.

Now we draw the lines.



OABC is the feasible region.

Corner points are

O(0, 0), A(1, 0), B(0, 4), C(x, y) = ?

Solving both equations to find corner point.

Adding (1) and (2)

$$\begin{array}{r} 2x + y = 4 \\ \underline{4x - y = 4} \\ 6x = 8 \end{array}$$

$$x = \frac{8}{6}$$

$$x = \frac{4}{3}$$

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Put $x = \frac{4}{3}$ in 1st equation.

$$2\left(\frac{4}{3}\right) + y = 4$$

$$\frac{8}{3} + y = 4$$

$$y = 4 - \frac{8}{3}$$

$$y = \frac{12-8}{3} = \frac{4}{3}$$

Corner point $C\left(\frac{3}{4}, \frac{4}{3}\right)$

Putting all corner points in $z = 2x + 3y$

$f(0, 0) = 2(0) + 3(0) = 0$

$f(1, 0) = 2(1) + 3(0) = 2$

$f(0, 4) = 2(0) + 3(4) = 12$

$$f\left(\frac{3}{4}, \frac{4}{3}\right) = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right)$$

$$= \frac{8}{3} + 4$$

$$= \frac{8+12}{3} = \frac{20}{3}$$

So maximum value of function is $\frac{20}{3}$ at corner point $\left(\frac{3}{4}, \frac{4}{3}\right)$

Q.4. Minimize $z = 2x + y$; subject to the constraints:

$x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$

Sol. Associated equations are

$$x + y = 3 \quad \ldots(1)$$

$$7x - 5y = 35 \quad \ldots(2)$$

x-intercept

Put $y = 0$

$$x + 0 = 3$$
$$x = 3$$

1st ordered pair is $(3, 0)$

y-intercept

Put $x = 0$

$$0 + y = 3$$
$$y = 3$$

2nd ordered pair is $(0, 3)$

Put $x = y = 0$ (origin test) in

$$x + y > 3$$
$$0 + 0 > 3$$
$$0 > 3$$

Which is false, so shading lies away from origin.

x-intercept

Put $y = 0$

$$7x + 5(0) = 35$$
$$7x = 35$$
$$x = \frac{35}{7}$$
$$x = 5$$

1st ordered pair is $(5, 0)$

y-intercept

Put $x = 0$

$$7(0) + 5y = 35$$
$$5y = 35$$
$$y = \frac{35}{5}$$
$$y = 7$$

2nd ordered pair is $(0, 7)$

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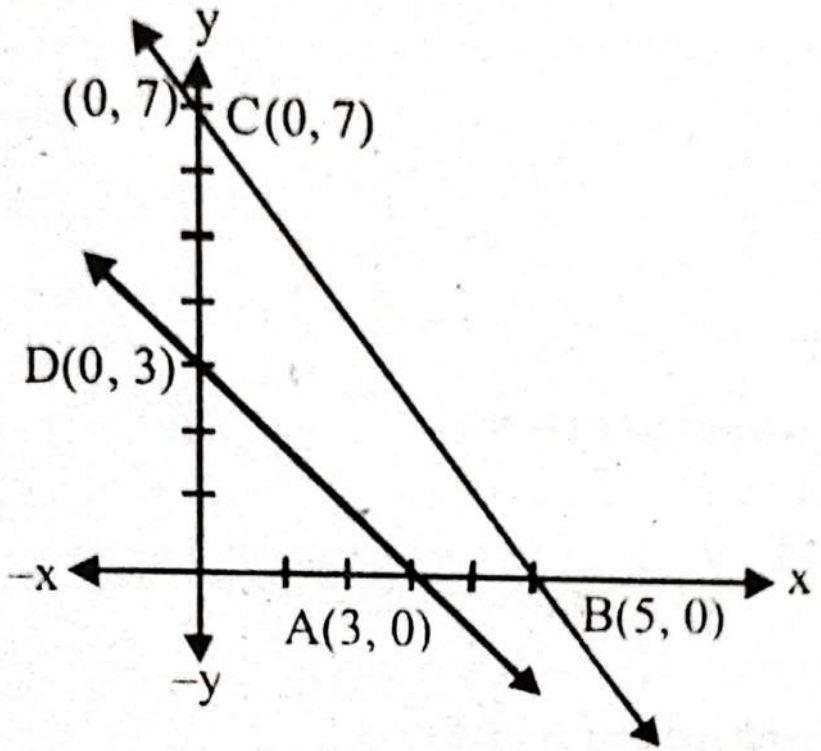
Put $x = y = 0$ in

$$7x + 5y < 35$$
$$7(0) + 5(0) < 35$$
$$0 < 35$$

Which is true, so shading lies towards the origin.

Now we draw these lines.



Common shaded region is ABCD

Corner points are A(3, 0), B(5, 0), C(0, 7), D(0, 3)

Function to be minimized is

$$z = 2x + y$$

Corner point A(3, 0)

$$z = 2(3) + 0 = 6$$

Corner point B(5, 0)

$$z = 2(5) + 0 = 10$$

Corner point C(0, 7)

$$z = 2(0) + 7 = 7$$

Corner point D(0, 3)

$$z = 2(0) + 3 = 3$$

We observe that z has minimum value 3 at corner point (0, 3)

Q.5. Maximize the function defined as; $f(x, y) = 2x +$ subject to the constraints:

$$2x + y \leq 8; \; x + 2y \leq 14; \qquad x \geq 0; \; y \geq 0$$

Sol. Associated equations are

$$2x + y = 8 \qquad \ldots(1)$$

$$x + 2y = 14 \qquad \ldots(2)$$

x-intercept

Put $y = 0$

$$2x + 0 = 8$$

$$2x = 8$$

$$x = \frac{8}{2} = 4$$

1st ordered pair is (4, 0)

y-intercept

Put $x = 0$

$$2(0) + y = 8$$

$$y = 8$$

2nd ordered pair is (0, 8)

Put $x = y = 0$ in

$$2x + y < 8$$

$$2(0) + 0 < 8$$

$$0 < 8$$

Which is true, so shading lies away from origin side.

x-intercept

Put $y = 0$

$$x + 2(0) = 14$$

$$x = 14$$

1st ordered pair is (14, 0)

y-intercept

Put $x = 0$

$$0 + 2y = 14$$

$$2y = 14$$

$$y = 7$$

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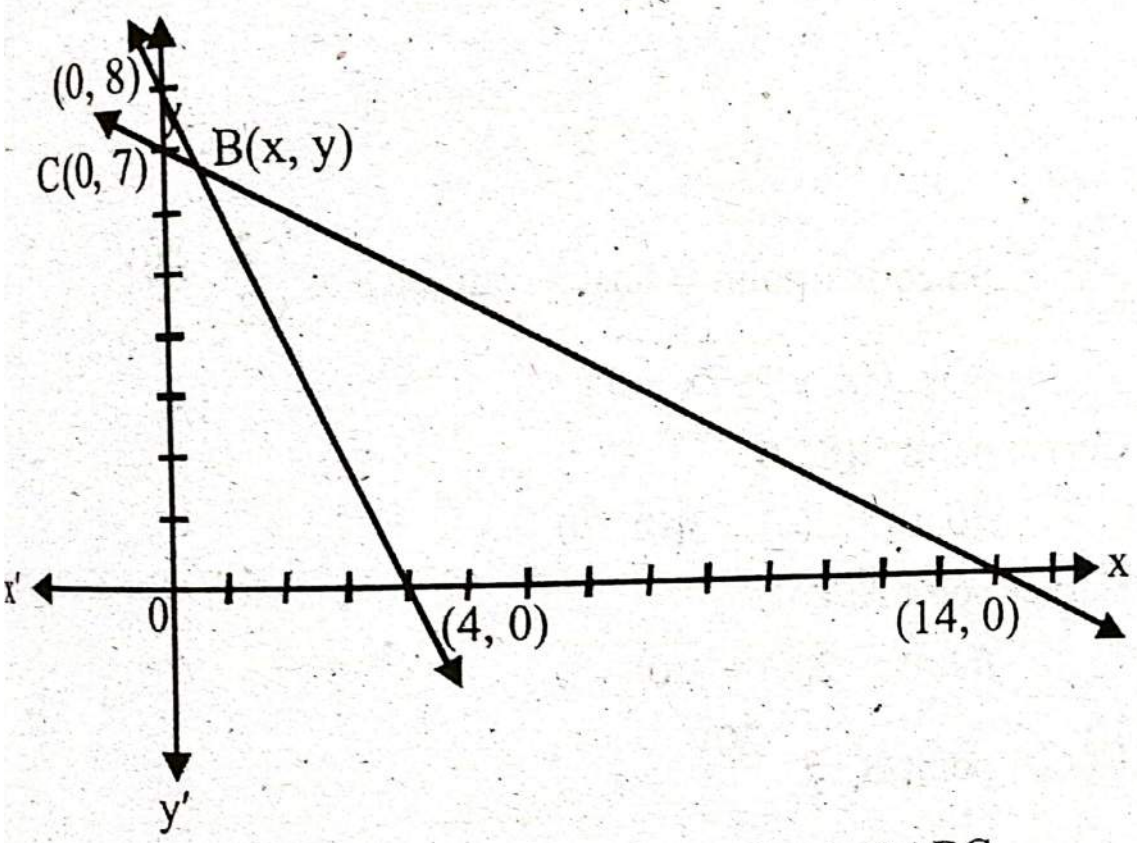
2nd ordered pair is (0, 7)

Put $x = y = 0$ in

$$x + 2y < 14$$
$$0 + 2(0) < 14$$
$$0 < 14$$

Which is true, so shading lies towards the origin side.

Now we draw the lines.



From diagram, common shaded region is OABC

Corner points are O(0, 0), A(4, 0), C(0, 7), B(x, y)

We find corner point B(x, y) by solving (1) and (2) as

Multiply (1) by (2) and subtract from (2)

$$x + 2y = 14$$
$$\underline{4x + 2y = 16}$$
$$3x = -2$$
$$x = \frac{2}{3}$$

Put $x = \frac{2}{3}$ in (1) for y.

$$2\left(\frac{2}{3}\right) + y = 8$$

$$\frac{4}{3} + y = 8$$

$$y = 8 - \frac{4}{3}$$

$$y = \frac{24 - 4}{3}$$

$$y = \frac{20}{3}$$

So corner point B has coordinates as $\left(\frac{2}{3}, \frac{20}{3}\right)$

Now $f(x, y) = 2x + 3y$

Corner point O(0, 0)

$f(0, 0) = 2(0) + 3(0) = 0$

Corner point A(4, 0)

$f(4, 0) = 2(4) + 3(0) = 8$

Corner point $B\left(\frac{2}{3}, \frac{20}{3}\right)$

$$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right)$$

$$= \frac{4}{3} + 20$$

$$= \frac{4 + 60}{3} = \frac{64}{3} = 21.33$$

Corner point C(0, 7)

$f(0, 7) = 2(0) + 3(7) = 21$

We observe that $f(x, y)$ has maximum value 21.3 at corner point $\left(\frac{2}{3}, \frac{20}{3}\right)$

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Q.6. Find minimum and maximum values of $z = 3x + y$; subject to the constraints:

$3x + 5y \geq 15$; $x + 6y \geq 9$; $x \geq 0$; $y \geq 0$

Sol. Associated equations are

$$3x + 5y = 15 \quad \ldots(1)$$

$$x + 6y = 9 \quad \ldots(2)$$

x-intercept

Put $y = 0$

$$3x + 5(0) = 15$$

$$3x = 15$$

$$x = \frac{15}{3} = 5$$

1st ordered pair is (5, 0)

y-intercept

Put $x = 0$

$$3(0) + 5y = 15$$

$$5y = 15$$

$$y = \frac{15}{5}$$

$$y = 3$$

2nd ordered pair is (0, 3)

Put $x = y = 0$ in

$$3x + 5y > 15$$

$$3(0) + 5(0) > 15$$

$$0 > 15$$

Which is false, so shading lies away from origin side.

x-intercept

Put $y = 0$

$$x + 6(0) = 9$$

$$x = 9$$

1st ordered pair is (9, 0)

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y-intercept

Put $x = 0$

$$0 + 6y = 9$$
$$6y = 9$$
$$y = \frac{9}{6} = \frac{3}{2}$$

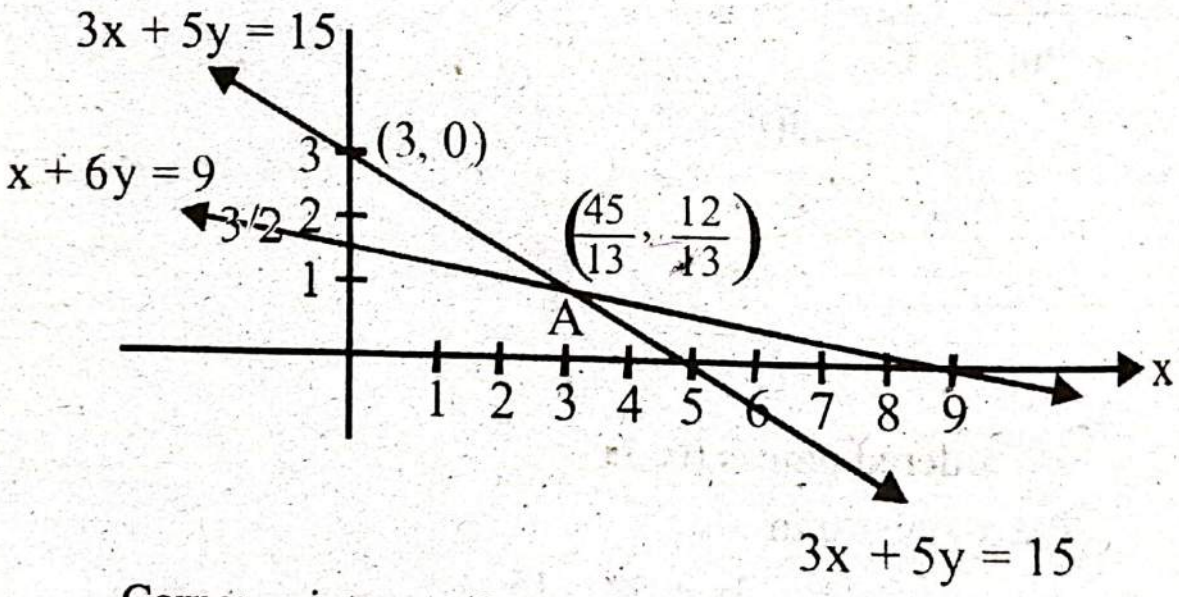
2nd ordered pair is $\left(0, \frac{3}{2}\right)$

Put $x = y = 0$ in

$$x + 6y > 9$$
$$0 + 6(0) > 9$$
$$0 > 9$$

Which is false, so shading lies away from origin side.

Now we draw the lines.



Corner points are (0, 3) and point A.

Solving $x + 6y = 9$ and $3x + 5y = 15$

$$3 \times (x + 6y) = 3 \times 9$$
$$3x + 18y = 27$$
$$\underline{3x + 5y = 15}$$
$$13y = 12$$
$$y = \frac{12}{13}$$

Put $y = \frac{12}{13}$ in $x + 6y = 9$ for x.

$$x + 6\left(\frac{12}{13}\right) = 9$$

$$x + \frac{72}{13} = 9$$

$$x = 9 - \frac{72}{13}$$

$$= \frac{117 - 72}{13} = \frac{45}{13}$$

Other corner point is $\left(\frac{45}{13}, \frac{12}{13}\right)$

To find minimum and maximum values

Corner point (0, 3)

$$z = 3(0) + 3 = 3$$

Corner point $\left(\frac{45}{13}, \frac{12}{13}\right)$

$$z = 3\left(\frac{45}{13}\right) + \frac{12}{13}$$

$$z = \frac{135}{13} + \frac{12}{13} = \frac{147}{13}$$

So minimum value of z is 3 at corner point (0, 3) and maximum value $\frac{147}{13}$ at corner point $\left(\frac{45}{13}, \frac{12}{13}\right)$

REVIEW EXERCISE 5

Q.1. Four options are given against each statement. Encircle the correct one.

(i) In the following, linear equation is:

(a) $5x > 7$ (b) $4x - 2 < 1$

(c) $2x + 1 = 1$ (d) $4 = 1 + 3$

Sol. Option (c) is correct.

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(ii) Solution of $5x - 10 = 10$ is:

(a) 0 (b) 50

(c) 4 (d) -4

Sol.

$$5x - 10 = 10$$
$$5x = 10 + 10$$
$$5x = 20$$
$$x = \frac{20}{5} = 4$$

Option (c) is correct.

(iii) If $7x + 4 < 6x + 6$, then x belongs to the interval:

(a) $(2, \infty)$ (b) $(2, \infty)$

(c) $(-\infty, 2)$ (d) $(-\infty, 2)$

Sol.

$$7x + 4 < 6x + 6$$
$$7x - 6x < 6 - 4$$
$$x < 2$$
$$(-\infty, 2)$$

Option (c) is correct.

(iv) A vertical line divides the plane into:

(a) left half plane (b) right half plane

(c) full plane (d) two half plane

Sol. A vertical line divides the plane into two half planes.

So option (d) is correct.

(v) The linear equation formed out of the linear inequality is called:

(a) linear equation (b) associated equation

(c) quadratic equal (d) none of these

Sol. Option (b) is correct.

(vi) $3x + 4 < 0$ is:

(a) equation (b) inequality

(c) not inequality (d) identity

Sol. $3x + 4 < 0$ is an inequality.

Option (b) is correct.

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(vii) Corner point is also called:

(a) code (b) vertex

(c) curve (d) region

Sol. Other name of corner point is vertex.

So, option (b) is correct.

(viii) (0, 0) is solution of inequality:

(a) $4x + 5y > 8$ (b) $3x + y > 6$

(c) $-2x + 3y < 0$ (d) $x + y > 4$

Sol. $4x + 5y > 8$

$4(0) + 5(0) > 8$

$(0 > 8$ False$)$

$3(0) + 0 > 0$

$(0 > 6$ False$)$

$-2(0) + 3(0) < 0$

$(0 < 0$ False$)$

$0 + 0 > 4$

$(0 > 4$ False$)$

No correct option provided in statement.

(ix) The solution region restricted to the first quadrant is called:

(a) objective region (b) feasible region

(c) solution region (d) constraints region

Sol. By definition, solution region restricted to the first quadrant is called feasible region.

So option (b) is correct.

(x) A function that is to be maximized or minimized is called:

(a) solution function (b) objective function

(c) feasible function (d) none of these

Sol. By definition "a function is to be maximized or minimized is called objective function.

So option (b) is correct.

Q.2. Solve and represent their solutions on real line.

(i) $\dfrac{x+5}{3} = 1 - x$

Sol. $x + 5 = 3(1 - x)$

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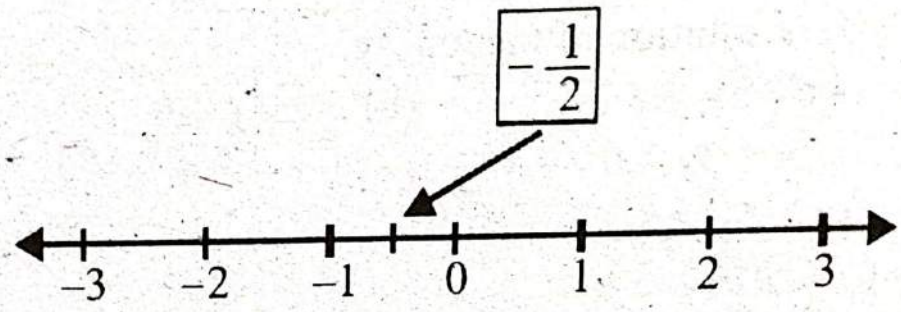
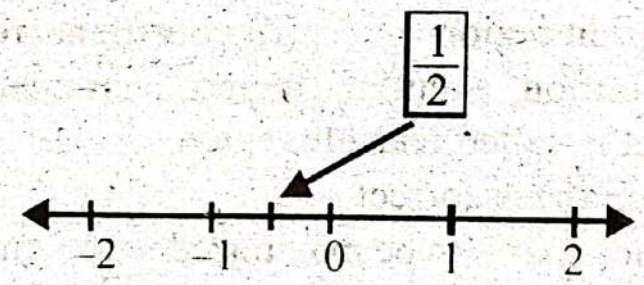
$$x + 5 = 3 - 3x$$
$$3x + x = 3 - 5$$
$$4x = -2$$
$$x = -\frac{1}{2}$$



(ii) $\frac{2x+1}{3} + \frac{1}{2} = 1 - \frac{x-1}{3}$

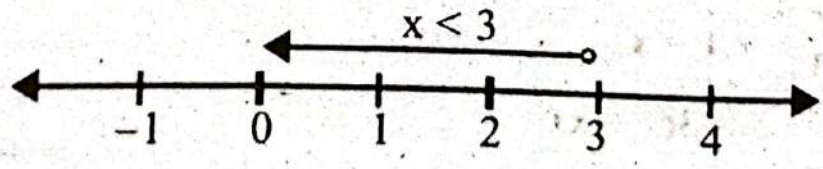
Sol. $\frac{2x+1}{3} + \frac{x-1}{3} = 1 - \frac{1}{2}$

$$\frac{2x+1+x-1}{3} = \frac{2-1}{2}$$
$$\frac{3x}{3} = \frac{1}{2}$$
$$x = \frac{1}{2}$$



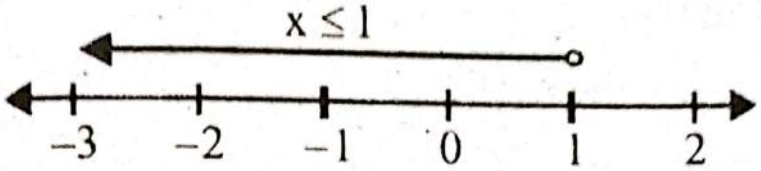
(iii) $3x + 7 < 16$

Sol.
$$3x < 16 - 7$$
$$3x < 9$$
$$x < \frac{9}{3}$$
$$x < 3$$



(iv) $5(x-3) \geq 26x - (10x+4)$

Sol.



$5x - 15 \geq 26x - 10x - 4$

$5x - 15 \geq 16x - 4$

$5x - 16x \geq -4 + 15$

$-11x \geq 11$

Or $11x \leq 11$

$x \leq 1$

No solution.

Q.3. Find the solution region of the following linear equalities.

(i) $3x - 4y \leq 12$; $3x + 2y \geq 3$

Sol. Associated equations are

$3x - 4y = 12$ ...(1)

and $3x + 2y = 3$ ...(2)

x-intercept

Put $y = 0$

$3x - 4(0) = 12$

$3x = 12$

$x = 4$

1st ordered pair is (4, 0)

y-intercept

Put $x = 0$

$3(0) - 4y = 12$

$-4y = 12$

$y = -3$

2nd ordered pair is (0, −3)

Take $x = y = 0$ (origin test)

$3(0) - 4(0) < 12$

$0 < 12$

Which is true. so shading lies towards origin side.

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x-intercept

Put $y = 0$

$$3x + 2(0) = 3$$
$$3x = 3$$
$$x = 1$$

1st ordered pair is $(1, 0)$

y-intercept

Put $x = 0$

$$3(0) + 2y = 3$$
$$2y = 3$$
$$y = \frac{3}{2}$$
$$y = 1.5$$

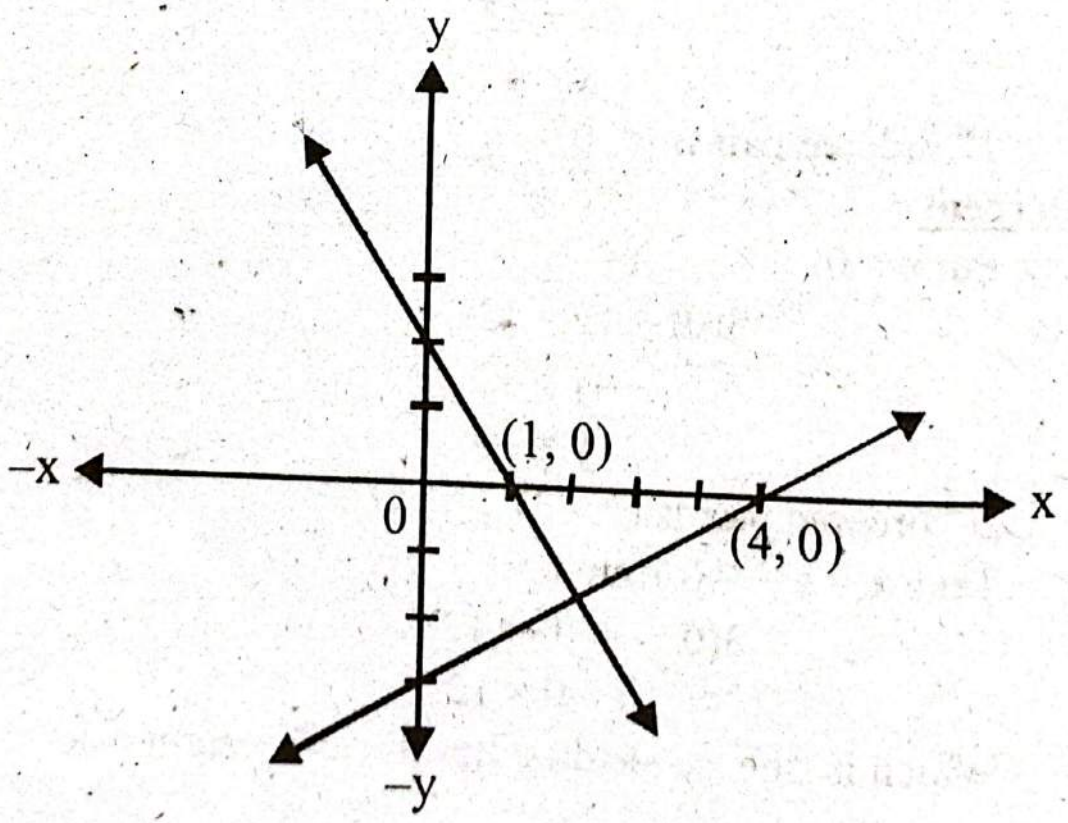
2nd ordered pair is $(0, 1.5)$

Take $x = y = 0$ (origin test)

$$3(0) + 2(0) < 3$$
$$0 < 3$$

Which is false, so shading lies away from origin side.

Now we draw the lines.



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(ii) $2x + y \leq 4$; $x + 2y \leq 6$

Sol. Associated equations are

$$2x + y = 4 \quad \ldots(1)$$

and $$x + 2y = 6 \quad \ldots(2)$$

x-intercept

Put $y = 0$

$$2x = 4$$
$$x = 2$$

1st ordered pair is (2, 0)

y-intercept

Put $x = 0$

$$2(0) + y = 4$$
$$y = 4$$

2nd ordered pair is (0, 4)

Put $x = y = 0$ (origin test)

$$2(0) + 0 < 4$$
$$0 < 4$$

Which is true. so shading lies towards origin side.

x-intercept

Put $y = 0$

$$x + 0 = 6$$
$$x = 6$$

1st ordered pair is (6, 0)

y-intercept

Put $x = 0$

$$0 + 2y = 6$$
$$2y = 6$$
$$y = 3$$

2nd ordered pair is (0, 3)

Take $x = y = 0$ (origin test)

$$0 + 2(0) < 6$$
$$0 < 6$$

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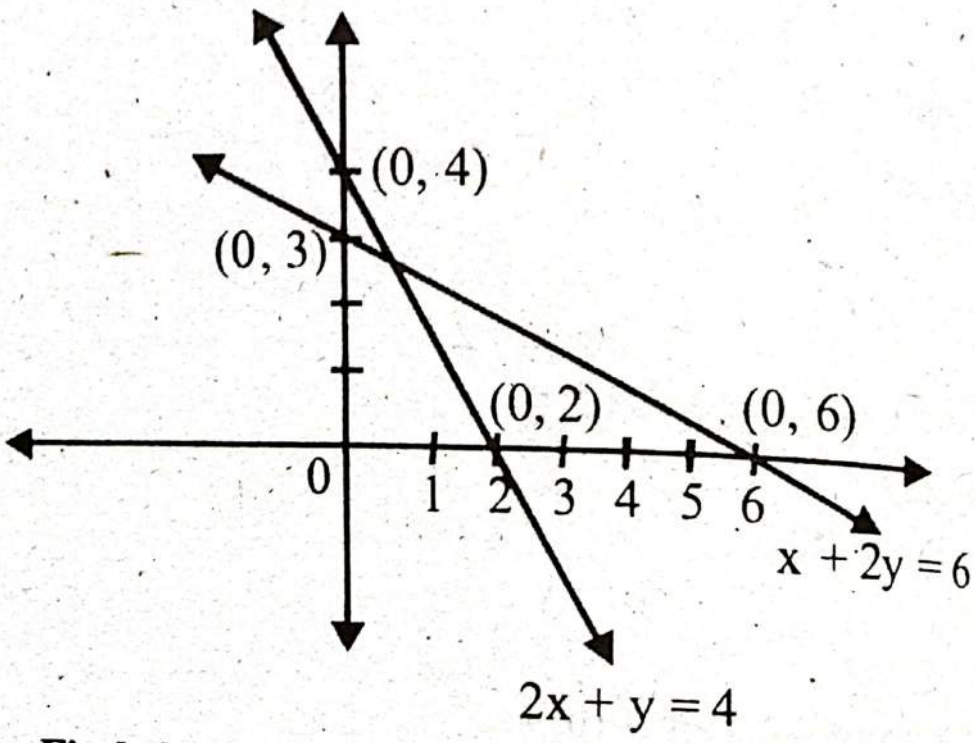
Which is true, so shading lies towards origin side.

Now we draw the lines.



Q.4. Find the maximum value of $g(x, y) = x + 4y$ subject to constraints $x + y \leq 4$, $x \geq 0$ and $y \geq 0$.

Sol. Associated equations are

$$x + y = 4 \qquad \ldots(1)$$

<u>x-intercept</u>

Put $y = 0$

$$x = 4$$

1st ordered pair is $(4, 0)$

<u>y-intercept</u>

Put $x = 0$

$$0 + y = 4$$

$$y = 4$$

2nd ordered pair is $(0, 4)$

Associated equations are $x = 0$, $y = 0$

Corner points are $(0, 0)$, $(4, 0)$, $(0, 4)$

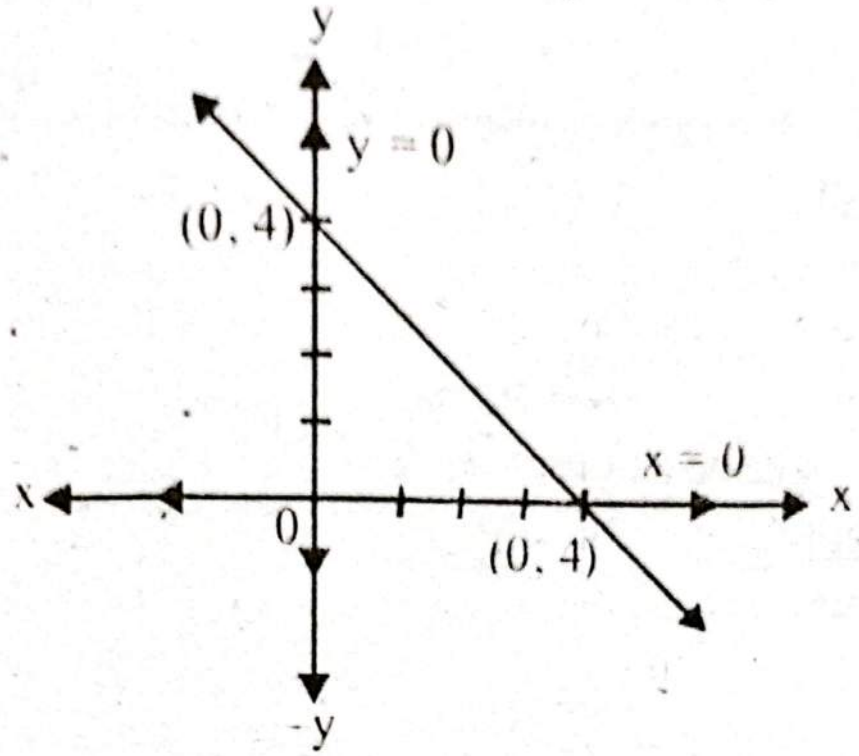
$g(x, y) = x + 4y$

$g(0, 0) = 0 + 4(0) = 0$

$g(4, 0) = 4 + 4(0) = 4$

$g(0, 4) = 0 + 4(4) = 16$

Maximum value is 16 at corner point (4, 0).



Q.5. Find the minimum value of $f(x, y) = 3x + 5y$ subject to constraints $x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$.

Sol. Associated equations are

$$x + 3y = 3 \quad \ldots(1)$$

$$x + y = 2 \quad \ldots(2)$$

x-intercept

Put $y = 0$

$$x + 3(0) = 3$$

$$x = 3$$

1st ordered pair is (3, 0)

y-intercept

Put $x = 0$

$$0 + 3y = 3$$

$$3y = 3$$

$$y = \frac{3}{3}$$

$$y = 1$$

2nd ordered pair is (0, 1)

Take $x = y = 0$ (origin test)

$$0 + 3(0) > 3$$

$$0 > 3$$

Which is false, so shading lies away from origin side.

-intercept

Put $y = 0$

$$x + 0 = 2$$

$$x = 2$$

1st ordered pair is (2, 0)

-intercept

Put $x = 0$

$$0 + y = 2$$

$$y = 2$$

2nd ordered pair is (0, 2)

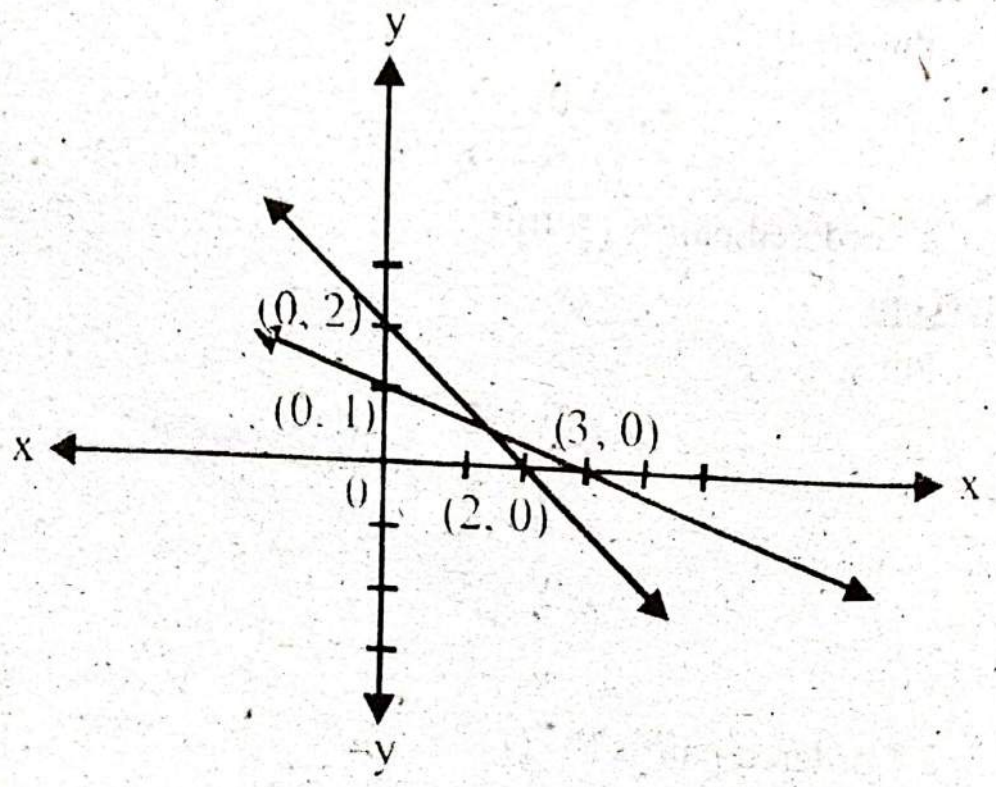
Take $x = y = 0$ (origin test)

$$0 + 0 > 2$$

$$0 > 2$$

Which is false, so shading lies towards the origin side.

Now we draw the lines.



Corner points are (0, 2), A(x, y)(3, 0)

We find corner point A. We solve (1) and (2)

Multiply (1) by (2) and subtract from (2)

$$x + 3y = 3$$
$$\underline{x + \ \ y = 2}$$
$$2y = 1$$
$$y = \frac{1}{2}$$

Put $x = \frac{1}{2}$ in (1) for x.

$$x + 3\left(\frac{1}{2}\right) = 3$$
$$x + \frac{3}{2} = 3$$
$$x = 3 - \frac{3}{2}$$
$$x = \frac{3}{2}$$

Corner point $A\left(\frac{3}{2}, \frac{1}{2}\right)$

Now $f(x, y) = 3x + 5y$

Corner point (0, 2)

$f(0, 2) = 3(0) + 5(2) = 10$

Corner point $\left(\frac{3}{2}, \frac{1}{2}\right)$

$$f\left(\frac{3}{2}, \frac{1}{2}\right) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right)$$
$$= \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$$

Corner point (3, 0)

$f(3, 0) = 3(3) + 5(0)$
$= 9$

Minimum value of function is 7 at corner point $\left(\frac{3}{2}, \frac{1}{2}\right)$

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