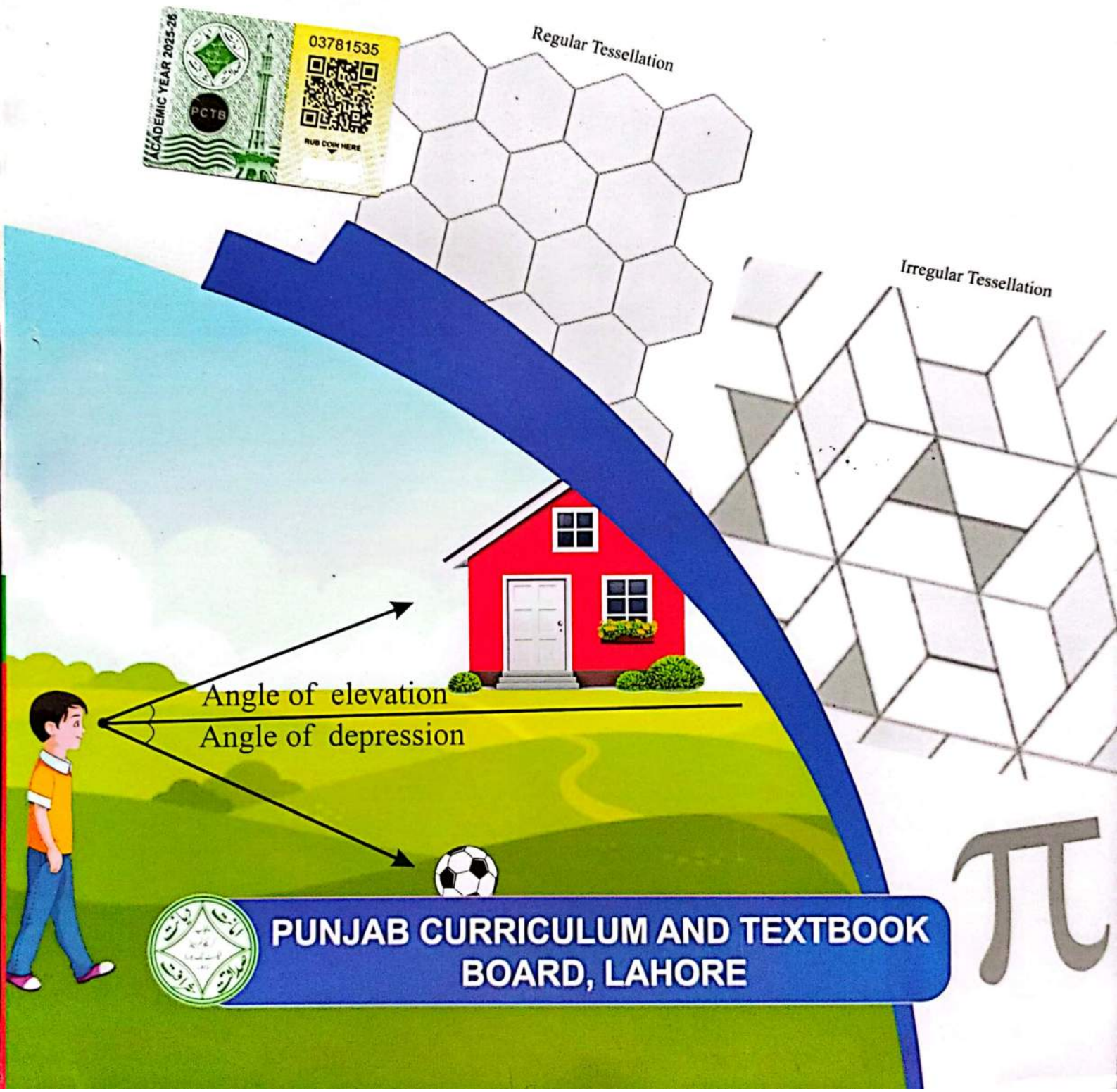
MATHEMATICS 9



ACADEMIC YEAR 2025-26
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PUNJAB CURRICULUM AND TEXTBOOK BOARD, LAHORE

UNIT 1 REAL NUMBER

EXERCISE 1.1

Q.1. Factorize by identifying common factors.

(i) 2.353535

Sol. 2.353535 is a rational number.

(ii) $\mathbf{0.\overline{6}}$

Sol. $0.\overline{6}$ is rational number.

(iii) 2.236067.....

Sol. 2.236067..... is irrational number.

(iv) $\sqrt{7}$

Sol. $\sqrt{7}$ is an irrational number.

(v) e

Sol. e is an irrational number.

(vi) π

Sol. π is an irrational number.

(vii) $5+\sqrt{11}$

Sol. $5+\sqrt{11}$ is an irrational number.

(viii) $\sqrt{3}+\sqrt{13}$

Sol. $\sqrt{3}+\sqrt{13}$ is an irrational number.

(ix) $\frac{15}{4}$

Sol. $\frac{15}{4}$ is a rational number.

(x) $(2-\sqrt{2})(2+\sqrt{2})$

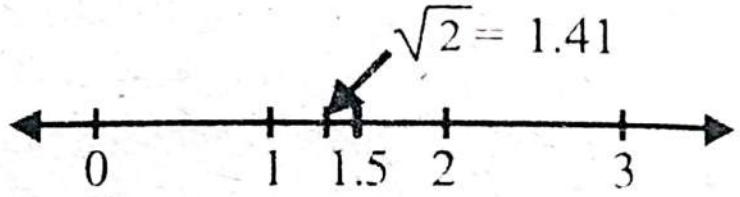
Sol. $(2-\sqrt{2})(2+\sqrt{2}) = 2^2 - (\sqrt{2})^2$

$= 4 - 2 = 2$ is a rational number.

Q.2. Represent the following numbers on number line:

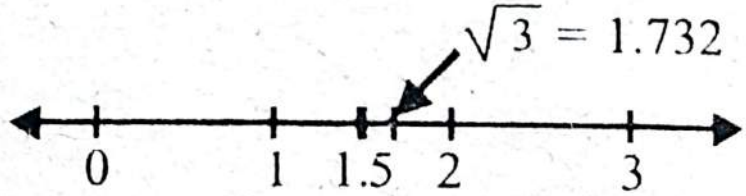
(i) $\sqrt{2}$

Sol.



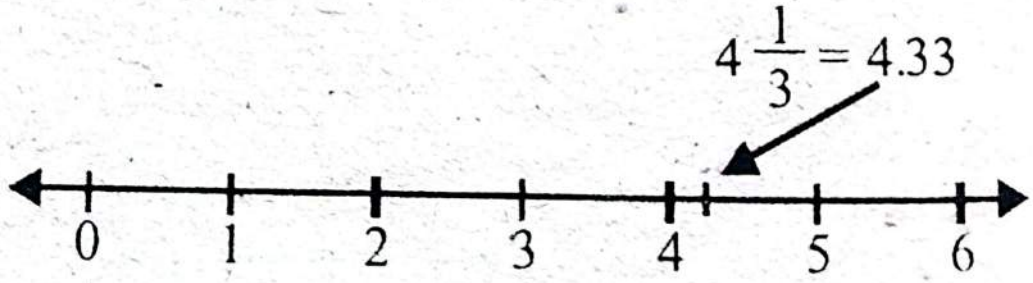
(ii) $\sqrt{3}$

Sol.



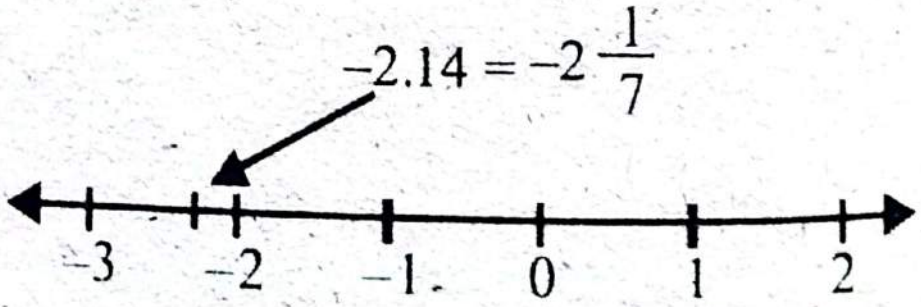
(iii) $4\frac{1}{3}$

Sol. $4\frac{1}{3} = \frac{13}{3} = 4.33$



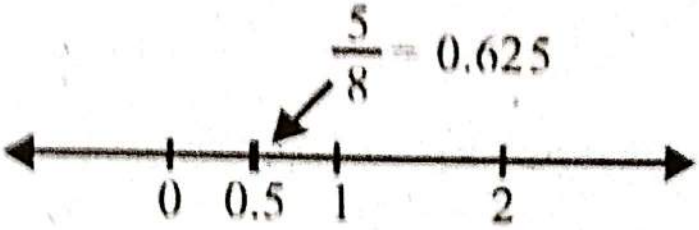
(iv) $-2\frac{1}{7}$

Sol. $-2\frac{1}{7} = -\frac{15}{7} = -2.14$



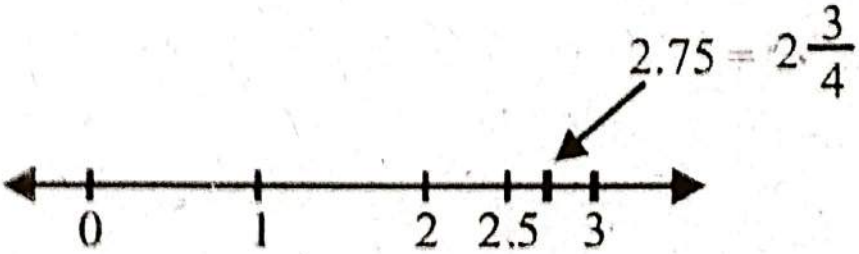
(v) $\frac{5}{8}$

Sol.



(vi) $2\frac{3}{4}$

Sol. $2\frac{3}{4} = 2.75$



Q.3. Express the following recurring decimals as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$

(i) $0.\overline{4}$

Sol. Let $x = 0.\overline{4}$

$x = 0.4444$ ...(i)

Multiply equation (i) by 10

$10x = 10 \times 0.44444...$

$10x = 4.4444...$ ...(ii)

Equation (ii) – Equation (i) $\Rightarrow$

$10x - x = 4.4444... - 0.4444...$

$9x = 4$

$x = \frac{4}{9}$

So, $\overline{4} = \frac{4}{9}$

(ii) $0.\overline{37}$

Sol. Let $x = 0.\overline{37}$

$x = 0.373737\ldots$ ...(i)

Multiply equation (i) by 100

$100x = 100 \times 0.373737\ldots$ ...(ii)

Equation (ii) – Equation (i)

$100x - x = 37.373737\ldots - 0.373737\ldots$

$99x = 37$

$x = \frac{37}{99}$

So, $0.\overline{37} = \frac{37}{99}$

(iii) $0.\overline{21}$

Sol. Let $x = 0.\overline{21}$

$x = 0.212121\ldots$ ...(i)

Multiply equation (i) by 100

$100x = 100 \times 0.212121\ldots$

$100x = 21.2121$ ...(ii)

Equation (ii) – Equation (i)

$100x - x = 21.212121\ldots - 0.212121\ldots$

$99x = 21$

$x = \frac{21}{99}$

So, $0.\overline{21} = \frac{21}{99}$

Q.4. Name the property used in the following:

(i) $(a + 4) + b = a + (4 + b)$

Sol. Associative property w.r.t addition.

(ii) $\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$

Sol. Commutative property w.r.t addition.

(iii) $x - x = 0$

Sol. Additive inverse.

(iv) $a(b + c) = ab + ac$

Sol. Distributive property w.r.t addition

(v) $16 + 0 = 16$

Sol. Additive identity.

(vi) $100 \times 1 = 100$

Sol. Multiplicative property.

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$

Sol. Associative property w.r.t. multiplication

(viii) $ab = ba$

Sol. Commutative property w.r.t multiplication

Q.5. Name the properties used in the following:

(i) $-3 < -1 \Rightarrow 0 < 2$

Sol. Adding 3 on both sides

$-3 + 3 < -1 + 3$

$0 < 2$

(Additive property)

(ii) **If $a < b$ then $\frac{1}{a} > \frac{1}{b}$**

Sol. Reciprocal property

(iii) **If $a < b$ then $a + c < b + c$**

Sol. Additive property

(iv) **If $ac < bc$ and $c > 0$ then $a < b$**

Sol. Cancellation property

(v) **If $ac < bc$ and $c < 0$ then $a > b$**

Sol. Cancellation property

(vi) **Either $a > b$ or $a = b$ or $a < b$**

Sol. Transitive property

Q.6. Insert two rational numbers between

(i) $\frac{1}{3}$ **and** $\frac{1}{4}$

Sol. There are infinite rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$.

We find two of them, 1st rational number between $\frac{1}{3}$ and $\frac{1}{4}$

$$= \frac{\frac{1}{3}+\frac{1}{4}}{2} = \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12 \times 2} = \frac{7}{24}$$

Other rational number between $\frac{7}{24}$ and $\frac{1}{4}$ is

$$= \frac{\frac{1}{4}+\frac{7}{24}}{2} = \frac{\frac{6+7}{24}}{2}$$

$$= \frac{13}{24 \times 2} = \frac{13}{48}$$

(ii) 3 and 7

Sol. One rational number between 3 and 7 is

$$= \frac{3+7}{2} = \frac{10}{2} = 5$$

Other rational number between 5 and 7 is

$$= \frac{5+7}{2} = \frac{12}{2} = 6$$

(iii) $\frac{3}{5}$ **and** $\frac{4}{5}$

Sol. One rational number between $\frac{3}{5}$ and $\frac{4}{5}$ is

$$= \frac{\frac{3}{5}+\frac{4}{5}}{2} = \frac{\frac{3+4}{5}}{2}$$

$$= \frac{7}{2 \times 5} = \frac{7}{10}$$

Other rational number between $\frac{7}{10}$ and $\frac{4}{5}$ is

$$= \frac{\frac{7}{10} + \frac{4}{5}}{2} = \frac{\frac{7+8}{10}}{2}$$

$$= \frac{15}{20} = \frac{3}{4}$$

EXERCISE 1.2

Q.1. Rationalize the denominator of following:

(i) $\frac{13}{4+\sqrt{3}}$

Sol. Multiply and divide by $4 - \sqrt{3}$

$$= \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$$

$$= \frac{13(4-\sqrt{3})}{4^2 - (\sqrt{3})^2}$$

$$= \frac{13(4-\sqrt{3})}{16-3}$$

$$= \frac{13(4-\sqrt{3})}{13}$$

$$= 4 - \sqrt{3}$$

(ii) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}}$

Sol. Multiply and divide by $\sqrt{3}$

$$= \frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}(\sqrt{2}+\sqrt{5})}{(\sqrt{3})^2}$$

$$= \frac{\sqrt{3\times 2}+\sqrt{3\times 5}}{(\sqrt{3})^2}$$

$$= \frac{\sqrt{6}+\sqrt{15}}{3} = \frac{1}{3}(\sqrt{6}+\sqrt{15})$$

(iii) $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

Sol. Multiply and divide by $\sqrt{5}$.

$$= \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}(\sqrt{2}-1)}{(\sqrt{5})^2}$$

$$= \frac{\sqrt{5\times 2}-\sqrt{5}}{5}$$

$$= \frac{\sqrt{10}-\sqrt{5}}{5} = \frac{1}{5}(\sqrt{10}-\sqrt{5})$$

(iv) $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Sol. Multiply and divide by $6-4\sqrt{2}$

$$= \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$

$$= \frac{(6-4\sqrt{2})^2}{6^2-(4\sqrt{2})^2}$$

$$= \frac{6^2+(4\sqrt{2})^2-2(6)(4\sqrt{2})}{36-16\times 2}$$

$$= \frac{68-48\sqrt{2}}{4}$$

$$= \frac{4(17-12\sqrt{2})}{4} = 17-12\sqrt{2}$$

(v) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Sol: Multiply and divide by $\sqrt{3}-\sqrt{2}$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{3}\sqrt{2}}{3-2}$$

$$= \frac{3+2-2\sqrt{6}}{1}$$

$$= 5-2\sqrt{6}$$

(vi) $\dfrac{4\sqrt{2}}{\sqrt{7}+\sqrt{5}}$

Sol. $= \dfrac{4\sqrt{2}}{\sqrt{7}+\sqrt{5}} \times \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$$= \frac{4\sqrt{2}\,(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5}$$

$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2}$$

$$= 2\sqrt{3}\,(\sqrt{7}-\sqrt{5})$$

Q.2. Simplify the following

(i) $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$

Sol. $= \left(\dfrac{3^4}{2^4}\right)^{-\frac{3}{4}}$ $\left(\begin{array}{l}\because\ 81 = 3\times3\times3\times3 \\ \quad\ \ 16 = 2\times2\times2\times2\end{array}\right)$

$$= \frac{3^{4 \times -\frac{3}{4}}}{2^{4 \times -\frac{3}{4}}}$$

$$= \frac{3^{-3}}{2^{-3}} = \frac{2^3}{3^3} = \frac{8}{27}$$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

Sol. $= \frac{3^{-2}}{4^{-2}} \div \frac{4^3}{9^3} \times \frac{16}{27}$

$$= \frac{4^2}{3^2} \times \frac{9^3}{4^3} \times \frac{16}{27}$$

$$= \frac{\overset{1}{\cancel{16}}}{\underset{1}{\cancel{9}}} \times \frac{\overset{3}{\cancel{729}}}{\underset{4}{\cancel{64}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{1}{\cancel{27}}}$$

$$= 3 \times 4 = 12$$

(iii) $(0.027)^{-\frac{1}{3}}$

Sol. $= \left(\frac{27}{100}\right)^{-\frac{1}{3}} = \left(\frac{3^3}{10^3}\right)^{-\frac{1}{3}}$

$$= \frac{3^{3 \times -\frac{1}{3}}}{10^{3 \times -\frac{1}{3}}} = \frac{3^{-1}}{10^{-1}} = \frac{10}{3} \text{ or } 3\frac{1}{3}$$

(iv) $\sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^{7}}}$

Sol. $= \left(\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^{7}}\right)^{1/7} = (x^{14} \times y^{21} \times y^{-14} \times z^{35} \times z^{-7})^{1/7}$

$$= (x^{14} \times y^{21-14} \times z^{35-7})^{1/7}$$

$$= (x^{14} \times y^{7} \times z^{28})^{1/7} = x^{14 \times 1/7} \times y^{7 \times 1/7} \times z^{28 \times 1/7}$$

$$= x^2 y z^4$$

(v) $\dfrac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$

Sol. $= \dfrac{5.(5^2)^{n+1} - 5^2.(5)^{2n}}{5.(5)^{2n+3} - (5^2)^{n+1}} = \dfrac{5.5^{2n+2} - 5^2(5)^{2n}}{5.5^{2n+3} - 5^{2n+2}}$

$= \dfrac{5.5^{2n+2} - 5^2.5^{2n}}{5.5^{2n+3} - 5^{2n+2}} = \dfrac{5.5^{2n+2} - 5^2.5^{2n}}{5.5^{2n}.5^2 - 5^{2n}.5^2}$

$= \dfrac{5.5^{2n+2} - 5^{2n+2}}{5^2.5^{2n+2} - 5^{2n+2}} = \dfrac{5^{2n+2}(5-1)}{5^{2n+2}(5^2-1)}$

$= \dfrac{4}{24} = \dfrac{1}{6}$

(vi) $\dfrac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$

Sol. $= \dfrac{(2^4)^{x+1} + (4+16).(2^2)^{2x}}{2^{x-3} \times (2^3)^{x+2}}$

$= \dfrac{2^{4x+4} + 4.2^{4x} + 16.2^{4x}}{2^{x-3} \times 2^{3x+6}}$

$= \dfrac{2^{4x+4} + 2^2.2^{4x} + 2^4.2^{4x}}{2^{x-3+3x+6}}$

$= \dfrac{2^{4x+2}(2^2 + 1 + 2^2)}{2^{4x+2+1}}$

$= \dfrac{2^{4x+2}(4+1+4)}{2^{4x+2}.2^1} = 2^{\cancel{4x}+2-\cancel{4x}-1}\left(\dfrac{9}{2}\right)$

$= 2^0\left(\dfrac{9}{2}\right) = 1 \times \dfrac{9}{2} = \dfrac{9}{2}$

(vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

Sol. $= (4^3)^{-\frac{2}{3}} \div (3^2)^{-\frac{3}{2}} = 4^{-2} \div 3^{-3}$

$= \dfrac{1}{4^2} \div \dfrac{1}{3^3} = \dfrac{1}{16} \div \dfrac{1}{27}$

$= \dfrac{1}{16} \times \dfrac{27}{1} = \dfrac{27}{16}$ or $1\dfrac{11}{16}$

(viii) $\dfrac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

Sol. $= \dfrac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}} = \dfrac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}}$

$= \dfrac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}} = \dfrac{3^{n+2n+2}}{3^{n-1+2n-2}}$

$= \dfrac{3^{3n+2}}{3^{3n-3}} = 3^{3n+2} \times 3^{-3n+3}$

$= 3^{3n+2-3n+3} = 3^5 = 243$

(ix) $\dfrac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^n \times 5^n}$

Sol. $= \dfrac{5^{n+1+2} - 6.5^{n+1}}{5^n(9 - 2^n)}$

$= \dfrac{5^{n+1}(5^2 - 6)}{5^n(9 - 2^n)} = \dfrac{5^n.5(25 - 6)}{5^n(9 - 2^n)}$

$= \dfrac{5^{n-n}.5 \times 19}{9 - 2^n} = \dfrac{5^0 \times 95}{9 - 2^n}$

$= \dfrac{1 \times 95}{9 - 2^n} = \dfrac{95}{9 - 2^n}$

Q.3. If $x = 3 + \sqrt{8}$ then find the value of

(i) $x + \dfrac{1}{x}$

Sol. $\dfrac{1}{x} = \dfrac{1}{3 + \sqrt{8}}$

$= \dfrac{1}{3 + \sqrt{8}} \times \dfrac{3 - \sqrt{8}}{3 - \sqrt{8}}$

$= \dfrac{3 - \sqrt{8}}{3^2 - (\sqrt{8})^2} = \dfrac{3 - \sqrt{8}}{9 - 8}$

$= \dfrac{3 - \sqrt{8}}{1} = 3 - \sqrt{8}$

Now $x + \dfrac{1}{x} = 3 + \sqrt{8} + 3 - \sqrt{8}$

$x + \dfrac{1}{x} = 6$

(ii) $x - \frac{1}{x}$

Sol. $x - \frac{1}{x} = (3+\sqrt{8}) + (3-\sqrt{8})$

$= 3 + \sqrt{8} - 3 + \sqrt{8}$

$= 2\sqrt{8}$ or $2\sqrt{4 \times 2}$

$= 2 \times 2\sqrt{2} = 4\sqrt{2}$

(iii) $x^2 + \frac{1}{x^2}$

Sol. Squaring $x + \frac{1}{x} = 6$

$$\left(x + \frac{1}{x}\right)^2 = 6^2$$

$$x^2 + \frac{1}{x^2} + 2 = 36$$

$$x^2 + \frac{1}{x^2} = 36 - 2$$

$$x^2 + \frac{1}{x^2} = 34$$

(iv) $x^2 - \frac{1}{x^2}$

Sol. $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= (6)(4\sqrt{2}) = 24\sqrt{2}$

(v) $\left(x^2 + \frac{1}{x^2}\right)^2 = 34^2$

Sol. $x^4 + \frac{1}{x^4} + 2 = 1156$

$$x^4 + \frac{1}{x^4} = 1156 - 2$$

$$x^4 + \frac{1}{x^4} = 1154$$

(vi) $\left(x-\frac{1}{x}\right)=(4\sqrt{2})$

Sol. $\left(x-\frac{1}{x}\right)^2=16\times 2=72$

Q.4. Find the rational numbers p and q such tha

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$$

Sol.

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$$

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}\times\frac{8-3\sqrt{2}}{4-3\sqrt{2}}=p+q\sqrt{2}$$

$$\frac{3^2-24\sqrt{2}-12\sqrt{2}+(3\sqrt{2})^2}{4^2-(3\sqrt{2})^2}=p+q\sqrt{2}$$

$$\frac{32-36\sqrt{2}+9\times 2}{16-18}=p+q\sqrt{2}$$

$$\frac{32-36\sqrt{2}+18}{-2}=p+q\sqrt{2}$$

$$\frac{50-36\sqrt{2}}{-2}=p+q\sqrt{2}$$

$$2\left(\frac{25-18\sqrt{2}}{-2}\right)=p+q\sqrt{2}$$

$$-25+18\sqrt{2}=p+q\sqrt{2}$$

$$\Rightarrow \quad p=-25$$

$$q=18$$

Q.5. Simplify the following:

(i) $$\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$$

Sol. $$=\frac{(5^2)^{\frac{3}{2}}\times(3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}}\times(2^3)^{\frac{4}{3}}}=\frac{5^3\times 3^3}{2^5\times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16} = \frac{3375}{512} = 6\frac{303}{512}$$

(ii) $\dfrac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

Sol. $= \dfrac{9 \times 6 \times [(27)^{2x}]^{1/3}}{9^{x+1} + 6 \times 6 \times 6(3^{2x-1})} = \dfrac{3^2 \times 2 \times 3 \times ((3^3)^{2x})^{1/3}}{(3^2)^{x+1} + (2 \times 3)^3 . 3^{2x-1}}$

$$= \frac{3^3 \times 2 \times 3^{2x}}{3^{2x+2} + 2^3 \times 3^3 . 3^{2x-1}} = \frac{2 \times 3^{2x+3}}{3^{2x+2} + 2^3 . 3^{2x-1+3}}$$

$$= \frac{2 \times 3^{2x+3}}{3^{2x+2} + 2^3 . 3^{2x+2}} = \frac{2 \times 3^{2x+2} . 3^1}{3^{2x+2} + 2^3 . 3^{2x+2}}$$

$$= \frac{3^{2x+2}(2 \times 3)}{3^{2x+2}(1 + 2^3)} = 3^{2x+2-2x-2} \times \frac{6}{1+8}$$

$$= 3^0 \times \frac{6}{9} = 1 \times \frac{2}{3} = \frac{2}{3}$$

(iii) $\sqrt{\dfrac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

Sol. $= \left[\dfrac{(6^3)^{2/3} \times (5^2)^{1/2}}{\left(\frac{4}{100}\right)^{-3/2}}\right]^{1/2} = \left[\dfrac{6^2 \times 5}{\left(\frac{1}{25}\right)^{-3/2}}\right]^{1/2}$

$$= \left[\frac{6^2 \times 5}{(5^{-2})^{-3/2}}\right]^{1/2} = \left[\frac{6^2 \times 5}{5^3}\right]^{1/2}$$

$$= \left(\frac{6^2 \times 5}{5^3}\right)^{1/2} = \left(\frac{6^2}{5^2}\right)^{1/2} = \frac{6^{2\times 1/2}}{5^{2\times 1/2}} = \frac{6}{5}$$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

Sol. **Formula:** $(a+b)(a^2 - ab + b^2) = a^3 + b^3$

$$= \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

$$= (a^{1/3})^3 + (b^{2/3})^3$$

$$= a + b^2$$

EXERCISE 1.3

Q.1. The sum of three consecutive integers is forty-two, find three integers.

Sol. Let the three consecutive integers be $x, x+1, x+2$

Then by given condition,

$$x + x + 1 + x + 2 = 42$$
$$3x + 3 = 42$$
$$3x = 42 - 3$$
$$3x = 39$$
$$x = \frac{39}{3}$$

1st integer $= x = 13$

2nd integer $= 13 + 1 = 14$

3rd integer $= 13 + 2 = 15$

Q.2. The diagram shows right angled ΔABC in which th length of AC is $(\sqrt{3} + \sqrt{5})$ cm. The area of ΔABC $(1 + \sqrt{15})cm^2$, Find the length AB in the for $(a\sqrt{3} + b\sqrt{5})$cm where a and b are integers.

Sol. Length of side AC $= (\sqrt{3} + \sqrt{5})$ cm

Area of triangle ABC $= (1 + \sqrt{15})\ cm^2$

Length of AB = Height of ΔABC

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$$1 + \sqrt{15} = \frac{1}{2} \times (\sqrt{3} + \sqrt{5}) \times \overline{AB}$$

$$2(1 + \sqrt{15}) = (\sqrt{3} + \sqrt{5})\ \overline{AB}$$

$$\overline{AB} = \frac{2(1 + \sqrt{15})}{\sqrt{3} + \sqrt{5}}$$

$$= \frac{2(1+\sqrt{15})}{\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$= \frac{2(\sqrt{3}-\sqrt{5}+\sqrt{45}-\sqrt{75})}{(\sqrt{3})^2-(\sqrt{5})^2}$$

$$= \frac{2(\sqrt{3}-\sqrt{5}+9\times\sqrt{5}-\sqrt{25\times 3})}{(\sqrt{3})^2-(\sqrt{5})^2}$$

$$= \frac{-4\sqrt{3}+2\sqrt{5}}{-1}$$

$$= 4\sqrt{3}-2\sqrt{5}$$

Which is of the form $a\sqrt{3}+b\sqrt{5}$

Q.3. A rectangle has sides of length $(2+\sqrt{18})$m and $\left(5-\frac{4}{\sqrt{2}}\right)$ m. Express the area of the rectangle in the form $a+b\sqrt{2}$ where a and b are integers.

Sol. Length of rectangle $= (2+\sqrt{18})$m

Width of rectangle $= \left(5-\frac{4}{\sqrt{2}}\right)$ m

Area of rectangle $=$ Length $\times$ width

$$= (2+\sqrt{18})\left(5-\frac{4}{\sqrt{2}}\right)$$

$$= 10-\frac{8}{\sqrt{2}}+5\sqrt{18}-\frac{4\sqrt{18}}{\sqrt{2}}$$

$$= 10-\frac{8}{\sqrt{2}}+5\sqrt{9\times 2}-\frac{4\sqrt{2\times 9}}{\sqrt{2}}$$

$$= 10-\frac{8}{\sqrt{2}}+5\times 3\sqrt{2}-\frac{4\sqrt{2}\times 3}{\sqrt{2}}$$

$$= 10-\frac{8}{\sqrt{2}}+15\sqrt{2}-12$$

$$= -2 - \frac{8}{\sqrt{2}} + 15\sqrt{2}$$

$$= -2 + \frac{-8 + 15(\sqrt{2})^2}{\sqrt{2}}$$

$$= -2 + \frac{-8 + 15 \times 2}{\sqrt{2}}$$

$$= -2 + \frac{-8 + 30}{\sqrt{2}}$$

$$= -2 + \frac{22}{\sqrt{2}}$$

$$= \frac{-2\sqrt{2} + 22}{\sqrt{2}}$$

$$= \frac{2(11 - \sqrt{2})}{\sqrt{2}}$$

$$= \sqrt{2}\,(11 - \sqrt{2})$$

$$= (11\sqrt{2} - 2)\ m^2$$

Q.4. Find two numbers whose sum is 68 and whose difference is 22.

Sol. Let two numbers be x and y then by given conditions.

$$x + y = 68 \qquad \ldots(i)$$

$$\& \qquad x - y = 22 \qquad \ldots(ii)$$

Adding (i) and (ii)

$$\begin{aligned} x + y &= 68 \\ \underline{x - y} &\underline{= 22} \\ \underline{2x} &\underline{= 100} \end{aligned}$$

$$x = \frac{100}{2}$$

$$x = 50$$

So other number = 68 – 50 = 18

So two numbers are 18 and 50.

Q.5. **The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperatures as high as 48°C. By using the formula, $\left(F = \frac{9}{5}C + 32\right)$ find the temperature as Fahrenheit scale.**

Sol. Temperature on Celsius scale = C = 48°C

$$F = \frac{9}{5}C + 32$$

Put $C = 48°C$ above

$$F = \frac{9}{5} \times 48 + 32$$

$$= 86.4 + 32$$

$$F = 118.4°F$$

Q.6. **The sum of the ages of the father and son is 72. Six years ago, the father's age was 2 times the age of the son. What was Son's age six years ago?**

Sol. Let son's age = x years

Six years ago father's age = 2x – 6

Sum of ages $= 2x - 6 + x$

$= 72$

$3x - 6 = 72$

$3x = 72 + 6$

$3x = 78$

$x = \frac{78}{3}$

$x = 26$

Son's age 6 years ago = 20 years

Q.7. **Mirha sells a toy for Rs.1520. What will the selling price be to get a 15% profit?**

Sol. Selling price of toy = Rs.1520

Since 15% profit to be earned. So

Selling price $= \frac{115}{100} \times 1520$

$= 23 \times 76$

$=$ Rs.1748

Other Question

Mirha bought a toy for Rs.1500 and sold for Rs.1520. What was her profit percentage?

Sol. C.P = Rs.1500, S.P = Rs.1520

Profit = 1520 – 1500 = Rs.20

Profit percentage $= \frac{20}{1500} \times 100\% = 1.33\%$

Q.8. The annual income of Tayab is Rs.9,60,000, while the exempted amount is Rs.1,30,000. How much tax would he have to pay at the rate of 0.75%.

Sol. Tayyab's annual income = Rs.9,60,000

Exempted amount = Rs.1,30,000

Taxable amount = 9,60,000 – 1,30,000

= Rs.8,30,000

Rate of tax = 0.75%

Tax amount $= 0.75\% \times 8,30,000$

$= \frac{75}{100} \times \frac{1}{100} \times 830000$

$= 75 \times 83$

= Rs.6225

Q.9. Find the compound markup on Rs.3,75,000 for one year at the rate of 14% compounded annually.

Sol. Principal = Rs.3,75,000

Time = T = 1 year

Rate = R = 14%

In compound interest.

$$A = P\left[1 + \frac{R}{100}\right]^n$$

$$= 375000\left[1 + \frac{14}{100}\right]^1$$

$$= 375000\,[1 + 0.14]$$

$$= 375000 \times 1.14$$

$$= \text{Rs.}4,27,500$$

Compound interest $= A - P$

$= 427500 - 375000$

$= \text{Rs.}52,500$

REVIEW EXERCISE 1

Q.1. Choose the correct option.

(i) $\sqrt{7}$ is:

(a) Integer (b) Rational number

(c) Irrational number (d) Natural number

Sol. (c)

(ii) π and e are:

(a) Natural number (b) Integers

(c) Rational number (d) Irrational number

Sol. (d)

(iii) If n is not a perfect square then $\sqrt{n}$ is:

(a) Rational number (b) Natural number

(c) Integer (d) Irrational number

Sol. (d)

(iv) $\sqrt{3} + \sqrt{5}$ is:

(a) Whole number (b) Integer

(c) Rational number (d) Irrational number

Sol. (d)

(v) For all $x \in R$, $x = x$ is called:

(a) Reflexive property (b) Transitive number

(c) Symmetric property (d) Irrational number

Sol. (a)

(vi) Let a, b, c $\in$ R then a > b and b > c $\Rightarrow$ a > c is called _____ property.

(a) Trichotomy (b) Transitive

(c) Additive (d) Trichotomy property

Sol. (b)

(vii) $2^x \times 8^x = 64$ then x =

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$

(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

Sol.

$$2^x \times 8^x = 64$$
$$2^x \times (2^3)^x = 2^6$$
$$2^x \times 2^{3x} = 2^6$$
$$2^{4x} = 2^6$$
$$\Rightarrow \quad 4x = 6$$
$$x = \frac{6}{4}$$
$$x = \frac{3}{2}$$

Option (a) is correct.

(viii) Let a, b, $\in$ R then a = b and b = a is called _____ property.

(a) Reflexive (b) Symmetric

(c) Transitive (d) Additive

Sol. (b)

(ix) $\sqrt{75} + \sqrt{27} =$

(a) $\sqrt{102}$ (b) $9\sqrt{3}$

(c) $5\sqrt{3}$ (d) $8\sqrt{3}$

Sol. $\sqrt{75} + \sqrt{27} = \sqrt{25 \times 3} + \sqrt{9 \times 3}$

$= 5\sqrt{3} + 3\sqrt{3}$

$= (5 + 3)\sqrt{3}$

$= 8\sqrt{3}$

Option (d) is correct.

(x) The product of $(3 + \sqrt{5})(3 - \sqrt{5})$ is:

(a) Prime number (b) odd number

(c) Irrational number (d) Rational number

Sol. $(3 + \sqrt{5})(3 - \sqrt{5}) = 3^2 - (\sqrt{5})^2$

$= 9 - 5$

$= 4$ (Rational number)

Option (d) is correct.

Q.2. If $a = \frac{3}{2}$, $b = \frac{5}{2}$ and $c = \frac{7}{2}$ then verify that

(i) $a(b + c) = ab + ac$

Sol. LHS $= a(b + c)$

$= \frac{3}{2}\left(\frac{5}{2} + \frac{7}{2}\right)$

$= \frac{3}{2}(6)$

$= 9$

RHS $= ab + ac$

$= \frac{3}{2} \cdot \frac{5}{2} + \frac{3}{2} \cdot \frac{7}{2}$

$= \frac{15}{4} + \frac{21}{4} = \frac{15 + 21}{4}$

$= \frac{36}{4} = 9$

So, LHS = RHS

(ii) $(a + b)c = ac + bc$

Sol. LHS $= (a + b)c$

$= \left(\frac{3}{2} + \frac{7}{2}\right)\frac{7}{2}$

$= \frac{8}{2} \times \frac{7}{2}$

$= 14$

RHS $= ac + bc$

$= \left(\frac{3}{2}\right)\left(\frac{7}{2}\right) + \frac{5}{2} \cdot \frac{7}{2}$

$= \frac{21}{4} + \frac{35}{4} = \frac{56}{4}$

$= 14$

So, LHS = RHS

Q.3. If $a = \frac{4}{3}$, $b = \frac{5}{2}$, $c = \frac{7}{4}$

Then, verify the Associative property of real numbers. w.r.t addition and multiplication.

Sol. $a = \frac{4}{3}$, $b = \frac{5}{2}$, $c = \frac{7}{4}$

Associative property of real numbers w.r.t addition:

$a + (b + c) = (a + b) + c$

LHS $= a + (b + c)$	RHS $= (a + b) + c$
$= \frac{4}{3} + \left(\frac{5}{2} + \frac{7}{4}\right)$	$= \left(\frac{4}{3} + \frac{5}{2}\right) + \frac{7}{4}$
$= \frac{4}{3}\left(\frac{10 + 7}{4}\right)$	$= \frac{8 + 15}{6} + \frac{7}{4}$
$= \frac{16 + 51}{4} = \frac{67}{2}$	$= \frac{23}{6} + \frac{7}{4}$
	$= \frac{46 + 21}{12} = \frac{67}{12}$

So, LHS = RHS

Associative property of real numbers w.r.t multiplication.

LHS $= a(bc)$	RHS $= (ab)c$
$= 4(5 \times 7)$	$= (4 \times 5)7$
$= 4 \times 35$	$= 20 \times 7$
$= 140$	$= 140$

So, LHS = RHS

Q.4. Is 0 a rational number? Explain.

Sol. Zero can be written as $\frac{0}{1}, \frac{0}{100}, \frac{0}{200}$ etc. etc.

So, 0 is a rational number.

Q.5. State trichotomy property of real numbers.

Sol. $\forall\ a, b \in R$, either $a = b$ or

$a > b$ or $a < b$ is called trichotomy property.

Q.6. Find two rational numbers between 4 and 5.

Sol. 4 and 5

1st rational number $= \frac{4+5}{2} = \frac{9}{2}$

2nd rational number $= \frac{\frac{9+10}{2}}{2} = \frac{19}{2 \times 2} = \frac{19}{4}$

So two rational numbers between 4 and 5 are $\frac{9}{2}$ and $\frac{19}{4}$

Q.7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$

Sol.

$$\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} = \left(\frac{x^{15}y^{35}}{z^{20}}\right)^{1/5}$$

$$= \frac{x^{15\times1/5} \times y^{35\times1/5}}{z^{20\times1/5}}$$

$$= \frac{x^3y^7}{z^4}$$

(ii) $\sqrt[3]{(27)^{2x}}$

Sol.

$$\sqrt[3]{(27)^{2x}} = [(3^3)^{2x}]^{1/3}$$

$$= (3^{6x})^{1/3} = 3^{6x\times1/3}$$

$$= 3^{2x}$$

(iii) $\frac{6(3)^{n+2}}{3^{n+1} - 3^n}$

Sol.

$$\frac{6(3)^{n+2}}{3^{n+1} - 3^n} = \frac{6.3^n.3^2}{3^n.3^1.3^n} = \frac{6.3^n.9}{3^n(3-1)}$$

$$= \frac{3^n \times 6 \times 9}{3^n \times 2} = 3^{n-n} \times \frac{54}{2}$$

$$= 3^0 \times 27 = 1 \times 27 = 27$$

Q.8. The sum of three consecutive odd integers is 51. Find the three integers.

Sol. Let x, x + 2, x + 4 be three consecutive odd integers.

Then $x + x + 2 + x + 4 = 51$

$$3x + 6 = 51$$
$$3x = 61 - 6$$
$$3x = 45$$
$$X = \frac{45}{3}$$

1st odd number = x = 15

2nd odd number = 15 + 2 = 17

3rd odd number = 15 + 4 = 19

Q.9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls are in each bucket?

Sol. Total number of balls = 96

Let no. of balls in 2nd bucket = x

No. of balls in 1st bucket = x + 28

Equation formed is

$$x + x + 28 = 96$$
$$2x = 96 - 28$$
$$2x = 68$$
$$x = \frac{68}{2}$$
$$x = 34$$

In 1st bucket, no. of balls = 34 + 28 = 62

In 2nd bucket, no. of balls = 96 – 62 = 34

Q.10. Salma invested Rs.3,50,000 in a bank which paid simple profit at a rate $7\frac{1}{4}$ % per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she has at the end of 7 years.

Sol. Principal = P = Rs.3,50,000

Rate of profit = R = $7\frac{1}{4}$ % p.a.

Time = T = 2 years

Simple interest = $\frac{PRT}{100}$

$$= \frac{350000 \times 7\frac{1}{4} \times 2}{100}$$

$$= \frac{350000}{100} \times \frac{29}{4} \times 2$$

$$= 3500 \times \frac{29}{\cancel{4}_{2}} \times \cancel{2}$$

= Rs.25375

Now

Total amount after 2 years = 350000 + 25375

= Rs.375375

Now

P = Rs.375375

T = 5 years

Rate of interest = R = 8% p.a.

Interest = $\frac{PRT}{100}$

$$= \frac{375375 \times \cancel{8}^{2} \times \cancel{5}^{1}}{\cancel{100}_{\cancel{20}_{5}}} = \frac{375375 \times 2}{5}$$

$$= \frac{750750}{5} = \text{Rs.}150150$$

Total amount after 5 years = 375375 + 150150

= Rs.5,25,525