

Unit No. 1

Real Numbers

Review Exercise No. 1

Question No. 1

Four options are given against each statement. Encircle the correct option.

(i). $\sqrt{7}$ is

- (a). integer
- (b). rational number
- (c). *irrational number*
- (d). natural number

(ii). π and e are

- (a). natural numbers
- (b). integers
- (c). rational numbers
- (d). *irrational numbers*

(iii). If n is not a perfect square, then \sqrt{n} is

- (a). rational number
- (b). natural number
- (c). integer
- (d). *irrational number*

(iv). $\sqrt{3} + \sqrt{5}$ is

- (a). whole number
- (b). integer
- (c). rational number
- (d). *irrational number*

(v). For all $x \in \mathbb{R}$, $x = x$ is called

- (a). *reflective property*
- (b). transitive property
- (c). symmetric property
- (d). trichotomy property

(vi). Let $a, b, c \in \mathbb{R}$ then $a > b$ and $b > c \Rightarrow a > c$ is called _____ property.

- (a). trichotomy
- (b). *transitive*

(c). additive

(d). multiplicative

(vii). $2^x \times 8^x = 64$ then $x =$

(a). $\frac{3}{2}$

(b). $\frac{3}{4}$

(c). $\frac{5}{6}$

(d). $\frac{2}{3}$

(viii). Let $a, b \in \mathbb{R}$ then $a=b$ and $b=a$ is called _____ property.

(a). *reflective*

(b). symmetric

(c). transitive

(d). additive

(ix). $\sqrt{75} + \sqrt{27} =$

(a). $\sqrt{102}$

(b). $9\sqrt{3}$

(c). $5\sqrt{3}$

(d). $8\sqrt{3}$

(x). The product of $(3 + \sqrt{5})(3 - \sqrt{5})$ is

(a). prime number

(b). odd number

(c). irrational number

(d). *rational number*

Question No. 2

If $a = \frac{3}{2}$, $b = \frac{5}{3}$ and $c = \frac{7}{5}$, then verify that

(i). $a(b + c) = ab + ac$

(ii). $(a + b)c = ac + bc$

Solving part (i):

Solution:

$$a(b + c) = ab + ac$$

by putting values:

$$\frac{3}{2} \left(\frac{5}{3} + \frac{7}{5} \right) = \frac{3}{2} \left(\frac{5}{3} \right) + \frac{3}{2} \left(\frac{7}{5} \right)$$

$$\frac{3}{2} \left(\frac{25+21}{15} \right) = \frac{15}{6} + \frac{21}{10}$$

$$\frac{3}{2} \left(\frac{46}{15} \right) = \frac{15 \times 5}{6 \times 5} + \frac{21 \times 3}{10 \times 3}$$

$$\frac{1}{1} \left(\frac{23}{5} \right) = \frac{75}{30} + \frac{63}{30}$$

$$\frac{23}{5} = \frac{75+63}{30}$$

$$\frac{23}{5} = \frac{138}{30}$$

$$\frac{23}{5} = \frac{23}{5}$$

Hence proved that

$$a (b + c) = ab + ac$$

(ii). (a + b) c = ac + bc

Solving part (ii):

$$a = \frac{3}{2}, b = \frac{5}{3} \text{ and } c = \frac{7}{5}$$

Solution:

$$(a + b) c = ac + bc$$

By putting values:

$$\left(\frac{3}{2} + \frac{5}{3} \right) \frac{7}{5} = \frac{3}{2} \left(\frac{7}{5} \right) + \frac{5}{3} \left(\frac{7}{5} \right)$$

$$\left(\frac{3 \times 3 + 5 \times 2}{6} \right) \frac{7}{5} = \frac{21}{10} + \frac{1}{3} \left(\frac{7}{1} \right)$$

$$\left(\frac{9+10}{6} \right) \frac{7}{5} = \frac{21}{10} + \frac{7}{3}$$

$$\left(\frac{19}{6} \right) \frac{7}{5} = \frac{21 \times 3}{10 \times 3} + \frac{7 \times 10}{3 \times 10}$$

$$\frac{133}{30} = \frac{63}{30} + \frac{70}{30}$$

$$\frac{133}{30} = \frac{63+70}{30}$$

$$\frac{133}{30} = \frac{133}{30}$$

Hence proved that

$$(a + b) c = ac + bc$$

Question No. 3

If $a = \frac{4}{3}$, $b = \frac{5}{2}$ and $c = \frac{7}{4}$, then verify the associative property of real numbers w.r.t addition and multiplication.

Data:

$$a = \frac{4}{3}$$

$$b = \frac{5}{2}$$

$$c = \frac{7}{4}$$

To find:

associative property of real numbers w.r.t addition:

$$a + (b + c) = (a + b) + c$$

associative property of real numbers w.r.t multiplication:

$$a(bc) = (ab)c$$

Solution:

associative property of real numbers w.r.t addition:

$$a + (b + c) = (a + b) + c$$

By putting values:

$$\frac{4}{3} + \left(\frac{5}{2} + \frac{7}{4}\right) = \left(\frac{4}{3} + \frac{5}{2}\right) + \frac{7}{4}$$

$$\frac{4}{3} + \left(\frac{5 \times 4}{2 \times 4} + \frac{7 \times 2}{4 \times 2}\right) = \left(\frac{4 \times 2}{3 \times 2} + \frac{5 \times 3}{2 \times 3}\right) + \frac{7}{4}$$

$$\frac{4}{3} + \left(\frac{20}{8} + \frac{14}{8}\right) = \left(\frac{8}{6} + \frac{15}{6}\right) + \frac{7}{4}$$

$$\frac{4}{3} + \left(\frac{20+14}{8}\right) = \left(\frac{8+15}{6}\right) + \frac{7}{4}$$

$$\frac{4}{3} + \frac{34}{8} = \frac{23}{6} + \frac{7}{4}$$

$$\frac{4 \times 8}{3 \times 8} + \frac{34 \times 3}{8 \times 3} = \frac{23 \times 4}{6 \times 4} + \frac{7 \times 6}{4 \times 6}$$

$$\frac{32}{24} + \frac{102}{24} = \frac{92}{24} + \frac{42}{24}$$

$$\frac{32+102}{24} = \frac{92+42}{24}$$

$$\frac{134}{24} = \frac{134}{24}$$

$$\frac{67}{12} = \frac{67}{12}$$

Hence proved that:

$$a + (b + c) = (a + b) + c$$

Solution:

associative property of real numbers w.r.t multiplication:

$$a(bc) = (ab)c$$

By putting values:

$$\frac{4}{3} \times \left(\frac{5}{2} \times \frac{7}{4}\right) = \left(\frac{4}{3} \times \frac{5}{2}\right) \times \frac{7}{4}$$

$$\frac{4}{3} \times \frac{35}{8} = \frac{20}{6} \times \frac{7}{4}$$

$$\frac{140}{24} = \frac{140}{24}$$

$$\frac{35}{6} = \frac{35}{6}$$

Hence proved that:

$$a(bc) = (ab)c$$

Question No. 4

Is 0 a rational number? Explain.

Answer:

Yes, Zero can be written in the form of

$$\frac{0}{1}, \frac{0}{2}, \frac{0}{3} \dots\dots \text{so } 0 \text{ is a rational number.}$$

Question No. 5

State trichotomy property of real numbers.

Answer:

For all $a, b \in \mathbb{R}$, either $a = b$ or $a > b$ or $a < b$ is called trichotomy property.

Question No. 6

Find two rational numbers between 4 and 5.

Finding 1st rational number:

$$\begin{aligned} (4 + 5) \div 2 \\ = \frac{9}{2} \end{aligned}$$

Finding 2nd rational number:

$$\begin{aligned} (5 + \frac{9}{2}) \div 2 \\ = (\frac{10 + 9}{2}) \div 2 \\ = \frac{19}{2 \times 2} \\ = \frac{19}{4} \end{aligned}$$

Question No. 7

Simplify the followings:

$$(i). \sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$$

Solution:

$$\begin{aligned} \sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} \\ = \frac{x^{\frac{15}{5}}y^{\frac{35}{5}}}{z^{\frac{20}{5}}} \\ = \frac{x^3y^7}{z^4} \end{aligned}$$

(ii). $\sqrt[3]{(27)^{2x}}$

Solution:

$$\begin{aligned}\sqrt[3]{(27)^{2x}} \\ &= \sqrt[3]{(3^3)^{2x}} \\ &= (3^3)^{\frac{2x}{3}} \\ &= 3^{2x}\end{aligned}$$

(iii). $\frac{6(3)^{n+2}}{3^{n+1} - 3^n}$

Solution:

$$\begin{aligned}\frac{6(3)^{n+2}}{3^{n+1} - 3^n} \\ &= \frac{3^n \cdot 6 \cdot 3^2}{3^n(3^1 - 1)} \\ &= \frac{6 \cdot 9}{3 - 1} \\ &= \frac{54}{2} \\ &= 27\end{aligned}$$

Question No. 8

The sum of three consecutive odd integers is 51. Find the three integers.

Data:

sum of three consecutive odd integers = 51

Let

1st odd integer = x

2nd odd integer = x + 2

3rd odd integer = x + 4

Solution:

Given Condition:

sum of three consecutive odd integers = 51

$$x + x + 2 + x + 4 = 51$$

$$3x + 6 = 51$$

$$3x = 51 - 6$$

$$3x = 45$$

$$x = \frac{45}{3}$$

$$x = 15$$

$$1^{\text{st}} \text{ odd integer} = x = 15$$

$$2^{\text{nd}} \text{ odd integer} = x + 2$$

$$= 15 + 2$$

$$= 17$$

$$3^{\text{rd}} \text{ odd integer} = x + 4$$

$$= 15 + 24$$

$$= 19$$

Question No. 9

Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

Data:

$$\text{Total balls} = 96$$

$$\text{Balls in 1}^{\text{st}} \text{ basket} = x$$

$$\text{Balls in 2}^{\text{nd}} \text{ basket} = x + 28$$

Solution:

$$\text{Total balls} = 96$$

$$x + x + 28 = 96$$

$$2x = 96 - 28$$

$$2x = 68$$

$$x = \frac{68}{2}$$

$$= 34$$

$$\text{Balls in 1}^{\text{st}} \text{ basket} = x = 34 \text{ balls}$$

$$\text{Balls in 2}^{\text{nd}} \text{ basket} = x + 28$$

$$= 34 + 28$$

$$= 62 \text{ balls}$$

Question No. 10

Salma invested Rs. 3,50,000 in a bank, which paid simple profit at the rate of $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years.

Data:

$$\text{Principal} = \text{Rs. } 3,50,000$$

$$\text{Rate} = R_1 = 7\frac{1}{4}\% = 7.25\%$$

$$\text{Rate} = R_2 = 8\%$$

$$\text{Time} = T_1 = 2 \text{ years}$$

$$\text{Time} = T_2 = 7 - 2 = 5 \text{ years}$$

To Find:**Total amount after 7 years = ?****Solution:****For 1st 2 years:**

$$\text{Simple Interest} = \frac{PRT}{100}$$

$$\text{Simple Interest} = \frac{(350000)(7.25)(2)}{100}$$

$$\text{Simple Interest} = \frac{(3500)(14.5)}{1}$$

$$\text{Simple Interest} = \text{Rs. } 50750$$

For next 5 years:

$$P = \text{Rs. } 350000$$

$$R = 8\%$$

$$T = 5 \text{ years}$$

For next 5 years:

$$\text{Simple Interest} = \frac{PRT}{100}$$

$$\text{Simple Interest} = \frac{(350000)(8)(5)}{100}$$

$$\text{Simple Interest} = \frac{(350000)(40)}{100}$$

$$\text{Simple Interest} = \frac{14000000}{100}$$

$$\text{Simple Interest} = \text{Rs. } 140000$$

Total:

$$\text{Total amount after 7 years} = 350000 + 50750 + 140000$$

$$\text{Total amount after 7 years} = \text{Rs. } 540750$$