Unit No. 4

Factorization and Algebraic Manipulation

Exercise No. 4.4

Question No. 1

Find the square root of the following polynomials by factorization method:

(i) $x^2 - 8x + 16$

Solution:

$x^2 - 8x + 16$

Factors of $x^2 - 8x + 16 = x^2 - 4x - 4x + 16$

$= x(x - 4) - 4(x - 4)$

$= (x - 4)(x - 4)$

$= (x - 4)^2$

Taking square root:

$= \pm (x - 4)$

(ii) $9x^2 + 12x + 4$

Solution:

$9x^2 + 12x + 4$

Factors of $9x^2 + 12x + 4 = 9x^2 + 6x + 6x + 4$

$= 9x^2 + 6x + 6x + 4$

$= 3x(3x + 2) + 2(3x + 2)$

$= (3x + 2)(3x + 2)$

$= (3x + 2)^2$

Taking square root:

$= \pm (3x + 2)$

(iii) $36a^2 + 84a + 49$

Solution:

$36a^2 + 84a + 49$

Factors of $36a^2 + 84a + 49 = (6a)^2 + 2(6a)(7) + (7)^2$

$= (6a + 7)^2$

Taking square root:

$= \pm (6a + 7)$

(iv) $64y^2 - 32y + 46$

Solution:

$64y^2 - 32y + 4$

Factors of $64y^2 - 32y + 4$

$= (8y)^2 - 2(8y)(2) + (2)^2$

$= (8y - 2)^2$

Taking square root:

$= \pm (8y - 2)$

(v) $200t^2 - 120\,t + 18$

Solution:

$200t^2 - 120\,t + 18$

Factors of $200t^2 - 120\,t + 18 = 2(100t^2 - 60t + 9)$

$= 2[(10t)^2 - 2(10t)(3) + (3)^2]$

$= 2\,(10t - 3)^2$

Taking square root:

$= \pm \sqrt{2}\,(10t - 3)$

vi) $40x^2 + 120x + 90$

Solution:

$40x^2 + 120x + 90$

Factors of $40x^2 + 120x + 90 = 10(4x^2 + 12x + 9)$

$= 10[(2x)^2 + 2(2x)(3) + (3)^2]$

$= 10(2x + 3)^2$

Taking square root:

$= \pm \sqrt{10}\,(2x + 3)$

Question No. 2

Find the square root of the following polynomials by division method:

(i) **$4x^4 - 28x^3 + 37x^2 + 42x + 9$**

Solution:

$2x^2 - 7x - 3$

	$4x^4 - 28x^3 + 37x^2 + 42x + 9$
$2x^2$	$\mp 4x^4$
	$-28x^3 + 37x^2$
$4x^2 - 7x$	$\mp 28x^3 \pm 49x^2$
	$-12x^2 + 42x + 9$
$4x^2 - 14x - 3$	$\mp 12x^2 \pm 42x \pm 9$
	0

$= \pm (2x^2 - 7x - 3)$

(ii) $\mathbf{121x^4 - 198x^3 - 183x^2 + 216x + 144}$

Solution:

	$11x^2 - 9x - 12$
	$121x^4 - 198x^3 - 183x^2 + 216x + 144$
$11x^2$	$\pm 121x^4$
	$-198x^3 - 183x^2$
$22x^2 - 9x$	$\mp 198x^3 \pm 81x^2$
	$-264x^2 + 216x + 144$
$22x^2 - 18x - 12$	$\mp 264x^2 \pm 216x \pm 144$
	0

$= \pm (11x^2 - 9x - 12)$

(iii) $\mathbf{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4}$

Solution:

	$x^2 - 5xy + y^2$
	$x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$
x^2	$\pm x^4$
	$-10x^3y + 27x^2y^2$
$2x^2 - 5xy$	$\mp 10x^3y \pm 25x^2y^2$
	$2x^2y^2 - 10xy^3 + y^4$
$2x^2 - 10xy + y^2$	$\pm 2x^2y^2 \mp 10xy^3 \pm y^4$
	0

$= \pm (x^2 - 5xy + y^2)$

(iii) $\mathbf{4x^4 - 12x^3 + 37x^2 - 42x + 49}$

Solution:

	$2x^2 - 3x + 7$
	$4x^4 - 12x^3 + 37x^2 - 42x + 49$
$2x^2$	$\pm 4x^4$
	$-12x^3 + 37x^2$
$4x^2 - 3x$	$\mp 12x^3 \pm 9x^2$
	$28x^2 - 42x + 49$
$4x^2 - 6x + 7$	$\pm 28x^2 \mp 42x \pm 49$
	0

$= \pm (2x^2 - 3x + 7)$

Question No. 3

An investor's return R(x) in rupees after investing x thousand rupees is given by the quadratic expression:

$R(x) = -x^2 + 6x - 8$

Factorize the expression and find the investment levels that result in zero return.

Solution:

$R(x) = -x^2 + 6x - 8$

$= -x^2 + 2x + 4x - 8$

$= -x(x - 2) + 4(x - 2)$

$= (x - 2)(-x + 4)$

Put $R(x) = 0$:

$(x - 2)(-x + 4) = 0$

$x - 2 = 0$ or $-x + 4 = 0$

$x = 2$ or $x = 4$

Question No. 4

A company's profit P(x) in rupees from selling x units of a product is modeled by the cubic expression:

$P(x) = x^3 - 15x^2 + 75x - 125$

Find the break-even point(s), where the profit is zero.

Solution:

$P(x) = x^3 - 15x^2 + 75x - 125$

$P(x) = (x)^3 - 3(x)^2(5) + 3(x)(5)^2 - (5)^3$

$P(x) = (x - 5)^3$

$P(x) = (x - 5)(x - 5)(x - 5)$

$x - 5 = 0$

$x = 5$

At $x = 5$ Profit is zero.

Question No. 5

The potential energy V(x) in an electric field varies as a cubic function of distance x, given by:

$V(x) = 2x^3 - 6x^2 + 4x$

Determine where the potential energy is zero.

Solution:

$V(x) = 2x^3 - 6x^2 + 4x$

$V(x) = 2x(x^2 - 3x + 2)$

$V(x) = 2x(x^2 - x - 2x + 2)$

$V(x) = 2x[x(x - 1) - 2(x - 1)]$

$V(x) = 2x(x - 1)(x - 2)$

$2x = 0$ or $x - 1 = 0$ or $x - 2 = 0$

$x = 0$ or $x = 1$ or $x = 2$

Question No. 6

In structural engineering, the deflection Y(x) of a beam is given by:

$Y(x) = 2x^2 - 8x + 6$

This equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection.

Solution:

$Y(x) = 2x^2 - 8x + 6$

$Y(x) = 2(x^2 - 4x + 3)$

$Y(x) = 2(x^2 - 3x - x + 3)$

$Y(x) = 2[x(x - 3) - 1(x - 3)]$

$Y(x) = 2(x - 1)(x - 3)$

Points of zero deflection:

$x - 1 = 0$ or $x - 3 = 0$

$x = 1$ or $x = 3$