Unit No. 1

Real Numbers

Exercise No. 1.3

Question No. 1

The sum of three consecutive integers is forty-two, find the three intgers.

Data:

Sum = 42

Let 1st integer = x

2nd integer = x + 1

3rd integer = x + 2

Solution:

Given Condition:

Sum of three integers = 42

$$x + x + 1 + x + 2 = 42$$

$$3x + 3 = 42$$
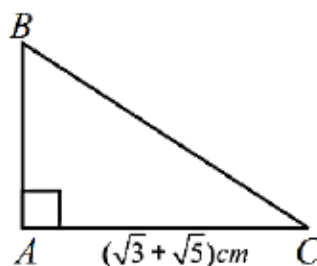
$$3x = 42 - 3$$

$$3x = 39$$

$$x = \frac{39}{3}$$

1st integer = $x = 13$

2nd integer = $x + 1 = 13 + 1 = 14$

3rd integer = $x + 2 = 13 + 2 = 15$

Question No. 2

The diagram shows the right angles $\triangle ABC$ in which the length of $\overline{AC}$ is $(\sqrt{3} + \sqrt{5})cm$. The area of $\triangle ABC$ is $(1 + \sqrt{15})cm^2$. Find the length of $\overline{AB}$ in the form $(a\sqrt{3} + b\sqrt{5})cm$, where a and b are integers.

Data:

Length of $\overline{AC} = (\sqrt{3} + \sqrt{5})cm$

Area of ΔABC $= \left(1 + \sqrt{15}\right) cm^2$

To Find:

Length of $\overline{AB}$ in the form $\left(a\sqrt{3} + b\sqrt{5}\right) cm = ?$

Solution:

Formula:

Area of $\Delta ABC = \frac{1}{2}$ length of $\overline{AC} \times$ length of $\overline{AB}$

$\left(1 + \sqrt{15}\right) = \frac{1}{2}\left(\sqrt{3} + \sqrt{5}\right) \times$ length of $\overline{AB}$

$\frac{\left(1 + \sqrt{15}\right)}{\left(\sqrt{3} + \sqrt{5}\right)} = \frac{1}{2}$ length of $\overline{AB}$

By rationalizing:

$\frac{1}{2}$ length of $\overline{AB} = \frac{\left(1 + \sqrt{15}\right)}{\left(\sqrt{3} + \sqrt{5}\right)} \times \frac{\left(\sqrt{3} - \sqrt{5}\right)}{\left(\sqrt{3} - \sqrt{5}\right)}$

$\frac{1}{2}$ length of $\overline{AB} = \frac{\left(1 + \sqrt{15}\right)\left(\sqrt{3} - \sqrt{5}\right)}{\left(\sqrt{3}\right)^2 - \left(\sqrt{5}\right)^2}$

$\frac{1}{2}$ length of $\overline{AB} = \frac{1\left(\sqrt{3} - \sqrt{5}\right) + \sqrt{15}\left(\sqrt{3} - \sqrt{5}\right)}{3 - 5}$

length of $\overline{AB} = 2 \times \frac{\sqrt{3} - \sqrt{5} + \sqrt{45} - \sqrt{75}}{-2}$

length of $\overline{AB} = \frac{\sqrt{3} - \sqrt{5} + \sqrt{(3)^2 \times 5} - \sqrt{(5)^2 \times 3}}{-1}$

length of $\overline{AB} = \frac{\sqrt{3} - \sqrt{5} + 3\sqrt{5} - 5\sqrt{3}}{-1}$

length of $\overline{AB} = -\left(-4\sqrt{3} + 2\sqrt{5}\right)$

length of $\overline{AB} = 4\sqrt{3} - 2\sqrt{5}$ cm

Question No. 3

A rectangle has sides of length $2 + \sqrt{18}\ m$ and $\left(5 - \frac{4}{\sqrt{2}}\right) m$. Express the area of rectangle in the form of $a + b\sqrt{2}$, where a and b are integers.

Data:

Length of side One $= 2 + \sqrt{18}\ m$

Length of side two $= \left(5 - \frac{4}{\sqrt{2}}\right) m$

To find:

Area of rectangle in the form of $a + b\sqrt{2} = ?$

Solution:

Formula:

Area of rectangle = Length of 1ˢᵗ side × Length of 2ⁿᵈ side

Area of rectangle $= \left(2 + \sqrt{18}\right) \times \left(5 - \frac{4}{\sqrt{2}}\right)$

Area of rectangle = $(2 + \sqrt{18}) \times \left(5 - [\frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}]\right)$

Area of rectangle = $(2 + \sqrt{18}) \times \left(5 - \frac{4\sqrt{2}}{(\sqrt{2})^2}\right)$

Area of rectangle = $(2 + \sqrt{18}) \times \left(5 - \frac{4\sqrt{2}}{2}\right)$

Area of rectangle = $(2 + \sqrt{18}) \times (5 - 2\sqrt{2})$

Area of rectangle = $2 \times (5 - 2\sqrt{2}) + \sqrt{18} \times (5 - 2\sqrt{2})$

Area of rectangle = $10 - 4\sqrt{2} + 5\sqrt{18} - 2\sqrt{36}$

Area of rectangle = $10 - 4\sqrt{2} + 5\sqrt{3 \times 3 \times 2} - 2\sqrt{6 \times 6}$

Area of rectangle = $10 - 4\sqrt{2} + 5\sqrt{(3)^2 \times 2} - 2\sqrt{(6)^2}$

Area of rectangle = $10 - 4\sqrt{2} + 3 \times 5\sqrt{2} - 6 \times 2$

Area of rectangle = $10 - 4\sqrt{2} + 15\sqrt{2} - 12$

Area of rectangle = $-2 + 11\sqrt{2}$

Area of rectangle = $(11\sqrt{2} - 2)\ m^2$

Question No. 4

Find two numbers whose sum is 68 and difference is 22.

Data:

Sum = 68

Difference = 22

To Find:

1ˢᵗ number = x = ?

2ⁿᵈ number = y = ?

Solution:

According to first condition of question:

Sum = 68

x + y = 68   …… (eq. i)

According to second condition of question:

difference = 22

x - y = 22   …… (eq. ii)

by adding both equations:

x + y = 68

x - y = 22
_____

2x = 90

x = $\frac{90}{2}$

x = 45

put the value of x in equation (i):

45 + y = 68

y = 68 – 45

y = 23

So,

1ˢᵗ number = 45 & 2ⁿᵈ number = 23

Question No. 5

The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperature as high as 48°C. By using the formula, ($°F = \frac{9}{5} °C + 32$) find the temperature as Fahrenheit scale.

Data:

°C = 48°C

$°F = \frac{9}{5} °C + 32$

To find:

?

Solution:

° + 32

By putting value:

$°F = \frac{9}{5} (48) + 32$

°F = 1.8 (48) + 32

°F = 86.4 + 32

°F = 118.4°F

Question No. 6

The sum of the ages of the father and son is 72 years. Six years ago, the father`s age was 2 times the age of the son. What was son`s age six years ago?

Given:

sum of the ages = 72 years

Six years ago:

father`s age = 2 times the age of the son

To find:

The age of the son = x = ?

Solution:

Six years ago father`s age = 2x – 6

Sum of ages = 2x – 6 + x

Sum of ages = 3x – 6

Given condition:

sum of the ages = 72 years

3x – 6 = 72

3x = 72 + 6

3x = 78

$x = \frac{78}{3}$

Son`s age = x = 26 years

Six years ago son`s age = 26 – 6 = 20 years

Question No. 7

Mirha bought a toy for Rs. 1500 and sold for Rs. 1520. What was her profit percentage?

Data:

Cost price = CP = Rs. 1500

Selling price = SP = Rs. 1520

To find:

Profit %age = ?

Solution:

Profit = SP – CP

Profit = 1520 – 1500

Profit = Rs. 20

Profit %age $= \frac{profit}{CP} \times 100\%$

Profit %age $= \frac{20}{1500} \times 100\%$

Profit %age = 1.33%

Question No. 8

The annual income of Tayyab is Rs. 9,60,000, while the exempted amount is Rs. 1,30,000. How much tax would he have to pay at the rate of 0.75%?

Data:

Annual income = Rs. 9,60,000

exempted amount = Rs. 1,30,000

Tax rate = 0.75%

To find:

Tax amount = ?

Solution:

Taxable amount = Rs. 9,60,000 - Rs. 1,30,000

Taxable amount = Rs. 8,30,000

Formula:

Tax amount = $\frac{rate}{100}$ × Taxable amount

Tax amount = $\frac{0.75}{100}$ × 8,30,000

Tax amount = Rs. 6225

Question No. 9

Find the compound mark up on Rs. 3,75,000 for one year at the rate of 14% compounded annually.

Data:

Principal = P = Rs. 3,75,000

Time = T = 1 year

Rate = R = 14%

To find:

Compound Interest = ?

Solution:

Formula:

$$A = P \times [1 + \frac{R}{100}]^T$$

By putting values:

$$A = 375000 \times [1 + \frac{14}{100}]^1$$

$$A = 375000 \times [1 + 0.14]^1$$

A = 375000 × 1.14

A = Rs. 427500

Compound Interest = A – P

Compound Interest = 427500 – 375000

Compound Interest = Rs. 52,500