

Unit No. 3

Set and Functions

Exercise No. 3.1

Question No. 1

Write the following sets in set builder notation:

(i). $\{1,4,9,16,25,36,\dots,484\}$

Solution:

$$\{1,4,9,16,25,36,\dots,484\}$$

Writing set in set builder notation:

$$= \{x|x = n^2, n \in \mathbb{N} \wedge 1 \leq x \leq 500\}$$

(ii). $\{2,4,8,16,\dots,256\}$

Solution:

$$\{2,4,8,16,\dots,256\}$$

Writing set in set builder notation:

$$= \{x|x = 2^n, n \in \mathbb{N} \wedge 1 \leq x \leq 256\}$$

(iii). $\{0,\pm 1,\pm 2,\dots,\pm 1000\}$

Solution:

$$\{0,\pm 1,\pm 2,\dots,\pm 1000\}$$

Writing set in set builder notation:

$$= \{x|x \in \mathbb{Z} \wedge -1000 \leq x \leq 1000\}$$

(iv). $\{6,12,18,\dots,120\}$

Solution:

$$\{6,12,18,\dots,120\}$$

Writing set in set builder notation:

$$= \{x|x = 6n, n \in \mathbb{N} \wedge 1 \leq n \leq 20\}$$

(v). $\{100,102,104,\dots,400\}$

Solution:

$$\{100,102,104,\dots,400\}$$

Writing set in set builder notation:

$$= \{x|x = 100 + 2n, n \in \mathbb{W} \wedge 0 \leq n \leq 150\}$$

(vi). $\{1,3,9,27,81,\dots\}$

Solution:

$$\{1,3,9,27,81,\dots\}$$

Writing set in set builder notation:

$$= \{ x | x = 3^n, n \in \mathbb{W} \}$$

(vii). $\{1,2,4,5,10,20,25,50,100\}$

Solution:

$$\{1,2,4,5,10,20,25,50,100\}$$

Writing set in set builder notation:

$$= \{ x | x \text{ is a divisor of } 100 \}$$

(viii). $\{5,10,15,\dots,100\}$

Solution:

$$\{5,10,15,\dots,100\}$$

Writing set in set builder notation:

$$= \{ x | x = 5n, n \in \mathbb{N} \wedge 1 \leq n \leq 20 \}$$

(ix). The set of all integers between -100 and 1000

Solution:

The set of all integers between -100 and 1000

Writing set in set builder notation:

$$= \{ x | x \in \mathbb{Z} \wedge -100 < x \leq 1000 \}$$

Question No. 2

Write each of the following sets in tabular forms:

(i). $\{x | x \text{ is a multiple of } 3 \wedge x \leq 35\}$

Solution:

$$\{x | x \text{ is a multiple of } 3 \wedge x \leq 35\}$$

Writing set in tabular form:

$$= \{3,6,9,\dots,33\}$$

(ii). $\{x | x \in \mathbb{R} \wedge 2x + 1 = 0\}$

Solution:

$$\{x | x \in \mathbb{R} \wedge 2x + 1 = 0\}$$

Writing set in tabular form:

$$= \left\{-\frac{1}{2}\right\}$$

(iii). $\{x|x \in P \wedge x < 12\}$

Solution:

$$\{x|x \in P \wedge x < 12\}$$

Writing set in tabular form:

$$= \{2,3,5,7,11\}$$

(iv). $\{x|x \text{ is a divisor of } 128\}$

Solution:

$$\{x|x \text{ is a divisor of } 128\}$$

Writing set in tabular form:

$$= \{1,2,4,8,16,32,64,128\}$$

(v). $\{x|x = 2^n, n \in N \wedge n < 8\}$

Writing set in tabular form:

$$= \{2,4,8,16,32,64,128\}$$

Solution:

$$\{x|x = 2^n, n \in N \wedge n < 8\}$$

Writing set in tabular form:

$$= \{2,4,8,16,32,64,128\}$$

(vi). $\{x|x \in N \wedge x + 4 = 0\}$

Solution:

$$\{x|x \in N \wedge x + 4 = 0\}$$

Writing set in tabular form:

$$= \Phi \text{ or } \{ \}$$

(vii). $\{x|x \in N \wedge x = x\}$

Solution:

$$\{x|x \in N \wedge x = x\}$$

Writing set in tabular form:

$$= \{1,2,3,4,5,\dots\}$$

(viii). $\{x|x \in Z \wedge 3x + 1 = 0\}$

Solution:

$$\{x|x \in Z \wedge 3x + 1 = 0\}$$

Writing set in tabular form:

$$= \{ \} \text{ or } \Phi$$

Question No. 3

Write two proper subsets of each of the following sets.

(i). $\{a,b,c\}$

Solution:

$$\{a,b,c\}$$

Two proper subsets of the given set:

$$= \Phi, \{a\}$$

(ii). $\{0,1\}$

Solution:

$$\{0,1\}$$

Two proper subsets of the given set:

$$= \Phi, \{0\}$$

(iii). \mathbb{N}

Solution:

$$\mathbb{N}$$

Two proper subsets of the given set:

$$= \Phi, \{1\}$$

(iv). \mathbb{Z}

Solution:

$$\mathbb{Z}$$

Two proper subsets of the given set:

$$= \Phi, \{-1\}$$

(v). \mathbb{Q}

Solution:

$$\mathbb{Q}$$

Two proper subsets of the given set:

$$= \Phi, \left\{-\frac{1}{2}\right\}$$

(vi). \mathbb{R}

Solution:

$$\mathbb{R}$$

Two proper subsets of the given set:

$$= \Phi, \{0\}$$

(vii). $\{x|x \in \mathbb{Q} \wedge 0 < x \leq 2\}$

Solution:

$$\{x|x \in \mathbb{Q} \wedge 0 < x \leq 2\}$$

Two proper subsets of the given set:

$$= \Phi, \left\{\frac{3}{2}\right\}$$

Question No. 4

Is there any set which has no proper subset? If so, name that set.

Answer:

Yes, there is a set which has no proper subset is Empty Set, $[\Phi \text{ or } \{\}]$.

Question No. 5

What is difference between $\{a,b\}$ and $\{\{a,b\}\}$?

Answer:

$\{a,b\}$ is a set containing two elements a and b while $\{\{a,b\}\}$ is a set containing only one element (singleton set) $\{a,b\}$.

Question No. 6

What is number of elements of the power set of each of following sets?

(i). $\{\}$

Solution:

$$\{\}$$

No. of elements of given set = $n = 0$

No. of elements of the power set = 2^n

No. of elements of the power set = 2^0

No. of elements of the power set = 1

(ii). $\{0, 1\}$

Solution:

$$\{0, 1\}$$

No. of elements of given set = $n = 2$

No. of elements of the power set = 2^n

No. of elements of the power set = 2^2

No. of elements of the power set = 4

(iii). $\{1,2,3,4,5,6,7\}$

Solution:

$$\{1,2,3,4,5,6,7\}$$

No. of elements of given set = $n = 7$

No. of elements of the power set = 2^n

No. of elements of the power set = 2^7

No. of elements of the power set = 128

(iv). $\{0,1,2,3,4,5,6,7\}$

Solution:

$\{0,1,2,3,4,5,6,7\}$

No. of elements of given set = $n = 8$

No. of elements of the power set = 2^n

No. of elements of the power set = 2^8

No. of elements of the power set = 256

(v). $\{a,\{b,c\}\}$

Solution:

$\{a,\{b,c\}\}$

No. of elements of given set = $n = 2$

No. of elements of the power set = 2^n

No. of elements of the power set = 2^2

No. of elements of the power set = 4

(vi). $\{\{a,b\},\{b,c\},\{d,e\}\}$

Solution:

$\{\{a,b\},\{b,c\},\{d,e\}\}$

No. of elements of given set = $n = 3$

No. of elements of the power set = 2^n

No. of elements of the power set = 2^3

No. of elements of the power set = 8

Question No. 7

Write down the power set of each of the following sets.

(i). $\{9,11\}$

Solution:

$\{9,11\}$

Let:

$A = \{9,11\}$

No. of elements of given set = $n = 2$

No. of elements of the power set = 2^n

No. of elements of the power set = $2^2 = 4$

$$P(A) = \{\Phi, \{9\}, \{11\}, \{9, 11\}\}$$

(ii). $\{+, -, \times, \div\}$

Solution:

$$\{+, -, \times, \div\}$$

Let:

$$B = \{+, -, \times, \div\}$$

No. of elements of given set = $n = 4$

No. of elements of the power set = 2^n

No. of elements of the power set = $2^4 = 16$

$$P(B) = \{\Phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$$

(iii). $\{\Phi\}$

Solution:

$$\{\Phi\}$$

Let:

$$C = \{\Phi\}$$

No. of elements of given set = $n = 1$

No. of elements of the power set = 2^n

No. of elements of the power set = $2^1 = 2$

$$P(C) = \{\Phi, \{\Phi\}\}$$

(iv). $\{a, \{b, c\}\}$

Solution:

$$\{a, \{b, c\}\}$$

Let:

$$D = \{a, \{b, c\}\}$$

No. of elements of given set = $n = 2$

No. of elements of the power set = 2^n

No. of elements of the power set = $2^2 = 4$

$$P(D) = \{\Phi, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$$