Unit No. 3

Set and Functions

Review Exercise No. 3

Question No. 1

Four options are given against each statement. Encircle the correct option.

(i) The set builder form of the set $\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\}$ is:

(a) $\{x|x = \frac{1}{n}, n \in W\}$

(b) *$\{x|x = \frac{1}{2n+1}, n \in W\}$*

(c) $\{x|x = \frac{1}{n+1}, n \in W\}$

(d) $\{x|x = 2n+1, n \in W\}$

(ii) If A = {}, then P(A) is:

(a) {}

(b) {1}

(c) *{{}}*

(d) ∅

(iii) If U = {1,2,3,4,5}, A = {1,2,3}, and B = {3,4,5}, then U − (A ∩ B) is:

(a) *{1,2,4,5}*

(b) {2,3}

(c) {1,3,4,5}

(d) {1,2,3}

(iv) If A and B are overlapping sets, then n(A−B) is equal to

(a) n(A)

(b) n(B)

(c) A∩B

(d) *n(A)−n(A∩B)*

(v) If A⊆B and B−A ≠ ∅, then n(B−A) is equal to:

(a) 0

(b) n(B)

(c) n(A)

(d) *n(B)−n(A)*

(vi) If n(A∪B)=50, n(A)=30 and n(B)=35, then n(A∩B)=:

(a) 23

(b) *15*

(c) 9

(d) 40

(vii) If A={1,2,3,4} and B={x,y,z}, then the Cartesian product of A and B contains exactly ______ elements.

(a) 13

(b) ***12***

(c) 10

(d) 6

(viii) If f(x)=x^2−3x+2, then the value of f(a+1) is equal to:

(a) a+1

(b) a^2+1

(c) a^2+2a+1

(d) a^2-a

(ix) Given that f(x)=3x+1, if f(x)=28, then the value of x is:

(a) 9

(b) 27

(c) 3

(d) 18

(x) Let A={1,2,3} and B={a,b} be two non-empty sets and f:A→B be a function defined as f={(1,a),(2,b),(3,b)}, then which of the following statement is True?

(a) f is injective

(b) f is surjective

(c) f is bijective

(d) f is into only

Question No. 2

Write each of the following sets in tabular forms:

(i). {x | x = 2n, n ∈ N}

Solution:

{x | x = 2n, n ∈ N}

In tabular form:

= {2,4,6,8,10,…}

(ii). {x|x = 2m+1, m ∈ N}

Solution:

{x|x = 2m+1, m ∈ N}

In tabular form:

= {3,5,7,9,11,…}

(iii). $\{x|x = 11n, n \in W \wedge n < 11\}$

Solution:

$\{x|x = 11n, n \in W \wedge n < 11\}$

In tabular form:

$= \{0,11,22,33,44,55,66,77,88,99,110\}$

(iv). $\{x|x \in E \wedge 4 < x < 6\}$

Solution:

$\{x|x \in E \wedge 4 < x < 6\}$

In tabular form:

$= \{\}$

(v). $\{x|x \in O \wedge 5 \leq x < 7\}$

Solution:

$\{x|x \in O \wedge 5 \leq x < 7\}$

In tabular form:

$= \{5\}$

(vi). $\{x|x \in Q \wedge x^2 = 2\}$

Solution:

$\{x|x \in Q \wedge x^2 = 2\}$

In tabular form:

$= \{\}$

(vii). $\{x|x \in Q \wedge x = -x\}$

Solution:

$\{x|x \in Q \wedge x = -x\}$

In tabular form:

$= \{0\}$

(viii). $\{x|x \in R \wedge x \notin Q'\}$

Solution:

$\{ x|x \in R \wedge x \notin Q'\}$

In tabular form:

$= Q$

Question No. 3

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,

$A = \{2, 4, 6, 8, 10\}$, $B = \{1,2,3,4,5\}$

and C = {1,3,5,7,9}

List the members of each of the following sets:

(i). A'

Solution:

A' = U – A

A' = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} - {2, 4, 6, 8, 10}

A' = {1,3,5,7,9}

(ii). B'

Solution:

B' = U – B

B' = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} - {1,2,3,4,5}

B ' = {6,7,8,9,10}

(iii). A ∪ B

Solution:

A ∪ B = {2, 4, 6, 8, 10} ∪ {1,2,3,4,5}

A ∪ B = {1, 2, 3, 4, 5, 6, 8, 10}

(iv). A - B

Solution:

A - B = {2, 4, 6, 8, 10} - {1,2,3,4,5}

A - B = {6, 8, 10}

(v). A ∩ C

Solution:

A ∩ C = {2, 4, 6, 8, 10} ∩ {1,3,5,7,9}

A ∩ C = {}

(vi). A' U C'

Solution:

A' U C' = {1,3,5,7,9} U {2, 4, 6, 8, 10}

A' U C' = {1,2,3,4,5,6,7,8,9,10}

(vii). A' U C

Solution:

A' U C = {1,3,5,7,9} U {1,3,5,7,9}

A' U C = {1,3,5,7,9}

(viii). U'

Solution:

U' = U – U = {1,2,3,4,5,6,7,8,9,10} - {1,2,3,4,5,6,7,8,9,10}

U' = {}

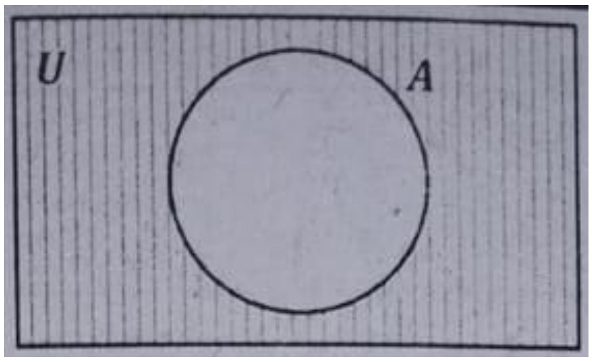
Question No. 4

Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i). A'

Solution:

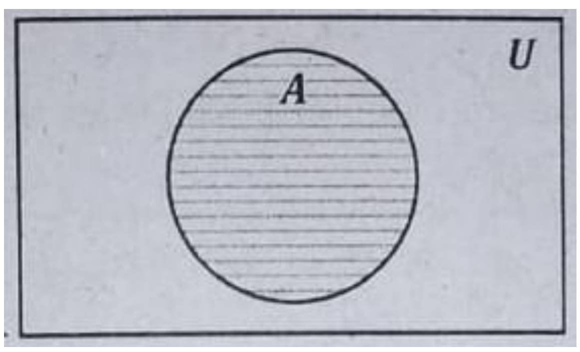
U – A = A'



(ii). A ∩ U

Solution:

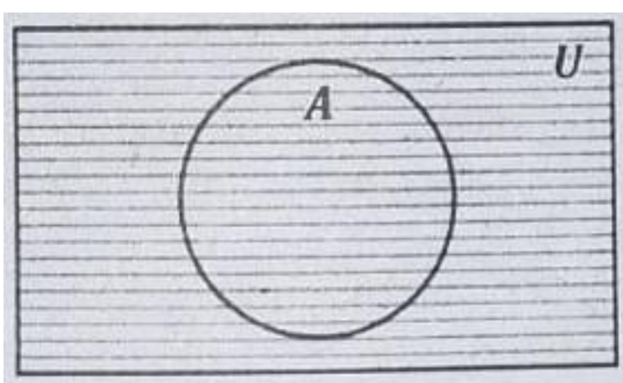
A ∩ U



(iii). A U U

Solution:

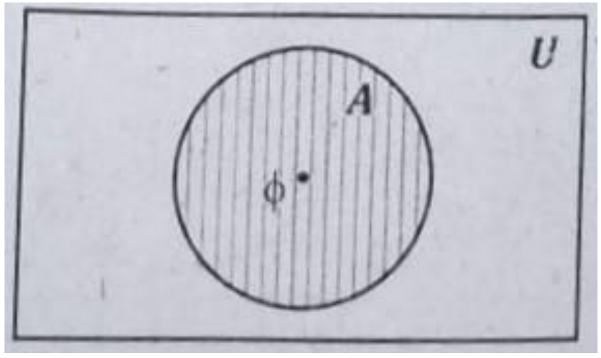
A U U



(iv). A U {}

Solution:

A U U



(v). {} ∩ {}

Solution:

{} ∩ {}

It has no Venn-Diagram.

Question No. 5

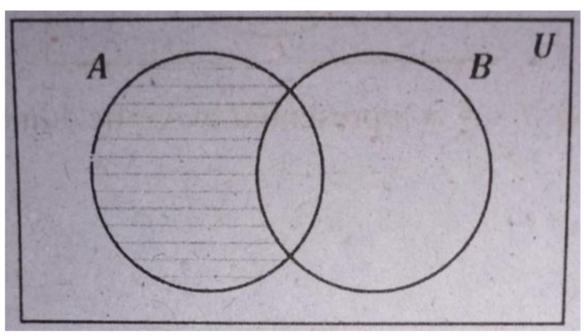
Use Venn diagrams to verify the following:

(i). A – B = A U B'

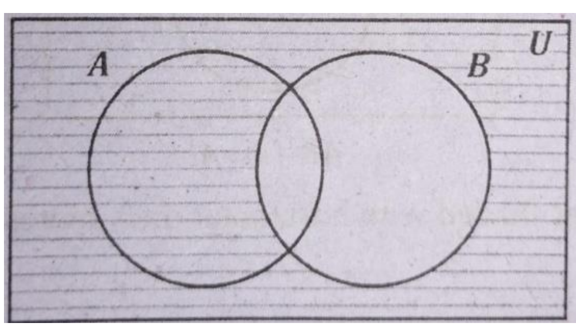
Solution:

A – B = A U B'

L.H.S: A – B



R.H.S: A U B'

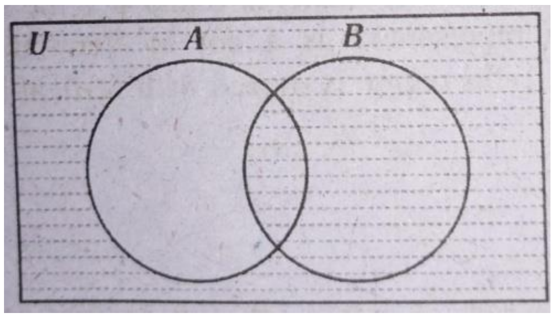


Hence proved that A – B = A U B'

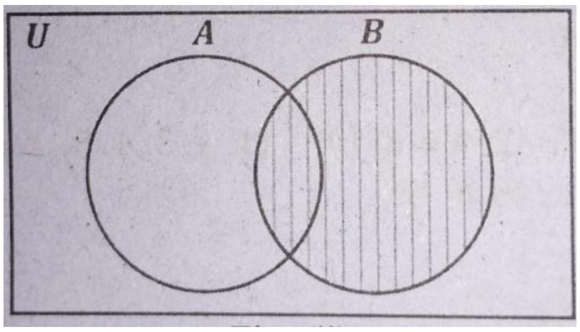
(ii). (A – B)' ∩ B = B

Solution:

L.H.S: (A – B)' ∩ B



R.H.S: B



Hence proved that (A – B)' ∩ B = B.

Question No. 6

Verify the properties for the sets A, B and C given below:

(i) Associativity of Union

(ii) Associativity of intersection.

(iii) Distributivity of Union over intersection.

(iv) Distributivity of intersection over union.

(a) A = {1,2,3,4}, B = {3, 4, 5, 6, 7, 8},

C = {5, 6, 7, 9, 10}

(b) A = {}, B = {0}, C = {0, 1,2}

(c) A = N, B = Z, C = Q

Solution:

(a).

1. Associativity of Union:

(A ∪ B) ∪ C = A ∪ (B ∪ C)

L.H.S: (A ∪ B) ∪ C

(A ∪ B) ∪ C = ({1,2,3,4} ∪ {3, 4, 5, 6, 7, 8}) ∪ {5, 6, 7, 9, 10}

(A ∪ B) ∪ C = {1, 2, 3, 4, 5, 6, 7, 8} ∪ {5, 6, 7, 9, 10}

(A ∪ B) ∪ C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

R.H.S: A ∪ (B ∪ C)

A ∪ (B ∪ C) = {1,2,3,4} ∪ ({3, 4, 5, 6, 7, 8} ∪ {5, 6, 7, 9, 10})

$A \cup (B \cup C) = \{1,2,3,4\} \cup \{3,4,5, 6, 7, 8,9, 10\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Therefore, $(A \cup B) \cup C = A \cup (B \cup C)$ is verified.

2. Associativity of Intersection:

$(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S: $(A \cap B) \cap C$

$(A \cap B) \cap C = (\{1,2,3,4\} \cap \{3, 4, 5, 6, 7, 8\}) \cap \{5, 6, 7, 9, 10\}$

$(A \cap B) \cap C = \{3,4\} \cap \{5, 6, 7, 9, 10\}$

$(A \cap B) \cap C = \emptyset$

R.H.S: $A \cap (B \cap C)$

$A \cap (B \cap C) = \{1,2,3,4\} \cap (\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 7, 9, 10\})$

$A \cap (B \cap C) = \{1,2,3,4\} \cap \{5, 6, 7\}$

$A \cap (B \cap C) = \emptyset$

Therefore, $(A \cap B) \cap C = A \cap (B \cap C)$ is verified.

3. Distributivity of Union over Intersection:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S: $A \cup (B \cap C)$

$A \cup (B \cap C)= \{1,2,3,4\} \cup (\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 7, 9, 10\})$

$A \cup (B \cap C) = \{1,2,3,4\} \cup \{5, 6, 7\}$

$A \cup (B \cap C) = \{1,2,3,4,5, 6, 7\}$

R.H.S: $(A \cup B) \cap (A \cup C)$

$(A \cup B) \cap (A \cup C) = (\{1,2,3,4\} \cup \{3, 4, 5, 6, 7, 8\}) \cap (\{1,2,3,4\} \cup \{5, 6, 7, 9, 10\})$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5, 6, 7\}$

Therefore, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is verified.

4. Distributivity of Intersection over Union:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S: $A \cap (B \cup C)$

$A \cap (B \cup C) = \{1,2,3,4\} \cap (\{3, 4, 5, 6, 7, 8\} \cup \{5, 6, 7, 9, 10\})$

$A \cap (B \cup C) = \{1,2,3,4\} \cap \{3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cap (B \cup C) = \{3, 4\}$

R.H.S: $(A \cap B) \cup (A \cap C)$

$A = \{1,2,3,4\}, B = \{3, 4, 5, 6, 7, 8\}, C = \{5, 6, 7, 9, 10\}$

$(A \cap B) \cup (A \cap C) = (\{1,2,3,4\} \cap \{3, 4, 5, 6, 7, 8\}) \cup (\{1,2,3,4\} \cap \{5, 6, 7, 9, 10\})$

$(A \cap B) \cup (A \cap C) = \{3,4\} \cup \emptyset$

$(A \cap B) \cup (A \cap C) = \{3,4\}$

Therefore, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is verified.

(b).

1. Associativity of Union:

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S $(A \cup B) \cup C$

$A \cup B = \{\} \cup \{0\} = \{0\}$

$(A \cup B) \cup C = \{0\} \cup \{0, 1, 2\} = \{0, 1, 2\}$

R.H.S $A \cup (B \cup C)$

$B \cup C = \{0\} \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$

$A \cup (B \cup C) = \{\} \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$

Therefore, $(A \cup B) \cup C = A \cup (B \cup C)$ is verified.

2. Associativity of Intersection:

$(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S $(A \cap B) \cap C$

$A \cap B = \{\} \cap \{0\}$

$= \{\}$ (empty set)

$(A \cap B) \cap C = \{\} \cap \{0, 1, 2\}$

$= \{\}$ (empty set)

R.H.S $A \cap (B \cap C)$

$B \cap C = \{0\} \cap \{0, 1, 2\}$

$= \{0\}$

$A \cap (B \cap C) = \{\} \cap \{0\}$

$= \{\}$ (empty set)

Therefore, $(A \cap B) \cap C = A \cap (B \cap C)$ is verified.

3. Distributivity of Union over Intersection:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $A \cup (B \cap C)$

$B \cap C = \{0\} \cap \{0, 1, 2\}$

$= \{0\}$

$A \cup (B \cap C) = \{\} \cup \{0\}$

$= \{0\}$

R.H.S $(A \cup B) \cap (A \cup C)$

$A \cup B = \{\} \cup \{0\}$

$= \{0\}$

$A \cup C = \{\} \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$

$(A \cup B) \cap (A \cup C) = \{0\} \cap \{0, 1, 2\}$

$= \{0\}$

Therefore, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is verified.

4. Distributivity of Intersection over Union:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $A \cap (B \cup C)$

$B \cup C = \{0\} \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$

$A \cap (B \cup C) = \{\} \cap \{0, 1, 2\}$

$= \{\}$ (empty set)

R.H.S $(A \cap B) \cup (A \cap C)$

$A \cap B = \{\} \cap \{0\}$

$= \{\}$ (empty set)

$A \cap C = \{\} \cap \{0, 1, 2\}$

$= \{\}$ (empty set)

$(A \cap B) \cup (A \cap C) = \{\} \cup \{\}$

$= \{\}$ (empty set)

Therefore, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is verified.

(c).

Given:

$A = N$, $B = Z$, $C = Q$

1. Associativity of Union:

$(A \cup B) \cup C = A \cup (B \cup C)$

$A \cup B = N \cup Z = Z$

(Since all natural numbers are integers)

$B) \cup C = Z \cup Q = Q$

(Since all integers are rational numbers)

$= Z \cup Q = Q$

(Since all integers are rational numbers)

$B \cup C) = N \cup Q = Q$

(Since all natural numbers are rational numbers)

Therefore, $(A \cup B) \cup C = A \cup (B \cup C)$ is verified.

2. Associativity of Intersection:

$(A \cap B) \cap C = A \cap (B \cap C)$

$A \cap B = N \cap Z = N$

(Since natural numbers are a subset of integers)

$(A \cap B) \cap C = N \cap Q = N$

(Since natural numbers are rational numbers)

$B \cap C = Z \cap Q = Z$

(Since integers are rational numbers)

$A \cap (B \cap C) = N \cap Z = N$

(Since natural numbers are integers)

Therefore, $(A \cap B) \cap C = A \cap (B \cap C)$ is verified.

3. Distributivity of Union over Intersection:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$B \cap C = Z \cap Q = Z$

$A \cup (B \cap C) = N \cup Z = Z$

$A \cup B = N \cup Z = Z$

$A \cup C = N \cup Q = Q$

$(A \cup B) \cap (A \cup C) = Z \cap Q = Z$

Therefore, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is verified.

4. Distributivity of Intersection over Union:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$B \cup C = Z \cup Q = Q$

$A \cap (B \cup C) = N \cap Q = N$

$A \cap B = N \cap Z = N$

$A \cap C = N \cap Q = N$

$(A \cap B) \cup (A \cap C) = N \cup N = N$

Therefore, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is verified.

Question No. 7

Verify De Morgan's Laws for the following sets:

$U = \{1,2,3, ..., 20\}$, $A = \{2, 4, 6, ..., 20\}$ and $B = \{1,3,5,...,19\}$.

Solution:

Given:

$U = \{1, 2, 3, ..., 20\}$

$A = \{2, 4, 6, ..., 20\}$

$B = \{1, 3, 5, ..., 19\}$

De Morgan's Laws:

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$

Solution Part (1):

L.H.S:

$A \cup B = \{2, 4, 6, ..., 20\} \cup \{1, 3, 5, ..., 19\}$

$A \cup B = \{1,2,3,4,..., 20\}$

$(A \cup B)' = \{1, 2, 3, ..., 20\} - \{1,2,3,4,..., 20\}$

$(A \cup B)' = \{\}$

R.H.S:

$A' = U - A = \{1, 2, 3, ..., 20\} - \{2, 4, 6, ..., 20\}$

$A' = \{1, 3, 5, ..., 19\}$

$B' = U - B = \{1, 2, 3, ..., 20\} - \{1, 3, 5, ..., 19\}$

$B' = \{2, 4, 6, ..., 20\}$

$A' \cap B' = \{1, 3, 5, ..., 19\} \cap \{2, 4, 6, ..., 20\}$

$A' \cap B' = \emptyset$

Hence proved $(A \cup B)' = A' \cap B'$

Solution Part (2):

$(A \cap B)' = A' \cup B'$

L.H.S:

$A \cap B = \emptyset$ (empty set)

$(A \cap B)' = U - \emptyset = U = \{1, 2, 3, ..., 20\}$

R.H.S:

$A' \cup B' = B \cup A = U = \{1, 2, 3, ..., 20\}$

Hence proved $(A \cap B)' = A' \cup B'$.

Question No. 8

Consider the set $P=\{x|x = 5m, m \in N\}$ and

$Q = \{x|x = 2m, m \in N\}$.

Find $P \cap Q$.

Given:

$P = \{x|x=5m, m \in N\}$

$P = \{5,10,15,20,25,30,35,\ldots\}$

$Q = \{x|x = 2m, m \in N\}$

$Q = \{2,4,6,8,10,12,14,16,18,20,22,24,\ldots\}$

To Find:

$\mathbf{P \cap Q = ?}$

Solution:

$P \cap Q = \{5,10,15,20,25,30,35,\ldots\} \cap \{2,4,6,8,10,12,14,16,18,20,22,24,\ldots\}$

$P \cap Q = \{10,20,30,40,\ldots\}$

Question No. 9

From suitable properties of union and intersection, deduce the following results:

(i). $A \cap (A \cup B) = A \cup (A \cap B)$

Solution:

By Solving L.H.S: $A \cap (A \cup B)$

$A \cap (A \cup B)$

By the **Distributive Law** of intersection over union:

$= (A \cap A) \cup (A \cap B)$

By the **Idempotent Law** of intersection:

$A \cap A = A$, So;

$= A \cup (A \cap B)$

Hence proved that:

$A \cap (A \cup B) = A \cup (A \cap B)$

(ii). $A \cup (A \cap B) = A \cap (A \cup B)$

Solution:

By Solving L.H.S: $A \cup (A \cap B)$

$A \cup (A \cap B)$

By the **Distributive Law** of union over intersection:

$= (A \cup A) \cap (A \cup B)$

By the **Idempotent Law** of union:

$A \cup A = A$, So;

$= A \cap (A \cup B)$

Hence proved that:

$A \cup (A \cap B) = A \cap (A \cup B)$

Question No. 10

If $g(x) = 7x - 2$ and $s(x) = 8x^2 - 3$ find:

(i). $g(0)$

(ii). $g(-1)$

(iii). $g(-\frac{5}{3})$

(iv). $s(1)$

(v). $s(-9)$

(vi). $s(\frac{7}{2})$

Solution:

(i). $g(0)$

$g(x) = 7x - 2$

$g(0) = 7(0) - 2$

$g(0) = 0 - 2$

$g(0) = -2$

(ii). $g(-1)$

$g(x) = 7x - 2$

$g(-1) = 7(-1) - 2$

$g(-1) = -7 - 2$

$g(-1) = -9$

(iii). $g(-\frac{5}{3})$

$g(x) = 7x - 2$

$g(-\frac{5}{3}) = 7(-\frac{5}{3}) - 2$

$g(-\frac{5}{3}) = -\frac{35}{3} - 2$

$g(-\frac{5}{3}) = \frac{-35-6}{3}$

$g(-\frac{5}{3}) = \frac{-41}{3}$

(iv). $s(1)$

$s(x) = 8x^2 - 3$

$s(1) = 8(1)^2 - 3$

$s(1) = 8(1) - 3$

$s(1) = 8 - 3$

$s(1) = 5$

(v). $s(-9)$

$s(x) = 8x^2 - 3$

$s(-9) = 8(-9)^2 - 3$

$s(-9) = 8(81) - 3$

$s(-9) = 648 - 3$

$s(-9) = 645$

(vi). $s(\frac{7}{2})$

$s(x) = 8x^2 - 3$

$s(\frac{7}{2}) = 8(\frac{7}{2})^2 - 3$

$s(\frac{7}{2}) = 8(\frac{49}{4}) - 3$

$s(\frac{7}{2}) = 2(49) - 3$

$s(\frac{7}{2}) = 98 - 3$

$s(\frac{7}{2}) = 95$

Question No. 11

Given that f(x) = ax + b, where a and b are constant numbers. If f(-2) =3 and f(4) = 10, then find the values of a and b.

Given function:

f(x) = ax + b

f(-2) =3

f(4) = 10

To Find:

a = ?

b = ?

Solution:

Step 1:

$f(x) = ax + b$

$3 = a(-2) + b$

$3 = -2a + b$ … (Equation 1)

Step 2:

$f(x) = ax + b$

$10 = a(4) + b$

$10 = 4a + b$ … (Equation 2)

Subtract Equation 1 from Equation 2

$(4a + b) - (-2a + b) = 10 - 3$

$4a + b + 2a - b = 7$

$6a = 7$

$a = \frac{7}{6}$

Find b:

Substituting $a = \frac{7}{6}$ into Equation 1:

$-2(\frac{7}{6}) + b = 3$

$-\frac{7}{3} + b = 3$

$b = 3 + \frac{7}{3}$

$b = \frac{9+7}{3}$

$b = \frac{16}{3}$

Question No. 12

Consider the function defined by k(x) = 7x -5. If k(x) = 100, find the value of x.

Given Function:

$k(x) = 7x - 5$

$k(x) = 100$

To Find:

$x = ?$

Solution:

$k(x) = 7x - 5$

$100 = 7x - 5$

$100 + 5 = 7x$

$105 = 7x$

$x = \frac{105}{7}$

$x = 15$ … (Either wrong value in book or wrong answer)

To solve according to book replace 5 by 3:

Question No. 13

Consider the function $g(x) = mx^2 + n$, where m and n are constant numbers. If $g(4)=20$ and $g(0) = 5$, find the values of m and n.

Given Function:

$g(x) = mx^2 + n$

$g(4) = 20$

$g(0) = 5$

To Find:

$m = ?$

$n = ?$

Solution:

$g(x) = mx^2 + n$

$20 = m(4)^2 + n$

$20 = 16m + n$ … Equation 1

$g(x) = mx^2 + n$

$5 = m(0)^2 + n$

$5 = n$

$n = 5$

Put in equation 1:

$20 = 16m + 5$

$16m = 20 - 5$

$16m = 15$

$m = \frac{15}{16}$

Question No. 14

A shopping mall has 100 products from various categories labeled 1 to 100, representing the universal set U. The products are categorized as follows:

• Set A: Electronics, consisting of 30 products labeled from 1 to 30.

• Set B: Clothing comprises 25 products labeled from 31 to 55.

• Set C: Beauty Products, comprising 25 products labeled from 76 to 100.

Write each set in tabular form, and find the union of all three sets.

Data:

Total products = 100

• Set A: Electronics, consisting of 30 products labeled from 1 to 30.

• Set B: Clothing comprises 25 products labeled from 31 to 55.

• Set C: Beauty Products, comprising 25 products labeled from 76 to 100.

To Find:

a). Write each set in tabular form:

b). Union of all three sets = A U B U C = ?

Solution (Part a):

Total products = n(U) = 100

In tabular form:

U = {1, 2, 3, …, 100}

• No. of Electronic products = n(A) = 30

In tabular form:

A = {1, 2, 3, …, 30}

• No. of Clothing products = n(B) = 25

In tabular form:

B = {31, 32, 33, …, 55}

• No. Of Beauty Products = n(C) = 25

In tabular form:

C = {76, 77, 78, …, 100}

Solution (Part b):

U = {1, 2, 3, …, 100}

A = {1, 2, 3, …, 30}

B = {31, 32, 33, …, 55}

C = {76, 77, 78, …, 100}

A U B U C = {1, 2, 3, …, 30} U {31, 32, 33, …, 55} U {76, 77, 78, …, 100}

A U B U C = {1, 2, 3, …, 30, 31, 32, 33, …, 55, 76, 77, 78, …, 100}

Number of elements in (A U B U C):

n(A U B U C) = 30 + 25 + 25 = 80

(Since no over lapping)

Question No. 15

Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.

(a) How many passed either the math or science test?

(b) How many did not pass either of the two tests?

(c) How many passed the science test but not the math test?

(d) How many failed the science test?

Data:

Total students = c(U) = 180

No. of students passed Math Test = n(M) = 120

No. of students passed Science Test = n(S) = 90

No. of students passed Math & Science Test = n(M∩S) = 60

(a) How many passed either the math or science test?

Solution:

The number of students who passed either the math or science test = $n(M \cup S)$ is given by the formula:

$n(M \cup S) = n(M) + n(S) - n(M \cap S)$

$n(M \cup S) = 120 + 90 – 60$

$n(M \cup S) = 210 – 60$

$n(M \cup S) = 150$ students passed either the math or science test.

(b) How many did not pass either of the two tests?

Solution:

Not passed either test

= total students - $n(M \cup S)$

= 180 – 150

= 30 students did not pass either test.

(c) How many passed the science test but not the math test?

Solution:

The number of students who passed the science test but not the math test:

$n(S – M) = n(S) - n(M \cap S)$

$n(S – M) = 90 – 60$

$n(S – M) = 30$ students passed the science test but not the math test.

(d) How many failed the science test?

Solution:

The number of students who failed the science test:

= $n(U) – n(S)$

= 180 – 90

= 90 students failed the science test.

Question No. 16

In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

- **150 developers like Python.**
- **130 developers like Java.**
- **120 developers like PHP.**
- **70 developers like both Python and Java.**
- **60 developers like both Python and PHP.**
- **50 developers like both Java and PHP.**
- **40 developers like all three languages: Python, Java and PHP.**

(a) How many developers use at least one of these languages?

(b) How many developers use only one of these languages?

(c) How many developers do not use any of these languages?

(d) How many developers use only PHP?

Given:

Total software developers = n(U) = 300

Developers who like Python = n(P) = 150

Developers who like Java = n(J) = 130

Developers who like PHP = n(H) = 120

Developers who like both Python and Java = $n(P \cap J) = 70$

Developers who like both Python and PHP = $n(P \cap H) = 60$

Developers who like both Java and PHP = $n(J \cap H) = 50$

Developers who like all three languages: Python, Java and PHP. = $n(P \cap J \cap H) = 40$

(a) How many developers use at least one of these languages?

Solution:

Using inclusion and exclusion rule:

$n(P \cup J \cup H) = n(P) + n(J) + n(H) - n(P \cap J) - n(P \cap H) - n(J \cap H) + n(P \cap J \cap H)$

$n(P \cup J \cup H) = 150 + 130 + 120 - 70 - 60 - 50 + 40$

$n(P \cup J \cup H) = 440 – 180$

$n(P \cup J \cup H) = 260$ developers use at least one of these languages.

(b) How many developers use only one of these languages?

Solution:

1. Developers who like Python only:

$n(P)$ only $= n(P) - n(P \cap J) - n(P \cap H) + n(P \cap J \cap H)$

$n(P)$ only $= 150 – 70 – 60 + 40$

$n(P)$ only $= 190 – 130$

$n(P)$ only $= 60$ Developers like Python only.

2. Developers who like Java only:

$n(J)$ only $= n(J) - n(P \cap J) - n(J \cap H) + n(P \cap J \cap H)$

$n(J)$ only $= 130 – 70 – 50 + 40$

$n(J)$ only $= 170 – 120$

$n(J)$ only $= 50$ Developers like Java only.

3. Developers who like PHP only:

$n(H)$ only $= n(H) - n(P \cap H) - n(J \cap H) + n(P \cap J \cap H)$

n(H) only = 120 – 60 – 50 + 40

n(H) only = 160 – 110

n(H) only = 50 Developers like PHP only.

Thus total number of developers who use only one languagevis:

= n(P) + n(J) + n(H)

= 60 + 50 + 50

= 160

(c) How many developers do not use any of these languages?

Solution:

Developers who did not use any of three languages:

= total developers – number of developers who use at least one language

= 300 – 260

= 40 did not use any of three languages.

(d) How many developers use only PHP?

Solution:

Developers who use only PHP:

n(H) only = n(H) - n(P∩H) - n(J∩H) + n(P∩J∩H)

n(H) only = 120 – 60 – 50 + 40

n(H) only = 160 – 110

n(H) only = 50 Developers like PHP only.