Unit No. 5

Linear Equations and Inequalities

Exercise No. 5.1

Question No. 1

Solve and represent the solution on a real line.

(i). 12x + 30 = −6

Solution:

$12x + 30 = -6$

$12x = -6 - 30$

$12x = -36$

$x = -\frac{36}{12}$

$x = -3$

Check:

Put (x = -3) in given equation:

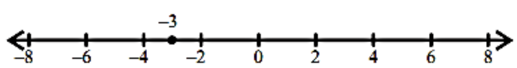
$12(-3) + 30 = -6$

$-36 + 30 = -6$

$-6 = -6$

So, Solution Set = {-3}

Real Line number:



(ii). $\frac{x}{3} + 6 = -12$

Solution:

$\frac{x}{3} + 6 = -12$

$\frac{x}{3} = -12 - 6$

$\frac{x}{3} = -18$

$x = -18 \times 3$

$x = -54$

Check:

Put (x = -54) in given equation:

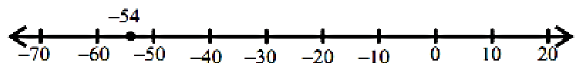
$\frac{-54}{3} + 6 = -12$

$-18 + 6 = -12$

-12 = -12

So, Solution Set = {-54}

Real Line number:



(iii). $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

Solution:

$\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

Multiply by 12 on both sides:

$12(\frac{x}{2}) - 12(\frac{3x}{4}) = 12(\frac{1}{12})$

6x – 9x = 1

-3x = 1

x = - $\frac{1}{3}$

Check:

Put (x = - $\frac{1}{3}$) in given equation:

$\frac{-1}{3(2)} - \frac{3(-1)}{4(3)} = \frac{1}{12}$

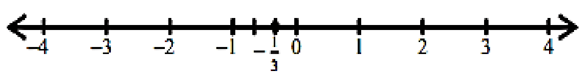
$\frac{-1}{6} + \frac{1}{4} = \frac{1}{12}$

$\frac{-2+3}{12} = \frac{1}{12}$

$\frac{1}{12} = \frac{1}{12}$

So, Solution Set = {- $\frac{1}{3}$}

Real Line number:



(iv). 2 = 7(2x + 4) + 12x

Solution:

2 = 7(2x + 4) + 12x

2 = 14x + 28 + 12x

2 - 28 = 26x

26x = - 26

x = - $\frac{26}{26}$

x = -1

Check:

Put (x = -1) in given equation:

2 = 7[2(-1) + 4] + 12(-1)

2 = 7[-2 + 4] - 12

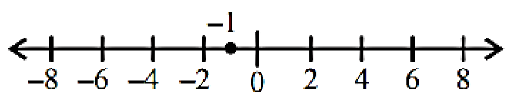
2 = 7[2] – 12

2 = 14 - 12

2 = 2

So, Solution Set = {-1}

Real Line number:



(v). $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$

Solution:

$\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$

Multiply by 12 on both sides:

$12(\frac{2x-1}{3}) - 12(\frac{3x}{4}) = 12(\frac{5}{6})$

$4(2x - 1) - 3(3x) = 2(5)$

$8x - 4 - 9x = 10$

$-x = 10 + 4$

x = - 14

Check:

Put (x = -14) in given equation:

$\frac{2(-14)-1}{3} - \frac{3(-14)}{4} = \frac{5}{6}$

$\frac{-28-1}{3} - \frac{-42}{4} = \frac{5}{6}$

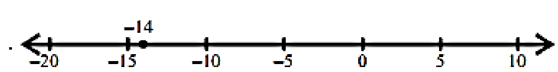
$\frac{-29}{3} + \frac{21}{2} = \frac{5}{6}$

$\frac{-58+63}{6} = \frac{5}{6}$

$\frac{5}{6} = \frac{5}{6}$

So, Solution Set = {-14}

Real Line number:



(vi). $\frac{-5x}{10} = 9 - \frac{10x}{5}$

Solution:

$$\frac{-5x}{10} = 9 - \frac{10x}{5}$$

Multiply by 10 on both sides:

$$10\left(\frac{-5x}{10}\right) = 10(9) - 10\left(\frac{10x}{5}\right)$$

$$-5x = 90 - 2(10x)$$

$$-5x = 90 - 20x$$

$$-5x + 20x = 90$$

$$15x = 90$$

$$x = \frac{90}{15}$$

$$x = 6$$

Check:

Put (x = 6) in given equation:

$$\frac{-5(6)}{10} = 9 - \frac{10(6)}{5}$$

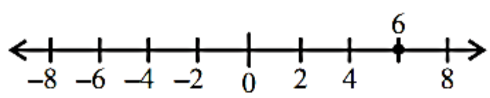
$$\frac{-30}{10} = 9 - \frac{60}{5}$$

$$-3 = 9 - 12$$

$$-3 = -3$$

So, Solution Set = {6}

Real Line Number:



Question No. 2

Solve each inequality and represent the solution on a real line.

(i). $x - 6 \leq -2$

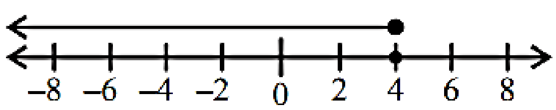
Solution:

$$x - 6 \leq -2$$

$$x \leq -2 + 6$$

$$x \leq 4$$

Real Line Number:



(ii). $-9 > -16 + x$

Solution:

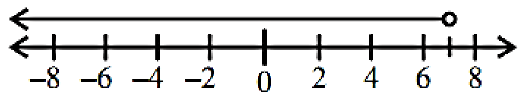
$-9 > -16 + x$

$-9 + 16 > x$

$7 > x$

$x < 7$

Real Line Number:



(iii). $3 + 2x \geq 3$

Solution:

$3 + 2x \geq 3$

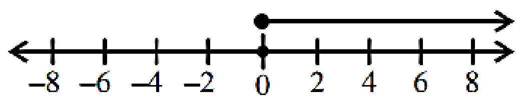
$2x \geq 3 - 3$

$2x \geq 0$

$x \geq 0/2$

$x \geq 0$

Real Line Number:



(iv). $6(x + 10) \leq 0$

Solution:

$6(x + 10) \leq 0$

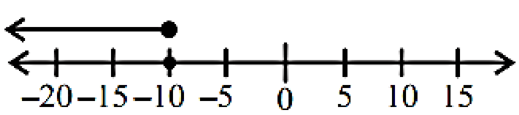
$6x + 60 \leq 0$

$6x \leq -60$

$x \leq \frac{-60}{6}$

$x \leq -10$

Real Line Number:



(v). $\frac{5}{3}x - \frac{3}{4} < -\frac{1}{12}$

Solution:

$\frac{5}{3}x - \frac{3}{4} < -\frac{1}{12}$

$12(\frac{5}{3}x) - 12(\frac{3}{4}) < 12(-\frac{1}{12})$

$4(5x) - 3(3) < 1(-1)$

$20x - 9 < -1$

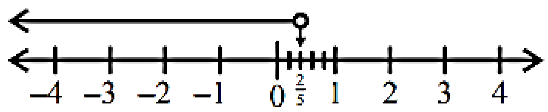
$20x < -1 + 9$

$20x < 8$

$x < \frac{8}{20}$

$x < \frac{2}{5}$

Real Line Number:



(vi). $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

Solution:

$\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

$4(\frac{1}{4}x) - 4(\frac{1}{2}) \leq 4(-1) + 4(\frac{1}{2}x)$

$1(1x) - 2(1) \leq -4 + 2(1x)$

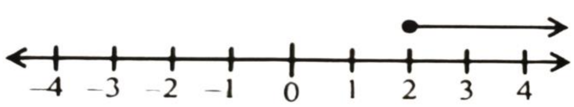
$x - 2 \leq -4 + 2x$

$x - 2x \leq -4 + 2$

$-x \leq -2$

$x \geq 2$

Real Line Number:



Question No. 3

Shade the solution region for the following linear inequalities in the xy-plane:

(i) $2x + y \leq 6$

Solution:

$2x + y \leq 6$

Associated equations:

$2x + y = 6$ … eq. i

- x-intercept: Set $y = 0$:

$2x + 0 = 6 \Longrightarrow x = 3$

So, the point is (3, 0).

- y-intercept: Set $x = 0$:

$2(0) + y = 6 \Longrightarrow y = 6$

$2(0) + y = 6 \Longrightarrow y = 6$

So, the point is (0, 6).

To check Region put (0, 0) in given eq.

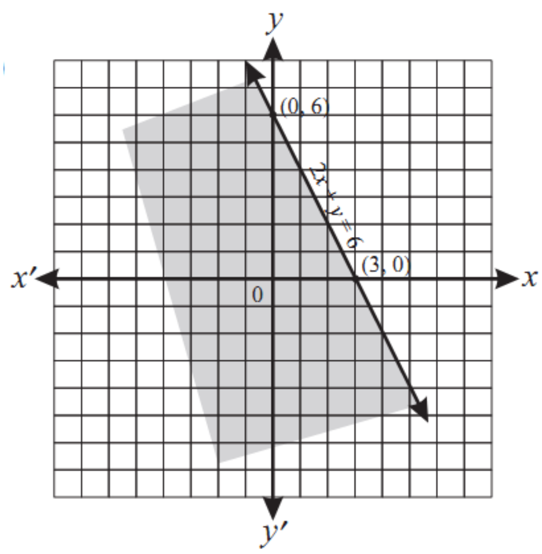
$2(0) + (0) \leq 6$

$0 + 0 \leq 6$

$0 \leq 6$ True

Graph lies towards the origin side.

Graphical representation:



(ii) $3x + 7y \geq 21$

Solution:

$3x + 7y \geq 21$

Associated equations:

$3x + 7y = 21$ … eq. i

- x-intercept: Set y = 0:

$3x + 7(0) \geq 21 \Longrightarrow 3x = 21 \Longrightarrow x = 21/3 \Longrightarrow x = 7$

So, the point is (7, 0).

- y-intercept: Set x = 0:

$3(0) + 7y = 21 \Longrightarrow 7y = 21 \Longrightarrow y = 21/7 \Longrightarrow y = 3$

So, the point is (0, 3).

To check Region put (0, 0) **in given eq.**

$3(0) + 7(0) \geq 21$

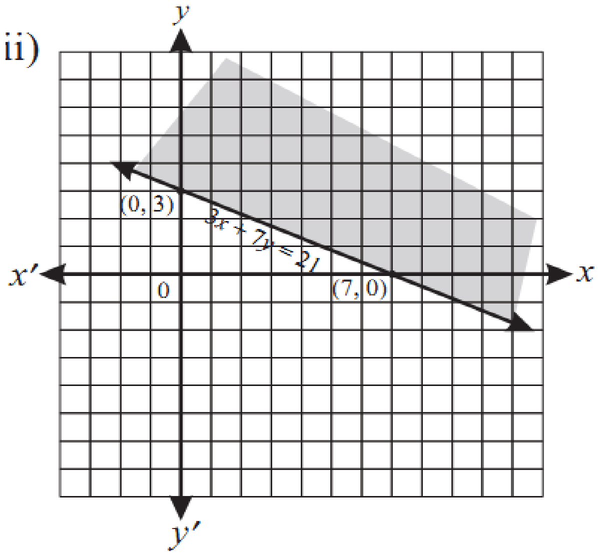
$0 + 0 \geq 21$

$0 \geq 21$ False

Graph away from the origin.

Graphical representation:

ii)



(iii) 3x − 2y ≥ 6

Solution:

$3x - 2y \geq 6$

Associated equations:

$3x - 2y = 6$ … eq. i

- x-intercept: Set y = 0:

$3x - 2(0) = 6 \Longrightarrow 3x = 6 \Longrightarrow x = 6/3 \Longrightarrow x = 2$

So, the point is (2, 0).

- y-intercept: Set x = 0:

$3(0) - 2y = 6 \Longrightarrow -2y = 6$
$\Longrightarrow y = 6/-2 \Longrightarrow y = -3$

So, the point is (0, -3).

To check Region put (0, 0) **in given eq.**

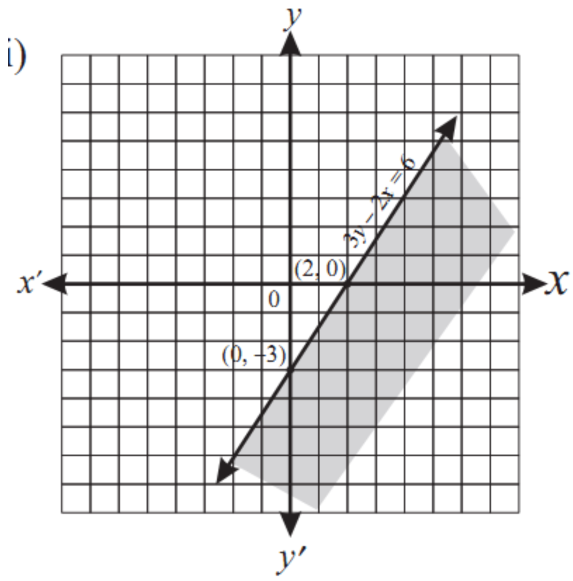
$3(0) - 2(0) \geq 6$

$0 - 0 \geq 6$

$0 \geq 6$ False

Graph away from the origin.

Graphical representation:



(iv) 5x − 4y ≤ 20

Solution:

$5x - 4y \leq 20$

Associated equations:

$5x - 4y = 20$ … eq. i

- x-intercept: Set y = 0:

$5x - 4(0) = 20 \implies 5x = 20 \implies x = 20/5 \implies x = 4$

So, the point is (4, 0).

- y-intercept: Set x = 0:

$5(0) - 4y = 20 \implies -4y = 20$

$\implies y = 20/-4 \implies y = -5$

So, the point is (0, -5).

To check Region put (0, 0) in given eq.

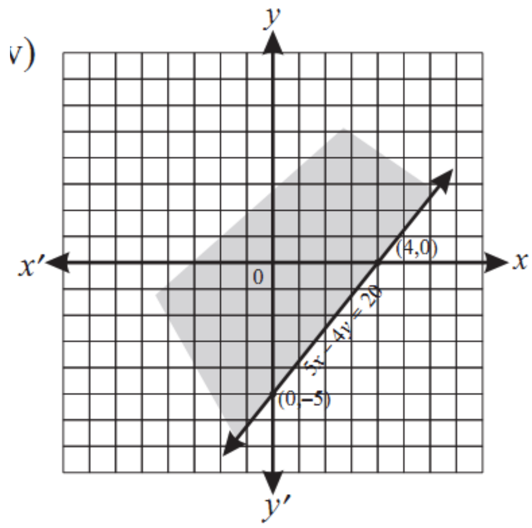
$5(0) - 4(0) \leq 20$

$0 - 0 \leq 20$

$0 \leq 20$ True

Graph towards the origin.

Graphical representation:



(v) 2x + 1 ≥ 0

Solution:

$2x + 1 \geq 0$

Associated equations:

$2x + 1 = 0$ … eq. i

Point:

$x = -1/2$

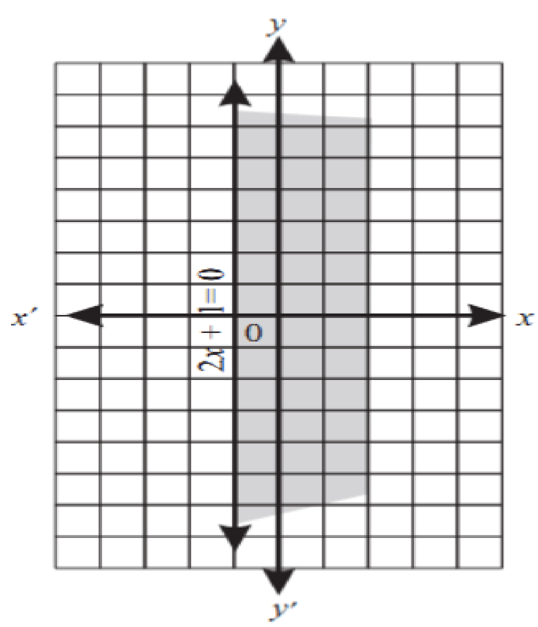
To check Region put x = 0:

$2(0) + 1 \geq 0$

$1 \geq 0$ True

Graph towards the origin.

Graphical representation:



(vi) 3y – 4 ≤ 0

Solution:

$3y - 4 \le 0$

Associated equations:

$3y - 4 = 0$ … eq. i

Point:

$y = 4/3$

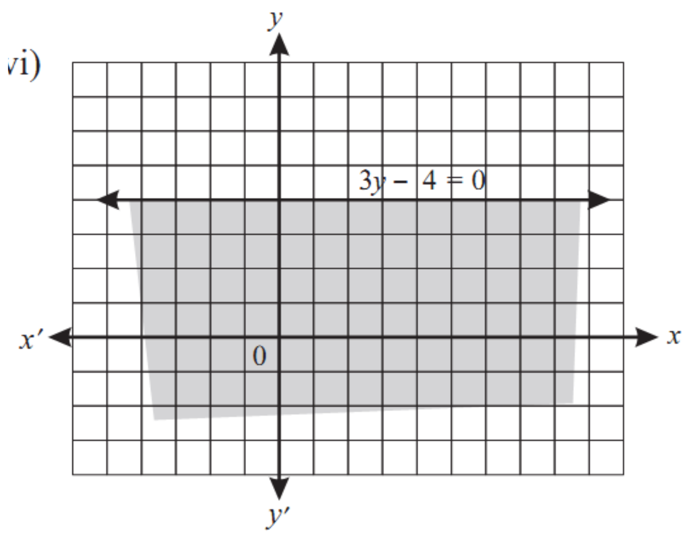
To check Region put x = 0:

$3(0) - 4 \le 0$

$-4 \le 0$ True

Graph towards the origin.

Graphical representation:



Question No. 4

Indicate the solution region of the following linear inequalities by shading:

(i) $2x - 3y \le 6$
$2x + 3y \le 12$

Solution:

$2x - 3y \le 6$ … (i)
$2x + 3y \le 12$ … (ii)

The associated equation of (i) is

$2x - 3y = 6$ … (iii)

For x-intercept, put y = 0 in (iii), we get

$2x - 3(0) = 6$

$2x - 0 = 6$
$x = \frac{6}{2}$

$x = 3$, so the point is (3, 0)

For y-intercept, put x = 0 in (iii), we get

$2(0) - 3y = 6$

$0 - 3y = 6$

$y = \frac{6}{-3}$

$y = -2$, so the point is (0, -2)

- Let's test the point (0, 0):
  $-3(0) \leq 6$
  $0 \leq 6$ (True)
- So, shading lies towards origin.

The associated equation of (ii) is

$2x + 3y = 12$ … (iv)

For x-intercept, put y = 0 in (iv), we get

$2x + 3(0) = 12$

$2x + 0 = 12$

$x = \frac{12}{2}$

$x = 6$, so the point is (6, 0)

For y-intercept, put x = 0 in (iv), we get

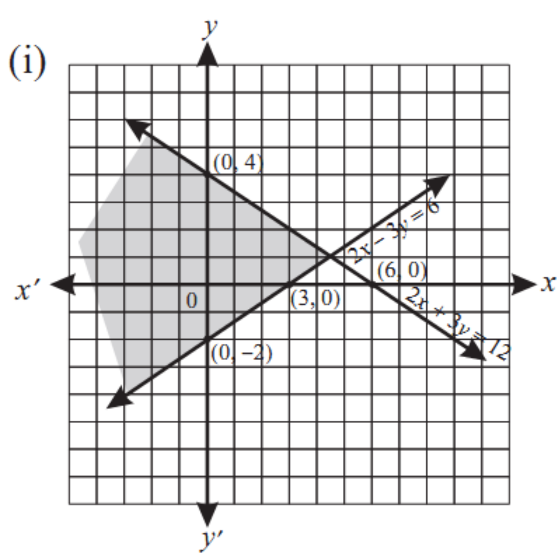
$2(0) + 3y = 12$

$0 + 3y = 12$

$y = \frac{12}{3}$

$y = 4$, so the point is (0, 4)

  - Let's test the point (0, 0):
    $2(0) + 3(0) \leq 12$
    $0 \leq 12$ (True)
- So, shading lies towards origin side.



(ii) $\mathbf{x + y \geq 5}$
$\mathbf{-y + x \leq 1}$

Solution:

$x + y \geq 5$ … (i)
$-y + x \leq 1$ … (ii)

The associated equation of (i) is

$x + y = 5$ … (iii)

For x-intercept, put $y = 0$ in (iii), we get

$x + 0 = 5$

$x = 5$, so the point is $(5, 0)$
For y-intercept, put $x = 0$ in (iii), we get

$0 + y = 5$

$y = 5$, so the point is $(0, 5)$

- Let's test the point $(0, 0)$:
  $0 + 0 \geq 5$

  $0 \geq 5$ (False)

- Since $(0, 0)$ does not satisfy the inequality, the region not containing the origin should be shaded for the first inequality.

The associated equation of (ii) is

$-y + x = 1$ … (iv)

For x-intercept, put $y = 0$ in (iv), we get

$-y + x = 1$

$-0 + x = 1$
$x = 1$, so the point is $(1, 0)$
For y-intercept, put $x = 0$ in (iv), we get

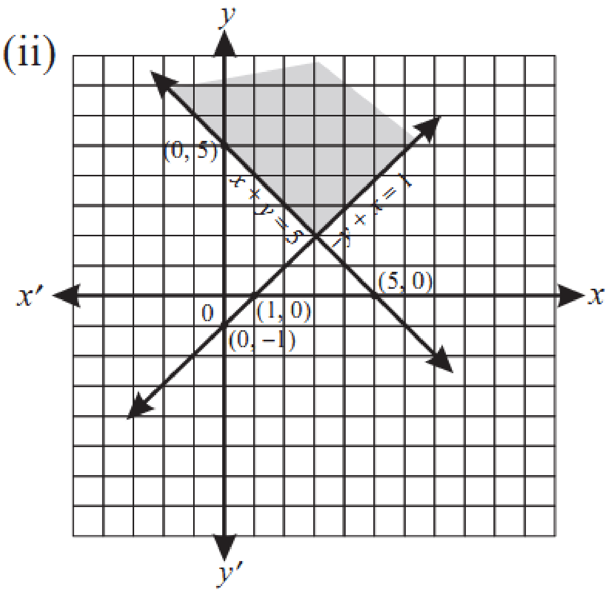
$-y + x = 1$

$-y + 0 = 1$

$y = -1$, so the point is $(0, -1)$

- Let's test the point $(0, 0)$:
  $-0 + 0 \leq 1$
  $0 \leq 1$ (True)
- Since $(0, 0)$ satisfies the inequality, the region containing the origin should be shaded for the second inequality.

(ii)



(iii) 3x + 7y ≥ 21

x – y ≤ 2

Solution:

3x + 7y ≥ 21 … (i)
x – y ≤ 2 … (ii)

The associated equation of (i) is

3x + 7y = 21 … (iii)

For x-intercept, put y = 0 in (iii), we get

3x + 7(0) = 21

3x + 0 = 21

$x = \frac{21}{3}$

x = 7, so the point is (7, 0)

For y-intercept, put x = 0 in (iii), we get

3(0) + 7y = 21

7y = 21

$y = \frac{21}{7}$

y = 3, so the point is (0, 3)

- Let's test the point (0, 0):
  3(0) + 7(0) ≥ 21
  0 ≥ 21 (False)
- Since (0, 0) does not satisfy the inequality, the region not containing the origin should be shaded for the first inequality.
  The associated equation of (ii) is

x – y = 2 … (iv)

For x-intercept, put y = 0 in (iv), we get

x – 0 = 2

x = 2, so the point is (2, 0)

For y-intercept, put x = 0 in (iv), we get

0 – y = 2

-y = 2

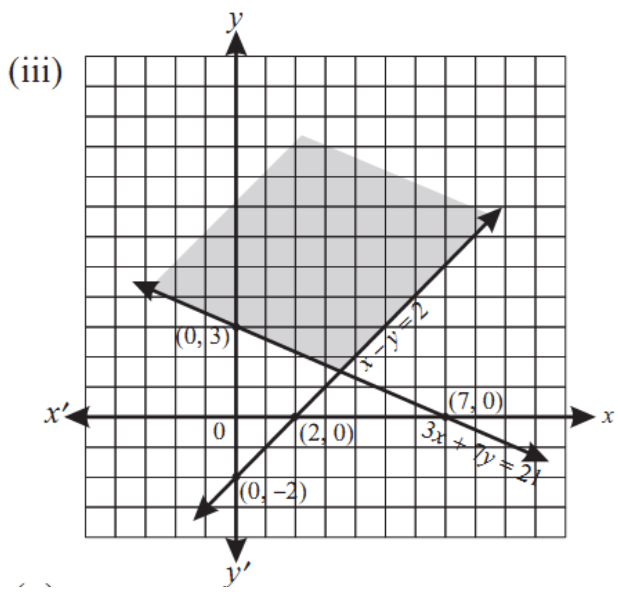
y = -2, so the point is (0, -2)

- Let's test the point (0, 0):

  0 – 0 ≤ 2

  0 ≤ 2 (True)

- Since (0, 0) satisfies the inequality, the region containing the origin should be shaded for the second inequality.



(iv) 4x - 3y ≤ 12

$x \geq -\frac{3}{2}$

Solution:

4x - 3y ≤ 12 … (i)

$x \geq -\frac{3}{2}$ … (ii)

The associated equation of (i) is

4x - 3y = 12 … (iii)

For x-intercept, put y = 0 in (iii), we get

4x – 3(0) = 12

4x + 0 = 12

x = $\frac{12}{4}$

x = 3, so the point is (3, 0)

For y-intercept, put x = 0 in (iii), we get

4(0) - 3y = 12

-3y = 12

y = $\frac{12}{-3}$

y = -4, so the point is (0, -4)

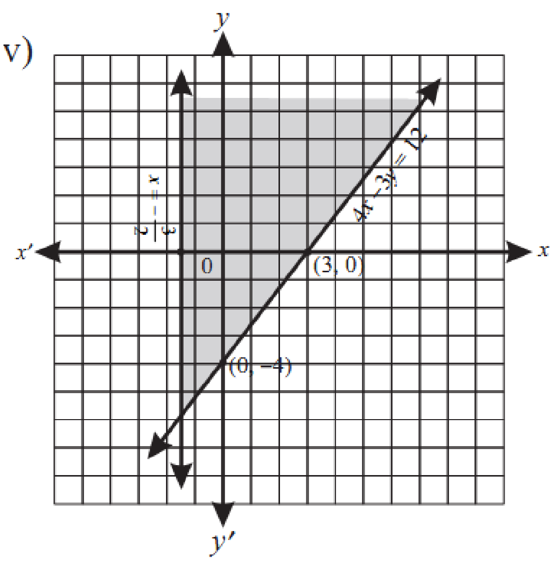
- Let's test the point (0, 0):
  4(0) - 3(0) ≤ 12
  0 ≤ 12 (True)
- Since (0, 0) satisfies the inequality, the region containing the origin should be shaded for the first inequality.

The associated equation of (ii) is

$x = -\frac{3}{2}$, so the point is $(-\frac{3}{2}, 0)$
$x = -\frac{3}{2}$ is a line parallel to y-axis put x = 0 in $\boldsymbol{x} = -\frac{3}{2}$

$0 > -\frac{3}{2}$ which is true. So, shading lies towards origin side.

v)



(v) 3x + 7y ≥ 21

y ≤ 4

Solution:

3x + 7y ≥ 21 … (i)

y ≤ 4 … (ii)

The associated equation of (i) is

3x + 7y = 21 … (iii)

For x-intercept, put y = 0 in (iii), we get

3x + 7(0) = 21

3x + 0 = 21

x = $\frac{21}{3}$

x = 7, so the point is (7, 0)
For y-intercept, put x = 0 in (iii), we get

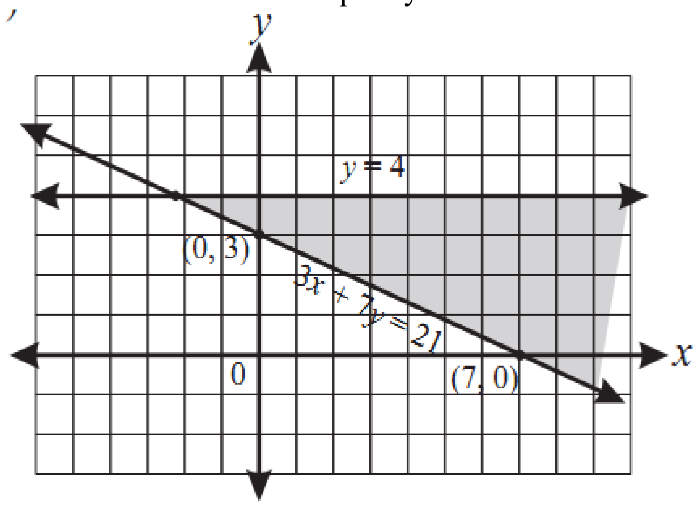
3(0) + 7y = 21

7y = 21

y = $\frac{21}{7}$

y = 3, so the point is (0, 3)

- Let's test the point (0, 0):
  3(0) + 7(0) ≥ 21
  0 ≥ 21 (False)
- Since (0, 0) does not satisfy the inequality, the region not containing the origin should be shaded for the first inequality.
  The associated equation of (ii) is
  y = 4, so the point is (0, 4)
- The associated equation is y = 4, which is a horizontal line passing through all points where the y-coordinate is 4.
- To determine the shaded region, we can test a point. Let's use (0, 0):
  0 ≤ 4 (True)
- Since (0, 0) satisfies the inequality, the region containing the origin (i.e., below the line y = 4) should be shaded for the second inequality.



(vi) 5x + 7y ≤ 35

x - 2y ≤ 2

Solution:

5x + 7y ≤ 35 … (i)
x - 2y ≤ 2 … (ii)

The associated equation of (i) is

5x + 7y = 35 … (iii)

For x-intercept, put y = 0 in (iii), we get

5x + 7(0) = 35

5x + 0 = 35

x = $\frac{35}{5}$

x = 7, so the point is (7, 0)
For y-intercept, put x = 0 in (iii), we get

5(0) + 7y = 35

7y = 35

$y = \frac{35}{7}$

y = 5, so the point is (0, 5)

- Let's test the point (0, 0):
  $5(0) + 7(0) \leq 35$
  $0 \leq 35$ (True)
- Since (0, 0) satisfies the inequality, the region containing the origin should be shaded for the first inequality.
  The associated equation of (ii) is

$x - 2y = 2$ … (iv)

For x-intercept, put y = 0 in (iv), we get

$x - 2(0) = 2$

x = 2, so the point is (2, 0)
For y-intercept, put x = 0 in (iv), we get

$0 - 2y = 2$

$y = -\frac{2}{2}$

y = -1, so the point is (0, -1)

- Let's test the point (0, 0):
  $0 - 2(0) \leq 2$
  $0 \leq 2$ (True)
- Since (0, 0) satisfies the inequality, the region containing the origin should be shaded for the second inequality