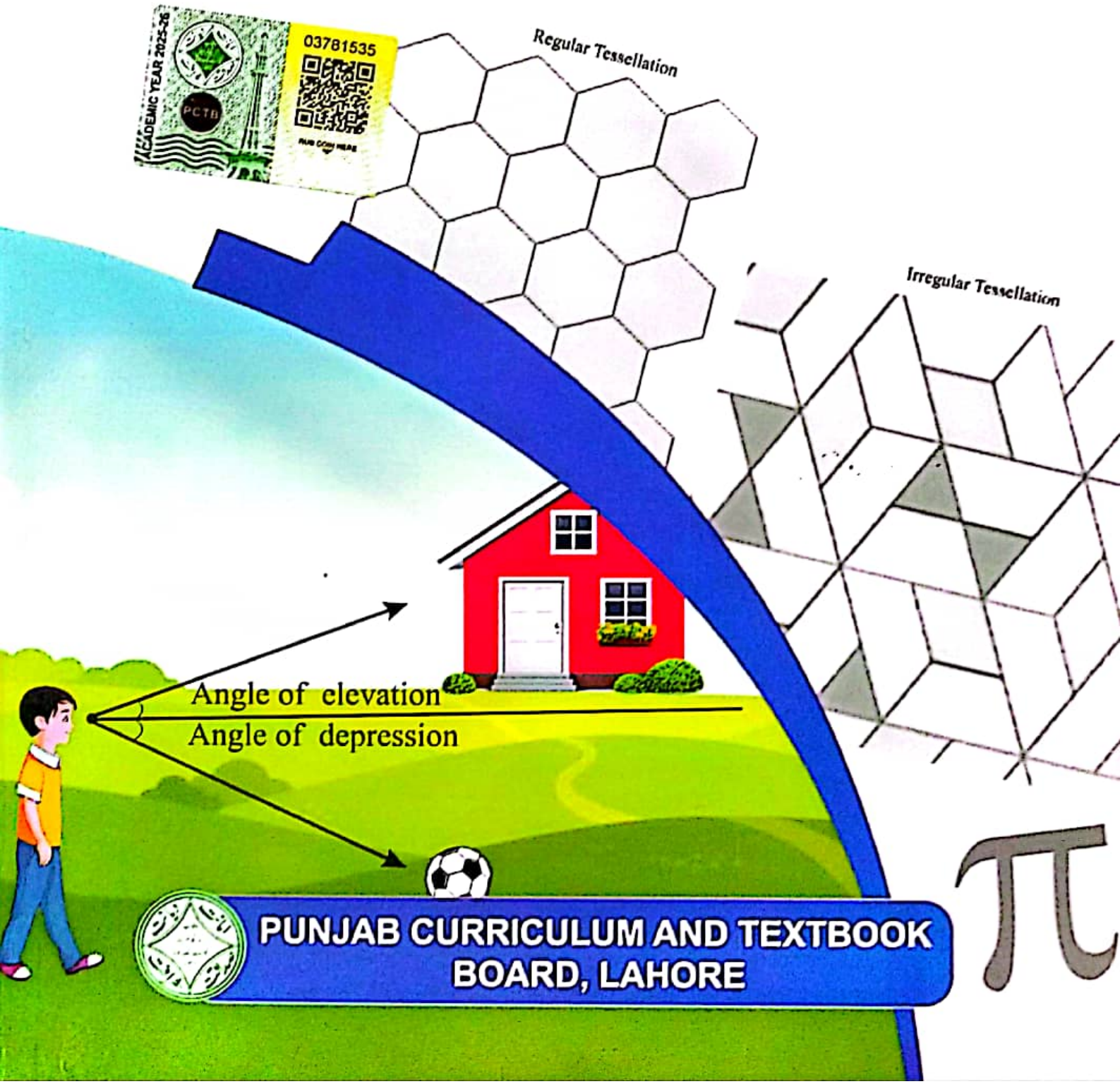
# MATHEMATICS 9




PUNJAB CURRICULUM AND TEXTBOOK BOARD, LAHORE

UNIT 13

# PROBABILITY

## EXERCISE 13.1

**Q.1.** **Arshad rolls a dice, with sides labelled L, M, O, P, U. What is the probability that the dice lands on consonant?**

**Sol.** Number of letters = n = 6 = n(S)

No. of consonants = n(A) = 6 – 2 = 4

(Since O, U are not consonants)

Using

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{6}$$

$$P(A) = \frac{2}{3}$$

**Q.2.** **Shazia throws a pair of fair dice. What will be the probability of getting:**

**(i)** **sum of dots is at least 4.**

**(ii)** **product of both dots is between 5 to 10.**

**(iii)** **the difference between both the dots is equal to 4.**

**(iv)** **number at least 5 on the first dice and the number at least 4 on the second dice.**

**Sol.** When a pair of fair dice is rolled, the sample space will be

| 1st / 2nd | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) | (1, 6) |
| 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) |
| 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) |
| 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) |
| 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) |
| 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |

(i) Sum of dots is at least 4.

(1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

No. of favourable outcomes = n(A) = 33

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{33}{36} = \frac{11}{12}$$

(ii) Product of both dots is between 5 and 10.

(1, 5), (1,6), (2, 3), (2, 4), (2, 5), (5, 1), (3, 2), (3, 3), (4, 2), (5, 2), (6, 1)

No. of favourable outcomes = n(A) = 11

$$n(S) = 36$$

So, $$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{11}{36}$$

(iii) the difference between both the dots is equal to 4.

Favourable outcomes are

(1, 5), (2, 6), (5, 1), (6, 2)

No. of favourable outcomes = n(A) = 4

$$n(S) = 36$$

$$\text{So, } P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{36} = \frac{1}{9}$$

(iv) There are six outcomes having at least 5 on the first dice and the number at least 4 on the second dice.

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{6}{36} = \frac{1}{6}$$

**Q.3. One alphabet is selected at random from the word "MATHEMATICS". Find the probability of getting:**

**(i) vowel (ii) consonant (iii) an E**

**(iv) an A (v) not M (vi) not T**

**Sol.** The word is MATHEMATICS

(i) Total letters in the word MATHEMATICS,

$$n(S) = 11$$

No. of vowels = n(A) = 4

$$\text{Probability of getting vowel} = P(A)$$

$$= \frac{n(A)}{n(S)}$$

$$= \frac{4}{11}$$

(ii) No. of consonants $= 11 - 4$

$$n(A) = 7$$

$$n(S) = 11$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{7}{11}$$

(iii) an "E"

Here $n(A) = 1$

$n(S) = 11$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{1}{11}$$

(iv) an "A"

Here A appears twice.

So, $n(A) = 2$

$n(S) = 11$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{2}{11}$$

(v) not "M"

Here M appears twice.

But not "M".

So, $n(A) = 11 - 2 = 9$

$n(S) = 11$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{9}{11}$$

(vi) Here T appears twice.

But not "T".

So, $n(A) = 11 - 2 = 9$

$n(S) = 11$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{9}{11}$$

**Q.4. Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of not getting the numbers 3 or 4.**

**Sol.** We have to find probability of getting the numbers 3 to 4.

Total outcomes = n(S) = 6

Favourable outcomes = n(A) = 2

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

Probability of not getting the numbers 3 or 4.

$$P(A') = 1 - P(A)$$

$$P(A') = 1 - \frac{1}{3} = \frac{2}{3}$$

**Q.5. Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:**

**(i) the number 25**

**Sol.** Total outcomes = n(S) = 30

Favourable outcomes = n(A) = 1 (the number 25)

Probability that selected card contains the number 25

$$= \frac{n(A)}{n(S)} = \frac{1}{30}$$

**(ii) number between 17 to 22**

**Sol.** Total outcomes = n(S) = 30

Favourable outcomes = n(A) = 6.

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{6}{30} = \frac{1}{5}$$

**(iii) number at least 20**

**Sol.** Total outcomes = n(S) = 30.

Favourable outcomes = n(A) = 11

(20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30)

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{11}{30}$$

**(iv) number not 27 and 29**

**Sol.** Total outcomes = n(S) = 30

Number not 27 and 29.

Favourable outcomes = 30 – 2 = 28 = n(A)

Using $$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{28}{30} = \frac{14}{15}$$

**(v) number not between 12 – 15**

**Sol.** Total outcomes = n(S) = 30

Favourable outcomes = n(A) = 30 – 4 = 26

(outcomes contain those numbers which are not between 12 – 15)

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{26}{30} = \frac{13}{15}$$

**Q.6. The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination?**

**Sol.** Probability that Ayesha will pass the examination = P(A) = 0.85

Probability that Ayesha will not pass examination = ?

$\because$ $P(A') + P(A) = 1$

$P(A') = 1 - P(A)$

$= 1 - 0.85$

$= 0.15$

Where A′ denotes

Ayesha will not pass examination.

**Q.7. Taabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:**

**(i) tail on coin and at least 4 on dice.**

**Sol.**

| For coin | For dice |
|---|---|
| Favourable outcome | Favourable outcome |
| $n(A) = 1$ | $n(B) = 3$ |
| $n(S) = 2$ | $n(S) = 6$ |
| $P(A) = \frac{1}{2}$ | $P(B) = \frac{3}{6} = \frac{1}{2}$ |

Probability of events $= P(A) \times P(B)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

**(ii) head on coin and the number 2,3 on dice.**

**Sol.**

| Head on coin, probability | On dice, the numbers 2, 3 |
|---|---|
| $= \frac{1}{2}$ | probability $= \frac{2}{6} = \frac{1}{3}$ |

Probability of events $= \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$

**(iii) head and tail on coin and the number 6 on dice.**

**Sol.** Head and tail on coin.

Favourable outcomes = 2

$$\text{Probability} = \frac{2}{2} = 1$$

Number 6 on dice

No. of favourable outcomes = 1

$$\text{Probability} = \frac{1}{6}$$

$$\text{Required probability} = 1 \times \frac{1}{6} = \frac{1}{6}$$

**(iv) not tail on coin and the number 5 on dice.**

**Sol.** Not-tail on coin.

$$\text{Probability} = \frac{1}{2}$$

Number 5 on dice

$$\text{Probability} = \frac{1}{6}$$

$$\text{Required probability} = \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

**(v) not head on coin and the number 5 and 2 on dice.**

**Sol.** Not head, probability $= \frac{1}{2}$

Number 5 and 2, probability $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Let A be the event, then

$$P(A) = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

**Q.8. A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting:**

**(i) a queen**

**Sol.** No. of favourable outcomes = n(A) = 4 (Queen)

Total possible outcomes = n(S) = 52

Probability of selecting a queen

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{52} = \frac{1}{13}$$

**(ii) neither a queen nor a jack**

**Sol.** No. of jacks in a deck of 52 cards = 4

No. of queen in a deck of 52 cards = 4

Total no. of queen and jack = 4 + 4 = 8

No. of cards which is neither a queen nor a jack

$$= 52 - 8 = 44$$

Probability P(E)

$$= \frac{\text{No. of outcoms favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

$$= \frac{44}{52}$$

$$P(E) = \frac{11}{13}$$

**Q.9. A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:**

**(i) a jack**

**Sol.** No. of jack = 4

So probability of getting a jack $= \frac{4}{52} = \frac{1}{13}$

(ii) no diamond

**Sol.** Given, a standard deck of 52 cards, we have to find the probability of getting no diamond. There are 13 cards of diamond in a 52-card deck.

So probability of getting no diamond $= 1 - \frac{13}{52}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

## EXERCISE 13.2

**Q.1. A researcher collected data on number of deaths from Horse-Ricks in Russian Army crops over to years. The table is as follows:**

| No. of death | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Frequency | 60 | 50 | 87 | 40 | 32 | 15 | 10 |

**Find the relative frequency of the given data.**

**Sol.**

| No. of deaths | Frequency | Relative Frequency |
|---|---|---|
| 0 | 60 | $\frac{60}{294} = \frac{30}{147}$ |
| 1 | 50 | $\frac{50}{294} = \frac{25}{147}$ |
| 2 | 87 | $\frac{87}{294} = \frac{29}{98}$ |
| 3 | 40 | $\frac{40}{294} = \frac{20}{147}$ |
| 4 | 32 | $\frac{32}{294} = \frac{16}{147}$ |
| 5 | 15 | $\frac{15}{294} = \frac{5}{98}$ |
| 6 | 10 | $\frac{10}{294} = \frac{5}{147}$ |
| | $\Sigma f = 294$ | |

**Q.2.** The frequency of defective products in 750 samples are shown in the following table. Find the relative frequency for the given table.

| No. of defectives per sample | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| No. of sample | 120 | 140 | 94 | 85 | 105 | 50 | 40 | 66 | 50 |

**Sol.**

| No. of defective per sample | No. of samples | Relative Frequency |
|---|---|---|
| 0 | 120 | $\frac{120}{750} = \frac{4}{25}$ |
| 1 | 140 | $\frac{140}{750} = \frac{14}{75}$ |
| 2 | 94 | $\frac{94}{750} = \frac{47}{375}$ |
| 3 | 85 | $\frac{85}{750} = \frac{17}{150}$ |
| 4 | 105 | $\frac{105}{750} = \frac{21}{150}$ |
| 5 | 50 | $\frac{50}{750} = \frac{1}{15}$ |
| 6 | 40 | $\frac{40}{750} = \frac{4}{75}$ |
| 7 | 66 | $\frac{66}{750} = \frac{33}{375}$ |
| 8 | 50 | $\frac{50}{750} = \frac{1}{15}$ |
| | $\Sigma f = 750$ | |

Q.3. A quiz competition on general knowledge is conducted. The number of corrected answers out of 5 questions for 100 sets of questions is given below.

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f$ | 10 | 23 | 15 | 25 | 18 | 9 |

Find the relative frequencies for the given data.

Sol.

| X | f | Relative Frequency |
|---|---|---|
| 0 | 10 | $\frac{10}{100} = \frac{1}{10}$ |
| 1 | 23 | $\frac{23}{100}$ |
| 2 | 15 | $\frac{15}{100} = \frac{3}{20}$ |
| 3 | 25 | $\frac{25}{100} = \frac{1}{4}$ |
| 4 | 18 | $\frac{18}{100} = \frac{9}{50}$ |
| 5 | 9 | $\frac{9}{100}$ |
| | $\Sigma f = 100$ | |

Q.4. A survey was conducted from the 50 students of a class and asked about their favourite food. The responses are as under:

| Name of food item | Biryani | Fresh Juice | Chicken | Bar.B.Q | Sweets |
|---|---|---|---|---|---|
| No. of Students | 40 | 07 | 21 | 15 | 25 |

i) how many percentages of students like biryani?

Sol. Total no. of students $= 40 + 07 + 21 + 15 + 25$

$= 108$

Percentage of students who like Biryani $= \frac{40}{108} \times 100\%$

$= 37.04\%$

**(ii) how many percentage of students like chicken?**

**Sol.** Percentage of student who like chicken $= \frac{21}{100} \times 100\%$

$= \frac{2100}{108}\%$

$= 19.44\%$

**(iii) which food is the least liked by the students?**

**Sol.** Fresh juice is liked by least no. of students.

**(iv) which food is the most prefer by the students?**

**Sol.** Biryani is most preferred by the students.

**Q.5. In 500 trials of a thrown of two dice, what is expected frequency that the sum will be greater than 8?**

**Sol.** Possible outcomes = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

No. of favourable outcomes (sum greater than 8)

$= n(A) = 10,$

Total possible outcomes $= n(S) = 36$

Expected frequency $= \frac{10}{36} \times 500$

$= \frac{5000}{36} = 138.88$

$\approx 139$

**Q.6. What is the expectation of a person who is to get Rs.120 if he obtains at least 2 heads in single toss of three coins?**

**Sol.** Sample space

HHH. HHT, HTH, THH, HTT, THT, TTH, TTT

Total no. of sample space is 8. Now you have to understand that what does at least mean. It means you have to consider, the value with 2 heads or more. For at least two heads

HHH, HHT, HTH, THH

Total no. of outcomes = 4

$$\text{Probability (at least two heads)} = \frac{\text{No. of outcomes}}{\text{Total no. of possible outcomes}}$$

$$= \frac{4}{8} = \frac{1}{2}$$

$$\text{Expectation of a person to get} = \frac{1}{2} \times 120$$

$$= \text{Rs.}60$$

**Q.7. Find the expected frequencies of the given data if the experiment is repeated 200 times.**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(x) | 0.11 | 0.21 | 0.17 | 0.18 | 0.09 | 0.17 | 0.07 |

**Sol.** Experiment is repeated 200 times.

| x | P(x) | Expected frequencies |
|---|---|---|
| 0 | 0.11 | $0.11 \times 200 = 22$ |
| 1 | 0.21 | $0.21 \times 200 = 42$ |
| 2 | 0.17 | $0.17 \times 200 = 34$ |
| 3 | 0.18 | $0.18 \times 200 = 36$ |
| 4 | 0.09 | $0.09 \times 200 = 18$ |
| 5 | 0.17 | $0.17 \times 200 = 34$ |
| 6 | 0.07 | $0.07 \times 200 = 14$ |

# REVIEW EXERCISE 13

**Q.1. Four options are given against each statement. Encircle the correct option.**

(i) Each element of the sample space is called:

(a) event (b) experiment

(c) sample point (d) outcomes

**Sol. (c)**

(ii) An outcome which represents how many times we expect the things to be happened is called:

(a) outcomes (b) favourable outcome

(c) sample space (d) sample point

**Sol. (b)**

(iii) Which one tells us how often a specific event occurs relative to the total number of frequency event or trials?

(a) expected frequency

(b) sum of relative frequency

(c) relative frequency (d) frequency

**Sol. (c)**

(iv) Estimated probability of an event occurring is also known as:

(a) relative frequency (b) expected frequency

(c) class boundaries

(d) sum of expected frequency

**Sol. (a)**

(v) The sum of all expected frequencies is equal to the fixed number of:

(a) trials (b) relative frequencies

(c) outcomes (d) events

**Sol. (a)**

(vi) The chance of occurrence of a particular event is called:

(a) sample space (b) estimated probability

(c) probability (d) expected frequency

**Sol.** **(c)**

(vii) An event which will probably occur. It has greater chance to occur is called:

(a) equally likely event (b) likely event

(c) unlikely event (d) certain event

**Sol.** **(b)**

(viii) Find out the total number of possible sample space when 4 dice are rolled:

(a) $6^2$ (b) $6^3$

(c) $6^4$ (d) $6^6$

**Sol.** **(c)**

(ix) While rolling a pair of dice, what will be the probability of double 2?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{5}{6}$ (d) $\frac{1}{36}$

**Sol.** **(d)**

(x) A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king:

(a) $\frac{2}{13}$ (b) $\frac{11}{13}$

(c) $\frac{2}{52}$ (d) $\frac{11}{52}$

**Sol.** **(b)**

**Q.2. Define the following:**

**(i) relative frequency**

**Sol.** Relative frequency is an estimated probability of an event occurring when an experiment is repeated a fixed number of times.

**(ii) expected frequency**

**Sol.** Expected frequency is a measure that estimate how often an event should be occurred depended on probability.

Expected frequency is found by using the following method.

Expected frequency = Total number of trials × probability of event

$$= N \times P(A)$$

**Q.3. An urn contain 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.**

**(i) a green ball (ii) a red ball**

**(iii) a blue ball (iv) not a red ball**

**(v) not a green ball**

**Sol.** No. of red balls $n(R) = 10$

No of green balls $= n(G) = 5$

No. of blue balls $= n(B) = 8$

(i) Probability of selecting at random a green ball = ?

Total no. of balls $= 10 + 5 + 8$

$$n(T) = 23$$

Probability of selecting green ball

$$P(G) = \frac{n(G)}{n(T)}$$

$$P(G) = \frac{5}{23}$$

(ii) a red ball

Probability of selecting red ball.

$$P(R) = \frac{n(R)}{n(T)}$$

$$P(R) = \frac{10}{23}$$

(iii) a blue ball.

Probability of blue ball.

$$P(B) = \frac{n(B)}{n(T)}$$

$$P(B) = \frac{8}{23}$$

(iv) Not a red

Probability that ball is not red

$$= 1 - P(R)$$

$$= 1 - \frac{10}{23}$$

$$= \frac{23 - 10}{23} = \frac{13}{23}$$

(v) Probability that ball is not green

$$= 1 - P(G)$$

$$= 1 - \frac{5}{23}$$

$$= \frac{23 - 5}{23} = \frac{18}{23}$$

**Q.4. Three coins are tossed together, what is the probability of getting:**

**(i) exactly three heads**

**Sol.** No. of favourable outcomes = n(A) = 1

Possible outcomes = $2^3 = 8 = n(S)$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{1}{8}$$

**HHH**
**HHT**
**HTH**
**THH**
**TTH**
**THT**
**HTT**
**TTT**

**(ii) at least two tails**

**Sol.** No. of favourable outcomes = n(A) = 4

Possible outcomes = n(S) = 8

$$\text{Probability} = \frac{n(A)}{n(S)}$$

$$= \frac{4}{8} = \frac{1}{2}$$

**(iii) not at least two heads**

**Sol.** No. of favourable outcomes = n(A) = 4

Possible outcomes = n(S) = 8

$$\text{Probability} = \frac{n(A)}{n(S)}$$

$$= \frac{4}{8} = \frac{1}{2}$$

**(iv) not exactly two heads**

**Sol.** Favourable outcomes = n(A) = 5

(Because outcomes in part (i) are HHH, TTH, THT, HTT, TTT)

Possible outcomes = n(S) = 8

Probability that not exactly two heads

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{5}{8}$$

**Q.5. A card is drawn from a well shuffled pack of 52 playing cards. What will be the probability of getting:**

**(i) king or jack of red colour**

**Sol.** There are 2 red jacks and 2 king out of 52 cards.

So $$\text{Probability} = \frac{4}{52}$$

$$= \frac{1}{13}$$

**(ii) not "2" of club and spade**

**Sol.** Probability of (not "2" to club and spade)

$$= \frac{50}{52} = \frac{25}{26}$$

**Q.6.** Six coins are tossed 600 times. The number of occurrence of tails are recorded and shon in the table given below:

| No. of tails | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Frequency | 110 | 90 | 105 | 80 | 76 | 123 | 16 |

**Find the relative frequency of given table.**

**Sol.**

| No. of tails | $f$ | Relative frequency |
|---|---|---|
| 0 | 110 | $\frac{110}{600} = \frac{11}{60}$ |
| 1 | 90 | $\frac{90}{600} = \frac{3}{20}$ |
| 2 | 105 | $\frac{105}{600} = \frac{7}{40}$ |
| 3 | 80 | $\frac{80}{600} = \frac{2}{15}$ |
| 4 | 76 | $\frac{76}{600} = \frac{19}{150}$ |
| 5 | 123 | $\frac{123}{600} = \frac{41}{200}$ |
| 6 | 16 | $\frac{16}{600} = \frac{2}{75}$ |
| | $\Sigma f = 600$ | |

**Q.7.** **From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non-defective items.**

**Sol.** No. of items = n(S) = 25

**No.** of defective items = n(A) = 8

**No.** of non-defective items = n(B) = 25 – 8 = 17

Relative frequency of non-defective items = $\frac{17}{25} = 0.68$

Expected frequency of non-defective items = 17