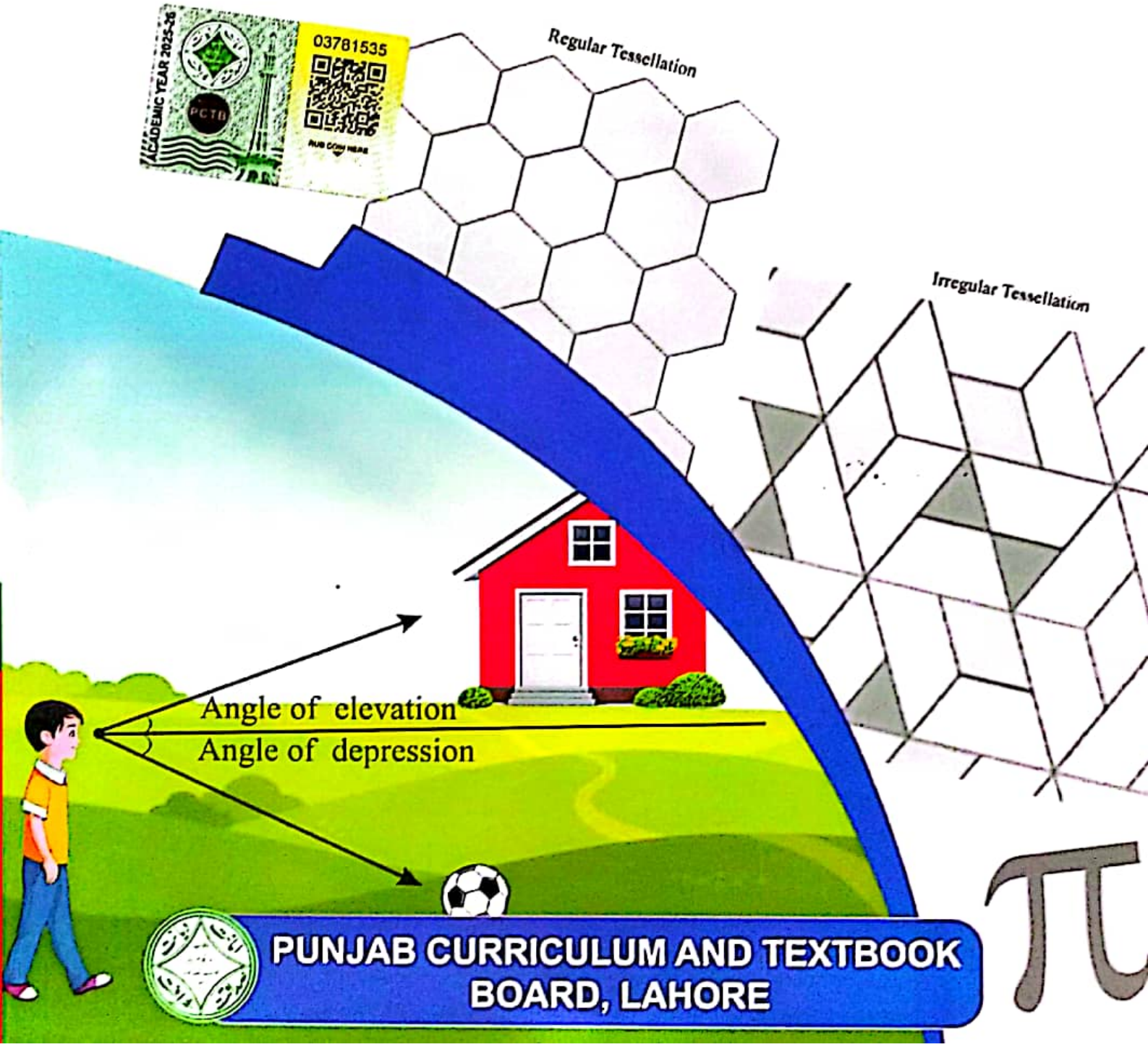
MATHEMATICS 9



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UNIT 6 TRIGONOMETRY

EXERCISE 6.1

Q.1. Find in which quadrant the following angles lie. Write co-terminal angle for each:

(i) 65° lies in 1st quadrant

(ii) 135° lies in 2nd quadrant

(iii) -40° lies in 4th quadrant

(iv) 210° lies in 3rd quadrant

(v) -150° lies in 3rd quadrant

Q.2. Convert the following into degrees, minutes, and seconds.

(i) 123.456°

Sol. $= 123^\circ + 0.456^\circ$

$1^\circ = 60$ minutes, $1' = 60''$

$= 123^\circ + 0.456 \times 60'$

$= 123^\circ + 27.36'$

$= 123^\circ + 27' + 0.36 \times 60''$

$= 123^\circ\ 27'\ 22''$

(ii) 58.7891°

Sol. $= 58^\circ + 0.781^\circ$

$= 58^\circ + 0.7891 \times 60'$

$= 58^\circ + 47.346'$

$= 58^\circ + 47' + 0.346'$

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$= 58° + 47' + 0.346 \times 60''$

$= 58° + 47' + 21''$

$= 58° \; 47' \; 21''$

(iii) **90.5678°**

Sol. $= 90° + 0.5678°$

$= 90° + 0.5678 \times 60'$

$= 90° + 34.068'$

$= 90° + 34' + 0.068'$

$= 90° + 34' + 4''$

$= 90° \; 34' \; 4''$

Q.3. Convert the following into decimal degrees.

(i) **65° 32′ 15″**

Sol. $= 65° \times 32' + 15''$

$= 65° + \left(\frac{32}{60}\right)^{\circ} + \left(\frac{15}{3600}\right)^{\circ} \qquad \left[1' = \left(\frac{1}{60}\right)^{''} ; 1° = \left(\frac{1}{3600}\right)^{\circ}\right]$

$= 65° + 0.5333° + 0.00417°$

$= 65.5375°$

(ii) **42° 18′ 45″**

Sol. $= 42° + 18' + 45''$

$= 42° + \left(\frac{18}{60}\right)^{\circ} + \left(\frac{45}{3600}\right)^{\circ}$

$= 42° + 0.3° + 0.0125°$

$= 42.3125°$

(iii) **78° 45′ 36″**

Sol. $= 78° + 45' + 36''$

$= 78° + \left(\frac{45}{60}\right)^{\circ} + \left(\frac{36}{3600}\right)^{\circ}$

$= 78° + 0.75° + 0.01°$

$= 78.76°$

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Q.4. Convert the following into radian.

(i) 36°

Sol. We know that

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

So, $$36^\circ = 36 \times \frac{\pi}{180} \text{ rad}$$

$$= \frac{\pi}{5} \text{ rad}$$

(ii) 22.5°

Sol. We know that

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

So, $$22.5^\circ = 22.5 \times \frac{\pi}{180}$$

$$= \frac{225}{10} \times \frac{\pi}{180}$$

$$= \frac{\pi}{8} \text{ rad}$$

(iii) 67.5°

Sol. We know that

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

So, $$67.5^\circ = 67.5 \times \frac{\pi}{180}$$

$$= \frac{675}{10} \times \frac{\pi}{180}$$

$$= \frac{3\pi}{8} \text{ rad}$$

Q.5. Convert the following into degrees.

(i) $\frac{\pi}{16}$ **rad**

Sol. We know that

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$$1 \text{ rad} = \left(\frac{\pi}{180}\right)^{\circ}$$

So, $$\frac{\pi}{16} \text{ rad} = \left(\frac{\pi}{16} \times \frac{180}{\pi}\right)^{\circ}$$

$$= \left(\frac{180}{16}\right)^{\circ}$$

$$= 11.25^{\circ}$$

or $$= 11^{\circ} + 0.25^{\circ}$$

$$= 11^{\circ} + (0.25 \times 60)'$$

$$= 11^{\circ} + 15'$$

$$11^{\circ}\ 15'$$

(ii) $\mathbf{\frac{11\pi}{5}}$ **rad**

Sol. We know that

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ}$$

So, $$\frac{11\pi}{5} \text{ rad} = \left(\frac{11\pi}{5} \times \frac{180}{\pi}\right)^{\circ}$$

$$= \left(\frac{11 \times 180}{5}\right)^{\circ}$$

$$= 11 \times 36$$

$$= 396^{\circ}$$

(iii) $\mathbf{\frac{7\pi}{6}}$ **rad**

Sol. We know that

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ}$$

So, $$\frac{7\pi}{6} \text{ rad} = \left(\frac{7\pi}{6} \times \frac{180}{\pi}\right)^{\circ}$$

$$= \left(\frac{7 \times 180}{6}\right)^{\circ}$$

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$= (7 \times 30)^{\circ}$

$= 210^{\circ}$

Q.6. Find the arc length and area of a sector if:

(i) $r = 6$ cm and central angle $\theta = \frac{\pi}{3}$ radians.

Sol. Arc length = ?

Area of sector = ?

$r = 6$cm and central angle $\theta = \frac{\pi}{3}$ radian.

Using

$$\ell = r\theta$$

$$= 6 \times \frac{\pi}{3} \text{ cm}$$

$$= 2\pi \text{ cm}$$

$$= 2 \times 3.14$$

Arc length $= \ell = 6.28$ cm

Now area of sector $= A = \frac{1}{2} r^2\theta$

$$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} \text{ cm}^2$$

$$= \frac{1}{2} \times 36 \times \frac{3.14}{3} \text{ cm}^2$$

$$= 18.84 \text{ cm}^2$$

(ii) $r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians.

Sol. Arc length $= \ell = ?$

Using

$$\ell = r\theta$$

$$= \frac{4.8}{\pi} \times \frac{5\pi}{6} \text{ cm}$$

$$= \frac{4.8 \times 5}{6} \text{ cm}$$

$$\ell = 4 \text{ cm}$$

Now area of sector = $A = \frac{1}{2} r^2 0$

$$= \frac{1}{2} \times \left(\frac{4.8}{\pi}\right)^2 \times \frac{5\pi}{6} \text{ cm}^2$$

$$= \frac{1}{2} \times \frac{23.04}{\pi^2} \times \frac{5\pi}{6}$$

$$= \frac{23.04 \times 5}{2 \times \pi \times 6} = \frac{115.2}{12 \times 3.14}$$

$$= \frac{115.2}{37.68}$$

$$A = 3.057 \text{ cm}^2$$

Q.7. If the central angle of a sector is 50° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it represents.

Sol. Central angle = $\theta = 60^\circ$

$$= 60 \times \frac{\pi}{180} \text{ rad}$$

$$= \frac{\pi}{3} \text{ rad}$$

Radius of the circle = r = 12cm

Area of sector = $\frac{1}{2} r^2 0$

$$= \frac{1}{2} \times (12)^2 \times 3\text{cm}^2$$

$$= \frac{1}{2} \times 12 \times 12 \times \frac{\pi}{3}$$

$$= 24\pi$$

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$= 24 \times 3.142$

$= 75.408 \text{ cm}^2$

Area of circle $= \pi r^2$

$= 3.14 \times 12^2$

3.14×144

$= 452.16 \text{ cm}^2$

Required percentage $= \dfrac{\text{area of sector}}{\text{area of circle}} \times 100\%$

$= \dfrac{75.408}{452.16} \times 100\%$

$= \dfrac{7540.8}{452.16}\%$

$= 16.68\%$

Q.8. Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.

Sol. Central angle $0 = \dfrac{\pi}{8}$ rad

Area of sector $= \dfrac{1}{2} r^2 0$

$= \dfrac{1}{2} r^2 \times \dfrac{\pi}{8}$

$= \dfrac{r^2\pi}{16}$

Area of circle $= \pi r^2$

Required percentage $= \dfrac{\frac{r^2\pi}{16}}{\pi r^2} \times 100\%$

$= \dfrac{100}{16}\%$

$= 6.25\%$

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Q.9. A circular sector of radius r = 12 has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?

Sol. Given radius of circle = 12 cm

Central angle $\theta = 150^\circ = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$ radian

Area of sector $= \frac{1}{2} r^2\theta$

$$= \frac{1}{2} \times 12^2 \times \frac{5\pi}{6}$$

$$= \frac{1}{2} \times 144 \times \frac{5\pi}{6}$$

$$= 60\pi \text{ cm}^2$$

Area of sector is nothing but the curved surface area of sector.

$$60\pi = \pi \times r \times \ell$$

$$\ell = \frac{60}{12}$$

$$\ell = 5 \text{ cm}$$

We know that

Slant height of cone $= \sqrt{\ell^2 + r^2}$

$$= \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169} = 13\text{cm}$$

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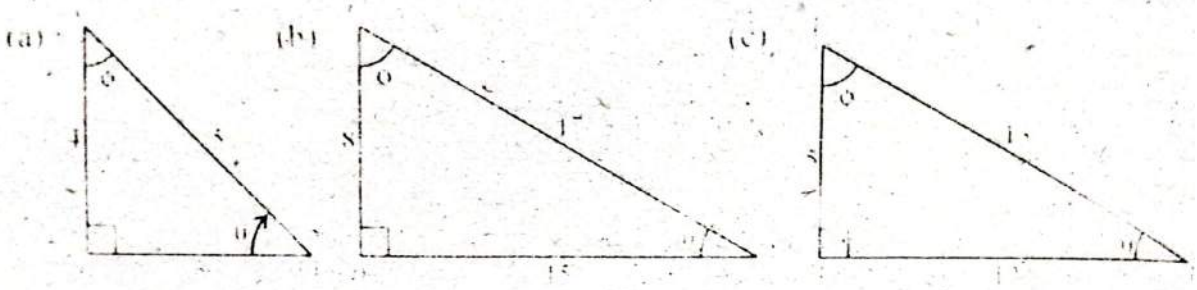
EXERCISE 6.2

Q.1. **For each of the following right-angled triangles, find the trigonometric ratios.**

(i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan\theta$ (iv) $\sec\theta$

(v) $\operatorname{cosec}\theta$ (vi) $\cot\phi$ (vii) $\tan\phi$ (viii) $\operatorname{cosec}\phi$

(ix) $\sec\phi$ (x) $\cos\phi$



Sol. First we consider figure (i) and answer these parts.

(i) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

$= \dfrac{\overline{BC}}{\overline{CA}} = \dfrac{4}{5}$

(ii) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$

$= \dfrac{\overline{AB}}{\overline{CA}} = \dfrac{3}{5}$

(iii) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

$= \dfrac{\overline{BC}}{\overline{AB}} = \dfrac{4}{3}$

(iv) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{3}{5}} = \dfrac{5}{3}$

(v) $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{4}{5}} = \dfrac{5}{4}$

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(vi) $\cot\phi = \frac{1}{\tan\phi} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

(vii) $\tan\phi = \frac{\text{Perpendicular}}{\text{Base}}$

$= \frac{3}{4}$

(viii) $\text{cosec}\,\phi = \frac{1}{\sin\phi} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

(ix) $\sec\phi = \frac{1}{\cos\phi} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

(x) $\cos\phi = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

Now we consider figure (ii) and answer these parts.

(i) $\sin\theta = \frac{8}{17}$

(ii) $\cos\theta = \frac{15}{17}$

(iii) $\tan\theta = \frac{8}{15}$

(iv) $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{15}{17}} = \frac{17}{15}$

(v) $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$

(vi) $\cot\phi = \frac{1}{\tan\phi} = \frac{1}{\frac{15}{8}} = \frac{8}{15}$

(vii) $\tan\phi = \frac{\text{Perpendicular}}{\text{Base}} = \frac{15}{8}$

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(viii) $\text{cosec}\ \phi = \dfrac{1}{\sin\phi} = \dfrac{1}{\frac{15}{17}} = \dfrac{17}{15}$

(ix) $\sec\phi = \dfrac{1}{\cos\phi} = \dfrac{1}{\frac{8}{17}} = \dfrac{17}{8}$

(x) $\cos\phi = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{15}{17}$

Now we consider figure (iii) and answer these parts.

(i) $\sin\theta = \dfrac{5}{13}$

(ii) $\cos\theta \quad \dfrac{12}{13}$

(iii) $\tan\theta = \dfrac{5}{12}$

(iv) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{12}{13}} = \dfrac{13}{12}$

(v) $\text{cosec}\ \theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{5}{13}} = \dfrac{13}{5}$

(vi) $\cot\phi \quad \dfrac{1}{\tan\phi} = \dfrac{1}{\frac{12}{5}} = \dfrac{5}{12}$

(vii) $\tan\phi = \dfrac{12}{15}$

(viii) $\text{cosec}\ \phi = \dfrac{1}{\sin\phi} = \dfrac{1}{\frac{12}{13}} = \dfrac{13}{12}$

(ix) $\sec\phi = \dfrac{1}{\cos\phi} = \dfrac{1}{\frac{5}{13}} = \dfrac{13}{5}$

(x) $\cos\phi = \dfrac{5}{13}$

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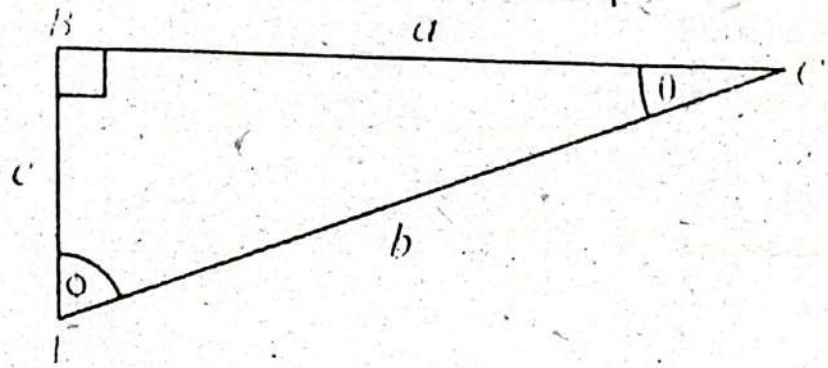
Q.2. **For each of the following right-angled triangles find the trigonometric ratios for which $\phi = m\angle A$, $\theta = m\angle C$.**

(i) $\sin\theta$ **(ii)** $\cos\theta$

(iii) $\tan\theta$ **(iv)** $\sin\phi$

(v) $\cos\phi$ **(vi)** $\tan\phi$



Sol. $\phi = m\angle A$, $\theta = m\angle C$

For figure (a)

(i) $\sin\theta = \dfrac{\overline{BC}}{\overline{AB}}$ (ii) $\cos\theta = \dfrac{\overline{AB}}{\overline{AC}}$

(iii) $\tan\theta = \dfrac{\overline{AB}}{\overline{BC}} = \dfrac{c}{a}$ (iv) $\sin\phi = \dfrac{b}{a}$

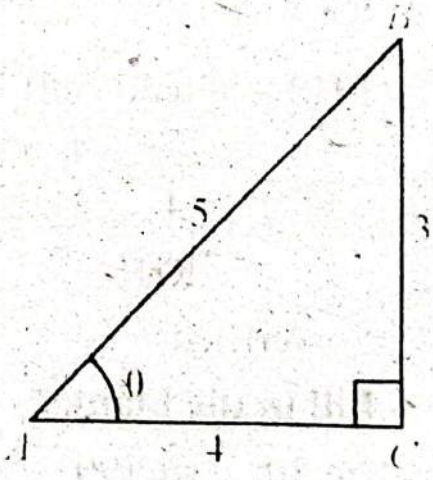
(v) $\cos\phi = \dfrac{c}{b}$ (vi) $\tan\phi = \dfrac{a}{c}$

Q.3. **Considering the adjoining triangle ABC, verify that:**

(i) $\sin\theta \operatorname{cosec}\theta = 1$

(ii) $\cos\theta \sec\theta = 1$

(iii) $\tan\theta \cot\theta = 1$



(i) $\sin\theta \operatorname{cosec}\theta = 1$

Sol. $\sin\theta = \dfrac{3}{5}$

$\text{cosec } \theta = \frac{5}{3}$

LHS $= \sin\theta \text{ cosec}\theta$

$= \frac{3}{5} \times \frac{5}{3} = 1$

$=$ RHS

So verified.

(ii) **$\cos \theta \sec \theta = 1$**

Sol. $\cos \theta = \frac{4}{5}$

$\sec \theta = \frac{5}{4}$

LHS $= \cos\theta \sec\theta$

$= \frac{4}{5} \times \frac{5}{4} = 1$

$=$ RHS

So verified.

(iii) **$\tan \theta \cot \theta = 1$**

Sol. $\tan \theta = \frac{3}{4}$

$\cot \theta = \frac{4}{3}$

LHS $= \tan\theta \cot\theta$

$= \frac{3}{4} \times \frac{4}{3} = 1$

$=$ RHS

So verified.

Q.4. **Fill in the blanks.**

(i) $\sin 30^\circ = \sin (90^\circ - 60^\circ) = \cos$ **60°**

(ii) $\cos 30^\circ = \cos (90^\circ - 60^\circ) = \sin$ **60°**

(iii) $\tan 60^\circ = \sin (90^\circ - 30^\circ) = \cot$ **30°**

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(iv) $\sin 60^\circ = \sin(90^\circ - 30^\circ) = \cos$ **30°**

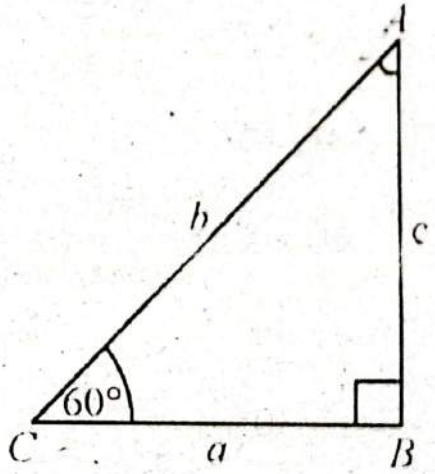
(v) $\cos 60^\circ = \cos(90^\circ - 30^\circ) = \sin$ **30°**

(vi) $\sin 45^\circ = \sin(90^\circ - 45^\circ) = \cos$ **45°**

(vii) $\tan 45^\circ = \tan(90^\circ - 45^\circ) = \cot$ **45°**

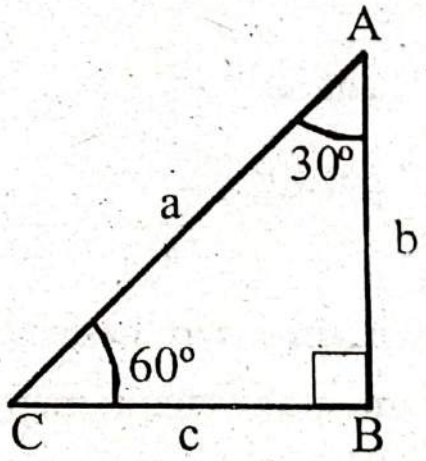
(viii) $\cos 45^\circ = \cos(90^\circ - 45^\circ) = \sin$ **45°**

Q.5. If in a right angled triangle ABC, $m\angle B = 90^\circ$ and C is an acute angle of 60°. Also $\sin m\angle A = \frac{a}{b}$, then find the following trigonometric ratios.



(i) $\dfrac{m\overline{BC}}{m\overline{AB}}$

Sol. $\dfrac{m\overline{BC}}{m\overline{AB}} = \dfrac{c}{a}$



(ii) cos 60°

Sol. $\cos 60^\circ = \dfrac{m\overline{BC}}{m\overline{AC}} = \dfrac{c}{b}$

(iii) tan 60°

Sol. $\tan 60^\circ = \dfrac{m\overline{AB}}{m\overline{BC}} = \dfrac{a}{c}$

(iv) cosec $\frac{\pi}{6}$

Sol. $\operatorname{cosec}\dfrac{\pi}{6} = \dfrac{1}{\frac{a}{b}} = \dfrac{b}{a}$

(v) cot 60°

Sol. $\cot 60^\circ = \dfrac{1}{\frac{a}{c}} = \dfrac{c}{a}$

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(vi) **sin 30°**

Sol. $\sin 30^\circ = \dfrac{m\overline{BC}}{m\overline{AC}} = \dfrac{c}{b}$

(vii) **cos 30°**

Sol. $\cos 30^\circ = \dfrac{m\overline{AB}}{m\overline{AC}} = \dfrac{a}{b}$

(viii) **tan 30°**

Sol. $\tan 30^\circ = \dfrac{m\overline{BC}}{m\overline{AB}} = \dfrac{c}{a}$

(ix) **sec 30°**

Sol. $\sec 30^\circ = \dfrac{1}{\frac{a}{b}} = \dfrac{b}{a}$

(x) **cot 30°**

Sol. $\cot 30^\circ = \dfrac{1}{\frac{c}{a}} = \dfrac{a}{c}$

Trigonometric Identities

Fundamental Trigonometric Identities

We shall consider some of the fundamental identities used in trigonometry. The key to these basic identities is the Pythagoras theorem in geometry.

"The square of the length of the hypotenuse of a right triangle is equal to the sum of the square of the lengths of the other two sides." (Fig. 1)

$$c^2 = a^2 + b^2$$

$$5^2 = 3^2 + 4^2$$

$$25 = 9 + 16$$

In figure (1) the opposite side equals to the length 'a' adjacent side equals to the length 'b', and hypotenuse equals to the length 'c'.

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By Pythagoras Theorem, we have

$a^2 + b^2 = c^2$ ...(i)

$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$ (Dividing by c^2)

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$a^2 + b^2 = c^2$

$\frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{c^2}{b^2}$ Dividing equation (i) by b^2), we have

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$

$\frac{a^2}{a^2} + \frac{b^2}{a^2} = \frac{c^2}{a^2}$ Dividing equation (i) by a^2), we have

$\boxed{1 + \cot^2\theta = \text{cosec}^2\theta}$ ...(iv)

→ $\sin^2\theta + \cos^2\theta = 1$

→ $\tan^2\theta + 1 = \sec^2\theta$

→ $1 + \cot^2\theta = \text{cosec}^2\theta$

Example

Show that $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Solution

L.H.S $= (\sec^2\theta - 1)\cos^2\theta$

$= \tan^2\theta \,.\, \cos^2\theta$ $(\because 1 + \tan^2\theta = \sin^2\theta)$

$= \frac{\sin^2\theta}{\cos^2\theta} \,.\, \cos^2\theta$ $\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \sin^2\theta =$ R.H.S

Hence, $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Example

Show that $\tan\theta + \cot\theta = \sec\theta\,\text{cosec}\theta$

Solution

L.H.S $= \tan\theta + \cot\theta$

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$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta.\sin\theta + \cos\theta.\cos\theta}{\cos\theta.\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$$

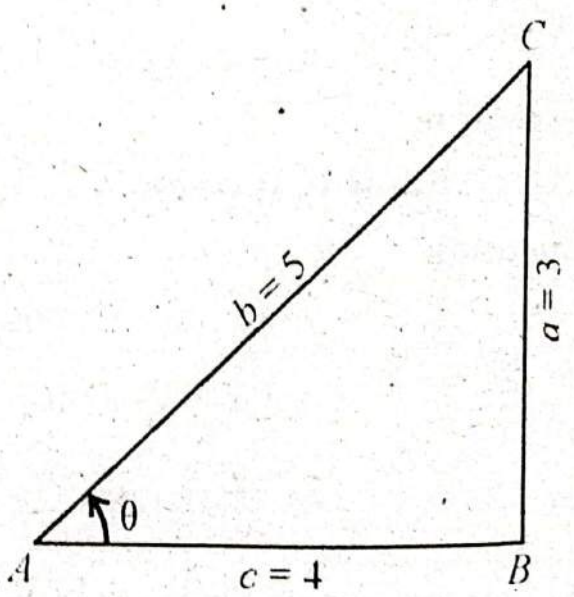
$$= \sec\theta.\text{cosec}\theta = \text{R.H.S}$$

Hence, $\tan\theta + \cot\theta = \sec\theta\ \text{cosec}\theta$

Example

If $\tan\theta = \frac{3}{4}$, find the remaining trigonometric ratios, when θ lies in first quadrant.

Solution



Given: $\tan\theta = \frac{3}{4} = \frac{a}{c}$,

Where, $a = 3$, $c = 4$, $b = ?$

By Pythagoras theorem, we have

$$b^2 = a^2 + c^2$$

$$= 9 + 16 = 25$$

$$\Rightarrow \quad b^2 = 5$$

Therefore, $\sin\theta = \frac{a}{b} = \frac{3}{5}$; $\text{cosec}\theta = \frac{b}{a} = \frac{5}{3}$

$\cos\theta = \frac{c}{b} = \frac{4}{5}$; $\sec\theta = \frac{b}{c} = \frac{5}{4}$

$\cot\theta = \frac{c}{a} = \frac{4}{3}$

EXERCISE 6.3

Q.1. If θ lies in first quadrant, find the remaining trigonometric ratios of θ.

(i) $\sin\theta = \frac{2}{3}$

Sol. $\sin\theta = \frac{2}{3} = \frac{a}{b}$

$a = 2, b = 3, c = ?$

Using Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$3^2 = 2^2 + c^2$$
$$9 - 4 = c^2$$
$$c^2 = 5$$
$$c = \sqrt{5}$$

$$\cos\theta = \frac{c}{b} = \frac{\sqrt{5}}{3}$$

$$\tan\theta = \frac{a}{c} = \frac{1}{\sqrt{5}}$$

$$\cot\theta = \frac{c}{a} = \frac{\sqrt{5}}{2}$$

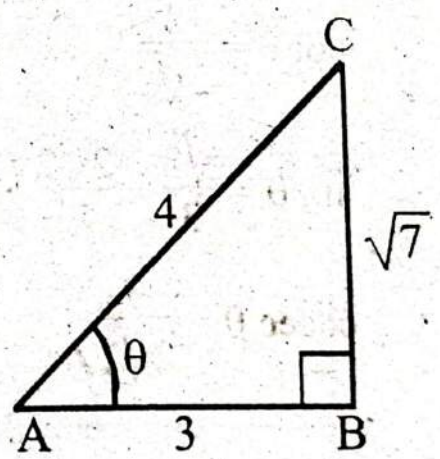
$$\operatorname{cosec}\theta = \frac{b}{a} = \frac{3}{2}$$

$$\sec\theta = \frac{b}{c} = \frac{3}{\sqrt{5}}$$

(ii) $\cos\theta = \frac{3}{4}$

Sol. $(\overline{AB})^2 + (\overline{BC})^2 = (\overline{AC})^2$

$$3^2 + (\overline{BC})^2 = 4^2$$
$$(\overline{BC})^2 = 16 - 9$$
$$(\overline{BC})^2 = 7$$
$$\overline{BC} = \sqrt{7}$$



$\sin\theta = \frac{\sqrt{7}}{4}$, $\text{cosec}\,\theta = \frac{4}{\sqrt{7}}$,

$\tan\theta = \frac{\sqrt{7}}{3}$, $\cot\theta = \frac{3}{\sqrt{7}}$, $\sec\theta = \frac{4}{3}$

(iii) $\tan\theta = \frac{1}{2}$

Sol. Hypotenuse $= \sqrt{2^2 + 1^2}$

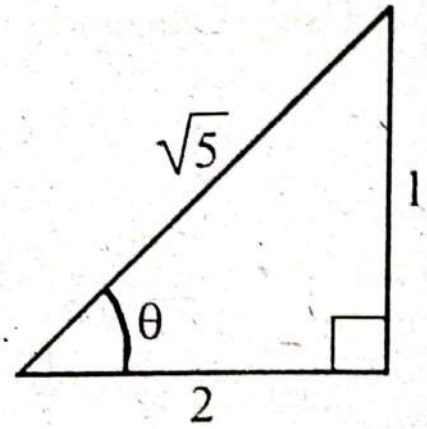
$\cot\theta = \frac{2}{1} = 2$

$\sin\theta = \frac{1}{\sqrt{5}}$

$\text{cosec}\,\theta = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\sec\theta = \frac{\sqrt{5}}{2}$



(iv) $\sec\theta = 3$

Sol. Hypotenuse $= \sqrt{3^2 - 1^2}$

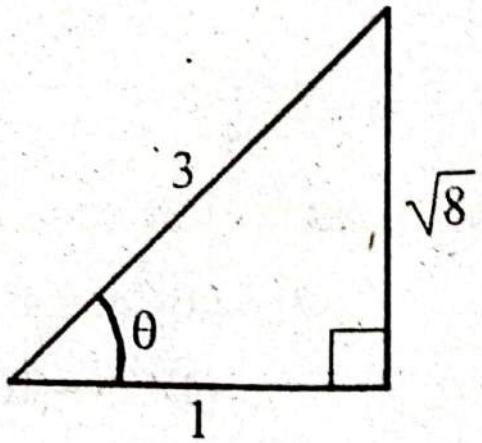
$= \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{\sqrt{8}}{1} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$

$\cot\theta = \frac{1}{2\sqrt{2}}$

$\sin\theta = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$

$\text{cosec}\,\theta = \frac{3}{2\sqrt{2}}$



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(v) $\cot\theta = \sqrt{\frac{3}{2}}$

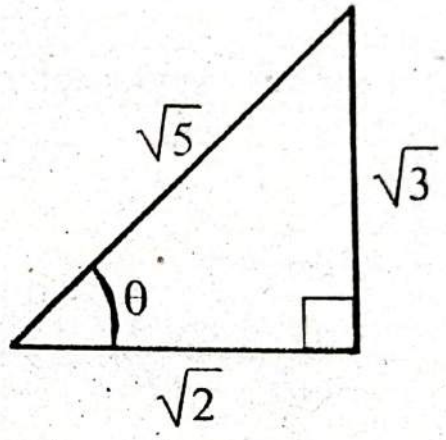
Sol. $\tan\theta = \sqrt{\frac{2}{3}}$

$\sin\theta = \frac{\sqrt{3}}{\sqrt{5}} = \sqrt{\frac{3}{5}}$

$\text{cosec}\,\theta = \sqrt{\frac{5}{3}}$

$\cos\theta = \frac{\sqrt{2}}{\sqrt{5}} = \sqrt{\frac{2}{5}}$

$\sec\theta = \sqrt{\frac{5}{2}}$



Prove the following identities:

Q.2. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Sol. L.H.S $= (\sin\theta + \cos\theta)^2$

Using $(a + b)^2 = a^2 + b^2 + 2ab$

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$=$ R.H.S

$[\because \sin^2\theta + \cos^2\theta = 1]$

<u>Additional Question</u>

Prove that $(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta$

L.H.S $= (\sin\theta - \cos\theta)^2$

Using $(a - b)^2 = a^2 + b^2 - 2ab$

$(\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$

$=$ R.H.S

$[\because \sin^2\theta + \cos^2\theta = 1]$

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Q.3. $\dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

Sol. R.H.S $= \dfrac{1}{\tan\theta}$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$$= \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$$

$$= \text{L.H.S}$$

So proved.

Q.4. $\dfrac{\sin\theta}{\operatorname{cosec}\theta} + \dfrac{\cos\theta}{\sec\theta} = 1$

Sol. L.H.S $= \dfrac{\sin\theta}{\operatorname{cosec}\theta} + \dfrac{\cos\theta}{\sec\theta}$

Since $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$

So $= \dfrac{\sin\theta}{\frac{1}{\sin\theta}} + \dfrac{\cos\theta}{\frac{1}{\cos\theta}}$

$$= \sin\theta \times \sin\theta + \cos\theta \times \cos\theta$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1 = \text{R.H.S}$$

So proved.

Q.5. $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

Sol. L.H.S $= \cos^2\theta - \sin^2\theta$

We know that

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

Put $\sin^2\theta = 1 - \cos^2\theta$ in L.H.S

$$= \cos^2\theta - (1 - \cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

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$= \cos^2\theta + \cos^2\theta - 1$

$= 2\cos^2\theta - 1 = \text{R.H.S}$

So proved.

Q.6. $\mathbf{\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta}$

Sol. L.H.S $= \cos^2\theta - \sin^2\theta$

We know that

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

Put $\cos^2\theta = 1 - \sin^2\theta$ in L.H.S

$= (1 - \sin^2\theta) - \sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta = \text{R.H.S}$

So proved.

Q.7. $\mathbf{\dfrac{1 - \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 + \sin\theta}}$

Sol. L.H.S $= \dfrac{1 - \sin\theta}{\cos\theta} \times \dfrac{1 + \sin\theta}{1 + \sin\theta}$

$= \dfrac{1^2 - \sin^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{\cos^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{\cos\theta}{1 + \sin\theta} = \text{RHS}$

So proved.

Q.8. $\mathbf{(\sec\theta - \tan\theta)^2 = \dfrac{1 - \sin\theta}{1 + \sin\theta}}$

Sol. L.H.S $= (\sec\theta - \tan\theta)^2$

$= \left(\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\dfrac{1 - \sin\theta}{\cos\theta}\right)^2 = \dfrac{(1 - \sin\theta)(1 - \sin\theta)}{1 - \sin^2\theta}$

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$$= \frac{(1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta}$$

$$= \text{R.H.S}$$

So proved.

Q.9. $\mathbf{(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta}$

Sol. L.H.S $= (\tan\theta + \cot\theta)^2$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}\right)^2 = \frac{1}{\cos^2\theta \sin^2\theta}$$

$$= \sec^2\theta \operatorname{cosec}^2\theta$$

$$= \text{R.H.S}$$

So proved.

Q.10. $\mathbf{\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta}$

Sol. L.H.S $= \dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta + \tan^2\theta - \sec^2\theta}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta + (\tan\theta + \sec\theta)(\tan\theta - \sec\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)[1 + \tan\theta - \sec\theta]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)}$$

$$= \tan\theta + \sec\theta$$

$$= \text{R.H.S}$$

So proved.

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Q.11. $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

Sol. L.H.S $= \sin^3\theta - \cos^3\theta$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

$=$ R.H.S

So proved.

Q.12. $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

Sol. L.H.S $= \sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)(1^2 - \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

$=$ R.H.S

So proved.

Values of Trigonometric Ratios of Angles

Trigonometric ratio of $45^\circ\left(\frac{\pi}{4}\text{ rad}\right)$

Consider a square ACBD of side length 1 unit.

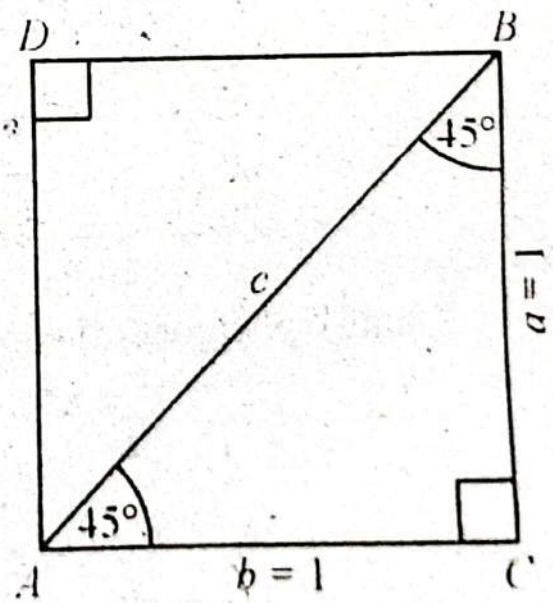
We know that the diagonal bisect the angles. So in the triangle ABC bisect the angles. So in triangle ABC

$m\angle A = m\angle B = 45^\circ$ and

$m\angle C = 90^\circ$.

Using Pythagoras theorem c^2

$= a^2 + b^2$



In ΔABC, $c^2 = 1 + 1$

$$c^2 = 2 \Rightarrow c = \sqrt{2}$$

Trigonometric ratio are:

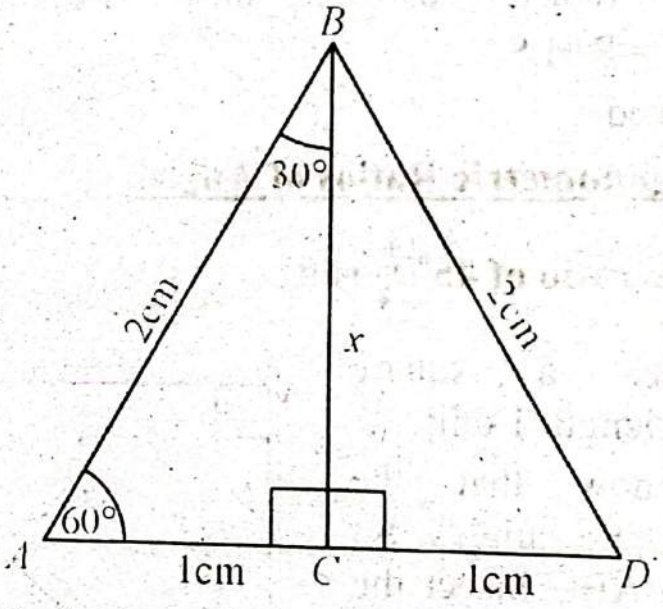
$$\sin 45^\circ = \frac{a}{c} = \frac{1}{\sqrt{2}} \quad ; \quad \operatorname{cosec} 45^\circ = \frac{c}{a} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cos 45^\circ = \frac{b}{c} = \frac{1}{\sqrt{2}} \quad ; \quad \sec 45^\circ = \frac{a}{b} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\tan 45^\circ = \frac{a}{b} = \frac{1}{1} = 1 \quad ; \quad \cot 45^\circ = \frac{b}{a} = \frac{1}{1} = 1$$

Trigonometric Ratios of $30^\circ \left(\frac{\pi}{6}\right)$ and $60^\circ \left(\frac{\pi}{3}\right)$

Consider an equilateral triangle ABD of side 2 units. Draw a perpendicular bisector AC on $\overline{BD}$. The point C bisects $\overline{BD}$. So, $m\overline{BC} = m\overline{CD}$ in which $m\angle B = 90^\circ$, $m\angle BAC = 30^\circ$, $m\angle ABC = 60^\circ$, $m\angle ACB = 90^\circ$, $m\overline{AC} = x$ units.



Using Pythagoras theorem in the ΔABC.

$$2^2 = 1^2 + x^2$$

$$4 - 1 = x^2$$

$$x^2 = 3$$

$$x = \sqrt{3}$$

$$m\overline{AC} = \sqrt{3}$$

Trigonometric Ratios of $30^\circ\left(\frac{\pi}{6}\text{ rad}\right)$

In ΔABC

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\cot 30^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\operatorname{cosec} 30^\circ = \frac{2}{1} = 2$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}}$$

Trigonometric Rations of $60^\circ\left(\frac{\pi}{6}\text{ rad}\right)$

In right angled ΔABC

$\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$

$\cos 60^\circ = \frac{1}{2}$, $\sec 60^\circ = \frac{2}{1} = 2$

$\tan 60^\circ = \sqrt{3}$, $\cot 60^\circ = \frac{1}{\sqrt{3}}$

θ	0	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

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EXERCISE 6.4

Q.1. Find the value of the following trigonometric ratios without using the calculator.

(i) **sin 30°**

Sol. $\sin 30^\circ = \frac{1}{2}$ or 0.5

(ii) **cos 30°**

Sol. $\cos 30^\circ = \frac{\sqrt{3}}{2}$ or 0.866

(iii) **$\tan \frac{\pi}{6}$**

Sol. $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ or 0.577

(iv) **tan 60°**

Sol. $\tan 60^\circ = \sqrt{3}$ or 1.732

(v) **sec 60°**

Sol. $\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2$

(vi) **$\cos \frac{\pi}{3}$**

Sol. $\cos \frac{\pi}{3} = \frac{1}{2}$ or 0.5

(vii) **cot 60°**

Sol. $\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}} = 0.577$

(viii) **sin 60°**

Sol. $\sin 60^\circ = \frac{\sqrt{3}}{2}$ or 0.866

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(ix) **sec 30°**

Sol. $\sec 30^\circ = \dfrac{1}{\cos 30^\circ} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$

(x) **cosec 30°**

Sol. $\text{cosec } 30^\circ = \dfrac{1}{\sin 30^\circ} = \dfrac{1}{\frac{1}{2}} = 2$

(xi) **sin 45°**

Sol. $\sin 45^\circ = \dfrac{1}{\sqrt{2}}$ or 0.707

(xii) $\cos \dfrac{\pi}{4}$

Sol. $\cos \dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ or 0.707

Q.2. **Find the values of each part:**

(i) **2 sin60° cos60°**

Sol. $= 2\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right)$

$= \dfrac{\sqrt{3}}{2}$

(ii) $2\cos\dfrac{\pi}{3} \ \sin\dfrac{\pi}{3}$

Sol. $= 2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{\sqrt{3}}{2}$

(iii) **2sin45° + 2cos45°**

Sol. $= 2(\sin 45^\circ + \cos 45^\circ)$

$= 2\left(\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\right)$

$= 2\left(\dfrac{1+1}{\sqrt{2}}\right) = 2 \times \dfrac{2}{\sqrt{2}} = \dfrac{4}{\sqrt{2}}$

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(iv) $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Sol. $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{3+1}{4} = \frac{4}{4} = 1$

(v) $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

Sol. $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= 0$

(vi) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

Sol. $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

(vii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Sol. $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{\sqrt{3}+\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

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(viii) $\tan\frac{\pi}{6}\cot\frac{\pi}{6}+1$

Sol. $=\left(\frac{1}{\sqrt{3}}\right).(\sqrt{3})+1$

$=1+1=2$

Q.3. If $\sin\frac{\pi}{4}$ and $\cos\frac{\pi}{4}$ equal to $\frac{1}{\sqrt{2}}$ each, then find the value of the followings:

(i) $2\sin45^\circ - 2\cos45^\circ$

Sol. $=2(\sin45^\circ-\cos45^\circ)$

$=2\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)$

$=2\left(\frac{1-1}{\sqrt{2}}\right)=2\times\frac{0}{\sqrt{2}}=0$

(ii) $3\cos45^\circ+4\sin45^\circ$

Sol. $=3\left(\frac{1}{\sqrt{2}}\right)+4\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{2}}$

$=\frac{3+4}{\sqrt{2}}=\frac{7}{\sqrt{2}}$

(iii) $5\cos45^\circ-3\sin45^\circ$

Sol. $=5\left(\frac{1}{\sqrt{2}}\right)-3\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{5}{\sqrt{2}}-\frac{3}{\sqrt{2}}$

$=\frac{5-3}{\sqrt{2}}=\frac{2}{\sqrt{2}}$

or $=\frac{2}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=\frac{2\sqrt{2}}{(\sqrt{2})^2}=\frac{2\sqrt{2}}{2}=\sqrt{2}$

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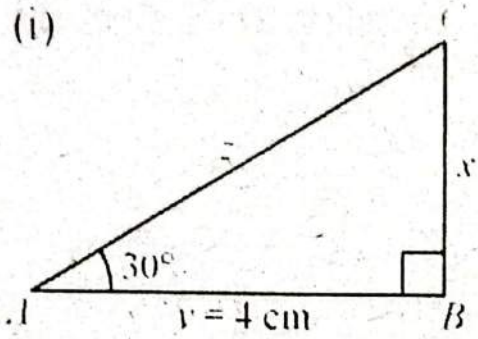
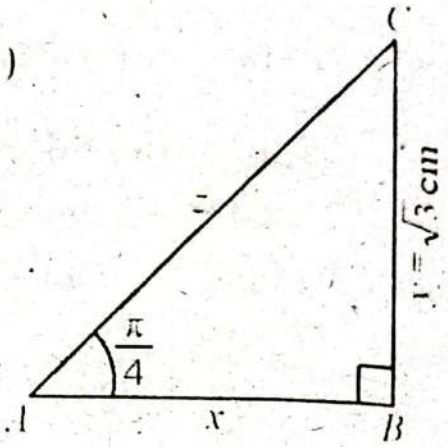
EXERCISE 6.5

Q.1. Find the values of x, y and z from the following right angled triangles.

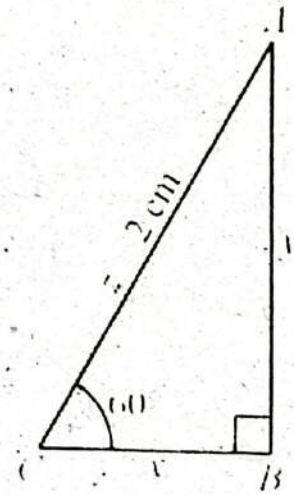
(i)



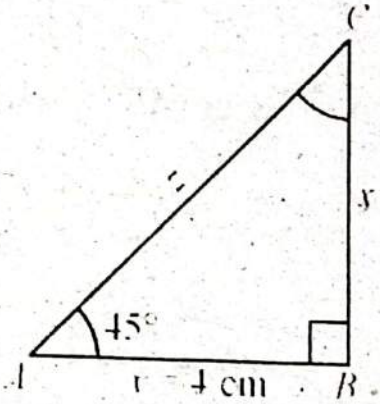
(ii)



(iii)



(iv)



Sol.(i) y = 4cm, x = ?, z = ?

Using $\tan 30^\circ = \dfrac{x}{y}$

$$\frac{1}{\sqrt{3}} = \frac{x}{4}$$

$$\sqrt{3}\,x = 4$$

$$x = \frac{4}{\sqrt{3}} \text{ cm}$$

Using Pythagoras theorem to find z.

$$z^2 = x^2 + y^2$$

$$= \left(\frac{4}{\sqrt{3}}\right)^2 + 4^2$$

$$z^2 = \frac{16+48}{3} = \frac{64}{3}$$

$$z = \sqrt{\frac{64}{3}}$$

$$z = \frac{8}{\sqrt{3}} \text{ cm}$$

(ii) $x = ?, y = \sqrt{3}$ cm, $z = ?$

This is an isosceles triangle.

$m\angle A = \frac{\pi}{4}$, $m\angle C = \frac{\pi}{4}$

Using $\tan\frac{\pi}{4} = \frac{y}{x}$

$$1 = \frac{\sqrt{3}}{x}$$

$$x = \sqrt{3} \text{ cm}$$

Using Pythagoras theorem to find z.

$$z^2 = x^2 + y^2$$

$$= (\sqrt{3})^2 + (\sqrt{3})^2$$

$$= 3 + 3$$

$$z^2 = 6$$

$$z = \sqrt{6} \text{ cm}$$

(iii) $x = ?, y = ?, z = 2$cm

$m\angle C = 60^\circ$

Using $\sin 60^\circ = \frac{y}{z}$

$$\frac{\sqrt{3}}{2} = \frac{y}{2}$$

$$2y = 2\sqrt{3}$$

$$y = \sqrt{3} \text{ cm}$$

Using $\tan 60^\circ = \frac{y}{x}$

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$$\sqrt{3} = \frac{\sqrt{3}}{x}$$

$$\sqrt{3}\,x = \sqrt{3}$$

$$x = 1 \text{ cm}$$

(vi) $y = 4\text{cm}, z = ?, x = ?$

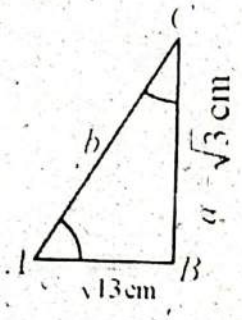
Using $\tan 45^\circ = \frac{x}{y}$

$$1 = \frac{x}{1}$$

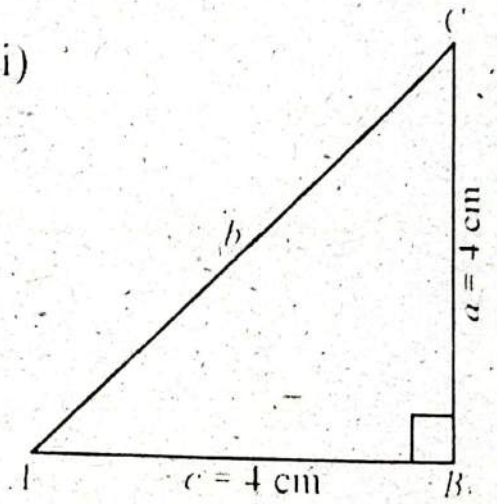
$$\Rightarrow \quad x = 1 \text{ cm}$$

Q.2. Find the unknown side and angles of the following triangles.

(i)



(ii)



Sol.(i) $c = \sqrt{13}\text{cm}, a = \sqrt{3}\text{ cm}, m\angle A = ?, m\angle C = ?$

Using Pythagoras theorem

$$b^2 = a^2 + c^2$$

$$= (\sqrt{3})^2 + (\sqrt{13})^2$$

$$= 3 + 13$$

$$b^2 = 16$$

$$b = 4 \text{ cm}$$

Using $\tan(m\angle A) = \frac{\sqrt{3}}{\sqrt{13}}$

$$m\angle A = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{13}}\right)$$

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$$m\angle A = 26.66^\circ$$

So $$m\angle C = 90^\circ - 26.66^\circ$$

$$m\angle C = 63.34^\circ$$

(ii) $c = 4cm$, $a = 4cm$, $b = ?$, $m\angle A = ?$, $m\angle C = ?$

Using Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$= 4^2 + 4^2$$
$$= 16 + 16$$
$$b^2 = 32$$
$$b^2 = 16 \times 2$$
$$b = \sqrt{16 \times 2}$$
$$b = 4\sqrt{2}\ cm$$

Using $\tan(m\angle A) = \frac{4}{4}$

$$\tan(m\angle A) = 1$$
$$m\angle A = \tan^{-1}(1)$$
$$m\angle A = 45^\circ$$

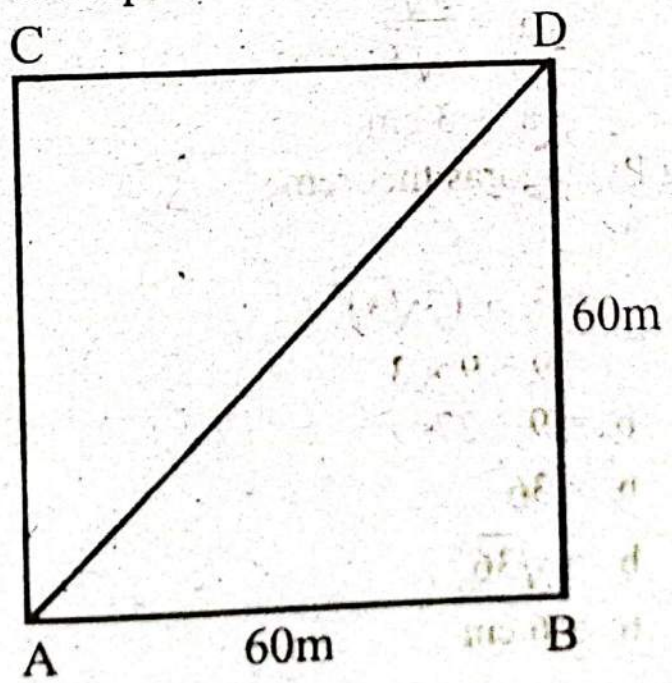
So $$m\angle C = 90^\circ - 45^\circ$$

$$m\angle C = 45^\circ$$

Q.3. Each side of a square field is 60m long. Find the lengths of the diagonals of the field.

Sol. Each side of square field = 60 m

ABCD is a square field of each side 60m.



CamScanner

ABC is the right angled triangle $\overline{AC}$ is the diagonal of the triangle.

Length of diagonal (by Pythagoras theorem) is

$$= \sqrt{60^2 + 60^2}$$
$$= \sqrt{3600 + 3600}$$
$$= \sqrt{7200}$$
$$= \sqrt{3600 \times 2}$$

Length of diagonal $= 60\sqrt{2}$ m

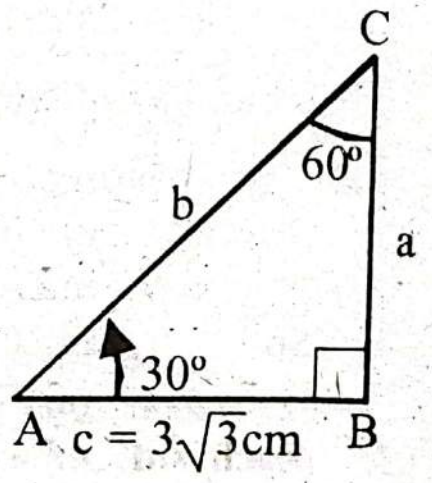
Solve the following triangles when:

Q.4. $\mathbf{m\angle C = 60^\circ, c = 3\sqrt{3}}$ **cm**

Sol. $m\angle C = 60^\circ$, $c = 3\sqrt{3}$ cm,

$m\angle B = 90^\circ$, $a = ?$,

$m\angle A = ?$, $b = ?$



$$m\angle A = 90^\circ - 60^\circ$$
$$m\angle A = 30^\circ$$

Using $\tan 30^\circ = \dfrac{a}{3\sqrt{3}}$

$$\frac{1}{\sqrt{3}} = \frac{a}{3\sqrt{3}}$$
$$a\sqrt{3} = 3\sqrt{3}$$
$$a = \frac{3\sqrt{3}}{\sqrt{3}}$$
$$a = 3 \text{ cm}$$

Using Pythagoras theorem,

$$b^2 = a^2 + c^2$$
$$= 3^2 + (3\sqrt{3})^2$$
$$= 9 + 9 \times 3$$
$$b^2 = 9 + 27$$
$$b^2 = 36$$
$$b = \sqrt{36}$$
$$\mathbf{b = 6 \text{ cm}}$$

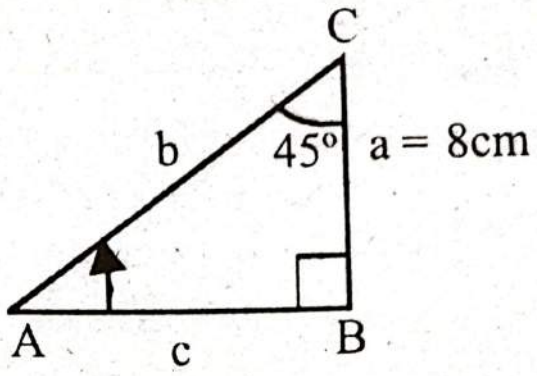
Q.5. $m\angle C = 45^\circ$, $a = 8$ cm

Sol. $m\angle C = 45^\circ$, $a = 8$cm,

$m\angle B = 90^\circ$, $b = ?$, $c = ?$

$m\angle A = 90^\circ - 45^\circ$

$m\angle A = 45^\circ$



Using $\tan(m\angle A) = \frac{a}{c}$

$$\tan 45^\circ = \frac{8}{c}$$

$$1 = \frac{8}{c}$$

$$c = 8\text{cm}$$

Using Pythagoras theorem,

$$b^2 = a^2 + c^2$$

$$= 8^2 + 8^2$$

$$= 64 + 64$$

$$b^2 = 128$$

$$b^2 = 64 \times 2$$

$$b = \sqrt{64 \times 2}$$

$$b = 8\sqrt{2}\text{ cm}$$

Q.6. $a = 12$cm, $c = 6$cm, $m\angle B = 90^\circ$

Sol. $(\overline{AC})^2 = 6^2 + 12^2$

$= 36 + 144$

$(\overline{AC})^2 = 180$

$|\overline{AC}| = \sqrt{180} = 6\sqrt{5}$cm

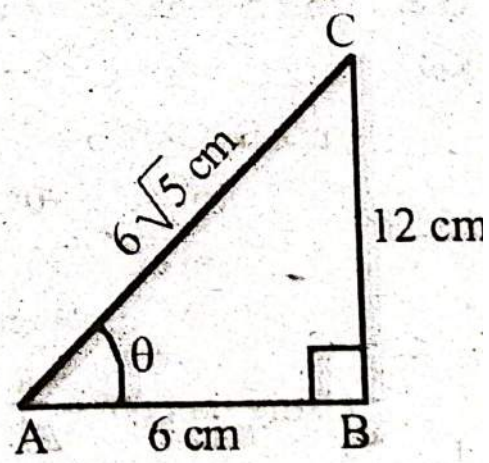
$\tan A = \frac{12}{6}$

$\tan A = 2$

$m\angle A = \tan^{-1}(2) = 63.4$

$m\angle C = 90^\circ - 63.4^\circ$

$m\angle C = 26.6^\circ$

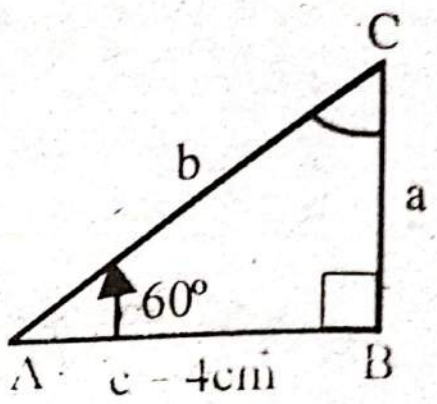


CamScanner

Q.7. **$m\angle A = 60^\circ$, $c = 4$ cm**

Sol. $m\angle C = 60^\circ$, $c = 4$cm,

$m\angle B = 90^\circ$, $b = ?$, $c = ?$



$$m\angle C = 90^\circ - 60^\circ$$

$$m\angle C = 30^\circ$$

$$c = 4 \text{ cm}$$

Using $\tan 60^\circ = \frac{a}{4}$

$$\sqrt{3} = \frac{a}{4}$$

$$a = 4\sqrt{3} \text{ cm}$$

Using Pythagoras theorem,

$$b^2 = a^2 + c^2$$

$$= (4\sqrt{3})^2 + (4)^2$$

$$= 16 \times 3 + 16$$

$$b^2 = 48 + 16$$

$$b^2 = 64$$

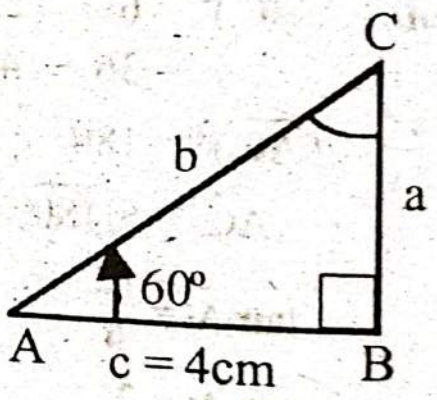
$$b = \sqrt{64}$$

$$b = 8 \text{ cm}$$

Q.8. **$m\angle A = 60^\circ$, $c = 4$ cm**

Sol. $m\angle C = 60^\circ$, $c = 4$cm,

$m\angle B = 90^\circ$, $b = ?$, $c = ?$



$$m\angle C = 90^\circ - 60^\circ$$

$$m\angle C = 30^\circ$$

$$c = 4 \text{ cm}$$

Using $\tan 60^\circ = \frac{a}{4}$

$$\sqrt{3} = \frac{a}{4}$$

$$a = 4\sqrt{3} \text{ cm}$$

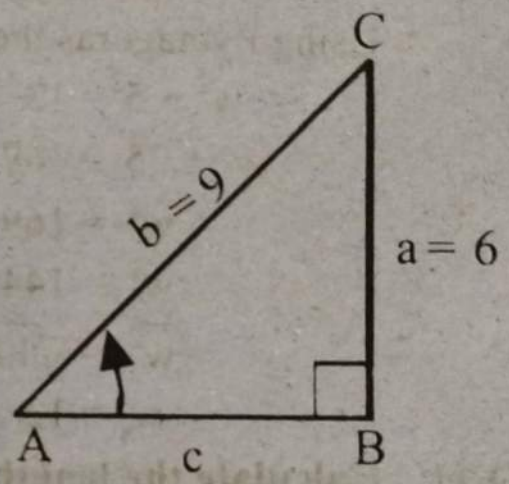
CamScanner

Using Pythagoras theorem,

$$b^2 = a^2 + c^2$$
$$= (4\sqrt{3})^2 + (4)^2$$
$$= 16 \times 3 + 16$$
$$b^2 = 48 + 16$$
$$b^2 = 64$$
$$b = \sqrt{64}$$
$$b = 8 \text{ cm}$$

Q.9. **b = 9 cm, a = 6 cm**

Sol. Using Pythagoras theorem to find c.



$$b^2 = c^2 + a^2$$
$$9^2 = c^2 + 6^2$$
$$c^2 = 9^2 - 6^2$$
$$c^2 = 81 - 36$$
$$c^2 = 45$$
$$c^2 = 9 \times 5$$
$$c = \sqrt{9 \times 5}$$
$$c = 3\sqrt{5} \text{ cm}$$

To find m∠A, using

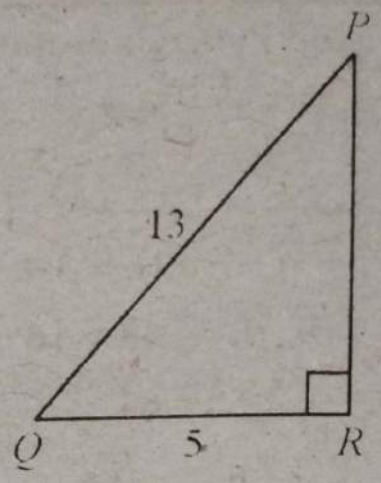
$$\sin A = \frac{6}{9} = \frac{2}{3}$$
$$m\angle A = \sin^{-1}\left(\frac{2}{3}\right)$$
$$= \sin^{-1}(0.6667)$$
$$m\angle A = 41.81^\circ$$

So $m\angle C = 90^\circ - 41.81^\circ$

$$m\angle C = 48.19^\circ$$

CamScanner

Q.10. Let Q and R be the two points on the same bank of a canal. The point P is placed on the other bank straight to point R. Find the width of the canal and the angle PQR.



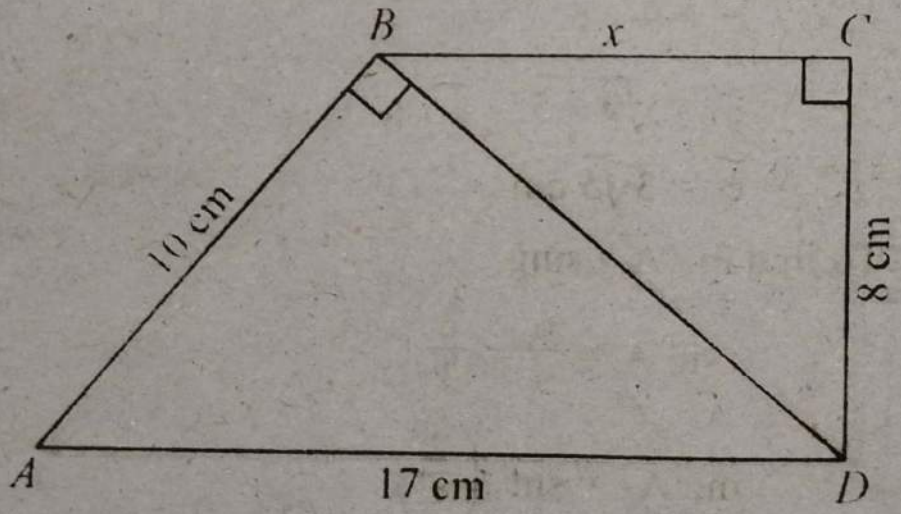
Sol. Width of canal = w = ?

PQR is a right angle triangle.

Using Pythagoras theorem

$$w^2 + 5^2 = 13^2$$
$$w^2 + 25 = 169$$
$$w^2 = 169 - 25$$
$$w^2 = 144$$
$$\sqrt{w^2} = \sqrt{144}$$
$$w = 12$$

Q.11. Calculate the length x in the adjoining figure.



Sol. Find we find BD in ΔABD.

Using Pythagoras theorem.

$$(\overline{BD})^2 + (AB)^2 = (\overline{AD})^2$$
$$(\overline{BD})^2 + 10^2 = 17^2$$

$$= 17^2 - 10^2$$
$$= 289 - 100$$
$$= 189$$
$$(\overline{BD})^2 = 9 \times 21$$
$$\overline{BD} = \sqrt{9 \times 21}$$
$$= 3\sqrt{21} \text{ cm}$$

Now consider ΔBCD

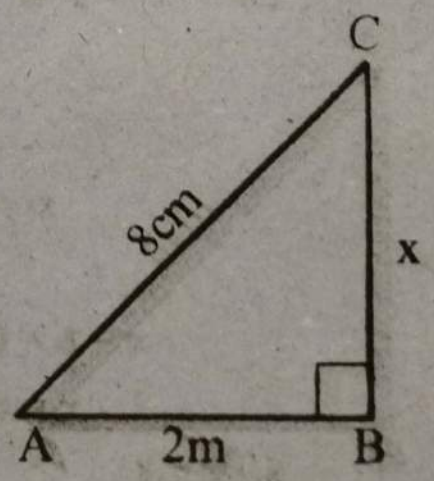
$$|\overline{BC}| = x = ?$$
$$|\overline{BD}| = 3\sqrt{21} \text{ cm}$$
$$|\overline{CD}| = 8 \text{ cm}$$

Using Pythagoras theorem.

$$x^2 + 8^2 = (3\sqrt{21})^2$$
$$x^2 + 64 = 9 \times 21$$
$$x^2 + 64 = 189$$
$$x^2 = 189 - 64$$
$$x^2 = 125$$
$$x^2 = 25 \times 5$$
$$x = \sqrt{25 \times 5}$$
$$x = 5\sqrt{5} \text{ cm}$$

Q.12. If the ladder is placed along the wall such that the foot of the ladder is 2 m away from the wall. If the length of the ladder is 8 m, find the height of the wall?

Sol. ABC is a right angled triangle.



$$m\overline{AB} = 2 \text{ m}$$
$$m\overline{AC} = 8 \text{ m}$$

Height of wall = $\overline{BC} = x = ?$

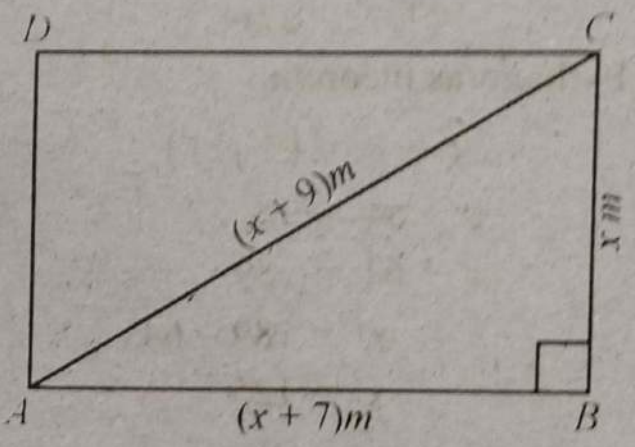
Using Pythagoras theorem.

$$x^2 + 2^2 = 8^2$$

$$x^2 + 4 = 64$$
$$x^2 = 64 - 4$$
$$x^2 = 60$$
$$x = \sqrt{4 \times 15}$$
$$x = 2\sqrt{15} \text{ m}$$

So height of wall $= 2\sqrt{15}$ m $= 7.75$ m

Q.13. The diagonal of a rectangular field ABCD is (x + 9)m and the sides are (x + 7)m and x m. Find the value of x.



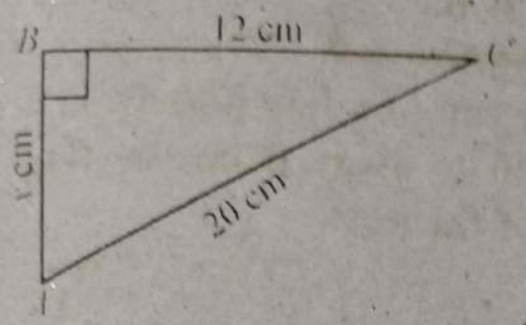
Sol. ABC is a right angled triangle.

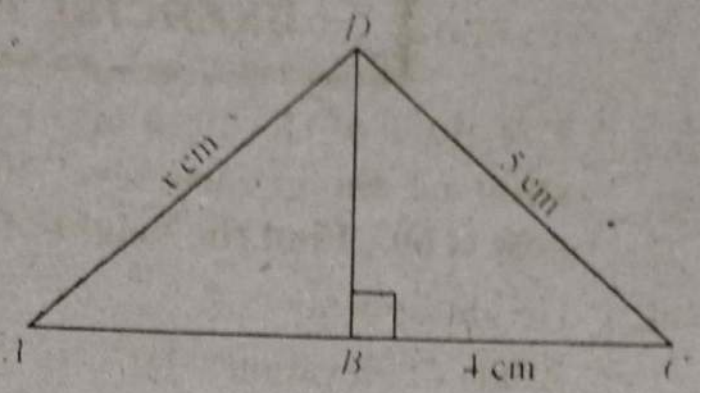
Using Pythagoras theorem,

$$(x + 9)^2 = x^2 + (x + 7)^2$$
$$x^2 + 18x + 81 = x^2 + (x + 7)^2$$
$$2x^2 + 14x + 49 = x^2 + 18x + 81$$
$$2x^2 - x^2 + 14x - 18x + 49 - 81 = 0$$
$$x^2 - 4x - 32 = 0$$
$$x^2 - 8x + 4x - 32 = 0$$
$$x(x - 8) + 4(x - 8) = 0$$
$$(x - 8)(x + 4) = 0$$
$$x - 8 = 0, x + 4 = 0$$
$$x = 8, \quad x = -4 \text{ (not possible)}$$

So $x = 8$

Q.14. Calculate the value of 'x' in each case.





Sol.(i) ABC is a right angled triangle m $\overline{AB}$ = x cm, m $\overline{BC}$ = 12 cm, m $\overline{AC}$ = 20 cm.

Using Pythagoras theorem.

$$(20)^2 = x^2 + 12^2$$
$$400 = x^2 + 144$$
$$x^2 = 480 - 144$$
$$x^2 = 256$$
$$\sqrt{x^2} = \sqrt{256}$$
$$x = 16 \text{ cm}$$

(ii) First we find m $\overline{BD}$ using Pythagoras theorem on ΔBCD.

$$|\overline{BD}|^2 = 5^2 - 4^2$$
$$|\overline{BD}|^2 = 25 - 16$$
$$|\overline{BD}|^2 = 9$$
$$|\overline{BD}| = 3 \text{ cm}$$

Since m $\overline{AB}$ = 4 cm, m $\overline{BD}$ = 3 cm.

Using Pythagoras theorem on ΔABD

$$x^2 = 4^2 + 3^2$$
$$x^2 = 16 + 9$$
$$x^2 = 25$$
$$x = \sqrt{25}$$
$$x = 5 \text{ cm}$$

EXERCISE 6.6

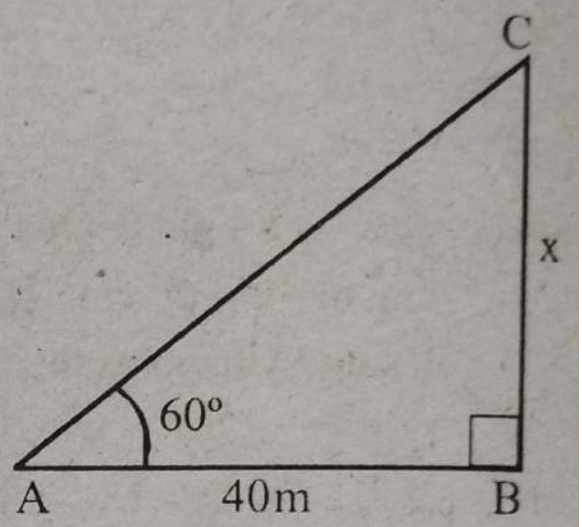
Q.1. The angle of elevation of the top of a flag post from a point on the ground level 40 m away from the flag post is 60°. Find the height of the post.

Sol. Height of post = x = ?

Angle of elevation = 60°

$m\overline{AB} = 40$ m

Using



$$\tan 60^\circ = \frac{x}{40}$$

$$\sqrt{3} = \frac{x}{40}$$

$$x = 40\sqrt{3}$$

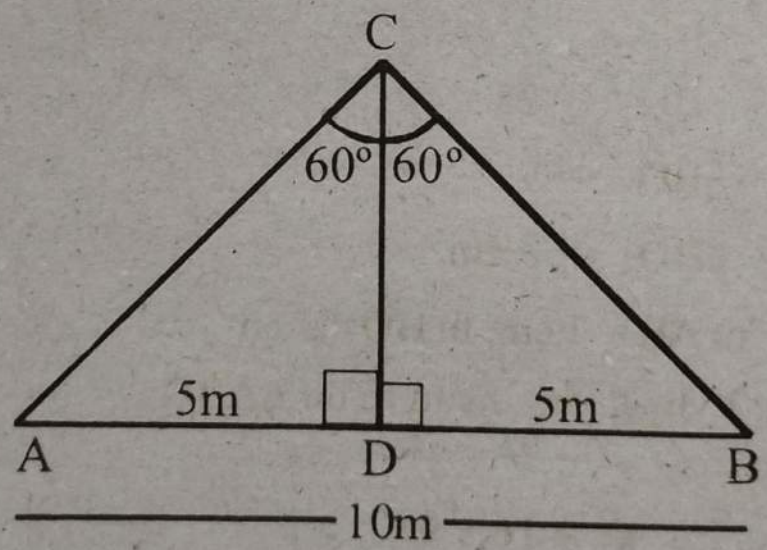
$$= 40 \times 1.732 \text{ m}$$

$$= 69.28 \text{ m}$$

Q.2. An isosceles triangle has a vertical angle of 120° and a base 10 cm long. Find the length of its altitude.

Sol. $m\angle A = 90^\circ - 60^\circ = 30^\circ$

Consider right angled ΔADC.



Using

$$\tan 30^\circ = \frac{\overline{CD}}{5}$$

$$0.577 = \frac{\overline{CD}}{5}$$

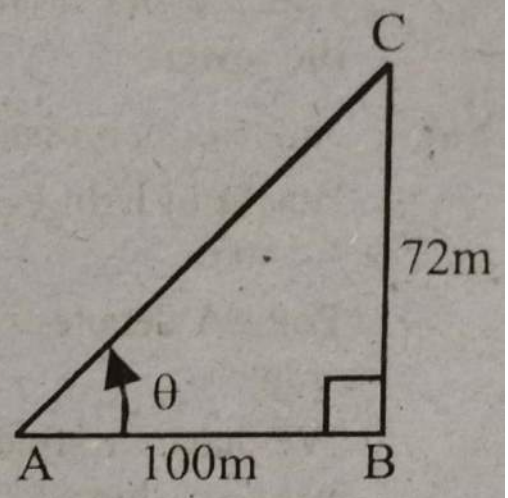
$$\overline{CD} = 5 \times 0.577$$

$$\overline{CD} = 2.885 \text{ cm}$$

Length of attitude = 2.885 cm = 2.89 cm

Q.3. A tree is 72 m high. Find the angle of elevation of its top 100 m away on the ground level.

Sol. Height of tree = 72 m



Angle of elevation = θ = ?

$$m\overline{AB} = 100 \text{ m}$$

ABC is a right angled triangle.

To find θ, using

$$\tan\theta = \frac{72}{100}$$

$$\tan\theta = 0.72$$

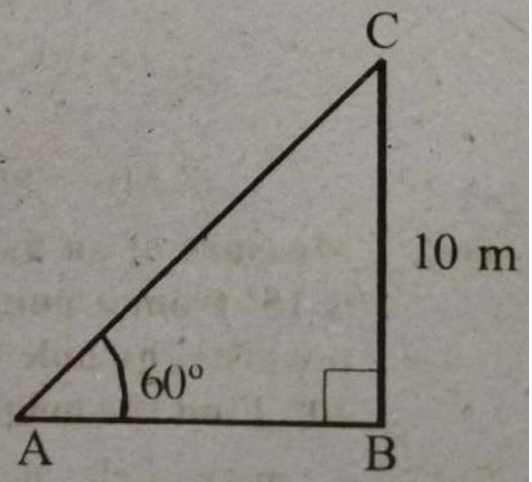
$$\theta = \tan^{-1}(0.72)$$

$$= 35^\circ\, 45'\, 14''$$

Angle of elevation of top = 35° 45′ 14″

Q.4. A ladder makes an angle of 60° with the ground and reaches a height of 10 m along the wall. Find the length of the ladder.

Sol. Height of the wall where ladder reaches = $m\overline{BC}$ = 10 m



Angle made by ladder with ground = 60°

Length of ladder = $m\overline{AC}$ = ?

Using

$$\cos 60^\circ = \frac{10}{m\overline{AC}}$$

$$\frac{1}{2} = \frac{10}{m\overline{AC}}$$

$$m\overline{AC} = 2 \times 10$$

$$m\overline{AC} = 20 \text{ m}$$

Length of ladder = 20 m

Q.5. A light house tower is 150 m high from the sea level. The angle of depression from the top of the tower to a ship is 60°. Find the distance between the ship and the tower.

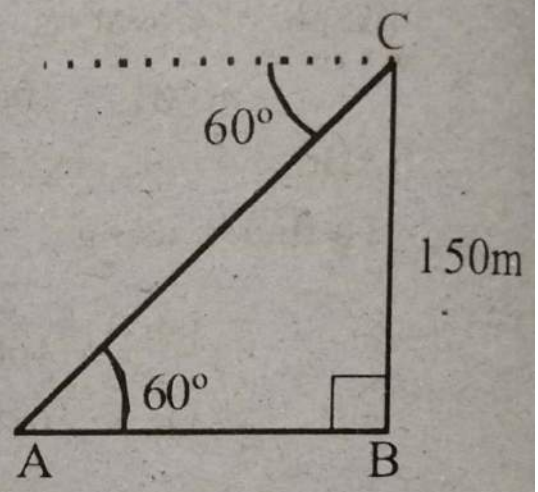
Sol. Angle of depression = 60°

Height of light house tower = 150 m

Point A denotes position of ship.

We have to find distance between ship and tower.

Using



$$\tan 60° = \frac{150}{m\overline{AB}}$$

$$\sqrt{3} = \frac{150}{m\overline{AB}}$$

$$m\overline{AB} = \frac{150}{\sqrt{3}}$$

$$= \frac{150}{1.732}$$

$$m\overline{AB} = 86.6 \text{ m}$$

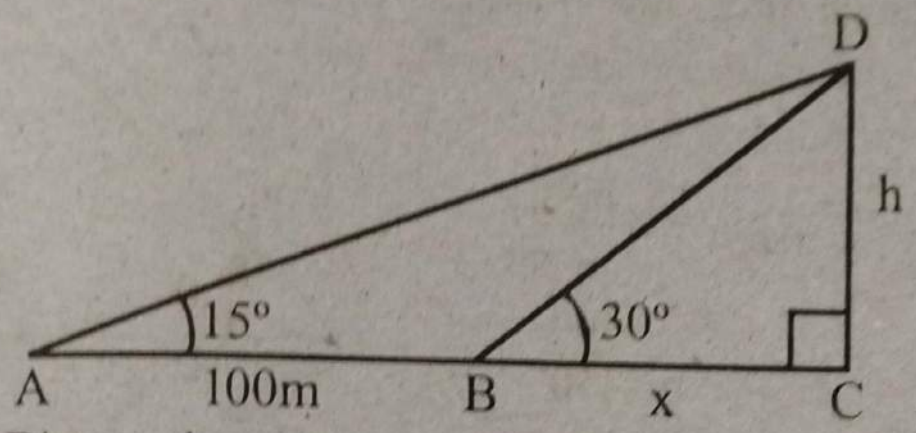
Q.6. Measure of an angle of elevation of the top of a pole is 15° from a point on the ground, in walking 100 m towards the pole the measure of angle is found to be 30°. Find the height of the pole.

Sol. 1st angle of elevation = 15°

On walking 100 m towards pole,

2nd angle of elevation = 30°

Let height of pole = h = ?



Distance between pole and point B = x m

Using

$$\tan 30^\circ = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = \sqrt{3}\,h$$

Now $$\tan 15^\circ = \frac{h}{x + 100} \quad (\because \overline{AC} = \overline{AB} + \overline{BC})$$

$$0.2679 = \frac{h}{\sqrt{3}h + 100}$$

$$0.2679(\sqrt{3}h + 100) = h$$

$$0.4640h + 26.79 = h$$

$$h - 0.4640\,h = 26.79$$

$$h(1 - 0.4640) = 26.79$$

$$h(0.536) = 26.79$$

$$h = \frac{26.79}{0.536}$$

$$h = 50 \text{ m}$$

So height of pole = 50 m

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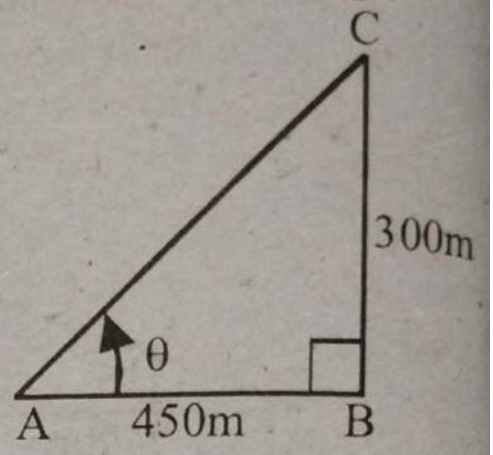
Q.7. Find the measure of an angle of elevation of the Sun, if a tower 300 m high casts a shadow 450 m long.

Sol. Height of tower = 300 m

Length of shadow = 450 m

Angle of elevation of sun = θ = ?



Using

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$= \frac{300}{450}$$

$$\tan\theta = \frac{2}{3}$$

$$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$

$$= \tan^{-1}(0.6667)$$

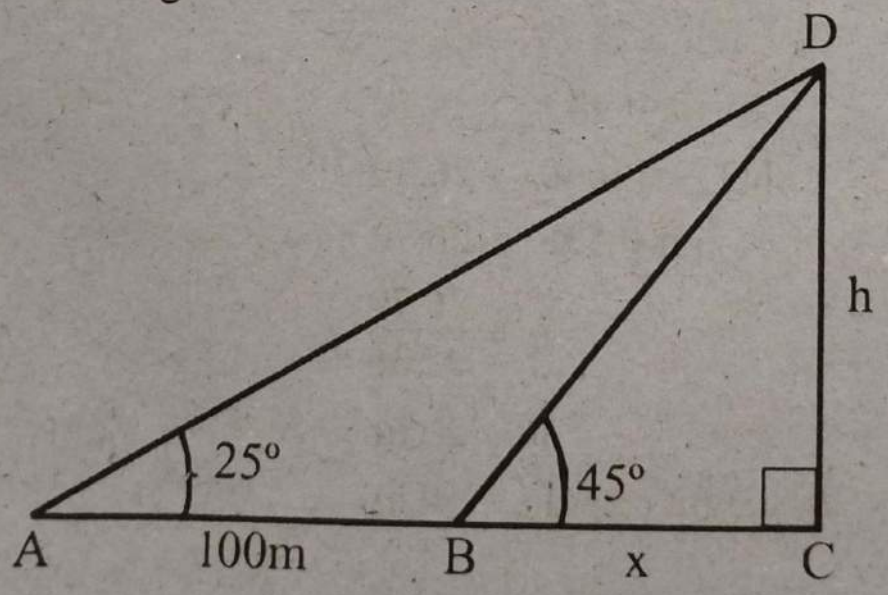
Angle of elevation of sun = 33.69°

Q.8. Measure of angle of elevation of the top of a cliff is 25°, on walking 100 metres towards the cliff, measure of angle of elevation of the top is 45°. Find the height of the cliff.

Sol. Angle of elevation of cliff at point A = 25°

On walking 100 m towards cliff, now angle of elevation of top of diff = 45°

Let height of cliff = h = ?



Consider right angled triangle BCD,

$$\tan 45^\circ = \frac{h}{x}$$

$$1 = \frac{h}{x}$$

$$\Rightarrow \quad x = h \qquad \ldots(1)$$

Now consider right angled triangle ACD, in which

Base = $\overline{AC} = (100 + x)$ m

Height = h

Using

$$\tan 25^\circ = \frac{h}{x + 100}$$

$$= \frac{h}{h + 100} \qquad (\because \ x = h)$$

$$0.4663 = \frac{h}{h + 100}$$

$$0.4663(h + 100) = h$$

$$0.4663\,h + 46.63 = h$$

$$46.63 = h - 0.4663\,h$$

$$= h(1 - 0.4663)$$

$$46.63 = 0.5337\,h$$

$$h = \frac{46.63}{0.5337}$$

$$h = 87.37 \text{ m}$$

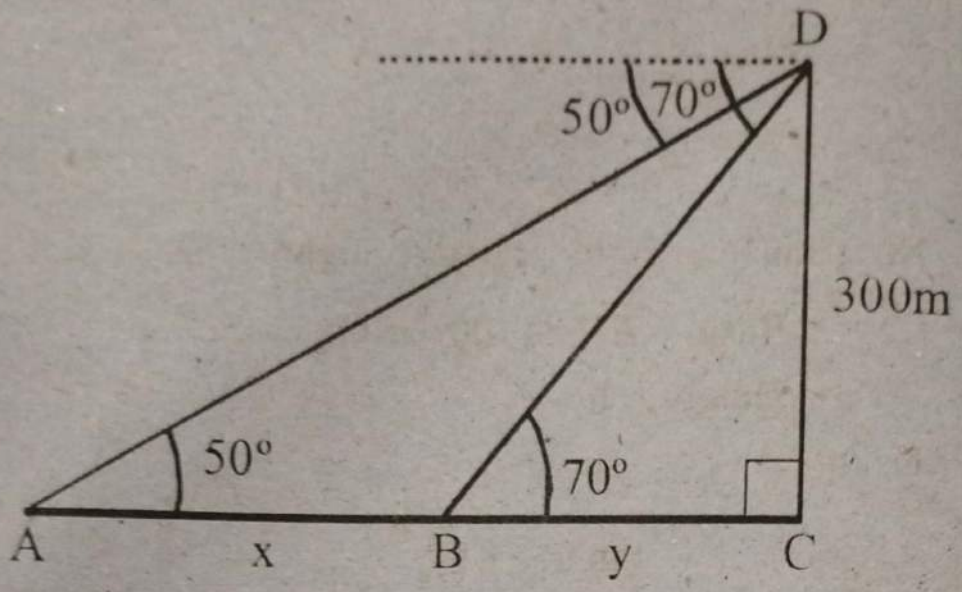
So height of cliff $= 87.37 \text{ m} \approx 87.4 \text{ m}$

Q.9. From the top of a hill 300 m high, the measure of the angle of depression of a point on the nearer shore of the river is 70° and measure of the angle of depression of a point, directly across the river is 50°. Find the width of the river. How far is the river from the foot of the hill?

Sol. Height of hill = 300 m

$\overline{AB} = x$ = width of river = ?

$\overline{BC} = y = ?$



$\overline{AC} = x + y$ = Distance of river from foot of the hill

Consider right angled triangle BCD.

Using

$$\tan 70^\circ = \frac{300}{y}$$

$$2.7475 = \frac{300}{y}$$

$$y\,(2.7475) = 300$$

$$y = \frac{300}{2.7475}$$

$$y = 109.19 \text{ m}$$

Now consider right angled triangle ACD.

Using

$$\tan 50^\circ = \frac{300}{x + y}$$

$$1.1918 = \frac{300}{x + 109.19}$$

$$1.1918(x + 109.19) = 300$$

$$1.1918\,x + 130.1326 = 300$$

$$1.1918\,x = 300 - 130.1326$$

$$= 169.8674$$

$$x = \frac{169.0674}{1.1918}$$

$$x = 142.53 \text{ m}$$

Width of river = 142.53 m

Distance of river from foot of hill = x + y

$$= 142.53 + 109.19$$

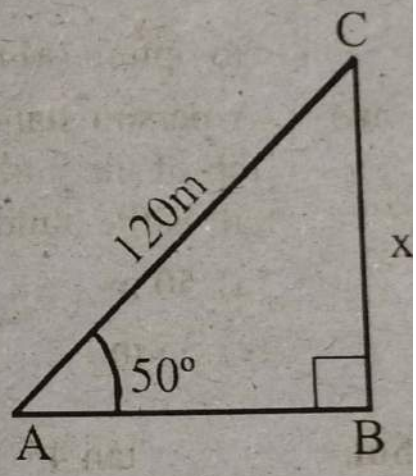
$$= 251.72 \text{ m}$$

Q.10. A kite has 120 m of string attached to it when at an elevation of 50°. How far is it above the hand holding it? (Assume that the string is stretched.)

Sol. Length of string = m $\overline{AC}$ = 120 m

Angle of elevation = 50°

To Find $\overline{BC}$, height of kite above hand = x = ?



Using

$$\sin 50° = \frac{x}{120}$$

$$0.7660 = \frac{x}{120}$$

$$x = 0.7660 \times 120$$

$$x = 91.92 \text{ m}$$

So height of kite above the hand holding it = 91.92 m

REVIEW EXERCISE 6

Q.1. Choose the correct option.

(i) The value of $\theta \tan^{-1} 2$ in radians?

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$

(c) $0.4636\,\pi$ (d) 0.4636

Sol. Option (c) is correct.

(ii) In a right triangle, the hypotenuse is 13 units and one of the angles is $\theta = 30^\circ$. What is the length of the opposite side?

(a) 6.5 units (b) 7.5 units

(c) 6 units (d) 5 units

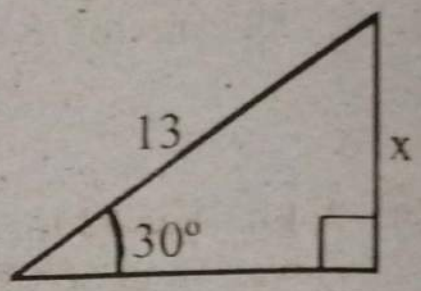
Sol.

$$\sin 30^\circ = \frac{x}{13}$$

$$0.5 = \frac{x}{13}$$

$$x = 0.5 \times 13$$

$$x = 6.5 \text{ units}$$



So option (a) is correct.

(iii) A person standing 50 m away from a building sees the top of the building at an angle of elevation of 45°. How tall is the building?

(a) 50 m (b) 25 m

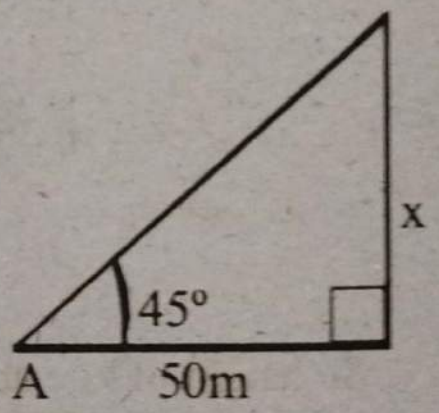
(c) 35 m (d) 70 m

Sol.

$$\tan 45^\circ = \frac{x}{50}$$

$$1 = \frac{x}{50}$$

$$x = 50 \text{ m}$$



So option (a) is correct.

(iv) $\sec^2\theta - \tan^2\theta =$ _____

(a) $\sin^2\theta$ (b) 1

(c) $\cos^2\theta$ (d) $\cot^2\theta$

Sol. We know that

$$1 + \tan^2\theta = \sec^2\theta$$

or $$\sec^2\theta - \tan^2\theta = 1$$

So option (b) is correct.

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(v) If $\sin\theta = \frac{3}{5}$, and θ is an acute angle, $\cos^2\theta =$ _____

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$

(c) $\frac{16}{25}$ (d) $\frac{4}{25}$

Sol.

$$\sin\theta = \frac{3}{5}$$

$$\Rightarrow \quad \sin^2\theta = \frac{9}{25}$$

$$\Rightarrow \quad 1 - \cos^2\theta = \frac{9}{25}$$

$$1 - \frac{9}{25} = \cos^2\theta$$

$$\frac{25-9}{25} = \cos^2\theta$$

$$\cos^2\theta = \frac{16}{25}$$

So option (c) is correct.

(vi) $\frac{5\pi}{24}$ rad = _____ degrees.

(a) 30° (b) 37.5°

(c) 45° (d) 52.5°

Sol.

$$\frac{5\pi}{24}\text{ rad} = \left(\frac{5\pi}{24} \times \frac{180}{\pi}\right)^\circ$$

$$= \left(\frac{5 \times 180}{24}\right)^\circ$$

$$= \left(\frac{900}{24}\right)^\circ$$

$$= 37.5^\circ$$

So option (b) is correct.

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(vii) $292.5^\circ =$ _____ rad.

(a) $\frac{17\pi}{6}$ (b) $\frac{17\pi}{4}$

(c) 1.6π (d) 1.625π

Sol.

$$= 292.5^\circ \times \frac{\pi}{180}$$

$$= \frac{2925}{10} \times \frac{\pi}{180}$$

$$= \frac{117 \times \pi}{2 \times 36} = \frac{13\pi}{8}$$

$$= 1.625\,\pi$$

So option (d) is correct.

(viii) Which of the following is a valid identity?

(a) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ (b) $\cos\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

(c) $\cos\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ (d) $\cos\left(\frac{\pi}{2} - \theta\right) = \text{cosec}\,\theta$

Sol. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

is valid identity.

$\cos(90^\circ - \theta) = \sin\theta$;	$\tan(90^\circ - \theta) = \tan\theta$
$\cot(90^\circ - \theta) = \tan\theta$;	$\sec(90^\circ - \theta) = \text{cosec}\,\theta$
$\text{cosec}(90^\circ - \theta) = \sec\theta$	

So option (a) is correct.

(ix) $\sin 60^\circ =$ _____.

(a) 1 (b) $\frac{1}{2}$

(c) $\sqrt{(3)^2}$ (d) $\frac{\sqrt{3}}{2}$

Sol. $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Option (d) is correct.

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(x) $\cos^2 100\pi + \sin^2 100\pi =$ _____.

(a) 1 (b) 2

(c) 3 (d) 4

Sol. $\cos^2 100\pi + \sin^2 100\pi = 1$

So option (a) is correct.

$[\because \cos^2\theta + \sin^2\theta = 1]$

Q.2. Convert the given angles from:

(a) degrees to radians giving answer in terms of π.

(i) 255° (ii) $75^\circ\, 45'$ (iii) 142.5°

Sol.(i) $225^\circ = 225 \times \frac{\pi}{180}$ $\left[\because 1^\circ = \frac{\pi}{180} \text{ rad}\right]$

$= \frac{5\pi}{4}$ rad

(ii) $75^\circ\, 45' = 75^\circ + \left(\frac{45}{60}\right)^\circ = \left(75 + \frac{3}{4}\right)^\circ$

$= \left(\frac{303}{4}\right)^\circ = \frac{303}{4} \times \frac{\pi}{180} = \frac{101\pi}{4 \times 60}$

$= \frac{101\pi}{240}$

(iii) $142.5^\circ = 142.5^\circ \times \frac{\pi}{180} = \frac{1425}{10} \times \frac{\pi}{180}$

$= \frac{57 \times \pi}{2 \times 36} = \frac{19\pi}{24}$ rad

(b) radians to degrees giving answer in degrees and minutes.

(i) $\frac{17\pi}{24}$ (ii) $\frac{7\pi}{12}$ (iii) $\frac{11\pi}{16}$

Sol.(i) $\frac{17\pi}{24} = \left(\frac{17\pi}{24} \times \frac{180}{\pi}\right)^\circ = \left(\frac{17}{24} \times 180\right)^\circ$

$= \left(\frac{17 \times 15}{2}\right)^\circ = \left(\frac{225}{2}\right)^\circ = 127.5^\circ$

$= 127^\circ\, 30'$

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(ii) $\frac{7\pi}{12} = \left(\frac{7\pi}{12} \times \frac{180}{\pi}\right)^{\circ} = \left(\frac{17}{12} \times 180\right)^{\circ}$

$= (7 \times 15)^{\circ} = 105^{\circ}$

(iii) $\frac{11\pi}{16} = \left(\frac{11\pi}{16} \times \frac{180}{\pi}\right)^{\circ} = \left(\frac{11}{16} \times 180\right)^{\circ}$

$= \left(\frac{11 \times 45}{4}\right)^{\circ} = \left(\frac{495}{4}\right)^{\circ} = 123.75^{\circ}$

$= 123^{\circ} + (0.75 \times 60)'$

$= 123^{\circ} + 45' = 123^{\circ}\, 45'$

Q.3. Prove the following trigonometric identities.

(i) $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$

Sol. L.H.S $= \frac{\sin\theta}{1-\cos\theta}$

Multiply and divide by $1 + \cos\theta$

$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{1^2-\cos^2\theta}$

$= \frac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta} = \frac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta}$

$= \frac{1+\cos\theta}{\sin\theta}$

$=$ R.H.S

So proved.

(ii) $\sin\theta\,(\operatorname{cosec}\theta - \sin\theta) = \frac{1}{\sec^2\theta}$

Sol. L.H.S $= \sin\theta\,(\operatorname{cosec}\theta - \sin\theta)$

$= \sin\theta \,.\, \operatorname{cosec}\theta - \sin\theta\sin\theta$

$= \sin\theta \,.\, \frac{1}{\sin\theta} - \sin^2\theta$

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$$1 - \sin^2\theta$$

$$= \cos^2\theta = \frac{1}{\frac{1}{\cos^2\theta}} = \frac{1}{\sec^2\theta}$$

$$= \text{R.H.S}$$

(iii) $$\frac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta} = \frac{1 - \cos\theta}{1 + \tan\theta}$$

Sol. L.H.S $$= \frac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta}$$

$$= \frac{\frac{1}{\sin\theta} - \frac{1}{\cos\theta}}{\frac{1}{\sin\theta} + \frac{1}{\cos\theta}} = \frac{\frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}}{\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

Divide in numerator and denominator by $\cos\theta$.

$$= \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} = \frac{1 - \tan\theta}{1 + \tan\theta}$$

$$\neq \text{R.H.S}$$

R.H.S must be $\frac{1 - \cos\theta}{1 + \tan\theta}$

So correction in R.H.S required here.

(iv) $$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

Sol. L.H.S $= \tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

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$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \text{R.H.S}$$

So proved.

(v) $$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{2}{1 - 2\sin^2\theta}$$

Sol. $$\text{L.H.S} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2\cos^2\theta + 2\sin^2\theta}{(1 - \sin^2\theta) - \sin^2\theta} = \frac{2(\cos^2\theta + \sin^2\theta)}{1 - 2\sin^2\theta}$$

$$= \frac{2(1)}{1 - 2\sin^2\theta} = \frac{2}{1 - 2\sin^2\theta}$$

$$= \text{R.H.S}$$

So proved.

(vi) $$\frac{1 + \cos\theta}{1 - \cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$$

Sol. $$\text{R.H.S} = (\operatorname{cosec}\theta + \cot\theta)^2$$

$$= \left(\frac{1 + \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 + \cos\theta)(1 + \cos\theta)}{\sin^2\theta}$$

$$= \frac{(1 + \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$\frac{1 + \cos\theta}{1 - \cos\theta}$$

$$= \text{L.H.S}$$

Q.4. If $\tan\theta = \frac{3}{\sqrt{2}}$ then find the remaining trigonometric ratios when θ lies in first quadrant.

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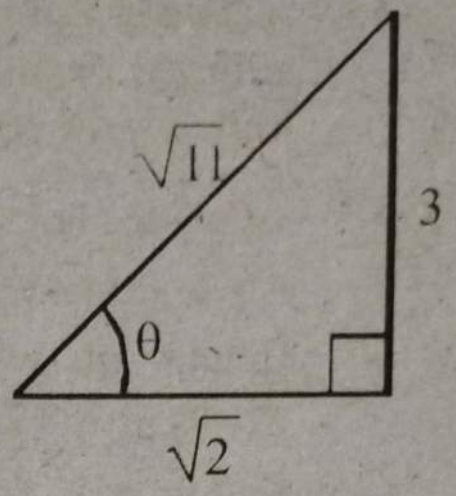
Sol. Hypotenuse $= \sqrt{3^2 + (\sqrt{2})^2}$

$= \sqrt{9+2} = \sqrt{11}$



$\sin\theta = \frac{3}{\sqrt{11}}$, $\operatorname{cosec}\theta = \frac{\sqrt{11}}{3}$,

$\cos\theta = \frac{\sqrt{2}}{\sqrt{11}} = \sqrt{\frac{2}{11}}$

$\sec\theta = \sqrt{\frac{11}{2}}$, $\cot\theta = \frac{\sqrt{2}}{3}$

All trigonometric ratios have positive sign. (θ lies in 1st quadrant.)

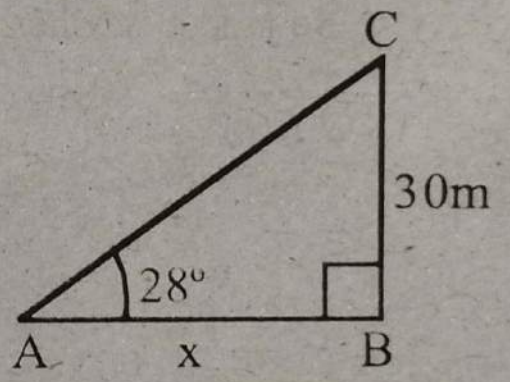
Q.5. From a point on the ground, the angle of elevation to the top of a 30-meter-high building is 28°. How far is the point from the base of the building?

Sol. Let A be the point from base of building.

Angle of elevation of top of building $= 28°$

Height of building = 30 m

We need distance of point A from base of building.



Using

$$\tan 28° = \frac{30}{x}$$

$$0.5317 = \frac{30}{x}$$

$$x \times 0.5317 = 30$$

$$x = \frac{30}{0.5317}$$

$$= 56.42 \text{ m}$$

So point A is 56.42 m far from base of the building.

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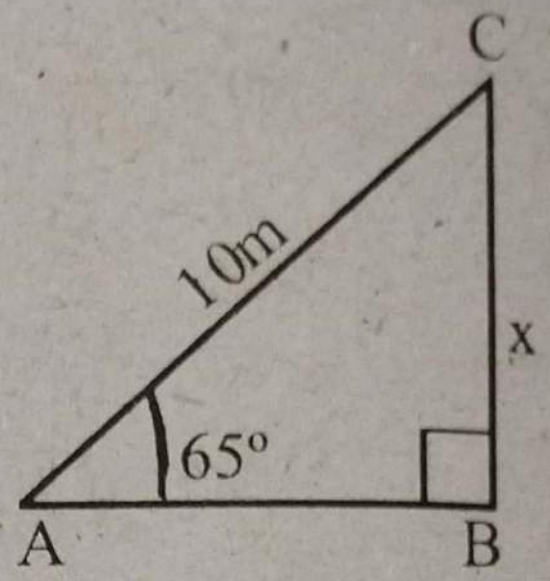
Q.6. A ladder leaning against a wall forms an angle of 65° with the ground. If the ladder is 10 meters long, how high does it reach on the wall?

Sol. $\overline{AC}$ is the ladder.



Angle formed with ground = 65°

Height of wall where ladder reaches = x = ?

Using

$$\sin 65^\circ = \frac{x}{10}$$

$$0.9063 = \frac{x}{10}$$

$$x = 10 \times 0.9063$$

$$x = 9.063 \text{ m} \approx 9.06 \text{ m}$$

So ladder reaches to a height of 9.06 m along the wall.

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