Unit No. 1

Real Numbers

Exercise No. 1.1

Question No. 1

Identify each of the following as a rational or irrational number:

(i). 2.353535

It is a Rational Number.

(ii). $0.\overline{6}$

It is a Rational Number.

(iii). 2.236067.....

It is an Irrational Number.

(iv). $\sqrt{7}$

It is an Irrational Number.

(v). e

It is an Irrational Number.

(vi). π

It is an Irrational Number.

(vii). $5 + \sqrt{11}$

It is an Irrational Number.

(viii). $\sqrt{3} + \sqrt{13}$

It is an Irrational Number.

(ix). $\frac{15}{4}$

It is a Rational Number.

(x). $(2 - \sqrt{2})(2 + \sqrt{2})$

It is a Irrational Number.

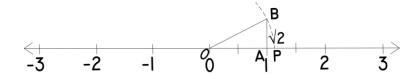
Solving further $(2 - \sqrt{2})(2 + \sqrt{2}) = 2^2 - \sqrt{2}^2 = 4 - 2 = 2$ which is rational number.

Question No. 2

Represent the following numbers on number line:

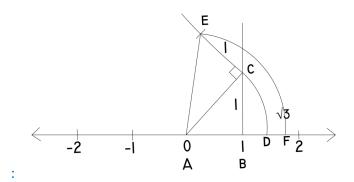
(i). $\sqrt{2}$

Solution:



(ii). $\sqrt{3}$

Solution:



(iii). $4\frac{1}{3}$

Solution:



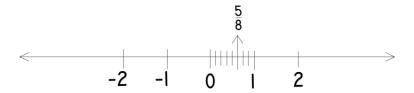
(iv). $-2\frac{1}{7}$

Solution:



(v). $\frac{5}{8}$

Solution:



(vi). $2\frac{3}{4}$

Solution:



Question No. 3

Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$:

(i). $0.\overline{4}$

Solution:

 $0.\overline{4}$

Let

$$x = 0.\overline{4}$$

 $x = 0.444....$ (eq. i)

Multiply by 10 on both sides:

$$10 \times x = 0.444..... \times 10$$

$$10x = 4.444...$$
 (eq. ii)

By subtracting eq. i from eq. ii

$$10x - x = 4.444.... - 0.444...$$

$$9x = 4$$

$$X = \frac{4}{9}$$

$$0.\,\overline{4}=\tfrac{4}{9}$$

(ii). $0.\overline{37}$

Solution:

 $0.\overline{37}$

Let

$$x = 0.\overline{37}$$

 $x = 0.3737....$ (eq. i)

Multiply by 100 on both sides:

$$100 \times x = 0.3737..... \times 100$$

$$100x = 37.3737...$$
 (eq. ii)

By subtracting eq. i from eq. ii

$$100x - x = 37.3737.... - 0.3737...$$

$$99x = 37$$

$$X = \frac{37}{99}$$

$$0.\overline{37} = \frac{37}{99}$$

(iii). $0.\overline{21}$

Solution:

 $0.\overline{21}$

Let

$$x = 0.\overline{21}$$

 $x = 0.2121....$ (eq. i)

Multiply by 100 on both sides:

$$100 \times x = 0.2121..... \times 100$$

$$100x = 21.2121.....$$
 (eq. ii)

By subtracting eq. i from eq. ii

$$100x - x = 21.2121.... - 0.2121...$$

$$99x = 21$$

$$X = \frac{21}{99}$$

$$0.\overline{21} = \frac{21}{99}$$

Question No. 4

Name the property used in the following:

(i).
$$(a + 4) + b = a + (4 + b)$$

Associative Property over addition.

(ii).
$$\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$$

Commutative Property over addition.

(iii).
$$x - x = 0$$

Additive Inverse

(iv).
$$a(b + c) = ab + ac$$

Left Distributive Property

(v).
$$16 + 0 = 16$$

Additive Identity

(vi).
$$100 \times 1 = 100$$

Multiplicative Identity

(vii).
$$4 \times (5 \times 8) = (4 \times 5) \times 8$$

Associative Property under Multiplication.

(viii).
$$a b = b a$$

Commutative Property under Multiplication.

Question No. 5

Name the property used in the following:

(i).
$$-3 < -1 \Rightarrow 0 < 2$$

Transitive Property

(ii). If
$$a < b$$
 then $\frac{1}{a} > \frac{1}{b}$

Reciprocal Property

(iii). If a < b then a + c < b + c

Additive Property

(iv). If ac < bc and c > 0 then a < b

Multiplicative Property

(v). If ac < bc and c < 0 then a > b

Multiplicative Property

(vi). Either a > b or a = b or a < b

Trichotomy Property

Question No. 6

Insert two rational numbers between:

(i).
$$\frac{1}{3}$$
 and $\frac{1}{4}$

Solution:

There are infinitive rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$,

1st Rational Number:

$$=\left(\frac{1}{3}+\frac{1}{4}\right)\div 2$$

$$= \left(\frac{4+3}{12}\right) \times \frac{1}{2}$$

$$=\frac{7}{12}\times\frac{1}{2}$$

$$= \frac{7 \times 1}{12 \times 2}$$
$$= \frac{7}{24}$$

2nd Rational Number between $\frac{7}{24}$ and $\frac{1}{4}$:

$$= \left(\frac{7}{24} + \frac{1}{4}\right) \div 2$$

$$= \left(\frac{(7 \times 1) + (1 \times 6)}{24}\right) \times \frac{1}{2}$$

$$= \left(\frac{7 + 6}{24}\right) \times \frac{1}{2}$$

$$= \frac{13}{24} \times \frac{1}{2}$$

$$= \frac{13 \times 1}{24 \times 2}$$

$$= \frac{13}{42}$$

(ii). 3 and 4

Solution:

There are infinitive rational numbers between 3 and 4,

1st Rational Number:

$$= (3+4) \div 2$$

$$= (7) \times \frac{1}{2}$$

$$= 7 \times \frac{1}{2}$$

$$= \frac{7 \times 1}{1 \times 2}$$

$$= \frac{7}{2}$$

2nd Rational Number between $\frac{7}{2}$ and 4:

$$= \left(\frac{7}{2} + 4\right) \div 2$$

$$= \left(\frac{(7 \times 1) + (4 \times 2)}{2}\right) \times \frac{1}{2}$$

$$= \left(\frac{7 + 8}{2}\right) \times \frac{1}{2}$$

$$= \frac{15}{2} \times \frac{1}{2}$$

$$= \frac{15 \times 1}{2 \times 2}$$

$$= \frac{15}{4}$$

(iii).
$$\frac{3}{5}$$
 and $\frac{4}{5}$

There are infinitive rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$,

1st Rational Number:

$$= \left(\frac{3}{5} + \frac{4}{5}\right) \div 2$$

$$= \left(\frac{3+4}{5}\right) \times \frac{1}{2}$$

$$= \frac{7}{5} \times \frac{1}{2}$$

$$= \frac{7 \times 1}{5 \times 2}$$

$$= \frac{7}{10}$$

2nd Rational Number between $\frac{7}{10}$ and $\frac{4}{5}$:

$$= \left(\frac{7}{10} + \frac{4}{5}\right) \div 2$$

$$= \left(\frac{(7 \times 1) + (4 \times 2)}{10}\right) \times \frac{1}{2}$$

$$= \left(\frac{7 + 8}{10}\right) \times \frac{1}{2}$$

$$= \frac{15}{10} \times \frac{1}{2}$$

$$= \frac{15 \times 1}{10 \times 2}$$

$$= \frac{15}{20}$$

$$= \frac{3}{4}$$