Unit No. 5

Linear Equations and Inequalities

Review Exercise No. 5

Question No. 1

Four options are given against each statement. Encircle the correct one.

i. In the following, linear equation is:

(a) $5x > 7$

(b) $4x - 2 < 1$

(c) $2x + 1 = 1$

(d) $4 = 1 + 3$

ii. Solution of $5x - 10 = 10$ is:

(a) 0

(b) 50

(c) 4

(d) -4

iii. If $7x + 4 < 6x + 6$, then x belongs to the interval

(a) $(2, \infty)$

(b) $[2, \infty)$

(c) $(-\infty, 2)$

(d) $(-\infty, 2]$

iv. A vertical line divides the plane into

(a) left half plane

(b) right half plane

(c) full plane

(d) two half planes

v. The equation formed from the linear inequality is called

(a) linear equation

(b) associated equation

(c) quadratic equation

(d) none of these

vi. $3x + 4 < 0$ is:

(a) equation

(b) inequality

(c) not inequality

(d) identity

vii. Corner point is also called:

(a) code

(b) vertex

(c) curve

(d) region

viii. (0,0) is solution of inequality:

(a) $4x + 5y > 8$

(b) $3x + y > 6$

(c) $-2x + 3y < 0$

(d) $x + y > 4$

None of given options are TRUE.

ix. The solution region restricted to the first quadrant is called:

(a) objective region

(b) feasible region

(c) solution region

(d) constraints region

x. A function that is to be maximized or minimized is called:

(a) solution function

(b) objective function

(c) feasible function

(d) none of these

Question No. 2

Solve and represent their solutions on real line.

(i) $\frac{(x+5)}{3} = 1 - x$

Solution:

$\frac{(x+5)}{3} = 1 - x$

Multiply by 3:

$\frac{3(x+5)}{3} = 3(1 - x)$

$x + 5 = 3(1 - x)$

$x + 5 = 3 - 3x$

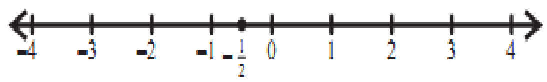
$x + 3x = 3 - 5$

$4x = -2$

$x = \frac{-2}{4}$

$x = -\frac{1}{2}$

Real Line:



(ii) $\frac{2x+1}{3} + \frac{1}{2} = 1 - \frac{x-1}{3}$

Solution:

$\frac{2x+1}{3} + \frac{1}{2} = 1 - \frac{x-1}{3}$

Multiply by 6 (L.C.M):

$6(\frac{2x+1}{3}) + 6(\frac{1}{2}) = 6(1) - 6(\frac{x-1}{3})$

$2(2x + 1) + 3(1) = 6(1) - 2(x - 1)$

$4x + 2 + 3 = 6 - 2x + 2$

$4x + 5 = 8 - 2x$

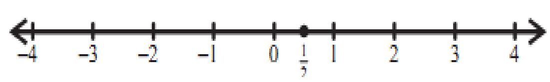
$4x + 2x = 8 - 5$

$6x = 3$

$x = \frac{3}{6}$

$x = \frac{1}{2}$

Real Line:



(iii) 3x + 7 < 16

Solution:

$3x + 7 < 16$

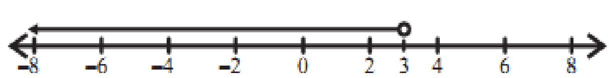
$3x < 16 - 7$

$3x < 9$

$x < 9/3$

$x < 3$

Real Line:



(iv) $5(x - 3) \geq 26x - (10x + 4)$

Solution:

$5(x - 3) \geq 26x - (10x + 4)$

$5x - 15 \geq 26x - 10x - 4$

$5x - 15 \geq 16x - 4$

$4 - 15 \geq 16x - 5x$

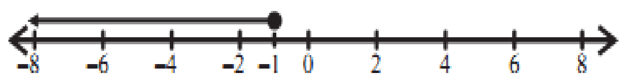
$-11 \geq x$

$-11/11 \geq x$

$-1 \geq x$

$x \leq -1$

Real Line:



Question No. 3

Find the solution region of the following linear inequalities:

(i) $3x - 4y \leq 12$; $3x + 2y \geq 3$

Solution:

$3x - 4y \leq 12$

$3x + 2y \geq 3$

Associated equations:

$3x - 4y = 12$ … eq. (i)

$3x + 2y = 3$ … eq. (ii)

- x-intercept for eq. (i): set $y = 0$:

$3x - 4(0) = 12$

$3x - 0 = 12$

$x = 12/3$

$x = 4$

So, the point is (4, 0).

- y-intercept for eq. (i): set $x = 0$:

$3(0) - 4y = 12$

$0 - 4y = 12$

$y = 12/-4$

$y = -3$

So, the point is (0, -3).

To check Region put (0, 0) in $3x - 4y \leq 12$:

$3(0) - 4(0) \leq 12$

$0 - 0 \leq 12$

$0 \leq 12$ True

Origin Side:

Graph lies towards the origin side.

- x-intercept for eq. (ii): set y = 0:

$3x + 2(0) = 3$

$3x + 0 = 3$

$3x = 3$

$x = 3/3$

$x = 1$

So, the point is (1, 0).

- y-intercept for eq. (ii): set x = 0:

$3(0) + 2y = 3$
$0 + 2y = 3$

$y = 3/2 = 1.5$

So, the point is (0, 1.5).

To check Region put (0, 0) in $3x + 2y \geq 3$:

$3(0) + 2(0) \geq 3$

$0 - 0 \geq 3$

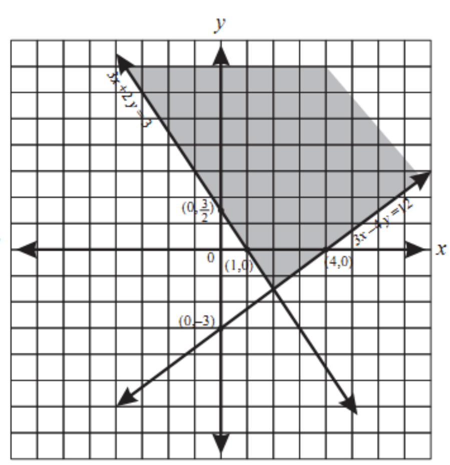
$0 \geq 3$ False

Origin Side:

Graph lies away from the origin side.

Graphical Representation:

(i)



(ii) $2x + y \leq 4$; $x + 2y \leq 6$

Solution:

$2x + y \leq 4$

$x + 2y \leq 6$

Associated equations:

$2x + y = 4$ … eq. (i)

$x + 2y = 6$ … eq. (ii)

- x-intercept for eq. (i): set $y = 0$:

$2x + 0 = 4$

$2x + 0 = 4$

$x = 4/2$

$x = 2$

So, the point is (2, 0).

- y-intercept for eq. (i): set $x = 0$:

$3(0) + 2y = 4$

$0 + 2y = 4$

$y = 4/2$

$y = 2$

So, the point is (0, 2).

To check Region put (0, 0) in $2x + y \leq 4$:

$2(0) + 2(0) \leq 4$

$0 + 0 \leq 4$

$0 \leq 4$ True

Origin Side:

Graph lies towards the origin side.

- x-intercept for eq. (ii): set $y = 0$:

$x + 2(0) = 6$

$x + 0 = 6$

$x = 6$

So, the point is (6, 0).

- y-intercept for eq. (ii): set $x = 0$:

$0 + 2y = 6$
$2y = 6$

$y = 6/2 = 3$

So, the point is (0, 3).

To check Region put (0, 0) in $x + 2y \leq 6$:

$0 + 2(0) \leq 6$

$0 + 0 \le 6$

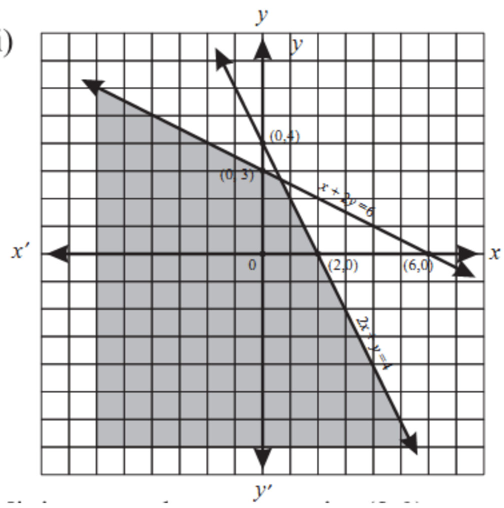
$0 \le 6$ True

Origin Side:

Graph lies towards the origin side.

Graphical Representation:

(ii)



Question No. 4

Find the maximum value of $g(x, y) = x + 4y$ subject to constraints $x + y \le 4$, $x \ge 0$ and $y \ge 0$.

Solution:

$x + y \le 4$

Associated equations:

$x + y = 4$

- x-intercept for eq. (i): set $y = 0$:

$x + 0 = 4$

$x = 4$

$x = 4$

So, the point is $(4, 0)$.

- y-intercept for eq. (i): set $x = 0$:

$0 + y = 4$

$y = 4$

So, the point is $(0, 4)$.

Associated equations: $x = 0$, $y = 0$

Corner Points are $(0, 0)$, $(4, 0)$, $(0, 4)$

$g(x, y) = x + 4y$

$g(0, 0) = 0 + 4(0) = 0 + 0 = 0$

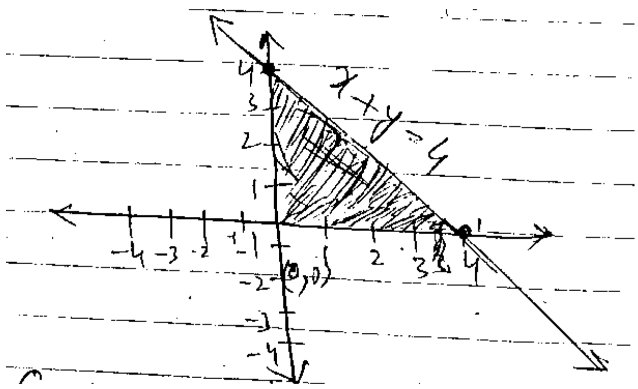
$g(4, 0) = 4 + 4(0) = 4 + 0 = 4$

g(0, 4) = 0 + 4(4) = 0 + 16 = 16

Maximum value:

Maximum value is 16 at point (0, 4).

Graphical Representation:



Question No. 5

Find the minimum value of f(x, y) = 3x + 5y subject to constraints

x + 3y ≥ 3, x + y ≥ 2, x ≥ 0, y ≥ 0.

Solution:

x + 3y ≥ 3

x + y ≥ 2

Associated Equations:

x + 3y = 3 … eq. (i)

x + y = 2 … eq. (ii)

- x-intercept for eq. (i): set y = 0:

x + 3(0) = 3

x = 3

So, the point is (3, 0).

- y-intercept for eq. (i): set x = 0:

0 + 3y = 3

3y = 3

y = 3/3

y = 1

So, the point is (0, 1).

To check Region put (0, 0) in x + 3y ≥ 3:

0 + 3(0) ≥ 3

0 + 0 ≥ 3

0 ≥ 3 False

Origin Side:

Graph lies away from the origin side.

- x-intercept for eq. (ii): set $y = 0$:

$x + 0 = 2$

$x = 2$

So, the point is (2, 0).

- y-intercept for eq. (ii): set $x = 0$:

$0 + y = 2$
$y = 2$

So, the point is (0, 2).

To check Region put (0, 0) in $x + y \geq 2$:

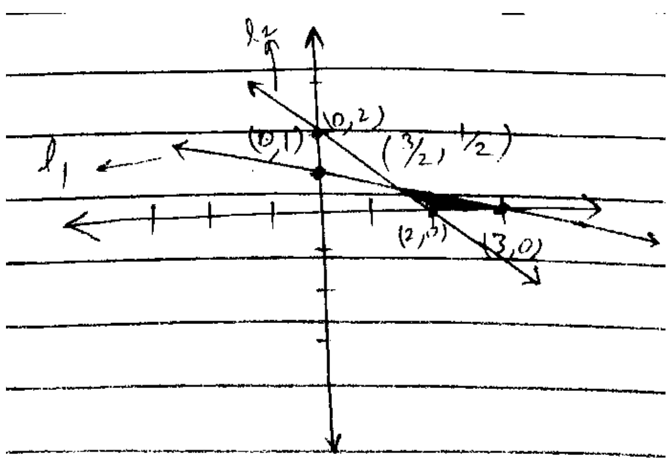
$0 + 0 \geq 2$

$0 \geq 2$ False

Origin Side:

Graph lies away from the origin side.

Graphical Representation:



Corner Points are (0, 2), A(x, y), (3, 0)

To find corner point A, Subtract eq. (i) from eq. (ii):

$(x + 3y) – (x + y) = (3) – (2)$

$x + 3y – x - y = 3 – 2$

$2y = 1$

$y = ½$

Put $y = ½$ in eq. (i) for x:

$x + 3(1/2) = 3$

$x = 3 – 3/2$

$x = 3/2$

Corner point A$(\frac{3}{2}, \frac{1}{2})$

Now;

$f(x, y) = 3x + 5y$

Corner Point (0, 2)

$f(0, 2) = 3(0) + 5(2)$

$f(0, 2) = 0 + 10 = 10$

Corner Point (3, 0)

$f(3, 0) = 3(3) + 5(0)$

$f(3, 0) = 9 + 0 = 9$

Corner Point $(\frac{3}{2}, \frac{1}{2})$

$f(\frac{3}{2}, \frac{1}{2}) = 3(\frac{3}{2}) + 5(\frac{1}{2})$

$f(\frac{3}{2}, \frac{1}{2}) = \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$

Minimum Value:

Minimum value is 7 at point $(\frac{3}{2}, \frac{1}{2})$.