Unit No. 9

Similar Figures

Basic Concepts

INTRODUCTION:

The concept of similarity dates back to ancient Greece, where Greek mathematicians, particularly Euclid, developed the fundamental principles of geometry. In his creative work, "The Elements", Euclid established the foundations of plane geometry, including the theory of similar triangles and polygons. Euclid's further work laid the groundwork for modern geometry and the concept of similarity remains central in many branches of mathematics, including trigonometry and algebra.

Similarity of Polygons:

Similar figures have same shape but not necessarily of same size. Two polygons are similar if their corresponding angles are equal and the corresponding sides are proportional (i.e., the ratios of the lengths of corresponding sides are equal). For example, all equilateral triangles are similar to each other because they have the same angles and the measure of the sides are proportional.

Identification of Similar Triangles:

(i) If two angles in one triangle are congruent to two corresponding angles in another triangle, the third angle in each triangle must be congruent. Similarity symbol is '~'. i.e., In the correspondence of the triangles △ABC and △DEF.

(ii) If the ratio of two corresponding sides and their included angle are equal, the triangles are similar.

(iii) If the ratio of all the corresponding sides are equal, then the triangles are similar.

Remember!

Three or more than three-sided closed figure is called polygon.

Need to Know!

Proportionality of sides means one side is k times of its corresponding side.

Area of Similar Figures:

The ratio of the areas of any two similar figures is equal to the square of the ratio of any two corresponding lengths of the figures.

$A_1 / A_2 = (l_1 / l_2)^2$

Volume of Similar Solids:

Two solids are said to be similar if they have same shape but possibly different sizes. Two solids are similar if lengths of the corresponding sides are proportional i.e., the ratio of the corresponding lengths are equal.

$r_1 / r_2 = h_1 / h_2$; $V_1 / V_2 = (r_1 / r_2)^3$

Geometrical Properties of Polygon and their Applications:

Regular Polygon:

A regular polygon has all sides and all angles equal. Some of the common regular polygons are equilateral triangles, squares, regular pentagons, regular hexagons, etc.

Sum of Interior Angles:

The formula for sum of interior angles of n-sided polygon is

$(n - 2) \times 180^\circ$.

Interior Angle:

For a regular n-sided polygon:

$$\text{Size of each Interior Angle} = \frac{(n-2) \times 180^\circ}{n}$$

Exterior Angle:

The sum of all exterior angles of any polygon is always 360° regardless of the number of sides.

The exterior angle of a regular n-sided polygon is:

Exterior Angle = $360^\circ / n$

The interior and exterior angles are supplementary at a vertex i.e.,

Interior + exterior angle = 180°

Diagonals:

The total number of diagonals in a regular polygon with n sides is $\frac{n(n-3)}{2}$

Symmetry:

A regular n-sided polygon has rotational symmetry and reflexive symmetry both of order n. e.g., a regular hexagon has six lines of symmetry and has rotational symmetry of order 6. A regular n-sided polygon can be rotated by $360^\circ / n$ and will look the same.

Geometrical Properties of Triangle:

A triangle is a polygon with three sides and three angles.

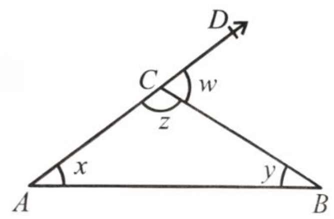
Angle sum:

The sum of the interior angles in any triangle is always 180°.

- In equilateral triangle, all sides are equal, and each angle is 60°. It has three lines of symmetry and rotational symmetry of order 3.
- In isosceles triangle, two sides are equal, and the angles opposite to the equal sides are also equal. It has one line of symmetry.

Exterior angle of a triangle:

The measure of an exterior angle in a triangle is equal to sum of the measures of two opposite interior angles i.e.,



In $\triangle ABC$, $m\angle A + m\angle B = m\angle BCD$

i.e., $x + y = w$

Geometrical Properties of Parallelogram:

A parallelogram is a quadrilateral whose opposite sides are parallel and equal in length and opposite angles are equal. Its adjacent angles are supplementary. The diagonals of a parallelogram bisect each other (they cross each other at the midpoint). They are not equal in length.

Rectangle:

All angles are 90° and diagonals are equal.

Rhombus:

All sides are equal, and diagonals bisect each other at right angles.

Square:

All sides are equal, all angles are 90° and diagonals are equal and bisect each other at right angles.

Applications of Polygons:

- Architects use polygons in building designs,
- Engineers rely on them to make strong structures like bridges.
- In art and design, polygons help create beautiful patterns and 3-D models.
- On maps, polygons show areas like cities or land boundaries.
- Polygons are also used in video games and animations to build characters and scenes.
- In science, they appear in molecular shapes, natural patterns like honeycombs and even in the design of telescope mirrors.

Tessellation:

A tessellation is a pattern of shapes that fit together perfectly, without any gaps or overlaps, covering a plane. These shapes can be repeated infinitely to create a repeating pattern. Tessellations can be created using a single shape or a combination of shapes. They can be regular or irregular and they can exhibit various symmetries and patterns.

Only three regular polygons can tessellate the plane on their own: equilateral triangles, squares, and regular hexagons. They have symmetries. Hexagons (interior angle 120°) can tessellate perfectly because three hexagons meet at each vertex to form a 360° angle with no space creating a natural look inspired by honeycombs.

Remember!

Equilateral triangles can tessellate perfectly because the internal angle of each equilateral triangle is 60°, and six of these triangles meet at a point to form a 360° angle, allowing them to fill space seamlessly. Squares can tessellate perfectly because each square has an internal angle of 90° and four squares meet at a point to form a 360° angle.

Remember!

Regular pentagons and other polygons with angles that don't add up to 360° at each vertex cannot form gap-free patterns. i.e., Tessellation is not possible.