Unit No. 4

Factorization and Algebraic Manipulation

Review Exercise No. 4

Question No. 1

Four options are given against each statement. Encircle the correct option.

i. The factorization of $12x + 36$ is:
(a) $12(x + 3)$
(b) $12(3x)$
(c) $12(3x + 1)$
(d) $x(12 + 36x)$

ii. The factors of $4x^2 - 12y + 9$ are:
(a) $(2x + 3)^2$
(b) $(2x - 3)^2$
(c) $(2x - 3)(2x + 3)$
(d) $(2 + 3x)(2 - 3x)^2$

iii. The HCF of a^3b^3 and ab^2 is:
(a) a^3b^3
(b) ab^2
(c) a^4b^5
(d) a^2b

Here is the properly formatted version of your text:

iv. The LCM of $16x^2$, $4x$, and $30xy$ is:
(a) $480x^3y$
(b) $240xy$
(c) $240x^2y$
(d) $120x^4y$

v. The product of LCM and HCF = _____ of two polynomials.
(a) sum
(b) difference
(c) product
(d) quotient

vi. The square root of $x^2 - 6x + 9$ is:
(a) $\pm(x - 3)$
(b) $\pm(x + 3)$
(c) $x - 3$
(d) $x + 3$

vii. The LCM of $(a - b)^2$ and $(a - b)^4$ is:
(a) $(a - b)^2$
(b) $(a - b)^3$
(c) $(a - b)^4$
(d) $(a - b)^6$

viii. Factorization of $x^3 + 3x^2 + 3x + 1$ is:
(a) $(x + 1)^3$
(b) $(x - 1)^3$

(c) $(x + 1)(x^2 + x + 1)$
(d) $(x - 1)(x^2 - x + 1)$

ix. Cubic polynomial has degree:
(a) 1
(b) 2
(c) 3
(d) 4

x. One of the factors of $x^3 - 27$ is:
(a) x - 3
(b) x + 3
(c) $x^2 - 3x + 9$
(d) Both a and c

Question No. 2

Factorize the following expressions:

(i) $4x^3 + 18x^2 - 12x$

Solution:

$4x^3 + 18x^2 - 12x$

$= 2x(2x^2 + 9x - 6)$

(ii) $x^3 + 64y^3$

Solution:

$x^3 + 64y^3$

$= (x)^3 + (4y)^3$

Using formula:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (x + 4y)[x^2 - (x)(4y) + (4y)^2]$

$= (x + 4y)(x^2 - 4xy + 16y^2)$

(iii) $x^3y^3 - 8$

Solution:

$x^3y^3 - 8$

$= (xy)^3 - (2)^3$

Using formula:

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (xy - 2)[(xy)^2 + (xy)(2) + (2)^2]$

$= (xy - 2)(x^2y^2 + 2xy + 4)$

(iv) $-x^2 - 23x - 60$

Solution:

$-x^2 - 23x - 60$

$= -(x^2 + 23x + 60)$

$= -(x^2 + 3x + 20x + 60)$

$= -[x(x + 3) + 20(x + 3)]$

$= -(x + 3)(x + 20)$

(v) $2x^2 + 7x + 3$

Solution:

$2x^2 + 7x + 3$

$= 2x^2 + x + 6x + 3$

$= x(2x + 1) + 3(2x + 1)$

$= (2x + 1)(x + 3)$

(vi) $x^4 + 64$

Solution:

$x^4 + 64$

$= (x^2)^2 + (8)^2$

$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$

$= (x^2 + 8)^2 - 16x^2$

$= (x^2 + 8)^2 - (4x)^2$

$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$

Re-arrainging:

$= (x^2 + 4x + 8)(x^2 - 4x + 8)$

(vii) $x^4 + 2x^2 + 9$

Solution:

$x^4 + 2x^2 + 9$

$= (x^2)^2 + (3)^2 + 2(x^2)(3) - 2(x^2)(3) + 2x^2$

$= (x^2 + 3)^2 - 6x^2 + 2x^2$

$= (x^2 + 3)^2 - 4x^2$

$= (x^2 + 3)^2 - (2x)^2$

$= (x^2 + 3 + 2x)(x^2 + 3 - 2x)$

Re-arranging:

$= (x^2 + 2x + 3)(x^2 - 2x + 3)$

(viii) $(x + 3)(x + 4)(x + 5)(x + 6) - 360$

Solution:

$(x + 3)(x + 4)(x + 5)(x + 6) - 360$

$= [(x + 3)(x + 6)][(x + 4)(x + 5)] - 360$

$= (x^2 + 6x + 3x + 18)(x^2 + 5x + 4x + 20) - 360$

$= (x^2 + 9x + 18)(x^2 + 9x + 20) - 360$

Let: $x^2 + 9x = y$

$= (y + 18)(y + 20) - 360$

$= y^2 + 20y + 18y + 360 - 360$

$= y^2 + 38y$

$= y(y + 38)$

Recall "y":

$= (x^2 + 9x)(x^2 + 9x + 38)$

$= x(x + 9)(x^2 + 9x + 38)$

(ix) $(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

Solution:

$(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

Let: $x^2 + 6x = y$

$= (y + 3)(y - 9) + 36$

$= y^2 - 9y + 3y - 27 + 36$

$= y^2 - 6y + 9$

$= y^2 - 3y - 3y + 9$

$= y(y - 3) - 3(y - 3)$

$= (y - 3)(y - 3)$

$= (y - 3)^2$

Recall "y":

$= (x^2 + 6x - 3)^2$

Question No. 3

Find LCM and HCF by prime factorization method:

(i) $4x^3 + 12x^2, 8x^2 + 16x$

Solution:

Factors of $4x^3 + 12x^2 = 4x^2(x + 3)$

$= 2.2.x.x(x + 3)$

Factors of $8x^2 + 16x = 8x(x + 2)$

$= 2.2.2.x(x + 2)$

C.F = 2.2.x

H.C.F = 4x

N.C.F = 2.x(x + 2)(x + 3)

L.C.M = C.F × N.C.F

L.C.M = 4x × 2.x(x + 2)(x + 3)

L.C.M = $8x^2(x + 2)(x + 3)$

(ii) $x^3 + 3x^2 - 4x$, $x^2 - x - 6$

Solution:

Factors of $x^3 + 3x^2 - 4x = x(x^2 + 3x - 4)$

$= x(x^2 + 4x – x - 4)$

$= x[x(x + 4) – 1(x + 4)]$

$= x(x + 4)(x – 1)$

Factors of $x^2 - x - 6 = x^2 - x - 6$

$= x^2 - 3x + 2x – 6$

$= x(x – 3) + 2(x – 3)$

$= (x – 3)(x + 2)$

C.F = 1

H.C.F = 1

N.C.F = x(x + 4)(x – 1)(x – 3)(x + 2)

L.C.M = C.F × N.C.F

L.C.M = 1 × x(x + 4)(x – 1)(x – 3)(x + 2)

L.C.M = x(x + 4)(x – 1)(x – 3)(x + 2)

… (Either wrong value in book or wrong answer)

(iii) $x^2 + 8x + 16$, $x^2 - 16$

Solution:

Factors of $x^2 + 8x + 16 = x^2 + 8x + 16$

$= x^2 + 4x + 4x + 16$

$= x(x + 4) + 4(x + 4)$

$= (x + 4)(x + 4)$

Factors of $x^2 - 16 = x^2 - 16$

$= (x)^2 - (4)^2$

$= (x + 4)(x – 4)$

C.F = (x + 4)

H.C.F = (x + 4)

N.C.F = (x + 4)(x – 4)

L.C.M = C.F × N.C.F

L.C.M = (x + 4) × (x + 4)(x – 4)

L.C.M = $(x + 4)^2(x - 4)$

(iv) $x^3 - 9x$, $x^2 - 4x + 3$

Solution:

Factors of $x^3 - 9x = x(x^2 - 9)$

$= x(x^2 - 3^2)$

$= x(x - 3)(x + 3)$

Factors of $x^2 - 4x + 3 = x^2 - x - 3x + 3$

$= x(x - 1) - 3(x - 1)$

$= (x - 1)(x - 3)$

C.F = $(x - 3)$

H.C.F = $(x - 3)$

N.C.F = $x(x + 3)(x - 1)$

L.C.M = C.F × N.C.F

L.C.M = $(x - 3) \times x(x + 3)(x - 1)$

L.C.M = $x(x - 3)(x + 3)(x - 1)$

L.C.M = $x(x^2 - 9)(x - 1)$

… (Either wrong value in book or wrong answer)

Question No. 4

Find square root by factorization and division method of the expression $16x^4 + 8x^2 + 1$.

Solution:

Finding square root by factorization:

$16x^4 + 8x^2 + 1$

Factors of $16x^4 + 8x^2 + 1$

$= (4x^2)^2 + 2(4x^2)(1) + (1)^2$

$= (4x^2 + 1)^2$

Taking square root:

$= \pm (4x^2 + 1)$

Finding square root by division method:

$$
\begin{array}{r|l}
 & 4x^2 + 1 \\
\hline
 & 16x^4 + 8x^2 + 1 \\
4x^2 & 16x^4 \\
\hline
 & 8x^2 + 1 \\
8x^2 + 1 & \pm 8x^2 \pm 1 \\
\hline
 & 0
\end{array}
$$

$= \pm (4x^2 + 1)$

Question No. 5

Huria is analyzing the total cost of her loan, modeled by the expression $C(x) = x^2 - 8x + 15$, where x represents the number of years. What is the optimal repayment period for Huria's loan?

Solution:

$C(x) = x^2 - 8x + 15$

$C(x) = x^2 - 3x - 5x + 15$

$C(x) = x(x - 3) - 5(x - 3)$

$C(x) = (x - 3)(x - 5)$

Let $C(x) = 0$

$(x - 3)(x - 5) = 0$

$x - 3 = 0$, $x - 5 = 0$

$x = 3$, $x = 5$

$x = 3$ years or $x = 5$ years