Unit No. 9

Similar Figures

Exercise No. 9.3

Question No. 1

The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes?

Given:

The ratio of radii of two spheres: $r_1 : r_2 = 3 : 4$

To Find:

The ratio of their volumes = ?

Solution:

$r_1 : r_2 = 3 : 4$

Ratio of volume of any two similar spheres is equal to the cube of the ratio of radii of the spheres.

$V_1 / V_2 = (r_1 / r_2)^3$ ...(i)

$V_1 / V_2 = (3 / 4)^3$

$V_1 / V_2 = 27 / 64$

$V_1 : V_2 = 27 : 64$

Question No. 2

Two regular tetrahedrons have volumes in the ratio 8 : 27. What is the ratio of their sides?

Given:

Ratio of volumes = $V_1 : V_2 = 8 : 27$

$V_1 / V_2 = (l_1 / l_2)^3$

To Find:

The ratio of their sides = ?

Solution:

$V_1 / V_2 = (l_1 / l_2)^3$

$8 / 27 = (l_1 / l_2)^3$

$(l_1 / l_2)^3 = (2 / 3)^3$

Taking cube root on both sides:

$l_1 / l_2 = 2 / 3$

$l_1 : l_2 = 2 : 3$

Question No. 3

Two right cones have volumes in the ratio 64 : 125. What is the ratio of:

(i) their heights

(ii) their base areas?

Given:

The ratio of the volumes of two similar cones is equal to the cube of the ratio of their heights.

$V_1 / V_2 = (h_1 / h_2)^3$

$V_1 : V_2 = 64 : 125$

To Find:

The ratio of their heights = ?

The ratio of their base areas = ?

Solution:

i). Finding the ratio of heights:

$V_1 / V_2 = (h_1 / h_2)^3$

$64 / 125 = (h_1 / h_2)^3$

$(h_1 / h_2)^3 = (4 / 5)^3$

Taking cube root on both sides:

$h_1 / h_2 = 4 / 5$

$h_1 : h_2 = 4 : 5$

ii). Finding the ratio of base areas:

$A_1 / A_2 = (l_1 / l_2)^2$

$A_1 / A_2 = (4 / 5)^2$

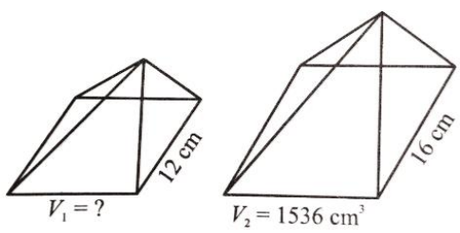
$A_1 / A_2 = 16 / 25$

$A_1 : A_2 = 16 : 25$

Question No. 4

Find the missing value in the following similar solids.

(i)



Given:

$V_2 = 1536\ cm^3$

l_1 = 12 cm

l_2 = 16 cm

To Find:

V_1 = ?

Relation:

$V_1 / V_2 = (l_1 / l_2)^3$

Solution:

$V_1 / V_2 = (l_1 / l_2)^3$

$V_1 / 1536 = (12 / 16)^3$

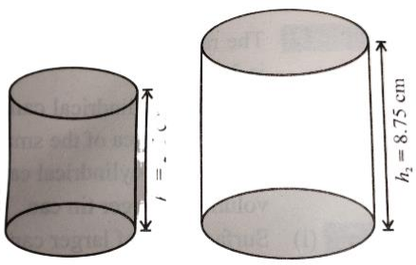
$V_1 = (3 / 4)^3 \times 1536$

$V_1 = 27 / 64 \times 1536$

$V_1 = 27 \times 24$

$V_1 = 648\ cm^3$

(ii)



Given:

$V_2 = 171.5\ cm^3$

$h_1 = 2.5$ cm

$h_2 = 8.75$ cm

To Find:

V_1 = ?

Relation:

$V_1 / V_2 = (h_1 / h_2)^3$

Solution:

$V_1 / V_2 = (h_1 / h_2)^3$

$V_1 / 171.5 = (2.5 / 8.75)^3$

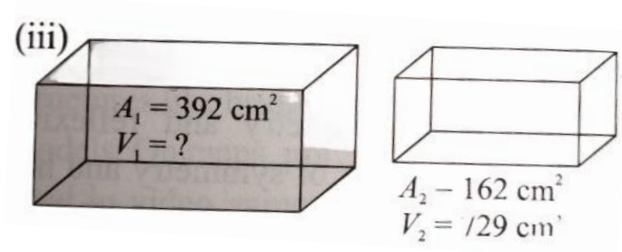
$V_1 = (1 / 3.5)^3 \times 171.5$

$V_1 = 1 / 42.875 \times 171.5$

$V_1 = 171.5 / 42.875$

$V_1 = 4\ cm^3$

(iii)



Given:

$A_1 = 392\ cm^2$

$A_2 = 162\ cm^2$

$V_2 = 729\ cm^3$

To Find:

$V_1 = ?$

Relation:

$A_1 / A_2 = (l_1 / l_2)^2$

Solution:

$A_1 / A_2 = (l_1 / l_2)^2$

$392 / 162 = (l_1 / l_2)^2$

$(l_1 / l_2)^2 = 196 / 81$

$(l_1 / l_2)^2 = (14 / 9)^2$

Taking square root on both sides:

$l_1 / l_2 = 14 / 9$

$l_1 : l_2 = 14 : 9$

Now;

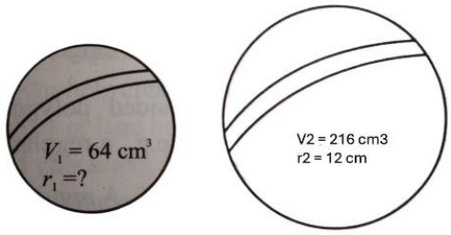
$V_1 / V_2 = (l_1 / l_2)^3$

$V_1 / 729 = (14 / 9)^3$

$V_1 = 2744 / 729 \times 729$

$V_1 = 2744\ cm^3$

(iv)



Given:

$r_2 = 12\ cm$

$V_1 = 64\ cm^3$

$V_2 = 216\ cm^3$

To Find:

$r_1 = ?$

Relation:

$V_1 / V_2 = (r_1 / r_2)^3$

Solution:

$V_1 / V_2 = (r_1 / r_2)^3$

$64 / 216 = (r_1 / 12)^3$

$8 / 27 = (r_1 / 12)^3$

$(2 / 3)^3 = (r_1 / 12)^3$

Taking cube root on both side:

$r_1 / 12 = 2 / 3$

$r_1 = 2 / 3 \times 12$

$r_1 = 8$ cm

Question No. 5

The ratio of the corresponding lengths of two similar canonical cans is 3 : 2.

(i) The larger canonical can have surface area of 96 m^2. Find the surface area of the smaller canonical can.

Given:

Area of larger can = $A_1 = 96\ m^2$

Ratio of corresponding lengths of cans = $l_1 / l_2 = 3 / 2$

To Find:

Area of smaller canonical can = $A_2 = ?$

Relation:

$A_1 / A_2 = (l_1 / l_2)^2$

Solution:

$A_1 / A_2 = (l_1 / l_2)^2$

$96 / A_2 = (3 / 2)^2$

$96 / A_2 = 9 / 4$

$96 \times 4 / 9 = A_2$

$A_2 = 384 / 9$

$A_2 = 42.67\ m^2$

(ii) The smaller canonical can have a volume of 240 m^3. Find the volume of larger canonical can.

Given:

Volume of smaller can = $V_2 = 240\ m^3$

Ratio of corresponding lengths of cans = $l_1 / l_2 = 3 / 2$

To Find:

Volume of larger can = $V_1 = ?$

Relation:

$V_1 / V_2 = (l_1 / l_2)^3$

Solution:

$V_1 / V_2 = (l_1 / l_2)^3$

$V_1 / 240 = (3 / 2)^3$

$V_1 = 27 / 8 \times 240$

$V_1 = 27 \times 30$

$V_1 = 810$

$V_1 = 810\ m^3$

Question No. 6

The ratio of the heights of two similar cylindrical water tanks is 5 : 3.

(i) If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank.

Given:

The ratio of the heights = $h_1 : h_2 = 5 : 3$

Surface area of the larger tank = $A_1 = 250\ m^2$

To Find:

Surface area of the smaller tank = $A_2 = ?$

Relation:

$A_1 / A_2 = (h_1 / h_2)^2$

Solution:

$A_1 / A_2 = (h_1 / h_2)^2$

$250 / A_2 = (5 / 3)^2$

$250 / A_2 = 25 / 9$

$250 \times 9 / 25 = A_2$

$A_2 = 10 \times 9$

$A_2 = 90\ m^2$

(ii) If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank.

Given:

The ratio of the heights = $h_1 : h_2 = 5 : 3$

Volume of the smaller tank = $V_2 = 270\ m^3$

To Find:

Volume of the larger tank = V_1= ?

Relation:

$V_1 / V_2 = (h_1 / h_2)^3$

Solution:

$V_1 / V_2 = (h_1 / h_2)^3$

$V_1 / 270 = (5 / 3)^3$

$V_1 = 125 / 27 \times 270$

$V_1 = 125 \times 10$

$V_1 = 1250\ m^3$