Unit No. 2

Logarithms

Exercise No. 2.4

Question No. 1

With out using calculator, evaluate the following.

(i). $\log_2 18 - \log_2 9$

Solution:

$\log_2 18 - \log_2 9$

Using the quotient rule of logarithms:

$= \log_2 \frac{18}{9}$

$= \log_2 2$

$= \frac{\log 2}{\log 2}$

$= 1$

(ii). $\log_2 64 + \log_2 2$

Solution:

$\log_2 64 + \log_2 2$

Using the product rule:

$= \log_2 (\ 64 \times 2\ )$

$= \log_2 (\ 128\ )$

$= \log_2 (\ 2^7\ )$

$= 7\log_2 (2)$

$= 7\ \frac{\log 2}{\log 2}$

$= 7\ (1)$

$= 7$

(iii). $\frac{1}{3}\log_3 8 - \log_3 18$

Solution:

$\frac{1}{3}\log_3 2^3 - \log_3 (2.3.3)$

$= \log_3 (2^3)^{\frac{1}{3}} - \log_3 (2.\ 3^2\ )$

$= \log_3 2 - \log_3 (2.\ 3^2\ )$

$= \log_3 \frac{2}{2.3^2}$

$= \log_3 \frac{1}{3^2}$

$= \frac{\log \frac{1}{3^2}}{\log 3}$

$= \frac{\log 3^{-2}}{\log 3}$

$= \frac{-2 \log 3}{\log 3}$

$= -2$

(iv). 2log2 + log25

Solution:

$2\log 2 + \log 25$

$= \log 2^2 + \log 25$

$= \log 4 + \log 25$

$= \log (4 \times 25)$

$= \log 100$

$= \log 10^2$

Using the power rule:

$= 2\log 10$

$= 2$

(v) $\frac{1}{3}\log_4 64 + 2 \log_5 25$

Solution:

$\frac{1}{3}\log_4 64 + 2 \log_5 25$

$= \frac{1}{3}\log_4 4^3 + 2 \log_5 5^2$

$= \log_4 (4^3)^{\frac{1}{3}} + \log_5 (5^2)^2$

$= \log_4 4 + \log_5 5^4$

$= \frac{\log 4}{\log 4} + \frac{\log 5^4}{\log 5}$

$= 1 + \frac{4 \log 5}{\log 5}$

$= 1 + 4$

$= 5$

(vi) $\log_3 12 + \log_3 0.25$

Solution:

$\log_3 12 + \log_3 0.25$

$= \log_3 (12 \times 0.25)$

$= \log_3 3$

$= \dfrac{\log 3}{\log 3}$

$= 1$

Question 2:

Write the following as a single logarithm

(i). $\mathbf{\frac{1}{2}\log 25 + 2\log 3}$

Solution:

$\frac{1}{2}\log 25 + 2\log 3$

$= \frac{1}{2}\log 5^2 + \log 3^2$

$= \log(5^2)^{\frac{1}{2}} + \log 9$

$= \log 5 + \log 9$

Using the product rule:

$= \log(5 \times 9)$

$= \log 45$

(ii). $\mathbf{\log 9 - \mathit{log}\frac{1}{3}}$

Solution:

$\log 9 - log\dfrac{1}{3}$

$= \log 3^2 - log\dfrac{1}{3}$

Using the quetient rule:

$= \log\dfrac{3^2}{\frac{1}{3}}$

$= \log 9 \times 3$

$= \log 27$

(iii). $\mathbf{\log_5 b^2.\ \log_a 5^3}$

Solution:

$\log_5 b^2.\ \log_a 5^3$

Using the power rule:

$= 2\log_5 \mathrm{b}.\log_a 5$

$= 6\log_5 \mathrm{b}.\log_a 5$

Using the change of base rule:

$= 6 \log_a b$

(iv). $2\log_3 x + \log_3 y$

Solution:

$2\log_3 x + \log_3 y$

Using the power rule:

$= \log_3 x^2 + \log_3 y$

Using the product rule:

$= \log_3 (x^2 \times y)$

$= \log_3 x^2 y$

(v). $4\log_5 x - \log_5 y + \log_5 z$

Solution:

$4\log_5 x - \log_5 y + \log_5 z$

Using the power rule:

$= \log_5 x^4 - \log_5 y + \log_5 z$

Using the quotient & the product rules:

$$= \log_5 \frac{x^4 z}{y}$$

(vi). $2\text{In}\, a + 3\text{In} b - 4\text{In} c$

Using the power rule:

$= \text{In}\, a^2 + \text{In} b^3 - \text{In} c^4$

Applying the product and quotient rules:

$$= In \frac{a^2 b^3}{c^4}$$

Question No. 3

Expand the following using laws of logarithms:

(i). $\log[\frac{11}{5}]$

Solution:

$\log[\frac{11}{5}]$

Using the quotient rule:

$= \log 11 - log 5$

(ii). $\log_5 \sqrt{8a^6}$

Solution:

$\log_5 \sqrt{8a^6}$

Rewriting the square root:

$$= \log_5(8a^6)^{\frac{1}{2}}$$

$$= \log_5(2^3\ a^6)^{\frac{1}{2}}$$

$$= \log_5 2^{\frac{3}{2}}\ a^{\frac{6}{2}}$$

$$= \log_5 2^{\frac{3}{2}}\ a^3$$

Using product rule:

$$= \log_5 2^{\frac{3}{2}} + \ \log_5 a^3$$

Using the power rule:

$$= \frac{3}{2}\log_5 2 + 3\log_5 a$$

(iii). $\boldsymbol{In[\frac{a^2 b}{c}]}$

Solution:

$$In[\frac{a^2\mathrm{b}}{\mathrm{c}}]$$

Using the product & the quotient rules:

$$= \ In\, a^2 + In\ b - In\ c$$

Using the power rule:

$$= \ 2In\,\mathrm{a} + In\ b - In\ c$$

(iv). $\boldsymbol{\log[\frac{xy}{z}]^{\frac{1}{9}}}$

Solution:

$$\log[\frac{\mathrm{xy}}{z}]^{\frac{1}{9}}$$

Using the power rule:

$$= \frac{1}{9}\log\frac{\mathrm{xy}}{z}$$

Using the product & the quotient rules:

$$= \frac{1}{9}(\log\mathrm{x} + \mathrm{logy} - \mathrm{logz})$$

(v). $\boldsymbol{\ln\sqrt[3]{16x^3}}$

Solution:

$$\ln\sqrt[3]{16x^3}$$

Rewriting the cube root:

$$= In\ (2^4.x^3)^{\frac{1}{3}}$$

$$= In\ 2^{\frac{4}{3}}.x^{\frac{3}{3}}$$

Applying the product rule:

$$= In\ 2^{\frac{4}{3}} + Inx^{\frac{3}{3}}$$

Using the power rule:

$$= \frac{4}{3} In\ 2 + In\ x$$

(vi). $\log_2[\frac{1-a}{b}]^5$

Solution:

$$\log_2\left[\frac{1-a}{b}\right]^5$$

Using the power rule:

$$= 5\ \log_2[\frac{1-a}{b}]$$

Using the quotient rule:

$$= 5\ \log_2[\frac{(1-a)}{b}]$$

$$= 5[\ \log_2(1-a) - \ \log_2 b]$$

Question No. 4

Find the value of x in the following questions:

(i). $\log 2 + \log x = 1$

Solution:

$$\log 2 + \log x = 1$$

In single log form:

$$\log 2x = 1$$

In exponential form:

$$10^1 = 2x$$

$$2x = 10$$

$$x = \frac{10}{2}$$

$$x = 5$$

(ii). $\log_2 x + \log_2 8 = 5$

Solution:

$$\log_2 x + \log_2 8 = 5$$

Using the product rule:

$$\log_2(x\,.\,8) = 5$$

$$\log_2(8x) = 5$$

In exponential form:

$$2^5 = 8x$$

$$32 = 8x$$

$$x = \frac{32}{8}$$

$$x = 4$$

(iii). $\mathbf{(81)^x = (243)^{x+2}}$

Solution:

$$(3^4)^x = (3^5)^{x+2}$$

$$3^{4x} = 3^{5x+10}$$

Bases are same so we can write:

$$4x = 5x + 10$$

$$4x - 5x = 10$$

$$-x = 10$$

$$x = -10$$

(iv). $\mathbf{[\frac{1}{27}]^{x-6} = 27}$

Solution:

$$\left[\frac{1}{27}\right]^{x-6} = 27$$

$$[27]^{-(x-6)} = 27$$

Since the bases are the same, we equate the exponents:

$$-(x-6) = 1$$

$$-x + 6 = 1$$

$$-1 + 6 = x$$

$$x = 5$$

(v). $\mathbf{\log(5x - 10) = 2}$

Solution:

$$\log(5x - 10) = 2$$

By writing in exponential form:

$$5x - 10 = 10^2$$

$$5x = 100 + 10$$

$$5x = 110$$

$$x = \frac{110}{5}$$

$x = 22$

(vi). $\mathbf{\log_2(x+1) - \log_2(x-4) = 2}$

Solution:

$$\log_2(x+1) - \log_2(x-4) = 2$$

Using quetient rule:

$$\log_2 \frac{(x+1)}{(x-4)} = 2$$

By writing in exponential form:

$$2^2 = \frac{(x+1)}{(x-4)}$$

$$4 = \frac{(x+1)}{(x-4)}$$

$$4(x-4) = x+1$$

$$4x - 16 = x+1$$

$$4x - x = 16+1$$

$$3x = 17$$

$$x = \frac{17}{3}$$

$$x = 5\frac{2}{3}$$

Question No. 5

Simple Steps For Solving this Question.

1. **Let: x = Values of question**
2. **Taking log on both sides:**
3. **Applying product, quetient & power rules:**
4. **Using log tables:**
5. **Calculating Values:**
6. **Taking antilog:**
7. **Simplifying:**

Find the value of following with the help of logarithm table:

(i). $\frac{3.68 \times 4.21}{5.234}$

Solution:

$$\frac{3.68 \times 4.21}{5.234}$$

Let:

$$x = \frac{3.68 \times 4.21}{5.234}$$

Taking log on both sides:

$$logx = \ log\frac{3.68 \ \times 4.21}{5.234}$$

Applying product & quetient rules:

$$logx = \ log3.68 + log4.21 - log5.234$$

Using log tables:

$logx = \ \log{(3.68 \times 10^0)} + \log{(4.21 \times 10^0)} - \log{(5.234 \times 10^0)}$

$logx = \ 0 + 0.5658 + 0.6243 + 0 - (0 + 0.7188)$

$logx = \ 0 + 0.5658 + 0.6243 + 0 - 0 - 0.7188$

$logx = \ 0.4713$

Taking antilog:

$$x = Antilog\ 0.4713$$

$$x = 2.960$$

(ii). $\mathbf{4.67 \ \times \ 2.11 \ \times \ 2.397}$

Solution:

$$4.67 \ \times \ 2.11 \ \times \ 2.397$$

Let:

$$x = \ 4.67 \ \times \ 2.11 \ \times \ 2.397$$

Taking log on both sides:

$$logx = \ log4.67 \ \times \ 2.11 \ \times \ 2.397$$

Applying product rule:

$$logx = \ log4.67 + \ log2.11 + \ log2.397$$

$logx = \ \log{(4.67 \ \times 10^0)} + \ \log{(2.11 \ \times 10^0)} + \ \log{(2.397 \ \times 10^0)}$

Using log tables:

$$logx = \ 0 + 0.6693 + 0 + 0.3243 + 0 + 0.3798$$

$$logx = \ 1.3734$$

Taking antilog:

$$x = Antilog\ 1.3734$$

$$x = 23.62$$

(iii). $\frac{\mathbf{(20.46)^2 \times (2.4122)}}{\mathbf{754.3}}$

Solution:

$$\frac{(20.46)^2 \ \times (2.4122)}{754.3}$$

Let:

$$x = \frac{(20.46)^2 \ \times (2.4122)}{754.3}$$

Taking log on both sides:

$$logx = log\frac{(20.46)^2 \times (2.4122)}{754.3}$$

Applying product & quetient rules:

$$logx = log(20.46)^2 + log(2.4122) - log754.3$$

Applying power rule:

$$logx = 2\log(20.46) + log(2.4122) - log754.3$$

$$logx = 2\log(2.046 \times 10^1) + log(2.4122 \times 10^0) - \log(7.543 \times 10^2)$$

Using log tables:

$$logx = 2[1 + 0.3109] + [0 + 0.3824] - [2 + 0.8776]$$

$$logx = 2[1.3109] + 0.3824 - [2.8776]$$

$$logx = 2.6218 + 0.3824 - 2.8776$$

$$logx = 0.1266$$

Taking antilog:

$$x = Antilog\ 0.1266$$

$$x = 1.339$$

(iv). $\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Solution:

$$\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$$

Let:

$$x = \frac{\sqrt[3]{9.364} \times 21.64}{3.21}$$

Taking log on both sides:

$$logx = log\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$$

Applying product & quetient rules:

$$logx = log\sqrt[3]{9.364} + log21.64 - log3.21$$

Applying power rule:

$$logx = \frac{1}{3}log9.364 + log21.64 - log3.21$$

$$logx = \frac{1}{3}\log(9.364 \times 10^0) + \log(2.164 \times 10^1) - \log(3.21 \times 10^0)$$

Using log tables:

$$logx = \frac{1}{3}(0 + 0.9715) + (1 + 0.3353) - (0 + 0.5065)$$

$$logx = \frac{1}{3}(0.9715) + 1.3353 - 0.5065$$

$$logx = 0.3238 + 1.3353 - 0.5065$$

$$logx = 1.1526$$

Taking antilog:

$$x = Antilog 1.1526$$

$$x = 14.21$$

Question No. 6

The formula to measure magnitude of earthquake is given by $M = \log_{10}\left[\frac{A}{A_0}\right]$. If amplitude (A) is 10,000 and reference amplitude (A_0) is 10. What is the magnitude of the earthquake?

Data:

$$M = \log_{10}\left[\frac{A}{A_0}\right]$$

$$A = 10000$$

$$A_0 = 10$$

To find:

$$M = ?$$

Solution:

$$M = \log_{10}\left[\frac{A}{A_0}\right]$$

By putting values:

$$M = \log_{10}\left[\frac{10000}{10}\right]$$

$$M = \log_{10}10000 - \log_{10}10$$

$$M = \log_{10}10^4 - \log_{10}10$$

$$M = 4\log_{10}10 - \log_{10}10 \qquad \text{as } \log_{10}10 = 1, \text{ so:}$$

$$M = 4 - 1$$

$$M = 3$$

So magnitude of the earthquake is 3.

Question No. 7

Abdullah invested Rs.100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modeled by an equation y = 100,000 $(1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.

Given:

Invested amount = Rs.100,000

Final amount after t years

= Rs. Y = 2 × 100000 = Rs. 200000

Rate = 5%

To find:

Time = t = ?

Solution:

$y = 100000\ (1.05)^t$

by putting values:

$200000 = 100000\ (1.05)^t$

Taking log on both sides:

$\log 200000 = \log 100000\ (1.05)^t$

$\log 200000 = \log 100000 + \log(1.05)^t$

$\log 200000 = \log 100000 + t\log(1.05)$

$\log(2\times 100000) = \log 100000 + t\log(1.05)$

$\log(2\times 10^5) = \log 10^5 + t\log(1.05)$

$\log 2 + \log 10^5 = \log 10^5 + t\log(1.05)$

$\log 2 + 5\log 10 = 5\log 10 + t\log(1.05)$

$0.3010 + 5 = 5 + t\ (0.021189)$

$0.3010 + 5 - 5 = t\ (0.021189)$

$$t = \frac{0.3010}{0.021189}$$

$t = 14.21$ years

$t = 14$ years

Question No. 8

Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature (T_i) at sea level is 20°C. Using the formula $T = T_i \times (0.97)^{\frac{h}{100}}$, calculate the temperature at an altitude of 500 metres.

Data:

Initial temperature (T_i) = 20°C

Altitude (h) = 500 m

$T = T_i \times (0.97)^{\frac{h}{100}}$

To find:

T = ?

Solution:

$T = T_i \times (0.97)^{\frac{h}{100}}$

By putting values:

$T = 20 \times (0.97)^{\frac{500}{100}}$

Taking log on both sides:

$\log T = \log 20 \times (0.97)^5$

$\log T = \log 20 + 5\log 0.97$

$\log T = \log(2 \times 10^1) + 5\log(9.7 \times 10^{-1})$

$\log T = (\log 2 + \log 10) + 5(\log 9.7 + (-1)\log 10)$

$\log T = (0.3010 + 1) + 5(0.9868 + (-1))$

$\log T = 1.3010 + 5(-0.0132)$

$\log T = 1.3010 - 0.066$

$\log T = 1.235$

Taking Antilog:

$T = \text{Antilog}\,1.235$

$T = 17.179°C$