Unit No. 7

Coordinate Geometry

Exercise No. 7.1

Question No. 1

Describe the location in the plane of the point P(x,y), for which

(i) $x > 0$

Solution:

$x > 0$

This means the x-coordinate of the point is positive.

Therefore, the point P(x, y) lies to the right of the y-axis.

It can be either the first quadrant (where y is also positive) or the fourth quadrant (where y is negative).

So, $x > 0$ lies in Right Half Plan.

(ii) $x > 0$ and $y > 0$

Solution:

$x > 0$ and $y > 0$

This means the x-coordinate of the point is positive, and the y-coordinate of the point is positive.

Therefore, the point P(x, y) lies to the right of the y-axis and above the x-axis.

It lies in the first quadrant.

So, $x > 0$ and $y > 0$ lies in the First Quadrant.

(iii) $x = 0$

Solution:

$x = 0$

This means the x-coordinate of the point is zero.

Therefore, the point P(x, y) lies on the y-axis.

So, $x = 0$ lies on the y-axis.

(iv) $y = 0$

Solution:

$y = 0$

This means the y-coordinate of the point is zero.

Therefore, the point P(x, y) lies on the x-axis.

So, $y = 0$ lies on the x-axis.

(v) x > 0 and y ≤ 0

Solution:

$x > 0$ and $y \leq 0$

This means the x-coordinate of the point is positive, and the y-coordinate of the point is less than or equal to zero.

Therefore, the point P(x, y) lies to the right of the y-axis and on or below the x-axis.

It lies in the fourth quadrant or on the positive x-axis.

So, $x > 0$ and $y \leq 0$ lies in the Fourth Quadrant and on the positive x-axis.

(vi) y = 0, x = 0

Solution:

$y = 0, x = 0$

This means the x-coordinate of the point is zero, and the y-coordinate of the point is zero.

Therefore, the point P(x, y) is at the origin.

So, $y = 0, x = 0$ lies at the Origin.

(vii) x = y

Solution:

$x = y$

This means the x-coordinate of the point is equal to the y-coordinate of the point.

Therefore, the point P(x, y) lies on the line that bisects the first and third quadrants.

So, $x = y$ lies on the line that bisects the first and third quadrants.

(viii) x ≥ 3

Solution:

$x \geq 3$

This means the x-coordinate of the point is greater than or equal to 3.

Therefore, the point P(x, y) lies on the vertical line $x = 3$ or to the right of it.

So, $x \geq 3$ lies on the vertical line $x = 3$ and to its right.

(ix) y > 0

Solution:

$y > 0$

This means the y-coordinate of the point is positive.

Therefore, the point P(x, y) lies above the x-axis.

It can be in either the first quadrant (where x is also positive) or the second quadrant (where x is negative).

So, $y > 0$ lies in the Upper Half Plane (above x-axis).

(x) x and y have opposite signs.

Solution:

x and y have opposite signs.

This means either x is positive and y is negative, or x is negative and y is positive.

Therefore, the point P(x, y) lies in the second quadrant or the fourth quadrant.

So, x and y have opposite signs lies in the Second and Fourth Quadrants.

Question No. 2

Find the distance between the points:

(i) A(6,7), B(0,–2)

Data:

A(6,7), B(0,–2)

Let $x_1 = 6$, $y_1 = 7$

$x_2 = 0$, $y_2 = -2$

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

$$|AB| = \sqrt{(0 - 6)^2 + (-2 - 7)^2}$$

$$|AB| = \sqrt{(-6)^2 + (-9)^2}$$

$$|AB| = \sqrt{36 + 81}$$

$$|AB| = \sqrt{117}$$

$$|AB| = \sqrt{3 \times 3 \times 13}$$

$$|AB| = 3\sqrt{13}$$

The distance between A and B is $3\sqrt{13}$.

(ii) C(−5,–2), D(3,2)

Data:

C(−5, −2), D(3, 2)

Let $x_1 = -5$, $y_1 = -2$

$x_2 = 3$, $y_2 = 2$

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

$$|CD| = \sqrt{(3 - -5)^2 + (2 - -2)^2}$$

$|CD| = \sqrt{(3+5)^2 + (2+2)^2}$

$|CD| = \sqrt{(8)^2 + (4)^2}$

$|CD| = \sqrt{64+16}$

$|CD| = \sqrt{80}$

$|CD| = \sqrt{4 \times 4 \times 5}$

$|CD| = 4\sqrt{5}$

The distance between C and D is $4\sqrt{5}$.

(iii) L(0,3), M(−2,−4)

Data:

L(0,3), M(−2,−4)

Let $x_1 = 0$, $y_1 = 3$

$x_2 = -2$, $y_2 = -4$

Formula:

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Solution:

$|LM| = \sqrt{(-2-0)^2 + (-4-3)^2}$

$|LM| = \sqrt{(-2)^2 + (-7)^2}$

$|LM| = \sqrt{4+49}$

$|LM| = \sqrt{53}$

The distance between L and M is $\sqrt{53}$.

(iv) P(−8,−7), Q(0,0)

Data:

P(−8,−7), Q(0,0)

Let $x_1 = -8$, $y_1 = -7$

$x_2 = 0$, $y_2 = 0$

Formula:

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Solution:

$|PQ| = \sqrt{(0 - -8)^2 + (0 - -7)^2}$

$|PQ| = \sqrt{(8)^2 + (7)^2}$

$|PQ| = \sqrt{64+49}$

$|PQ| = \sqrt{113}$

The distance between P and Q is $\sqrt{113}$.

Question No. 3

Find in each of the following:

(i) The distance between the two given points.

(ii) Midpoint of the line segment joining the two points:

(a) A(3,1), B(−2,−4)

(i). Finding Distance between A & B.

Data:

A(3,1), B(−2,−4)

Let $x_1 = 3$, $y_1 = 1$

$x_2 = -2$, $y_2 = -4$

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

$$AB = \sqrt{(-2 - 3)^2 + (-4 - 1)^2}$$

$$AB = \sqrt{(-5)^2 + (-5)^2}$$

$$AB = \sqrt{25 + 25}$$

$$AB = \sqrt{50}$$

$$AB = \sqrt{5 \times 5 \times 2}$$

$$AB = 5\sqrt{2}$$

The distance between A and B is $5\sqrt{2}$.

(b) A(−8,3), B(2,−1)

Data:

A(−8,3), B(2,−1)

Let $x_1 = -8$, $y_1 = 3$

$x_2 = 2$, $y_2 = -1$

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

$$AB = \sqrt{(2 - -8)^2 + (-1 - 3)^2}$$

$$AB = \sqrt{(2 + 8)^2 + (-4)^2}$$

$$AB = \sqrt{(10)^2 + (-4)^2}$$

$AB = \sqrt{100 + 16}$

$AB = \sqrt{116}$

$AB = \sqrt{2 \times 2 \times 29}$

$AB = 2\sqrt{29}$

The distance between A and B is $2\sqrt{29}$.

(c) $A(-\sqrt{5}, -\frac{1}{3})$, $B(-3\sqrt{5}, 5)$

Data:

$A(-\sqrt{5}, -\frac{1}{3})$, $B(-3\sqrt{5}, 5)$

Let $x_1 = -\sqrt{5}$, $y_1 = -\frac{1}{3}$

$x_2 = (-3\sqrt{5}$, $y_2 = 5$

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

$$AB = \sqrt{\left(-3\sqrt{5} - -\sqrt{5}\right)^2 + \left(5 - -\frac{1}{3}\right)^2}$$

$$AB = \sqrt{\left(-3\sqrt{5} + \sqrt{5}\right)^2 + \left(5 + \frac{1}{3}\right)^2}$$

$$AB = \sqrt{\left(-2\sqrt{5}\right)^2 + \left(\frac{16}{3}\right)^2}$$

$$AB = \sqrt{20 + \frac{256}{9}}$$

$$AB = \sqrt{\frac{436}{9}}$$

$$AB = \sqrt{\frac{2 \times 2 \times 109}{3 \times 3}}$$

$$AB = \frac{2\sqrt{109}}{3}$$

(ii). Finding Mid-Point between A & B.

(a) A(3,1), B(−2,−4)

Data:

A(3,1), B(−2,−4)

Let $x_1 = 3, \quad y_1 = 1$

$x_2 = -2, \quad y_2 = -4$

Formula:

$$\boldsymbol{Mid - Point = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})}$$

Solution:

$$Mid - Point(AB) = (\frac{3-2}{2}, \frac{1-4}{2})$$

$$Mid - Point(AB) = (\frac{1}{2}, \frac{-3}{2})$$

The midpoint between the points A(3, 1) and B(−2, −4) is $(\frac{1}{2}, \frac{-3}{2})$.

(b) A(−8, 3), B(2, −1)

Data:

A(−8, 3), B(2, −1)

Let $x_1 = -8, \quad y_1 = 3$

$x_2 = 2, \quad y_2 = -1$

Formula:

$$\boldsymbol{Mid - Point = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})}$$

Solution:

$$Mid - Point(AB) = (\frac{-8+2}{2}, \frac{3-1}{2})$$

$$Mid - Point(AB) = (\frac{-6}{2}, \frac{2}{2})$$

$$Mid - Point(AB) = (-3, 1)$$

The midpoint between the points A(−8, 3) and B(2, −1) is $(-3, 1)$.

(c) $A(-\sqrt{5}, -\frac{1}{3}), B(-3\sqrt{5}, 5)$

Data:

$A(-\sqrt{5}, -\frac{1}{3}), B(-3\sqrt{5}, 5)$

Let $x_1 = -\sqrt{5}, \qquad y_1 = -\frac{1}{3}$

$x_2 = (-3\sqrt{5}, \quad y_2 = 5$

Formula:

$$\boldsymbol{Mid - Point = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})}$$

Solution:

$$Mid - Point(AB) = (\frac{-\sqrt{5} - 3\sqrt{5}}{2}, \frac{-\frac{1}{3} + 5}{2})$$

$$Mid - Point(AB) = (\frac{-4\sqrt{5}}{2}, \frac{14}{2 \times 3})$$

$$Mid - Point(AB) = (-2\sqrt{5}, \frac{7}{3})$$

The midpoint between the points A$(-\sqrt{5}, -\frac{1}{3})$ and B$(-3\sqrt{5}, 5)$ is $(-2\sqrt{5}, \frac{7}{3})$.

Question No. 4

Which of the following points are at a distance of 15 units from the origin?

(i) $(\sqrt{176}, 7)$

Solution:

A$(\sqrt{176}, 7)$, O$(0, 0)$

Let $x_1 = \sqrt{176}$, $y_1 = 7$

$x_2 = 0$, $y_2 = 0$

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

$$AO = \sqrt{\left(0 - \sqrt{176}\right)^2 + (0 - 7)^2}$$

$$AO = \sqrt{\left(\sqrt{176}\right)^2 + (7)^2}$$

$$AO = \sqrt{176 + 49}$$

$$AO = \sqrt{225}$$

$$AO = 15\ units$$

The point $(\sqrt{176}, 7)$ is at a distance of 15 units from the origin.

(ii) $(10, -10)$

Solution:

A$(10, -10)$, O$(0, 0)$

Let $x_1 = 10$, $y_1 = -10$

$x_2 = 0$, $y_2 = 0$

Formula:

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Solution:

$$AO = \sqrt{(0 - 10)^2 + (0 - -10)^2}$$

$AO = \sqrt{(-10)^2 + (10)^2}$

$AO = \sqrt{100 + 100}$

$AO = \sqrt{200}$

$AO = 10\sqrt{2}\,units$

The point (10, −10) is not at a distance of 15 units from the origin.

(iii) (1, 15)

Solution:

A(1, 15), O(0, 0)

Let $x_1 = 1$, $y_1 = 15$

$x_2 = 0$, $y_2 = 0$

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

$AO = \sqrt{(0 - 1)^2 + (0 - 15)^2}$

$AO = \sqrt{(-1)^2 + (-15)^2}$

$AO = \sqrt{1 + 225}$

$AO = \sqrt{226}\,units$

The point (1, 15) is not at a distance of 15 units from the origin.

Question No. 5

Show that:

(i) The points A(0, 2),B($\sqrt{3}$, 1) and C(0, −2) are vertices of a right triangle.

Data:

A = (0, 2)

B = ($\sqrt{3}$, 1)

C = (0, −2)

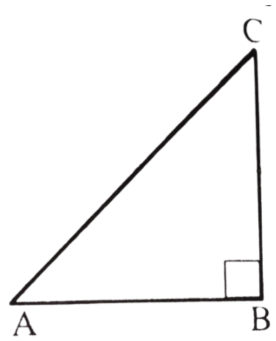
To Find:

|AB|, |BC|, |AC| = ?

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Pictorial Form:



Solution:

1. Length of AB;

$|AB| = \sqrt{\left(\sqrt{3} - 0\right)^2 + (1 - 2)^2}$

$|AB| = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}$

$|AB| = \sqrt{3 + 1}$

$|AB| = \sqrt{4}$

$|AB| = 2$

2. Length of BC;

$|BC| = \sqrt{\left(0 - \sqrt{3}\right)^2 + (-2 - 1)^2}$

$|BC| = \sqrt{\left(-\sqrt{3}\right)^2 + (-3)^2}$

$|BC| = \sqrt{3 + 9}$

$|BC| = \sqrt{12}$

3. Length of AC;

$|AC| = \sqrt{(0 - 0)^2 + (-2 - 2)^2}$

$|AC| = \sqrt{(0)^2 + (-4)^2}$

$|AC| = \sqrt{0 + 16}$

$|AC| = \sqrt{16}$

$|AC| = 4$

$|AB|^2 + |BC|^2 = |AC|^2$

$(2)^2 + (\sqrt{12})^2 = (4)^2$

$4 + 12 = 16$

$16 = 16$

The points A(0, 2), B($\sqrt{3}$, 1), and C(0, -2) are the vertices of a right triangle, with the right angle at vertex B.

(ii) The points A(3,1),B(−2,−3) and C(2,2) are vertices of an isosceles triangle.

Data:

A = (3, 1)

B = (-2, -3)

C = (2, 2)

To Find:

|AB|, |BC|, |AC| = ?

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

1. Length of AB:

$$|AB| = \sqrt{(-2 - 3)^2 + (-3 - 1)^2}$$

$$|AB| = \sqrt{(-5)^2 + (-4)^2}$$

$$|AB| = \sqrt{25 + 16}$$

$$|AB| = \sqrt{41}$$

2. Length of BC:

$$|BC| = \sqrt{(2 - -2)^2 + (2 - -3)^2}$$

$$|BC| = \sqrt{(2 + 2)^2 + (2 + 3)^2}$$

$$|BC| = \sqrt{(4)^2 + (5)^2}$$

$$|BC| = \sqrt{16 + 25}$$

$$|BC| = \sqrt{41}$$

3. Length of AC:

$$|AC| = \sqrt{(2 - 3)^2 + (2 - 1)^2}$$

$$|AC| = \sqrt{(-1)^2 + (1)^2}$$

$$|AC| = \sqrt{1 + 1}$$

$$|AC| = \sqrt{2}$$

Since |AB|=|BC| = 41, the triangle has two sides of equal length. Therefore, the points A(3, 1), B(-2, -3), and C(2, 2) are the vertices of an isosceles triangle.

(iii) The points A(5, 2),B(−2, 3),C(−3, −4) and D(4, −5) are vertices of a parallelogram.

Data:

A = (5, 2)

B = (−2, 3)

C = (−3, −4)

D = (4, −5)

To Find:

|AB|, |BC|, |CD|, |DA| = ?

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution:

1. Length of AB:

$$|AB| = \sqrt{(-2 - 5)^2 + (3 - 2)^2}$$

$$|AB| = \sqrt{(-7)^2 + (1)^2}$$

$$|AB| = \sqrt{49 + 1}$$

$$|AB| = \sqrt{50}$$

2. Length of BC:

$$|BC| = \sqrt{(-3 - -2)^2 + (-4 - 3)^2}$$

$$|BC| = \sqrt{(-3 + 2)^2 + (-7)^2}$$

$$|BC| = \sqrt{(-1)^2 + (7)^2}$$

$$|BC| = \sqrt{1 + 49}$$

$$|BC| = \sqrt{50}$$

3. Length of CD:

$$|CD| = \sqrt{(-3 - 4)^2 + (-4 + 5)^2}$$

$$|CD| = \sqrt{(-7)^2 + (1)^2}$$

$$|CD| = \sqrt{49 + 1}$$

$$|CD| = \sqrt{50}$$

4. Length of DA:

$$|DA| = \sqrt{(4 - 5)^2 + (-5 - 2)^2}$$

$$|DA| = \sqrt{(-1)^2 + (-7)^2}$$

$$|DA| = \sqrt{1 + 49}$$

$$|DA| = \sqrt{50}$$

Since;

$$|AB| = |BC| = |CD| = |DA| = \sqrt{50}$$

Now;

$$|AC| = \sqrt{(-3 - 5)^2 + (-4 - 2)^2}$$

$$|AC| = \sqrt{(-8)^2 + (-6)^2}$$

$|AC| = \sqrt{64 + 36}$

$|AC| = \sqrt{100}$

$|AC| = 10$

Using Pythagoras theorem:

$|AB|^2 + |BC|^2 = |AC|^2$

$|\sqrt{50}|^2 + |\sqrt{50}|^2 = |10|^2$

$50 + 50 = 100$

$100 = 100$

Since Pythagoras thereom is satisfied, So ABCD is a square.

Question No. 6

Find h such that the points A(3, −1),

B(0, 2) and C(h, −2) are vertices of a right triangle with right angle at the vertex A.

Data:

A = (3, −1)

B = (0, 2)

C = (h, −2)

To Find:

h = ?

Solution:

1. Length of AB:

$|AB| = \sqrt{\left(0 - \sqrt{3}\right)^2 + (2 - 1)^2}$

$|AB| = \sqrt{\left(-\sqrt{3}\right)^2 + (1)^2}$

$|AB| = \sqrt{3 + 1}$

$|AB| = \sqrt{4}$

$|AB| = 2$

2. Length of AC:

$|AC| = \sqrt{\left(h - \sqrt{3}\right)^2 + (-2 - 1)^2}$

$|AC| = \sqrt{\left(h - \sqrt{3}\right)^2 + (-3)^2}$

$|AC| = \sqrt{h^2 - 2(h)\left(\sqrt{3}\right) + (\sqrt{3})^2 + 9}$

$|AC| = \sqrt{h^2 - 2h\sqrt{3} + 3 + 9}$

$|AC| = \sqrt{h^2 - 2h\sqrt{3} + 12}$

3. Length of CD:

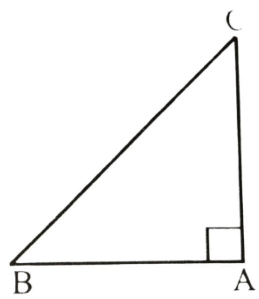
$|BC| = \sqrt{(h - 2)^2 + (-2 - 2)^2}$

$|BC| = \sqrt{h^2 - 2(h)(2) + (2)^2 + (-4)^2}$

$|BC| = \sqrt{h^2 - 4h + 4 + 16}$

$|BC| = \sqrt{h^2 - 4h + 20}$

Pictorial Form:



Using Pythagoras theorem:

$|AB|^2 + |AC|^2 = |BC|^2$

$|2|^2 + |\sqrt{h^2 - 2h\sqrt{3} + 12}|^2 = |\sqrt{h^2 - 4h + 20}|^2$

$4 + h^2 - 2h\sqrt{3} + 12 = h^2 - 4h + 20$

$4 + h^2 - 2h\sqrt{3} + 12 - h^2 + 4h - 20 = 0$

$4h - 2h\sqrt{3} - 8 = 0$

$2(2h - h\sqrt{3}) = 8$

$2h - h\sqrt{3} = 8/2$

$2h - 3.64h = 4$

$0.36h = 4$

$h = \dfrac{4}{0.36}$

$h = 11.1$ Either value is wrong or Answer is wrong.

Question No. 7

Find h such that A(−1, h),B(3, 2) and C(7, 3) are collinear.

Data:

A = (−1, h)

B = (3, 2)

C = (7, 3)

To Find:

h = ?

Solution:

Slope of AB:

$$mAB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - h}{3 - (-1)} = \frac{2 - h}{4}$$

Slope of BC:

$$mBC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{7 - 3} = \frac{1}{4}$$

For collinearity;

$$mAB = mBC$$

$$\frac{2 - h}{4} = \frac{1}{4}$$

$$4(2 - h) = 4$$

$$8 - 4h = 4$$

$$4h = 8 - 4$$

$$h = 4 / 4$$

$$h = 1$$

Therefore, the value of (h) for which the points A, B, and C are collinear is 1.

Question No. 8

The points A(−5,−2) and B(5,−4) are ends of a diameter of a circle. Find the centre and radius of the circle.

Data:

A = (-5, -2)

B = (5, -4)

To Find:

Center of the circle = ?

Radius of the circle = ?

Solution:

1. Finding the Center:

The center of a circle is the midpoint of its diameter.

$$Mid - Point = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

$$Mid - Point = (\frac{-5+\ 5}{2}\ , \frac{-2+\ -4}{2})$$

$$Mid - Point = (\frac{0}{2}\ , \frac{-6}{2})$$

$$Mid - Point = (0\ , -3)$$

So, the center of the circle is (0, -3).

2. Finding the Radius:

The radius of the circle is half the length of the diameter. We can find the length of the diameter AB using the distance formula:

$$d\ =\ \sqrt{(x_2 - \ x_1)^2 +\ (y_2 - \ y_1)^2}$$

$$|AB| =\ \sqrt{(5 - \ -5)^2 +\ (-4 - \ -2)^2}$$

$$|AB| =\ \sqrt{(10)^2 +\ (-2)^2}$$

$$|AB| =\ \sqrt{100 +\ 4}$$

$$|AB| =\ \sqrt{104} =\ 2\sqrt{26}$$

The length of the diameter is $\sqrt{104}$. The radius (r) is half of the diameter:

$$r = \frac{2\sqrt{26}}{2} =\ \sqrt{26}$$

The center of the circle is (0, -3) and the radius of the circle is $\sqrt{26}$.

Question No. 9

Find h such that the points A(h, 1),

B(2, 7) and C (−6, −7) are vertices of a right triangle with right angle at the vertex A.

Data:

A = (h, 1)

B = (2, 7)

C = (-6, -7)

To Find:

h = ?

Solution:

1. Finding |AB|:

$$d\ =\ \sqrt{(x_2 - \ x_1)^2 +\ (y_2 - \ y_1)^2}$$

$$|AB|\ =\ \sqrt{(2-h)^2 +\ (7 - \ 1)^2}$$

$$|AB|\ =\ \sqrt{(2)^2 - 2(2)(h) + (h)^2 +\ 6^2}$$

$$|AB|\ =\ \sqrt{4 - 4h + h^2 +\ 36}$$

$$|AB|\ =\ \sqrt{h^2 - 4h + 40}$$

2. Finding |BC|:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|BC| = \sqrt{(-6-2)^2 + (-7-7)^2}$$

$$|BC| = \sqrt{(-8)^2 + (-14)^2}$$

$$|BC| = \sqrt{64 + 196}$$

$$|BC| = \sqrt{260}$$

3. Finding |AC|:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AC| = \sqrt{(-6-h)^2 + (-7-1)^2}$$

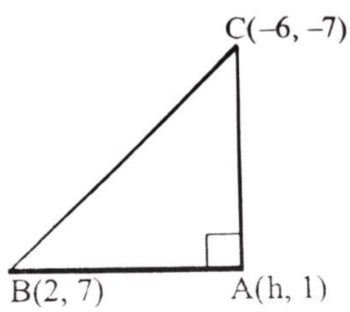
$$|AC| = \sqrt{(6+h)^2 + (-8)^2}$$

$$|AC| = \sqrt{(6)^2 + 2(6)(h) + (h)^2 + 64}$$

$$|AC| = \sqrt{36 + 12h + h^2 + 64}$$

$$|AC| = \sqrt{h^2 + 12h + 100}$$

Pictorial Form:



Using the Pythagorean theorem:
$|BC|^2 = |AB|^2 + |AC|^2$

$|\sqrt{260}|^2 = |\sqrt{h^2 - 4h + 40}|^2 + |\sqrt{h^2 + 12h + 100}|^2$

$260 = h^2 - 4h + 40 + h^2 + 12h + 100$

$260 = 2h^2 + 8h + 140$

$0 = 2h^2 + 8h + 140 - 260$

$2h^2 + 8h - 120 = 0$

$2(h^2 + 4h - 60) = 0$

$h^2 + 4h - 60 = 0/2$

$h^2 - 6h + 10h - 60 = 0$

$h(h - 6) + 10(h - 6) = 0$

$(h - 6)(h + 10) = 0$

$h - 6 = 0 \qquad ; \qquad h + 10 = 0$

$h = 6 \quad ; \quad h = -10$

The possible values of (h) are **6** and **-10**.

Question No. 10

A quadrilateral has the points A (9, 3), B(−7, 7), C (−3, −7) and D (5, −5) as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.

Data:

A = (9, 3)

B = (-7, 7)

C = (-3, -7)

D = (5, -5)

To Find:

1. **Midpoints of the sides AB, BC, CD, and DA.**
2. **Show that the quadrilateral formed by these midpoints is a parallelogram.**

Solution:

1. Finding the Midpoints:

$$Mid - Point = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

- **Midpoint of AB (E):**

$$Mid - Point(AB) = (\frac{9 - 7}{2}, \frac{3 + 7}{2})$$

$$E = (\frac{2}{2}, \frac{10}{2})$$

$$E = (1, 5)$$

- **Midpoint of BC (F):**

$$Mid - Point(BC) = (\frac{-3 - 7}{2}, \frac{-7 + 7}{2})$$

$$F = (\frac{-10}{2}, \frac{0}{2})$$

$$F = (-5, 0)$$

- **Midpoint of CD (G):**

$$Mid - Point(CD) = (\frac{5 - 3}{2}, \frac{-5 - 7}{2})$$

$$G = (\frac{2}{2}, \frac{-12}{2})$$

$$G = (1, -6)$$

- **Midpoint of DA (H):**

$$Mid - Point(DA) = (\frac{5+9}{2}, \frac{-5+3}{2})$$

$$H = (\frac{14}{2}, \frac{-2}{2})$$

$$H = (7, -1)$$

2. Finding the Distance:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of EF:

$$|\mathrm{EF}| = \sqrt{(-5-1)^2 + (0-5)^2}$$

$$|\mathrm{EF}| = \sqrt{(-6)^2 + (-5)^2}$$

$$|\mathrm{EF}| = \sqrt{36 + 25}$$

$$|\mathrm{EF}| = \sqrt{61}$$

Distance of FG:

$$|\mathrm{FG}| = \sqrt{(1+5)^2 + (-6-0)^2}$$

$$|\mathrm{FG}| = \sqrt{(6)^2 + (6)^2}$$

$$|\mathrm{FG}| = \sqrt{36 + 36}$$

$$|\mathrm{FG}| = \sqrt{72}$$

Distance of GH:

$$|\mathrm{GH}| = \sqrt{(7-1)^2 + (-1+6)^2}$$

$$|\mathrm{GH}| = \sqrt{(6)^2 + (5)^2}$$

$$|\mathrm{GH}| = \sqrt{36 + 25}$$

$$|\mathrm{GH}| = \sqrt{61}$$

Distance of HE:

$$|\mathrm{HE}| = \sqrt{(1-7)^2 + (5+1)^2}$$

$$|\mathrm{HE}| = \sqrt{(-6)^2 + (6)^2}$$

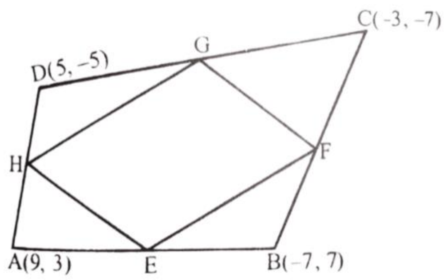
$$|\mathrm{HE}| = \sqrt{36 + 36}$$

$$|\mathrm{HE}| = \sqrt{72}$$

As; $|\mathrm{EF}| = |\mathrm{GH}| = \sqrt{61}$

$|\mathrm{FG}| = |\mathrm{HE}| = \sqrt{72}$

Pictorial Form:



Conclusion:

The midpoints of the sides of the quadrilateral ABCD are E(1, 5), F(-5, 0), G(1, -6), and H(7, -1). The quadrilateral formed by joining these midpoints consecutively, EFGH, is a parallelogram because its diagonals EG and FH bisect each other at the point (1,−21).

This result is a specific case of the Midpoint Theorem for quadrilaterals, which states that the figure formed by joining the midpoints of the sides of any quadrilateral is always a parallelogram.