Unit No. 9

Similar Figures

Exercise No. 9.4

Question No. 1

(i) What is the sum of the interior angles of a decagon (10-sided polygon)?

Solution:

For a regular n-sided polygon:

Size of each interior angle = $(n - 2) / n \times 180^\circ$

For decagon n = 10

Size of each interior angle of decagon = $(10 - 2) / 10 \times 180^\circ = 8 / 10 \times 180 = 144^\circ$

Sum of interior angles of decagon $= 144^\circ \times 10 =$ **1440°** $(n = 10)$

(ii) Calculate the measure of each interior angle of a regular hexagon.

Solution:

For a regular n-sided polygon:

Size of each interior angle = $(n - 2) / n \times 180^\circ$

For regular hexagon n = 6

Size of each interior angle of regular hexagon = $(n - 2) / n \times 180^\circ$

$= (6 - 2) / 6 \times 180^\circ = 4 \times 30^\circ = 120^\circ$

Hence size of each interior angle of regular hexagon is 120°

(iii) What is each exterior angle of a regular pentagon?

Solution:

The exterior angle of each side of a regular n-sided polygon is:

Exterior angle = $360^\circ / n$

No. of sides of a regular pentagon = 5

Exterior angle of regular pentagon = $360^\circ / 5 = 72^\circ$

Hence, each exterior angle of a regular pentagon is 72°

(iv) If the sum of the interior angles of a polygon is 1260°, how many sides does the polygon have?

Given:

Sum of interior angles of polygon $= 1260^\circ$

To Find:

No. of sides of the polygon = ?

Solution:

Sum of interior angles of polygon $=1260^\circ$

We know that:

Sum of interior angles = Size of each interior angle × (no. of sides)

$= [(n - 2) \times 180^\circ / n] \times n$

Sum of interior angles $= (n - 2) \times 180^\circ$

By putting value:

$1260^\circ = (n - 2) \times 180^\circ$

$1260^\circ / 180^\circ = n - 2$

$7 = n - 2$

$7 + 2 = n$

$9 = n$

Hence, the given polygon has 9 sides.

Question No. 2

In a parallelogram ABCD, m AB = 10 cm, m AD = 6 cm and m ∠BAD = 45°.

Calculate the area of ABCD.

Given:

m AB = m CD = 10 cm

m AD = m BC = 6 cm

m ∠BAD = 45°

To Find:

Area of parallelogram ABCD = ?

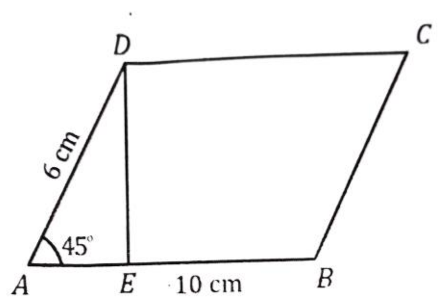
Solution:

We know that

Area of parallelogram = length × height = m AB × m ED…... eq. (i)

To find height, draw a perpendicular DE on base AE;

such that m ∠AED=90°



Now consider right angle triangle AED:

Sin m ∠EAD = m ED / m AD

Sin 45° = m ED / 6

$\frac{1}{\sqrt{2}} \times 6 =$ m ED

m ED = $\frac{6}{\sqrt{2}}$ m

Area of parallelogram = $10 \times \frac{6}{\sqrt{2}}$

Area of parallelogram = $30\sqrt{2}$ m^2 = 42.43 cm^2

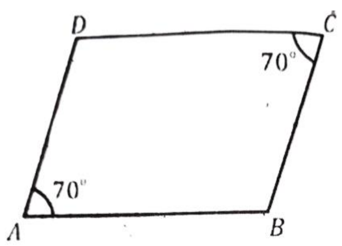
Question No. 3

In a parallelogram ABCD if m ∠DAB = 70°, find the measures of all other angles in the parallelogram.

Given:

m ∠DAB = 70°

In a parallelogram, opposite angles are equal, and adjacent angles are supplementary (add up to 180°).



Solution:

m ∠DAB = 70°

So, m ∠DAB = m ∠BCD = 70° (Opposite angles are equal)

m ∠DAB + m ∠ABC = 180° (∵ Adjacent angles are supplementary)

⇒ m ∠ABC = 180° − m ∠ DAB

m ∠ ABC = 180° − 70°

m ∠ ABC = 110°

Also; m ∠ABC =m ∠ADC = 110° (∵ Opposite angles are equal)

Hence

m ∠DAB = 70° (Given)

m ∠ABC = 110°

m ∠BCD = 70°

m ∠ADC = 110°

Question No. 4

A shape is created by cutting a square in half diagonally and then attaching a right-angled triangle to the hypotenuse of each half. Explain why this shape can tessellate and calculate the interior angle of the new shape.

Solution:

- A square is cut in half diagonally, creating two right-angled triangles.
- A right-angled triangle is attached to the hypotenuse of each half.

The New Shape:

The new shape consists of two right angled triangles. The angles of a right angled triangle are 90°, 45°, 45° (= 180°) because it is formed by cutting a square in half. Since two right angled triangles are attached, so the interior angles will be summing to (180° + 180°) = 360° A shape can tessellate if it can fit together without gaps when repeated.

Since the given shape consists of right angled triangles and triangles tessellate naturally, so the new shape square (made of two triangles) will also tessellate.

Question No. 5

A tessellation is created by repeatedly reflecting a basic shape. The basic shape is a right-angled triangle with sides of length 3, 4, and 5 units. Find:

The minimum number of reflections needed to create a tessellation that covers a square with an area of 3600 square units.

Given:

Basic shape = Right angled triangle

Let

Base of triangle = 3 units

Height: Perpendicular = 4 units

Hypotenuse = 5 units

$(3)^2 + (4)^2 = 5^2$; $9 + 16 = 25$

To Find:

The minimum number of reflections = ?

Solution:

We know that;

Area of triangle = 1 / 2 (base)×(height)

By Putting values:

Area of triangle = 1 / 2 (3)×(4)

Area of triangle = 6 units

Area to be covered = 3600 units

No. of reflections of right angled triangle required = 3600 / 6 = 600

Hence; 600 reflections are needed to create a tessellation that covers a square with an area of 3600 square units.

Question No. 6

A tessellation is created using regular hexagons. Each hexagon has a side length of 5 cm. Find the total area of the tessellation if it consists of 25 hexagons and total perimeter of the outer edge of the tessellation, assuming it's a perfect

hexagon.

Given:

Basic shape = hexagon (6 sides)

Length of one side of hexagon = 5 cm

No. of hexagons = 25

To Find:

Total area of tessellation = ?

Total perimeter of outer edge of tessellation = ?

Solution:

We know that:

Area of hexagon = $\frac{3\sqrt{3}}{2} \times (\text{side})^2$

Area of hexagon = $\frac{3\sqrt{3}}{2} \times (5)^2 = \frac{3\sqrt{3}}{2} \times 25 = \frac{75\sqrt{3}}{2}$

Area of hexagon = 64.95 cm^2

Total area of tessellation = (Area of one hexagon) × (No. of hexagons)

Total area of tessellation = (64.95) × 25

Total area of tessellation = 1623.8 cm^2

Perimeter of tessellation of a perfect hexagon = No. of hexagons × no. of sides

Perimeter of tessellation of a perfect hexagon = 25 × 6

Perimeter of tessellation of a perfect hexagon = 150 cm

Question No. 7

A rectangular floor is 12 m by 15 m. How many square tiles, each 1 m by 1 m, are needed to cover the floor?

Given:

Dimensions of floor = 12 m × 15 m

Dimensions of square of tile = 1 m × 1 m

To Find:

No. of tiles required = ?

Solution:

Formula for Area of square/rectangle = length × breadth

Area of square tile = 1 × 1 = 1 m^2

Total area of floor to be covered = 12 × 15 = 180 m^2

No. of tiles required = Area of floor / Area of tile

No. of tiles required = 180 / 1

No. of tiles required = 180 tiles

Question No. 8

A rectangular wall is 10 m tall and 120 m wide. How many gallons of paint are needed to cover the wall, if one gallon covers 35 m^2?

Given:

Length of rectangular wall = 10 m

Width of rectangular wall = 120 m

Area of wall covered with one gallon of paint = 35 m^2 per gallon

Volume of paint = 1 gallon (covers 35 m^2)

To Find:

No. of gallons of paint required to cover the wall = ?

(Note: The solution calculates in gallons, not litres as stated in the "Required" section)

Solution:

Formula for Area of rectangle = length × width

Area of wall = 120 × 10 = 1200 m^2

Paint required to cover 35 m^2 of wall = 1 gallon

Paint required to cover 1 m^2 of wall = 1 / 35 gallon

Paint required to cover 1200 m^2 of wall = 1 / 35 × 1200 gallon

Paint required to cover 1200 m^2 of wall = 34.28 gallons

So practically, 35 gallons of paint is required to paint the wall.

Question No. 9

A rectangular wall has a length of 10 m and a width of 4 meters. If 1 litre of paint covers 7 m^2, how many liters of paint are needed to cover the wall?

Given:

Length of wall = 10 m

Width of wall = 4 m

Area of wall covered with paint per litre = 7 m^2

Paint used = 1 litre (covers 7 m^2)

To Find:

No. of litres of paint required to paint whole wall = ?

Solution:

Formula for Area of rectangle = length × width

Area of rectangular wall = $10 \times 4 = 40\ m^2$

Paint required to cover $7\ m^2$ of wall = 1 litre

Paint required to cover $1\ m^2$ of wall = 1 / 7 litre

Paint required to cover $40\ m^2$ of wall = $1 / 7 \times 40 = 40 / 7 = 5.71$ litres

So practically, 6 litres of paint is required to paint the wall.

Question No. 10

A window has a trapezoidal shape with parallel sides of 3 m and 1.5 m and a height of 2 m. Find the area of the window.

Given:

Shape of window = trapezoidal

Length of one parallel side = 3 m

Length of other parallel side = 1.5 m

Height of trapezoid = 2 m

To Find:

Area of window = ?

Solution:

Formula for Area of trapezoid = 1 / 2 × (sum of parallel sides) × height

Area of window = $1 / 2 \times (3 + 1.5) \times (2)$

Area of window = $0.5 \times (4.5) \times (2)$

Area of window = 0.5×9

Area of window = $4.5\ m^2$

Hence, area of trapezoidal window = $4.5m^2$