Unit No. 10

Graphs of Functions

Review Exercise No. 10

Question No.1

Four options are given against each statement. Encircle the correct option.

(i) x=5 represents:

(a) x-axis

(b) y-axis

(c) line ∥ to x-axis

(d) line ∥ to y-axis

(ii) Slope of the line y=5x+3 is:

(a) 3

(b) -3

(c) 5

(d) -5

(iii) The y- intercepts of $y = -2x - 1$ is:

(a) -2

(b) 2

(c) -1

(d) 1

(iv) The graph of y=x2, cuts the x-axis at:

(a) $x = 0$

(b) $x = 1$

(c) $x = -1$

(d) $x = 2$

(v) The graph of 3x represents:

(a) growth

(b) decay

(c) both (a) and (b)

(d) a line

(vi) The graph of $y = -x 2 + 5$ opens:

(a) upward

(b) downward

(c) left side

(d) right side

(vii) The graph of $y = x2 - 9$ opens:

(a) upward

(b) downward

(c) left side

(d) right side

(viii) $y = 5x$ is ____________ function.

(a) linear

(b) quadratic

(c) cubic

(d) exponential

(ix) Reciprocal function is:

(a) $y = 7x$

(b) $y = x2$

(c) $y = 2x^2$

(d) $y = 5x^3$

(x) $y = -3x^3 + 7$ is ____________ function.

(a) exponential

(b) cubic

(c) linear

(d) reciprocal

Question No. 2

Plot the graph of the following functions:

(i) $y = 3^{-x}$ for x from -2 to 4

Solution:

Type: Exponential function.

Here x = -2, -1, 0, 1, 2, 3, 4

By putting these values in $y = 3^{-x}$, we can find y:

If $x = -2$; $y = 3^{-(-2)} = 3^2 = 9$

If $x = -1$; $y = 3^{-(-1)} = 3^1 = 3$

If $x = 0$; $y = 3^{-(0)} = 1$

If $x = 1$; $y = 3^{-(1)} = 3^{-1} = 1/3$

If $x = 2$; $y = 3^{-(2)} = 3^{-2} = 1/9$

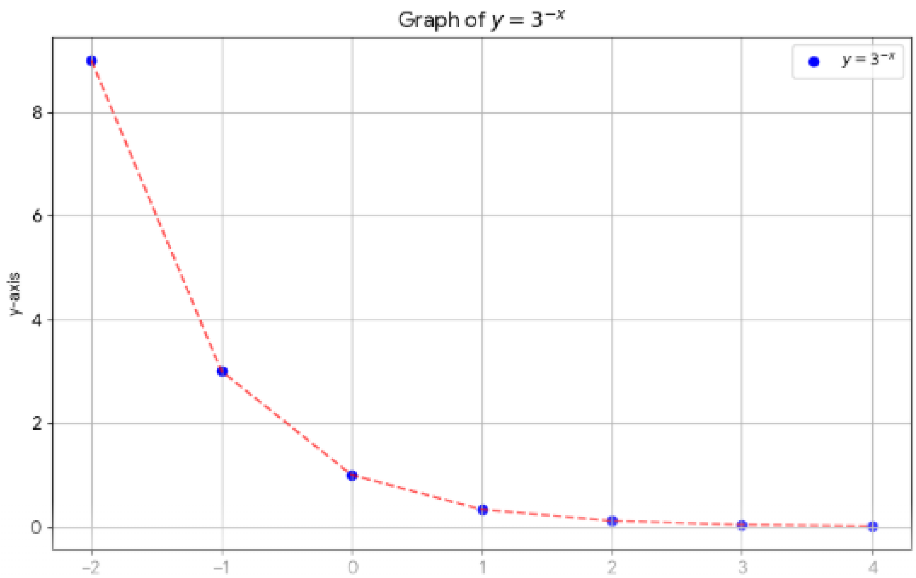
If x = 3 ; $y = 3^{-(3)} = 3^{-3} = 1/27$

If x = 4 ; $y = 3^{-(4)} = 3^{-4} = 1/81$

Table:

	A	B	C	D	E	F	G
x-axis	-2	-1	0	1	2	3	4
y-axis	9	3	1	1/3	1/9	1/27	1/81

Graphical Representation:



(ii) y = 2/x, x ≠ 0

Solution:

Type: Quadratic function.

Here x = -2, -1, 1, 2, 3, 4

By putting these values in $y = 3^{-x}$, we can find y:

If x = - 2; y = 2 / -2 = -1

If x = - 1; y = 2 / -1 = -2

If x = 1 ; y = 2 / 1 = 2

If x = 2 ; y = 2 / 2 = 1

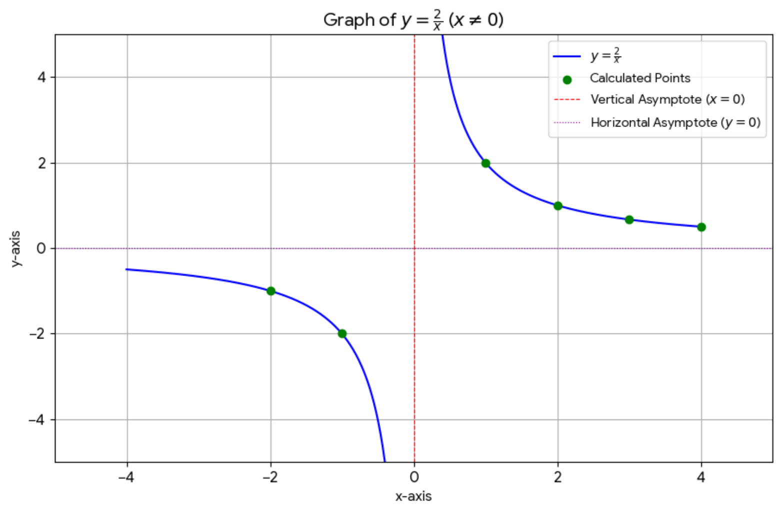
If x = 3 ; y = 2 / 3

If x = 4 ; y = 2 / 4 = 1/2

Table:

	A	B	C	D	E	F
x-axis	-2	-1	1	2	3	4
y-axis	-1	-2	2	1	2/3	1/2

Graphical Representation:



Question No.3

Sales for a new magazine are expected to grow according to the equation:

$S = 200000\ (1 - e^{-0.05t})$, where t is given in weeks.

(a) Plot graph of sales for the first 50 weeks.

(b) Calculate the number of magazines sold, when t = 5 and t = 35.

Solution (a):

$S = 200000\ (1 - e^{-0.05t})$

Given Weeks = 1, 2, 3, … 50

To solve, we assume weeks = t = 0, 10, 20, 30, 40, 50

Now, calculating Sale:

i). S when t = 0

$S = 200000\ (1 - e^{-0.05(0)})$

$S = 200000\ (1 - e^{0}) = 200000\ (1 - 1) = 200000\ (0) = 0$

ii). S when t = 10

$S = 200000\ (1 - e^{-0.05(10)})$

$S = 200000\ (1 - e^{-0.5}) = 200000\ (1 - 0.6065) = 200000\ (0.3935) = 78700$

iii). S when t = 20

$S = 200000\ (1 - e^{-0.05(20)})$

$S = 200000\ (1 - e^{-1}) = 200000\ (1 - 0.3679) = 200000\ (0.6321) = 126420$

v). S when t = 30

$S = 200000\ (1 - e^{-0.05(30)})$

S = 200000 (1 – $e^{-1.5}$) = 200000 (1 – 0. 2231) = 200000 (0. 7769) = 155374

vi). S when t = 40

S = 200000 (1 − $e^{-0.05(40)}$)

S = 200000 (1 – e^{-2}) = 200000 (1 – 0.1353) = 200000 (0.8647) = 172940

vii). S when t = 50

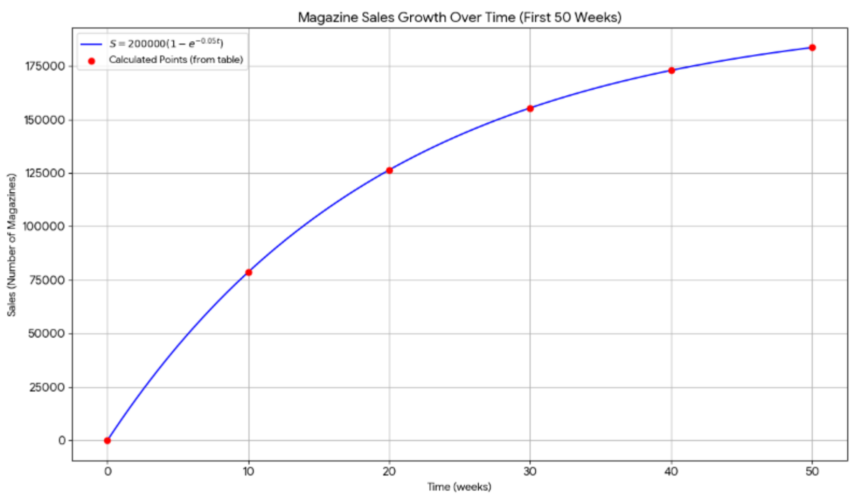
S = 200000 (1 − $e^{-0.05(50)}$)

S = 200000 (1 – $e^{-2.5}$) = 200000 (1 – 0.2183) = 200000 (0.7817) = 156340

Table:

	A	B	C	D	E	F
x-axis t.(weeks)	0	10	20	30	40	50
y-axis S.(Sales)	0	78700	126420	155374	172940	156340

Graphical Representation:



Solution (b): Calculating the number of magazines sold, when t = 5 and t = 35.

I). S when t = 5

S = 200000 (1 − $e^{-0.05(5)}$)

S = 200000 (1 – $e^{-0.25}$) = 200000 (1 – 0.7788) = 200000 (0.2212) = 44240

II). S when t = 35

S = 200000 (1 − $e^{-0.05(35)}$)

S = 200000 (1 – $e^{-1.75}$) = 200000 (1 – 0.1738) = 200000 (0.8262) = 165240

Question No.4

Plot the graph of following for x from −5 to 5:

(i) y = x^2 − 3

(ii) y = 15 − x^2

Solution (i):

y = x^2 – 3

Take x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5

Now, calculating for y:

$y = (-5)^2 – 3 = 25 – 3 = 22$

$y = (-4)^2 – 3 = 16 – 3 = 13$

$y = (-3)^2 – 3 = 9 – 3 = 6$

$y = (-2)^2 – 3 = 4 – 3 = 1$

$y = (-1)^2 – 3 = 1 – 3 = - 2$

$y = (0)^2 – 3 = 0 – 3 = - 3$

$y = (1)^2 – 3 = 1 – 3 = - 2$

$y = (2)^2 – 3 = 4 – 3 = 1$

$y = (3)^2 – 3 = 9 – 3 = 6$

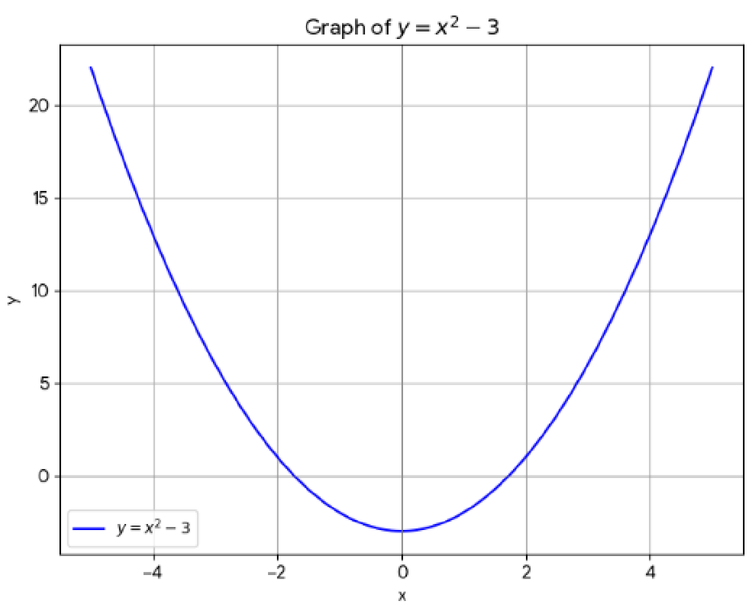
$y = (4)^2 – 3 = 16 – 3 = 13$

$y = (5)^2 – 3 = 25 – 3 = 22$

Table:

	A	B	C	D	E	F	G	H	I	J	K
x-axis	-5	-4	-3	-2	-1	0	1	2	3	4	5
y-axis	22	13	6	1	-2	-3	-2	1	6	13	22

Graphical Representation:



Solution (ii):

$y = 15 - x^2$

Take x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5

Now, calculating for y:

$y = 15 – (-5)^2 = 15 – 25 = -10$

$y = 15 – (-4)^2 = 15 – 16 = -1$

$y = 15 – (-3)^2 = 15 – 9 = 6$

$y = 15 – (-2)^2 = 15 – 4 = 11$

$y = 15 – (-1)^2 = 15 – 1 = 14$

$y = 15 – (0)^2 = 15 – 0 = 15$

$y = 15 – (1)^2 = 15 – 1 = 14$

$y = 15 – (2)^2 = 15 – 4 = 11$

$y = 15 – (3)^2 = 15 – 9 = 6$

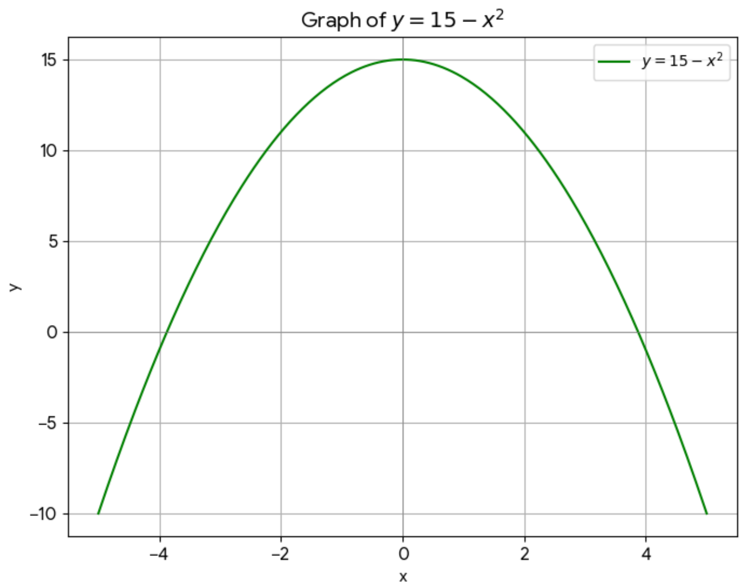
$y = 15 – (4)^2 = 15 – 16 = -1$

$y = 15 – (5)^2 = 15 – 25 = -10$

Table:

	A	B	C	D	E	F	G	H	I	J	K
x-axis	-5	-4	-3	-2	-1	0	1	2	3	4	5
y-axis	-10	-1	6	11	14	15	14	11	6	-1	-10

Graphical Representation:



Question No.5

Plot the graph of $y = \frac{1}{2}(x + 4)(x - 1)(x - 3)$ from -5 to 4.

Solution:

$y = \frac{1}{2}(x + 4)(x - 1)(x - 3)$

Take x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4

Now, calculating for y:

I). when x = - 5

$y = \frac{1}{2}(-5 + 4)(-5 - 1)(-5 - 3) = \frac{1}{2}(-1)(-6)(-8) = \frac{1}{2}(-48) = -24$

II). when x = - 4

$y = \frac{1}{2}(-4 + 4)(-4 - 1)(-4 - 3) = \frac{1}{2}(0)(-5)(-7) = \frac{1}{2}(0) = 0$

III). when x = - 3

$y = \frac{1}{2}(-3 + 4)(-3 - 1)(-3 - 3) = \frac{1}{2}(1)(-4)(-6) = \frac{1}{2}(24) = 12$

IV). when x = - 2

$y = \frac{1}{2}(-2 + 4)(-2 - 1)(-2 - 3) = \frac{1}{2}(2)(-3)(-5) = \frac{1}{2}(30) = 15$

V). when x = - 1

$y = \frac{1}{2}(-1 + 4)(-1 - 1)(-1 - 3) = \frac{1}{2}(3)(-2)(-4) = \frac{1}{2}(24) = 12$

VI). when x = 0

$y = \frac{1}{2}(0 + 4)(0 - 1)(0 - 3) = \frac{1}{2}(4)(-1)(-3) = \frac{1}{2}(12) = 6$

VII). when x = 1

$y = \frac{1}{2}(1 + 4)(1 - 1)(1 - 3) = \frac{1}{2}(5)(0)(-2) = \frac{1}{2}(0) = 0$

VIII). when x = 2

$y = \frac{1}{2}(2 + 4)(2 - 1)(2 - 3) = \frac{1}{2}(6)(1)(-1) = \frac{1}{2}(-6) = -3$

IX). when x = 3

$y = \frac{1}{2}(3 + 4)(3 - 1)(3 - 3) = \frac{1}{2}(7)(2)(0) = \frac{1}{2}(0) = 0$

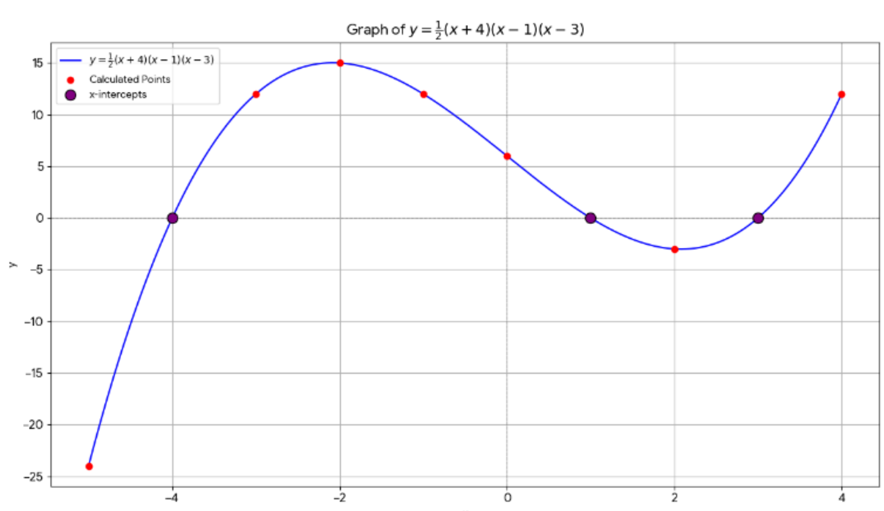
X). when x = 4

$y = \frac{1}{2}(4 + 4)(4 - 1)(4 - 3) = \frac{1}{2}(8)(3)(1) = \frac{1}{2}(24) = 12$

Table:

	A	B	C	D	E	F	G	H	I	J
x-axis	-5	-4	-3	-2	-1	0	1	2	3	4
y-axis	-24	0	12	15	12	6	0	-3	0	12

Graphical Representation:



Question No.6

The supply and demand functions for a particular market are given by the equations: $Ps = Q^2 + 5$ and $Pd = Q^2 - 10Q$, where P represents price and Q represents quantity, Sketch the graph of each function over the interval

$Q = -20$ to $Q = 20$.

Solution:

$Ps = Q^2 + 5$

Given: $Q = -20$ to $Q = 20$

Take Q = -20, -15, -10, -5, 0, 5, 10, 15, 20

Now, calculating for Ps:

$Q = -20; Ps = (-20)^2 + 5 = 400 + 5 = 405$

$Q = -15; Ps = (-15)^2 + 5 = 225 + 5 = 230$

$Q = -10; Ps = (-10)^2 + 5 = 100 + 5 = 105$

$Q = -5; Ps = (-5)^2 + 5 = 25 + 5 = 30$

$Q = 0; Ps = (0)^2 + 5 = 0 + 5 = 5$

$Q = 5; Ps = (5)^2 + 5 = 25 + 5 = 30$

$Q = 10; Ps = (10)^2 + 5 = 100 + 5 = 105$

$Q = 15; Ps = (15)^2 + 5 = 225 + 5 = 230$

$Q = 20; Ps = (20)^2 + 5 = 400 + 5 = 405$

Table:

	A	B	C	D	E	F	G	H	I
x-axis Q	-20	-15	-10	-5	0	5	10	15	20
y-axis Ps	405	230	105	30	5	30	105	230	405

$Pd = Q^2 - 10Q$

Given: $Q = -20$ to $Q = 20$

Take Q = -20, -15, -10, -5, 0, 5, 10, 15, 20

Now, calculating for Pd:

$Q = -20; Pd = (-20)^2 - 10(-20) = 400 + 200 = 600$

$Q = -15; Ps = (-15)^2 - 10(-15) = 225 + 150 = 375$

$Q = -10; Ps = (-10)^2 - 10(-10) = 100 + 100 = 200$

$Q = -5; Ps = (-5)^2 - 10(-5) = 25 + 50 = 75$

$Q = 0; Ps = (0)^2 - 10(0) = 0 - 0 = 0$

$Q = 5; Ps = (5)^2 - 10(5) = 25 - 50 = -25$

$Q = 10; Ps = (10)^2 - 10(10) = 100 - 100 = 0$

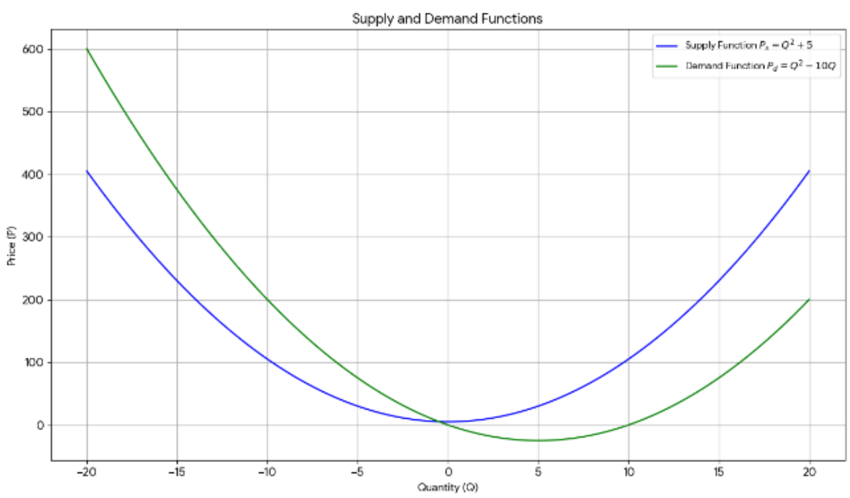
$Q = 15; Ps = (15)^2 - 10(15) = 225 - 150 = 75$

$Q = 20; Ps = (20)^2 - 10(20) = 400 - 200 = 200$

Table:

	A	B	C	D	E	F	G	H	I
x-axis Q	-20	-15	-10	-5	0	5	10	15	20
y-axis Ps	600	375	200	75	0	-25	0	75	200

Graphical Representation:



The Supply function $Ps = Q^2 + 5$ is plotted in blue.

The Demand function $Pd = Q^2 - 10Q$ is plotted in green.

Question No. 7

A television manufacturer company make 40 inches LEDs. The cost of manufacturing x LEDs is C(x) = 60,000 + 250 x and the revenue from selling x LEDs is R(x) = 1200 x. Find the break-even point and find the profit or loss when 100 LEDs are sold. Identify the break-even point graphically.

Solution:

C(x) = 60,000 + 250 x

Take; x = 0, 20, 40, 60, 80

Now, calculating for C(x):

x = 0; C(x) = 60,000 + 250 (0) = 60,000 + 0 = Rs. 60,000

x = 20; C(x) = 60,000 + 250 (20) = 60,000 + 5000 = Rs. 65,000

x = 40; C(x) = 60,000 + 250 (40) = 60,000 + 10000 = Rs. 70,000

x = 60; C(x) = 60,000 + 250 (60) = 60,000 + 15000 = Rs. 75,000

x = 80; C(x) = 60,000 + 250 (80) = 60,000 + 20000 = Rs. 80,000

Table:

	A	B	C	D	E
x-axis x	0	20	40	60	80
y-axis C(x) Rs.	60000	65000	70000	75000	80000

R(x) = 1200 x

Take; x = 0, 20, 40, 60, 80

Now, calculating for C(x):

x = 0; R(x) = 1200 (0) = 0

x = 20; R(x) = 1200 (20) = 24000

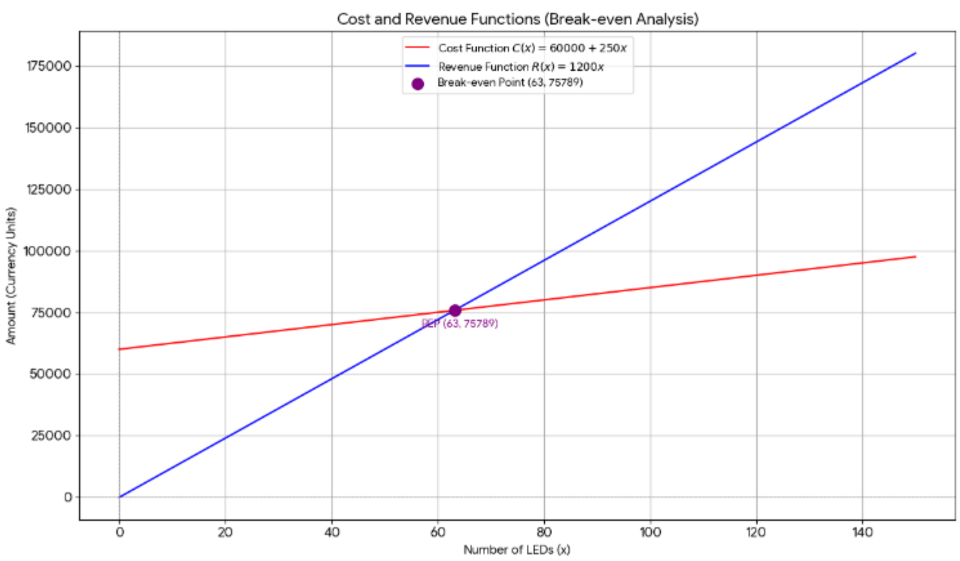
x = 40; R(x) = 1200 (40) = 48000

x = 60; R(x) = 1200 (60) = 72000

x = 80; R(x) = 1200 (80) = 96000

	A	B	C	D	E
x-axis x	0	20	40	60	80
y-axis C(x) Rs.	0	24000	48000	72000	96000

Graphical Representation:



Break-even point is (63, 75790)

If profit is realized then S.P > C.P

Profit = 1200x - (60000 + 250x)

Profit = 1200x-60000-250x

Profit = 1200x-250x-60000

Profit = 950x-60000

For 100 LEDs, x = 100

Profit = 950 x 100-60000

Profit = 95000-60000

Profit = Rs.35000