Unit No. 6

Trigonometry

Review Exercise No. 6

Question No. 1

Four options are given against each statement. Encircle the correct one.

(i). The value of $\tan^{-1} 2$ in radians is:

(a) $\pi/2$

(b) $3\pi/2$

(c) 0.4636π

(d) 0.4636

True option is NOT given:

(ii). In a right triangle, the hypotenuse is 13 units and one of the angles is θ=30°. The length of the opposite side is:

(a) 6.5 units

(b) 7.5 units

(c) 6 units

(d) 5 units

(iii). A person standing 50 m away from a building sees the top of the building at an angle of elevation of 45°. Height of the building is:

(a) 50 m

(b) 25 m

(c) 35 m

(d) 70 m

(iv). $\sec^2\theta - \tan^2\theta$ = _____.

(a) $\sin^2\theta$

(b) 1

(c) $\cos^2\theta$

(d) $\cot^2\theta$

(v). If $\sin\theta = \frac{3}{5}$ and θ is an acute angle,

$\cos^2\theta$ = _____ .

(a) $\frac{7}{25}$

(b) $\frac{24}{25}$

(c) $\frac{16}{25}$

(d) $\frac{4}{25}$

(vi). $\frac{5\pi}{24}$ **rad = ________________ degrees.**

(a) 30°

(b) 37.5°

(c) 45°

(d) 52.5°

(vii). 292.5° = __________________ rad.

(a) $\frac{17\pi}{6}$

(b) $\frac{17\pi}{4}$

(c) 1.6π

(d) 1.625π

(viii). Which of the following is a valid identity?

(a) $\cos(\frac{\pi}{2} - \theta) = \sin\theta$

(b) $\cos(\frac{\pi}{2} - \theta) = \cos\theta$

(c) $\cos(\frac{\pi}{2} - \theta) = \sec\theta$

(d) $\cos(\frac{\pi}{2} - \theta) = \mathrm{cosec}\theta$

(ix). sin 60° = _____ .

(a) 1

(b) $\frac{1}{2}$

(c) $\sqrt{(3)^2}$

(d) $\frac{\sqrt{3}}{2}$

(x). $\cos^2 100\pi$ **= ______ .**

(a) 1

(b) 2

(c) 3

(d) 4

Question No. 2

Convert the given angles from:

(a) degrees to radians giving answer in terms of π:

(i) 255°

Solution:

255°

Radians = 255 × $\frac{\pi}{180}$

Radians = $\frac{255\pi}{180}$ π rad

Radians = $\frac{17\pi}{12}$ rad

(ii) 75° 45'

Solution:

75° 45'

= 75° + $\frac{45}{60}^\circ$

= 75° + $\frac{3}{4}^\circ$

= $75\frac{3}{4}^\circ$

= $\frac{303}{4}^\circ$

Radians = $\frac{303}{4}$ × $\frac{\pi}{180}$ rad

Radians = $\frac{303\pi}{720}$ π rad

Radians = $\frac{101\pi}{240}$ rad

(iii) 142.5°

Solution:

142.5°

Radians = 142.5 × $\frac{\pi}{180}$

Radians = $\frac{142.5\pi}{180}$ π rad

Radians = $\frac{1425\pi}{1800}$ π rad

Radians = $\frac{19\pi}{24}$ rad

(b) radians to degrees giving answer in minutes:

(i) $\frac{17\pi}{24}$

Solution:

$\frac{17\pi}{24}$

= $\frac{17\pi}{24} \times \frac{180}{\pi}$

= $\frac{17 \times 15}{2}$

= $\frac{255}{2}$

= 127.5°

= 127° (0.5 × 60)'

= 127° 30'

(ii) $\frac{7\pi}{12}$

Solution:

$\frac{7\pi}{12}$

$= \frac{7\pi}{12} \times \frac{180}{\pi}$

= 7 × 15°

= 105°

(iii) $\frac{11\pi}{16}$

Solution:

$\frac{11\pi}{16}$

$= \frac{11\pi}{16} \times \frac{180}{\pi}$

$= \frac{11 \times 45}{4}$

$= \frac{495}{4}$

= 123.75°

= 123° (0.75 × 60)'

= 123° 45'

Question No. 3

Prove the following trigonometric identities:

(i). $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$

Solution:

By Solving L.H.S:

$= \frac{\sin\theta}{1-\cos\theta}$

Multiply the numerator and the denominator by (1 + sinθ):

$= \frac{\sin\theta}{1-\cos\theta} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$

$= \frac{\sin\theta(1+\cos\theta)}{1^2 - \cos^2\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{1 - \cos^2\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$

$= \frac{(1+\cos\theta)}{\sin\theta}$

This is equal to the right-hand side.

Therefore, $\frac{sin\theta}{1-cos\theta} = \frac{1+cos\theta}{sin\theta}$ is proven.

(ii). $\mathbf{\sin\theta(\mathrm{cosec}\theta - \sin\theta) = \frac{1}{\sec^2\theta}}$

Solution:

$\sin\theta(\mathrm{cosec}\theta - \sin\theta) = \frac{1}{\sec^2\theta}$

By Solving L.H.S:

$= \sin\theta(\mathrm{cosec}\theta \; - \sin\theta)$

$= \sin\theta\cdot \mathrm{cosec}\theta - \sin\theta\cdot\sin\theta$

We know that $\mathrm{cosec}\theta = \frac{1}{sin\theta}$:

$= \sin\theta\cdot \frac{1}{sin\theta} - sin^2\theta$

$= 1 - \; sin^2\theta$

We know that, $1 - \; sin^2\theta = \; cos^2\theta$:

$= cos^2\theta$

$= \frac{1}{\sec^2\theta}$

This is equal to the right-hand side.

Therefore, $\sin\theta(\mathrm{cosec}\theta - \sin\theta) = \frac{1}{\sec^2\theta}$ is proven.

(iii). $\boldsymbol{\frac{cosec\theta - sec\theta}{cosec\theta + sec\theta} = \frac{1 - tan\theta}{1 + tan\theta}}$

Solution:

$\frac{cosec\theta - sec\theta}{cosec\theta + sec\theta} = \frac{1 - tan\theta}{1 + tan\theta}$

By Solving L.H.S:

$= \frac{cosec\theta - sec\theta}{cosec\theta + sec\theta}$

$= (cosec\theta \; - \; sec\theta) \div (cosec\theta \; + \; sec\theta)$

$= (\frac{1}{sin\theta} \; - \; \frac{1}{cos\theta}) \div (\frac{1}{sin\theta} \; + \; \frac{1}{cos\theta})$

$= (\frac{cos\theta - \; sin\theta}{sin\theta \; . \; cos\theta}) \div (\frac{cos\theta + \; sin\theta}{sin\theta \; . \; cos\theta})$

$= (\frac{cos\theta - \; sin\theta}{sin\theta \; . \; cos\theta}) \times (\frac{sin\theta \; . \; cos\theta}{cos\theta + \; sin\theta})$

$= \frac{cos\theta - \; sin\theta}{cos\theta + \; sin\theta}$

$= (cos\theta \; - \; sin\theta) \div \; (cos\theta \; + \; sin\theta)$

Divide by $cos\theta$:

$= (\frac{cos\theta - \; sin\theta}{cos\theta}) \div (\frac{cos\theta + \; sin\theta}{cos\theta})$

$= (\frac{cos\theta}{cos\theta} - \frac{sin\theta}{cos\theta}) \div (\frac{cos\theta}{cos\theta} + \frac{sin\theta}{cos\theta})$

$= (1 - \tan\theta) \div (1 + \tan\theta)$

$= \frac{1 - \tan\theta}{1 + \tan\theta}$

This is equal to the right-hand side.

Therefore, secθtanθ+cotθ=cscθ is proven.

(iv). $\tan\theta + \cot\theta = \frac{1}{sin\theta cos\theta}$

Solution:

$\tan\theta + \cot\theta = \frac{1}{sin\theta cos\theta}$

By Solving L.H.S:

$= \tan\theta + \cot\theta$

Expressing tanθ and cotθ in terms of sinθ and cosθ:

$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}$

$= \frac{sin^2\theta + cos^2\theta}{cos\theta\ .\ sin\theta}$

$= \frac{sin^2\theta + cos^2\theta}{cos\theta\ .\ sin\theta}$

Use the Pythagorean identity

$\sin^2\theta + \cos^2\theta = 1$:

$= \frac{1}{cos\theta\ .\ sin\theta}$

This is equal to the right-hand side.

Therefore, $\tan\theta + \cot\theta = \frac{1}{sin\theta cos\theta}$ is proven.

(v). $\frac{cos\theta + sin\theta}{cos\theta - sin\theta} + \frac{cos\theta - sin\theta}{cos\theta + sin\theta} = \frac{2}{1-2\sin^2\theta}$

Solution:

$\frac{cos\theta + sin\theta}{cos\theta - sin\theta} + \frac{cos\theta - sin\theta}{cos\theta + sin\theta} = \frac{2}{1-2\sin^2\theta}$

By Solving L.H.S:

$= \frac{cos\theta + sin\theta}{cos\theta - sin\theta} + \frac{cos\theta - sin\theta}{cos\theta + sin\theta}$

$= \frac{(cos\theta + sin\theta)^2 + (cos\theta - sin\theta)^2}{(cos\theta - sin\theta)(cos\theta + sin\theta)}$

$= \frac{cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta + cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta}{cos^2\theta - \sin^2\theta}$

$= \frac{cos^2\theta + \sin^2\theta + cos^2\theta + \sin^2\theta}{cos^2\theta - \sin^2\theta}$

$= \frac{2cos^2\theta + 2\sin^2\theta}{cos^2\theta - \sin^2\theta}$

Use the Pythagorean identity

$\cos^2\theta = 1 - \sin^2\theta$:

$= \frac{2(cos^2\theta + \sin^2\theta)}{1 - sin^2\theta - \sin^2\theta}$

$= \frac{2(1)}{1-2sin^2\theta}$

$= \frac{2}{1-2sin^2\theta}$

This is equal to the right-hand side.

Therefore, $\frac{cos\theta + sin\theta}{cos\theta - sin\theta} + \frac{cos\theta - sin\theta}{cos\theta + sin\theta} = \frac{2}{1-2\sin^2\theta}$ is proven.

(vi). $\frac{1 + cos\theta}{1 - cos\theta} = (\text{cosec}\theta + \cot\theta)^2$

Solution:

$\frac{1 + cos\theta}{1 - cos\theta} = (\text{cosec}\theta + \cot\theta)^2$

By Solving R.H.S:

$(\text{cosec}\theta + \cot\theta)^2$

Use the Pythagorean identities

i). cosec$\theta = \frac{1}{sin\theta}$; ii). cot$\theta = \frac{cos\theta}{sin\theta}$

$= (\frac{1}{sin\theta} + \frac{cos\theta}{sin\theta})^2$

$= (\frac{1 + cos\theta}{sin\theta})^2$

$= \frac{(1 + \cos\theta)^2}{\sin^2\theta}$

Use the Pythagorean identity

$\sin^2\theta = 1 - \cos^2\theta$:

$= \frac{(1 + \cos\theta)^2}{1 - \cos^2\theta}$

$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{(1+\cos\theta)}{(1-\cos\theta)}$

This is equal to the right-hand side.

Therefore, $\frac{1 + cos\theta}{1 - cos\theta} = (\text{cosec}\theta + \cot\theta)^2$ is proven.

Question No. 4

If tan$\theta = \frac{3}{\sqrt{2}}$ then find the remaining trigonometric ratios when θ lies in first quadrant.

Data:

Tan$\theta = \frac{3}{\sqrt{2}}$

To Find:

Sinθ, Cosθ, Cosecθ, Secθ, Cotθ = ?

Solution:

Tanθ = $\frac{P}{B} = \frac{a}{b} = \frac{3}{\sqrt{2}}$

a = 3, b = $\sqrt{2}$, c = ?

Pythagorean theorem:

$c^2 = a^2 + b^2$

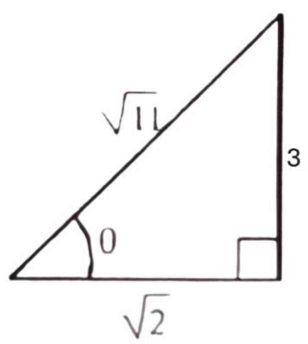
$c^2 = (3)^2 + (\sqrt{2})^2$

$c^2 = 9 + 2$

$c^2 = 11$

$c = \sqrt{11}$

Pictorial Form:



$\sin\theta = \frac{a}{c} = \frac{3}{\sqrt{11}}$, $\operatorname{cosec}\theta = \frac{c}{a} = \frac{\sqrt{11}}{3}$

$\cos\theta = \frac{b}{c} = \frac{\sqrt{2}}{\sqrt{11}} = \sqrt{\frac{2}{11}}$, $\sec\theta = \frac{c}{b} = \frac{\sqrt{11}}{\sqrt{2}} = \sqrt{\frac{11}{2}}$

$\tan\theta = \frac{a}{b} = \frac{3}{\sqrt{2}}$, $\cot\theta = \frac{b}{a} = \frac{\sqrt{2}}{3}$

Question No. 5

From a point on the ground, the angle of elevation to the top of a 30 m high building is 28°. How far is the point from the base of the building?

Data:

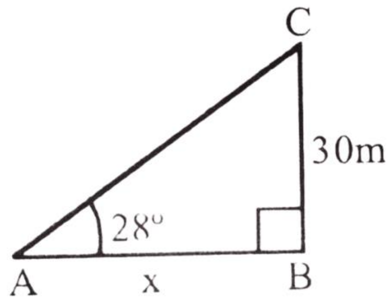
Height of the building = a = 30 m

Angle of elevation = θ = 28°

To Find:

Distance from point to building = b = ?

Pictorial Form:



Solution:

$$\tan\theta = \frac{BC}{AB} = \frac{a}{b}$$

$$\tan 28 = \frac{30}{AB}$$

$$AB = \frac{30}{\tan 28}$$

$$AB = \frac{30}{0.5317}$$

$$AB = 56.42\ m$$

The point on the ground is approximately 56.42 meters away from the base of the building.

Question No. 6

A ladder leaning against a wall forms an angle of 65∘ with the ground. If the ladder is 10 m long, how high does it reach on the wall?

Data:

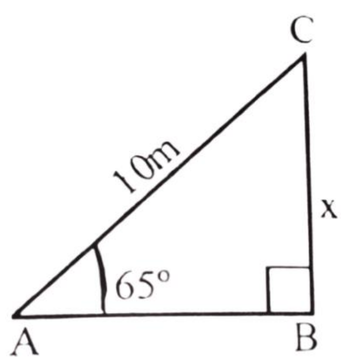
Length of the ladder = c = 10 m

Angle between ladder and ground = θ = 65°

To Find:

Height of ladder = a (x) = ?

Pictorial Form:



Solution:

$$\sin\theta = \frac{BC}{AC} = \frac{a}{c}$$

$$\sin 65 = \frac{BC}{10}$$

$BC = \sin 65 \times 10$

$BC = 0.9063 \times 10$

$BC = 9.063\ m$

The ladder reaches approximately 9.063 meters high on the wall.