Markov Chains Clearly Explained! Part - 1

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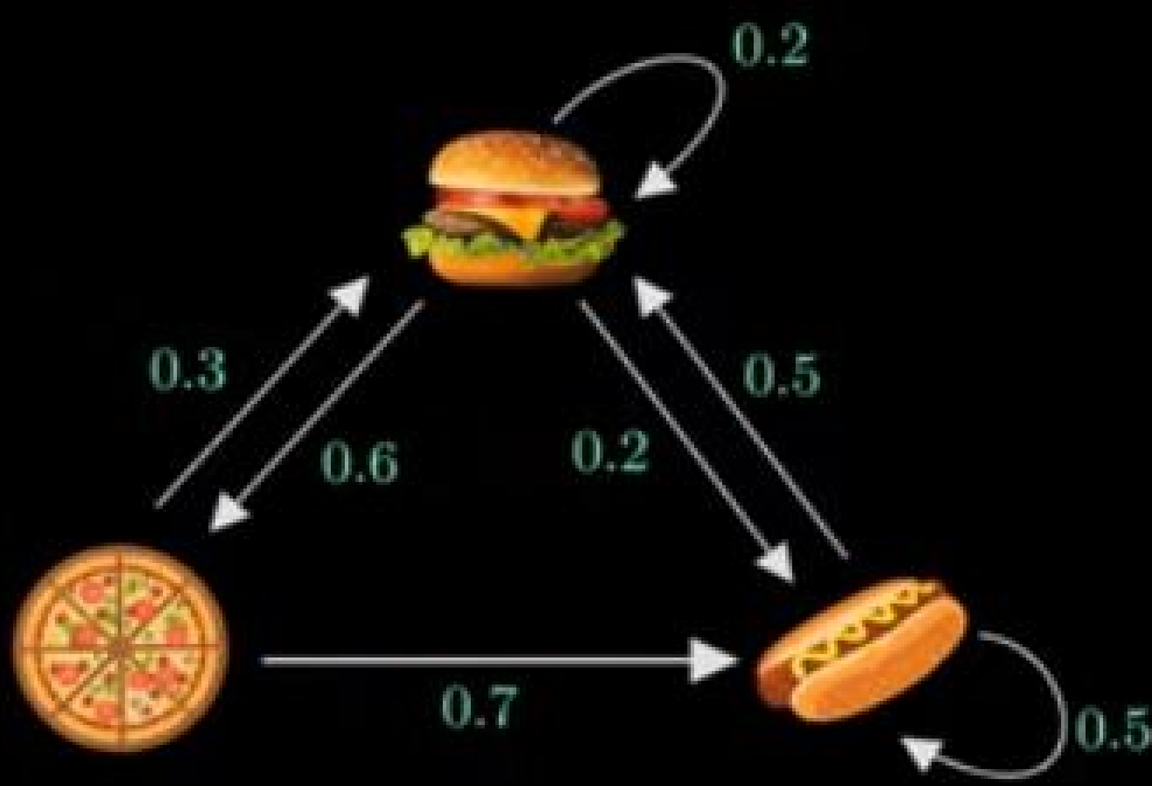
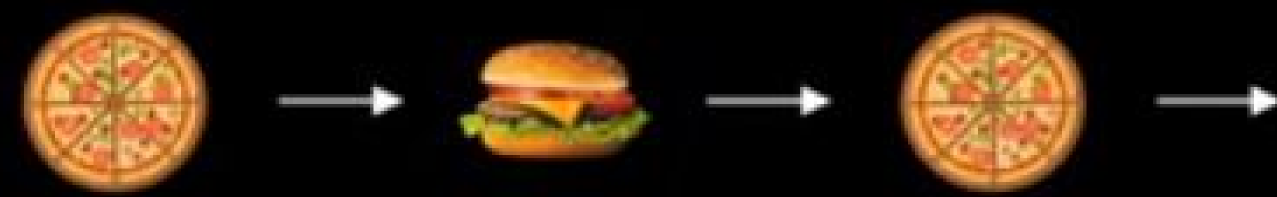
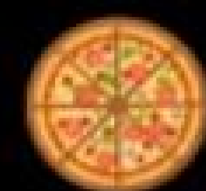
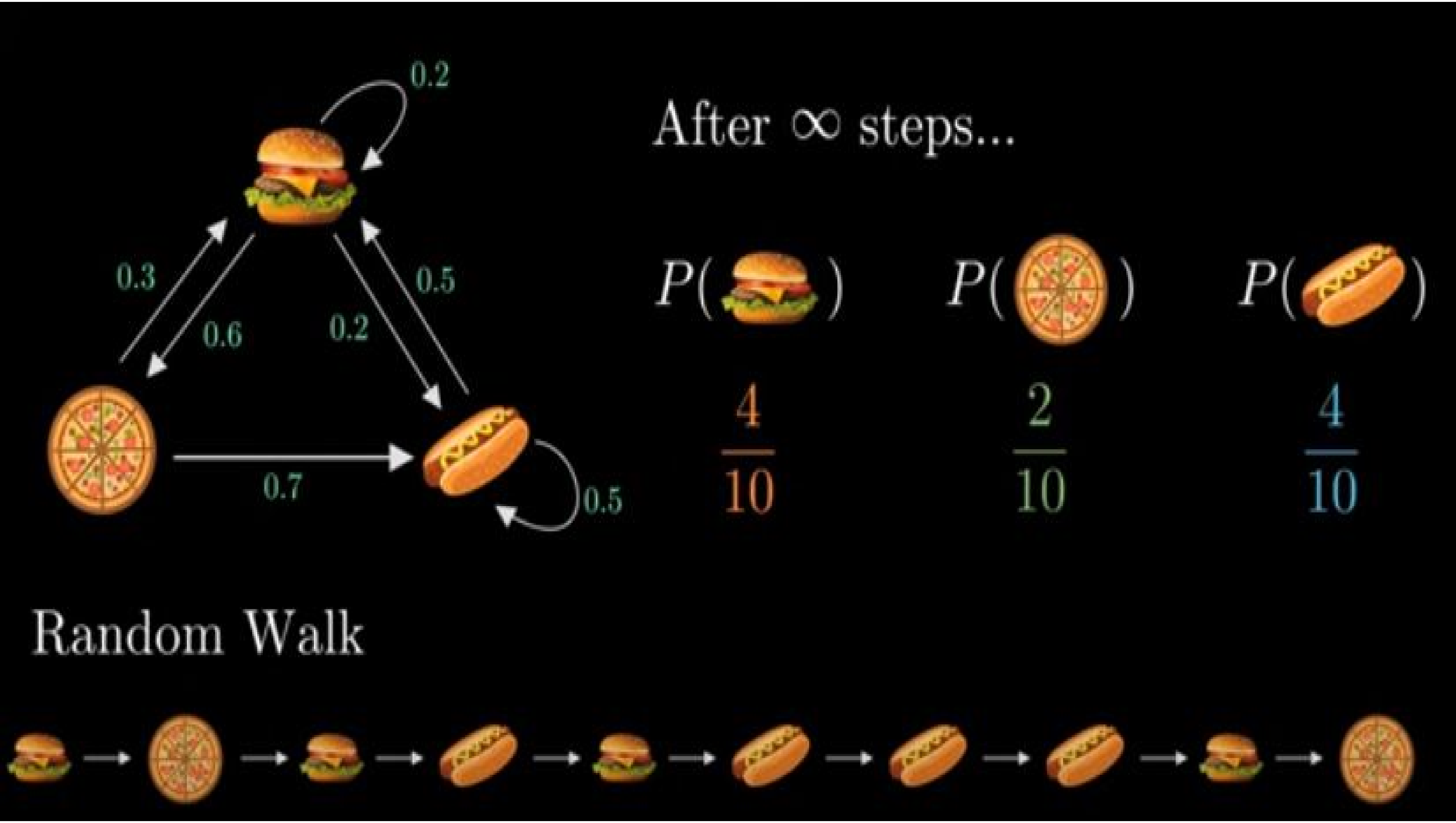
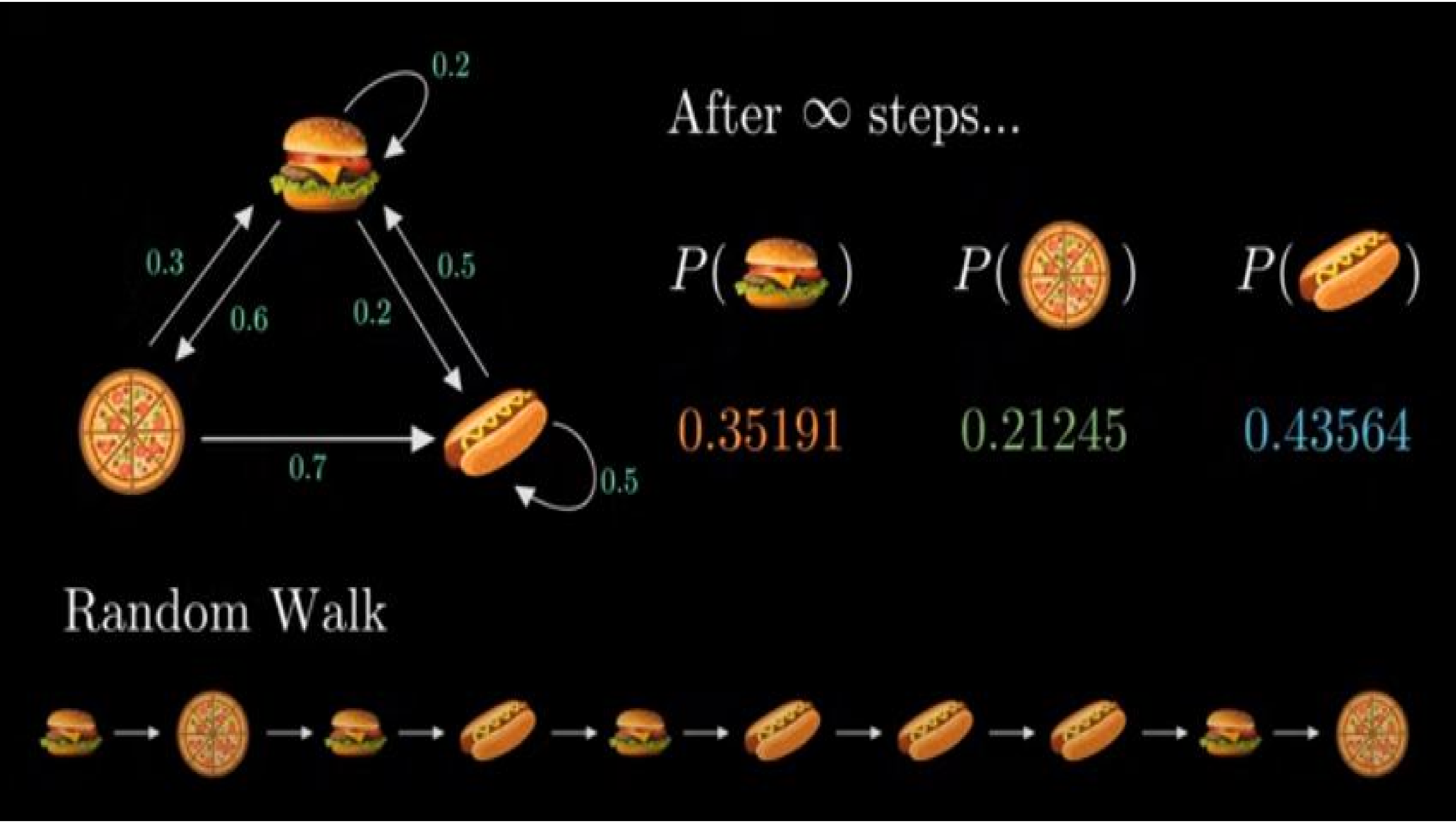


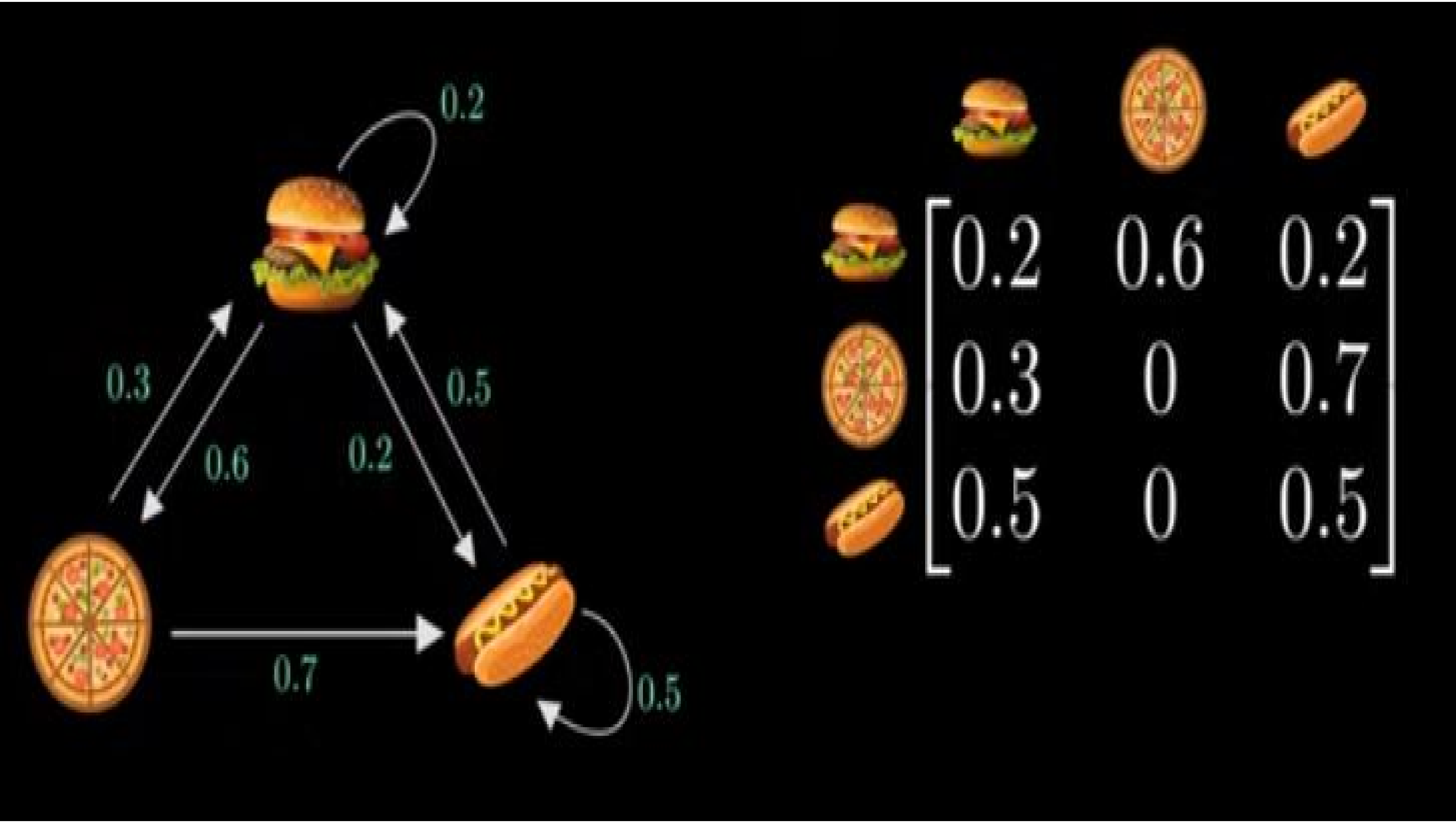


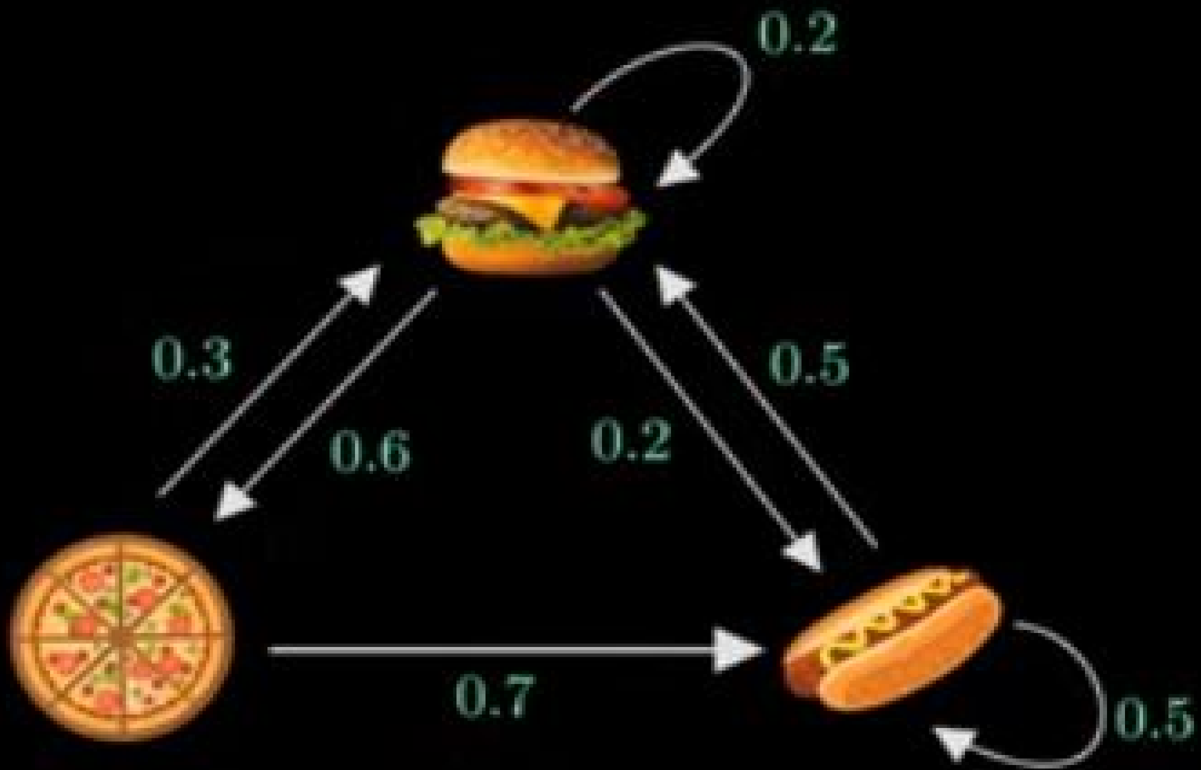
$$P(X_{n+1} = x \mid X_n = x_n)$$

$P(X_4 =$  $\mid X_3 =$  $) = 0.7$


0.2
0.3
0.6
0.2
0.5
0.7
0.5
After ∞ steps...
P( )
P( )
P( )
4/10
2/10
4/10
Random Walk


0.2
0.3
0.6
0.2
0.5
0.7
0.5
After ∞ steps...
P( )
P( )
P( )
0.35191
0.21245
0.43564
Random Walk

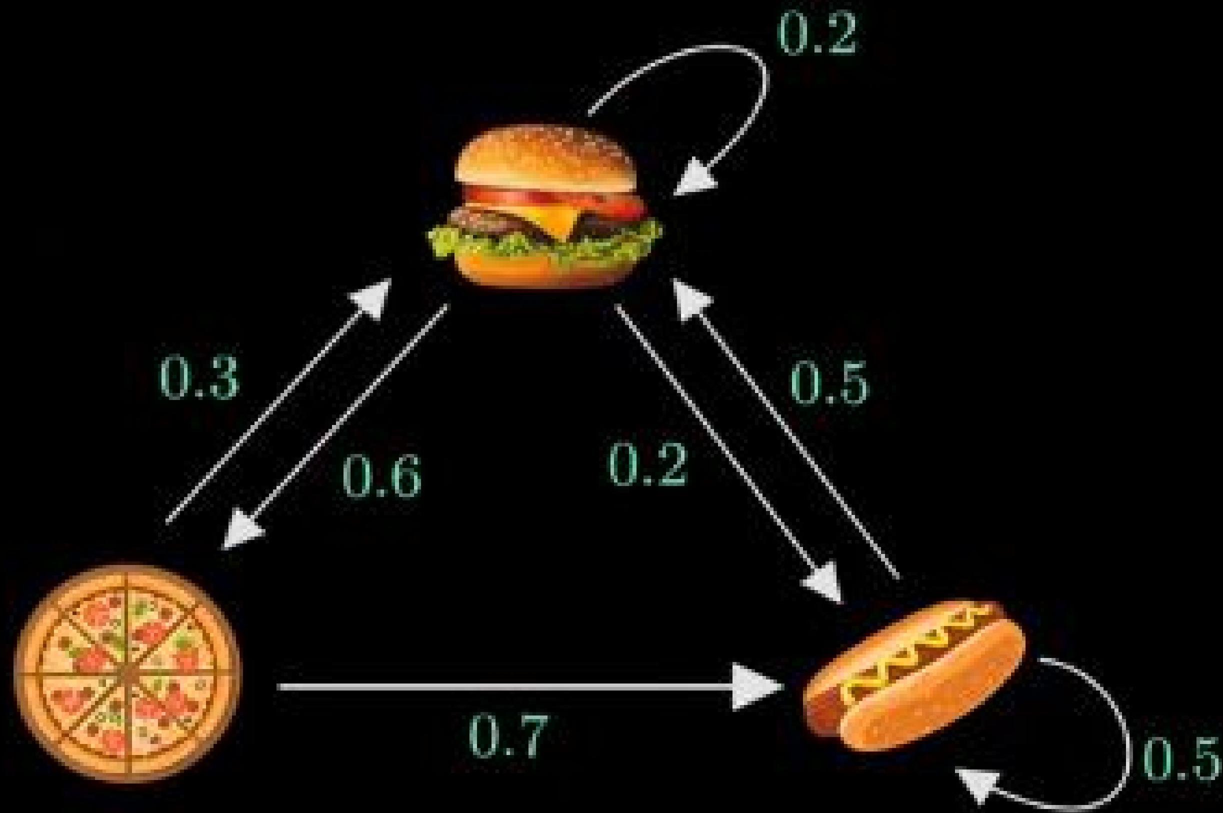

0.2
0.3
0.6
0.2
0.5
0.7
0.5
0.2 0.6 0.2
0.3 0 0.7
0.5 0 0.5



$$A = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

$$\pi_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

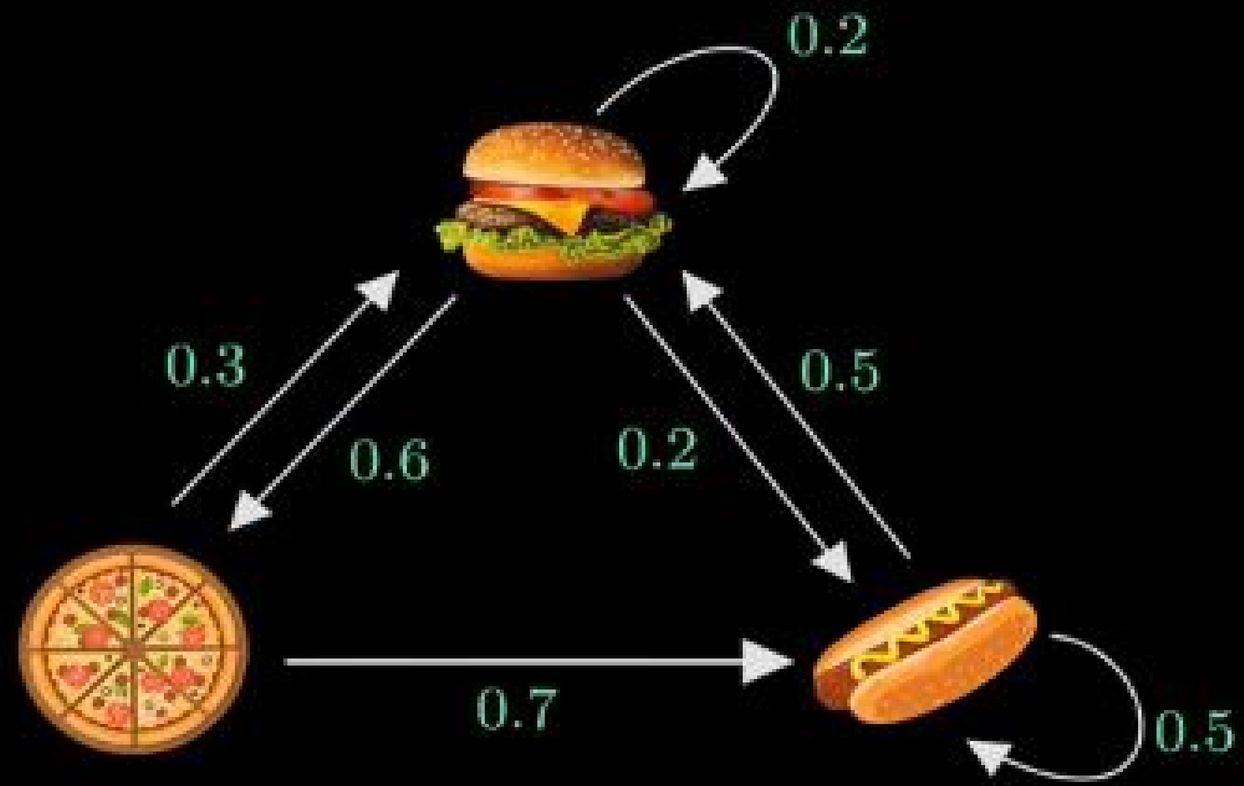
$$\pi_0 A = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.3 & 0 & 0.7 \end{bmatrix}$$



$$A = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

$$\pi_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

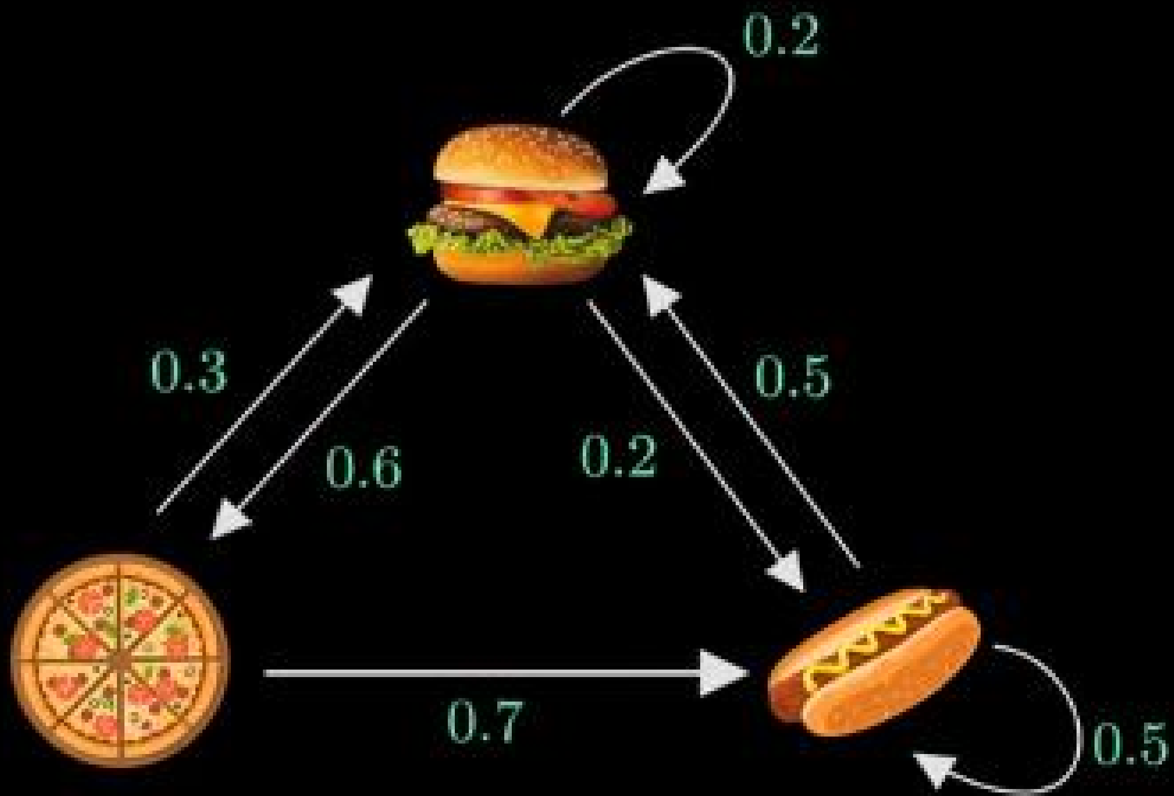
$$\pi_1 A = [0.3 \quad 0 \quad 0.7] \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix} = [0.41 \quad 0.18 \quad 0.41]$$



$$A = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

$$\pi_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\pi_2 A = \begin{bmatrix} 0.41 & 0.18 & 0.41 \end{bmatrix} \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.34 & 0.25 & 0.41 \end{bmatrix}$$



$$A = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

$$\pi_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

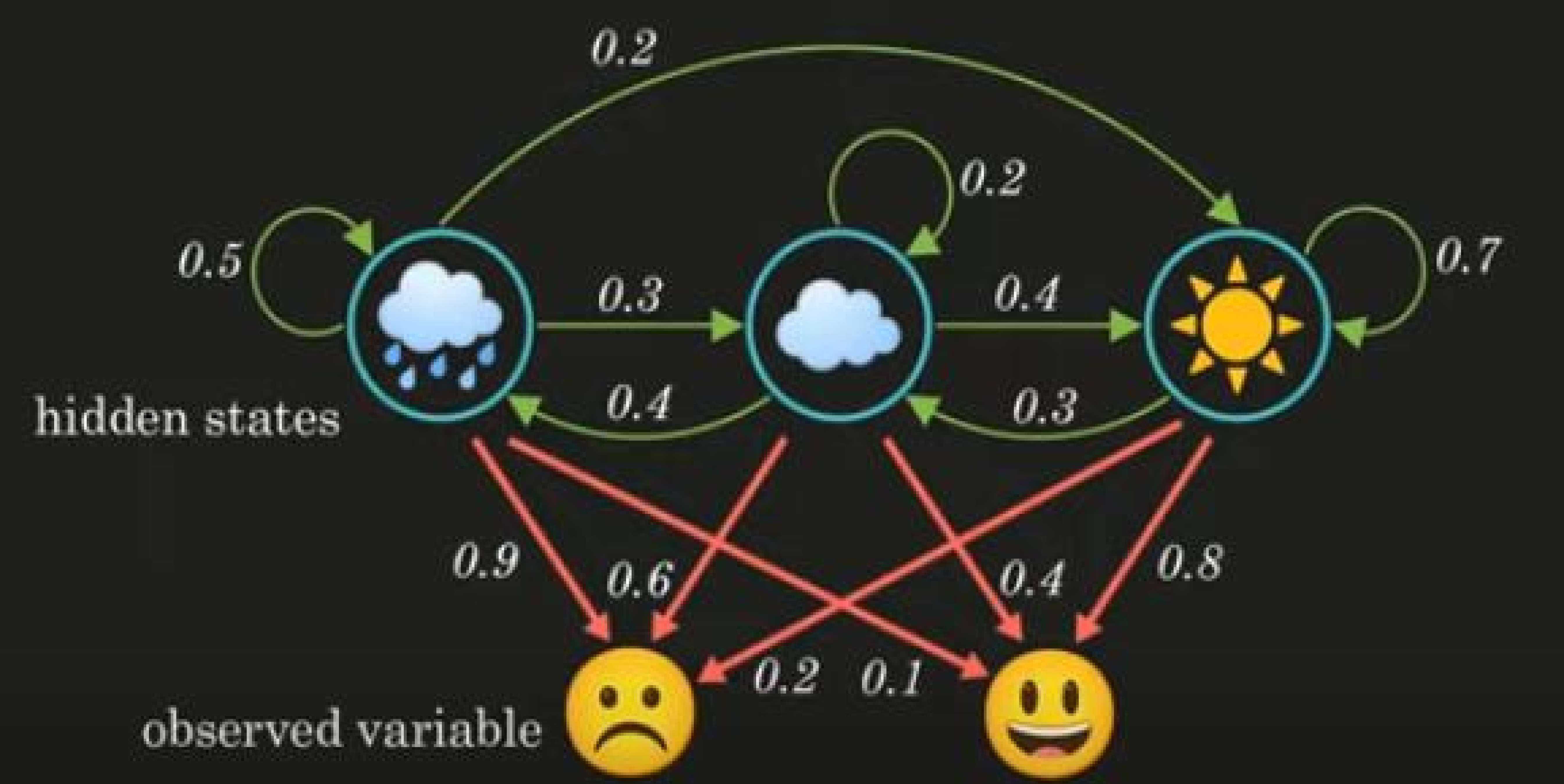
Method 1

$$\pi A = \pi$$

Method 2

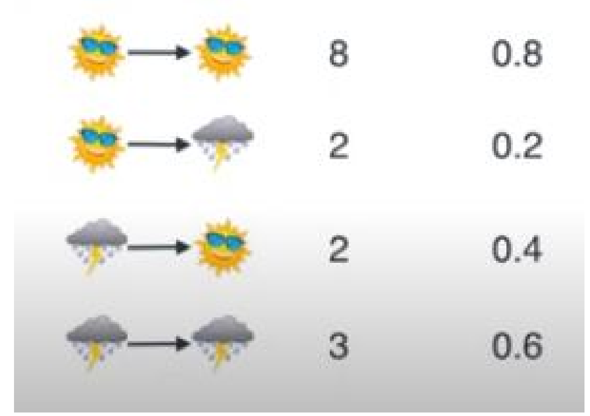
$$Av = \lambda v$$

Hidden Markov Model



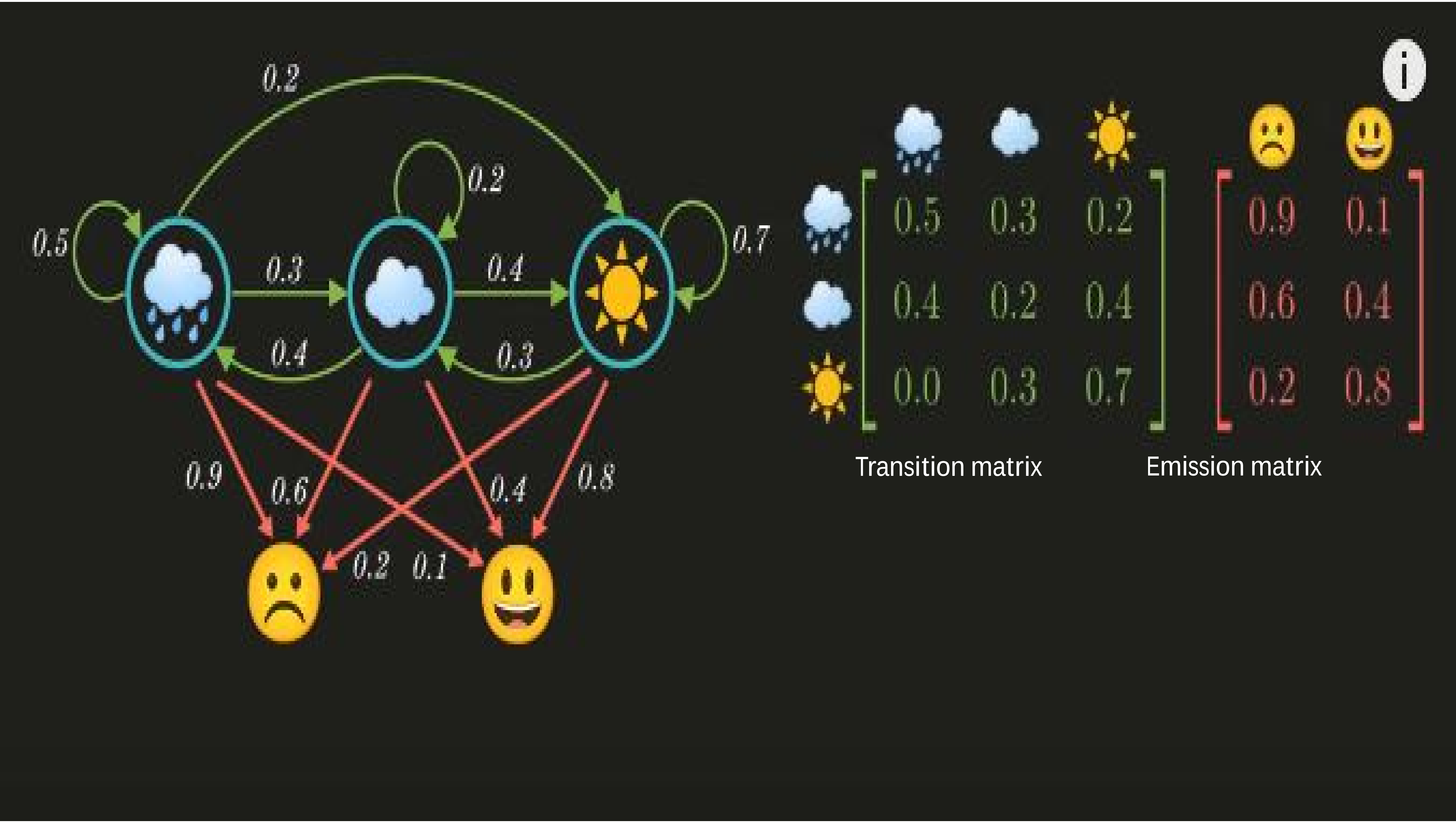
- HMM = HMC + observed variables

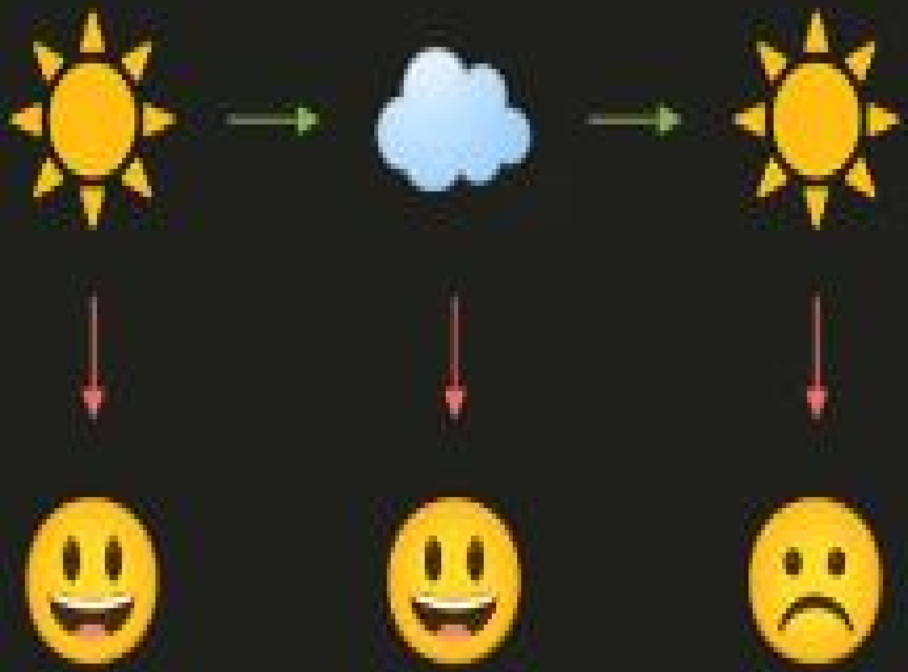
How did we find the probabilities?



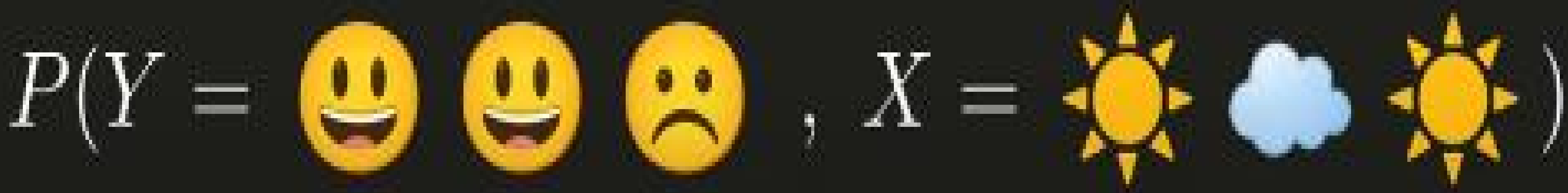
- P(Sunny)=10/15, P(Rainy)=5/15
- P(Sunny/Sunny)=0.8, P(Rainy/Sunny)=0.2, P(Sunny/Rainy)=0.4, P(Rainy/Rainy)=0.6
- P(Happy/Sunny)=0.8, P(Happy/Rainy)= 0.4, P(Angry/Sunny)=0.2, P(Angry/Rainy)=0.6

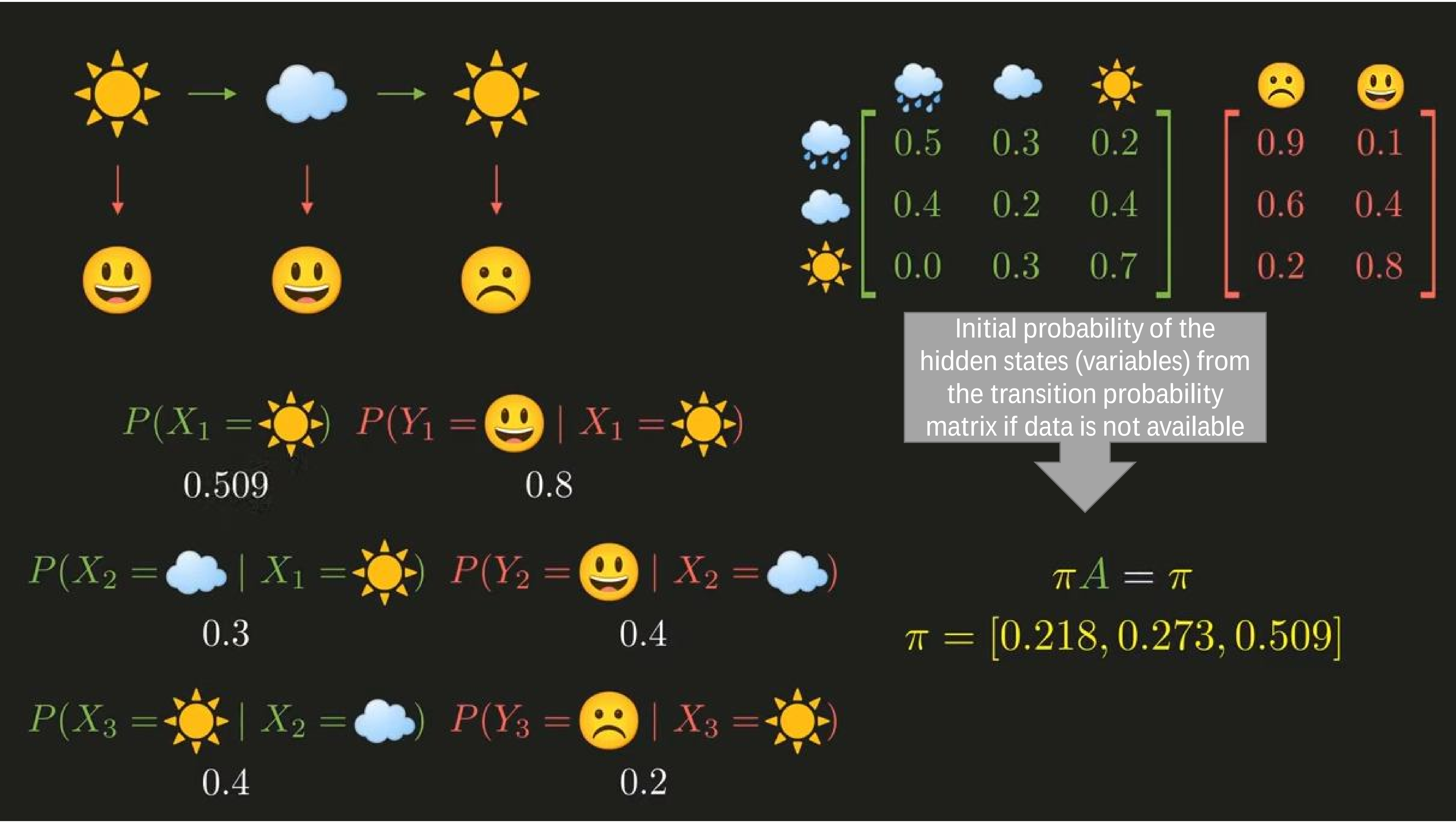
If the data is given to find out the initial hidden state, transition, and emission probabilities.

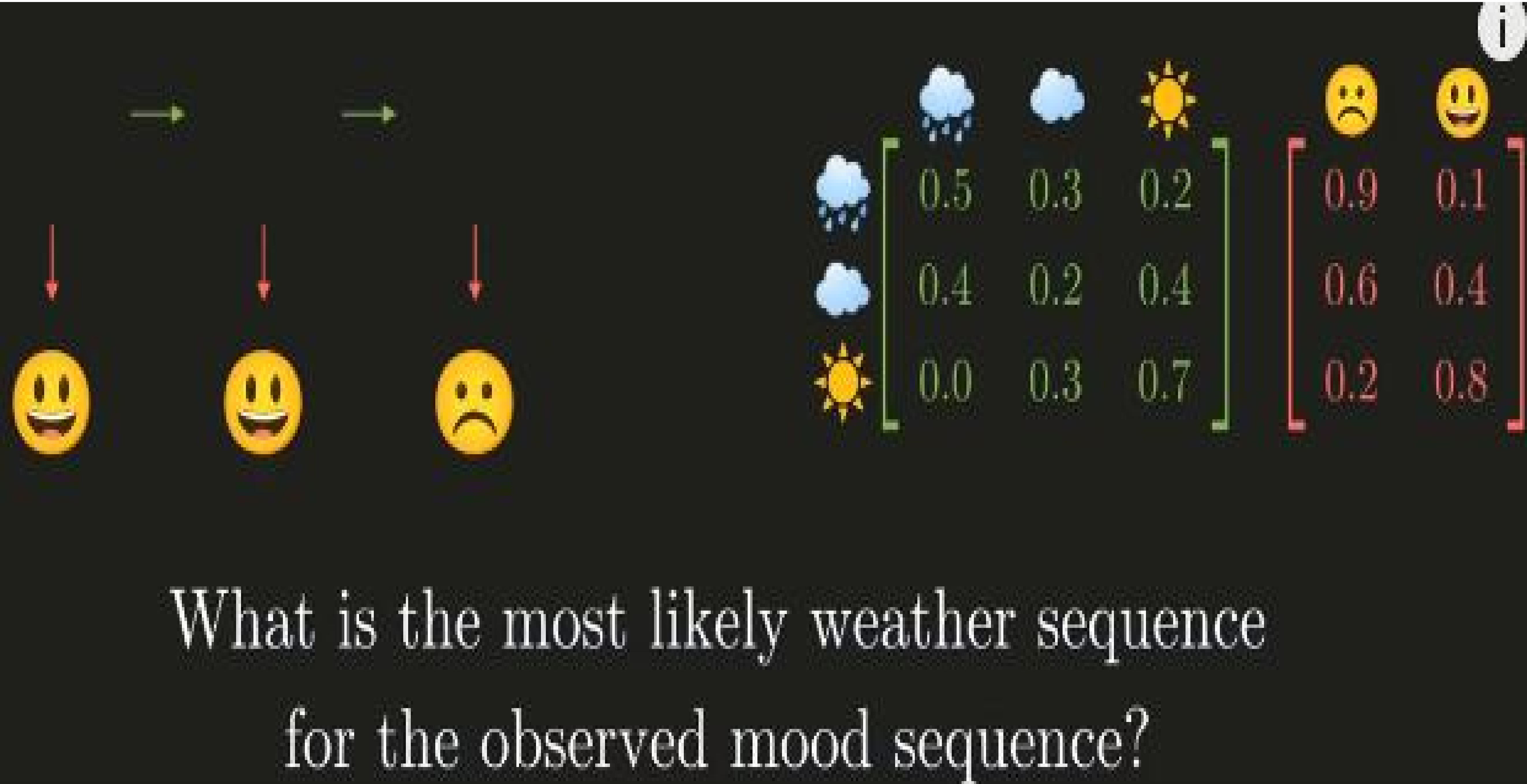

0.2
0.2
0.5
0.7
0.3
0.4
0.4
0.3
0.9
0.6
0.4
0.8
0.2
0.1
0.5 0.3 0.2
0.4 0.2 0.4
0.0 0.3 0.7
0.9 0.1
0.6 0.4
0.2 0.8
Transition matrix
Emission matrix



- The joint probability of the observed mood sequence and the weather sequence

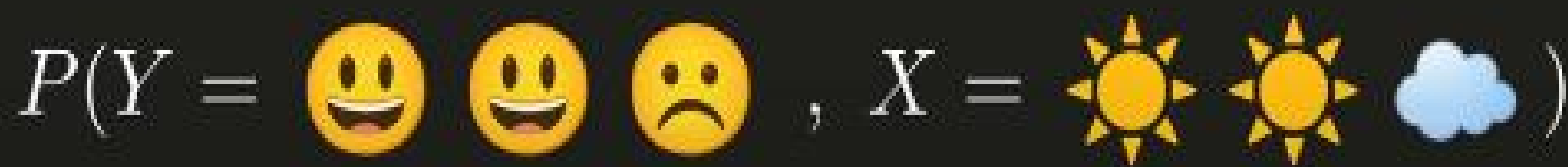



0.5 0.3 0.2
0.4 0.2 0.4
0.0 0.3 0.7
0.9 0.1
0.6 0.4
0.2 0.8
Initial probability of the hidden states (variables) from the transition probability matrix if data is not available
$\pi A = \pi$
$\pi = [0.218, 0.273, 0.509]$
$P(X_1 =$ $)$ $P(Y_1 =$ $\mid X_1 =$ $)$
0.509
0.8
$P(X_2 =$ $\mid X_1 =$ $)$ $P(Y_2 =$ $\mid X_2 =$ $)$
0.3
0.4
$P(X_3 =$ $\mid X_2 =$ $)$ $P(Y_3 =$ $\mid X_3 =$ $)$
0.4
0.2

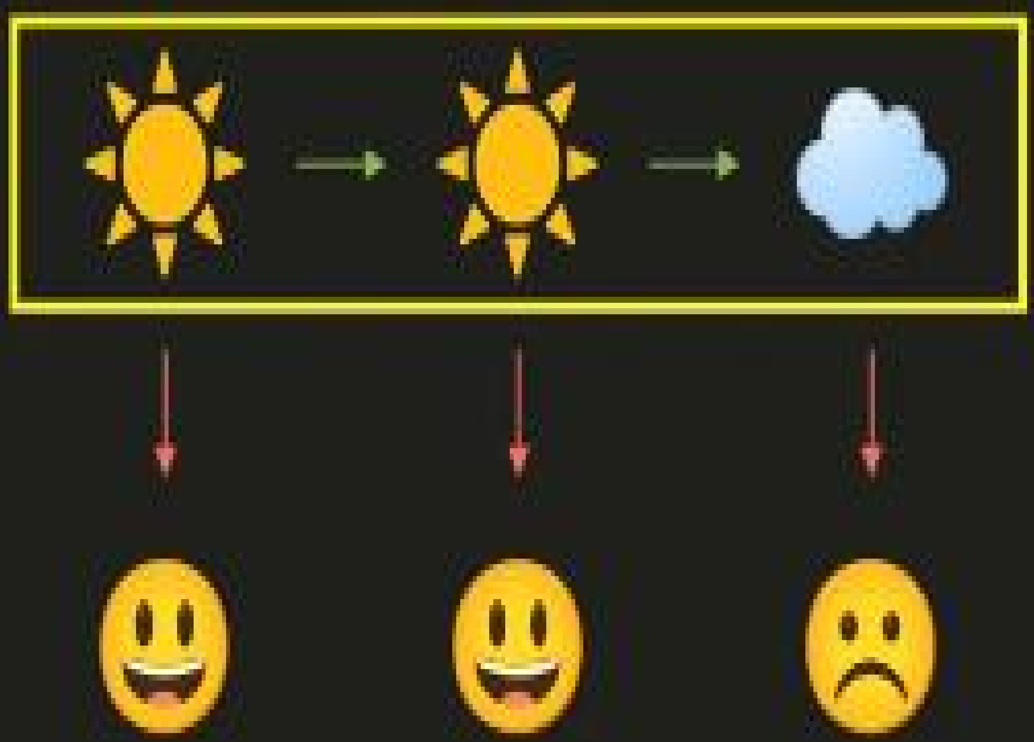


- There are many permutation of the weather sequence, 3^3=9 (#hidden variables)^(#observed variables)

- Select the sequence that maximize the joint probability.



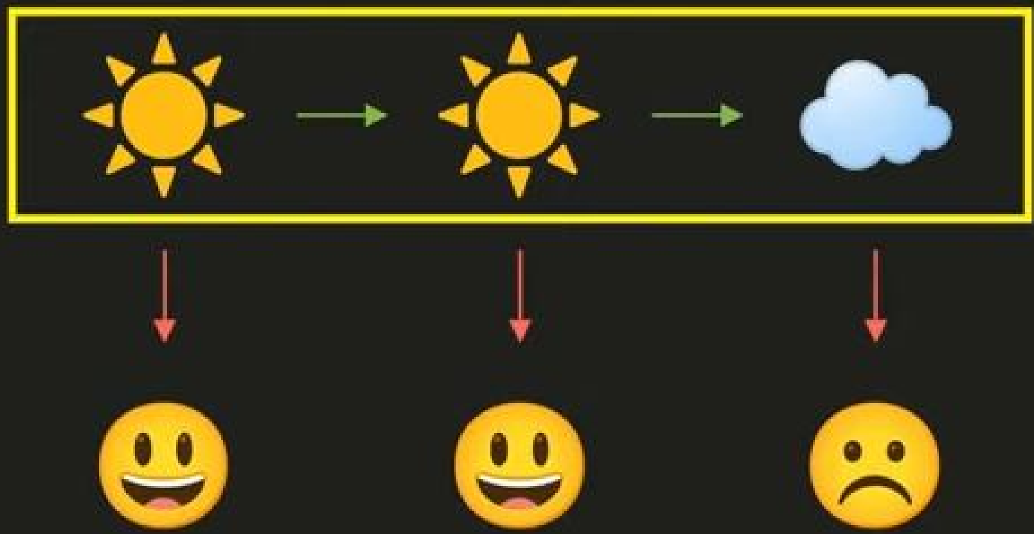
$$= 0.04105$$



$$
\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \\ 0.0 & 0.3 & 0.7 \end{bmatrix}
\qquad
\begin{bmatrix} 0.9 & 0.1 \\ 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}
$$

Find that particular sequence of X, for which the probability of X given Y is maximum.

$$
\underset{X=X_1,X_2,\ldots X_n}{\arg\max} \quad P(X = X_1, X_2, \ldots X_n \mid Y = Y_1, Y_2, \ldots Y_n)
$$



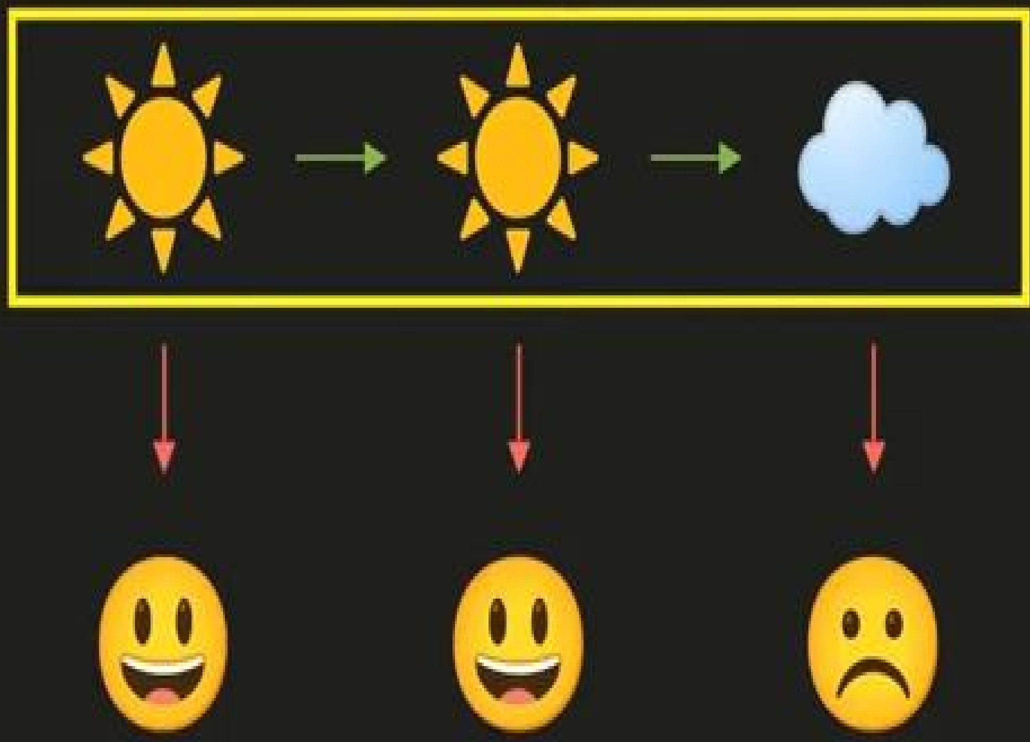
	🌧	☁	☀
🌧	0.5	0.3	0.2
☁	0.4	0.2	0.4
☀	0.0	0.3	0.7

	☹	😃
🌧	0.9	0.1
☁	0.6	0.4
☀	0.2	0.8

$$\underset{X=X_1,X_2,\ldots X_n}{\arg\max} \frac{P(Y|X)P(X)}{P(Y)}$$

$$P(Y|X) = \prod P(Y_i \mid X_i)$$

$$P(X) = \prod P(X_i \mid X_{i-1})$$



$$
\begin{array}{c|ccc}
 & 🌧️ & ☁️ & ☀️ \\
\hline
🌧️ & 0.5 & 0.3 & 0.2 \\
☁️ & 0.4 & 0.2 & 0.4 \\
☀️ & 0.0 & 0.3 & 0.7
\end{array}
\qquad
\begin{array}{cc}
☹️ & 😃 \\
\hline
0.9 & 0.1 \\
0.6 & 0.4 \\
0.2 & 0.8
\end{array}
$$

$$
\underset{X=X_1,X_2,\ldots X_n}{\arg\max} \prod P(Y_i \mid X_i)\, P(X_i \mid X_{i-1})
$$

Task

- Consider a scenario where a student can be in one of three states during a class: "attentive" (A), "distracted" (D), or "sleeping" (S). The teacher, who cannot directly observe the student's state, can only see two behaviors: "raising hand" (H) or "yawning" (Y). Based on the observations of the teacher, we want to determine the most likely sequence of states the student went through during the class.
- Assume the following probabilities:
- The initial probabilities for the states are: P(A) = 0.4, P(D) = 0.3, P(S) = 0.3.
- The transition probabilities between states are:
  - P(A → A) = 0.7, P(A → D) = 0.2, P(A → S) = 0.1
  - P(D → A) = 0.4, P(D → D) = 0.4, P(D → S) = 0.2
  - P(S → A) = 0.1, P(S → D) = 0.3, P(S → S) = 0.6
- The emission probabilities for the observations are:
  - P(H | A) = 0.8, P(Y | A) = 0.2
  - P(H | D) = 0.5, P(Y | D) = 0.5
  - P(H | S) = 0.1, P(Y | S) = 0.9
- Given this information and a sequence of observations by the teacher [H --> Y --> H], can you determine the most likely sequence of student states? /*Number of permutations here = 3^3=27*/