

## HMM:-

Hidden states = Rainy (R), Sunny (S).

Observed states = walk (W), shop ( $\bar{S}$ ), clean (C)

Transition matrix:-

|   | R   | S   |
|---|-----|-----|
| R | 0.7 | 0.3 |
| S | 0.4 | 0.6 |

Emission Matrix:-

|   | W   | $\bar{S}$ | C   |
|---|-----|-----------|-----|
| R | 0.1 | 0.4       | 0.5 |
| S | 0.6 | 0.3       | 0.1 |

Initial Prob:-

$$\pi = \begin{bmatrix} P(R) & P(S) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}$$

Given sequence:-

W  $\bar{S}$   $\bar{S}$

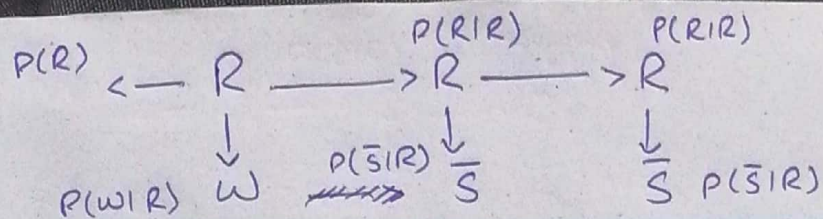
Total permutation = hidden states  $\wedge$  observed states

$$= 2^3$$

$$= |8|$$

|       |       |
|-------|-------|
| R R R | S R R |
| R R S | S R S |
| R S R | S S R |
| R S S | S S S |





$$1) \begin{array}{c} W \quad \bar{S} \quad \bar{S} \\ \uparrow \quad \uparrow \quad \uparrow \\ \leftarrow R \rightarrow R \rightarrow R \end{array}$$

$$\begin{aligned}
 P(RRR) &= P(R) \times P(W|R) \times P(R|R) \times P(\bar{S}|R) \times P(R|R) \\
 &\quad \times P(\bar{S}|R) \\
 &= 0.6 \times 0.1 \times 0.7 \times 0.4 \times 0.7 \times 0.4 \\
 &= 0.0047
 \end{aligned}$$

$$2) \begin{array}{c} W \quad \bar{S} \quad \bar{S} \\ \uparrow \quad \uparrow \quad \uparrow \\ \leftarrow R \rightarrow R \rightarrow S \end{array}$$

$$\begin{aligned}
 P(RRS) &= P(R) \times P(W|R) \times P(R|R) \times P(\bar{S}|R) \times P(S|R) \\
 &\quad \times P(\bar{S}|R) \\
 &= 0.6 \times 0.1 \times 0.7 \times 0.4 \times 0.3 \times 0.4 \\
 &= 0.0015
 \end{aligned}$$

$$3) \begin{array}{c} W \quad \bar{S} \quad \bar{S} \\ \uparrow \quad \uparrow \quad \uparrow \\ \leftarrow R \rightarrow S \rightarrow R \end{array}$$

$$\begin{aligned}
 P(RSR) &= P(R) \times P(W|R) \times P(S|R) \times P(\bar{S}|S) \times P(R|S) \\
 &\quad \times P(\bar{S}|R) \\
 &= 0.6 \times 0.1 \times 0.3 \times 0.3 \times 0.4 \times 0.4 \\
 &= 0.00086
 \end{aligned}$$

$$4) \begin{array}{c} W \quad \bar{S} \quad \bar{S} \\ \uparrow \quad \uparrow \quad \uparrow \\ \leftarrow R \rightarrow S \rightarrow S \end{array}$$

$$\begin{aligned}
 P(RSS) &= P(R) \times P(W|R) \times P(S|R) \times P(\bar{S}|S) \\
 &\quad \times P(S|S) \times P(\bar{S}|S) \\
 &= 0.6 \times 0.1 \times 0.3 \times 0.3 \times 0.6 \times 0.3 \\
 &= 0.00097
 \end{aligned}$$

$$5) \begin{array}{c} W \quad \bar{S} \quad \bar{S} \\ \uparrow \quad \uparrow \quad \uparrow \\ \leftarrow S \rightarrow R \rightarrow R \end{array}$$

$$\begin{aligned}
 P(SRR) &= P(S) \times P(W|S) \times P(R|S) \times P(\bar{S}|R) \\
 &\quad \times P(R|R) \times P(\bar{S}|R) \\
 &= 0.4 \times 0.6 \times 0.4 \times 0.4 \times 0.7 \times 0.4 \\
 &= 0.010
 \end{aligned}$$



6)  $\overset{w}{\uparrow} \overset{\bar{s}}{\uparrow} \overset{\bar{s}}{\uparrow}$   
 $\leftarrow SRS$

$$P(SRS) = P(S) \times P(W|S) \times P(R|S) \times P(\bar{S}|R) \\ \times P(S|R) \times P(\bar{S}|S)$$

$$= 0.4 \times 0.6 \times 0.4 \times 0.4 \times 0.3 \times 0.4$$

$$= 0.0069$$

$$= 0.4 \times 0.6 \times 0.4 \times 0.4 \times 0.3 \times 0.3$$

$$= 0.0034$$

7) SSR.

$$P(SSR) = P(S) \times P(W|S) \times P(S|S) \times P(\bar{S}|S) \times P(R|S) \\ \times P(\bar{S}|R)$$

$$= 0.4 \times 0.6 \times 0.6 \times 0.3 \times 0.4 \times 0.4$$

$$= 0.0069$$

8) SSS.

$$P(SSS) = P(S) \times P(W|S) \times P(S|S) \times P(\bar{S}|S) \\ \times P(S|S) \times P(\bar{S}|S)$$

$$= 0.4 \times 0.6 \times 0.6 \times 0.3 \times 0.6 \times 0.3$$

$$= 0.0077$$

$$\max \{ P(RRR), P(RRS), P(RSR), P(RSS), P(SRR), \\ P(SRS), P(SSR), P(SSS) \}$$

$$= \max(0.0047, 0.0015, 0.00086, 0.00097, \\ 0.010, 0.0034, 0.0069, 0.0077)$$

$$= 0.010$$

Maximum prob:-

$$P(SRR) = 0.010$$

So the most likely sequence for the given sequence of observations  $[w \bar{s} \bar{s}]$  is  $(SRR) \rightarrow$  sunny, Rainy, Rainy

