HIDDEN MARKOV MODEL

TASK:-

- Hidden States = Attentive(A), Distracted(D), Sleeping(S)
- Observed States = Raising Hand(H), Yawning(Y)
- Given Sequence = [H, Y, H] = ?

$$\begin{array}{c} \\ A \\ D \\ S \end{array}\begin{array}{c} \begin{array}{ccc} A & D & S \end{array} \\ \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.4 & 0.4 & 0.2 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \end{array}$$

Transition Matrix

$$\begin{array}{c} \begin{array}{cc} H & Y \end{array} \\ \begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \\ 0.1 & 0.9 \end{bmatrix} \end{array}$$

Emission Matrix

→ Initial Probability (π) = $\begin{array}{c} \begin{array}{ccc} P(A) & P(D) & P(S) \end{array} \\ \begin{bmatrix} 0.4 & 0.3 & 0.3 \end{bmatrix} \end{array}$

By adding the prob of H & Y for each states we will get total prob 1.

→ for (A) = 0.8 + 0.2 = 1

→ for (D) = 0.5 + 0.5 = 1

→ for (S) = 0.1 + 0.9 = 1

Total Sequences = (Num of possible states)^(length of sequence)

$= 3^3$

$= 27$

Now, we have to make combinations of these sequences and the total of them will be 27. Also find probability for each sequence

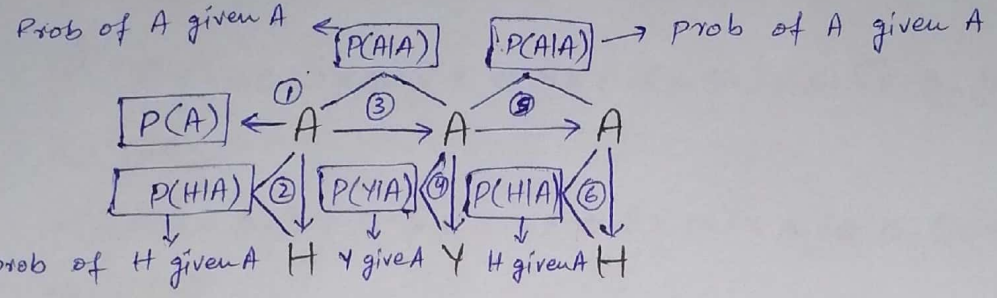
1. AAA

$P(AAA) = P(A) \times P(H|A) \times P(A|A) \times P(Y|A) \times P(A|A) \times P(H|A)$

$= 0.4 \times 0.8 \times 0.7 \times 0.2 \times 0.7 \times 0.8$

$= 0.039$

The formola for finding probability is derived as:-



This should be written as,

→ $P(A) \times P(H|A) \times P(A|A) \times P(Y|A) \times P(A|A) \times P(H|A)$

So, the logic will be same for all other sequences.

2. AAD

$P(AAD) = 0.4 \times 0.8 \times 0.7 \times 0.2 \times 0.2 \times 0.5 = 0.0067$

3. AAS

$P(AAS) = 0.4 \times 0.8 \times 0.7 \times 0.2 \times 0.1 \times 0.1 = 0.0022$

4. ADA

$P(ADA) = 0.4 \times 0.8 \times 0.2 \times 0.5 \times 0.4 \times 0.8 = 0.025$

5. ADD

$P(ADD) = 0.4 \times 0.8 \times 0.2 \times 0.5 \times 0.4 \times 0.5 = 0.008$

6. ADS

$P(ADS) = 0.4 \times 0.8 \times 0.2 \times 0.5 \times 0.3 \times 0.1 = 0.0024$

7. ASA

$P(ASA) = 0.4 \times 0.8 \times 0.1 \times 0.9 \times 0.1 \times 0.8 = 0.0023$

8. ASD

$P(ASD) = 0.4 \times 0.8 \times 0.1 \times 0.9 \times 0.3 \times 0.5 = 0.0043$

9. ASS

$P(ASS) = 0.4 \times 0.8 \times 0.1 \times 0.9 \times 0.6 \times 0.1 = 0.0043$

10. DAA

$P(DAA) = 0.3 \times 0.5 \times 0.4 \times 0.8 \times 0.7 \times 0.8 = 0.033$

11. DAD

$P(DAD) = 0.3 \times 0.5 \times 0.4 \times 0.8 \times 0.2 \times 0.5 = 0.012$

12. DAS

$P(DAS) = 0.3 \times 0.5 \times 0.4 \times 0.8 \times 0.1 \times 0.1 = 0.0012$

13. DDA

$P(DDA) = 0.3 \times 0.5 \times 0.4 \times 0.5 \times 0.4 \times 0.8 = 0.012$

14. DDD

$P(DDD) = 0.3 \times 0.5 \times 0.4 \times 0.5 \times 0.4 \times 0.5 = 0.006$

15. DDS

$P(DDS) = 0.3 \times 0.5 \times 0.4 \times 0.5 \times 0.3 \times 0.1 = 0.0022$

16. DSA

$P(DSA) = 0.3 \times 0.5 \times 0.3 \times 0.9 \times 0.1 \times 0.8 = 0.0048$

17. DSD

$P(DSD) = 0.3 \times 0.5 \times 0.3 \times 0.9 \times 0.6 \times 0.5 = 0.020$

18. DSS

$P(DSS) = 0.3 \times 0.5 \times 0.3 \times 0.9 \times 0.6 \times 0.1 = 0.0027$

19. SAA

$P(SAA) = 0.3 \times 0.1 \times 0.1 \times 0.8 \times 0.7 \times 0.8 = 0.0013$

20. SAD

$P(SAD) = 0.3 \times 0.1 \times 0.1 \times 0.8 \times 0.2 \times 0.5 = 0.00012$

21. SAS

$P(SAS) = 0.3 \times 0.1 \times 0.1 \times 0.8 \times 0.1 \times 0.1 = 0.00024$

22. SDA

$P(SDA) = 0.3 \times 0.1 \times 0.6 = 0.5 \times 0.4 \times 0.8 = 0.0036$

23. SDD

$P(SDD) = 0.3 \times 0.1 \times 0.6 \times 0.5 \times 0.4 \times 0.5 = 0.0018$

24. SDS

$P(SDS) = 0.3 \times 0.1 \times 0.6 \times 0.5 \times 0.3 \times 0.1 = 0.00027$

25. SSA

$P(SSA) = 0.3 \times 0.1 \times 0.6 \times 0.9 \times 0.1 \times 0.8 = 0.0019$

26. SSD

$P(SSD) = 0.3 \times 0.1 \times 0.6 \times 0.9 \times 0.6 \times 0.5 = 0.0032$

27. SSS

$P(SSS) = 0.3 \times 0.1 \times 0.6 \times 0.9 \times 0.1 = 0.0001$

Now, from all the probabilities, find the maximum probability which is $P(AAA) = 0.0039$

Therefore, the most likely sequence for the given sequence of observations [H, Y, H] is:-

Attentive → Attentive → Attentive (AAA)