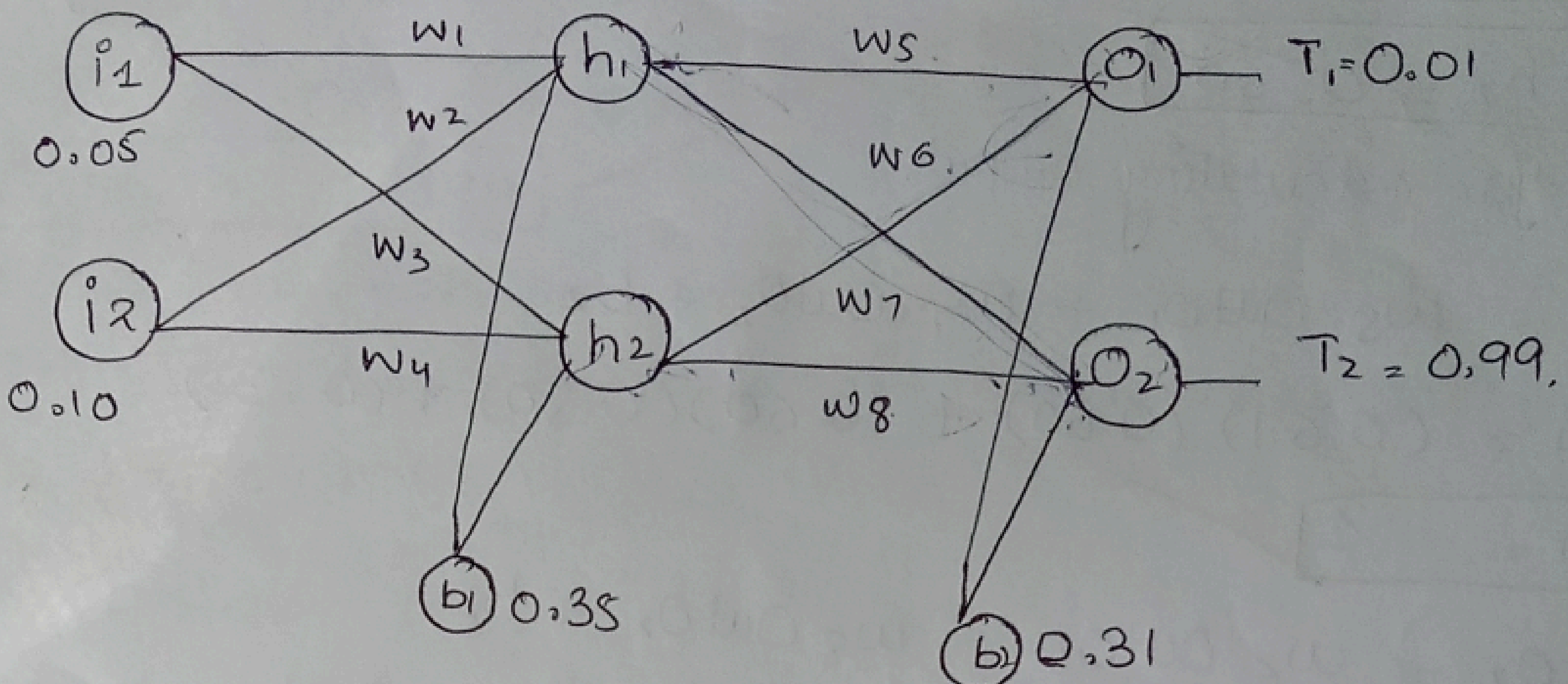
Question No 01 :-

BACK PROPAGATION :-



→ Given that :-

$i_1 = 0.05$; $b_1 = 0.35$ $T_1 = 0.01$

$i_2 = 0.10$ $b_2 = 0.31$ $T_2 = 0.99$

→ Initial Weights are :-

$w_1 = 0.7$ $w_5 = 0.67$

$w_2 = 0.8$ $w_6 = 0.45$

$w_3 = 0.35$ $w_7 = 0.50$

$w_4 = 0.30$ $w_8 = 0.55$

- $net_{h_1} = i_1 \times w_1 + i_2 \times w_3 + b_1$

$= (0.7)(0.05) + (0.35)(0.10) + 0.35$

$\boxed{net_{h_1} = 0.42}$

- $net_{h_2} = i_1 \times w_2 + i_2 \times w_4 + b_1$

$= (0.8)(0.05) + (0.30)(0.10) + 0.35$

$\boxed{net_{h_2} = 0.42}$.

Sigmoid Function is $= \dfrac{1}{1+e^{-x}}$.

$$Out\,h_1 = \frac{1}{1+e^{-net\,h_1}} = \frac{1}{1+(2.71)^{-0.42}}$$

$$\boxed{Out\,h_1 = 0.60}$$

$$\boxed{Out\,h_2 = 0.60}$$

Now for calculating outputs.

→ $net_{O_1} = w_5 \cdot Out\,O_1 + w_7 \cdot Out\,O_2 + b_2$

$net\,O_1 = (0.67)(0.60) + (0.60)(0.50) + (0.31)$

$$\boxed{net\,O_1 = 1}$$

→ $net\,O_2 = w_6 \cdot Out\,O_1 + w_8\, Out\,O_2 + b_2$

$net\,O_2 = (0.45)(0.60) + (0.60)(0.55) + 0.31$

$net\,O_2 = 0.27 + 0.33 + 0.31$

$$\boxed{net\,O_2 = 0.91}$$

- $Out\,O_1 = \dfrac{1}{1+e^{-1}} = \boxed{0.73}$

- $Out\,O_2 = \dfrac{1}{1+e^{-0.91}} = \boxed{0.71}$

⇒ Values of Outputs O_1 and O_2 not Matching with the Target Value So we calculate the error.

$$E_{Total} = \Sigma \frac{1}{2}(\text{Target} - \text{Output}_o)^2$$

- $EO_1 = \dfrac{1}{2}(\text{Target} - O\,out\,O_1)^2$

$EO_1 = \dfrac{1}{2}(0.73 - 0.01)^2$

$$\boxed{EO_1 = 0.25}$$

$EO_2 = \frac{1}{2}(0.71 - 0.99)^2$

$\boxed{EO_2 = 0.039}$

★ $E_{Total} = EO_1 + EO_2$

$= 0.25 + 0.039$

$\boxed{E_{Total} = 0.289}$

→ Now we get the error and on the basis of theses errors we Back Propagate it by updated Weight.

BACK PROPAGATE :-

First Consider w5 :-

Error at w5 is $= \frac{\partial E_{Total}}{\partial w5}$.

$$\frac{\partial E_{Total}}{\partial w5} = \frac{\partial E_{Total}}{\partial Out O_1} \cdot \frac{\partial Out O_1}{\partial net O_1} \cdot \frac{\partial net O_1}{\partial w5}$$

→ $\frac{\partial E_{Total}}{\partial Out O_1} = -(Target - Out_{O_1})$

$= -(0.73 - 0.01)$

$= -0.73 + 0.01$

$= -0.72.$

→ $\frac{\partial Out O_1}{\partial net O_1} = Out O_1 (1 - Out O_1)$

$= 0.73(1 - 0.73)$

$= 0.19.$

→ $\frac{\partial net O_1}{\partial w5} = Out h_1 = 0.60.$

$$\frac{\partial E_{Total}}{\partial w_5} = (-0.72)(0.19)(0.60)$$

$$\frac{\partial E_{Total}}{\partial w_5} = -0.08 \longrightarrow \text{change in } w_5.$$

→ Now, we have to update the value of w_5 with $\alpha = 0.5$

$$w_5^* = w_5 - \alpha \frac{\partial E_{Total}}{\partial w_5}$$

$$w_5^* = 0.67 - (0.5)(-0.08)$$

$$\boxed{w_5^* = 0.71}$$ New w_5

→ Consider w_6 :-

$$\frac{\partial E_{Total}}{\partial w_6} = \frac{\partial E_{Total}}{\partial out_{O_2}} \times \frac{\partial out_{O_2}}{\partial net_{O_2}} \times \frac{\partial net_{O_2}}{\partial w_6}.$$

→ $$\frac{\partial E_{Total}}{\partial out_{O_2}} = (out_{O_2} - Target_{O_2})$$

$$= (0.71 - 0.99)$$

$$= 0.28.$$

→ $$\frac{\partial out_{O_2}}{\partial net_{O_2}} = out_{O_2}(1 - out_{O_2})$$

$$= (0.71)(1 - 0.71)$$

$$= 0.2059.$$

→ $$\frac{\partial net_{O_2}}{\partial w_6} = out_{h_2} = 0.60.$$

$$\frac{\partial E_{total}}{\partial w_6} = (0.28)(0.2059)(0.60)$$

↳ $= 0.034 \longrightarrow$ change in w_6.

, updating w_6 :-

$$W_6^* = W_6 - \alpha \frac{\partial E_{Total}}{\partial w_6}.$$

$$W_6^* = 0.45 - (0.5)(0.034)$$

$$\boxed{W_6^* = 0.43.} \rightarrow \text{updated weight } w_6.$$

→ Consider w_8 :-

$$\frac{\partial E_{Total}}{\partial w_8} = \frac{\partial E_{Total}}{\partial Out\,O_2} \times \frac{\partial Out\,O_2}{\partial net\,O_2} \times \frac{\partial net\,O_2}{\partial w_8}.$$

$$= (0.28)(0.2059)(0.60)$$

$$\frac{\partial E_{Total}}{\partial w_8} = 0.034 \longrightarrow \text{Change in } W_8.$$

Now, updating W_8 :-

$$W_8^* = W_8 - \alpha \frac{\partial E_{Total}}{\partial w_8}.$$

$$W_8^* = 0.55 - (0.5)(0.034)$$

$$W_8^* = 0.55 - 0.0179$$

$$\boxed{W_8^* = 0.53.} \longrightarrow \text{updated } W_8.$$

→ consider W_7 :-

$$\frac{\partial E_{Total}}{\partial W_7} = \frac{\partial E_{Total}}{\partial Out\,O_1} \times \frac{\partial Out\,O_1}{\partial net\,O_1} \times \frac{\partial net\,O_1}{\partial w_7}$$

$$= (-0.72)(0.19)(0.60)$$

$$= -0.08.$$

$$W_7^* = W_7 - \alpha \frac{\partial E_{Total}}{\partial W_7}$$

$$W_7^* = 0.50 - (0.5)(-0.08)$$

$$\boxed{W_7^* = 0.54} \rightarrow \text{updated } W_7.$$

HIDDEN LAYERS:-

→ Consider W_1 :-

$$\frac{\partial E_{Total}}{\partial W_1} = \frac{\partial E_{TOTAL}}{\partial Out h_1} \times \frac{\partial Out h_1}{\partial net h_1} \times \frac{\partial net h_1}{\partial W_1}.$$

$$\Rightarrow \frac{\partial E_{total}}{\partial Out h_1} = \frac{EO_1}{\partial Out h_1} + \frac{EO_2}{\partial Out h_1}$$

$$= 0.0068 + 0.0291$$

$$= 0.0178.$$

$$\Rightarrow \frac{EO1}{\partial Out h_1} = \frac{EO1}{net O1} \cdot \frac{Net O1}{\partial Out h_1}$$

$$= (-0.136)(0.05) = -0.0068$$

$$\rightarrow \frac{EO1}{net O1} = \frac{EO1}{Out O1} \times \frac{Out O_1}{net O1}.$$

$$= (-0.72)(0.19) = -0.1368.$$

$$\Rightarrow \frac{EO_2}{\partial Out h_1} = \frac{EO_2}{net O_2} \times \frac{net O_2}{\partial Out h_1}$$

$$= (0.057)(0.43) = 0.02451.$$

$$\frac{EO_2}{net O_2} = \frac{\cdot EO_2}{Out O_2} \times \frac{Out O_2}{net O_2}$$

$$= (0.28)(0.205) = 0.0574$$

$\rightarrow \frac{Out h_1}{net h_1} = Out h_1 (1 - Out h_1)$

$$= (0.60)(1-0.60) = 0.24.$$

$\rightarrow \frac{net h_1}{w_1.} = 0.05.$

$\frac{\partial ETotal}{\partial w_1.} = 0.000213. \longrightarrow$ Change in w_1.

Now updating w_1.

$$W_1^* = w - \alpha \frac{\partial ETotal}{\partial w_1}.$$

$$W_2^* = 0.7 - (0.5)\cancel{(0.000213)}$$

$\boxed{W_1^* = 0.69} \longrightarrow$ update W_1.

Consider W_3 :-.

$$\frac{\partial Etotal}{\partial W_3} = \frac{ETotal}{Out h_1} \times \frac{Out h_1}{net h_1} \times \frac{net h_1}{w_3}.$$

$\rightarrow \frac{\partial ETotal}{Out h_1.} = \frac{EO_1}{Out h_1} + \frac{EO_2}{Out h_1.}$

• $\frac{EO_1}{Out h_1.} = -0.0068 + 0.00246. = 0.178.$

- $\frac{out h_1}{net h_1} = 0.24.$

- $\frac{net h_1}{w_3} = 0.10$

$\frac{\partial ETotal}{\partial w_3} = 0.000408. \rightarrow$ change in W_3.

Now updating W_3

$W_3^* = W_3 - \alpha \frac{\partial Etotal}{\partial W_3}.$

$W_3^* = 0.35 - (0.5)(0.000408)$

$\boxed{W_3^* = 0.34} \rightarrow$ updated W_3.

Now $\rightarrow$ consider W_4 :-

$\frac{\partial Etotal}{\partial W_4} = \frac{\partial ETotal}{\partial outh_2} \times \frac{\partial out h_2}{\partial neth_2} \times \frac{\partial neth_2}{\partial w_4}.$

$\rightarrow \frac{\partial Etotal}{\partial out h_2} = \frac{EO_1}{Outh_2} + \frac{EO_1}{Out h_2}$

$= 0.0302 + -0.07$

$= -0.0428$

- $\frac{EO_1}{Outh_2} = \frac{EO1}{net O_2} \times \frac{net O_1}{Out h_1}$

$= 0.057 \times 0.53 = 0.0302 \Rightarrow \frac{EO1}{Outh_2}.$

- $\frac{EO1}{netO_2} = \frac{EO1}{Out O_1} \times \frac{Out O1}{net O_2}$

$$\frac{EO1}{netO_2} = (0.28)(0.205)$$

$$= 0.057$$

$\rightarrow$ $$\frac{EO1}{Outh_2} = \frac{EO_2}{netO_1} \times \frac{netO_1}{outh_2}$$

$$= -0.136 \times 0.54$$

$$= -0.07344.$$

$$\frac{EO_1}{netO_1} = \frac{EO1}{OutO_1} \times \frac{OutO1}{netO_1}$$

$$= 0.136.$$

$\rightarrow$ $$\frac{\partial Outh_2}{\partial neth_2} = Outh_2(1 - Outh_2)$$

$$= 0.024.$$

$\rightarrow$ $$\frac{\partial neth_2}{\partial w_4} = 0.10.$$

$$\frac{\partial ETotal}{\partial w_4} = (-0.0428)(0.024)(0.10)$$

$= -0.000102$ $\rightarrow$ change in w_4.

Nowupdating w_4 :-

$$W_4^* = W_4 - \alpha \frac{\partial Etotal}{\partial w_4}.$$

$$W_4^* = 0.30 - (0.5)(-0.000102)$$

$W_4^* = 0.30 - 0.000251$

$\boxed{W_4^* = 0.29}$

→ Now consider W_2 :-

$$\frac{\partial E_{total}}{\partial W_2} = \frac{\partial E_{total}}{\partial out_{h_2}} \times \frac{\partial out_{h_2}}{\partial net_{h_2}} \times \frac{\partial net_{h_2}}{\partial w_2}.$$

→ $\frac{\partial E_{total}}{\partial out_{h_2}} = -0.0428.$

→ $\frac{\partial out_{h_2}}{\partial net_{h_2}} = 0.024$

→ $\frac{\partial net_{h_2}}{\partial w_2} = 0.05.$

$$\frac{\partial E_{total}}{\partial w_2} = (-0.0428)(0.024)(0.05)$$

$= -0.00005136$ → change in W_2.

Now updating W_2

$$W_2^* = W_2 - \alpha \frac{\partial E_{total}}{\partial w_2}$$

$W_2^* = (0.8) - (0.5)(-0.0005136)$

$W_2^* = 0.8 - 0.00002568.$

$\boxed{W_2^* = 0.79}$