(PCA)

Steps

- → Mean
- → Covarience $\begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$
- → Eigen Values $\det(S - \lambda I) \rightarrow \lambda_1, \lambda_2$
- → Eigen Vectors $(S - \lambda I)U \rightarrow e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \begin{matrix} \rightarrow \lambda_1 \\ \rightarrow \lambda_2 \end{matrix}$
- → Normalization
- → New Dataset

X	Y	PCA1	PCA2
1	1	0.929	-0.407
2	1	1.099	0.583
1	4	-2.011	-0.077

Step1 Calculate Mean

$$\bar{X} = \frac{1+2+1}{3} = 1.3$$

$$\bar{Y} = \frac{1+1+4}{3} = 2$$

$X - \bar{X}$	$Y - \bar{Y}$
-0.3	-1
+0.7	-1
-0.3	2

Step2 Covariance Matrix

$$cov(X,X) = (X_1-\bar{X})^2 + (X_2-\bar{X})^2 + (X_3-\bar{X})^2$$

$$= 0.09 + 0.49 + 0.09$$

$$= 0.67$$

$$cov(X,Y) = (X_1-\bar{X})*(Y_1-\bar{Y}) + (X_2-\bar{X})*(Y_2-\bar{Y}) + (X_3-\bar{X})*(Y_3-\bar{Y})$$

$$= (-0.3)(-1)+(0.7)(-1)+(-0.3)(2)$$

$$= 0.3+(-0.7)+(-0.6)$$

$$= -1$$

$$cov(Y,Y) = (Y_1-\bar{Y})^2+(Y_2-\bar{Y})^2+(Y_3-\bar{Y})^2$$

$$= (-1)^2+(-1)^2+(2)^2$$

$$= 1+1+4$$

$$= 6$$

$$cov(S) = \begin{bmatrix} 0.67 & -1 \\ -1 & 6 \end{bmatrix}$$

Eigen Values :-

$$det(S - \lambda I)$$

$$det\begin{bmatrix} 0.67 & -1 \\ -1 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$det\begin{bmatrix} 0.67 & -1 \\ -1 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$det\begin{bmatrix} 0.67-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

$$= (0.67-\lambda)(6-\lambda) - (-1)(-1)$$

$$= (4.02-0.67\lambda-6\lambda+\lambda^2) - 1$$

$$= \lambda^2 - 6.67\lambda + 3.02$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6.67) \pm \sqrt{(-6.67)^2 - 4(1)(3.02)}}{2(1)}$$

$$= \frac{6.67 \pm \sqrt{44.49 - 12.08}}{2}$$

$$= \frac{6.67 \pm \sqrt{32.41}}{2}$$

$$= \frac{6.67 \pm 5.69}{2}$$

$$= \frac{6.67 + 5.69}{2} \quad ; \quad \frac{6.67 - 5.89}{2}$$

$$= 6.18 \Rightarrow \lambda_1 \quad , \quad 0.49 \Rightarrow \lambda_2$$

<u>Eigen Vectors</u> :-

$$(S - \lambda I)U$$

$$\begin{bmatrix} 0.67-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$(0.67-\lambda)U_1 + (-1)U_2 \longrightarrow (1)$$

$$(-1)U_1 + (6-\lambda)U_2 \longrightarrow (2)$$

from (1)

$$(0.6-\lambda)U_1 + (-1)U_2$$

$$(0.6-\lambda)U_1 = U_2$$

$$U_1 = \frac{U_2}{0.6-\lambda}$$

$$v_1 = \frac{v_2}{0.6-\lambda} = t$$

$$v_1 = t \quad , \quad \frac{v_2}{0.6-\lambda} = t$$

$$\therefore t = 1$$

$$\boxed{v_1 = 1} \quad , \quad \boxed{v_2 = 0.6-\lambda}$$

for eigen vector λ_1

$$e_1 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 1 \\ 0.6-\lambda_1 \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 1 \\ 0.6-6.18 \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 1 \\ -5.58 \end{bmatrix}$$

for λ_2

$$e_2 = \begin{bmatrix} 1 \\ 0.6-\lambda_2 \end{bmatrix} \quad e_2 = \begin{bmatrix} 1 \\ 0.11 \end{bmatrix}$$

Normalization:-

$$e_1 = \begin{bmatrix} \dfrac{1}{\sqrt{(1)^2+(-5.58)^2}} \\ \dfrac{-5.58}{\sqrt{(1)^2+(-5.58)^2}} \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 0.17 \\ -0.98 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} \dfrac{1}{\sqrt{(1)^2+(0.11)^2}} \\ \dfrac{0.11}{\sqrt{(1)^2+(0.11)^2}} \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0.99 \\ 0.10 \end{bmatrix}$$

New Dataset :-

$$P_{11} = e_1 \begin{bmatrix} X_1 - \bar{X} \\ Y_1 - \bar{Y} \end{bmatrix}$$

$$= \begin{bmatrix} 0.17 & -0.98 \end{bmatrix} \begin{bmatrix} -0.3 \\ -1 \end{bmatrix}$$

$$= (0.17)(-0.3) + (-0.98)(-1)$$

$$P_{11} = 0.929$$

$$P_{12} = e_1 \begin{bmatrix} X_2 - \bar{X} \\ Y_2 - \bar{Y} \end{bmatrix}$$

$$P_{12} = \begin{bmatrix} 0.17 & -0.98 \end{bmatrix} \begin{bmatrix} 0.7 \\ -1 \end{bmatrix}$$

$$= (0.17)(0.7) + (-0.98)(-1)$$

$$P_{12} = 1.099$$

$$P_{13} = e_1 \begin{bmatrix} X_3 - \bar{X} \\ Y_3 - \bar{Y} \end{bmatrix}$$

$$= \begin{bmatrix} 0.17 & -0.98 \end{bmatrix} \begin{bmatrix} -0.3 \\ 2 \end{bmatrix}$$

$$= (0.17)(-0.3) + (-0.98)(2)$$

$$P_{13} = -2.011$$

$$P_{21} = e_2 \begin{bmatrix} X_1 - \bar{X} \\ Y_2 - \bar{Y} \end{bmatrix}$$

$$= \begin{bmatrix} 0.99 & 0.11 \end{bmatrix} \begin{bmatrix} -0.3 \\ -1 \end{bmatrix}$$

$$= (0.99)(-0.3) + (0.11)(-1)$$

$$P_{21} = -0.407$$

$$P_{22} = (0.99)(0.7) + (0.11)(-1)$$

$$P_{22} = 0.583$$

$$P_{23} = (0.99)(-0.3) + (0.11)(2)$$

$$= -0.077$$