Chapter 1

Probability

1.1 Introduction

One of the most influential of seventeenth-century mathematicians, Fermat earned his living as a lawyer and administrator in Toulouse. He shares credit with Descartes for the invention of analytic geometry, but his most important work may have been in number theory. Fermat did not write for publication, preferring instead to send letters and papers to friends. His correspondence with Pascal was the starting point for the development of a mathematical theory of probability.

Pierre de Fermat (16011665)

Pascal was the son of a nobleman. A prodigy of sorts, he had already published a treatise on conic sections by the age of sixteen. He also invented one of the early calculating machines to help his father with accounting work. Pascals contributions to probability were stimulated by his correspondence, in 1654, with Fermat. Later that year he retired to a life of religious meditation.

Blaise Pascal (16231662)

Probability theory provides a mathematical model for the study of randomness and uncertainty. The concept of probability occupies an important role in the decision-making process, whether the problem is one faced in business, in engineering, in government, in sciences, or just in ones own everyday life. Most decisions are made in the face of uncertainty. The mathematical models of probability theory enable us to make predictions about certain mass phenomena from the necessarily incomplete information derived from sampling techniques. It is the probability theory that enables one to proceed from descriptive statistics to inferential statistics. In fact, probability theory is the most important tool in statistical inference.

The origin of probability theory can be traced to modeling of games of chances such as dealing from a deck of cards, or spinning a roulette wheel. The earliest results on probability arose from the collaboration of the eminent mathematicians Blaise Pascal and Pierre Fermant and a gambler, Chevalier de Mr. They were interested in what seemed to be contradictions between mathematical calculations and actual games of chance, such as throwing dice, tossing coin, or spinning a roulette wheel. For example, in repeated throws of a die, it was observed that each number, 1 to 6, appeared with a frequency of approximately 1/6. However, if two dice are rolled, the sum of numbers showing on two dice, that is, 2 to 12, did not appear equally often. It was then recognized that, as the number of throws increased, the frequency of these possible results could be predicted by following some simple rules. Similar basic experiments were conducted using other games of chance, which resulted in the establishment of various basic rules of probability. Probability theory was developed solely to be applied to games of chance until the 18th century, when Pierre Laplace and Karl F. Gauss applied the basic probabilistic rules to other physical problems. Modern probability theory owes much to the 1933 publicationFoundations of Theory of Probabilityby the Russian mathematician Andrei N. Kolmogorov. He developed the probability theory from an axiomatic point of view.

1.2 Random experiment

Intuitively by an experimentone pictures a procedure being carried out under a certain set of conditions whereby the procedure can be repeated any number of times under the same set of conditions, and upon completion of the procedure certain results are observed. An experiment is a deterministic experiment if, given the conditions under which the experiment is carried out, the outcome is completely determined. If, for example, a container of pure water is brought to a temperature of 100C and 760 mmHg of atmospheric pressure the outcome is that the water will boil. Also, a certificate of deposit of \$1,000 at the annual rate of 5% will yield \$1,050 after one year, and \$(1.05) n1,000 after n years when the (constant) interest rate is compounded. An experiment for which the outcome cannot be determined, except that it is known to be one of a set of possible outcomes, is called a random experiment. Examples of random experiments are tossing a coin, rolling a die, drawing a card from a standard deck of playing cards, recording the number of telephone calls which arrive at a telephone exchange within a specified period of time, counting the number of defective items produced by a certain manufacturing process within a certain period of time, recording the heights of individuals in a certain class, etc.

1.3 Sample Space and Event

The set of all possible outcomes of a random experiment is called a sample space and is denoted by S. The elements of S are called sample points and number of sample point in sample space is denoted by n(S). Event is a subset of a sample space. The set of all possible subsets of a sample space is known as event space.

Example 1

Consider the experiment of flipping a coin three times. What is the sample space? Which sample outcomes make up the event A: Majority of coins show heads?

Think of each sample outcome here as an ordered triple, its components representing the outcomes of the first, second, and third tosses, respectively. Altogether, there are eight different triples, so those eight comprise the sample space:

S=HHH,HHT,HTH,THH,HTT,THT,TTH,TTT

By inspection, we see that four of the sample outcomes in S constitute the event A:

A=HHH,HHT,HTH,THH

EXAMPLE 2

A local TV station advertises two news casting positions. If three women(W1,W2, W3) and two men (M1,M2) apply, the experiment of hiring two coanchors generates a sample space of ten outcomes:

S=(W1,W2), (W1,W3), (W2,W3), (W1,M1), (W1,M2), (W2,M1), (W2,M2), (W3,M1), (W3,M2), (M1,M2)

EXAMPLE 3

1.4 Types of Events

Null event.
Sure Event.
Simple Event and Compound Event
An event is said to be simple if it consists of exactly one outcome and compound if it consists of more than one outcome. Compound event consists of more than one simple events.
Example
Consider an experiment in which each of three vehicles taking a particular freeway exit turns left (L) or right (R) at the end of the exit ramp. The eight possible outcomes that comprise the sample space S=$\{LLL, RLL, LRL, LLR, LRR, RLR, RRL, RR$
Thus there are eight simple events, among which are $E_1 = \{LLL\}$ and $E_5 = \{LRR\}$. Some compound events include:
A=the event that exactly one of the three vehicles turns right= $\{RLL, LRL, LLR\}$
B=the event that at most one of the vehicles turns right=$\{LLL, RLL, LRL, LLR\}$
C=the event that all three vehicles turn in the same direction=$\{LLL, RRR\}$
Disjoint or Mutually Exclusive Events.
Events are said to be Mutually exclusive (M.E) if these events cannot happen together. Mathematically, Two events A and Ba re said to be mutually exclusive or disjoint if $A \cap B = \phi$. In above example Event E_1 and E_5 are M.E, event A and C are M.E
Collectively Exhaustive (C.E) Events.
Two or more Events are C.E if their union is the entire sample space. Mathematically, Two event A and B are C.E if $A \cup B = S$. In tossing a balance coin head and tail are C.E events.
Equally Likely (E.L) Events.
Two Events A and B are said to be equally likely if one event is as likely to occur as other. In other word, event should occur in equal number of times in repeated trials. In rolling a die even numbers as likely to occur as odd number.

1.5 Definition of probability

Probability theory mathematically formulates incomplete knowledge pertaining to the likelihood of an occurrence. For example, a meteorologist might say there is a 60% chance that it will rain tomorrow. This means that in 6 of every 10 times when the world is in the current state, it will rain.

A probability is a real number $p \in [0, 1]$. In everyday speech, the number is usually expressed as a percentage (between 0% and 100%) rather than a decimal (i.e., a probability of 0.25 is expressed as 25%).A probability of 1 means that we are 100% sure of the occurrence of an event, and a probability of 0 means that we are 100% sure of the nonoccurrence of the event. The probability of any event A in the sample space S is denoted by P(A).

Probability theory can be classified as objective approach of the probability of an event and subjective approach of probability of en event.

1.6 Objective approach

The probability that an event will occur based an analysis in which each measure is based on a recorded observation is objective probability approach. Over the years, objective definition of probability has undergone several revisions. There is nothing contradictory in the multiple definitionsthe changes primarily reflected the need for greater generality and more mathematical rigor. Objective definition of probability following three categories

1.6.1 Classical or Prior definition of probability

The first formulation (often referred to as the classical definition of probability) is credited to Gerolamo Cardano. It applies only to situations where (1) the number of possible outcomes is finite and (2) all outcomes are equally likely and mutually exclusive. Under those conditions, the probability of an event comprised of m outcomes is the ratio $\frac{m}{n}$, where n is the total number of (equally likely and Mutually exclusive) outcomes. For example Tossing a fair, six-sided die, the probability of rolling an even number (that is, either 2, 4, or 6)is given as:

$$p(A) = \frac{m}{n} = \frac{3}{6}$$

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Question-1 Three nuts with metric threads have been accidentally mixed with twelve nuts with U.S. threads. To a person taking nuts from a bucket, all fifteen nuts seem to be the same. One nut is chosen randomly. What is the probability that it will be metric?

Question-2 Two fair coins are tossed. What is the probability of getting one heads and one tails?

Question-3 A coin is tossed twice. What is the probability that at least one head occurs?

Question-4 A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If E is the event that a number less than 4 occurs on a single toss of the die, find P(E)

Question-5 A statistics e:lass for engineers consists of 25 industrial, 10 mechanical, 10 electrical, and 8 civil engineering students. If a person is randomly selected by the instructor to answer a question, find the probability that the student chosen is

(a) an industrial engineering major,

(b) a civil engineering or an electrical engineering major

Question-6 In a poker hand consisting of 5 cards, find the probability of holding 2 aces and 3 jacks

Question-7 A bag contains 6 red balls, 5 yellow balls and 3 green balls. A ball is drawn at random. What is the probability that the ball is: (a) green, (b) not yellow, (c) red or yellow?

Question-8 2. A pilot plant has produced metallurgical batches which are summarized as follows:

	Low strength	High strength
Low in impurities	2	27
High in impurities	12	4

If these results are representative of full-scale production, find estimated probabilities that a production batch will be:

i) low in impurities

ii) high strength

iii) both high in impurities and high strength

iv) both high in impurities and low strength

Question-9 If the numbers of dots on the upward faces of two standard six-sided dice give the score for that throw, what is the probability of making a score of 7 in one throw of a pair of fair dice?

Question-10 A fair coin is tossed three times. What is the probability that the sequence will be heads, tails, heads?

Question-11 A boxcar contains six complex electronic systems. Two of the six are to be randomly selected for thorough testing and then classified as defective or not defective.
a- If two of the six systems are actually defective, find the probability that at least one of the two systems tested will be defective. Find the probability that both are defective.
b- If four of the six systems are actually defective, find the probabilities indicated in part (a).
Question-12 A retailer sells only two styles of stereo consoles, and experience shows that these are in equal demand. Four customers in succession come into the store to order stereos. The retailer is interested in their preferences.
a- List the possibilities for preference arrangements among the four customers (that is, list the sample space).
b- Assign probabilities to the sample points.
c- Let A denote the event that all four customers prefer the same style. Find P(A)
Question-13 A committee of 3 persons is to be selected randomly from a group of 5 men and 3 women.
(a) Find the probability that the committee consists of 1 men and women.
(b) Find the probability that the committee consists of all women
Problem 14 In a tank containing 10 fishes, there are three yellow and seven black fishes. We select three fishes at random.
(a) What is the probability that exactly one yellow fish gets selected?
(b) What is the probability that at most one yellow fish gets selected?
(c) What is the probability that at least one yellow fish gets selected?

Question-15 A package of 15 apples contains two defective apples. Four apples are selected at random.
(a) Find the probability that none of the selected apples is defective.
(b) Find the probability that at least one of the selected apples is defective.
Question-16 A homeowner wants to repaint her home and install new carpets (no store where she live sells both paint and carpet). She plans to get the services from the stores where she buys the paint and carpet. Suppose there are 12 paint stores with painting service available and 15 carpet stores with installation services available in that city. In how many ways can she choose these two stores?
Question-17 From an urn containing 15 white, 7 black, and 8 yellow balls

a sample of 3 balls is drawn at random. Find the probability that
(a) All three balls are yellow.
(b) All three balls are of the same color.
(c) All three balls are of different colors.
Question-18 A box of manufactured items contains 12 items, of which four are defective. If three items are drawn at random without replacement, what is the probability that
(a) The first one is defective and the rest are good?
(b) Exactly one of the three is defective?

1.7 Rule of Probability

Addition law of probability

If A and B are event defined in a sample space S. Probability that event A or event B (at least on of them) will occur is denoted by $p(A \cup B)$ defined as

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

Proof

Question-19 A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?
Question-20 A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?
Question-21 A spinner has 4 equal sectors colored yellow, blue, green, and red. What is the probability of landing on red or blue after spinning this spinner?
Question-22 In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?
Question-23 A bag contains 18 coloured marbles: 4 are coloured red, 8 are coloured yellow and 6 are coloured green. A marble is selected at random. What is the probability that the ball chosen is either red or green?
Question-24 A lot consists of 10 good articles, 4with minor defects, and 2

with major defects. One article is chosen at random. Find the probability that:
(a) it has no defects
(b) it has no major defects
(c) it is either good or has major defects
Question-25 A bag contains 20 balls, 3 are coloured red, 6 are coloured green, 4 are coloured blue, 2 are coloured white and 5 are coloured yellow. One ball is selected at random. Find the probabilities of the following events.
(a) A red marble
(b) A green marble
(c) A white marble
(d) red or white marble
(e) green or white marble
(f) green, red or white marble
Question-26 In a group of 101 students 40 are juniors, 50 are female, and 22 are female juniors. Find the probability that a student picked from this group at random is either a junior or female.
Question-27 A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a king or a club?
Question-28 Hydraulic landing assemblies coming from an aircraft rework facility are each inspected for defects. Historical records indicate that 8% have defects in shafts only, 6% have defects in bushings only, and 2% have defects in both shafts and bushings. One of the hydraulic assemblies is selected randomly. What is the probability that the assembly has
(a) a bushing defect?
(b) a shaft or bushing defect?
(c) exactly one of the two types of defects?
(d) neither type of defect?
Question-29 If $P(A) = 0.24, P(B) = 0.67, and P(AB) = 0.09$, find
(a) $P(A \cup B)$
(b) $P((A \cup B)^c$
(c) $P(A^c \cup B^c)$

Question-30 A sample of four electronic components is taken from the output of a production line. The probabilities of the various outcomes are calculated to be: Pr [0 defectives] = 0.6561, Pr [1 defective] = 0.2916, Pr [2 defectives] = 0.0486, Pr [3 defectives] = 0.0036, Pr [4 defectives] = 0.0001. What is the probability of at least one defective?

Question-31 If one card is drawn from a well-shuffled bridge deck of 52 playing cards (13 of each suit), what is the probability that the card is a queen or a heart?

Question-32 If one card is drawn from a well-shuffled bridge deck of 52 playing cards (13 of each suit), what is the probability that the card is a queen or a heart?

Question-33 Assign: The class registrations of 120 students are analyzed. It is found that: 30 of the students do not take any of Applied Mechanics, Chemistry, or Computers. 15 of them take only Applied Mechanics. 25 of them take Chemistry and Computers but not Applied Mechanics. 20 of them take Applied Mechanics and Computers but not Chemistry. 10 of them take all three of Applied Mechanics, Chemistry, and Computers. A total of 45 of them take Chemistry. 5 of them take only Chemistry.

a) How many of the students take Applied Mechanics and Chemistry but not Computers?

b) How many of the students take only Computers?

c) What is the total number of students taking Computers?

d) If a student is chosen at random from those who take neither Chemistry nor Computers, what is the probability that he or she does not take Applied Mechanics either?

e) If one of the students who take at least two of the three courses is chosen at random, what is the probability that he or she takes all three courses?

Question-34 Past records show that 4 of 135 parts are defective in length, 3 of 141 are defective in width, and 2 of 347 are defective in both. Use these figures to estimate probabilities of the individual events assuming that defects occur independently in length and width. a) What is the probability that a part produced under the same conditions will be defective in length or width or both?

b) What is the probability that a part will have neither defect?

c) What are the fair odds against a defect (in length or width or both)?

Question-35 In a group of 72 students, 14 take neither English nor chemistry, 42 take English and 38 take chemistry. What is the probability that a student chosen at random from this group takes:

a) both English and chemistry?

b) chemistry but not English?

Question-36 A fair six-sided die is tossed twice. What is the probability that a five will occur at least once?

1.8 Conditional Probability

Introduction.
The probabilities assigned to various events depend on what is known about the experimental situation when the assignment is made. Subsequent to the initial assignment, partial information about or relevant to the outcome of the experiment may become available. Such information may cause us to revise some of our probability assignments. For a particular event A, we have used P(A) to represent the probability assigned to A; we now think of P(A) as the original or unconditional probability of the event A. We examine how the information an event B has occurred affects the probability assigned to A. For example, A might refer to an individual having a particular disease in the presence of certain symptoms. If a blood test is performed on the individual and the result is negative (B negative blood test), then the probability of having the disease will change (it should decrease, but not usually to zero, since blood tests are not infallible). We will use the notation $P(A \mid B)$ to represent the conditional probability of A given that the event B has occurred.

Example We toss two balanced dice, and let A be the event that the sum of the face values of two dice is 8, and B be the event that the face value of the first one is 3. Calculate probability of A given B.
Solution
The elements of the events A and B are A = (2, 6), (6, 2), (3, 5), (5, 3), (4, 4).
and
B = (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6).
Now $A \cap B = (3, 5)$
P(A) = 5/36, $P(B) = \frac{6}{36}$ and $P(A \cap B) = \frac{1}{36}$. Therefore,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$$

. **Question-37** A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?
Question-38 A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble

on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?
Question-39 At Kennedy Middle School, the probability that a student takes Technology and Spanish is 0.087. The probability that a student takes Technology is 0.68. What is the probability that a student takes Spanish given that the student is taking Technology?
A laboratory blood test is 98% effective in detecting a certain disease if the person has the disease (sensitivity). However, the test also yields a false positive result for 0.5% of the healthy persons tested. (That is, if a healthy person is tested, then, with probability 0.005, the test result will show positive.) Assume that 2% of the population actually has this disease (prevalence). What is the probability a person has the disease given that the test result is positive?

1.9 Dependent and Independent Event

Two events are said to be independent if the result of the second event is not affected by the result of the first event. Otherwise, events are called dependent events. **Question-40** A card is elected at random from an ordinary deck of 52 playing cards. If E is the event that the selected card is an ace and F is the event that it is a spade, show that E and F are independent events

1.10 Multiplication Rule

A fruit basket contains 30 apples, of which five are bad. If you pick two apples at random, what is the probability that both are good apples?
2.4.9. Two students are to be selected at random from a class with 10 girls and 12 boys. What is the probability that both will be girls?

Assignment Question- 1 The U.S. Congress, Joint Committee on Printing, provides information on the composition of the Congress in the Congressional Directory. For the 110th Congress, 18.7% of the members are senators and 49% of the senators are Democrats. What is the probability that a randomly selected member of the 110th Congress is a Democratic senator?

Assignment Question- 2 Consider again the experiment of randomly selecting one card from a deck of 52 playing cards. Let
F = event a face card is selected,
K = event a king is selected, and
H = event a heart is selected.
a. Determine whether event K is independent of event F.
b. Determine whether event K is independent of event H.

Assignment Question- 3

An American roulette wheel contains 38 numbers, of which 18 are red, 18 are black, and 2 are green. When the roulette wheel is spun, the ball is equally likely to land on any of the 38 numbers. In three plays at a roulette wheel, what is the probability that the ball will land on green the first time and on black the second and third times?

Assignment Question- 4 Cards numbered 1, 2, 3,..., 10 are placed in a box. The box is shaken, and a blindfolded person selects two successive cards without replacement.
a. What is the probability that the first card selected is numbered 6?
b. Given that the first card is numbered 6, what is the probability that the second is numbered 9?
c. Find the probability of selecting first a 6 and then a 9.
d. What is the probability that both cards selected are numbered over 5?

Assignment Question- 5 The National Sporting Goods Association collects and publishes data on participation in selected sports activities. For Americans 7 years old or older, 17.4% of males and 4.5% of females play golf. According to the U.S. Census Bureau publication Current Population Reports, of Americans 7 years old or older, 48.6% are male and 51.4% are female. From among those who are 7 years old or older, one is selected at random. Find the probability that the person selected
a. plays golf.
b. plays golf, given that the person is a male.

c. is a female, given that the person plays golf.

Assignment Question- 6 In a certain county, 40% of registered voters are Democrats, 32% are Republicans, and 28% are Independents. Sixty percent of the Democrats, 80% of the Republicans, and 30% of the Independents favor increased spending to combat terrorism. If a person chosen at random from this county favors increased spending to combat terrorism, what is the probability that the person is a Democrat?

Assignment Question- 7

Textbook publishers must estimate the sales of new (first-edition) books. The records of one major publishing company indicate that 10% of all new books sell more than projected, 30% sell close to the number projected, and 60% sell less than projected. Of those that sell more than projected, 70% are revised for a second edition, as are 50% of those that sell close to the number projected and 20% of those that sell less than projected.

a. What percentage of books published by this publishing company go to a second edition?

b. What percentage of books published by this publishing company that go to a second edition sold less than projected in their first edition?

Assignment Question- 8

One box contains six red balls and four green balls, and a second box contains seven red balls and three green balls. A ball is randomly chosen from the first box and placed in the second box. Then a ball is randomly selected from the second box and placed in the first box.

a. What is the probability that a red ball is selected from the first box and a red ball is selected from the second box?

b. At the conclusion of the selection process, what is the probability that the numbers of red and green balls in the first box are identical to the numbers at the beginning

Assignment Question- 9

Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared.

a. If it has an emergency locator, what is the probability that it will not be discovered?

b. If it does not have an emergency locator, what is the probability that it will be discovered?

Assignment Question- 10 One percent of all individuals in a certain population are carriers of a particular disease. A diagnostic test for this disease has a 90carriers and a 5Suppose the test is applied independently to two different blood samples from the same randomly selected individual.
a. What is the probability that both tests yield the same result?
b. If both tests are positive, what is the probability that the selected individual is a carrier?
Assignment Question- 11

In a certain region of the country it is known from (last, experience that the: probability of selecting an adult over 40 years of age: with cancer is 0.05, If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the: probability of incorrectly diagnosing a person without cancer as having the disease is 0.1)6, what is the probability that, a person is diagnosed as having cancer?

1.11 Binomial Probability Distribution

Question-41 It is known that screws produced by a certain machine will be defective with probability 0.01 independently of each other. If we randomly pick 10 screws produced by this machine, what is the probability that at least two screws will be defective?
Question-42 Compute the following binomial probabilities directly from the formula for b(x; n, p):
a. b(3; 8, .6)
b. b(5; 8, .6)
c. $P(3 \leq X \leq 5)$ when n = 8 and p = 0.6
$d.P(1 \leq X)$ when n =12 and p = 0.1
Question-43 When circuit boards used in the manufacture of compact disc players are tested, the long-run percentage of defectives is 5%. Let X = the number of defective boards in a random sample of size n =25, so $X \sim Bin(25, .05)$.
a. Determine $P(X \leq 2)$.
b. Determine $P(X \geq 5)$.
c. Determine $P(1 \leq X \leq 4)$.
d. What is the probability that none of the 25 boards is defective?
Question-44 A company that produces fine crystal knows from experience

that 10% of its goblets have cosmetic flaws and must be classified as seconds.
a. Among six randomly selected goblets, how likely is it that only one is a second?
b. Among six randomly selected goblets, what is the probability that at least two are seconds?
c. If goblets are examined one by one, what is the probability that at most five must be selected to find four that are not seconds?
Question-45 Suppose that only 25% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when no other cars are visible. What is the probability that, of 20 randomly chosen drivers coming to an intersection under these conditions,
a. At most 6 will come to a complete stop?
b. Exactly 6 will come to a complete stop?
c. At least 6 will come to a complete stop?
d. How many of the next 20 drivers do you expect to come to a complete stop?
Question-46 A particular type of tennis racket comes in a midsize version and an oversize version. Sixty percent of all customers at a store want the oversize version.
a. Among ten randomly selected customers who want this type of racket, what is the probability that at least six want the oversize version?
b. Among ten randomly selected customers, what is the probability that the number who want the oversize version is within 1 standard deviation of the mean value?
c. The store currently has seven rackets of each version. What is the probability that all of the next ten customers who want this racket can get the version they want from current stock?
Question-47 AHarris Interactive survey for InterContinental Hotels & Resorts asked respondents, When traveling internationally, do you generally venture out on your own to experience culture, or stick with your tour group and itineraries? The survey found that 23% of the respondents stick with their tour group (USA Today, January 21, 2004).
a. In a sample of six international travelers, what is the probability that two will stick with their tour group?
b. In a sample of six international travelers, what is the probability that at least two will stick with their tour group?
c. In a sample of 10 international travelers, what is the probability that none will stick with the tour group?

Question-48 In San Francisco, 30% of workers take public transportation daily (USA Today, December 21, 2005).
a. In a sample of 10 workers, what is the probability that exactly three workers take public transportation daily?
b. In a sample of 10 workers, what is the probability that at least three workers take public transportation daily?
Question-49 When a new machine is functioning properly, only 3% of the items produced are defective. Assume that we will randomly select two parts produced on the machine and that we are interested in the number of defective parts found. a. Describe the conditions under which this situation would be a binomial experiment. b. Draw a tree diagram similar to Figure 5.3 showing this problem as a two-trial experiment. c. How many experimental outcomes result in exactly one defect being found? d. Compute the probabilities associated with finding no defects, exactly one defect, and two defects.
Question-50 Nine percent of undergraduate students carry credit card balances greater than $7000 (Readers Digest, July 2002). Suppose 10 undergraduate students are selected randomly to be interviewed about credit card usage.
a. Is the selection of 10 students a binomial experiment? Explain.
b. What is the probability that two of the students will have a credit card balance greater than $7000?
c. What is the probability that none will have a credit card balance greater than $7000?
d. What is the probability that at least three will have a credit card balance greater than $7000?
Question-51. Military radar and missile detection systems are designed to warn a country of an enemy attack. A reliability question is whether a detection system will be able to identify an attack and issue a warning. Assume that a particular detection system has a .90 probability of detecting a missile attack. Use the binomial probability distribution to answer the following questions.
a. What is the probability that a single detection system will detect an attack?
b. If two detection systems are installed in the same area and operate independently, what is the probability that at least one of the systems will detect the attack?
c. If three systems are installed, what is the probability that at least one of

the systems will detect the attack?
d. Would you recommend that multiple detection systems be used? Explain.
Question-51 Fifty percent of Americans believed the country was in a recession, even though technically the economy had not shown two straight quarters of negative growth (BusinessWeek, July 30, 2001). For a sample of 20 Americans, make the following calculations.
a. Compute the probability that exactly 12 people believed the country was in a recession.
b. Compute the probability that no more than five people believed the country was in a recession.
Question-52 The Census Bureaus Current Population Survey shows 28% of individuals, ages 25 and older, have completed four years of college (The New York Times Almanac, 2006). For a sample of 15 individuals, ages 25 and older, answer the following questions:
a. What is the probability four will have completed four years of college?
b. What is the probability three or more will have completed four years of college?

1.12 Hypergeometric Probability Distribution

Question-53 Suppose N = 10 and k = 3. Compute the hypergeometric probabilities for the following values of n and x.
a. n = 4, x = 1.
b. n = 2, x = 2.
c. n = 2,= x =0.
d. n = 4, x =2.
e. n = 4, x = 4.
Question-54 Suppose N = 15 and r = 4. What is the probability of x = 3 for n = 10?
Question-54 An instructor who taught two sections of statistics last term, the first with 20 students and the second with 30, decided to assign a term project. After all projects had been turned in, the instructor randomly ordered them before grading. Consider the first 15 graded projects.
a. What is the probability that exactly 10 of these are from the second section?
b. What is the probability that at least 10 of these are from the second

section?
c. What is the probability that at least 10 of these are from the same section?
d. What are the mean value and standard deviation of the number among these 15 that are from the second section?

Question-55 A bookstore has 15 copies of a particular textbook, of which 6 are first printings and the other 9 are second printings (later printings provide an opportunity for authors to correct mistakes). Suppose that 5 of these copies are randomly selected, and let X be the number of first printings among the selected copies.
a. What kind of a distribution does X have (name and values of all parameters)?
b. Compute $P(X = 2), P(X \leq 2), and P(X \geq 2)$. c. Calculate the mean value and standard deviation of X.

Question-55-A Axline Computers manufactures personal computers at two plants, one in Texas and the other in Hawaii. The Texas plant has 40 employees; the Hawaii plant has 20. A random sample of 10 employees is to be asked to fill out a benefits questionnaire.
a. What is the probability that none of the employees in the sample work at the plant in Hawaii?
b. What is the probability that one of the employees in the sample works at the plant in Hawaii?
c. What is the probability that two or more of the employees in the sample work at the plant in Hawaii?
d. What is the probability that nine of the employees in the sample work at the plant in Texas?

Question-55-B An important problem encountered by personnel directors and others faced with the selection of the best in a finite set of elements is exemplified by the following scenario. From a group of 20 Ph.D. engineers, 10 are randomly selected for employment. What is the probability that the 10 selected include all the 5 best engineers in the group of 20?

Question-55-C An urn contains ten marbles, of which five are green, two are blue, and three are red. Three marbles are to be drawn from the urn, one at a time without replacement. What is the probability that all three marbles drawn will be green?

Question-55-D A warehouse contains ten printing machines, four of which are defective. A company selects five of the machines at random, thinking all are in working condition. What is the probability that all five of the machines are nondefective?

Question-55-D In southern California, a growing number of individuals pursuing teaching credentials are choosing paid internships over traditional student teaching programs. A group of eight candidates for three local teaching positions consisted of five who had enrolled in paid internships and three who enrolled in traditional student teaching programs. All eight candidates appear to be equally qualified, so three are randomly selected to fill the open positions. Let Y be the number of internship trained candidates who are hired.
(a)- Does Y have a binomial or hypergeometric distribution? Why?
(b)- Find the probability that two or more internship trained candidates are hired. (c) What are the mean and standard deviation of Y ?

1.13 Negative Binomial Probability Distribution

Question-56 Suppose that 30% of the applicants for a certain industrial job possess advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool.
(a)- Find the probability that the third applicant with advanced training in programming is found on the Eight interview (b)-Find the probability that the first applicant with advanced training in programming is found on the fifth interview.
Question-57 An oil prospector will drill a succession of holes in a given area to find a productive well. The probability that he is successful on a given trial is 0.2.
a)- What is the probability that the fifth hole drilled is the third to yield a productive well?
(b)- What is the probability that the third hole drilled is the first to yield a productive well?
Question-58 A certified public accountant (CPA) has found that nine of ten company audits contain substantial errors. If the CPA audits a series of company accounts, what is the probability that the first account containing substantial errors
(a) is the third one to be audited?
(b) will occur on or after the third audited account?

Question-59
The employees of a firm that manufactures insulation are being tested for indications of asbestos in their lungs. The firm is requested to send three employees who have positive indications of asbestos on to a medical center for further testing. If 40% of the employees have positive indications of asbestos in their lungs, find the probability that ten employees must be tested in order to find three positives.
Question-60
Ten percent of the engines manufactured on an assembly line are defective. If engines are randomly selected one at a time and tested, what is the probability that the first nondefective engine will be found on the second trial?
Question-61
The telephone lines serving an airline reservation office are all busy about 60% of the time. If you are calling this office, what is the probability that you will complete your call on the first try? The second try? The third try?
Question-62

A geological study indicates that an exploratory oil well should strike oil with probability 0.2
(a)- What is the probability that the first strike comes on the third well drilled?
(b)– What is the probability that the third strike comes on the seventh well drilled?
(c)- What assumptions did you make to obtain the answers to parts (a) and (b)?
(d)- Find the mean and variance of the number of wells that must be drilled if the company wants to set up three producing wells.

1.14 Poisson Distribution

Question-62
Let X, the number of flaws on the surface of a randomly selected carpet of a particular type, have a Poisson distribution with parameter $\mu = 5$. Compute the following probabilities:
$a.P(X \leq 8)$
$b.P(X = 8)$

$c.P(9 \geq X)$
$d.P(5 \leq X \leq 8)$
$e.P(5 < X < 8)$

Question-63

Suppose the number X of tornadoes observed in a particular region during a 1-year period has a Poisson distribution with $\theta = 8$.
a. Compute$P(X \leq 5)$. b. Compute $P(6 \leq X < 9)$. c. Compute $P(10 \leq X)$.

Question-64

Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has a Poisson distribution with parameter l 20 (suggested in the article Dynamic Ride Sharing: Theory and Practice, J. Transp. Engrg., 1997: 308312). What is the probability that the number of drivers will
a. Be at most 10?
b. Exceed 20?
c. Be between 10 and 20, inclusive? Be strictly between 10 and 20?
d. Be within 2 standard deviations of the mean value?

Question-65

Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter $\lambda = 0.20$. (Suggested in Average Sample Number for Semi-Curtailed Sampling Using the Poisson Distribution, J. Qual. Tech., 1983: 126129.)
a. What is the probability that a disk has exactly one missing pulse?
b. What is the probability that a disk has at least two missing pulses?

Question-66

A certain type of tree has seedlings randomly dispersed in a large area, with the mean density of seedlings being approximately five per square yard. If a forester randomly locates ten 1-square-yard sampling regions in the area, find the probability that none of the regions will contain seedlings.

Question-67

Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that
a no more than three customers arrive?

b at least two customers arrive?
c exactly five customers arrive?
Question-68
The number of typing errors made by a typist has a Poisson distribution with an average of four errors per page. If more than four errors appear on a given page, the typist must retype the whole page. What is the probability that a randomly selected page does not need to be retyped?

1.15 Exponential probability distribution

1.16 Normal Probability Distribution