Date 26th April

Day

Image Enhancement

what is Image Enhancement?

is process of making the image more useful.

why we do image enhancement

we do image enhancement to
- Highlighting interesting detail in image
- Remove noise from image
- Making images more visually apealing

Categories

⇒ Spatial Domain
⇒ Frequency Domain

Technique

Image Histogram:

Shows the distribution of gray levels in the image

- Gray level is b/w 0.0 to 1.0

Date: ____________ Day: M T W T F S

0 represents the black colo[r]
1 " " white "

Histogram Equalisation

Equalising the image.

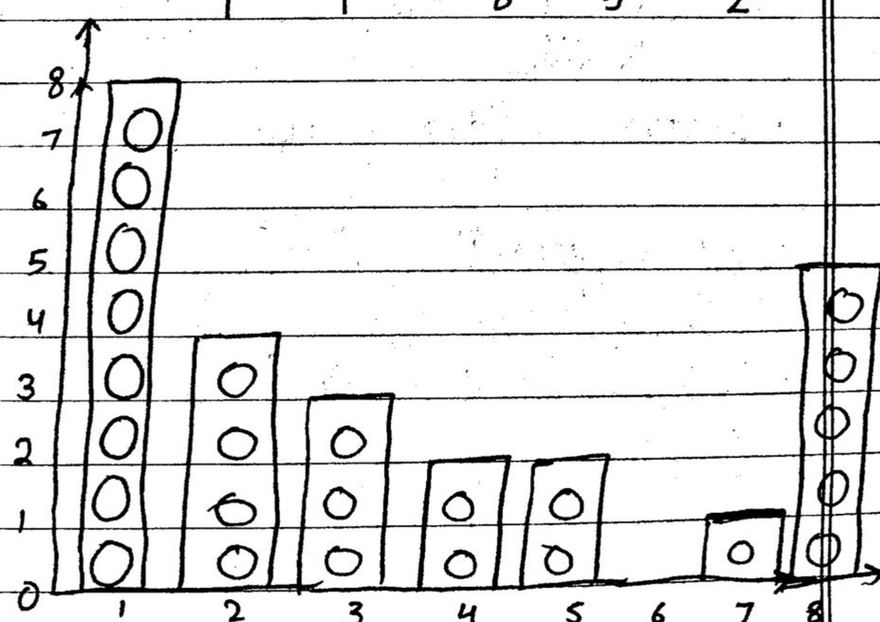
formula $= S_K = T(r_K)$

$$= \sum_{j=1}^{K} P_r(r_j)$$

$$= \sum_{j=1}^{k} \frac{n_j}{n}$$

e.g:

1	8	4	3	4
1	1	1	7	8
8	8	3	3	1
2	2	1	5	2
1	1	8	5	2



Date ______ Day: M T W T F S

Original Histogram	Normalised Histogram Function
$h(r_1) = 8$	$p(r_1) = 8/25 = 0.32$
$h(r_2) = 4$	$p(r_2) = 4/25 = 0.16$
$h(r_3) = 3$	$p(r_3) = 3/25 = 0.12$
$h(r_4) = 2$	$p(r_4) = 2/25 = 0.08$
$h(r_5) = 2$	$p(r_5) = 2/25 = 0.08$
$h(r_6) = 0$	$p(r_6) = 0 = 0$
$h(r_7) = 1$	$p(r_7) = 1/25 = 0.04$
$h(r_8) = 5$	$p(r_8) = 5/25 = 0.20$

Formula of Normalised Histogram function

$$p(r_k) = \frac{h(r_k)}{n} = \frac{n_k}{n}$$

The Sum of all the NHF over the range of all intensities is 1.

Intensity Transformation function

$T(r_1) = p(r_1)$

$T(r_2) = p(r_1) + p(r_2)$

$T(r_3) = p(r_1) + p(r_2) + (p r_3)$

$\vdots \qquad \vdots$

$T(r_1) = 0.32$

$T(r_2) = 0.32 + 0.16 = 0.48$

$T(r_3) = 0.32 + 0.16 + 0.12 = 0.60$

$T(r_4) = 0.32 + 0.16 + 0.12 + 0.08 = 0.68$

$T(r_5) = 0.32 + 0.16 + 0.12 + 0.08 + 0.08 = 0.76$

$T(r_6) = \quad " \quad + 0 = 0.76$

$T(r_7) = \quad " \quad + 0.04 = 0.80$

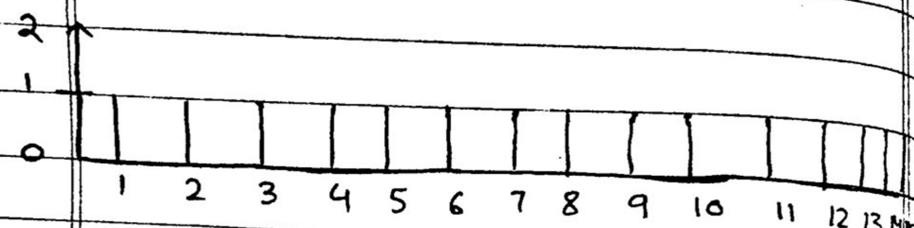
$T(r_8) = 0.32 + 0.16 + 0.12 + 0.08 + 0.08 + 0 + 0.04 + 0.20 = 1.0$

Date

Day

Setting Lower Order bit to Zero

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15



Binary Code

	0	0	0	0	0
	1	0	0	0	1
	2	0	0	1	0
	3	0	0	1	1
	4	0	1	0	0
	5	0	1	0	1
	6	0	1	1	0
	7	0	1	1	1
	8	1	0	0	0
	9	1	0	0	1
A	10	1	0	1	0
B	11	1	0	1	1
C	12	1	1	0	0
D	13	1	1	0	1
E	14	1	1	1	0
F	15	1	1	1	1

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

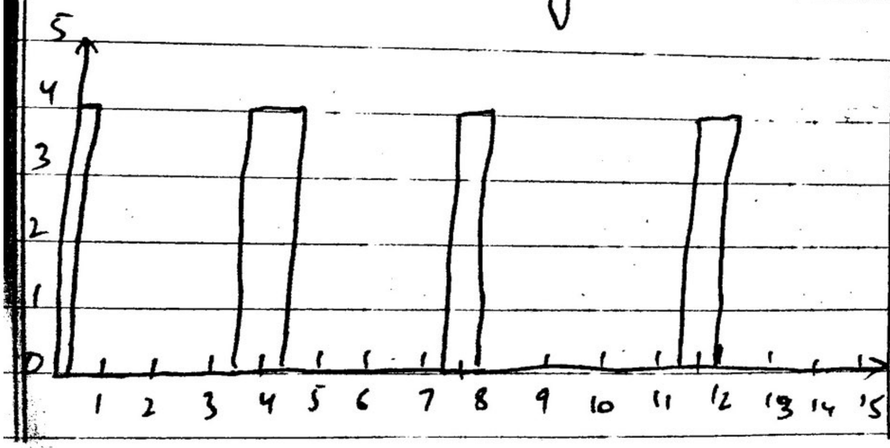
⇓ Set lower bits to 0

0000	0000	0000	0000
0100	0100	0100	0100
1000	1000	1000	1000
1100	1100	1100	1100

⇓ convert Binary code to Original no.s

0	0	0	0
4	4	4	4
8	8	8	8
12	12	12	12

make Histogram

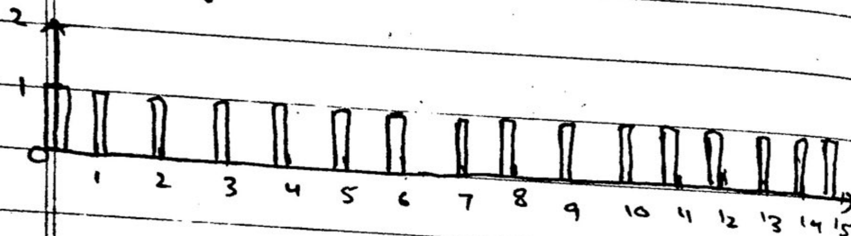


Setting Higher Order Bit to Zero

e.g:

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Histogram



Convert no.s into Binary Code

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

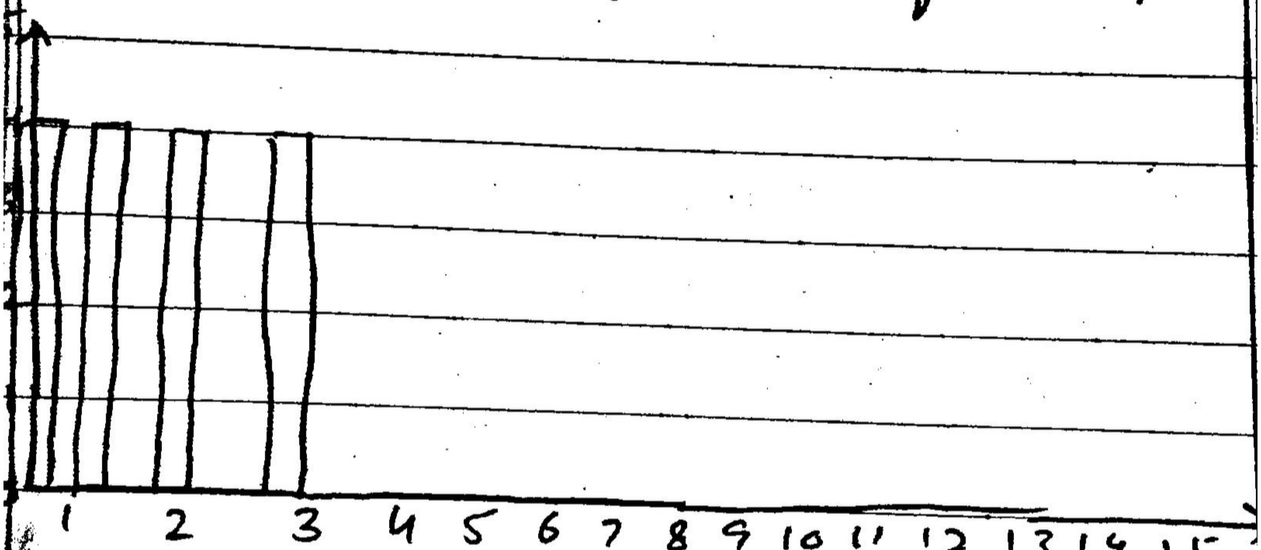
⇓ Convert higher bit to Zero

0000	0001	0010	0011
0000	0001	0010	0011
0000	0001	0010	0011
0000	0001	0010	0011

⇓ Convert into original numbers.

0	1	2	3
0	1	2	3
0	1	2	3
0	1	2	3

make histogram of output

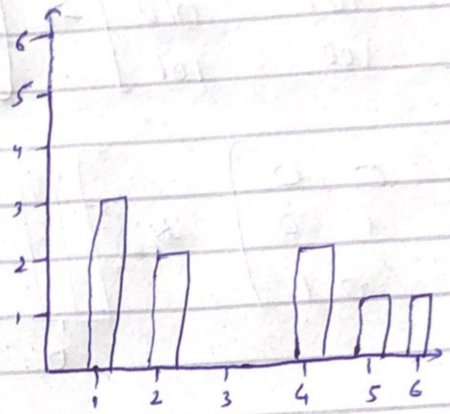


Histogram

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 5 & 4 \\ 4 & 6 & 1 \end{bmatrix}$$

$h(r_1) = 3$
$h(r_2) = 2$
$h(r_3) = 0$
$h(r_4) = 2$
$h(r_5) = 1$
$h\ r_6 = 1$

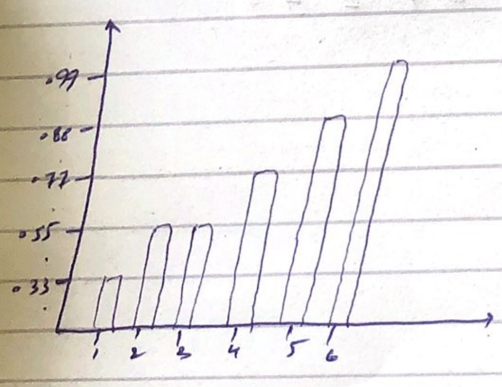
$\overline{9.}$



$3/9 = 0.33$
$2/9 = 0.22$
$0/9 = 0$
$2/9 = 0.22$
$1/9 = 0.11$
$1/9 = 0.11$

$\rightarrow$

0.33
$0.33 + 0.22 = 0.55$
$0.55 + 0 = 0.55$
$0.55 + 0.22 = 0.77$
$0.77 + 0.11 = 0.88$
$0.88 + 0.11 = 0.99.$



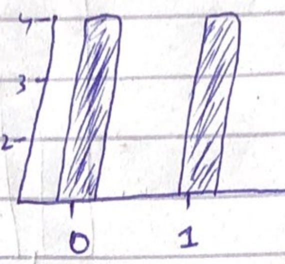
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

$$\begin{bmatrix} 1 & 6 & 3 \\ 2 & 5 & 2 \\ 4 & 4 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 001 & 110 & 011 \\ 0010 & 101 & 010 \\ 100 & 100 & 101 \end{bmatrix}$$

for higher bits

$$\begin{bmatrix} 001 & 110 & 110 \\ 001 & 101 & 010 \\ 100 & 100 & 101 \end{bmatrix} \rightarrow \begin{bmatrix} 001 & 000 & 000 \\ 001 & 001 & 000 \\ 000 & 000 & 001 \end{bmatrix}$$

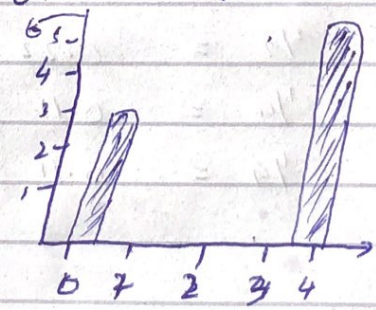
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$$



for lower bits

$$\begin{bmatrix} 001 & 110 & 110 \\ 001 & 101 & 010 \\ 100 & 100 & 101 \end{bmatrix} \rightarrow \begin{bmatrix} 000 & 100 & 100 \\ 000 & 100 & 000 \\ 100 & 100 & 100 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & 4 \\ 0 & 4 & 0 \\ 4 & 4 & 4 \end{bmatrix} \rightarrow$$



histogram →

TRANSLATION

→ change Place.

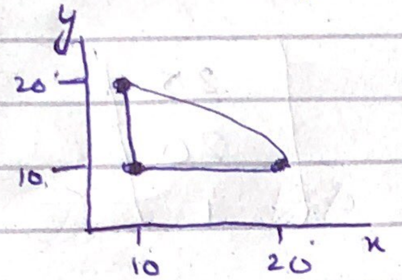
$x' = x + tx$

$y' = y + ty$

(10, 20) , (10, 10) , (20, 10)

$tx = 5$

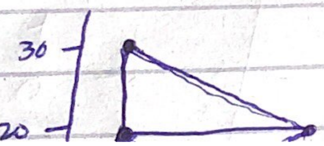
$ty = 10.$



$x' = 10 + 5 = 15$

$y' = 20 + 10 = 30$ → (15, 30), (15, 20), (25, 20)

and so on



TRANSLATION

→ change place.

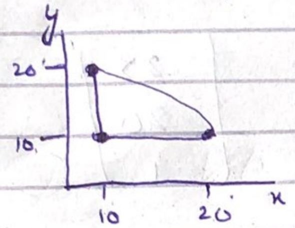
$x' = x + tx$

$y' = y + ty$

(10, 20) , (10, 10) , (20, 10)

$tx = 5$

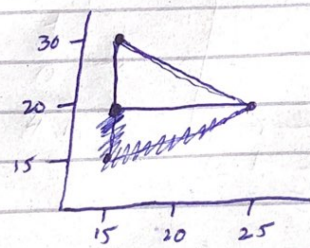
$ty = 10.$



$x' = 10 + 5 = 15$

$y' = 20 + 10 = 30$ → (15, 30), (15, 20), (25, 20)

and so on



Using Matrix method

$$\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 10 + 0 + 5 \\ 0 + 20 + 10 \\ 0 + 0 + 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 15 \\ 30 \\ 1 \end{bmatrix}$$

for second point (10,10) & 3rd point (20,10)

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix} \quad \& \quad \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix} \quad \& \quad \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 15 \\ 20 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 25 \\ 20 \\ 1 \end{bmatrix}$$

So, the new points are

(15, 30), (15, 20) & (25, 20)

Re-translation. (1)

(2,2), (4,2), (2,4), (4,4)

Apply Scaling

$S_x = 2$

$S_y = 2$

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 \\ 8 \\ 1 \end{bmatrix}$$

min $x = 2$; $y = 2$

max $x = 4$; $y = 4$

$t_x = \dfrac{\text{min}x + \text{max}X}{2}$; $t_y = \dfrac{\text{min}x + \text{max}X}{2}$

$t_x = \dfrac{2+4}{2} = 3$; $t_y = \dfrac{2+4}{2} = 3$

Scaling

(10, 20), (10, 10), (20, 10)

$S_x = 2, \; S_y = 1.5$

$$\left.\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix}\right\}$$

formulla

$x' = x * S_x$

$y' = y * S_y$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 20 \\ 30 \\ 1 \end{bmatrix}$ also for $(10, 10) \Rightarrow \begin{bmatrix} 20 \\ 15 \\ 1 \end{bmatrix}$ and $(20, 10) \Rightarrow \begin{bmatrix} 40 \\ 15 \\ 1 \end{bmatrix}$

So, the new points are (20, 30) (20, 15), (40, 15)

Using formulle

$$\left.\begin{aligned} x' &= 10 * 2 = 20 \\ y' &= 20 * 1.5 = 30 \end{aligned}\right\}$$

for Point 2

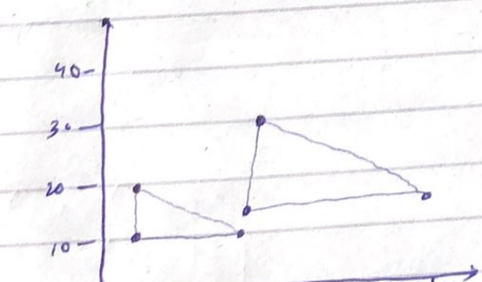
$= 10 \times 2$

$= 10 \times 1.5$

for Point 3

$= 20 \times 2$

$= 10 \times 1.5$



Re-translation

②

-c+2
-c

translation   scaling   re-trans

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} S_x & 1 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Put values

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Resultant matric}$$

$$\begin{bmatrix} 2 & 0 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

new Points

{(1,1) (5,1) (1,5) (5,5)

old point

(4,4) (8,4) (4,8) (8,8)