Probability

A single 6-sided die is rolled. What is the probability of rolling an even number?

Sample Space = {1, 2, 3, 4, 5, 6}
$n(S) = 6$.

Even numbers = 3

$$P(\text{even number}) = \frac{3}{6_2} = \frac{1}{2}$$

(19) same.

(20) A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5.

Sample Space = {1, 2, 3, 4, 5, 6}
$n(S) = 6$.

$P(2) = 1/6$
$P(5) = 1/6$.

$$P(2 \text{ or } 5) = P(2) + P(5)$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1+1}{6} = \frac{2}{6_3} = \frac{1}{3}$$

(21) A spinner has 4 equal sectors colored yellow, blue, green and red. What is the probability of landing on red or blue after spinning this spinner?

Total colors = 4.
Yellow, blue, green, red

$P(\text{red}) = 1/4$

$P(\text{blue}) = 1/4$

$P(\text{red or blue}) = \frac{1}{4} + \frac{1}{4}$

$= \frac{1+1}{4} = \frac{2}{4_2}\frac{1}{1} = \frac{1}{2}$

(22) In a math class of 30 students, 17 are boys, 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is choosen at random from the class, what is the probability of choosing a girl or an A student.

Total students = 30.
Boys = 17
Girls = 13

A Grade = 4 boys.
A Grade = 5 girls

P(Girls or A grade) = ?

P(Girls) = 13/30.

P(A Grade) = 4/30.

$$P(\text{Girls or A Grade}) = \frac{13}{30} + \frac{4}{30}$$

$$= \frac{17}{30}$$

(23) A bag contain 18 coloured marbles; 4 are coloured red, 8 are coloured Yellow and 6 are coloured green. A marble is selected at random. What is the probability that the ball chosen is either red or green?

Total = 18.
Red = 4.
Yellow = 8
Green = 6.

P(Red) = 4/18.
P(Green) = 6/18.

$$P(\text{Red or Green}) = \frac{4}{18} + \frac{6}{18}$$

$$= \frac{10}{18} \Rightarrow \frac{5}{9}$$

(24) A lot consists of 10 good articles, 4 with minor defects, and 2 with major defects. One article is chosen at random. Find the probability that:

(a) it has no defects.

(b) it has no major defects.

(c) it is either good or has major defects.

Total = 10+4+2 = 16.

Good = 10 } Non-defect

Minor Defects = 4 } Defect

Major Defects = 2 }

(c) it is either good or has major defects.

14/16

P(Good) = 10/16

P(Major) = 2/16.

$$P(\text{Good or Major}) = \frac{10}{16} + \frac{2}{16}$$

$$= \frac{12^{3}}{16_{4}} = \frac{3}{4}.$$

(b) it has no major defects.

$$\frac{10+4}{16}$$

(a) it has no defects

$\frac{10}{16}$

(25) A bag contains 20 balls, 3 are coloured red, 6 are coloured green, 4 are coloured blue, 2 are coloured white and 5 are coloured yellow. One ball is ~~colour~~ selected at random. Find the probabilities of the following events.

(a) a red marble.
(b) a green marble.
(c) a white marble.
(d) red or white marble.
(e) green or white marble.
(f) green, red or white marble.

Total = 20.
Red = 3
Green = 6
Blue = 4
White = 2.
Yellow = 5

(a) a red marble

$$\frac{\binom{3}{1}\binom{17}{0}}{\binom{20}{1}}$$

$$P(\text{Red}) = \frac{3}{20}$$

b) a green marble.

$$\frac{\binom{6}{1}\binom{14}{0}}{\binom{20}{1}}$$

$$P(\text{Green}) = \frac{6}{20}$$

(c) a white marble

$$\frac{\binom{2}{1}\binom{18}{0}}{\binom{20}{1}} \quad \Big| \quad P(\text{white}) = \frac{2}{20}$$

(d) Red or white marble

$$P(\text{Red}) = \frac{3}{20}$$

$$P(\text{White}) = \frac{2}{20}$$

$$P(\text{Red or white}) = \frac{3}{20} + \frac{2}{20}$$

$$= \frac{3+2}{20}$$

$$= \frac{5}{20_4} \Rightarrow \frac{1}{4}$$

(e) green or white marble

$$P(\text{Green}) = \frac{6}{20}$$

$$P(\text{white}) = \frac{2}{20}$$

$$P(\text{Green or white}) = \frac{6}{20} + \frac{2}{20}$$

$$= \frac{6+2}{20} \Rightarrow \frac{8^2}{20_5}$$

$$= \frac{2}{5}$$

(f) green, red or white marble.

$$P(\text{Green}) = \frac{6}{20}$$

$$P(\text{Red}) = \frac{3}{20}$$

$$P(\text{White}) = \frac{2}{20}$$

$$P(\text{Green or Red or White}) = \frac{6}{20} + \frac{3}{20} + \frac{2}{20}$$

$$= \frac{6+3+2}{20}$$

$$= \frac{11}{20}$$

(26). In a group of 101 students 40 are juniors, 50 are female, and 22 are female juniors. Find the probability that a student picked from this group at random is either a junior or female.

Total = 101

Juniors = 40

Females = 50.

Female Juniors = 22

$$P(\text{Juniors}) = \frac{40}{101}$$

$$P(\text{Females}) = \frac{50}{101}$$

$$P(\text{Female Juniors}) = \frac{22}{101}$$

$$\frac{40}{101} + \frac{50}{101} - \frac{22}{101}$$

$$= \frac{40+50-22}{101}$$

$$= \frac{90-22}{101}$$

$$= \frac{68}{101}$$

(27) A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of chosing a king or a club?

Total cards = 52

King = 4.

Club = 13.

$$P(\text{King}) = \frac{4}{52} \qquad P(\text{King of Club}) = \frac{1}{52}$$

$$P(\text{Club}) = \frac{13}{52}$$

$$P(\text{King or Club}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{4+13-1}{52} \Rightarrow \frac{17-1}{52} \Rightarrow \frac{16}{52}^{4}_{13}$$

$$= \frac{4}{13}$$

(29) If $P(A) = 0.24$, $P(B) = 0.67$, and $P(AB) = 0.09$, find.

(a) $P(A \cup B)$.

(b) $P((A \cup B)^c)$

(c) $P(A^c \cup B^c)$

(a) $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(\overset{AB}{A \cap B})$$

$$P(A \cup B) = 0.24 + 0.67 - 0.09$$

$$P(A \cup B) = 0.82$$

(b) $P(A \cup B)^c$.

$$P(A \cup B)^c = 1 - P(A \cup B)$$

$$P(A \cup B)^c = 1 - 0.82$$

$$P(A \cup B)^c = 0.18$$

(c). $P(A^c \cup B^c)$

$$A^c = 1 - P(A)$$
$$A^c = 1 - 0.24$$
$$A^c = 0.76$$

$B^c = 1 - P(B)$

$B^c = 1 - 0.67$

$B^c = 0.33$.

$P(A^c B^c) = 0.76 \times 0.33$

$P(A^c B^c) = 0.25$

$P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c B^c)$

$P(A^c \cup B^c) = 0.76 + 0.33 - 0.25$

$P(A^c \cup B^c) = 0.84$

(30) A sample of four electronic components is taken from the output of a production line. The probabilities of the various outcomes are calculated to be:

Pr[0 defectives] = 0.6561,

Pr[1 defectives] = 0.2916,

Pr[2 defectives] = 0.0486,

Pr[3 defectives] = 0.0036,

Pr[4 defectives] = 0.0001.

What is the probability of at least one defective?

P(At least one defective)

Pr[1 + 2 + 3 + 4 defective] = At least one

$0.2916 + 0.0486 + 0.0036 + 0.0001$

$= 0.3439$ //

P(At least one defective) = $1 - 0.6561 \Rightarrow 0.3439$ (No defective)

(31) and (32) same

If one card is drawn from a well-shuffled bridge deck of 52 playing cards (13 of each suit), what is the probability that the card is a queen or a heart?

Total = 52.

Queen = 4.

Heart = 13.

$P(\text{Queen}) = \frac{4}{52}$ $P(\text{Queen of heart}) = \frac{1}{52}$

$P(\text{heart}) = \frac{13}{52}$

$P(\text{Queen or heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{4 + 13 - 1}{52} \Rightarrow \frac{17-1}{52}$

$= \frac{16}{52}$

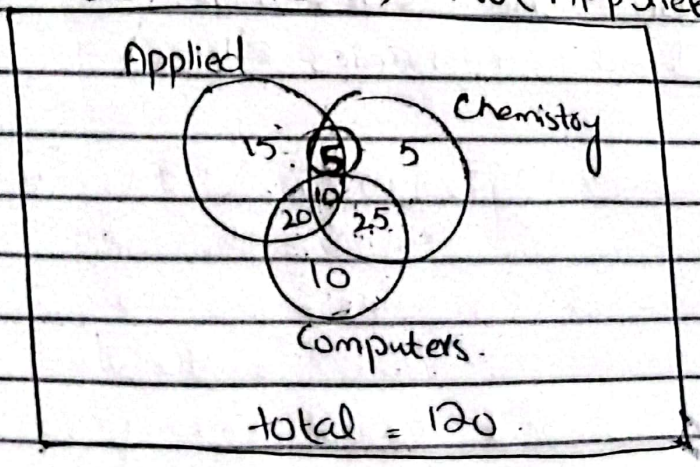
$= \frac{4}{13}$

(33) The class registrations of 120 students are analyzed. It is found that: 30 of the students do not take any of Applied Mechanics, Chemistry or Computers. 15 of them take only Applied Mechanics. 25 of them take Chemistry and computers but not applied mechanics. 20 of them take Applied Mechanics and computers but not chemistry. 10 of them take all three of Applied Mechanics, Chemistry and Computers. A total of 45 of them take chemistry. 5 of them only Chemistry.

(a) How many of the students take applied mechanics and chemistry but not computers?

$= 5$.

$120 - 30 = 90 \rightarrow$ Not Applied all three



Total chemistry = 45

$45 - (10 + 5 + 25)$

$45 - 40 \Rightarrow 5$.

(b) How many of the students take only computers?

$= 90 - [15 + 20 + 5 + 10 + 5 + 25]$

$= 90 - [80]$

$= 10$

(c) What is the total number of students taking computers?

$= 20 + 10 + 25 + 10$

$= 65$

(d) If a student is chosen at random from those who take neither chemistry nor computers, what is the probability that he or she does not take applied mechanics either?

Total chemistry $= 5 + 10 + 5 + 25$

$= 45$

Total computer $= 20 + 10$

$= 30$

Total student who belong to chemistry or computer

$= 45 + 30$

$= 75$

Neither chemistry nor computers =
$120 - 75 \Rightarrow 45$

Not applied mechanics = 30.

$$P(\text{Does not take}) = \frac{30}{45} \Rightarrow \frac{2}{3}$$

(e) If one of the students who take at least two of the three courses is choosen at random, what is the probability that he or she takes all three courses?

at least two of three courses
$= 20 + 10 + 5 + 25$
$= 60$

All three courses = 10.

$$P(\text{She take}) = \frac{10}{60} \Rightarrow \frac{1}{6}$$