KNAPSACK PROBLEM

i item	w_i weight	v_i Value
1	1	10
2	3	15
3	2	25
4	4	30

capacity = 5 => j

F1:- $V[i,j] = \max(v[i-1,j], v_i + V[i-1, j-w_i]) \Rightarrow$ if $(j - w_i \geq 0)$

F2 :- $V[i,j] = V[i-1, j] \Rightarrow (j - w_i < 0)$

i \ j	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	10	10	10	10	10
2	0	10	10	15	25	25
3	0	10	25	35	35	40
4	0	10	25	35	35	40

initially $v[0,j]$ & $v[i,0] = 0$

$\Rightarrow i = 1$, $j = 1$ $(j - w_i \Rightarrow j - w_1 \Rightarrow 1 - 1 = 0)$

F1:- $V[1,1] = \max(V[0,1], 10 + V[0,0]$]

$V[1,1] = \max(0, 10+0)$

$V[1,1] = 10$

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$\Rightarrow i = 1 , j = 2 \; (j - w_1 \Rightarrow 2 - 1 = 1)$

F1:- $V[1,2] = \max(V[0,2], 10 + V[0,1])$

$V[1,2] = \max(0, 10)$

$V[1,2] = 10$

$\Rightarrow i = 1 , j = 3 \; (3 - 1 \Rightarrow 2)$

F1:- $V[1,3] = \max(V[0,3], 10 + V[0,2])$

$V[1,3] = \max(0, 10)$

$V[1,3] = 10$

$\Rightarrow i = 1 , j = 4 \; (4 - 1 \Rightarrow 3)$

F1:- $V[1,4] = \max(V[0,4], 10 + V[0,3])$

$V[1,4] = 10$

$\Rightarrow i = 1 , j = 5 \; (5 - 1 = 4)$

F1:- $V[1,5] = \max(V[0,5], 10 + V[0,4])$

$V[1,5] = 10$

$\Rightarrow i = 2 , j = 1 \; (j - w_2 \Rightarrow 1 - 3 = -2 > 0)$

F2:- $V[2,1] = V[1,1]$

$V[2,1] = 10$

$\Rightarrow i = 2 \; j = 2 \; (2 - 3 = -1 > 0)$

F2:- $V[2,2] = V[1,2]$

$V[2,2] = 10$

$\Rightarrow i = 2 , j = 3 \; (3 - 3 = 0)$

F1:- $V[2,3] = \max(V[1,3], 15 + V[1,0])$

$V[2,3] = \max(10, 15)$

$\Rightarrow i=2 \quad , \quad j=4 \quad (4-3=1)$

$F1,-\ V[2,4] = \max(V[1,4], 15+V[1,1])$

$\max(10, 15+10)$

$\max(10, 25)$

$V[2,4] = 25$

$\Rightarrow i=2 \quad , \quad j=5 \quad (5-3=2)$

$F1_2-\ V[2,5] = \max(V[\cancel{1},5], 15+V[1,2])$

$= \max(10, 15+10)$

$V[2,5] = 25$

$\Rightarrow i=3 \quad , \quad j=1 \quad (1-2=-1 \not< 0)$

$F2,-\ V[3,1] = V[2,1]$

$V[3,1] = 10$

$\Rightarrow i=3 \quad , \quad j=2 \quad (2-2=0)$

$F1,-\ V[3,2] = \max(V[2,2], 25+V[2,0])$

$V[3,2] = \max(10, 25)$

$V[3,2] = 25$

$\Rightarrow i=3 \quad , \quad j=3 \quad (3-2=1)$

$F1,-\ V[3,3] = \max(V[2,3], 25+V[2,1])$

$\max(15, 25+10)$

$V[3,3] = 35$

$\Rightarrow i=3 \quad , \quad j=4 \quad (4-2=2)$

$F1,-\ V[3,4] = \max(V[2,4], 25+V[2,2])$

$= \max(25, 25+10)$

$V[3,4] = 35$

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$\Rightarrow i = 3 \quad , \quad j = 5 \quad (5 - 2 = 3)$

$F1 :- V[3,5] = \max(V[2,5], 25 + V[2,3])$

$= \max(25 \ , 25 + 15)$

$V(3,5) = 40$

$\Rightarrow i = 4 \ , \ j = 1 \quad (1 - 4 = -3 < 0)$

$F2 :- V[4,1] = V[3,1]$

$= 10$

$\Rightarrow i = 4 \ , \ j = 2 \quad (2 - 4 = -2 < 0)$

$F2 :- V[4,2] = V[3,2]$

$= 25$

$\Rightarrow i = 4 \ , \ j = 3 \quad (3 - 4 = -1 < 0)$

$F2 :- V[4,3] = V[3,3]$

$= 35$

$\Rightarrow i = 4 \ , \ j = 4 \quad (4 - 4 = 0)$

$F1 :- V[4,4] = \max(V[3,4], 30 + V[3,0])$

$= \max(35 \ , 30)$

$V[4,4] = 35$

$\Rightarrow i = 4 \ , \ j = 5 \quad (5 - 4 = 1)$

$F1 :- V[4,5] = \max(V[3,5], 30 + V[3,1])$

$V[4,5] = \max(40, 30 + 10)$

$V[4,5] = 40$

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Back Tracking

	0	1	2	3	4	5
0	0	$\boxed{0}$	0	0	0	0
1	0	10	10	10	10	10
2	0	10	10	$\boxed{15}$	25	25
3	0	10	25	35	35	$\boxed{40}$
4	0	10	25	35	35	40

$40 - 25 = 15$

$15 - 15 = 0$

Optimal Solution includes items [3, 2].

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