

Given the following information, find optimal BST using DP.

Value	60	30	20	50	40
Probability	0.15	0.25	0.17	0.13	0.30

$$C[i, j] = \min \{ C[i, k-1] + C[k+1, j] \} + \sum_{s=i}^j p_s$$

$$i \leq k \leq j$$

$$C[i, j] = p[i] \quad \text{for } 1 \leq i \leq n$$

0.15	0.25	0.17	0.13	0.30
1	2	3	4	5

i \ j	0	1	2	3	4	5
1	0	0.15	0.55	0.89	1.28	2.11
2		0	0.25	0.62	0.93	1.66
3			0	0.17	0.43	1.03
4				0	0.13	0.56
5					0	0.30
6						0

i \ j	0	1	2	3	4	5
1	0	1	2	2	2	3
2		0	2	2	3	3
3			0	3	3	5
4				0	4	5
5					0	5
6						0

$$\begin{aligned}
 C[i, j] &= \min [1, 1-1] + [1+1, 2] + 0.4 \\
 &= C[1, 0] + [2, 2] + 0.4 \\
 &= 0 + 0.25 + 0.4 \\
 C[1, 2] &= 0.65
 \end{aligned}$$

$$\begin{aligned}
 C[i, j] &= \min [1, 2-1] + [2+1, 2] + 0.4 \\
 &= C[1, 1] + [3, 2] + 0.4 \\
 &= 0.15 + 0 + 0.4 \\
 C[1, 2] &= 0.55
 \end{aligned}$$

$$\begin{aligned}
 C[i, j] &= \min [2, 2-1] + [2+1, 3] + 0.42 \\
 &= C[2, 1] + [3, 3] + 0.42 \\
 &= 0 + 0.17 + 0.42 \\
 C[2, 3] &= 0.62
 \end{aligned}$$

$$\begin{aligned}
 C[i, j] &= \min [2, 3-1] + [3+1, 3] + 0.42 \\
 &= C[2, 2] + [4, 3] + 0.42 \\
 &= 0.25 + 0 + 0.42 \\
 &= 0.67
 \end{aligned}$$

$$\begin{aligned}
 C[i, j] &= \min [3, 3-1] + [3+1, 4] + 0.3 \\
 &= C[3, 2] + [4, 4] + 0.3 \\
 &= 0 + 0.13 + 0.3 \\
 C[3, 4] &= 0.43
 \end{aligned}$$

$$k=4$$

$$\begin{aligned} C[i, j] &= \min[3, 4-1] + [4+1, 4] + 0.3 \\ &= C[3, 3] + [5, 4] + 0.3 \\ &= 0.17 + 0 + 0.3 \\ C[3, 4] &= 0.47 \end{aligned}$$

$$k=4$$

$$\begin{aligned} C[i, j] &= \min[4, 4-1] + [4+1, 5] + 0.43 \\ &= C[4, 3] + [5, 5] + 0.43 \\ &= 0 + 0.30 + 0.43 \\ C[4, 5] &= 0.73 \end{aligned}$$

$$k=5$$

$$\begin{aligned} C[i, j] &= \min[4, 5-1] + [5+1, 5] + 0.43 \\ &= C[4, 4] + [6, 5] + 0.43 \\ &= 0.13 + 0 + 0.43 \end{aligned}$$

$$C[4, 5] = 0.56$$

$$k=1$$

$$\begin{aligned} C[i, j] &= \min[1, 1-1] + [1+1, 3] + 0.57 \\ &= C[1, 0] + [2, 3] + 0.57 \\ &= 0 + 0.62 + 0.57 \\ C[1, 3] &= 1.19 \end{aligned}$$

$$k=2$$

$$\begin{aligned} C[i, j] &= \min[1, 2-1] + [2+1, 3] + 0.57 \\ &= C[1, 1] + [3, 3] + 0.57 \\ &= 0.15 + 0.17 + 0.57 \end{aligned}$$

$$C[1, 3] = 0.89$$

$$k=3$$

$$\begin{aligned} C[1,3] &= \min[1, 3-1] + [3+1, 3] + 0.57 \\ &= C[1,2] + [4,3] + 0.57 \\ &= 0.55 + 0 + 0.57 \end{aligned}$$

$$C[1,3] = 1.12$$

$$k=2$$

$$\begin{aligned} C[2,4] &= \min[2, 2-1] + [2+1, 4] + 0.55 \\ &= C[1,1] + [3,4] + 0.55 \\ &= 0.15 + 0.43 + 0.55 \end{aligned}$$

$$C[2,4] = 1.13$$

$$k=3$$

$$\begin{aligned} C[2,4] &= \min[2, 3-1] + [3+1, 4] + 0.55 \\ &= C[2,2] + [4,4] + 0.55 \\ &= 0.25 + 0.13 + 0.55 \end{aligned}$$

$$C[2,4] = 0.93$$

$$k=4$$

$$\begin{aligned} C[2,4] &= \min[2, 4-1] + [4+1, 4] + 0.55 \\ &= C[2,3] + [5,4] + 0.55 \\ &= 0.62 + 0 + 0.55 \end{aligned}$$

$$C[2,4] = 1.17$$

$$k=3$$

$$\begin{aligned} C[3,5] &= \min[3, 3-1] + [3+1, 5] + 0.6 \\ &= C[3,2] + [4,5] + 0.6 \\ &= 0 + 0.56 + 0.6 \end{aligned}$$

$$C[3,5] = 1.16$$



$$[K=4]$$

$$C[i, j] = \min[3, 4-1] + [4+1, 5] + 0.6$$

$$= C[3, 3] + [5, 5] + 0.6$$

$$= 0.17 + 0.30 + 0.6$$

$$C[3, 5] = 1.07$$

$$[K=5]$$

$$C[i, j] = \min[3, 5-1] + [5+1, 5] + 0.6$$

$$= C[3, 4] + [6, 5] + 0.6$$

$$= 0.43 + 0 + 0.6$$

$$C[3, 5] = 1.03$$

$$[K=1]$$

$$C[i, j] = \min[1, 1-1] + [1+1, 4] + 0.7$$

$$= C[1, 0] + [2, 4] + 0.7$$

$$= 0 + 0.93 + 0.7$$

$$C[1, 4] = 1.63$$

$$[K=2]$$

$$C[i, j] = \min[1, 2-1] + [2+1, 4] + 0.7$$

$$= C[1, 1] + [3, 4] + 0.7$$

$$= 0.15 + 0.43 + 0.7$$

$$C[1, 4] = 1.28$$

$$[K=3]$$

$$C[i, j] = \min[1, 3-1] + [3+1, 4] + 0.7$$

$$= C[1, 2] + [4, 4] + 0.7$$

$$= 0.55 + 0.13 + 0.7$$

$$C[1, 4] = 1.38$$

$$k=4$$

$$\begin{aligned} C[i,j] &= \min[1, 4-1] + [4+1, 4] + 0.7 \\ &= C[1,3] + [5,4] + 0.7 \\ &= 0.89 + 0 + 0.7 \\ C[1,4] &= 1.59 \end{aligned}$$

$$\begin{aligned} C[i,j] &= \min[2, 2-1] + [2+1, 5] + 0.85 \\ &= C[2,1] + [3,5] + 0.85 \\ &= 0 + 1.03 + 0.85 \\ C[2,5] &= 1.88 \end{aligned}$$

$$k=3$$

$$\begin{aligned} C[i,j] &= \min[2, 3-1] + [3+1, 5] + 0.85 \\ &= C[2,2] + [4,5] + 0.85 \\ &= 0.25 + 0.56 + 0.85 \end{aligned}$$

$$C[2,5] = 1.66$$

$$k=4$$

$$\begin{aligned} C[i,j] &= \min[2, 4-1] + [4+1, 5] + 0.85 \\ &= C[2,3] + [5,5] + 0.85 \\ &= 0.62 + 0.30 + 0.85 \\ C[2,5] &= 1.77 \end{aligned}$$

$$k=5$$

$$\begin{aligned} C[i,j] &= \min[2, 5-1] + [5+1, 5] + 0.85 \\ &= C[2,4] + [6,5] + 0.85 \\ &= 0.93 + 0 + 0.85 \\ C[2,5] &= 1.78 \end{aligned}$$

$$\begin{aligned}
 C[1,5] &= \min[1,1-1] + [1+1,5] + 1 \\
 &= C[1,0] + [2,5] + 1 \\
 &= 0 + 1.66 + 1 \\
 C[1,5] &= 2.66
 \end{aligned}$$

$$\begin{aligned}
 C[1,5] &= \min[1,2-1] + [2+1,5] + 1 \\
 &= C[1,1] + [3,5] + 1 \\
 &= 0.15 + 1.03 + 1 \\
 C[1,5] &= 2.18
 \end{aligned}$$

$$\begin{aligned}
 C[1,5] &= \min[1,3-1] + [3+1,5] + 1 \\
 &= C[1,2] + [4,5] + 1 \\
 &= 0.55 + 0.56 + 1 \\
 C[1,5] &= 2.11
 \end{aligned}$$

$$\begin{aligned}
 C[1,5] &= \min[1,4-1] + [4+1,5] + 1 \\
 &= C[1,3] + [5,5] + 1 \\
 &= 0.89 + 0.30 + 1 \\
 C[1,5] &= 2.19
 \end{aligned}$$

$$\begin{aligned}
 C[1,5] &= \min[1,5-1] + [5+1,5] + 1 \\
 &= C[1,4] + [6,5] + 1 \\
 &= 1.28 + 0 + 1 \\
 C[1,5] &= 2.28
 \end{aligned}$$