L#2.1

Recurrence Relations

10 20 30 40 50 60 70

i ↗ (10) ... ↑ j (70)

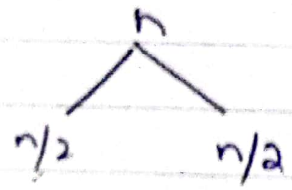
→ BS(a, i, j, x) — x: 30

i = first pointer
j = last pointer
x = search element

→ mid = $\frac{(i+j)}{2}$

$= \frac{1+7}{2} \Rightarrow \frac{8^4}{2} \Rightarrow 4$

∴ Problem divide in two parts



→ if (a[mid] = = x)
→ return (mid);
40 = = 30 False

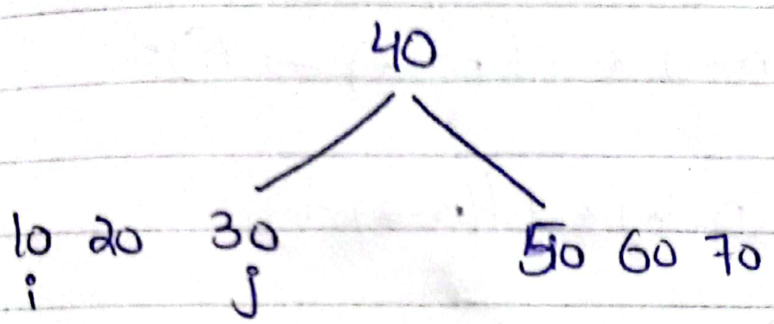
→

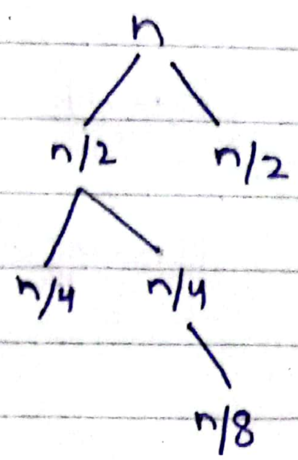
else
if (a[mid] > x) — a[mid]: 40, x: 30

∴ Agar condition true ho to 1st wala run ho ga.

→ BS(a, i, mid-1, x)
else
BS(a, mid+1, J, x)

10 20 30 40 50 60 70
i Midpoint J





$\frac{i+j}{2} \Rightarrow \frac{1+3}{2} \Rightarrow \frac{4}{2} \Rightarrow 2$

mid == x → 2 == 30

mid > x.

20>30

∴ Condition false hai to 2nd wala.

BS(a, mid+1, J, x).

L# 2.2

→ Substitution Method:-

$$T(n) = \begin{cases} T(n/2) + c & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$$

$T(n) = T\left(\frac{n}{2}\right) + c \rightarrow ①$

Decrease by division

$T(n/2) = T\left(\frac{n}{4}\right) + c \rightarrow ②$

back Substitution

$T(n/4) = T\left(\frac{n}{8}\right) + c \rightarrow ③$

⋮

$T(n) = T\left(\frac{n}{4}\right) + c + c$

$T(n) = T\left(\frac{n}{2^2}\right) + 2c$

$T(n) = T\left(\frac{n}{8}\right) + 2c + c$

$T(n) = T\left(\frac{n}{2^3}\right) + 3c$

⋮

$T(n) = T\left(\frac{n}{2^k}\right) + kc$

$\therefore \quad n = 2^k$

$T(n) = T(\cancel{n}/\cancel{n}) + kc$

$T(n) = T(1) + kc$

$T(n) = 1 + kc \rightarrow (4)$

$1 + kc$.

$\because n = 2^k$.

Apply log

$\log n = \log 2^k$

$\log n = k \log 2$

$\log n = k(1)$

$\boxed{\log n = k}$ put in (4).

$T(n) = 1 + \log n\, c$

$\because$ 1 and c are constant

so $O(\log_2 n)$

$= O \log_2 n$.

Time complexity of this recurrence Ya Binary search.

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Substitution Method:

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ n \times T(n-1) & \text{if } n > 1 \end{cases}$$

$T(n) = n \times T(n-1) \rightarrow ①$

$\therefore (n-1)$ function decrease by subtraction.
$\rightarrow$ Next pr $(n-2)$
$\rightarrow (n-3)$
$\rightarrow (n-4)$

$T(n-1) = (n-1) \times T((n-1)-1)$

$= (n-1) \times T(n-1-1)$

$T(n-1) = (n-1) \times T(n-2) \rightarrow ②$

$T(n-2) = (n-2) \times T((n-2)-1)$

$= (n-2) \times T(n-2-1)$

$T(n-2) = (n-2) \times T(n-3) \rightarrow ③$

Use substitution method.

eq ② in ①

$T(n) = n \times (n-1) \times T(n-2) \rightarrow ④$

③ Put in ④.

$T(n) = n \times (n-1) \times (n-2) \times T(n-3)$

Decrement krtay ja

$(n-1)$ steps tk chlay ga.

$$T(n) = n \times (n-1) \times (n-2) \times T(n-3) \ldots T(n-(n-1))$$

$$= n \times (n-1) \times (n-2) \times T(n-3) \ldots T(\cancel{n}-\cancel{n}+1)$$

$$= n \times (n-1) \times (n-2) \times T(n-3) \ldots T(1)$$

$\therefore T(1) = 1$

$$= n \times (n-1) \times (n-2) \times T(n-3) \ldots 1$$

common liya hai

$$= n \times n\left(1-\frac{1}{n}\right) \times n\left(1-\frac{2}{n}\right) \times n\left(1-\frac{3}{n}\right) \ldots n\left(\frac{1}{n}\right)$$

$\therefore$ n kitni time aa raha hai nth time

$= n^n$

$O(n^n)$

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Recurrence Relation:

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

Using back substitution Method

$$T(n) = 2T(n/2) + n \rightarrow (1)$$

$$T\left(\frac{n}{2}\right) = 2T\left(\underset{2\times 2}{\frac{n}{4}}\right) + \frac{n}{2} \rightarrow (2)$$

$$T\left(\frac{n}{4}\right) = 2T\left(\underset{2\times 4}{\frac{n}{8}}\right) + \frac{n}{4} \rightarrow (3)$$

eq (2) put in (1)

$$= 2\left(2T\left(\frac{n}{4}\right) + \frac{n}{2}\right) + n$$

$$= 4T\left(\frac{n}{4}\right) + \cancel{2}\left(\frac{n}{\cancel{2}}\right) + n$$

$$= 4T\left(\frac{n}{4}\right) + n + n$$

$$= 4T\left(\frac{n}{4}\right) + 2n \rightarrow (4)$$

eq (3) put in eq (4)

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$$= 4\left[2T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n$$

$$= 8T\left(\frac{n}{8}\right) + \cancel{4}\left(\frac{n}{\cancel{4}}\right) + 2n$$

$$= 8T\left(\frac{n}{8}\right) + n + 2n$$

$$= 8T\left(\frac{n}{8}\right) + 3n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$\vdots$$

$$= 2^k T\left(\frac{n}{2^k}\right) + kn \rightarrow 5$$

$\therefore \frac{n}{2^k} = 1$

$$n = 2^k$$

$$\therefore \log n = \log 2^k$$

$$\log n = k \log 2$$

$$\log n = k(1)$$

$$\log n = k$$

$$= 2^k T(1) + kn$$

$$= nT(1) + kn$$

$$= n(1) + kn$$

$$= n + \log n \cdot n \Rightarrow n + n\log n \Rightarrow O(n\log n)$$

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L#25

$$T(n) = \begin{cases} 1 & , \text{ if } n = 1 \\ T(n-1) + \log n & , \text{ if } n > 1 \end{cases}$$

$$T(n) = T(n-1) + \log n \rightarrow ①$$

$$T(n-1) = T(n-1-1) + \log n-1$$

$$T(n-1) = T(n-2) + \log(n-1) \rightarrow ②$$

$$T(n-2) = T(n-2-1) + \log(n-2)$$

$$T(n-2) = T(n-3) + \log(n-2) \rightarrow ③$$

eq ② put in ①

$$T(n) = T(n-2) + \log(n-1) + \log n \rightarrow ④$$

eq ③ put in ④

$$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n$$

$$\vdots$$

$$T(n) = T(n-k) + \log(n-(k-1)) + \log(n-(k-2)) + \log(n-(k-3)) + \dots + \log n$$

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$\therefore \quad n-k = 1$

$k = n$

$= T(1) + \log(n-(n-1)) + \log(n-(n-2)) + \log(n-(n-3)) + \dots \log n$

$= 1 + \log 1 + \log 2 + \log 3 + \dots \log n$

$1 + \log(1.2.3 \dots n)$

$\therefore \log m + \log n$

$\log(m \cdot n)$

$1 + \log(n!)$

$n \times (n-1) \times (n-2) \times (n-3)!$

$1 + \log(n^n)$

$1 + \log n^n$

$1 + n\log n$

$= O(n\log n)$

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L#2.6

Master Theorem

$$\boxed{T(n) = aT\left(\frac{n}{b}\right) + f(n)}$$

$a \geq 1$, $b > 1$

$T(n) = T(n-1) + 1$

$a = 1$, $b = 1$

false because $b > 1$

hona chaiye.

using substiting for this.

$$\boxed{T(n) = n^{\log_b a} [u(n)]}$$

$u(n)$ depends on $h(n)$

$$h(n) = \frac{f(n)}{n^{\log_b a}}$$

Relation between $h(n)$ and $u(n)$ is.

if $h(n)$	$u(n)$
$n^r, r>0$	$O(n^r)$
$n^r, r<0$	$O(1)$
$(\log n)^i \; i \geq 0$	$\frac{(\log_2 n)^{i+1}}{i+1}$

$$\boxed{T(n) = 8T(n/2) + n^2}$$

$a = 8$, $b = 2$, $f(n) = n^2$

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$T(n) = n^{\log_b a}\, u(n) \rightarrow$ n power log

$= n^{\log_2 8}\, u(n)$

$\therefore \log_2 8$

$= n^3\, u(n) \Rightarrow n^3\, \Theta(1) = \Theta(n^3)$

$u(n)$ depends on $h(n)$, so find $h(n)$.

$h(n) = \dfrac{f(n)}{n^{\log_b a}}$

$= \dfrac{n^2}{n^{\log_2 8}}$

$= \dfrac{n^2}{n^3} = \dfrac{1}{n} = n^{-1}$

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Master Theorem:

$$T(n) = T\left(\frac{n}{2}\right) + c$$

$$a = 1, \; b = 2, \; f(n) = c$$

$$\therefore \quad a \geq 1, \; b > 1$$

$$T(n) = n^{\log_b^a} \, u(n)$$

$$= n^{\log_2^1} \, u(n)$$

$$= n^0 \cdot u(n)$$

$$= 1 \cdot u(n)$$

$$= u(n)$$

$u(n)$ depends on $h(n)$

$$h(n) = \frac{f(n)}{n^{\log_b^a}}$$

$$h(n) = \frac{c}{n^{\log_2^1}}$$

$$= \frac{c}{1}$$

$$h(n) = c$$

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$$= (\log_2 n)^0 \cdot c$$

$$= \frac{(\log_2 n)^{0+1}}{0+1} \cdot c$$

$$= \log_2 n \cdot c$$

$$= O(\log_2 n)$$

$h(n)$	$u(n)$
n^r, $r>0$	$O(n^r)$
n^r, $r<0$	$O(1)$
$(\log n)^i$, $i \geq 0$	$\frac{(\log_2 n)^{i+1}}{i+1}$

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L#2.8

$$T(n) = \begin{cases} T(\sqrt{n}) + \log n, & \text{if } n > 2 \\ O(1) & \text{else} \end{cases}$$

$\rightarrow T(n) = T(\sqrt{n}) + \log n$

$\therefore T(n) = aT\left(\frac{n}{b}\right) + n^k \log^p n$

Suppose $n = 2^m$

$T(2^m) = T(\sqrt{2^m}) + \log 2^m$

$T(2^m) = T(2^{m/2}) + \log 2^m$

$= T(2^{m/2}) + m \log 2$

$= T(2^{m/2}) + m(1)$

$T(2^m) = T(2^{m/2}) + m \rightarrow (1)$

Suppose $T(2^m) = S(m)$

$S(m) = S\left(\frac{m}{2}\right) + m$

$a = 1, \quad b = 2, \quad f(n) = m$

$T(n) = n^{\log_b^a} \, U(n)$

$= n^{\log_2^1} \, U(n)$

$= n^0 \cdot u(n)$

$= 1 \cdot u(n)$

$= u(n)$

$u(n)$ depends on $h(n)$

$$h(n) = \frac{f(n)}{n^{\log_b^a}}$$

$$= \frac{m}{n^{\log_2^1}}$$

$$= \frac{m}{1}$$

$= m$

$= \log n$

$n = 2^m$

$\log n = \log 2^m$

$\log n = m \log 2$

$\log n = m(1)$

$\log n = m$

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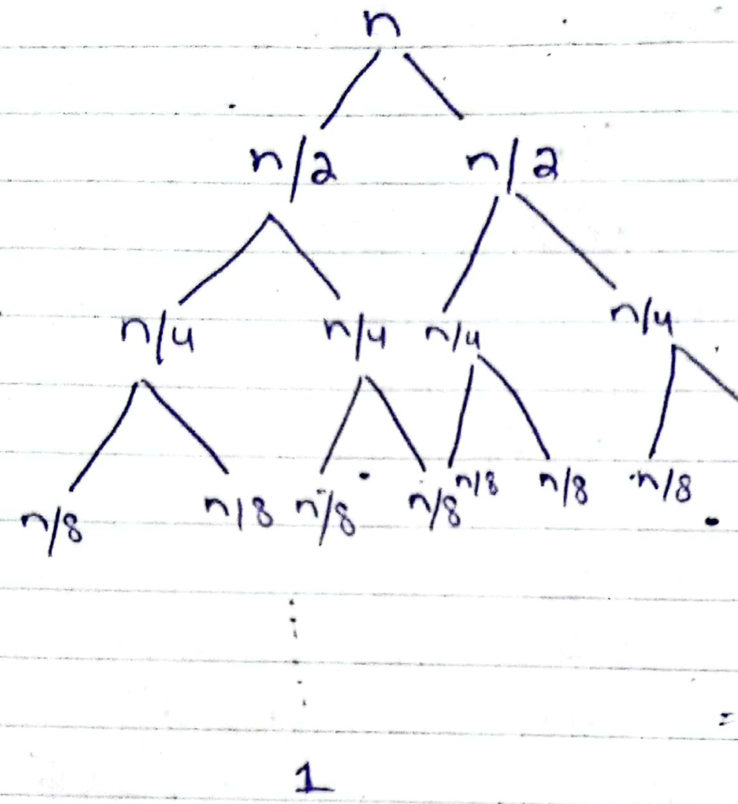
L#2.9

Recursive Tree:

$T(n) = 2T\left(\frac{n}{2}\right) + cn$



Cn

Cn

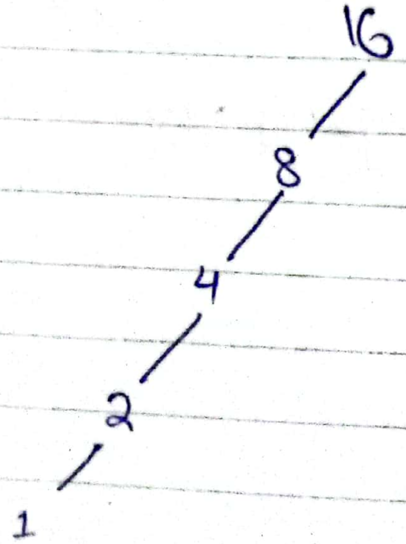
Cn

$= 3cn$

$= cn \cdot \log n$

$= O(n\log n)$

→ Depending on the height

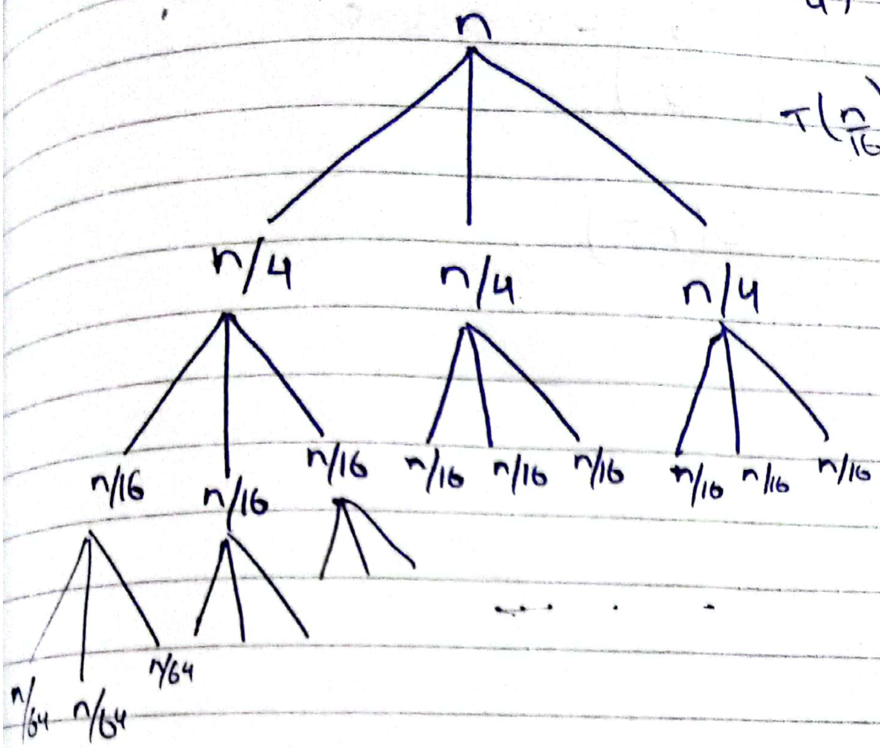


$\log_2 16$

$$T(n) = 3T\left(\frac{n}{4}\right) + cn^2$$

$$T\left(\frac{n}{4}\right) = \frac{n}{4} \times \frac{1}{4} = \frac{n}{16}$$

$$T\left(\frac{n}{16}\right) = \frac{n}{4} \times \frac{1}{16} = \frac{n}{64}$$



cn^2

$$\frac{3cn^2}{16}$$

$$\frac{9cn^2}{64}$$

$$cn^2 + \frac{3}{16}cn^2 + \frac{9}{64}cn^2 + \ldots$$

$$cn^2\left[1 + \frac{3}{16} + \left(\frac{3}{16}\right)^2 + \ldots\right]$$

$$cn^2\left[1 + r + r^2 + r^3 + \ldots\right]$$

$$\frac{1}{1-r} \qquad r < 1$$

$$cn^2\left[\frac{1}{1 - \frac{3}{16}}\right] \Rightarrow cn^2\left[\frac{1}{\frac{16-3}{16}}\right]$$

$$cn^2\left[\frac{1}{\frac{13}{16}}\right]$$

$$cn^2\left[\frac{16}{13}\right] \to \text{constant.}$$

$$O(n^2)$$

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