

①  
Show how to multiply this matrix chain optimally.

$$A_1 = 30 \times 35$$

$$A_2 = 35 \times 15$$

$$A_3 = 15 \times 5$$

$$A_4 = 5 \times 10$$

$$A_5 = 10 \times 20$$

$$A_6 = 20 \times 25$$

	1	2	m	3	4	5	6
1	0	15750	7875	9375	11875	15125	
2		0	2625	4375	7125	10500	
3			0	750	2500	5375	
4				0	1000	3500	
5					0	5000	
6							0

		S				
	0	1	1	3	3	3
		0	2	3	3	3
			0	3	3	3
				0	4	5
					0	5
						0

$$A_1 \cdot A_2 = m[1,2]$$

$$30 \times 35 \cdot 35 \times 15 = 30 \times 35 \times 15 = 15750$$

$$A_2 \cdot A_3 = m[2,3]$$

$$35 \times 15 \cdot 15 \times 5 = 35 \times 15 \times 5 = 2625$$

$$A_3 \cdot A_4 = m[3,4]$$

$$15 \times 5 \cdot 5 \times 10 = 15 \times 5 \times 10 = 750$$

$$A_4 \cdot A_5 = m[4,5]$$

$$5 \times 10 \cdot 10 \times 20 = 5 \times 10 \times 20 = 1000$$

$$A_5 \cdot A_6 = m[5,6]$$

$$10 \times 20 \cdot 20 \times 25 = 10 \times 20 \times 25 = 5000$$

$$m[i,j] = \min \{ m[i,k] + m[k+1,j] + P_{i-1} P_k P_j \}$$

j	i							
		30	35	15	5	10	20	25
		0	1	2	3	4	5	6

$$(A_1 \cdot A_2) A_3$$

$$m[i,k] + m[k+1,j] + P_{i-1} P_k P_j$$

$$15750 + 0 + 30 \times 5 \times 15$$

$$= 18000$$

$$A_1 (A_2 \cdot A_3)$$

$$m[i,k] + m[k+1,j] + P_{i-1} P_k P_j$$

$$0 + 2625 + 30 \times 35 \times 5$$

$$= 7875$$

$$(A_2 \cdot A_3) A_4$$

$$m[i,k] + m[k+1,j] + P_{i-1} P_k P_j$$

$$2625 + 0 + 35 \times 5 \times 10$$

$$= 4375$$

$$A_2 (A_3 \cdot A_4)$$

$$m[i,k] + m[k+1,j] + P_{i-1} P_k P_j$$

$$0 + 750 + 35 \times 15 \times 10$$

$$= 6000$$

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$$(A_3 \cdot A_1) A_5$$

$$m \begin{bmatrix} i & k \\ 3 & 4 \end{bmatrix} + m \begin{bmatrix} k+1 & j \\ 5 & 5 \end{bmatrix} + p_{2-1}^i p_4^k p_5^j$$

$$750 + 0 + 15 \times 10 \times 20$$

$$= 3750$$

$$A_3 (A_4 \cdot A_5)$$

$$m \begin{bmatrix} i & k \\ 3 & 3 \end{bmatrix} + m \begin{bmatrix} k+1 & j \\ 4 & 5 \end{bmatrix} + p_a^i p_3^k p_5^j$$

$$0 + 600 + 15 \times 5 \times 20$$

$$= 2500$$

$$(A_4 A_5) A_6$$

$$m \begin{bmatrix} i & k \\ 4 & 5 \end{bmatrix} + m \begin{bmatrix} k+1 & j \\ 6 & 6 \end{bmatrix} + p_{3-1}^i p_5^k p_6^j$$

$$1000 + 0 + 5 \times 20 \times 25$$

$$= 3500$$

$$A_4 (A_5 \cdot A_6)$$

$$m \begin{bmatrix} i & k \\ 4 & 4 \end{bmatrix} + m \begin{bmatrix} k+1 & j \\ 5 & 6 \end{bmatrix} + p_3^i p_4^k p_6^j$$

$$0 + 5000 + 5 \times 10 \times 25$$

$$= 6250$$

$$\left. \begin{array}{l} \rightarrow A_1 (A_2 \cdot A_3 \cdot A_4) \\ \rightarrow (A_1 A_2) (A_3 A_4) \\ \rightarrow (A_1 \cdot A_2 \cdot A_3) A_4 \end{array} \right\} 3 \text{ ways}$$

$$\rightarrow A_1 (A_2 \cdot A_3 \cdot A_4)$$

$$\therefore (A_2 \cdot A_3 \cdot A_4) = 4375$$

$$4375 + 30 \times 35 \times 10$$

$$= 14875$$

$$\rightarrow (A_1 \cdot A_2) (A_3 \cdot A_4)$$

$$15750 + 750 + 30 \times 15 \times 10$$

$$= 21000$$

$$\rightarrow (A_1 \cdot A_2 \cdot A_3) A_4$$

$$\therefore (A_1 A_2 A_3) = 7875$$

$$7875 + 30 \times 5 \times 10$$

$$= 9375$$

$$A_3 \cdot A_4$$

$$15 \times 5 \cdot 5 \times 10$$

$$15 \times 10$$

$$A_2 \cdot (15 \times 10)$$

$$35 \times 15 \cdot 15 \times 10$$

$$(35 \times 10)$$

$$A_1 \cdot (35 \times 10)$$

$$30 \times 35 \cdot 35 \times 10$$

$$\left. \begin{aligned} \rightarrow A_2(A_3 \cdot A_4 \cdot A_5) \\ \rightarrow (A_2 \cdot A_3)(A_4 \cdot A_5) \\ \rightarrow (A_2 \cdot A_3 \cdot A_4)A_5 \end{aligned} \right\} 3 \text{ ways}$$

$$\begin{aligned} \rightarrow A_2(A_3 \cdot A_4 \cdot A_5) \\ \therefore (A_3 \cdot A_4 \cdot A_5) = 2500 \\ 2500 + 35 \times 15 \times 20 \\ = 13000 \end{aligned}$$

$$\begin{aligned} \rightarrow (A_2 \cdot A_3)(A_4 \cdot A_5) \\ 2625 + 1000 + 35 \times 5 \times 20 \\ = 7125 \end{aligned}$$

$$\begin{aligned} \rightarrow (A_2 \cdot A_3 \cdot A_4)A_5 \\ \therefore (A_2 \cdot A_3 \cdot A_4) = 4375 \\ 4375 + 35 \times 10 \times 20 \\ = 11375 \end{aligned}$$

$$\rightarrow A_3(A_4 \cdot A_5 \cdot A_6)$$

$$\rightarrow (A_3 \cdot A_4)(A_5 \cdot A_6)$$

$$\rightarrow (A_3 \cdot A_4 \cdot A_5)A_6$$

$$\begin{aligned} \rightarrow A_3(A_4 \cdot A_5 \cdot A_6) \\ \therefore (A_4 \cdot A_5 \cdot A_6) = 3500 \\ 3500 + 15 \times 5 \times 25 \\ = 5375 \end{aligned}$$

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$$\rightarrow (A_3 \cdot A_4) (A_5 \cdot A_6)$$

$$750 + 5000 + 15 \times 10 \times 25 \\ = 9500$$

$$\rightarrow (A_3 \cdot A_4 \cdot A_5) A_6$$

$$\therefore (A_3 \cdot A_4 \cdot A_5) = 2500$$

$$2500 + 15 \times 20 \times 25 \\ = 10000$$

$$\begin{array}{l} \rightarrow A_1(A_2 \cdot A_3 \cdot A_4 \cdot A_5) \\ \rightarrow (A_1 \cdot A_2 \cdot A_3)(A_4 \cdot A_5) \\ \rightarrow (A_1 \cdot A_2 \cdot A_3 \cdot A_4) \cdot A_5 \\ \rightarrow (A_1 \cdot A_2)(A_3 \cdot A_4 \cdot A_5) \end{array} \left. \vphantom{\begin{array}{l} \rightarrow A_1(A_2 \cdot A_3 \cdot A_4 \cdot A_5) \\ \rightarrow (A_1 \cdot A_2 \cdot A_3)(A_4 \cdot A_5) \\ \rightarrow (A_1 \cdot A_2 \cdot A_3 \cdot A_4) \cdot A_5 \\ \rightarrow (A_1 \cdot A_2)(A_3 \cdot A_4 \cdot A_5) \end{array}} \right\} 4 \text{ ways}$$

$$\rightarrow A_1(A_2 \cdot A_3 \cdot A_4 \cdot A_5)$$

$$\therefore (A_2 \cdot A_3 \cdot A_4 \cdot A_5) = 7125$$

$$7125 + 30 \times 35 \times 20 \\ = 28125$$

$$\rightarrow (A_1 \cdot A_2 \cdot A_3)(A_4 \cdot A_5)$$

$$7875 + 1000 + 30 \times 5 \times 20$$

$$\boxed{= 11875}$$

$$\rightarrow (A_1 \cdot A_2 \cdot A_3 \cdot A_4) A_5$$

$$\therefore (A_1 \cdot A_2 \cdot A_3 \cdot A_4) = 9375$$

$$= 9375 + 30 \times 10 \times 20$$

$$= 15375$$

$$\rightarrow (A_1 \cdot A_2)(A_3 \cdot A_4 \cdot A_5)$$

$$15750 + 2500 + 30 \times 15 \times 20$$

$$= 27250$$



$$\begin{aligned}
 &\rightarrow A_2 (A_3 \cdot A_4 \cdot A_5 \cdot A_6) \\
 &\rightarrow (A_2 \cdot A_3 \cdot A_4) (A_5 \cdot A_6) \\
 &\rightarrow (A_2 \cdot A_3 \cdot A_4 \cdot A_5) A_6 \\
 &\rightarrow (A_2 \cdot A_3) (A_4 \cdot A_5 \cdot A_6) \\
 &\rightarrow A_2 (A_3 \cdot A_4 \cdot A_5 \cdot A_6)
 \end{aligned}
 \left. \vphantom{\begin{aligned} &\rightarrow A_2 (A_3 \cdot A_4 \cdot A_5 \cdot A_6) \\ &\rightarrow (A_2 \cdot A_3 \cdot A_4) (A_5 \cdot A_6) \\ &\rightarrow (A_2 \cdot A_3 \cdot A_4 \cdot A_5) A_6 \\ &\rightarrow (A_2 \cdot A_3) (A_4 \cdot A_5 \cdot A_6) \\ &\rightarrow A_2 (A_3 \cdot A_4 \cdot A_5 \cdot A_6) \end{aligned}} \right\} 4 \text{ ways}$$

$$\therefore (A_3 A_4 A_5 A_6) = 5375$$

$$5375 + 35 \times 15 \times 25$$

$$= 18500$$

$$\rightarrow (A_2 \cdot A_3 \cdot A_4) (A_5 \cdot A_6)$$

$$4375 + 5000 + 35 \times 10 \times 25$$

$$= 18125$$

$$\rightarrow (A_2 \cdot A_3 \cdot A_4 \cdot A_5) A_6$$

$$\therefore (A_2 A_3 A_4 A_5) = 7125$$

$$7125 + 35 \times 20 \times 25$$

$$= 24625$$

$$\rightarrow (A_2 \cdot A_3) (A_4 \cdot A_5 \cdot A_6)$$

$$2625 + 3500 + 35 \times 5 \times 25$$

$$= 10500$$

$$\rightarrow A_1 (A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6)$$

$$\rightarrow (A_1 \cdot A_2 \cdot A_3) (A_4 \cdot A_5 \cdot A_6)$$

$$\rightarrow (A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5) A_6$$

$$\left. \vphantom{\begin{aligned} &\rightarrow A_1 (A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6) \\ &\rightarrow (A_1 \cdot A_2 \cdot A_3) (A_4 \cdot A_5 \cdot A_6) \\ &\rightarrow (A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5) A_6 \end{aligned}} \right\} 3 \text{ ways.}$$

$$\rightarrow A_1 (A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6)$$

$$18500 + 30 \times 35 \times 25$$

$$= 44750$$

$$\rightarrow (A_1 A_2 A_3) (A_4 A_5 A_6)$$

$$7875 + 3500 + 30 \times 5 \times 25$$

$$= 15125$$

$$\rightarrow (A_1 A_2 A_3 A_4 A_5) A_6$$

$$11875 + 30 \times 20 \times 25$$

$$= 26875$$