Variable is a characteristic that vary from one element to another element. For example, Income, score, temp. etc

Types of Variable:-

There are two types of Variable

Variable
- Qualitative
- Quantitative
  - Countinuous
  - Discrete

Data:- The set of information is called data.

e.g marks (X)

(i) X = { 20, 15, 30, 40, 45 - - }

=> X is quantitative

(ii) X = {green, red, yellow}

=> X is qualitative

Data
- Quantitative
- Qualitative

Discrete or continuous	Primary data or Secondary data	time series data or cross section data or Pannal data

Primary Data:-

The information collected first time is called primary data

Secondary Data:-

The ordered primary data is called secondary data.

Time Series Data:-

Time dependent data is called time series data.

e.g weather forcast (hour, day, week month)

Cross Section Data:-

To give information at a time from different particles.

Pannal Data:-

A Pannal data consists of the qualities of time series data and cross section data.

(i): Nominal Scale:→ =, ≠
e.g math, mgt, CS, EE.

ii): ORdinal Scale:- =, ≠, <, >
e.g strong disagree < disagree < agree < strong agree

(Nominal scale and ordinal scale are qualitative.)

iii): Interval:- =, ≠, <, >, +, -
(In interval scale 0 is meaningless)
e.g Temperature

iv): Ratio Scale:- =, ≠, <, >, +, -, ×, ÷
(In Ratio Scale 0 is meaningfull)

(Interval and Ratio scales are quantitative.)

Statistics:-

Statistics is a Science of data.

Mathematical Statistics is a branch of mathematics that consists of methods and techniques used in collection, presentation, analysis and interpretation of data to make some decision.

Q.

The following data show the waiting time recorded over one-hour
2, 5, 10, 12, 4, 4, 5, 17, 11, 8, 9, 8, 12, 21, 6, 8, 7, 13, 18, 3.

a) Make frequency distribution, (b): R.f, %f,
(C): C.f + C.R.f + C.%f.

i) Decide the number of classes?
when K denotes number of classes and N is total number of values then

$$K = 1 + 3.222 \log(N)$$
$$K = 1 + 3.222 \log(20)$$
$$K = 5.19$$
$$K \approx 5$$

2): Decide width of each class.

$$h = \frac{max - min}{K}$$
$$= \frac{21 - 2}{5}$$
$$h = \frac{19}{5}$$
$$h = 3.8$$
$$h \approx 4$$

3). Decide the lower class limit of first class

Frequency Distribution Table:

classes	Tally	f	R.f	% f	C.F	C.R.f
1 – 4	IIII	4	$\frac{4}{20} = 0.2$	$\frac{4}{20} \times 100 = 20$	4	$\frac{4}{20} = .2$
5 – 8	~~IIII~~ II	7	$\frac{7}{20} = 0.35$	$\frac{7}{20} \times 100 = 35$	$4+7=11$	$\frac{11}{20} = .55$
9 – 12	~~IIII~~	5	$\frac{5}{20} = 0.25$	$\frac{5}{20} \times 100 = 25$	$11+5=16$	$\frac{16}{20} = .8$
13 – 16	I	1	$\frac{1}{20} = .05$	$\frac{1}{20} \times 100 = 5$	$16+1=17$	$\frac{17}{20} = .85$
17 – 20	II	2	$\frac{2}{20} = 0.1$	$\frac{2}{20} \times 100 = 10$	$17+2=19$	$\frac{19}{20} = .95$
21 – 24	I	1	$\frac{1}{20} = .05$	$\frac{1}{20} \times 100 = 5$	$19+1=20$	$\frac{20}{20} = 1$

R.f denotes relative frequency

% f → Percentage frequency distribution
C.F → commulative frequency dis—
C.R.f → commulative relative frequency
C.P.f → commulative Percentage frequency distribution

Consider the following data
14, 21, 23, 21, 16, 19, 22, 25, 16, 16, 24, 24, 25, 1
16, 19, 18, 19, 21, 12, 16, 17, 18, 23, 25, 20, 23,
16, 20, 19, 24, 26, 15, 22, 24, 20, 22, 24, 22, 20

(a) Develop frequency distribution.

i) Decide the number of classes

$$K = 1 + 3.222 \log(N)$$
$$K = 1 + 3.222 \log(40)$$
$$K = 6.16$$
$$K \approx 6$$

ii):

$$h = \frac{max - min}{K}$$
$$h = \frac{26 - 12}{6} = \frac{14}{6} = \frac{7}{3}$$
$$h = 2.33$$
$$h \approx 3$$

Classes	Tally	f	R.f	% f	C.F
11 – 13	I	1	$\frac{1}{40} = .025$	$\frac{1}{40} \times 100 = 2.5$	1
14 – 16	卌III	8	$\frac{8}{40} = .2$	$\frac{8}{40} \times 100 = 20$	1+8=9
17 – 19	卌III	8	$\frac{8}{40} = .2$	$\frac{8}{40} \times 100 = 20$	9+8=17
20 – 22	卌卌I	11	$\frac{11}{40} = .275$	$\frac{11}{40} \times 100 = 27.5$	17+11=28
23 – 25	卌卌I	11	$\frac{11}{40} = .275$	$\frac{11}{40} \times 100 = 27.5$	28+11=39
26 – 28	I	1	$\frac{1}{40} = .025$	$\frac{1}{40} \times 100 = 2.5$	39+1=40

Exclusive method:-

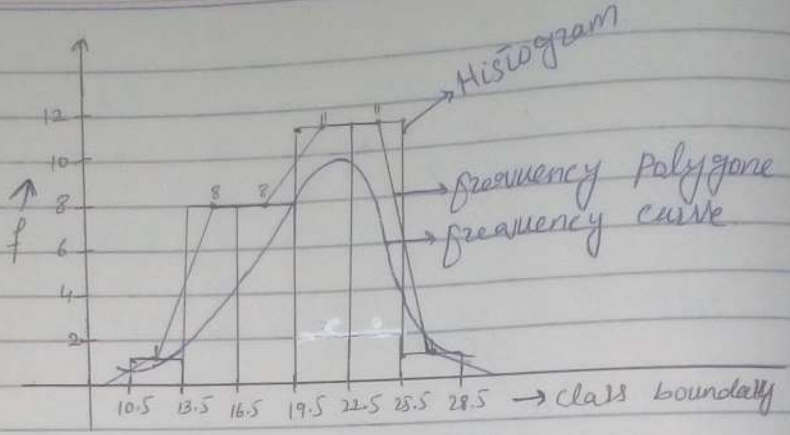
If data is continuous (data in points) then upper C.L of 1st class and lower limit of 2nd class are same.

e.g. $12 \le x < 14$

$14 \le x < 16$

$16 \le x < 18$

This method will be used if data is continuous, to draw graphical representation or histogram etc



Question. 1.6

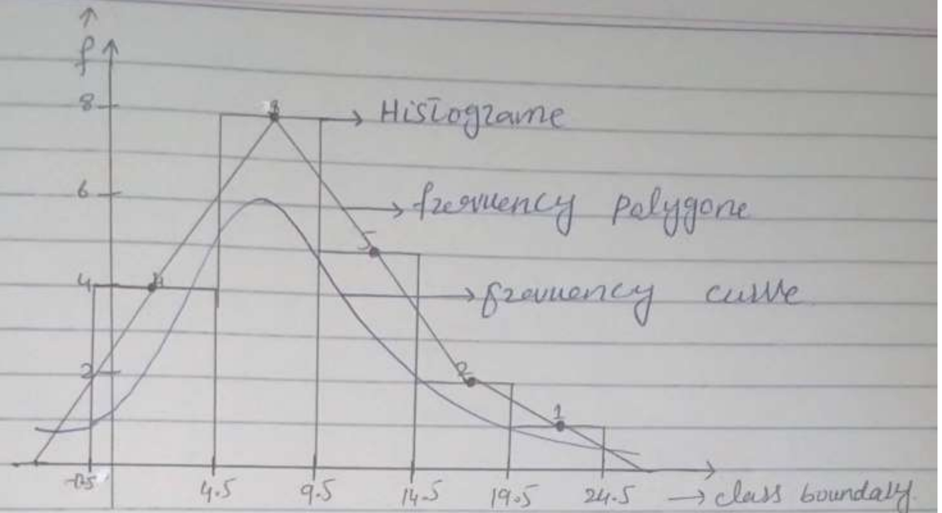
Consider the following data.

2, 5, 10, 12, 4, 4, 5, 17, 11, 8, 9, 8, 12, 21, 6, 8, 7, 13, 18, 3.

use classes of 0–4, 5–9, and so on in the following:

Classes	Tally	f (a)	R.f (b)	C.F (c)	C.R.f (d)	% f	P.c.f
0 – 4	IIII	4	$\frac{4}{20} = .2$	4	$\frac{4}{20} = .2$	20%	20
5 – 9	卌III	8	$\frac{8}{20} = .4$	4+8=12	$\frac{12}{20} = .6$	40	60
10 – 14	卌	5	$\frac{5}{20} = .25$	12+5=17	$\frac{17}{20} = .85$	25	85
15 – 19	II	2	$\frac{2}{20} = .1$	17+2=19	$\frac{19}{20} = .95$	10	95
20 – 24	I	1	$\frac{1}{20} = .05$	19+1=20	$\frac{20}{20} = 1$	5	100
		20					

e):- what proportion of Patie...



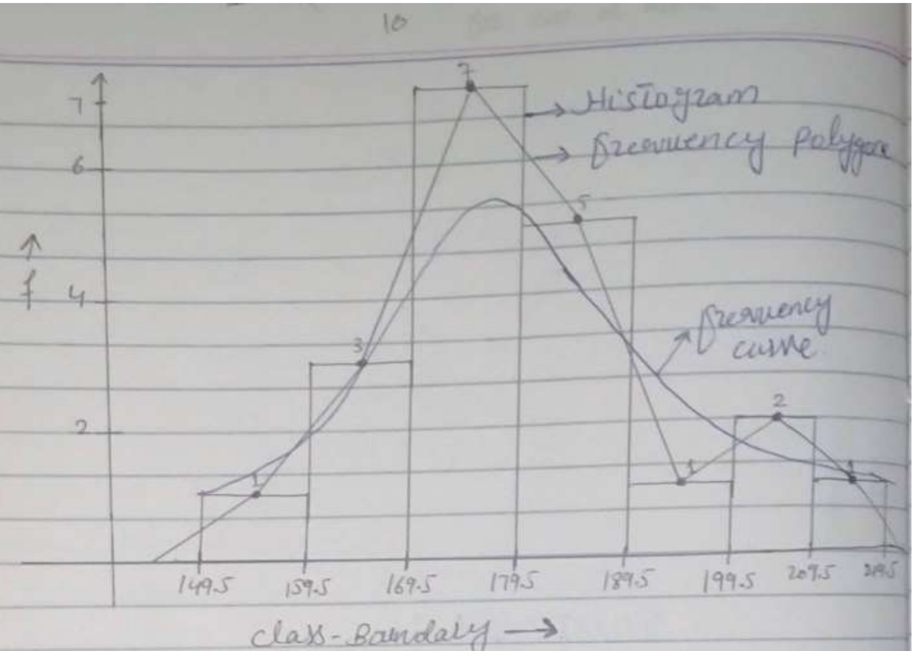
Question: 1.7.

The following data.

187, 184, 174, 185, 175, 172, 202, 197, 165, 208, 215, 164, 162, 172, 182, 156, 172, 175, 170, 183.

use classes of 150 – 159, 160 – 169 so on.

Class	Tally	f	% f	c.f	c.P.f.
150 – 159	I	1	$\frac{1}{20} \times 100 = 5$	1	$\frac{1}{20} \times 100 = 5$
160 – 169	III	3	$\frac{3}{20} \times 100 = 15$	1+3=4	$\frac{4}{20} \times 100 = 20$
170 – 179	卌II	7	$\frac{7}{20} \times 100 = 35$	4+7=11	$\frac{11}{20} \times 100 = 55$
180 – 189	卌	5	$\frac{5}{20} \times 100 = 25$	11+5=16	$\frac{16}{20} \times 100 = 80$
190 – 199	I	1	$\frac{1}{20} \times 100 = 5$	16+1=17	$\frac{17}{20} \times 100 = 85$
[illegible]	II	2	$\frac{2}{20} \times 100 = 10$	17+2=19	$\frac{19}{20} \times 100 = 95$



Question: 1.8.

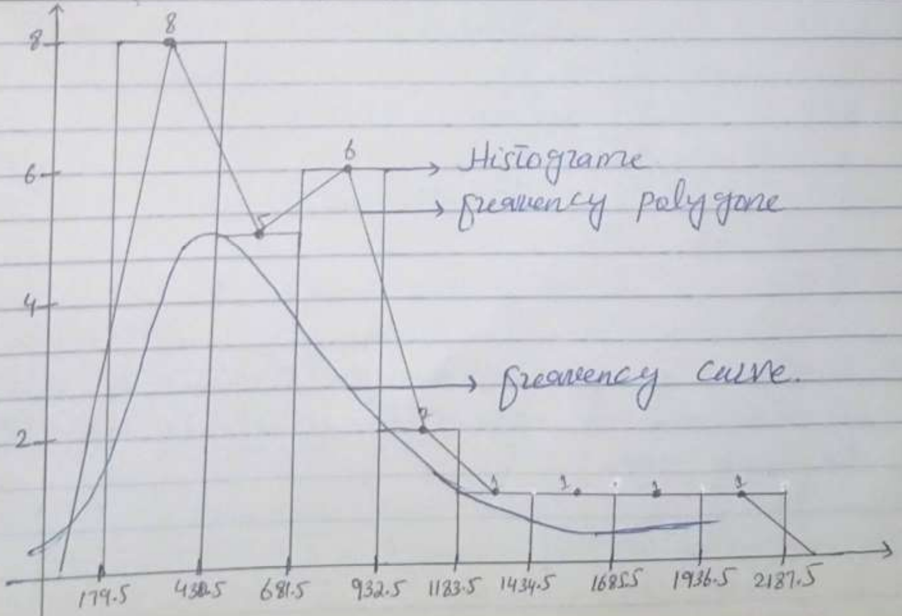
The following data.

1200, 850, 740, 590, 340, 450, 890, 260, 610, 350, 1780, 180, 850, 2050, 770, 800, 1090, 510, 520, 220, 1450, 280, 1120, 200, 350.

(a): what is the lowest holiday spending? The highest?

The lowest holiday spending is 180 and highest holiday spending is 2050.

classes	Tally	f	% f
180 – 430	~~IIII~~ III	8	$\frac{8}{25} \times 100 = 32$
431 – 681	~~IIII~~	5	$\frac{5}{25} \times 100 = 20$
682 – 932	~~IIII~~ I	6	$\frac{6}{25} \times 100 = 24$
933 – 1183	II	2	$\frac{2}{25} \times 100 = 8$
1184 – 1434	I	1	$\frac{1}{25} \times 100 = 4$
1435 – 1685	I	1	$\frac{1}{25} \times 100 = 4$
1686 – 1936	I	1	$\frac{1}{25} \times 100 = 4$
1937 – 2187	I	1	$\frac{1}{25} \times 100 = 4$
		25	



19 **Question: 1.9**

The following data:
2, 4, 8, 4, 8, 1, 2, 32, 12, 1, 5, 7, 5, 5, 3, 4, 24, 19, 4, 14.

Decide the number of classes

$K = 1 + 3.222 \log(20) = 5.19297$

$K \approx 5$

Decide the length of each interval

$h = \frac{max - min}{k} \Rightarrow \frac{32 - 1}{5} \Rightarrow 6.2$

$h \approx 6$

Classes	Tally	f	R.f	c.f	c.R.f	% f
1 – 5	~~IIII~~ ~~IIII~~ II	12	$\frac{12}{20} = 0.6$	12	0.6	$\frac{12}{20} \times 100 = 60$
6 – 10	III	3	$\frac{3}{20} = 0.15$	12+3=15	0.6+0.15=.75	15
11 – 15	II	2	$\frac{2}{20} = 0.1$	15+2=17	.75+.1=.85	0.1×100 = 10
16 – 20	I	1	$\frac{1}{20} = 0.05$	17+1=18	.85+.05=.9	5
21 – 25	I	1	$\frac{1}{20} = 0.05$	18+1=19	.9+.05=.95	5
26 – 30	–	0	$\frac{0}{20} = 0$	19+0=19	.95+0=.95	0
31 – 35	I	1	$\frac{1}{20} = 0.05$	19+1=20	.95+.05=1	5
		20				

e): Percentage of office workers for 5 minutes or less is 60% and for more than 10 minutes are 25%

f): See above page.

Question: 1.9. (Extra)

The following data.

2, 4, 8, 4, 8, 1, 2, 3, 2, 1, 2, 1, 5, 7, 5, 5, 3, 4, 24, 19, 4, 14.

Q): Decide the number of classes.

$K = 1 + 3.222 \log(N)$

$K = 1 + 3.222 \log(22)$

$K = 5.3253$

$K = 5$

Decide the length of each interval

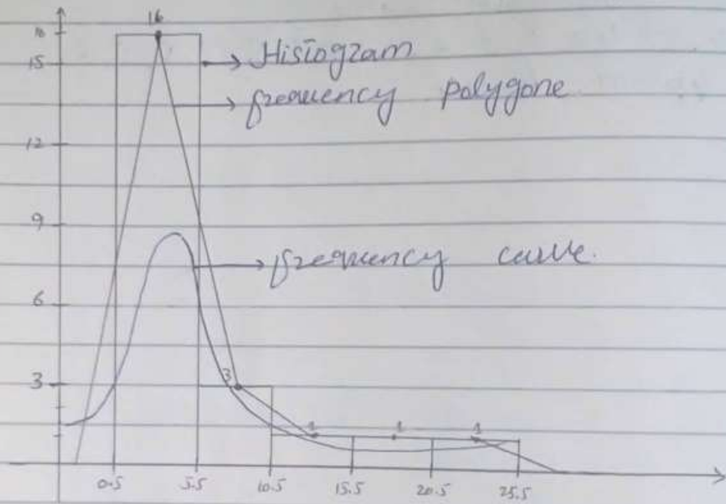
$h = \frac{max - min}{K}$

$h = \frac{24 - 1}{5} \Rightarrow \frac{23}{5}$

$h = 4.6$

$h \approx 5$

Classes	Tally	f	R.f	C.f	C.R.f	%.f
1 – 5	~~IIII~~ ~~IIII~~ ~~IIII~~ I	16	$\frac{16}{22} = .73$	16	$\frac{16}{22} = .73$	$\frac{16}{22} \times 100 = 72.7\%$
6 — 10	III	3	$\frac{3}{22} = .14$	16+3=19	$\frac{19}{22} = .86$	$\frac{3}{22} \times 100 = 13.6$
11 — 15	I	1	$\frac{1}{22} = .045$	19+1=20	$\frac{20}{22} = .91$	$\frac{1}{22} \times 100 = 4.5$
16 — 20	I	1	$\frac{1}{22} = .045$	20+1=21	$\frac{21}{22} = .95$	$\frac{1}{22} \times 100 = 4.5$
21 — 25	I	1	$\frac{1}{22} = .045$	21+1=22	$\frac{22}{22} = 1$	$\frac{1}{22} \times 100 = 4.5$
		22				



Question: 10.

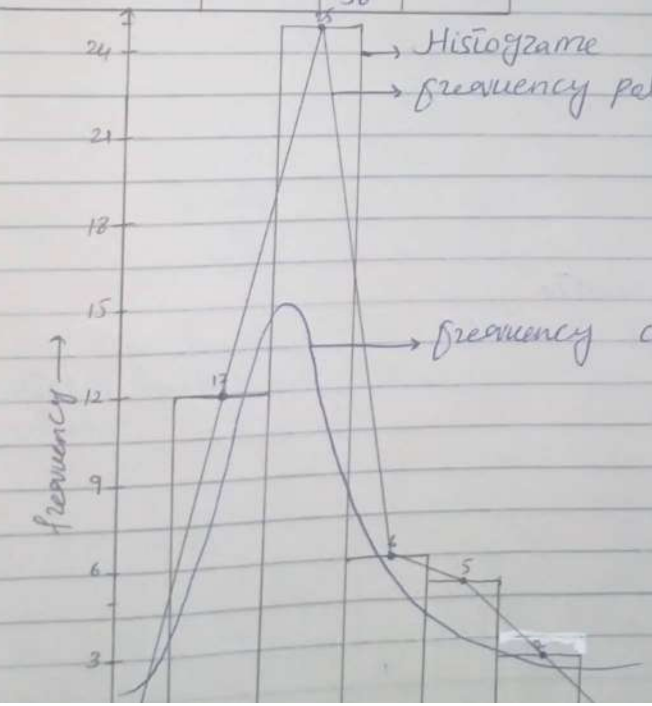
The given data.

4.1, 1.5, 10.4, 5.9, 3.4, 5.7, 1.6, 6.1, 3.0, 3.7, 3.1, 4.8, 2, 14.8, 5.4, 4.2, 3.9, 4.1, 11.1, 3.5, 4.1, 4.1, 8.8, 5.6, 4.3, 3.3, 7.1, 10.3, 6.2, 7.6, 10.8, 2.8, 9.5, 12.9, 12.1, 0.7, 4, 9.2, 4.4, 5.7, 7.2, 6.1, 5.7, 5.9, 4.7, 3.9, 3.7, 3.1, 6.1, 3.1

$N = 50.$

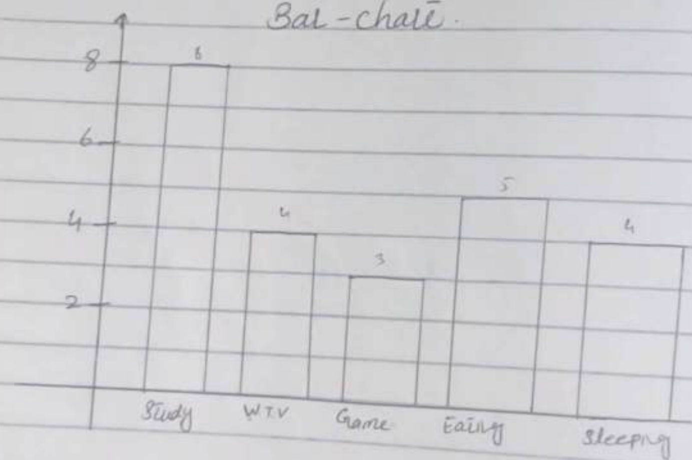
$h = 3$

classes	Tally	f	R.f
$0.7 \le x < 3.7$	~~||||~~ ~~||||~~ ||	12	$\frac{12}{50} = .24$
$3.7 \le x < 6.7$	~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~	25	$\frac{25}{50} = .5$
$6.7 \le x < 9.7$	~~||||~~ |	6	$\frac{6}{50} = .12$
$9.7 \le x < 12.7$	~~||||~~	5	$\frac{5}{50} = .1$
$12.7 \le x < 15.7$	||	2	$\frac{2}{50} = .04$
		50	



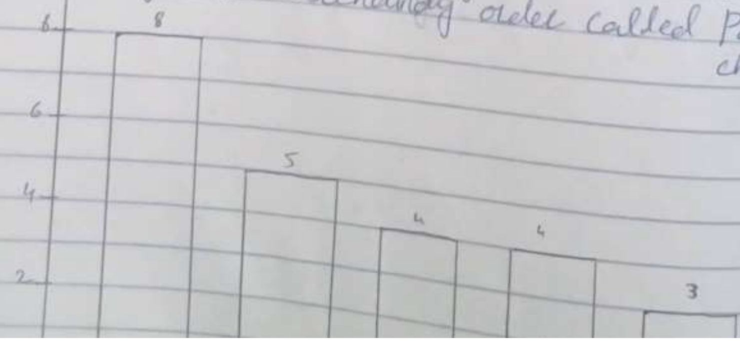
Bar chart.

categories	Values
Study	8
watching T.V	4
Games	3
Eating	5
Sleeping	4
	24



Pareto-chart:-

If Bar-chart is vertically ordered in descending or ascending order called Pareto chart.



Pie chart:-

For Study:- $\frac{8}{24} \times 360° = 120°$

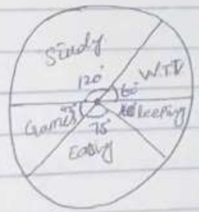
For W.T.V:- $\frac{4}{24} \times 360° = 60°$

For Games:- $\frac{3}{24} \times 360 = 45°$

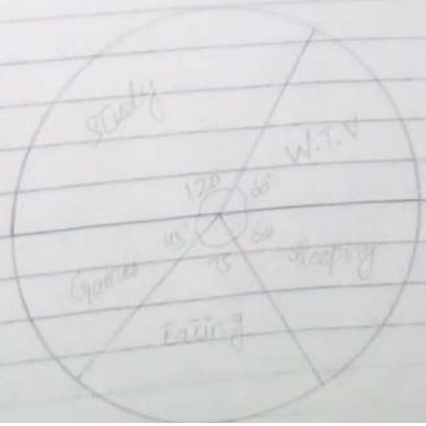
For Eating:- $\frac{5}{24} \times 360° = 75°$

For Sleeping:- $\frac{4}{24} \times 360° = 60$

Pie-chart



Pie-chart:



Stem And Leaf Method:

(Data in ascending or descending order called Array data.)

Question: 1.2

Construct a stem-and-leaf display for the following data. 11.3, 9.6, 10.4, 7.5, 8.3, 10.5, 10.0, 9.3, 8.1, 7.7, 7.5, 8.4, 6.3, 8.8.

Stem-and-leaf plot:-

Stem	Leaves
6	3
7	5, 7, 5
8	1, 3, 4, 8
9	6, 3
10	4, 5, 0
11	3

Question: 1.4

(a):- Show a stretched stem-and-leaf display

Stem unit	Leaves
2	4, 7, 6, 1
3	3, 7, 2, 0, 7, 1, 6, 5, 1, 1
4	9, 0, 4, 6, 3, 0, 6, 3, 3, 4, 3, 3, 7
5	6, 7, 5, 0, 2, 0, 2, 0, 9
6	4, 1, 6
7	2

(b): What age group had the largest number of runners?

Ans! 40—49

(c): What age occurred most frequently?

43 (3 is most repeated leave in stem 4)

(d): Percentage of 20-something is $\frac{4}{40} \times 100 = 10$

Assignment:

odd Q:S:- 1.1, 1.3, 1.5 .... 1.13
Quiz on monday.

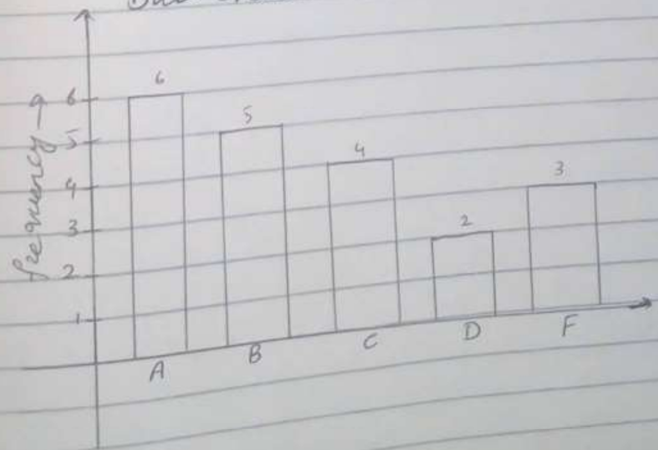
Question: 1.12

The following data give the letter grades of 20 students enrolled in a statistics course. A, B, F, A, C, C, D, A, B, F, C, D, B, A, B, A, F, B, C, A.

(a): construct a bar graph.
(b) construct a pie chart.

class	Tally	f	R.f	%f
A	卌 I	6	$\frac{6}{20} = .3$	30
B	卌	5	0.25	25
C	IIII	4	0.2	20
D	II	2	0.1	10
F	III	3	0.15	15
		20		

Bar-chart



Pie-chart:

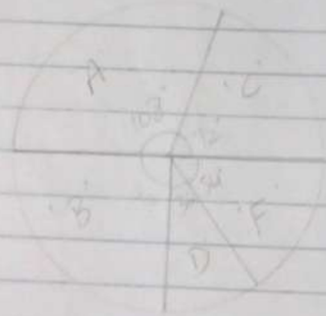
For A:- $\frac{6}{20} \times 360^\circ = 108^\circ$

For B:- $\frac{5}{20} \times 360^\circ = 90^\circ$

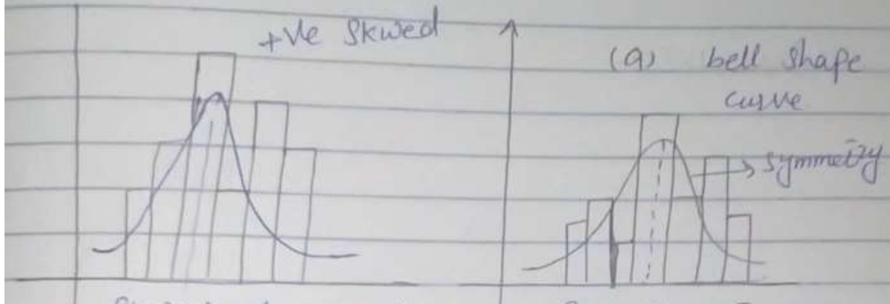
For C:- $\frac{4}{20} \times 360^\circ = 72^\circ$

For D:- $\frac{2}{20} \times 360^\circ = 36^\circ$

For F:- $\frac{3}{20} \times 360^\circ = 54^\circ$

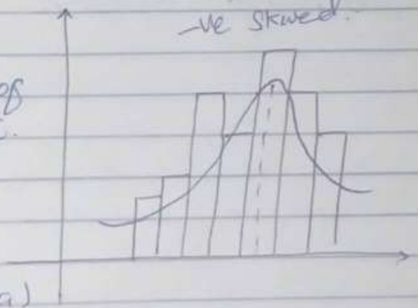


Graphical Presentation:



Skwnes: Any departure from symmetry

Symmetry:-
(a) Same data lies in the sides of the maximum point.



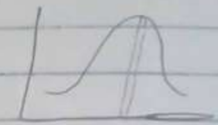
-Ve Skwed:-
(b) left side (data) is more than right side of maximum point

(b) +Ve Skewed:-
Right side (data) is more than left side of maximum pt.

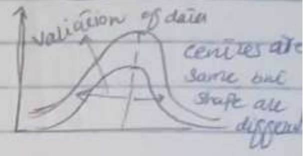
Presentation of Data: (merits): (i) It is convenient in comparison b/w two or more data sets. from book
(ii) Provide base for other statistical measure
(iii) Help in model selection
(iv) condensation of data.

Two subject. Comparison is impossible without graph.

Demerits:-
(i) find central position of data



(ii) Variation of data



(iii) shape of distribution

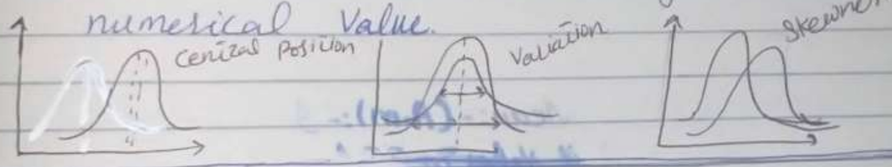
Although Graphical presentation condense the raw data in meaningfull form. But it unable to describe three major properties of data in precise form which provide base for statistical analysis. The three properties are
(i): central position within set of data around which other values of data set show a tendency to cluster or group. called central tendency.

The method of computing the central tendency / location is called measure of central tendency.

(ii): The extent to which numerical values are despersed / scatter around the central value

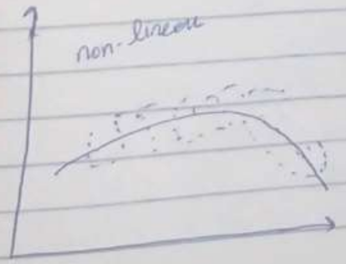
The method used to measure the Variation / dispersion / scatterness are called measure of dispersion & variation.

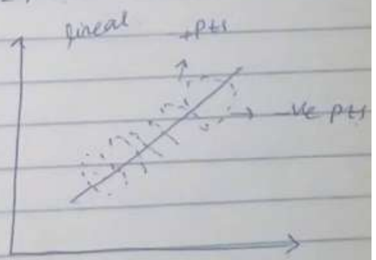
iii): The departure of data from Symmetry, called Skewness measure of Skewness consist of different methods used to summarize / describe skewness in single numerical Value.



$$qd = a - bP$$

$$= a - b_1 P_1 + b_2 P^2$$





Sum of +ve & -ve are zero.

Methods of measure of central Tendency

(i): Arithematic mean

ii): Geometric mean

iii): Harmonic mean

iv): Median

v): Mode

Grouped Data . ungrouped data
eg frequency data . eg raw data

Arithematic Mean:- (A.m):- It is defined as the ratio of sum of all Values to total number of Values, i.e

$$\text{Population mean} = \mu = \frac{\text{Sum of all Values}}{\text{Total no. of Values}}$$

$$\mu = \frac{X_1 + X_2 + \cdots + X_N}{N}$$

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

which represent the A.m of X_i.

$$\text{Sample mean} = \bar{x} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

$$= \frac{\sum_{i=1}^{n} X_i}{n}$$

Geometric Mean (G.m):- It is defined as antilog of Arithematic mean of $\log x_i$

$$G.m = \text{Antilog}\left[\frac{1}{n}\sum_{i=1}^{n} \log x_i\right]$$

Harmonic Mean:- It is reciprocal of Arithematic mean of $1/x_i$ i.e $H.m = \frac{n}{\ldots}$

Measures of central tendency is a numerical value which describe the centre of distribution.

Prof. Bowley defined the measure of central tendency as a statistical constant which enables us to comprehend in a single effort the significance of whole (data)

Objective:-

(i): To summarize the characteristic of whole data set in a precise form (single value)
e.g CGPA = 3.5 which represents the marks of (CGPA) of 7-subjects with only one value

(ii): To make comparison between two or more data sets e.g CGPA of S=3.5
A=3.0

(iii): It offers a base for other statistical measures

Criteria for good measure of central tendency:-

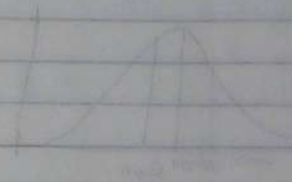
(i): It should be easy to understand

(ii): It should based on all observations.

(iii): It should have sampling stability

(iv): It should be capable of further algebraic treatment.

(v): Dont effected by extreme values. Pop 2, 4, 6



$N = 3, \quad n = 2$

2, 4	3
2, 6	4
4, 6	5

$\left.\begin{matrix}3\\4\\5\end{matrix}\right\}$ → almost equal

sampling stability

Un-grouped data.

Example

The following data show marks of students

5, 3, 8, 5, 7

Find A.m, G.m, Harmonic mean, median and mode?

(i) $$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{5+3+8+5+7}{5} = 5.6$$

(ii) $$G.m = [x_1 . x_2 . x_3 \ldots x_n]^{1/n}$$

$$\log G.m = \frac{1}{n} \sum_{i=1}^{n} \log x_i$$

$$G.m = \text{Antilog}\left[\frac{1}{n} \sum_{i=1}^{n} \log x_i\right]$$

Now

$$G.m = [5 \times 3 \times 8 \times 5 \times 7]^{1/5} = 5.30457$$

$\sum_{i=1} \left(\frac{1}{x_i}\right)$

$$H.m. = \frac{5}{\frac{1}{5}+\frac{1}{3}+\frac{1}{8}+\frac{1}{5}+\frac{1}{7}}$$

(iv):

$$\text{median} = \begin{cases} \left(\frac{n+1}{2}\right)^{th} \text{ Value in array data if } n \text{ is odd} \\ \frac{1}{2}\left(\frac{n}{2}^{th} + \left(\frac{n}{2}+1\right)^{th}\right) \text{ Value in array data if } n \text{ is even} \end{cases}$$

e.g (i) array data
3, 5, 5, 7, 8

median $\left(\frac{5+1}{2}\right) = 3rd$ Value $= 5$

(ii) 3, 5, 5, 7, 8, 10

$$\frac{1}{2}\left[\frac{6}{2} + \left(\frac{6}{2}+1\right)\right]$$

$$= \frac{1}{2}\left[3rd + 4th \text{ Value}\right]$$

$$= \frac{1}{2}\left[5+7\right] = 6$$

mode = most repeated value in data

$\hat{x} = 5$

Q. Consider the following data
4, 1, 3, 1, 6, 9, 2, 5, 6.
Find Arithematic mean, Geometric mean, Harmonic mean, median and mode

$n = 9$

$$A.m = \frac{4+1+3+1+6+9+2+5+6}{9}$$

$$= \frac{37}{9}$$

$$\boxed{A.m = 4.1111}$$

$$G.m = (4\times1\times3\times1\times6\times9\times2\times5\times6)^{1/9}$$

$$\boxed{G.m = 3.2357}$$

$$H.m = \frac{n}{\sum_{i=1} \left(\frac{1}{x_i}\right)}$$

$$H.m = \frac{9}{\frac{1}{4}+\frac{1}{1}+\frac{1}{3}+\frac{1}{1}+\frac{1}{6}+\frac{1}{9}+\frac{1}{2}+\frac{1}{5}+\frac{1}{6}}$$

$$= \frac{9}{.25+1+.33+1+.167+.11+.5+.2+.17}$$

$$= \frac{9}{3.73}$$

$H.m = 2.4129$

Array data

$\text{median} = \left(\frac{n+1}{2}\right)th \Rightarrow \left(\frac{9+1}{2}\right)th \Rightarrow 5th$

median = (4)

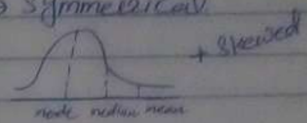
$\boxed{\text{median} = 4}$

$\boxed{\text{mode} = 6,1}$

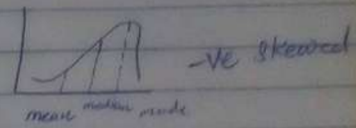
Relation b/w A.m, G.m and H.m

(1) A.m > G.m > H.m

ii) A.m = median = mode → Symmetrical



iii) A.m > median > mode
+ Skewed



iv) A.m < median < mode
-ve Skewed

Exercise:-

Question: 1

Find the mean, median, mode and range of each set of numbers below.

(a): 3, 4, 7, 3, 5, 2, 6, 10.

$n = 8$

$$A.m = \frac{3+4+7+3+5+2+6+10}{8}$$

$$= \frac{40}{8}$$

$\boxed{A.m = 5}$

$$G.m = \left[(3\times4\times7\times3\times5\times2\times6\times10)\right]^{1/8}$$

$\boxed{G.m = 4.490623}$

$$H.m = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)}$$

$$H.m = \frac{8}{\frac{1}{3}+\frac{1}{4}+\frac{1}{7}+\frac{1}{3}+\frac{1}{5}+\frac{1}{2}+\frac{1}{6}+\frac{1}{10}}$$

$$= \frac{8}{.33+.25+.14+.33+.2+.5+.17+.1}$$

$$= \frac{8}{2.02}$$

$\boxed{H.m = 3.96}$

Array data

2, 3, 3, 4, 5, 6, 7, 10

$$\text{median} = \frac{1}{2}(4+5) = \frac{9}{2} = 4.5$$

$\boxed{\text{median} = 4.5}$

$\boxed{\text{mode} = 3}$

Question: 2

Twenty students were asked their shoe sizes. The results are given below.
8, 6, 7, 6, 5, 4.5, 7.5, 6.5, 8.5, 10, 7, 5, 5.5, 8, 9, 7, 5, 6, 8.5, 6.
Find

(a) mean?

$n = 20$

$$A.m = \frac{8+6+7+6+5+4.5+7.5+6.5+8.5+10+7+5+5.5+8+9+7+5+6+8.5+6}{20}$$

$$= \frac{136}{20}$$

$\boxed{A.m = 6.8}$

$$G.m = \text{antilog}\left[\frac{1}{n}\sum_{i=1}^{n} \log x\right]$$

$$= \text{antilog}\left[\frac{1}{20}\log(8\times6\times7\times6\times\cdots\times8.5\times6)\right]$$

$$= \text{antilog}\left[0.82228\right]$$

$\boxed{G.m = 6.64171}$

(b): Median = ?

when data is arranged.
4.5, 5, 5, 5, 5.5, 6, 6, 6, 6, 6.5, 7, 7, 7, 7.5, 8, 8, 8.5, 8.5, 9, 10

$$\text{median} = \frac{1}{2}\left\{\frac{n}{2} + \left(\frac{n}{2}+1\right)\right\}th$$

$$= \frac{1}{2}\left\{\frac{20}{2} + \left(\frac{20}{2}+1\right)\right\}th$$

$$= \frac{1}{2}\left\{10 + (10+1)\right\}th$$

$$= \frac{1}{2}\left\{10th + 11th\right\}$$

$$= \frac{1}{2}\left\{6.5 + 7\right\}$$

$$= \frac{13.5}{2}$$

$\boxed{\text{median} = 6.75}$

$\boxed{\text{mode} = 6}$

Range = max − min = 10 − 4.5 => 5.5 Ans!

Question: 8

A gas station owner records the no. of cars which visit his premises on 10 days. The numbers are
204, 310, 279, 314, 257, 302, 232, 261, 308, 217.

(a): Find mean number of cars per day?

$n = 10$

$$A.m = \frac{204+310+279+314+257+302+232+261+308+217}{10}$$

$$A.m = \frac{2684}{10}$$

$$\boxed{A.m = 268.4}$$

(b): The owner hopes that mean will increase if he includes the no. of cars on the next day. If 252 cars use the gas station on next day, will the mean increase or decrease?

Now data is with 252 cars i.e so

$$A.m = \frac{2684+252}{11}$$

$$\boxed{A.m = 266.9}$$

Hence when number of cars increases on gas station then mean will ~~in~~ also decrease. (mean except 252 is 268.4 < 243.6)

Question: 9

The students in a class state how many children there are in their family. The no.s they state are given below:

1, 2, 1, 3, 2, 1, 2, 4, 2, 2, 1, 3, 1, 2, 2, 2, 1, 1, 7, 3, 1, 2, 1, 2, 2, 1, 2, 3.

(a): Find mean, median and mode for this data.

$n = 28$

$$A.m = \frac{1+2+1+3+2+1+2+4+2+2+1+3+1+2+2+2+1+1+7+3+1+2+1+2+2+1+2+3}{28}$$

$$A.m = \frac{57}{28}$$

$$\boxed{A.m = 2.0357}$$

$$G.m = \text{Antilog}\left[\frac{1}{n} \log(9289728)\right]$$

$$= \text{Antilog}\left[\frac{1}{28}(6.968003)\right]$$

$$G.m = \text{Antilog}(0.248857)$$

$$\boxed{G.m = 1.773605}$$

$n = 28$ even

$$\text{median} = \frac{1}{2}\left\{\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right\}$$

$$= \frac{1}{2}\left\{\frac{28}{2}\text{th} + (28+1)\text{th}\right\}$$

$$= \frac{1}{2}\left\{14\text{th} + 15\text{th}\right\}\text{th}$$

$$= \frac{1}{2}(2+2) = \frac{4}{2}$$

$$\boxed{\text{median} = 2}$$

mode = most repeated value

$$\boxed{\text{mode} = 2}$$

(b): What is most sensible average to use in this case?

The most sensible average is A.m = 2.0357 b/c it based on all observations

Data	frequency (or time)	Average
1	10	35.7143
2	12	42.8571
3	4	14.2857
4	1	$\frac{1}{28} \times 100 = 3.5714$
7	1	3.5714

~~The most sensible average is 3.5714 and is of 4 and 7.~~

Question: 10.

In a beauty contest the scores awarded by eight judges were:
5.9, 6.7, 6.8, 6.5, 6.7, 8.2, 6.1, 6.3

(i). Using the eight scores, determine

(a). mean?

$$A.m = \frac{5.9+6.7+6.8+6.5+6.7+8.2+6.1+6.3}{8}$$

$$= \frac{53.2}{8}$$

$$\boxed{A.m = 6.65}$$

$$G.m = (5.9 \times 6.7 \times 6.8 \times 6.5 \times 6.7 \times 8.2 \times 6.1 \times 6.3)^{1/8}$$

$$= (3688995.481)^{1/8}$$

$$\boxed{G.m = 6.62008}$$

Arranged data

5.9, 6.1, 6.3, 6.5, 6.7, 6.7, 6.8, 8.2

$$\text{median} = \frac{1}{2}\left(\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right) \text{term}$$

$$= \frac{1}{2}\left(\frac{8}{2}\text{th} + (4+1)\text{th}\right)$$

$$= \frac{1}{2}(4\text{th} + 5\text{th})$$

$$= \frac{1}{2}(6.5+6.7)$$

$$\boxed{\text{median} = 6.6}$$

$$\boxed{\text{mode} = 6.7}$$

(ii) Only six scores are to be used. Which two scores may be omitted to leave the value of the median the same?

Arrange data is

5.9, 6.1, 6.3, 6.5, 6.7, 6.7, 6.8, 8.2

The median remain same (as of 6.6) if we omitted two values that is 5.9 and 8.2 (i.e minimum and most maximum scores).

Measures of central tendency.

Notations:-

A.m : ($\bar{x}$, μ)

For sample mean $= \bar{x}$, Population mean $= \mu$

G.m : G.m

H.m : H.m

median : $\tilde{x}$

mode : $\hat{x}$

Merits & Demerits:-

Measures

A.m		G.m		H.m		Median		Mode	
merits	Demerits	M	demerits	m	de.	m	de	m	de.
1	3	2	1	2	1	1	2	1	2
2		4	3	3		3	4	3	4
4		5		4			5		5
5		6		5			6		6
6				6					
	no.s of good measures of central tendency								

- A.m is always good for sampling stability.
- median and mode does not use for Inferential statistic.

weighted mean $= \dfrac{\Sigma w_i x_i}{\Sigma w_i}$

x	w
500 →	10
800	10
800	15
2000	25
450	15
	$\Sigma w_i = 75$

$= \dfrac{\Sigma 10(500)}{75}$

A.Vange $= \dfrac{500+800+800+2000+450}{5}$

e.g.1

when customer ~~by~~ buy any object

Average cost

$\bar{x}_c = 150$

$\bar{x}_D = 100$

$\bar{x}_D = 100$ is better

e.g2 player : A B

marks { μ_A 150 μ_B 100

μ_A = 150 is better.

	Prefer
1): Extreme Value	Median & mode
2): Skewness	Median & mode
3): Measurement Scale	Median & mode (Qualitative (Nominal & ordinal
4): Inference	

Pop.
$$\text{Sample} \leftarrow \bar{X} = \frac{\Sigma X}{n} = \boxed{\text{numerical Value.}}$$

Statistic — estimator — estimate

Properties of Arithematic Mean:-

(i): Sum of deviations of observation from Arithematic mean is zero.

i.e $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

$\Rightarrow \sum_{i=1}^{n} x_i - n\bar{x} = 0$

$\Rightarrow \sum_{i=1}^{n} x_i - \cancel{n}\frac{\sum_{i=1}^{n} x_i}{\cancel{n}} = 0$

$\Rightarrow \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i = 0$

(ii): $\sum_{i=1}^{n}(x_i - \bar{x})^2 < \sum_{i=1}^{n}(x_i - a)^2$
where $\bar{x} \neq a$, and a is constant.

(iii): Let $\bar{x}_1$ is A.m of x_1 with n_1 Values and $\bar{x}_2$ is A.m of x_2 with n_2 Values. combine A.m for x_1 and x_2 is.

$$\bar{X}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$$

$$\bar{X}_c = \frac{\sum_{i=1}^{k} n_i\bar{x}_i}{\sum_{i=1}^{k} n} \rightarrow \text{will confirm}$$

(4): $\bar{X} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$ Given, $n \rightarrow$ Given

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i + K}{n}$$

($\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$

correct $x_2 = 60$, incorrect $x_2 = 76$

then

$\Sigma x = (\Sigma x_i + \text{correct} - \text{incorrect})$

Question: 4

Two people work in a factory making parts for cars. The table shows how many complete parts they make in one week.

worker	Mon	Tue	wed	Thu	Fri
Randy	20	21	22	20	21
John	30	15	12	36	28

(a): Find mean & Range for Randy & John?

$$A.m(\text{Randy}) = \frac{20+21+22+20+21}{5}$$

$$\boxed{A.m(\text{Randy}) = \overset{20.8}{\cancel{34.67}}}$$

Range (Randy) $= 22 - 20 = 2$

$$A.m(\text{John}) = \frac{30+15+12+36+28}{5}$$

$$\boxed{A.m(\text{John}) = 24.2}$$

Range (John) $= 36 - 12 = 24$

(b): who is more consistant?

work of ~~Randy~~ John is more consistant because average (A.m) of ~~Randy~~ John is greater than ~~John~~ Randay, implies ~~Randy~~ John make most parts of cars in one week.

(C): who makes the most parts in a week?

John ~~Randy~~ makes most parts in a week.

Question: 5.

A gardener buys 10 packets of seeds from two different companies. Each Pack contains 20 seeds and he records the number of plants which grow from each pack.

(a): Find mean, median and mode for each company's seeds.

Company A	20	5	20	20	20	6	20	20	20	8
Company B	17	18	15	16	18	18	17	15	17	18

$$A.m(A) = \frac{20+5+20+20+20+6+20+20+20+8}{10}$$

$$\boxed{A.m(A) = 15.9}$$

$$A.m(B) = \frac{17+18+15+16+18+18+17+15+17+18}{10}$$

$$\boxed{A.m(B) = 16.9}$$

For company A:- $n = 10$

Array data: 5, 6, 8, 20, 20, 20, 20, 20, 20, 20

(A). $\text{median} = \frac{1}{2}\left\{\frac{n}{2} + \left(\frac{n}{2}+1\right)\right\}th$

$= \frac{1}{2}\left\{\frac{10}{2} + \left(\frac{10}{2}+1\right)\right\}th \Rightarrow \frac{1}{2}\left\{5 + 6\right\}th$

$= \frac{1}{2}\left\{20 + 20\right\}$

$\boxed{\text{median} = 20}$

mode = most repeated value.

$\boxed{\text{mode} = 20}$

company (B). $n = 10$

Array data: 15, 15, 16, 17, 17, 17, 18, 18, 18, 18

$\text{median}(B) = \frac{1}{2}\left\{\frac{n}{2} + \left(\frac{n}{2}+1\right)\right\}th$ term

$= \frac{1}{2}\left\{17 + 17\right\}$

$\boxed{\text{median}(B) = 17}$

$\boxed{\text{mode}(B) = 18}$

(b): Which company does the mode suggest is best?

Company 'A' does the mode suggest is best. (b/c $\text{mod}(A) = 20 > \text{mod}(B) = 18$).

(C): which company does the mean suggest is best?
Company (B) (b/c A.m(B) = 16.9 > A.m(A) = 15.9)
(d): Find Range for each company's seeds?
Range(A) = 20 − 5 => 15
Range (B) = 18 − 15 => 3

Question: 6.

Lionel takes four tests & scores following marks.
65, 72, 58, 77.
(a): what are median and mean score?

$A.m = \frac{65+72+58+77}{4}$, $n=4$

$\boxed{A.m = 68}$

Array data: 58, 65, 72, 77
As $n=4$ → even

$median = \frac{1}{2}\{\frac{n}{2} + (\frac{n}{2}+1)\}th$

$= \frac{1}{2}(2+3)th$ term

$= \frac{1}{2}(65+72)$

$\boxed{median = 67}$

(b): If he scores 70 in his next test, does his mean score increase or decrease? Find his new mean score?
New data will be
65, 72, 58, 77, 70

$A.m = \frac{65+72+58+77+70}{5}$

$\boxed{A.m = 68.4}$ implies his ~~bis~~ mean score increase.

New data in array form
58, 65, 70, 72, 77 , $n=5$

$median = (\frac{n+1}{2})th$

$= (\frac{5+1}{2})th => (\frac{6}{2})th => 3^{rd}$ term

$\boxed{median = 70}$

Hence his median score increased most. b/c
(70 − 67 = 3 > 68.4 − 68 = 0.4)

Question: 15

Students in Grade 8 are arranged in eleven classes.
The class sizes are 23, 24, 24, 26, 27, 28, 30, 24, 29, 24, 27
(a): what is the model class size?

class size	Tally	f
23	I	1
24	IIII	4
26	I	1
27	II	2
28	I	1
29	I	1
30	I	1

Model class size is 24 (b/c its 'f' is most)
(b): calculate mean class size?

$A.m = \frac{23+24+26+27+28+29+30+24+24+24+27}{11}$

$\boxed{A.m = 26}$

(c): Range (Grade 9) = 3
Range (Grade 8) = 30 − 23 => 7
The class size for Grade 9 is better b/c his range is less than grade 8.

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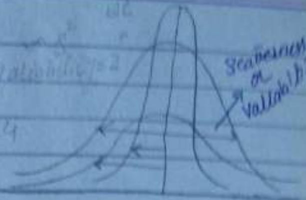
Measures of Dispersion:-



A : 4 6 8 Variability = 2

B : 2 6 10 V = 4

$\bar{X}_A = 6$ but max dif – (Range) (8-4) = 4

$\bar{X}_B = 6$ => 10 − 2 => 8.

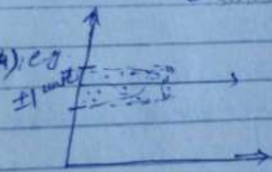
Objectives:-

1): Test the reliability of estimates (averages)
2): Compare two or more data set with respect to variability / dispersion or scatteredness.
3): Facilitates the use of other statistical techniques
4): Control the variability.

3) $Z = \frac{X - \mu}{\sigma}$ σ = standard of deviation

1): $X \rightarrow \mu$ (Pop—) fixed
$\rightarrow \bar{x}$ (Sample) Variable

4) e.g. ±1 unit

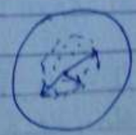


Sampling distribution of mean: $\bar{x}$, x_1, x_2, ⋮ | f | $\bar{x}_1$, $\bar{x}_2$, ⋮

other wise should control

i)

d_1
d_2
⋮



calculate mean then

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Criteria for good measures of Dispersion.

Same as measure of central tendency

class	units	
A-	kg	5, 6, 8
B-	meter	0.5, 1, 1.2
C-	liter	1, 2.1, 2.5

Measure of Dispersion

- Absolute Measure — Relative Measure
  Data have same (unit) — unit free (or have different unit)
- Measure of central tendency include only absolute measure.

Measure of Dispersion:-

1) $$\text{Range} = \begin{cases} X_{max} - X_{min} & \text{(Absolute)} \\ \frac{X_{max} - X_{min}}{X_{max} + X_{min}} & \text{(Relative)} \end{cases}$$

"Co-efficient of Range" implies Relative measure (unite free) (when co-efficient word is used implies be unit free)

mean absolute deviation

2): Mean Deviation: M.D is the A.m of absolute deviations of observations from mean / median

M.D from mean $= \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$ → better (M.A.D)

OR

M.D from median $= \frac{\sum_{i=1}^{n} |x_i - \text{median}(\tilde{x})|}{n}$

e.g. A = 4, 6, 8

Range = 8 − 4 = 4

co-efficient M.D $= \frac{M.D}{\bar{x} \text{ or } \tilde{x}}$

$x_i - \bar{x}$

$d_1 = 4 - 6 = -2$ (can solved

$d_2 = 8 - 6 = 2$

$d_3 = 6-6=0$, $\bar{X}_A = 6$

3). Variance:

(Sample) $S^2 = \frac{\sum (X-\bar{x})^2}{n}$ biased

Population Variance $= \delta^2 = \frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$

Sample Variance $= S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$ unbiased

co-efficient of Varience (C.V) $= \frac{S.D}{mean} \times 100$

$\bar{X} = \frac{\sum X}{n} = \square$

Estimator

Propert

i) unbiased (ii) Efficiency (iii) sufficiency

iv) consistency.

4): Standard deviation:

the square root of Variance.

$$\delta = \sqrt{\delta^2} = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}}$$

For Quiz Preparation.

Q: 1.1, 1.2, 1.3, 1.6, 1.7, 1.9, 1.12

Find A.m, G.m, H.m, median, mod., range, m.D, Variance, S.D, co-efficient of range, m.D, co-efficient of Variation

Example:-

Police records show the following number of daily crime reported for sample number of days during winter months 18, 20, 15, 16, 21, 20, 12, 16, 19, 20

Find

(1): Range

(2): Mean deviation from mean

(3): Variance

(4): Standard deviation.

Sol:-

Range $= X_{max} - X_{min} = 21-12 = 9$

i) $\bar{X} = \frac{\sum X}{n} = \frac{18+20+15+ \cdots +19+20}{10} = 17.7$

ii) $m.D = \frac{|18-17.7|+|20-17.7|+|15-17.7|+|16-17.7|+|21-17.7|+|20-17.7|+|12-17.7|+|16-17.7|+|19-17.7|+|20-17.7|}{10}$

$\boxed{m.D = 2.36}$

iii) $S^2 = \frac{(18-17.7)^2+(20-17.7)^2+(15-17.7)^2+(16-17.7)^2+(21-17.7)^2+(20-17.7)^2+(12-17.7)^2+(16-17.7)^2+(19-17.7)^2+(20-17.7)^2}{10}$

$S^2 = \frac{(0.3)^2+(2.3)^2+(-2.7)^2+(-1.7)^2+(3.3)^2+(2.3)^2+(-5.7)^2+(-1.7)^2+(1.3)^2 \ldots}{10}$

$\boxed{S^2 = 7.41}$

(iv): $S.D = \sqrt{S^2}$

$S.D = \sqrt{7.41}$

$\boxed{S.D = 2.72213}$

Question:

The following data, 7, 4, 9, 7, 3, 12
(a) compute mean median and mode
(b). Compute Range, Variance, Standard deviation, and co-efficient of Variation.

(a) compute mean for your self

median:-

Array data: 3, 4, 7, 7, 9, 12

$n = 6$

$$\text{median} = \frac{1}{2}\left\{\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right\} \text{term}$$

$$= \frac{1}{2}\{\text{3rd } \text{4th}\} \text{ term}$$

$$= \frac{1}{2}(7+7)$$

$\boxed{\text{median} = 7}$

mode ↔ most repeated value

$\boxed{\text{Mode} = 7}$

(b): Range $= X_{max} - X_{min}$

$= 12 - 3$

$\boxed{\text{Range} = 9}$

$$\text{Variance} = S^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\bar{x} = \frac{7+4+9+7+3+12}{6}$$

$\boxed{\bar{x} = 7}$

$$S^2 = \frac{(7-7)^2+(4-7)^2+(9-7)^2+(3-7)^2+(12-7)^2+(7-7)^2}{6}$$

$$S^2 = \frac{0+9+4+16+25}{6} \Rightarrow$$

$\boxed{S^2 = 9}$

$$S.D = \sqrt{S^2}$$

$$= \sqrt{9}$$

$\boxed{S.D = 3}$

$$\text{co-efficient of variation} = \frac{S.D}{\text{mean}} \times 100$$

$$= \frac{3}{7} \times 100$$

$= 42.86\,\%$ Ans:

Question:

The following data.

7, −5, −8, 7, 9 , compute
(a) mean, median, mode?
(b) Range, Variance, S.D & co-efficient of Variation?

$n = 5$

(a) $$\text{Mean} = \bar{x} = \frac{7-5-8+7+9}{5}$$

$\boxed{\bar{x} = 2}$

median:- Array data

−8, −5, 7, 7, 9

$$\text{median} = \frac{(n+1)}{2}\text{th} \Rightarrow \left(\frac{5+1}{2}\right)\text{th} \Rightarrow \left(\frac{6}{2}\right)\text{th}$$

= 3rd term

51

$\boxed{median = 7}$

$\boxed{mode = 7}$

(b): Range $= 9-(-8)$
$= 17$

Variance $= S^2 = \frac{\Sigma(x_i-\bar{x})^2}{n}$

$$S^2 = \frac{(7-2)^2+(-5-2)^2+(-8-2)^2+(7-2)^2+(9-2)^2}{5}$$

$$S^2 = \frac{25+49+100+25+49}{5} \quad \because S^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1}$$

$\boxed{S^2 = 49.6}$ $\Rightarrow \boxed{S^2 = 62}$

$S.D = \sqrt{S^2} \Rightarrow \sqrt{49.6}$

$\boxed{S.D = 7.0427}$

$C.V = \frac{S.D}{mean} \times 100$

$= \frac{7.0427}{2} \times 100$

$\boxed{C.V = 352.136}$

Question

Data: 11.3, 9.6, 10.4, 7.5, 8.3, 10.5, 10, 9.3, 8.1, 7.7, 7.5, 8.4, 6.3, 8.8

Find A.m, G.m, H.m, median, mode, range, MAD, Variance, S.D, co-efficient of range, co-efficient of mean deviation, co-efficient of Variance? ~~co-efficient of range~~

52

n = 14

$$A.m = \frac{11.3+9.6+10.4+7.5+8.3+10.5+10+9.3+8.1+7.7+7.5+8.4+6.3+8.8}{14}$$

$\boxed{A.m = 8.8357}$

G.m = Antilog (0.94102)

$\boxed{G.m = 8.73011}$

$$H.m = \frac{14}{\frac{1}{11.3}+\frac{1}{9.6}+\frac{1}{10.4}+\frac{1}{7.5}+\frac{1}{8.3}+\frac{1}{10.5}+\frac{1}{10}+\frac{1}{9.3}+\frac{1}{8.1}+\frac{1}{7.7}+\frac{1}{7.5}+\frac{1}{8.4}+\frac{1}{6.3}+\frac{1}{8.8}}$$

$$= \frac{14}{.09+.10+.10+.13+.12+.10+.1+.12+.12+.13+.13+.12+.16+.11}$$

$H.m = \frac{14}{1.634}$

$\boxed{H.m = 8.568}$

Median:-

Array data:

6.3, 7.5, 7.5, 7.7, 8.1, 8.3, 8.4, 8.8, 9.3, 9.6, 10.4, 10.5, 11.3

n = 14

median $= \frac{1}{2}\left(\frac{n}{2}th + \left(\frac{n}{2}+1\right)th\right)$

$= \frac{1}{2}\left(\frac{14}{2}th + \left(\frac{14}{2}+1\right)th\right)$

$= \frac{1}{2}(7th + 8th)$

$median = \frac{1}{2}(8.4+8.8)$

$\boxed{median = 8.6}$

$\boxed{mode = 7.5}$

$Range = X_{max} - X_{min}$
$= 11.3 - 6.3$

$\boxed{Range = 5}$

$$\text{co-efficient of Range} = \frac{X_{max} - X_{min}}{X_{max} + X_{min}}$$

$$= \frac{11.3-6.3}{11.3+6.3} = \frac{5}{17.6}$$

$$= 0.28409$$

$$MAD = \frac{\Sigma|x_i - \bar{x}|}{n}$$

$$MAD = \frac{|6.3-8.8|+|7.5-8.8|+|7.5-8.8|+|7.7-8.8|+|8.1-8.8| + |8.3-8.8|+|8.4-8.8|+|8.8-8.8|+|9.3-8.8|+|9.6-8.8| + |10.4-8.8|+|10.5-8.8|+|11.3-8.8|}{14}$$

$$MAD = \frac{2.5+1.3+1.3+1.1+.7+.5+.4+0+.5+.8+1.6+1.7+2.5}{14}$$

$\boxed{MAD = 1.06}$

$$Variance = S^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$S^2 = \frac{(6.3-8.8)^2+(7.5-8.8)^2+ \cdots}{14}$$

$$S^2 = \frac{24.33}{14}$$

$\boxed{S^2 = 1.738}$

$S.D = \sqrt{S^2}$

$S.D = \sqrt{1.738}$

$\boxed{S.D = 1.318}$

$$\text{Co-efficient of mean deviation} = \frac{MAD \text{ or } m.D}{\bar{x}}$$

$$= \frac{1.06}{8.8}$$

$C.m.D = 0.1205$

$$\text{co-efficiente of Variance} = \frac{S.D}{\bar{x}} \times 100$$

$$C.V = \frac{1.318 \times 100}{8.8}$$

$\boxed{C.V = 14.98}$

Sessional (I):

1): Variable and its types.
Data
Measurement scales
Statistics and its Branches
Parameter, Statistic
Statistical Population, Sample.
Sampling, census and sample survey.

2): Presentation of data.
objectives, Tabular method, Graphical presentation.
Frequency distribution and its types
Histogram, Frequency Polygon.
Frequency curve, Bar-chart.
Pareto-chart, Pie-chart, Stem & leaf method.
Demerits of Presentation of data.
(Three properties of data that does not measure by presentation of data)

3): Measures of central tendency:-
Introduction, objectives, criteria
A.m
G.m — merits demerits applications.
H.m
median
mode

4): Measure of Dispersion.
Introduction, objectives, criteria for good measure,
Range, m.D, Variance, S.D, co-efficient of Variation

---

outlier. 2, 3, 0, 1, 3, 4, 5 say (20) is outlier.

Measures Of Skewness:

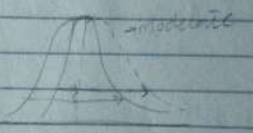
Pearson's Co-efficient Of Skewness

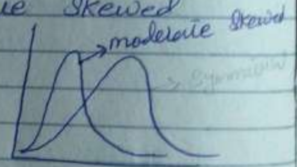
$$SK_P = \frac{3(\text{mean} - \text{median})}{\text{Standard deviation}}$$

$SK_P = 0$ => distribution is symmetrical
$SK_P < 0$ => distribution is -vely skewed
$SK_P > 0$ => distribution is +vely skewed

Other Measures.
β_1 and β_2, called moment Ratios, are used to measure the Skewness and kurtosis respectively.
study of Problems of curve

- $-1 \le SK_P \le 1$ => moderate skewed





- If difference is small

Moment: Sometimes the expected values of integer powers of x and $x - \bar{x}$ are called moments
(1) mean
(2) S.D
(3) Skewness
(4) Kurtosis

If data "x" has same values of that distribution as data "y" then 'x' and 'y' has same distribution otherwise they are different.

(1) Moments About Mean:-

Let $x_1, x_2, \dots x_n$ be the values of a data set with mean $\bar{x}$. The r^{th} moment about mean is denoted by μ_r defined as

$$\mu_r = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^r \quad (1),\ r = 1,2,3,\dots$$

Definition:- It is the A.m of r^{th} deviation of data from sample mean.

(2): Moment About Origin [Raw moments].

is denoted by μ_r' defined as

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{n}(x_i-0)^r$$

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{n}x_i^r$$

if $r = 1$

$$\mu_1' = \frac{1}{n}\sum x_i = \bar{x} = \text{mean}$$

- In (1), if put $r = 2$

$$\mu_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 = S^2 = \text{Variance}$$

- and if put $r = 1$ then

$$\mu_1 = \frac{1}{n}\sum(x_i-\bar{x}) = 0 \quad \text{(by A.m property.)}$$

Moment About Constant A.- is defined as

$$\mu_r = \frac{1}{n}\sum_{i=1}^{n}(x_i-A)^r$$

Relationship among raw moments and moments about mean: (μ_r', μ_r).

$$\mu_r = \mu_r' - \binom{r}{1}\mu_1'\mu_{r-1}' + \binom{r}{2}(\mu_1')^2\mu_{r-2}' + \dots + (-1)^r(\mu_1')^r$$

put $r = 1,2,3,4$, we get

$$\mu_1 = \mu_1' - \mu_1' = 0$$
$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

Moment Ratios:

$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$ → used to measure Skewness. $\beta_1 = 0 \Rightarrow$ Symmetrical

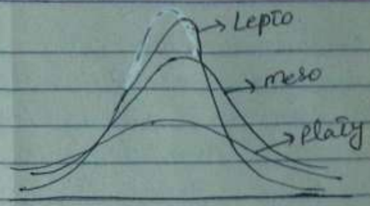
$\beta_1 > 0 \Rightarrow$ +vely skewed. $\beta_1 < 0 \Rightarrow$ -vely skewed

$\beta_2 = \frac{\mu_4}{(\mu_2)^2}$ → used to measure the kurtosis

if $\beta_2 = 3 \Rightarrow$ distribution is mesokurtic

$\beta_2 > 3 \Rightarrow$ distribution is Leptokurtic

$\beta_2 < 3 \Rightarrow$ distribution is Platykurtic



Statistical Population

Real population
⇓
unit exist physically
e.g marks of student
X = weight, Prices, income etc.
It (x) has distribution
(i) mean
(ii) Variance
(iii) Skewness
(iv) kurtosis

Hypothetical population
⇓
Physically unit does not exist. It consist of the result of random experiment. e.g set of outcomes of tossing a coin
$S = \{H, T\}$
$S = \{A, A^-, B^+, B, C, F\}$

R.E → r.v → P.d
Random experiment Random variable

Now we are in Hypothetical population.
Probability: → To measure chance.
Blaise Pascal
Pierre Fermant
chevalier
Andrei N - Kolmogoro
Fisher

These are mathematician whose play role in probability's developement.

uncertainty → see from book (Probability)

Probability Theory:-

1). Introduction
2). History of Probability Theory
3). Role of mathematicians in development of Probability Theory (7)(5+2)
4). Basic Terms that used in probability
5). Mathematical methods / Techniques used in pure sciences and social analysis

Mathematical Models:-

Real world Problem
↓
mathematical model
↓
Solution
↓
Implementation

(1) Deterministic Models:

dependent $y = f(x)$ → independent

$y = \underbrace{\alpha + \beta}_{\text{Parameter}} x$

demand $y = f(x)$ prices

(2): Probabalistic / Stochastic models:

$y = \underbrace{\alpha + \beta x}_{\text{deterministic}} + \varepsilon_i$ → Random

used in uncertain situation

Random Experiment:- any activity that can ... generate set of ...

↓
result
Predictable → deterministic experiment
unpredictable → environment same; long run repeat

e.g a_1 = Predict Value
a_2 = ...
a_3 = ...
⋮
a_{10} = ...
⋮
a_n = □ → if we can not introduce general term then it is unpredictable.

~~(when Personal Biase is removed totaly then it is)~~

Examples of random experiments are tossing a coin, rolling a die etc.

Trial: Single Performance of an experiment.

outcome: The result of trial

Sample Space: set of all possible outcomes. and is denoted by S

e.g $S = \{H, T\}$ ⇒ where H, T are sample points.

$n(S) = 2$

It may be finite or infinite.

Sample Point:- The elements of Sample Space

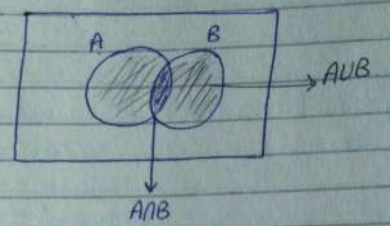
Set :- $\{H, T\}$

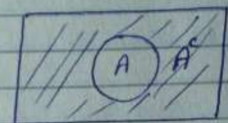
Subsets:

$A_1 = \phi$	$n(A_1) = 0$
$A_2 = \{H\}$	$n(A_2) = 1$
$A_3 = \{T\}$	$n(A_3) = 1$
$A_4 = \{H, T\}$	$n(A_4) = 2$

Power Set = $\{\phi, \{H\}, \{T\}, \{H,T\}\}$

Union And Intersection:





Venn-diagrame → such graphical presentation of union

Event:-

✓ Event is Subset of Sample Space.

e.g A_1, A_2, A_3, A_4

Event Space:- The set of all possible subsets of a Sample space.

e.g Power Set is called event space.

event Space = $\{\phi, \{H\}, \{T\}, \{H,T\}\}$

Set of all possible events.

Types of Events:-

1): null event / impossible event. e.g (in previous example,) $A_1 = \phi$
2). Sure event e.g $A_4 = \{T, H\}$
3). Simple event e.g A_2, A_3 (has → 1 Sample point)
4). compound event. e.g A_4
5). mutually exclusive events. (A_2 & A_3) ∵ $A_2 \cap A_3 = \phi$
6). collectively exhaustive events. ($A \cup B = S$) e.g $A_2 \cup A_3 = \{H, T\} = S$

Next → Probability of event

7) Equally likely events. (have same number of sample pt eg $A_1 A_2$ i.e $n(A_2)=n(A_1)$)
8) dependent events
9) Independent events

(if occurance of an event is effected by the occurance of other event then it is dependent event.)

For example:

$S=A=\{a,b,c\}$. Find events?

Event Subsets of $A = \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$

null event $= \phi$
Sure event $= \{a,b,c\}$
Simple event $= \{a\}, \{b\}, \{c\}$
Compound event $= \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$
Mutually exclusive Events are

$A_1 = \{a\}$ and $\{b\}$ because $\{a\} \cap \{b\} = \phi$

$A_2 = \{a\}$ and $\{c\}$ because $\{a\} \cap \{c\} = \phi$
$A_3 = \{b\}$ and $\{c\}$ because $\{b\} \cap \{c\} = \phi$

Collectively exhaustive events:-

$A_1 = \{a\} \cup \{b,c\} = A$
$A_2 = \{b\} \cup \{a,c\}$
$A_3 = \{c\} \cup \{a,b\}$
$A_4 = \phi \cup \{a,b,c\}$

Equally likely Events:

(i): $\{a\}, \{b\}$ and $\{c\}$ b/c $n(\{a\}) = n(\{b\}) = n(\{c\}) = 1$
(ii): $\{a,b\}, \{a,c\}$ and $\{b,c\}$ b/c $n(\{a,b\}) = n(\{a,c\}) = n(\{b,c\}) = 2$.
(i) and (ii) are equally likely events

Introduction :- A Probability is the numeric value representing the chance, likelihood, or possibility a particular event will occur, such as the price of a stock increasing, a rainy day or the outcome five in a single toss of a die. In all these instances, the probability involved is a proportion or fraction whose value ranges b/w 0 and 1, inclusive. An event that has no chance of occuring has a probability of 0. An event that is sure to occur has 1. Probability theory provides a mathematical model for the study of randomness and uncertainty.

Role of Mathematicians In development of Probability Theory:-

Blaise Pascal, Pierre Fermat and chevalier:-

The earlier results on probability arose from the collaboration of the eminent mathematicians Blaise pascal, Pierre Fermat and chevalier. They were intrested in what seemed to be contradictions b/w mathematical calculations and actual games of chance, such as throwing dice, tossing coins. Fermat shares credit for invention of analytic geometry. His correspondence with Pascal was the starting point for the development of a mathematical theory of probability. Pascal invented one of early calculating machines to accounting work. 65

Andrei N. Kolmogorov:-

Modern probability theory owes much to the 1933 publications Foundations of Theory of Probability by the Russian mathematician Andrei N. Kolmogorov. He developed the probability theory from an axiomatic point of view.

Fisher's:- Fisher would work in statistics at all. He had a brilliant career as a mathematics student at cambridge university and published two student paper at that time. Fisher's memoir introduced ideas as basic as the parametric family of distributions. Fisher was continuing to think a bit about the correlation co-efficient. Fishers most of work is in probability and wrote the 1096 and 97.5 percentile point of normal distribution used in probability and statistic also originated in the book.

History:- Infact, probability theory is the most important tool in statistical inference. The earliest results on probability arose from the collaboration of the eminent mathematicians Blaise pascal, Fermat glamber and chevalier de Mr. They were intrested in what seemed to be

(See above page)

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Probability of Event:

uncertainty in numerical form called Probability of event.

e.g $S = \{Q, H, A\}$, $A_1 = \{Q\}$, $P(A_1) = 50\%, 80\%$

chance of possibility is may be different

Subjective approach:-

By personal Judgment, Probability of same event is different. It can not be generalize.

objective approach:-

It is based on mathematical model. Probability of same event is same. It can be generalized and used for future.

our concern is on objective approach.

(i) classical statistics

(ii) Bayesian statistics

In objective approach there are three method to calculate probability of event.

(1) Classical / Prior definition of Probability of Event.

for finite sample space

$S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

$A = \{1, 2, 4\}$, $n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ → General

(a): finite sample space

(b): mutually exclusive events e.g. $A = \{..\}$ $B = \{..\}$ then $A \cap B = \phi$.

$$P(A) = \frac{\text{number of outcomes in event } A}{\text{number of outcomes in sample space } S}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{m}{n} \quad \star$$

∴ $n(A) = m$

∴ $n(S) = n$

Example:- In tossing two coins Find probability that

(i): both are heads
(ii): both are tail
(iii): Exactly one head
(iv): At least one head
(v): At most one head

} events

Solution:- **Step (I)**:- Define event

Let we define the events such that

A = both are Heads

B = both are Tail

C = Exactly one head

D = Atleast one head

E = At most one head

Step (II): Given information i.e.

Tossing two coins

Sample space $S = \{HH, HT, TH, TT\}$

$A = \{HH\}$

$B = \{TT\}$

$C = \{HT, TH\}$

$D = \{HT, TH, HH\}$ → At least 1 head/heads

$E = \{HT, TH, TT\}$ → At most 1 head/heads

$n(S) = 4$
$n(A) = 1$
$n(B) = 1$
$n(C) = 2$
$n(D) = 3$
$n(E) = 3$

Step (III):- Find

$P(A), P(B), P(C), P(D), P(E)$

Step (IV): Solution procedure

we know that

$$P(\cdot) = \frac{n(\cdot)}{n(S)}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{4} \Rightarrow \frac{1}{2}$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{4}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

Note:-

Total Cards = 52

Black Cards (26) , Red Cards (26)

Spade (13) , club (13) . heart (13) , Diamond (13)

1, 2, 3, ... ,10, K, Q, J . K, Q, J, 1, 2, 3, ..., 10

i.e: (9 - numbers card
1 - Ace card
3 - face card) → in Each = 13

Out of 52:
Total face Cards = 12
Ace cards = 4
Number cards = 36

Example:- One card is drawn at random from ordinary deck of 52 playing cards. Find the probability that

(i) It will be red
(ii) It will be King of Black Cards
(iii) It will be face card

Solution:-

Step (I):
A = Selected card is a red.
B = Selected card is king of Black card.
C = Selected card is a face card.

Step (II):
$n(S) = 52$
$n(A) = 26$
$n(B) = 2$
$n(C) = 12$

Step (III): Find
$P(A)$, $P(B)$, $P(C)$ = ?

Step (IV):- Solution Procedure

$\because P(\cdot) = \frac{n(\cdot)}{n(S)}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{12}{52} = \frac{6}{26} \Rightarrow \frac{3}{13}$$

$P(\text{null event / impossible event}) = \frac{0}{n(S)} = 0$

$P(\text{Sure even}) = 1$ $= \frac{n(S)}{n(S)}$

(2): Posterior / Relative frequency definition of Probability event:-

If an experiment is repeated a large number of time, say n and out of which event 'A' occure m times. The probability of event A is defined as

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

(3) Axiomatic definition:-

Let A be any event defined in a sample space S, P(A) is called probability of event A if it satisfied three conditions

(1): $0 \le P(A) \le 1$

(2): $P(S) = 1$

(3) $P(A \cup B) = P(A) + P(B)$ if A & B are mutually exclusive (i.e $A \cap B = \phi$)

e.g. $S = \{H, T\}$, $A = \{H\}$

$P(A) = \frac{1}{2}$

$n \to \infty \Rightarrow P(A) = 1/2$

Counting The Sample Points:

Multiplication Rule.

Multistep Experiment:-

In an experiment consist of sequence of K trials which produce $n_1, n_2, n_3 \dots n_k$ mutually exclusive outcomes on first, 2^{nd}, 3^{rd} ---- and k^{th} trial respectively. The total number of outcomes in this experiment is $n_1 \times n_2 \times n_3 \times \dots \times n_k$.

e.g. Tossing a coin in 3-times

$2.2.2 = 8$

Possibility of 1-Time = 2

Example:(1):

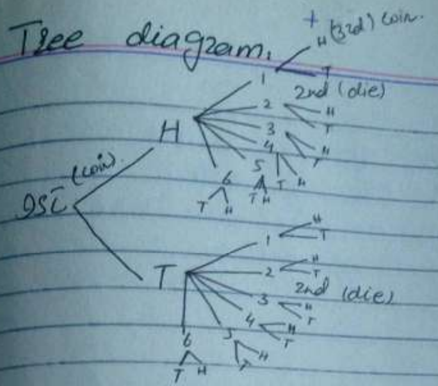
An experiment consist of tossing a coin first, then rolling a die. How many number of sample point?

Sol:-

	1st	2nd
	coin	die
Possibility:	2	6

Total Possibilities = Number of outcomes $= 2 \times 6$
or Sample Points = 12 $= 12$

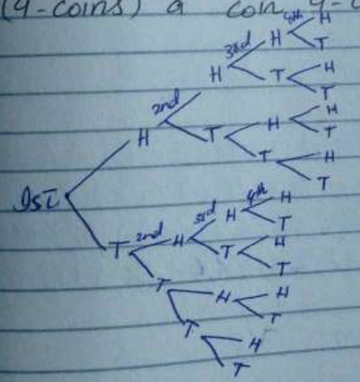
Tree diagram.



$S_1 = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

$S_2 = \{(H,1,H), (H,1,T), (H,2,H), \dots\}$

Make the sample space of tossing (4-coins) a coin 4-time?



$S = \{(H,H,H,H), (H,H,H,T), (H,H,T,H), (H,H,T,T), (H,T,T,H), (H,T,T,T),$
$(H,T,H,H), (H,T,H,T), (T,H,T,H), (T,H,T,T),$
$(T,H,H,H), (T,H,H,T), (T,T,T,H), (T,T,T,T)\}$
$(T,T,H,H), (T,T,H,T)$

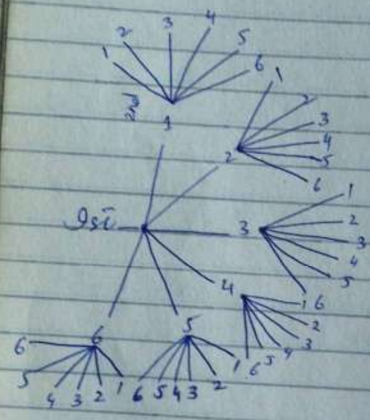
$n(S) = 16$

Example.

(a) Make sample space of rolling 2 dies

(b): make sample space of rolling a die 3-times. See next two page.

(a): Tree diagram of rolling 2-die.



$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3),$
$(2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3),$
$(5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Also

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Permutation:

It is an ordered arrangement

The number of permutations of 'r' objects selected from n objects is defined by ^nP_r

defined as $$^nP_r = \frac{n!}{(n-r)!}$$

Combination:- (95% will be used in [illegible])

It is an arrangements without ordered (Position). The number of combinations of r object selected at a time from n objects is ^nC_r

$$^nC_r = \frac{n!}{r!\,(n-r)!} \qquad n \geq r$$

Selection:-

Total	Select	Position meaningful	Method
n	r	yes	Permutation
n	r	No	Combination

e.g. 1: A, B $\neq$
2: B, A

Example:- A, B, C
$n = 3$, $r = 2$

$^3P_2 = 6$: Permutation (5%) Combination (used 95%)

Permutation	Combination
A, B	A, B = B, A
B, A	A, C = C, A
A, C	B, C = C, B
C, A	
B, C	
C, B	

Committee
1 → President
2 → Vice President
Position

Example:- Two lottery tickets are draw at random from 20 tickets for first and 2nd prize. Find number of sample points in sample space? or How many ways in which these two prizes are awarded. $n(S) = ?$

$\Rightarrow$ 1st prize, 2nd prize $\Rightarrow$ Permutation will be used

Sol:-

$n = 20$, $r = 2$

$$^nP_r = {}^{20}P_2 = \frac{n!}{(n-r)!} = \frac{20!}{(20-2)!}$$

$$n(S) = {}^{20}P_2 = \frac{20 \cdot 19 \cdot \cancel{18!}}{\cancel{18!}} \Rightarrow 20 \cdot 19$$

$$n(S) = 380$$

Example:- Two lottery tickets are draw at random from 20 tickets for same prize. Find number of sample points in sample space? or How many ways in which these two prizes are awarded.

Quiz on monday

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$$^{n}C_{r} = {}^{20}C_{2} = \frac{20!}{2!\,(20-2)!}$$

$$= \frac{20!}{2.\,18!} \Rightarrow \frac{\overset{10}{\cancel{20}}.\,19.\,\cancel{18!}}{\cancel{2}.\,\cancel{18!}} \Rightarrow$$

$$= 190.$$

Question: 13.

A commitee of 5 persons is to be selected randomly from a group of 5 men and 10 women (a): Find the Probability that the commitee consists of 2 men & 3 women.
(b): Find Probability that commitee consists of all women.

men	women	total	select
5	10	15	5

(a) A = 2 men and 3 women

$$\Rightarrow {}^{5}C_{2} = \binom{5}{2}$$

$$n(S) = {}^{15}C_{5} \text{ ways} = 3003$$

$$n(A) = \binom{5}{2}\binom{10}{3} = (10)\,(120) \Rightarrow 1200$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1200}{3003} \Rightarrow 0.3996$$

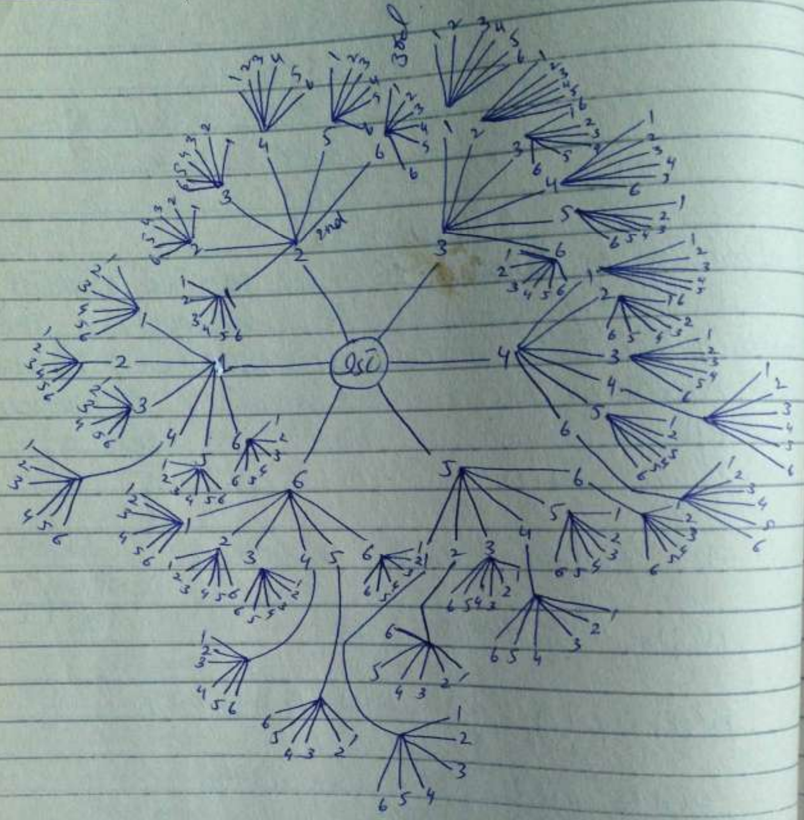
(b). B = all women

$$n(B) = \binom{10}{5}\binom{5}{0} = (252)\,(1) \Rightarrow 252$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{252}{3003} \Rightarrow$$

$$P(B) = 0.084.$$

(b). Make Sample space of rolling a die 3-times?



S = {(1,1,1), (1,1,2), (1,1,3), (1,1,4), (1,1,5), (1,1,6)
(1,2,1), (1,2,2) (1,2,3), (1,2,4), (1,2,5), (1,2,6),
(1,3,1), (1,3,2), (1,3,3), (1,3,4), (1,3,5), (1,3,6), (1,4,1)
(1,4,2), (1,4,3) (1,4,4) (1,4,5) (1,4,6) (1,5,1), (1,5,2),
(1,5,3), (1,5,4), (1,5,5), (1,5,6), (1,6,1), (1,6,2), (1,6,3),
(1,6,4), (1,6,5), (1,6,6), (2,1,1) (2,1,2), (2,1,3), (2,1,4),
(2,1,5), (2,1,6), (2,2,1) (2,2,2) (2,2,3) (2,2,4), (2,2,5),
(2,2,6), (2,3,1), (2,3,2), (2,3,3), (2,3,4) (2,3,5), (2,3,6), (2,4,1)
(2,4,2), (2,4,3), (2,4,4), (2,4,5), (2,4,6), (2,5,1), (2,5,2), (2,5,3),
(2,5,4), (2,5,5), (2,5,6), (2,6,1), (2,6,2), (2,6,3), (2,6,4), (2,6,5).

(2,6,6), (3,1,1), (3,1,2), (3,1,3), (3,1,4), (3,1,5), (3,1,6), (3,2,1)
(3,2,2), (3,2,3), (3,2,4), (3,2,5), (3,2,6), (3,3,1), (3,3,2), (3,3,3)
(3,3,4), (3,3,5), (3,3,6), (3,4,1), (3,4,2), (3,4,3), (3,4,4),
(3,4,5), (3,4,6), (3,5,1), (3,5,2), (3,5,3), (3,5,4), (3,5,5), (3,5,6)
(3,6,1), (3,6,2), (3,6,3), (3,6,4), (3,6,5), (3,6,6), (4,1,1)
(4,1,2), (4,1,3), (4,1,4), (4,1,5), (4,1,6), (4,2,1), (4,2,2),
(4,2,3), (4,2,4), (4,2,5), (4,2,6), (4,3,1), (4,3,2), (4,3,3),
(4,3,4), (4,3,5), (4,3,6), (4,4,1), (4,4,2), (4,4,3), (4,4,4),
(4,4,5), (4,4,6), (4,5,1), (4,5,2), (4,5,3), (4,5,4), (4,5,5),
(4,5,6), (4,6,1), (4,6,2), (4,6,3), (4,6,4), (4,6,5),
(4,6,6), (5,1,1), (5,1,2), (5,1,3), (5,1,4), (5,1,5), (5,1,6),
(5,2,1), (5,2,2), (5,2,3), (5,2,4), (5,2,5), (5,2,6), (5,3,1),
(5,3,2), (5,3,3), (5,3,4), (5,3,5), (5,3,6), (5,4,1), (5,4,2),
(5,4,3), (5,4,4), (5,4,5), (5,4,6), (5,5,1), (5,5,2), (5,5,3),
(5,5,4), (5,5,5), (5,5,6), (5,6,1), (5,6,2), (5,6,3), (5,6,4),
(5,6,5), (5,6,6), (6,1,1), (6,1,2), (6,1,3), (6,1,4), (6,1,5)
(6,1,6), (6,2,1), (6,2,2), (6,2,3), (6,2,4), (6,2,5), (6,2,6),
(6,3,1), (6,3,2), (6,3,3), (6,3,4), (6,3,5), (6,3,6), (6,4,1),
(6,4,2), (6,4,3), (6,4,4), (6,4,5), (6,4,6), (6,5,1), (6,5,2),
(6,5,3), (6,5,4), (6,5,5), (6,5,6), (6,6,1), (6,6,2), (6,6,3),
(6,6,4), (6,6,5), (6,6,6). }

$n(S) = 216$.

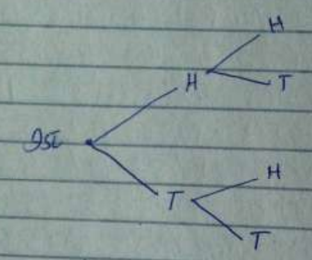
Ex- Q.No.2 Chap: Probability

Two fair coins are tossed. what is the probability of getting one heads and one tails?

Step(I):- Define events

A = one heads and one tails

Step(II):-



$S = \{HH, HT, TH, TT\}$

$A = \{HT, TH\}$

$n(S) = 4$

$n(A) = 2$

Step(III):- Find P(A)

Step(IV):- Solution procedure

$P(A) = \frac{n(A)}{n(S)}$

$P(A) = \frac{2}{4} \Rightarrow \frac{1}{2} \Rightarrow 0.5$

Q.No. 3.

A coin is tossed twice. what is the probability that at least one head occure?

Step(I):-

A = At least one head occure

(II):-

$S = \{HH, HT, TH, TT\}$

$A = \{HH, HT, TH\}$

$n(S) = 4$

$n(A) = 3$

(III):- Find $P(A)$.

(IV):-

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4} \Rightarrow 0.75$$

Q.No.4.

A die is loaded in such a way that an even number is twice as likely to occure as an odd number. If E is the event that a number less than 4 occurs on a single toss of die, find P(E)?

(I):

E = A no. less than 4 occurs on a single toss of die.

(II): $S = \{1, 2, 2, 3, 4, 4, 5, 6, 6\}$

$n(S) = 9$

$E = \{1, 2, 2, 3\}$

$n(E) = 4$

$$P(E) = \frac{4}{9}$$

Q.No.5.

Industrial, mechanical, electrical,
25, 10, 10

Civil engineering, Total,
8, 53

(a): A = An industrial engineering major

$$n(S) = \binom{53}{25}$$

$$= \frac{53!}{25!\,(53-25)!}$$

$n(S) = 9.03936 \times 10^{14}$

$$n(A) = \binom{25}{1} \Rightarrow 25$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{25}{9.03936 \times 10^{14}}$$

$P(A) = 2.76568 \times 10^{-14}$

(b): A = a civil engineering or an electrical engineering major.

Electrical engineering major

$$n(S) = \binom{53}{10}$$

$n(S) = 1.9499 \times 10^{10}$

$$n(A) = \binom{10}{1} = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{1.9499 \times 10^{10}}$$

$P(A) = 5.2086 \times 10^{-10}$

A= civil

$n(S) = \binom{53}{8} = 886322710$

$n(A) = \binom{8}{1} = 8$

$P(A) = \frac{8}{886322710} \Rightarrow 0.00000000902$

Q.No. 6

Aces	Jacks	Total	Select
2	3	52	5

(a): A= 2 aces and 3 Jacks

$n(S) = {}^{52}C_5 = 2598960$

$n(A) = \binom{4}{2}\binom{4}{3}$

$= (6)(4)$

$n(A) = 24$

$P(A) = \frac{n(A)}{n(S)} = \frac{24}{2598960}$

$P(A) = 0.0000092345$ Ans!

Q. No. 7

Red balls	Yellow	Green	Total
6	5	3	14

(a): A = Green (ball)

$n(S) = {}^{14}C_3 = 364$

$n(A) = {}^{3}C_1 = 3$

$P(A) = \frac{3}{364} = 0.0082418$

Question: 8

A: Production batch have low in impurities

B: Production batch have high strength

C: Both high in impurities and high strength

D: Both high in impurities and low strength

	Low Strength	High Strength	Total
Low in impurities	2	27	29
High in impurities	12	4	16
Total	14	31	45

(a): $n(S) = 45$

$n(A) = 29$

$P(A) = \frac{29}{45}$

(b) $n(B) = 31$

$P(B) = \frac{31}{45}$

(c): $n(C) = 4$

$P(C) = \frac{4}{45}$

(d): $n(D) = 12$

$P(D) = \frac{12}{45} = \frac{4}{15}$

Q. No. 7

Event G: Green balls
B: not Yellow
C: Red or Yellow

Green, Yellow, Red balls, Total
$n(G) = 3$, $n(Y) = 5$, $n(R) = 6$, $n(S) = 14$

$n(S) = 14$

(a). $P(G) = \frac{n(G)}{n(S)} = \frac{3}{14}$

(b). B = not Yellow
B^c : Yellow
$P(B^c) = \frac{n(Y)}{n(S)} = \frac{5}{14}$

~~as~~ compliment law
$P(B^c) = 1 - P(B)$
$P(B) = 1 - P(B^c)$
$P(B) = 1 - \frac{5}{14} \Rightarrow \frac{14-5}{14} \Rightarrow$

$$\boxed{P(B) = \frac{9}{14}}$$

(c). C: Red or Yellow
C_1 = Red, C_2 = Yellow
$n(C_1) = 6$, $n(C_2) = 5$
$P(C_1) = \frac{6}{14}$, $P(C_2) = \frac{5}{14}$

$P(C_1 \cup C_2) = P(C_1) + P(C_2) - P(C_1 \cap C_2)$
$P(\text{Red or Yellow}) = \frac{6}{14} + \frac{5}{14} - \frac{0}{14}$

$$\boxed{P(C) = \frac{11}{14}}$$

Q. Two fair die are rolled and recorded the upper faces turn up. Find Probability that
(i) Same numbers will be appear.
(ii). Different numbers will be appeal
(iii): Sum of two number is less than 5
(iv) Product of two no. is 8.
(V) Product of two no. is mutiple of 2.

Sol: B: Same no.s will be appeal.
D: different no.s will be appeal
A: Sum of two no. is less than 5.
P = Product of two no. is 8.
C: Product of two nos is multiple of 2.

Elements of Sample Space =

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	[illegible]
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	[illegible]
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	[illegible]
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	[illegible]
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	[illegible]
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	[illegible]

$n(S) = 6 \times 6 = 36$

(i) $B = \{(1,1),(2,2)(3,3).(4,4)(5,5)(6,6)\}$
$n(B) = 6$

(ii) $n(D) = 30$

(iii) $A = \{(1,3)(3,1)(2,1)(1,2)(2,2)(1,1)\}$
$n(A) = 6$

(iv): $P = \{(2,4)(4,2)\}$
$n(D) = 2$

(v): ~~$C = \{(1,2)(1,4)(1,6)$~~

$P(B) = \frac{6}{36} = \frac{1}{6} = 0.167$ $\because P(B) = \frac{n(B)}{n(S)}$

$P(D) = \frac{30}{36} = \frac{5}{6} = .833$ $\because P(D) = \frac{n(D)}{n(S)}$

$P(A) = \frac{6}{30} = \frac{1}{5} = 0.2$ $P(A) = \frac{n(A)}{n(S)}$

$P(P) = \frac{2}{36} = 0.056$

(v): $C = \{(1,2)(1,4)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(3,2)(5,4)$
$(3,6)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)(5,2)(5,4)(5,6)(6,1)$
$(6,2)(6,3)(6,4)(6,5)(6,6)\}$

$n(C) = 27$

$P(C) = \frac{n(C)}{n(S)} = \frac{27}{36} \Rightarrow \frac{9}{12} = \frac{3}{4} = 0.75$

Law of Probability:-

(i): **complement law of Probability:-**

(used for Guess, mostly)

Statement:- Let A be any event defined in Sample space S. Then

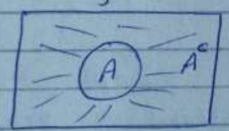
$A \subseteq U$

$P(A^c) = 1 - P(A)$

$A^c = U - A$ sample space

Proof:-

S



Since A and A^c are two mutually exclusive events

i.e $A \cap A^c = \phi$

From above diagrame

$A \cup A^c = S$

$P(A \cup A^c) = P(S)$

$P(A) + P(A^c) = 1$

$\Rightarrow P(A^c) = 1 - P(A)$

Example:- A coin is tossed 6-times what is probability that at least one head occure.

Sol.- $n(S) = 2.2.2.2.2.2 = 64$

A = at least one head.

P[at least one head or no Head] = 1. — Imp. Step

$P(A \text{ or } A^c) = P(A) + P(A^c) = 1$

$P[A] = 1 - P(A^c)$

$P[A] = 1 - \frac{n(A^c)}{n(S)}$ — ★

$A^c = \{TTTTTT\} \Rightarrow n(A^c) = 1$

Now ★ become

$P[A] = 1 - \frac{1}{64} \Rightarrow \frac{63}{64}$

Example:-

A Student "omama" attempt to Pass 5 papers in Sessional-2 Smester Fall 15. Her Previous record show that she passed any exame in 4 attempt out of 6. what is the probability that she will pas at least one paper in sessional 2

Sol:-

A = Omama can pass at least one paper

A : 1st Paper, 2^{nd} Paper, 3^{rd} Paper, 4^{th}, 5^{th}

$P[A] + P[A^c] = 1$

$P[A] = 1 - P[A^c]$

$P[A] = 1 - P[\text{all Paper are fail}]$

$= 1 - P[\frac{2}{6}]$ ×

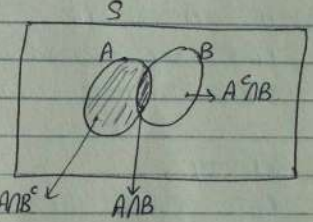
$P[A] = \frac{4}{6}$

Theorem:-

Addition law of Probability:-

Let A and B be the two events defined in Same sample space. Probability that event A or event B or Both (at least one of them) will occure is denoted by $P[A \cup B]$ defined as

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$



Proof:- From above diagrame, we note that three events $A \cap B^c$, $A \cap B$ and $A^c \cap B$ are mutually exclusive.

Now, we can write event A as union of two mutually exclusive event.

$$A = (A \cap B^c) \cup (A \cap B)$$

$$P(A) = P[(A \cap B^c) \cup (A \cap B)]$$

$$P[A] = P(A \cap B^c) + P(A \cap B) \quad \text{— (1)}$$

Now.

$$B = (A^c \cap B) \cup (A \cap B)$$

$$P[B] = P[(A^c \cap B) \cup (A \cap B)]$$

$$P[B] = P(A^c \cap B) + P(A \cap B) \quad \text{— (2)}$$

adding (1) and (2)

$$P(A) + P(B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B) + P(A \cap B)$$

$$= \underbrace{P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)}_{= A \cup B} + P(A \cap B)$$

from diagram

$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$\therefore P[A \cup B] = P[A^c \cap B] + P[A \cap B^c] + P[A \cap B]$

$= P[(A^c \cap B) \cup (A \cap B^c) \cup (A \cap B)]$

Q.19: A single die is rolled. what is the probability that number 2 or 5 will occure? $P(2 \text{ or } 5) = P(A \cup B)$

Q.20: A single 6-sided die is rolled. Find the probability that 2 or even number will occure?

Sol: 19

let Event A: 2 occure on rolling a die

Event B: 5 occure on rolling a die

$P(2 \text{ or } 5) = P[A \cup B]$

As we know

$P[A \cup B] = P[A] + P[B] - P[A \cap B]$

$A = \{2\}$ $\quad n(A) = 1$

$B = \{5\}$ $\quad n(B) = 1$

$A \cap B = \phi$ $\quad n(A \cap B) = 0.$

$n(S) = 6$

$P[A\cup B] = \frac{1}{6} + \frac{1}{6} - \frac{0}{6}$

$= \frac{2}{6} \Rightarrow \frac{1}{3}$

Q.20 A Single 6-sided die is rolled. what is Probability of rolling a 2 or an even number?

Events are defined

A: 2 occure on rolling a die.

B: Even no. will occure on rolling a die

$n(S) = 6$

$P(2 \text{ or even no.}) = P[A\cup B]$

$A = \{2\}$ $\quad n(A) = 1$

$B = \{2,4,6\}$ $\quad n(B) = 3$

$A\cap B = \{2\}$ $\quad n(A\cap B) = 1$

As addition law is defined as

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

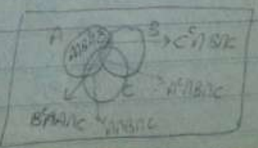
$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A\cap B)}{n(S)}$

$= \cancel{\frac{1}{6}} + \frac{3}{6} - \cancel{\frac{1}{6}}$

$P(A\cup B) = \frac{1}{2}$

Assignment:- State and prove addition law of Probability for three event.

i.e $P[A\cup B\cup C] = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C) + P(A\cap B\cap C)$.



1—38 H.W

Q.No.23.

Red	Yellow	Green	Total	select
4	8	6	18	1

what is the Probability that a randomly selected is red or green?

A: Selected marble is red

B: Selected marble is Green

C: Selected marble is Yellow

$P(\text{red or Green}) = P[\text{red} \cup \text{Green}]$

$n(S) = \binom{18}{1} = 18$

$n(A) = \binom{4}{1}\binom{8}{0}\binom{6}{0} = 4$

$n(B) = \binom{4}{0}\binom{8}{0}\binom{6}{1} = 6$

$P[A\cup B] = P[A] + P[B] - P[A\cap B]$

$= \frac{4}{18} + \frac{6}{18} - \frac{0}{18}$

$= \frac{10}{18}$

$P[A\cup B] = \frac{5}{9}$ Ans!

2nd method:-

at a time red + green

$P[A\cup B] = P[A] + P[B] - P[A\cap B]$

$= \frac{\binom{4}{1}}{\binom{18}{1}} + \frac{\binom{6}{1}}{\binom{18}{1}} - 0$

$= \frac{4}{18} + \frac{6}{18} - 0$

$= \frac{5}{9}$

If collection word is used then we will use combination.

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Question.

Red	Yellow	Green	Total	Select
4	8	6	18	3

what is the probability that all three are Red marbles

Sol:

R: Red marbles

$n(S) = \binom{18}{3} = 816$

$n(R) = \binom{4}{3} = 4$

$P(R) = \frac{\binom{4}{3}}{\binom{18}{3}} = \frac{4}{816}$ Ans!

Question: 5

Industrial Students	Mechanical	Electrical	Civil engineering	Total
25	10	10	8	53

(a): A = Industrial engineering major

$n(A) = 25$

$P(A) = \frac{n(A)}{n(S)} = \frac{25}{53}$

(b). B = A civil engineering or an electrical engineering major

B_1 = A civil engineering

$n(B_1) = 8$

$P(B_1) = \frac{8}{53}$

B_2 = An electrical engineering major

$n(B_2) = 10$

$P(B_2) = \frac{n(B_2)}{n(S)} = \frac{10}{53}$

$P(B_1 \cup B_2) = P(B_1) + P(B_2) - P(B_1 \cap B_2)$

$P(B_1 \cup B_2) = \frac{8}{53} + \frac{10}{53} - 0$

$\Rightarrow \boxed{P(B) = \frac{18}{53}}$

Q.No.1.

S = {3 nuts with metric thread, 12 nuts with U.S threads}

$n(S) = 15$.

Event A: it will be metric

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{15} = \frac{1}{5}$ Ans!

Question: 9.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

S = {(1,1).(1,2)(1,3)(1,4)(1,5).(1,6).(2,1).(2,2).(2,3).
(2,4).(2,5).(2,6).(3,1).(3,2).(3,3).(3,4).(3,5).(3,6)
(4,1).(4,2).(4,3).(4,4).(4,5).(4,6).(5,1).(5,2).(5,3)(5,4)
(5,5)(5,6).(6,1).(6,2)(6,3).(6,4).(6,5).(6,6)}

die ↓ 6, 2nd die ↓ 6 = 6×6 = 36

$n(S) = 36$.

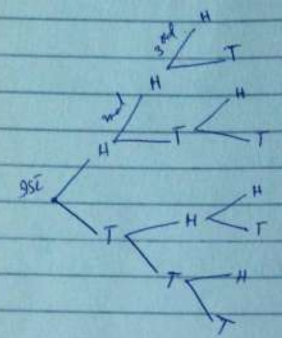
Event A: Score of 7 in one throw of a pair of fair dice

A = {(1,6),(2,5)(3,4)(4,3),(5,2)(6,1)}.

$n(A) = 6$.

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} \Rightarrow \frac{1}{6}$.

Q.No. 10.



S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$n(S) = 8$

Event A: head, tail head = {HTH}

$n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$ Ans!

Q.No. 11.

(a):

Defective	Not defective	Total	Selected
2	4	6	2

$n(S) = {}^{6}C_2 = 15$.

(b) A: At least one of two systems will be defective

$n(A) = \binom{2}{1}\binom{4}{1} + \binom{2}{2}\binom{4}{0}$

$= (2)(4) + 1$

$n(A) = 9$

$\boxed{P(A) = \frac{9}{15} = \frac{3}{5}}$

(B): Both are defective

$n(B) = \binom{2}{2}\binom{4}{0} = 1$

$\boxed{P(B) = \frac{1}{15}}$

(b): Defective , Not defective , Total , select

4 , 2 , 6 , 2

Event C : At least one system will be defective

$n(C) = \binom{4}{1}\binom{2}{1} + \binom{4}{2}\binom{2}{0}$

$= (4)(2) + (6)(1) \Rightarrow 8 + 6 \Rightarrow$

$n(C) = 14$

$\boxed{P(C) = \frac{14}{15}}$

Event D: Both are defective

$n(D) = \binom{4}{2}\binom{2}{0} = (6)(1)$

$n(D) = 6$

$P(D) = \frac{6}{15} \Rightarrow \frac{2}{5}$

$\boxed{P(D) = \frac{2}{5}}$ Ans!

Q.NO.12

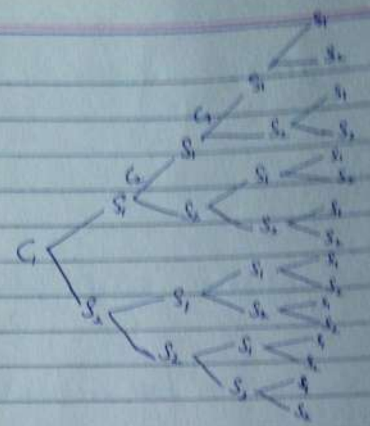
Type = 2 (i.e style 1 , style 2)

Customers = 4 (C_1, C_2, C_3, C_4)

Each customer has possibility = 2 (i.e S_1 and S_2).

So

$n(S) = 2.2.2.2 = 16$



(a): $S = \{(S_1S_1S_1S_1)(S_1S_1S_1S_2)(S_1S_1S_2S_1)(S_1S_1S_2S_2)(S_1S_2S_1S_1)$
$(S_1S_2S_1S_2)(S_1S_2S_2S_1)(S_1S_2S_2S_2)(S_2S_1S_1S_1)(S_2S_1S_1S_2)$
$(S_2S_1S_2S_1)(S_2S_1S_2S_2)(S_2S_2S_1S_1)(S_2S_2S_1S_2)(S_2S_2S_2S_1)$
$(S_2S_2S_2S_2)\}$

$n(S) = 16$

(c): Event A: All 4 customers prefer same style

$A = \{(S_1S_1S_1S_1), (S_2S_2S_2S_2)\}$

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{16} = \frac{1}{8}$

(b): Assign probabilities to the sample pts

$P(\text{each sample pt.}) = \frac{1}{16}$

Q.No. 14.

Yellow fishes, Black fishes, Total, Selected
3, 7, 10, 3

$n(S) = {}^{10}C_3 = 120$

Events.

A: Exactly one yellow fish gets selected
B: At most one yellow fish gets selected
C: At least one yellow fish gets selected

$n(A) = \binom{3}{1}\binom{7}{2} = 3(21) \Rightarrow$

$n(A) = 63$

$\boxed{P(A) = \frac{63}{120}}$

$n(B) = \binom{3}{1}\binom{7}{2} + \binom{3}{0}\binom{7}{3} = 3(21) + 1(35)$

$n(B) = 98$

$\boxed{P(B) = \frac{98}{120}}$

$n(C) = \binom{3}{1}\binom{7}{2} + \binom{3}{2}\binom{7}{1} + \binom{3}{3}\binom{7}{0}$

$= 3(21) + 3(7) + (1)(1)$

$n(C) = 85$

$\boxed{P(C) = \frac{85}{120}}$

Q.No:15

Defective, Not defective, Total, Selected
2, 13, 15, 4

(a): A: None of selected apple is defective

$n(A) = \binom{2}{0}\binom{13}{4} = 715$

As $n(S) = {}^{15}C_4 = 1365$

$P(A) = \frac{n(A)}{n(S)} = \frac{715}{1365}$

B: At least one of selected apple is defective

$n(B) = \binom{2}{1}\binom{13}{3} + \binom{2}{2}\binom{13}{2}$

$= 2(286) + 1(78)$

$= 572 + 78$

$\boxed{n(B) = 650}$

$P(B) = \frac{n(B)}{n(S)} = \frac{650}{1365}$ Ans!

Q.No. 16

Paint stores, Carpet stores, Total
12, 15, 27

$n_1 = 12$ $\qquad n_1 + n_2 = n$

$n_2 = 15$ $\qquad \frac{n!}{n_1! \, n_2!}$

$n = n_1 + n_2 = 27$

Total ways $= \frac{n!}{n_1! \, n_2!} = \frac{27!}{12! \, 15!}$

$= \frac{27.26.25.24.23.22.21.20.19.18.17.16.15!}{12! \quad 15!}$

$= \frac{4475671200 \times 1860480}{479001600}$

$= 17383860$. Ans!

Q.No. 17

white balls, black balls, Yellow, Total, select
15 , 7 , 8 , 30 , 3

$n(S) = {}^{30}C_3 = 4060$

Events are:

A : All three balls are Yellow

B : All three balls are of same colour

C : All three balls are of dif— colour.

$n(A) = \binom{15}{0}\binom{7}{0}\binom{8}{3} = 56$

$P(A) = \frac{56}{4060}$

$n(B) = \binom{15}{3}\binom{7}{0}\binom{8}{0} + \binom{15}{0}\binom{7}{3}\binom{8}{0} + \binom{15}{0}\binom{7}{0}\binom{8}{3}$

$n(B) = 455 + 35 + 56$

$n(B) = 546$

$P(B) = \frac{546}{4060}$

$n(C) = \binom{15}{1}\binom{7}{1}\binom{8}{1}$

$= (15)(7)(8)$

$n(C) = 840$

$P(C) = \frac{84\cancel{0}}{406\cancel{0}}$

$\boxed{P(C) = 0.20690}$ Ans

Q.NO. 18.

Defective , Not defective , Total , Select
4 , 8 , 12 , 3

$n(S) = {}^{12}C_3 = 220$

(a): A: First one is defective & rest are good.

A_1 : First one is defective

Q.NO. 22

	Boys	Girls	Total
	17	13	30
Grad A	4	5	9

P[girl or Grad A] = ?

Event G: Girls

$n(S) = \binom{30}{1} = 30$

$n(G) = \binom{13}{1} = 13$

$P(G) = \frac{13}{30}$

A: Grad A

$n(A) = \binom{9}{1} = 9$

$P(A) = \frac{9}{30}$

As $P[G \cup A] = P(G) + P(A) - P(G \cap A)$ ——— ★

$n(G \cap A) = \binom{5}{1} = 5$

$P(G \cap A) = \frac{5}{30}$

Now ★ become

$P[G \cup A] = \frac{13}{30} + \frac{9}{30} - \frac{5}{30}$

$P[G \cup A] = \frac{17}{30}$

Q.NO. 24.

good Articles	minor defect	major defect	Total	Select
10	4	2	16	1

$n(S) = {}^{16}C_1 = 16$

Event are

(a) A: It has not defect.

B: It has no major defect.

C: It is either good or has major defect.

$n(A) = \binom{10}{1} = 10$

$P(A) = \frac{10}{16} = \frac{5}{8}$

$n(B) = \binom{10}{1}\binom{4}{0} + \binom{10}{0}\binom{4}{1} = 10 + 4$

$n(B) = 14$ or $P(B) = 1 - \frac{2}{16} \Rightarrow \frac{14}{16}$

$P(B) = \frac{14}{16} = \frac{7}{8}$

Event C_1: It is good

C_2: It has major defect

$n(C_1) = \binom{10}{1} = 10$

$P(C_1) = \frac{10}{16} = \frac{5}{8}$

$n(C_2) = \binom{2}{1} = 2$

$P(C_2) = \frac{2}{16} = \frac{1}{8}$

$C_1 \cap C_2 = \phi$ $\quad n(C_1 \cap C_2) = 0$

As

$P(C_1 \cup C_2) = P(C_1) + P(C_2) - P(C_1 \cap C_2)$

$= \frac{5}{8} + \frac{1}{8} - \frac{0}{16}$

$= \frac{6}{8}$

$= \frac{3}{4}$

Q.NO.25

Red, Green, Blue, white, Yellow, Total, Select.
3, 6, 4, 2, 5, 20, 1

$n(S) = {}^{20}C_1 = 20$

Event A: Red marble
$n(A) = 3$
$P(A) = \frac{3}{20}$

B: A Green marble
$n(B) = 6$
$P(B) = \frac{6}{20}$

C: A white marble
$n(C) = 2$
$P(C) = \frac{2}{20}$

D: Red or white marble
$D = A \cup C$
$P(D) = P(A \cup C)$
$= P(A) + P(C) - P(A \cap C)$
$= \frac{3}{20} + \frac{2}{20} - \frac{0}{20}$

$P(D) = \frac{5}{20}$

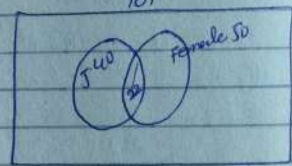
E: Green or white marble
$E = B \cup C$
$P(E) = P(B \cup C)$
$= P(B) + P(C) - P(B \cap C)$
$= \frac{6}{20} + \frac{2}{20} - \frac{0}{20}$
$= \frac{8}{20}$

F: Green, red or white marble
$n(F) = \binom{6}{1} + \binom{3}{1} + \binom{2}{1} \Rightarrow 6+3+2 \Rightarrow 11$

$P(F) = \frac{11}{20}$

Q.NO.26.

$n(S) = 101$
Events A: Female students, $n(A) = 50$
B: Junior students, $n(B) = 40$
$A \cup B$: Female or Junior



$n(A) = 50$
$P(A) = \frac{50}{101}$

$n(B) = 40$
$P(B) = \frac{40}{101}$

$n(A \cap B) = 22$
$P(A \cap B) = \frac{22}{101}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{50}{101} + \frac{40}{101} - \frac{22}{101}$

$P(A \cup B) = \frac{68}{101}$

(a): a Junior $= 40/101$
(b) a Female $= 50/101$
(c) a Junior or Female $= \frac{40}{101} + \frac{50}{101} - \frac{22}{101}$
(d). a junior and Female $= 22/101$

Conditional Probability

Q.No.27

$n(S) = 52$

A: King , $n(A) = 4$

B: club , $n(B) = 13$

$n(A \cap B) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ — ★

$P(A) = \frac{4}{52}$

$P(B) = \frac{13}{52}$

$P(A \cap B) = \frac{1}{52}$

Now ★ become

$P(A \cup B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$P(A \cup B) = \frac{16}{52}$

Q.No. 29.

$P(A) = .24$, $P(B) = .67$, $P(AB) = .09$

$P(A \cap B) = .09$

(a): $P(A \cup B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= .24 + .67 - .09$

$= \frac{24}{100} + \frac{67}{100} - \frac{9}{100} \Rightarrow \frac{82}{100} \Rightarrow$

$P(A \cup B) = \frac{41}{50}$

(c): $P(A^c \cup B^c) = ?$

$A^c \cup B^c = (A \cap B)^c$ → By De-margin's law

$P(A^c \cup B^c) = P((A \cap B)^c)$ — (i)

By complement law

$P((A \cap B)^c) = 1 - P(A \cap B)$

$= 1 - .09$

$= 1 - \frac{9}{100} = \frac{91}{100}$

put it in (i)

$P(A^c \cup B^c) = \frac{91}{100}$

(b): $P((A \cup B)^c) = ?$

$P((A \cup B)^c) = 1 - P(A \cup B)$

$= 1 - \frac{41}{50} \Rightarrow \frac{50-41}{50} \Rightarrow \frac{9}{50}$

$P((A \cup B)^c) = \frac{9}{50}$ Ans!

Q.NO.30.

P[At least 1 defective] = ?

A: At least 1 defective

P[A] = 1 − P[0-defective]

= 1 − .6561

P[A] = .3439 Ans!

or P(A) = P(1-defective) + P(2-def...) + P(3-def...) + P(4-defective)

Q.NO.31.

$n(S) = 52$

Event A: Card is Queen

B: card is heart.

$n(A) = 4$

$n(B) = 13$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$

$n(A \cap B) = 1$

$P(\text{Queen or heart}) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{16}{52} \Rightarrow \frac{4}{13}$ Ans!

Q.No.34

$P(\text{defective in length}) = \frac{4}{135}$

$P(\text{defective in width}) = \frac{3}{141}$

where Event A: defective in length
B: defective in width

$P(A \cap B) = \frac{2}{347}$

Let $D = A \cap B$

(a) P(defective in length or width or both) = ?

$P(A \cup B \cup D) = P(A) + P(B) + P(D) - P(B \cap D) - P(A \cap B) - P(A \cap D) + P(A \cap B \cap D)$

$= \cancel{P(A)} + \cancel{P(B)} + P(A \cap B) - \cancel{P(B)} - P(A \cap B) - \cancel{P(A)} + P(A \cap B)$

$P(A \cup B \cup D) = P(A \cap B)$

$= \frac{2}{347}$

(b): E: Part will have neither defect

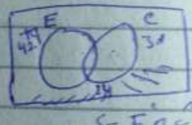
$P(E) = 1 - P(E^c)$

$= 1 - P(A \cap B)$

$= 1 - \frac{2}{347} \Rightarrow \frac{347-2}{347} \Rightarrow \frac{345}{347}$

(c): Find odds against a defect (in length or width or both) = ?

Q.No.35



$n(S) = 72$

Event E: Take english
C: Take chemistry

$n(E^c \cap C^c) = 14 \Rightarrow n((E \cup C)^c) = 14$

$n(E) = 42$

$\therefore P(E \cup C) = 1 - P((E \cup C)^c)$

$n(C) = 38$

$= 1 - \frac{14}{72} \Rightarrow \frac{58}{72}$

(a). D: Both english and chemistry

$P(D) = P(E \cap C)$

As $P(E \cup C) = P(E) + P(C) - P(E \cap C)$

$\frac{58}{72} = \frac{42}{72} + \frac{38}{72} - P(E \cap C)$

$P(E \cap C) = \frac{22}{72} = \frac{11}{36}$

(b): F: chemistry but not english.

$P(F) = P(C \cap E^c) = \frac{38}{72}$

Q.No. 36

$n(S) = 6.6 = 36$

$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)$
$(2,6)(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)$
$(4,4)(4,5)(4,6)(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)$
$(6,2)(6,3)(6,4)(6,5)(6,6)\}$

Event A: 5 will occur at least once

$A = \{(1,5)(2,5)(3,5)(4,5)(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,5)\}$

$n(A) = 11$

$P(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$ Ans!

Q.No. 33

$n(S) = 120$

A: Applied Mechanics
B: chemistry
C: computer

Events are:-

D: do not take any of A, B, C

$n(D) = n(A^c \cap B^c \cap C^c) = 30$

$n(A) = 15$

$n(A^c \cap B \cap C) = 25$

$n(A \cap C \cap B^c) = 20$

$n(A \cap B \cap C) = 10$

$n(B) = 5$

E: Take B and C but not A. $\Rightarrow n(E) = 25$

F: Take A and C but not B. $\Rightarrow n(F) = 20$

G: Take A, B and C. $\Rightarrow n(G) = 10$

$\Rightarrow E = B \cap C \cap A^c$

$\Rightarrow F = A \cap C \cap B^c$

$E \cap F = (B \cap C \cap A^c) \cap (A \cap C \cap B^c)$

$= B \cap C \cap (A^c \cap A) \cap C \cap B^c$

$= (B \cap C) \cap (\phi) \cap (C \cap B^c)$

$= (B \cap C) \cap (C \cap B^c)$

$= (C \cap B) \cap (B^c \cap C)$

$= C \cap (B \cap B^c) \cap C$

$E \cap F = C$

$n(E \cap F) = n(C)$

$20 = n(C)$

(b) $\Rightarrow$ $P(C) = \frac{n(C)}{n(S)} = \frac{20}{120} \Rightarrow \frac{1}{6}$

($\cancel{d}$): H: Total no. of students taking computer

$H = A \cup B \cup C$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$
$- P(B \cap C) + P(A \cap B \cap C)$

$= \frac{15}{120} + \frac{5}{120} + \frac{20}{120} - \frac{5}{120} - \frac{20}{120} - \frac{25}{120} + \frac{10}{120}$

$= 0$

(d): $P(A^c \cap B^c \cap C^c) = ?$

$$P(A^c \cap B^c \cap C^c) = \frac{30}{120} \Rightarrow \frac{1}{4}$$

(e): $P(A \cap B \cap C) = ?$

$$P(A \cap B \cap C) = \frac{10}{120} \Rightarrow \frac{1}{12}$$

Theorem:-

Statement:- If A and B, C are any three events then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

Proof:- we may write as

$A \cup B \cup C = A \cup (B \cup C)$

let $B \cup C = D$

$A \cup B \cup C = A \cup D$

$P(A \cup B \cup C) = P(A \cup D)$

$P(A \cup B \cup C) = P(A) + P(D) - P(A \cap D)$ — (1)

$D = B \cup C$

$P(D) = P(B \cup C)$

$P(D) = P(B) + P(C) - P(B \cap C)$ — (2)

Now

$(A \cap D) = A \cap (B \cup C)$

$A \cap D = (A \cap B) \cup (A \cap C)$

Taking Probability on both sides

$P(A \cap D) = P((A \cap B) \cup (A \cap C))$

$= P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (B \cap C))$

$P(A \cap D) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$ — (3)

using (2), (3) in (1)

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - \{P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)\}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

Hence Proved.

Conditional Probability:-

Sample space is the set of all possible outcome of random experiment. Sometime, we recieve additional information about the experiment before it occurence. The set of possible outcome (Sample Space) is reduced due to additional information. The probability of any event related to this reduced sample space is known as conditional probability.

e.g $S = \{1, 2, 3, 4, 5, 6\}$

$P[2] = \frac{1}{6}$

Information is just even no. will occure then

$S = \{2, 4, 6\}$

$P[2] = \frac{1}{3}$

If A and B are two events defined in a sample space S. The probability of event A given that event B has already occured is denoted by $P[A|B]$ defined as

$$P[A|B] = \frac{P[A \cap B]}{P[B]}, \quad P[B] \neq 0$$

$$= \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$$

$$P[A|B] = \frac{n(A \cap B)}{n(B)}$$

Example:- In rolling 2-dies

$n(S) = 36$

A = At least one five

$A = \{(1,5)(2,5)(3,5)(4,5)(5,5)(5,6)(5,4)(5,3)(5,2)(5,1)(6,5)\}$

B : Sum is 7

$B = \{(1,6)(6,1)(2,5)(5,2)(3,4)(4,3)\}$

$A \cap B = \{(2,5)(5,2)\}$

$$P[B|A] = \frac{n[A \cap B]}{n(A)} = \frac{2}{11}$$

Q. A student is selected at random to nominate as sports representative what is probability that the selected student is

	Male	Female	Total
M = Participant	2	10	12
N = Not participant	5	20	25
	7	30	37

(i): is a Female

(ii): is a female given that sports students are selected.

Sol:

F: Selected student is a Female

$P(F) = \frac{30}{37}$

m: Selected student is Female Sports man

$$P[F|m] = \frac{P[F\cap m]}{P(m)}$$

$$= \frac{n(F\cap m)}{n(m)} = \frac{10}{12} = \frac{5}{6}$$

Q. A class of MS-I consist of 41 students. 15 students study Peitabation and 20 students Viscous-I, 8 student study both Petrabation and viscous-I. A student is selected at random from the class. what is the probability that the selected student take the class of

(i) Peitabation

(ii) given that the student study peitabation. what is the probability that student study viscous-I

$n(S) = 41$

A: student study Peite

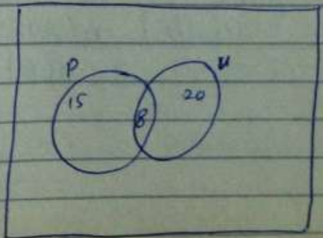
B: " " Viscous I

C: given that student study Petra

$n(A) = 15$

$n(B) = 20$

$n(A\cap B) = 8$



$$P(A) = \frac{15}{41}$$

$$P[B|A] = \frac{n(B\cap A)}{n(A)}$$

$$= \frac{8}{15}$$

$$P(B) = \frac{20}{41}$$

Q.

class	favour	against	Total
MSC-III	15	22	37
MSC-IV	8	5	13
Total	23	27	50

A student is selected at random from the group. what is probability that the selected student

(a) belong to msc III

(b) belong to msc III given that student is in favour of multimedia

A: Selected student is belong to msc III

$$P(A) = \frac{37}{50}$$

F: belong to msc III given that student is in favour of multimedia

$$P[A|F] = \frac{n(A\cap F)}{n(F)} = \frac{15}{23}.$$

$P(\lambda) = \frac{3}{10}$ → unconditional probability

$P[\lambda | S_{snow}] = \frac{2}{7}$ → conditional probability.

⇒ $P(\lambda) \neq P[\lambda | S_{snow}]$

⇒ Then it is called dependent probability.

If

$P(\lambda) = \frac{3}{10}$, $P[\lambda | S_{snow}] = \frac{3}{10}$

⇒ $P(\lambda) = P[\lambda | S_{snow}]$

⇒ Then it is called Independent Probability.

Independent Events:-

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

Two events are said to be independent if the probability of 2nd event is not effected by the probability of first event. Otherwise, event are called dependent events.

mathematically,

$P[A|B] = P[A]$ or $P[B|A] = P[B]$

III ⇒ A and B are independent.

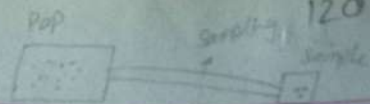
If $P[A|B] \neq P[A]$ or $P[B|A] \neq P[B]$

⇒ A and B are dependent.

With replacement Sampling

(A sampling in which size of population remain same)

unit is observed and return to population before selecting next unit.



without replacement sampling:-

(In which population size decreases)

unit is selected and not replace to population before selecting next unit. In this sampling, population size is decreases.

Example:-

R	G	W	total	Select
5	6	8	19	1

$P[R] = \frac{5}{19}$

$P[G] = \frac{6}{19}$

$P[R \cap G] = 0$

$P[R \cup G] = \frac{5}{19} + \frac{6}{19}$

Multiplication Law Of Probability:-

If A and B are two event defined in a sample space 'S'. The probability that event A and event B [Both event] will occure is denoted by $P[A \cap B]$ defined as

$P[A \cap B] = P[A]\, P[B|A]$

If A and B are independent $P[B|A] = P[B]$

$P[A \cap B] = P[A]\, P[B]$

Q: R, G, W, total, select
5, 6, 8, N 19, n 2
Find Prob that 1st marble is Red colour and 2nd is white if the Sampling is done by
(i): with replacement
(ii): without replacement.

Sol:-
Let R: Red marble is selected
W: white
$P[R \cap W] = ?$

As $P[R \cap W] = P[R]\, P[W|R]$

I). with replacement:- → Events are independent

$P[R] = \frac{5}{19}$

$P[W|R] = \frac{8}{19}$

$P[R \cap W] = \frac{5}{19} \cdot \frac{8}{19}$

II): Without replacement:-

$P[R] = \frac{5}{19}$

$P[W|R] = \frac{8}{18}$

$P[R \cap W] = \frac{5}{19} \cdot \frac{8}{18}$

(b): First Red[R] and 2nd Red[R_2]

$P[R_1 \cap R_2] = P[R_1]\, P[R_2|R_1]$:(without replacement)

$= \left(\frac{5}{19}\right)\left(\frac{5-1}{19-1}\right) \Rightarrow \frac{5}{19} \cdot \frac{4}{18}$

with replacement

$P[R_1 \cap R_2] = \left(\frac{5}{19}\right)\left(\frac{5}{19}\right)$

In without replacement → so n(S) decreases
Selection one by one

(c): Both are red (2 selected at a time)

$P[R] = \frac{\binom{5}{2}}{\binom{19}{2}}$

Example:-
What is Prob. of getting 2-heads in two flips of a balance coin?
A: 1st head and 2nd head

$P[H_1 \cap H_2] = P[H_1]\, P[H_2|H_1]$

$= P[H_1]\, P[H_2]$

$= \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{1}{4}$

chap → problem
6.40 — 6.57 (Page# 224-225)

Or $P(A) = \frac{\binom{2}{2}}{n(S)} \rightarrow \frac{1}{4}$ $\because n(S) = 4$

Chap-6.

Q.No. 40.

E_1: 6 appears on atleast one die
E_2: 5 appears on exactly one die
E_3: Same no. appear on both dice

$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)$
$(2,5)(2,6)(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)$
$(4,4)(4,5)(4,6)(5,1)(5,2)(5,3)(5,4)(5,5)(6,1)(6,2)(6,3)$
$(6,4)(6,5)(6,6)\}$ $\quad n(S) = 36$

$E_1 = \{(1,6)(2,6)(3,6)(4,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)(5,6)\}$
$n(E_1) = 11 \Rightarrow P(E_1) = 11/36$
$E_2 = \{(1,5)(2,5)(3,5)(4,5)(5,1)(5,2)(5,3)(5,4)(5,6)(6,5)\}$
$n(E_2) = 10 \Rightarrow P(E_2) = 10/36$
$E_3 = \{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$
$n(E_3) = 6 \Rightarrow P(E_3) = \frac{6}{36} = \frac{1}{6}$

(i) If two events are independent then
$P(E_1 \cap E_2) = P(E_1)\,P(E_2)$ — (1)

$E_1 \cap E_2 : \{(5,6)(6,5)\}$, $n(E_1 \cap E_2) = 2$
$P(E_1 \cap E_2) = \frac{2}{36}$

Equ. (1) $\Rightarrow \frac{2}{36} \neq \frac{11}{36} \cdot \frac{10}{36}$

$\Rightarrow E_1$ and E_2 are dependent.

(ii) Are E_2 and E_3 independent.
$E_2 \cap E_3 = 0$, $n(E_2 \cap E_3) = 0$
$P(E_2 \cap E_3) = P(E_2)\,P(E_3)$
$\frac{0}{36} \neq \frac{10}{36} \cdot \frac{1}{6}$
$\Rightarrow E_2$ and E_3 are dependent.

· If two events are independent then their complements are also independent.

$E_1 \cap E_3 = \{(6,6)\}$
$P(E_1 \cap E_3) = P(E_1)\,P(E_3)$
$\frac{1}{36} \neq \frac{11}{36} \cdot \frac{1}{6}$
$\Rightarrow E_1$ and E_3 are dependent.

Q.No. 42.

$P(A) = \frac{5}{7}$ A: A will be alive
B: B will be alive
$P(B) = \frac{7}{9}$, $P(B^c) = 1 - P(B) = 1 - \frac{7}{9} = \frac{2}{9}$
$P(A^c) = 1 - P(A) = 1 - \frac{5}{7} = \frac{2}{7}$

C: both of them will die
D: A will be alive and B dead
E: B will be alive and A dead
F: both of them will be alive

$F = A \cap B$
$P(F) = P(A \cap B)$
$= P(A)\,P(B) = \frac{5}{\cancel{7}} \cdot \frac{\cancel{7}}{9} \Rightarrow \frac{5}{9}$

$C = A^c \cap B^c = (A \cup B)^c$ OR $P(C) = P(A^c \cap B^c)$
$= P(A^c) \cdot P(B^c)$
$P(C) = \frac{2}{7} \cdot \frac{2}{9} \Rightarrow \frac{4}{63}$

$P(C) = P((A \cup B)^c)$
$= 1 - P(A \cup B)$
As $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{5}{7} + \frac{7}{9} - \frac{5}{9} \Rightarrow \frac{45 + 49 - 35}{63} \Rightarrow \frac{59}{63}$

$P(C) = 1 - \frac{59}{63} \Rightarrow \frac{63 - 59}{63} \Rightarrow \frac{4}{63}$

$P[D/F] = \frac{P[D \cap F]}{P[F]}$

· If two events are independent then their complements are also independent.

$E_1 \cap E_3 = \{(6,6)\}$

$P(E_1 \cap E_3) = P(E_1)\,P(E_3)$

$\frac{1}{36} \neq \frac{11}{36} \cdot \frac{1}{6}$

$\Rightarrow E_1$ and E_3 are dependent.

Q.No. 42

$P(A) = \frac{5}{7}$ A: A will be alive

B: B will be alive

$P(B) = \frac{7}{9}$, $P(B^c) = 1 - P(B) = 1 - \frac{7}{9} = \frac{2}{9}$

$P[A^c] = 1 - P[A] = 1 - \frac{5}{7} = \frac{2}{7}$

C: both of them will die

D: A will be alive and B dead.

E: B will be alive and A dead.

F: both of them will be alive

$F = A \cap B$

$P(F) = P(A \cap B)$

$= P(A)\,P(B) = \frac{5}{\cancel{7}} \cdot \frac{\cancel{7}}{9} \Rightarrow \frac{5}{9}$

$C = A^c \cap B^c = (A \cup B)^c$ OR $P(C) = P(A^c \cap B^c)$

$P(C) = P((A \cup B)^c)$ $= P(A^c) \cdot P(B^c)$

$= 1 - P(A \cup B)$ $P(C) = \frac{2}{7} \cdot \frac{2}{9} \Rightarrow \frac{4}{63}$

As $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{7} + \frac{7}{9} - \frac{5}{9} \Rightarrow \frac{45 + 49 - 35}{63} \Rightarrow \frac{59}{63}$

$P(C) = 1 - \frac{59}{63} \Rightarrow \frac{63 - 59}{63} \Rightarrow \frac{4}{63}$

$P[D|F] = \frac{P[D \cap F]}{P[F]}$

$P[D|F] = \frac{P[A \text{ will be alive}]}{P[F]}$

$P[D|F] = \frac{5/7}{5/9} \Rightarrow \frac{9}{7}$

$P[E|F] = \frac{P[E \cap F]}{P[F]} \Rightarrow \frac{P[B \text{ will be alive}]}{P[F]}$

$P[E|F] = \frac{7/9}{5/9} \Rightarrow$

$P(D) = P[A \cap B^c] = P[A]\,P[B^c]$

$= \frac{5}{7} \cdot \frac{2}{9} \Rightarrow \frac{10}{63}$

$P(E) = P[B \cap A^c] = P[B]\,P[A^c]$

$= \frac{7}{9} \cdot \frac{2}{7} \Rightarrow \frac{14}{63} \Rightarrow \frac{2}{9}$ Ans!

Q.No. 43

(b): U_1: 3 white balls, 2 black U_2: 5 white, 3 black

$E = (U_1 \cap W)$ or $(U_2 \cap W)$

$P(E) = P[U_1 \cap W] + P[U_2 \cap W]$

$= P[U_1]\,P[W|U_1] + P[U_2]\,P[W|U_2]$

$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{5}{8}$

$= \frac{3}{10} + \frac{5}{16} \Rightarrow \frac{48 + 50}{160} \Rightarrow \frac{98}{160} \Rightarrow \frac{49}{80}$

$P[D|F] = \frac{P[A \text{ will be alive}]}{P[F]}$

$P[D|F] = \frac{5/7}{3/9} \Rightarrow \frac{9}{7}$

$P[E|F] = \frac{P[E \cap F]}{P[F]} \Rightarrow \frac{P[B \text{ will be alive}]}{P[F]}$

$P[E|F] = \frac{7/9}{5/9} \Rightarrow$

$P(D) = P[A \cap B^c] = P[A]\,P[B^c]$

$= \frac{5}{7} \cdot \frac{2}{9} \Rightarrow \frac{10}{63}$

$P(E) = P[B \cap A^c] = P[B]\,P[A^c]$

$= \frac{7}{9} \cdot \frac{2}{7} \Rightarrow \frac{14}{63} \Rightarrow \frac{2}{9}$ Ans!

Q.No. 43

(b):

U_1	U_2
3 white ball, 2 black	5 white, 3 black

$E = (U_1 \cap W)$ or $(U_2 \cap W)$

$P(E) = P[U_1 \cap W] + P[U_2 \cap W]$

$= P[U_1].P[W|U_1] + P[U_2].P[W|U_2]$

$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{5}{8}$

$= \frac{3}{10} + \frac{5}{16} \Rightarrow \frac{48+50}{160} \Rightarrow \frac{98}{160} \Rightarrow \frac{49}{80}$

42, 43

Q.No. 44.

A: containing 5 white and 8 black balls

white	black	total	select
5	8	13	3

$n(S) = {}^{13}C_3 = 286$

A: white balls are selected, $n(A) = \binom{5}{3} = 10$

B: Black, $n(B) = \binom{8}{3} = 56$

$P[A \cap B] = ?$

$P(A) = \frac{10}{286}$

due to without replacement

$P[B|A] = \frac{56}{\binom{10}{3}} \Rightarrow \frac{56}{120}$

$P[A \cap B] = P[A]\,P[B|A] = \frac{10}{286} \cdot \frac{56}{120} \Rightarrow \frac{7}{429}$

Q.No. 43

(a)

	bag 1	bag 2	Total
Red	3	4	7
black	5	7	12
Total	8	11	19

$P[R] = \frac{7}{19}$

(b):

	Q.A		total
white	3	5	8
black	2	3	5
total	5	8	13

$P[W] = \frac{8}{13}$

42, 43

Q.No.44.

A: containing 5 white and 8 black balls

white	black	total	select
5	8	13	3

$n(S) = {}^{13}C_3 = 286$

A: white balls are selected, $n(A) = \binom{5}{3} = 10$

B: Black, $n(B) = \binom{8}{3} = 56$

$P[A \cap B] = ?$

$P(A) = \frac{10}{286}$

due to without replacement

$P[B|A] = \frac{56}{\binom{10}{3}} \Rightarrow \frac{56}{120}$

$P[A \cap B] = P[A]\, P[B|A] = \frac{10}{286} \cdot \frac{56}{120} \Rightarrow \frac{7}{429}$

Q.No.43

(a)

	bag 1	bag 2	Total
Red	3	4	7
black	5	7	12
Total	8	11	19

$P[R] = \frac{7}{19}$

(b):

	Q.A		total
white	3	5	8
black	2	3	5
total	5	8	13

$P[W] = \frac{8}{13}$

Q.No.46

	girls	boys	total	select
G_1	3	1	4	3
G_2	2	2	4	
G_3	1	3	4	
total	6	6	12	

E_1 : 1 girl and 2 boys

Possibilities:
- A = 1 girl from 1st, 1 boy from 2^{nd}, 1 boy from 3^{rd}
- B = 1 boy from 1st, 1 girl from 2^{nd}, 1 boy from 3^{rd}
- C = 1 boy from 1st, 1 boy from 2^{nd}, 1 girl from 3^{rd}

$P[1 \text{ girl and } 2 \text{ boys}] = P[(g|A)]P(b|B)P(b|C)] + P(b|A)P(g|B)P(b|C) + P(b|A)P(b|B)P(g|C)$

$P(A) = \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} = \frac{9}{32}$ $\quad = P[E_1] = P[A] + P[B] + P[C]$

$P(B) = \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} = \frac{3}{32}$

$P(C) = \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{1}{4} = \frac{1}{32}$

$P(E) = P(A) + P(B) + P(C)$

$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} \Rightarrow \frac{13}{32}$ Ans.

Q.No.45

(a): $P(A \cap B \cap C) = P(A|B \cap C)\, P(B|C)\, P(C)$

$A \cap B \cap C = (B \cap C) \cap A$

let $D = B \cap C$

$A \cap B \cap C = D \cap A$

$P(A \cap B \cap C) = P(D \cap A)$

$= P(D)\, P(A|D)$ $\quad P(A) \cdot P(D|A)$

$= P(B \cap C)\, P(A|B \cap C)$

$= P(B)\, P(C|B)\, P(A|B \cap C)$

Q.NO.43

(a):

B_1

R	B
3	5

B_2

R	B
4	7

$E = (B_1 \text{ and } R) \text{ or } (B_2 \text{ and } R)$

$P[E] = P(B_1 \cap R] + P[B_2 \cap R]$

$= P[B_1] \, P[R|B_1] + P[B_2] \, P[R|B_2]$

$P[E] = \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{11} \Rightarrow \frac{3}{16} + \frac{2}{11} \Rightarrow \frac{33+32}{176}$

$P(E) = \frac{65}{176}$

Q.No.49

(a): $P(A) = \frac{4}{5}$ $\qquad P(A^c) = 1 - \frac{4}{5} \Rightarrow \frac{1}{5}$

$P(B) = \frac{3}{4}$ $\qquad P(B^c) = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$

$P(C) = \frac{2}{3}$ $\qquad P(C^c) = 1 - \frac{2}{3} = \frac{1}{3}$

E: at least two hits
or A hit and B hit and C not hit
or A hit and B not hit and C hit
or A not hit and B hit and C hit
or A hit B hit & C hit

$P[E] = P(A)P(B)P(C^c) + P(A)P(B^c)P(C) + P(A^c)P(B)P(C) + P(A)\,P(B)\,P(C)$

$P[E] = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{\cancel{3}}{\cancel{4}_2} \cdot \frac{\cancel{2}}{\cancel{3}} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3}$

$= \frac{1}{5} + \frac{2}{15} + \frac{1}{10} + \frac{2}{5} \Rightarrow \frac{6+4+3+12}{30} \Rightarrow \frac{25}{30} \Rightarrow$

$P(E) = \frac{5}{6}$

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Q.NO.43

(a):

B_1

R	B
3	5

B_2

R	B
4	7

$E = (B_1 \text{ and } R) \text{ or } (B_2 \text{ and } R)$

$P[E] = P(B_1 \cap R] + P[B_2 \cap R]$

$= P[B_1] \, P[R|B_1] + P[B_2] \, P[R|B_2]$

$P[E] = \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{11} \Rightarrow \frac{3}{16} + \frac{2}{11} \Rightarrow \frac{33+32}{176}$

$P(E) = \frac{65}{176}$

Q.No.49

(a): $P(A) = \frac{4}{5}$ $\qquad P(A^c) = 1 - \frac{4}{5} \Rightarrow \frac{1}{5}$

$P(B) = \frac{3}{4}$ $\qquad P(B^c) = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$

$P(C) = \frac{2}{3}$ $\qquad P(C^c) = 1 - \frac{2}{3} = \frac{1}{3}$

E: at least two hits
or A hit and B hit and C not hit
or A hit and B not hit and C hit
or A not hit and B hit and C hit
or A hit B hit & C hit

$P[E] = P(A)P(B)P(C^c) + P(A)P(B^c)P(C) + P(A^c)P(B)P(C) + P(A)\,P(B)\,P(C)$

$P[E] = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{\cancel{3}}{\cancel{4}_2} \cdot \frac{\cancel{2}}{\cancel{3}} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3}$

$= \frac{1}{5} + \frac{2}{15} + \frac{1}{10} + \frac{2}{5} \Rightarrow \frac{6+4+3+12}{30} \Rightarrow \frac{25}{30} \Rightarrow$

$P(E) = \frac{5}{6}$

50 (ii) $P(A)P(B^c)P(C^c)$

Q.No.52

$P(A) = \frac{6}{12} \Rightarrow \frac{1}{2} \Rightarrow P[A^c] = 1 - \frac{1}{2} \Rightarrow \frac{1}{2}$

$P(B) = \frac{4}{12} \Rightarrow \frac{1}{3} \Rightarrow P(B^c) = 1 - \frac{1}{3} \Rightarrow \frac{2}{3}$

$P(T) = \frac{2}{12} \Rightarrow \frac{1}{6} \Rightarrow P(T^c) = 1 - \frac{1}{6} \Rightarrow \frac{5}{6}$

(i) P(A wins all three games) = ?

- $P(A \cap A \cap A) = P(A)\,P(A)\,P(A)$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \Rightarrow \frac{1}{8}$

(ii) P(Two games end in a tie) = ?

$= P(T^c \cap T \cap T) + P(T \cap T^c \cap T) + P(T \cap T \cap T^c)$

$= P(T^c)\,P(T)\,P(T) + P(T)\,P(T^c)\,P(T) + P(T)\,P(T)\,P(T^c)$

$= \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6}$

$= \frac{5}{216} + \frac{5}{216} + \frac{5}{216} \Rightarrow \frac{15}{216} \Rightarrow$

$= \frac{5}{72}$

(iii) P(A and B wins alternately)

$= P(A \cap B \cap A) + P(B \cap A \cap B)$

$= P(A)\,P(B)\,P(A) + P(B)\,P(A)\,P(B)$

$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3}$

$= \frac{1}{12} + \frac{1}{18} \Rightarrow \frac{3+2}{36} \Rightarrow$

$= \frac{5}{36}$

(iv): P(B wins atleast one game)

$= 1 - P(\text{B wins no game})$

$= 1 - P(B^c \cap B^c \cap B^c) \Rightarrow 1 - \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \Rightarrow 1 - \frac{8}{27} \Rightarrow \frac{19}{27}$

Sessional II → (Probability) (Q.5 → 45)

Q.No.47

	boys(B)	girls(G)	total
F_1 :	2	2 → G_1	4
F_2 :	3	1 → G_2	4
F_3 :	1	3 → G_3	4

E_1: only girls turn up for party

E_2: Two girls and one boy turn up for party

$P(E_1) = P(G_1 \cap G_2\, G_3)$

$P(E_1) = P(G_1) \cdot P(G_2) \cdot P[G_3]$

$= \frac{\cancel{2}}{\cancel{4}_2} \cdot \frac{1}{4} \cdot \frac{3}{4}$

$P[E_1] = \frac{3}{32}$

E_2 : two girls and one boy turn up for party.

Possibilities are

A: 1 boy from F_1, 1 girls from F_2, 1 girl from F_3

B: 1 girl from F_1, 1 boy from F_2, 1 girl from F_3

C: 1 girl from F_1, 1 girl from F_2, 1 boy from F_3

$P(A) = P[b|F_1]\,P[g|F_2]\,P[g|F_3]$

$= \frac{\cancel{2}}{\cancel{4}_2} \cdot \frac{1}{4} \cdot \frac{3}{4} \Rightarrow \frac{3}{32}$

$P[B] = P[g|F_1]\,P[b|F_2]\,P[g|F_3]$

$= \frac{\cancel{2}}{\cancel{4}_2} \cdot \frac{3}{4} \cdot \frac{3}{4} \Rightarrow \frac{9}{32}$

$P[C] = P[g|F_1]\,P[g|F_2]\,P[b|F_3]$

$= \frac{\cancel{2}}{\cancel{4}_2} \cdot \frac{1}{4} \cdot \frac{1}{4} \Rightarrow \frac{1}{32}$

$P[E] = P[A] + P[B] + P[C]$

$= \frac{3}{32} + \frac{9}{32} + \frac{1}{32} \Rightarrow \frac{3+9+1}{32} \Rightarrow \frac{13}{}$

Q.No.50

A: 1st man hits the target
B: 2nd man
C: 3rd man

$P(A) = \frac{1}{6}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{3}$

$P(A^c) = 1 - \frac{1}{6}$, $P(B^c) = 1 - \frac{1}{4}$, $P(C^c) = 1 - \frac{1}{3}$

$P(A^c) = \frac{5}{6}$, $P(B^c) = \frac{3}{4}$, $P(C^c) = \frac{2}{3}$

(i) P(Exactly one hits the target)
D: Exactly one hits the target

$P(D) = P(A \cap B^c \cap C^c) + P(A^c \cap B \cap C^c) + P(A^c \cap B^c \cap C)$

$= \frac{1}{6} \cdot \frac{3}{4_2} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{1}{4_2} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{3}$

$= \frac{1}{12} + \frac{5}{36} + \frac{5}{24} \Rightarrow$

$P(D) = \frac{31}{72}$

(ii) P(if only one hits target, then it was 1st man)
i.e $P[A|D] = \frac{P(A \cap D)}{P(D)}$

$= \frac{P(A \cap B^c \cap C^c)}{P(D)} \Rightarrow \frac{\frac{1}{6} \cdot \frac{3}{4_2} \cdot \frac{2}{3}}{31/72}$

$= \frac{1/12}{31/72_6} \Rightarrow \frac{6}{31}$ Ans!

Q.No.51

Let m_1 : 1st missile hits the target
m_2 : 2nd missile " "
m_3 : 3rd missile " "

$P(m_1) = 0.4 \Rightarrow P(m_1^c) = 1 - .4 = 0.6 \Rightarrow P(m_1^c) = P(m_3)$

$P(m_2) = 0.5 \Rightarrow P(m_2^c) = 1 - 0.5 = 0.5 \Rightarrow P(m_2^c) = P(m_2)$

$P(m_3) = 0.6 \Rightarrow P(m_3^c) = 1 - 0.6 = .4 \Rightarrow P(m_3^c) = P(m_1)$

Events are
(i) A: All missiles hits the target

$P(A) = P(m_1 \cap m_2 \cap m_3)$
$= P(m_1) P(m_2) P(m_3)$
$= (0.4)(0.5)(0.6)$
$= 0.12$

(ii): B: At least one of the three hits target

$P(B) = 1 - P(\text{no missile hits the target})$
$= 1 - P(m_1^c \cap m_2^c \cap m_3^c)$
$= 1 - P(m_1^c) P(m_2^c) P(m_3^c)$
$= 1 - (0.6)(0.5)(0.4) \Rightarrow 1 - 0.12$

$P(B) = 0.88$

(iii): C: Exactly one hits the target.
Possibilities are

C: $(m_1 \cap m_2^c \cap m_3^c)$ or $(m_1^c \cap m_2 \cap m_3^c)$ or $(m_1^c \cap m_2^c \cap m_3)$

$P(C) = P(m_1 \cap m_2^c \cap m_3^c) + P(m_1^c \cap m_2 \cap m_3^c) + P(m_1^c \cap m_2^c \cap m_3)$
$= (0.4)(0.5)(0.4) + (.6)(.5)(.4) + (.6)(.5)(.6)$

$P(C) = 0.38$

(iv): D: Exactly 2 missiles hit the target.
Possibilities are

D: $(m_1 \cap m_2 \cap m_3^c)$ or $(m_1 \cap m_2^c \cap m_3)$ or $(m_1^c \cap m_2 \cap m_3)$

$P(D) = P(m_1 \cap m_2 \cap m_3^c) + P(m_1 \cap m_2^c \cap m_3) + P(m_1^c \cap m_2 \cap m_3)$
$= (.4)(.5)(.4) + (.4)(.5)(.6) + (.6)(.5)(.6)$

$P(D) = 0.38$ Ans!

Q.No.41

(a): A: A can solve problem
B: B can solve problem

$P(A) = 75\% = \frac{75}{100}$

$P(B) = 70\% = \frac{70}{100}$

P[A or B can solve problem] = $P[A \cup B] = ?$

$P[A\cap B] = P[A]\,P[B]$

$= \frac{75}{100} \cdot \frac{70}{100} \Rightarrow \frac{21}{40}$

$P[A\cup B] = P[A] + P(B) - P(A\cap B)$

$= \frac{75}{100} + \frac{70}{100} - \frac{21}{40} \Rightarrow \frac{30+28-21}{40} \Rightarrow$

$P(A\cup B) = \frac{37}{40}$

(b):

$n(S) = 52$

A: 1st card is red ace , $n(A) = 2$

B: 2nd is 10 or Jack , $n(B) = 4+4 = 8$

C: 3rd card is $3 < \text{card} < 7$, $n(C) = 12$

$P(A) = \frac{2}{52}$

$P(B|A) = \frac{8}{51}$

$P[C|A\cap B] = \frac{12}{50}$

$P(A\cap B\cap C) = P(A)\,P(B|A)\,P[C|A\cap B]$

$= \frac{2}{52} \cdot \frac{8}{51} \cdot \frac{12}{50}$

$P(A\cap B\cap C) = \frac{24}{16575}$

Q. No. 45

(b): A : Spotted egg is choosen in 1st draw

B : Spotted egg is choosen in 2nd draw

C : clear egg is choosen

$P(A) = \frac{5}{30} \Rightarrow \frac{1}{6}$

$P(B|A) = \frac{5-1}{30-1} \Rightarrow \frac{4}{29}$

$P[C|A\cap B] = \frac{30-5}{280} \Rightarrow \frac{25}{28}$

$P(A\cap B\cap C) = P(A)\,P(B|A)\,P(C|A\cap B)$

$= \frac{1}{6} \cdot \frac{4}{29} \cdot \frac{25}{28}$

$= \frac{25}{1218}$ Ans!

Q. No. 56.

(a):

$S = \{6 \text{ coins are tossed}\}$

$n(S) = 2^6 = 64$

A: Exactly 4 heads obtained

$n(A) = \binom{6}{4} = 15$

$P(A) = \frac{15}{64}$

(b):

$S = \{16 \text{ coins are tossed}\}$

$n(S) = 2^{16} = 65536$

B: Exactly 8 heads are obtained

$n(B) = \binom{16}{8} = 12870$

$P(B) = \frac{12870}{65536} \Rightarrow \frac{6435}{32768}$

C: Exactly 11 heads are obtained

$n(C) = \binom{16}{11} = 4368$

$P(C) = \frac{4368}{65536} \Rightarrow \frac{273}{4096}$

Assignment (Prove that thm)

Theorem.-

Let $\{C_n\}$ be the nondecreasing sequence of events, then

$$\lim_{n\to\infty} P(C_n) = P(\lim_{n\to\infty} C_n) = P(\bigcup_{n=1}^{\infty} C_n)$$

Theorem:- Let $\{C_n\}$ be the decreasing sequence of events, then

$$\lim_{n\to\infty} P(C_n) = P(\lim_{n\to\infty} C_n) = P(\bigcap_{n=1}^{\infty} C_n)$$

Theorem :- Let $\{C_n\}$ be an arbitrary sequence of events Then

$$P(\bigcup_{n=1}^{\infty} C_n) \leq \sum_{n=1}^{\infty} P(C_n)$$

Q.No.49

(b):

$P(A) = .05$, $P(B) = .05$, $P(C) = .10$

Let each member vote independently

$P(A^c) = .95$, $P(B^c) = .95$, $P(C^c) = .90$

Let E be event that wrong dicision on basis of majority vote

$$P(E) = P(A\cap B\cap C^c) + P(A\cap B^c\cap C) + P(A^c\cap B\cap C) + P(A\cap B\cap C)$$

$= .00225 + .00475 + .00475 + .00025$

$= .012$

This indicates (must) that committee will be wrong in 1.2% of its dicision.

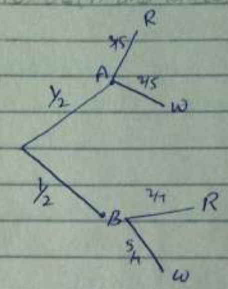
Q.No.53

A	
red	white
3	2

B	
red	white
2	5

Select A or B

(A and R) or (A and white) OR $(B\cap R)$ or $(B\cap W)$



Let A be the event that urn A is selected

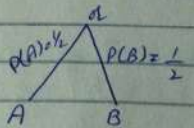
, B , , , urn B is .

Let W_1 be event that white ball is drawn from selected urn.

Let R_1 , , Red ball , .

Let W_2 ..... , from other urn

Let R_2 " " ... " ..



R_1	W_1	T
3	2	5

2/5 W_1 3/5 R_1

R	W	T
2	6	8

$W_2 = 6/8$

R	W	T
3	5	8

$R = 3/8$

R	W	T
2	5	7

2/7 R_1 5/7 W_1

R	W	T
4	2	6

$R_2 = 4/6$

R	W	T
3	3	6

$W_2 = 3/6$

P(both balls are of same colour) = ?

$= P(A \cap W_1 \cap W_2) + P(A \cap R_1 \cap R_2) + P(B \cap R_1 \cap R_2) + P(B \cap W_1 \cap W_2)$

$= P(A).P(W_1|A).P(W_2|A \cap W_1) + P(A)\,P(R_1|A_1).P(R_2|A \cap R_1)$
$+ P(B)\,P(R_1|B).P(R_2|R_1 \cap B) + P(B).P(W_1|B).P(W_2|B \cap W_1)$

$= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)\left(\frac{6}{8}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{5}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{7}\right)\left(\frac{4}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{8}{7}\right)\left(\frac{3}{6}\right)$

$= \frac{12}{80} + \frac{9}{80} + \frac{8}{84} + \frac{15}{84} \Rightarrow \frac{901}{1680}$

Q.No.54

$n(S) = (30)^6 = 729000000$

A : None of friends born on same day

$n(A) = 30 \times 29 \times 29 \times 27 \times 26 \times 25 = 427518000$

$P(A) = \frac{n(A)}{n(S)} = \frac{427518\cancel{000}}{729\cancel{000000}} \Rightarrow 0.586$ Ans!

Q.No. 55

(a): Probability of throwing a head with a coin $= 1/2$

A : 1st , 3rd , 5th $\to p = 1/2$ $\;p = 1 - q$

B : 2nd , 4th , 6th $\to q = 1/2$

$P(A\text{'s winning}) = \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$

$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\right]$

$= \frac{1}{2}\left[\frac{1}{1-\left(\frac{1}{2}\right)^2}\right] \Rightarrow \frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right] \Rightarrow$

$= \frac{1}{2}\left[\frac{1}{\frac{3}{4}}\right] \Rightarrow \frac{1}{2}\left[\frac{4}{3}\right] \Rightarrow$

$= \frac{2}{3}$

$P(B\text{'s winning}) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^6 + \ldots$

$= \left(\frac{1}{2}\right)^2\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\right]$

$= \frac{1}{4}\left[\frac{1}{1-\left(\frac{1}{2}\right)^2}\right] \Rightarrow \frac{1}{4}\left[\frac{4}{3}\right] \Rightarrow$

$= \frac{1}{3}$ Ans.

(b):-

$p = P(\text{success}) = P(\text{head}) = 1/2$

$q = 1 - p = 1 - 1/2 = 1/2$

$P(\text{success in } k^{th} \text{ attempt}) = q^{k-1}p \quad \to (1)$

let A, B, C be three men. Now

A can win in 1st, 4th, 7th attempts + ---
B " " in 2nd, 5th, 8th attempts + ---
C " " 3rd, 6th, 9th ---

using (1)

$P(A) = q^{1-1}p + q^{4-1}p + q^{7-1}.p + \ldots$

$P(A) = q^0 p + q^3 p + q^6 p + \ldots \quad -(*)$

The finite geometric series

$a = p \quad r = \frac{q^3 p}{p} = q^3$

$P(A) = \frac{a}{1-r} \Rightarrow \frac{p}{1-q^3} \Rightarrow \frac{1/2}{1-(1/2)^3} \Rightarrow \frac{1/2}{1-1/8} \Rightarrow$

$P(A) = \frac{4}{7}$

Again using (1)

$P(B) = q^{2-1}p + q^{5-1}.p + q^{8-1}.p + \ldots$

$= q(p + q^3 p + q^6 p + \ldots)$

$P(B) = q(P(A))$ using ★

$P(B) = \frac{1}{2}\left(\frac{4}{7}\right) \Rightarrow \frac{2}{7}$

We know that $P(A)+P(B)+P(C)=1$

$P(C) = 1 - P(A) - P(B)$

$P(C) = 1 - \frac{4}{7} - \frac{2}{7} \Rightarrow \frac{1}{7}$ Ans.

Q.No. 57

Let P denote the Prob. of passing an examination. Then $P = 0.4$ & $q = 1 - P = 0.6$.

Here $n = 6$.

(i) P (2 Candidates will pass).

$\binom{6}{2}(0.4)^2(0.6)^{6-2}$

$= 15(0.4)^2(0.6)^4 = 0.31104$

(ii) P(5 candidates will pass) =

$= \binom{6}{5}(0.4)^5(0.6)^{6-5}$

$= 6.(0.4)^5(0.6) = 0.036864 = 0.04$.

Now P(all candidates pass) $= P^6$

P(all " fail) $= q^6$

But $P = 0.4$ and $q = 0.6$. Prob are not equal.

The Prob of all passing would be equal to prob of all failing when

$P = q = 0.5$

2nd Sessional

Random Variable:-

Population
↙ ↘
Real pop — Hypothetical pop
↓ ↓
Graphical — Random experiment
measure of C.T ↓
" " Dispersion — Random Variable
" " Skewness
" " Kurtosis

$S = \{H, T\}$ ∴ X is no. of head

events	X	Prob
Null event		0
H	1	½
T	0	½
Sure event		1

$\therefore P(H) = P(X=1) = \frac{1}{2}$

$P(T) = P(X=0) = \frac{1}{2}$

(Domain belong to sample space and Range belong [0,1])

Definition (Random Variable):-

"A Variable whose values are determined from random experiment."

e.g no. of Heads in tossing 2 coin

$S = \{HH, HT, TH, TT\}$

X = no. of Heads

X = 0, 1, 2

Another Definition:- A random variable is a fun whose domain from sample space and Range (counter domain) in [0,1] e.g.(i)

x	$P(x)$
0	$1/2$
1	$1/2$

e.g. (ii) $X \to$ no. of head
Prob distribution or $p(x)$

Event	X	$f(x)$	$F(x) = P(X \le x)$
TT	0	$1/4$	$1/4 = P(X \le 0)$
TH, HT	1	$2/4$	$3/4 = P(X \le 1)$
HH	2	$1/4$	$4/4 = P(X \le 2)$
		1	

- $F(x) = \sum^{x} f(x)$

$P(X=2) = P(HH) = 1/4$
$P(X=0) = P(TT) = 1/4$
$P(X=1) = P(TH, HT) = 2/4$

Prob. fun.

$$\boxed{P(x) = \frac{\binom{2}{x}}{4}, \quad x = 0, 1, 2}$$

- $F(x)$ is distribution function. and $F(x) = \sum^{x} f(x)$

- $P(X \le 1) = P(X=0) + P(X=1)$

$F(0) = P(X=0) = 1/4$
$F(1) = P(X \le 1) = 3/4$
$F(2) = P(X \le 2) \Rightarrow P(X=0) + P(X=1) + P(X=2) = \frac{4}{4}$
$= \frac{1}{4} + \frac{2}{4} + \frac{1}{4}$

Example:

R	W	T	n
5	10	15	2

X = no. of Red balls
$X = 0, 1, 2$, $n(S) = \binom{15}{2} =$

X	
0	$\binom{10}{2} / \binom{15}{2}$
1	$\binom{5}{1}\binom{10}{1} / \binom{15}{2}$
2	$\binom{5}{2}\binom{10}{0} / \binom{15}{2}$

7.4(4) Example:- Three balls are drawn from a bag containing 5 white and 3 black balls. If X denotes the no. of white balls draw from the bag. Then find the prob. distribution of X.

W	B	Total	Select
5	3	8	3

X = no. of white balls
$X = 0, 1, 2, 3$
$n(S) = \binom{8}{3}$

X	
0	$\binom{5}{0}\binom{3}{3} / \binom{8}{3}$
1	$\binom{5}{1}\binom{3}{2} / \binom{8}{3}$
2	$\binom{5}{2}\binom{3}{1} / \binom{8}{3}$
3	$\binom{5}{3}\binom{3}{0} / \binom{8}{3}$

Extra

3): A coin is tossed twice. what is probability that atleast one tail occur.
$S = \{HH, TH, HT, TT\}$
X = no. of tails occur
$X = 0, 1, 2$
$n(S) = 4$

X	Event	$f(x)$	$F(x)$
1	HT, TH	$2/4$	$2/4$
0	HH	$1/4$	$3/4$
2	TT	$1/4$	$4/4$

$P(X=2) = P(TT) = 1/4$
$P(X=1) = P(HT, TH) = 2/4$
$P(X=0) = P(HH) = 1/4$

Probability Function:-

Any function $f(x)$ that satisfies the following conditions is called probability function. The probability fun. for discrete random variable (d.r.v) is known as Probability mass function (P.m.f). The probability fun. for continuous random variable (c.r.v) is known as Probability density function

data can measure which is for (d.r.v)

countable data which is for (c.r.v)

$1 < x < 2 \rightarrow$ continous

$x = 1, 2 \rightarrow$ discrete

- If fun. pmf and $1 \le x \le 3$ then it means $x = 1, 2, 3$

Conditions (Probability function)

(1) $f(x) \geq 0 \quad \forall x$

(2) $\sum_x f(x) = 1$ → for d.r.v $\int_x f(x)dx = 1$ → for c.r.v

Example:-

X	P(X)	P(X)
-1	3C	$3(\frac{1}{12}) = \frac{1}{4}$
0	3C	$= \frac{1}{4}$
1	6C	$= \frac{1}{2}$
	$=\frac{1}{12C}$	

find values of C , $P(X \le 0) = ?$ $\quad \therefore 12C = 1$ $\quad \boxed{C = \frac{1}{12}}$

$P(X \le 0) = P(X = 0) + P(X = -1)$

$= \frac{3}{12} + \frac{3}{12} = \frac{6}{12} = \frac{1}{2}$

7.7) Example:- A continuous random variable has p.d.f.

$$f(x) = \begin{cases} c(2-x)(2+x) & ; \quad 0 < x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Check $f(x) = \begin{cases} 2(3-x^2) & \text{if} \quad 0 \le x \le 5 \\ 0 \end{cases}$

$f(x)$ is p.d.f ? then find $\int_0^5 f(x)\,dx$

find $P[x = \frac{1}{2}]$, $P[x \le 1]$, $P[x \ge 2]$, $P[1 \le x \le 2]$, $P[x \ge \frac{1}{2}]$

$$\int_0^2 c(2-x)(2+x)\,dx = 1$$

$$\int_0^2 c(2-x)(2+x)dx = \int_0^2 c(4-x^2)dx$$

$$= c\left\{4|x|_0^2 - \left|\frac{x^3}{3}\right|_0^2\right\}$$

$$= c\left\{4(2-0) - \frac{1}{3}(8-0)\right\}$$

$$= c\left\{8 - \frac{8}{3}\right\} \Rightarrow c\left\{\frac{24-8}{3}\right\} \Rightarrow$$

$$= c\left(\frac{16}{3}\right)$$

$$\Rightarrow c\left(\frac{16}{3}\right) = 1 \Rightarrow \boxed{C = \frac{3}{16}}$$

(i): $P[x = \frac{1}{2}] = 0 = \int_{1/2}^{1/2} f(x)\,dx$

$= 0$

(ii): $P(x \le 1) = \int_{-\infty}^{0} \cancel{f(x)}^{0} dx + \int_0^1 f(x)\,dx$

$= \int_0^1 c(4-x^2)dx \Rightarrow \int_0^1 \frac{3}{16}(4-x^2)dx \Rightarrow \frac{3}{16}\left\{4|x|_0^1 - \left|\frac{x^3}{3}\right|_0^1\right\} \Rightarrow$

$= \frac{3}{16}\left\{4 - \frac{1}{3}\right\} \Rightarrow \frac{3}{16}\left\{\frac{12-1}{3}\right\} \Rightarrow \frac{11}{16}$

(iii): $P(x \ge 2) = \int_2^{\infty} f(x)\,dx = 0$

(iv): $P(1 \le x \le 2) = \int_1^2 f(x)\,dx$

$= \int_1^2 \frac{3}{16}(4-x^2)dx \Rightarrow \frac{3}{16}\left\{4|x|_1^2 - \left|\frac{x^3}{3}\right|_1^2\right\} \Rightarrow$

$= \frac{3}{16}\left\{4(2-1) - \frac{1}{3}(8-1)\right\} \Rightarrow \frac{3}{16}\left\{4 - \frac{7}{3}\right\} \Rightarrow \frac{3}{16}\left\{\frac{12-7}{3}\right\} \Rightarrow \frac{5}{16}$

• If ${}_{a}\int f(x)\,dx = 1$ then $f(x)$ is p.d.f.

v) $P(X \geq \frac{1}{2}) = \int_{1/2}^{2} f(x)\,dx + \int_{2}^{\infty} \overset{0}{\cancel{f(x)}}\,dx$

$= \int_{1/2}^{2} \frac{3}{16}(4-x^2)\,dx$

$= \frac{3}{16}\left\{ \left| 4x - \frac{x^3}{3} \right|_{1/2}^{2} \right\}$

$= \frac{3}{16}\left\{ \left(8 - \frac{8}{3}\right) - \left(\frac{4}{2} - \frac{1}{24}\right) \right\}$

$= \frac{3}{16}\left\{ \frac{16}{3} - \left(\frac{48-1}{24}\right) \right\} \Rightarrow \frac{3}{16}\left\{ \frac{128-47}{248} \right\}$

$= \frac{1}{16}\left\{ \frac{81}{8} \right\} \Rightarrow \frac{81}{128}$

e.g. $f(x) = \binom{3}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x}$. $x = 0, 1, 2, 3.$

$f(x) > 0.$

$f(x) = \frac{3}{16}(4-x^2)$. $0 \leq x \leq 2$

$f(x) \geq 0.$

Example: Let X denotes the number of Head and Y denotes the number of tail in Tossing a coin. Find P.d of (X, Y).

tail

X \ Y (head)	0	1	P(X)
0	0	1/2	1/2
1	1/2	0	1/2
P(Y)	1/2	1/2	1

P(X=0 and Y=0)
P(no head & no tail) = 0
p(no head & 1 tail] = 1/2
p(1 head & no tail] = 1/2
p(1 head & 1 tail] = 0

where P(X) and P(Y) are marginal probability fun

Bivariate (Joint) Probability distribution:-

The P.d of one variable is known as univariate P.d. The P.d of two variable is known as Bivariate (joint) P.d.

• Marginal prob of X, we will add values of Y.

Conditions for Joint Probability Function:-

$f(X, Y)$ joint P.f.

(1) $f(X, Y) \geq 0 \quad \forall X \,\&\, Y$

(2) $\sum_{X}\sum_{Y} f(X, Y) = 1$ for d.r.Vs

or $\int_X\int_Y f(X, Y)\,dY\,dX = 1$ for c.r.V

d.r.v

• $f(X, Y)$ is a joint P.m.f, marginal prob fun.s of X:-

$g(X) = \sum_{Y} f(X, Y)$

marginal prob fun of Y:-

$h(y) = \sum_{X} f(X, Y)$

• $f(X, Y)$ is a joint Prob. density fun marginal Prob. fun.s of X.

$g(X) = \int_Y f(x, y)\,dy$

marginal prob fun.s of Y

$h(Y) = \int_X f(x, y)\,dx.$

Conditional Probability Function:- $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$f(X|Y) = \frac{f(X,Y)}{h(y)}$$ → marginal prob fun of Y

$$f(Y|X) = \frac{f(X,Y)}{g(X)}$$ → marginal prob fun of X

Example: Let X, Y have joint prob fun.

$$f(X,Y) = \frac{xy}{66}, \quad x = 2,4,5, \quad y = 1,2,3$$

(i). Find marginal prob. fun. of X & Y.
(ii). Conditional prob. func. of X given Y.

Sol:-

(i) $P(X) = \sum_y f(x,y) = \sum_{y=1}^{3} \frac{xy}{66}$

$$P(X) = \frac{x}{66} + \frac{2x}{66} + \frac{3x}{66} \Rightarrow \frac{6x}{66} \Rightarrow \frac{x}{11}, \quad x = 2,4,5$$

$$h(Y) = \sum_x f(x,y) = \sum_{x=2}^{5} \frac{xy}{66}$$

$$h(Y) = \frac{2y}{66} + \frac{3y}{66} + \frac{4y}{66} + \frac{5y}{66} = \frac{14y}{66}$$

(ii) $f(x|y) = \frac{f(x,y)}{h(Y)} \Rightarrow \frac{xy/66}{14y/66} \Rightarrow \frac{x}{14}$

- $\sum_{x=2}^{5}\sum_{y=1}^{3} \frac{xy}{A} = 1$ if any constant is given in fun. (then 1st find constant)

$$\sum_{x=2}^{5}\left[\frac{x}{A} + \frac{2x}{A} + \frac{3x}{A}\right] = 1 \Rightarrow \sum_{x=2}^{5}\left[\frac{6x}{A}\right] = 1 \Rightarrow$$

$$\frac{6(2)}{A} + \frac{\cancel{6(3)}}{\cancel{A}} + \frac{6(4)}{A} + \frac{6(5)}{A} = 1$$

$$\frac{6}{A}(2 + 4 + 5) = 1 \Rightarrow \frac{6(11)}{A} = 1 \Rightarrow \boxed{A = 66}$$

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Example: Let X, Y have joint prob. func.

$$f(x,y) = \frac{xy}{66}, \quad 2 \le X \le 5, \quad 1 \le Y \le 3$$

(i) Find marginal prob func of X & Y
(ii): Conditional prob. fun. of X given Y

$$P(X) = \int_y f(x,y)\,dy = \int_1^3 \frac{xy}{66}\,dy \Rightarrow \frac{x}{66}\left|\frac{y^2}{2}\right|_1^3 \Rightarrow \frac{x}{66(2)}(9-1) \Rightarrow \frac{x}{66}(4) \Rightarrow \frac{2x}{33}$$

$$h(Y) = \int_x f(x,y)\,dx = \int_2^5 \frac{xy}{66}\,dx \Rightarrow \frac{y}{66}\left|\frac{x^2}{2}\right|_2^5 \Rightarrow \frac{y}{2(66)}(25-4) \Rightarrow \frac{21y}{2(66)} \Rightarrow \frac{7y}{44}$$

$$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{xy/66}{7y/44} \Rightarrow \frac{xy}{33}\cdot\frac{22}{7y} \Rightarrow \frac{22x}{231}$$

Chap 7:

Q.No. 3

(b):

Red balls	Black	total	select
4	6	10	4

X: no. of red balls

X = 0, 1, 2, 3, 4

$n(S) = \binom{10}{4} = 210$

X	P(X)	F(X)
0	$\binom{4}{0}\binom{6}{4}/210 = \frac{(1)(15)}{210} = \frac{3}{42} = \frac{1}{14}$	$\frac{3}{42}$
1	$\binom{4}{1}\binom{6}{3}/206 = \frac{4(20)}{206} = \frac{40}{103}$	$\frac{3}{42} + \frac{40}{103} = \frac{663}{1442}$
2	$\binom{4}{2}\binom{6}{2}/202 = \frac{6(15)}{202} = \frac{45}{101}$	$\frac{663}{1442} + \frac{45}{101} = .905$
3	$\binom{4}{3}\binom{6}{1}/198 = \frac{4(6)}{198} = \frac{12}{99} = \frac{4}{33}$	$.905 + \frac{4}{33} = 1.0265$
4	$\binom{4}{4}\binom{6}{0}/194 = \frac{(1)(1)}{194} = \frac{1}{194}$	$1.0265 + \frac{1}{194} = 1.0317$

Ex: 7

Q No. 4

(a): defective radios 5, Nt def. 10, Total 15, Select 3

defective radios	Nt def.	Total	Select
5	10	15	3

X : no. of defective radios ;

$X = 0, 1, 2, 3$

$n(S) = \binom{15}{3} = 455$

X	P(x)	F(x)
0	$\binom{5}{0}\binom{10}{3}/455 = \frac{1(120)}{455}$	$\frac{120}{455}$
1	$\binom{5}{1}\binom{10}{2}/455 = \frac{5(45)}{455}$	$\frac{120}{455}+\frac{225}{455}=\frac{345}{455}$
2	$\binom{5}{2}\binom{10}{1}/455 = \frac{10(10)}{455}$	$\frac{345}{455}+\frac{100}{455}=\frac{445}{455}$
3	$\binom{5}{3}\binom{10}{0}/455 = \frac{10(1)}{455}$	$\frac{445}{455}+\frac{10}{455}=\frac{455}{455}=1$

Q No. 5

(b). $f(x) = \begin{cases} A(4x-2x^2), & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

$A = ?$ as $f(x)$ is p.d.f.

$$\int_0^2 A(4x-2x^2)\,dx = 1$$

$$A\left\{4\left|\frac{x^2}{2}\right|_0^2 - 2\left|\frac{x^3}{3}\right|_0^2\right\} = 1$$

$$A\left\{2(4-0) - \frac{2}{3}(8-0)\right\} = 1$$

$$A\left\{8 - \frac{2}{3}(8)\right\} = 1 \Rightarrow 8A\left\{1 - \frac{2}{3}\right\} = 1 \Rightarrow 8A\left\{\frac{1}{3}\right\} = 1 \Rightarrow$$

$$\boxed{A = \frac{3}{8}}$$

Q.No.6

(b) A c.r.v, X has p.d.f as follows

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x \le 1 \\ \frac{1}{4}(3-x), & 1 < x \le 2 \\ \frac{1}{4}, & 2 < x \le 3 \\ \frac{1}{4}(4-x), & 3 < x \le 4 \\ & \text{elsewhere} \end{cases}$$

Compute $P(X \ge 3)$, $P(X=2)$, $P(|X| < 1.5)$ and $P(1 < X < 3)$.

$P(X \ge 3) = \int_3^\infty f(x)\,dx$

$P(x \ge 3) =$

$= \int_3^\infty f(x)\,dx$

$= \int_3^4 f(x)\,dx + \int_4^\infty f(x)\,dx$

$= \int_3^4 \frac{1}{4}(4-x)\,dx + 0$

$= \frac{1}{4}\left\{|4x|_3^4 - \left|\frac{x^2}{2}\right|_3^4\right\}$

$= \frac{1}{4}\left\{4(4-3) - \frac{1}{2}(16-9)\right\} \Rightarrow \frac{1}{4}\left\{4 - \frac{7}{9}\right\}$

$= \frac{29}{36}$

$P(X=2) = \int_2^2 f(x)\,dx = 0$

$P(|x| < 1.5) = P(-1.5 < x < 1.5)$

$= \int_{-1.5}^0 f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^{1.5} f(x)\,dx$

$= 0 + \int_0^1 \frac{x}{2}\,dx + \int_1^{1.5} \frac{1}{4}(3-x)\,dx$

$= \frac{1}{2}\left|\frac{x^2}{2}\right|_0^1 + \frac{1}{4}\left\{3x\Big|_1^{1.5} - \left|\frac{x^2}{2}\right|_1^{1.5}\right\}$

$= \frac{1}{4}(1-0) + \frac{1}{4}\left\{3(1.5-1) - \frac{1}{2}(2.25-1)\right\}$

$= .46875$

$P(1 < X < 3) = \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx$

$= \int_1^2 \frac{1}{4}(3-x)\,dx + \int_2^3 \frac{1}{4}\,dx$

$= \frac{1}{4}\left\{3x\Big|_1^2 - \frac{x^2}{2}\Big|_1^2\right\} + \frac{1}{4}|x|_2^3$

$= \frac{1}{4}\{3(2-1) + \frac{1}{2}(4-1)\} + \frac{1}{4}(3-2)$

$= \frac{1}{4}\{3 - \frac{3}{2}\} + \frac{1}{4} \Rightarrow \frac{3}{8} + \frac{1}{4} \Rightarrow \frac{3+2}{8} \Rightarrow$

$= \frac{5}{8}$

Q.No.10

(b): Given joint p.d of two r.v's X & Y, whose values f(x, y)

X \ Y	1	2	3	g(x)
1	$\frac{6}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{8}{30}$
2	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{1}{30}$	$\frac{10}{30}$
3	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{12}{30}$
h(y)	$\frac{12}{30}$	$\frac{10}{30}$	$\frac{8}{30}$	1

$f(x/y) = \frac{f(x,y)}{h(y)}$

$f(x=1/y=1) = \frac{f(1,1)}{h(1)} \Rightarrow \frac{6/30}{12/30} \Rightarrow \frac{1}{2}$

$f[x=1 / Y=2] = \frac{P[x=1 \text{ and } y=2]}{h(Y=2)}$

$= \frac{1/30}{10/30} \Rightarrow \frac{1}{10}$

- - - - -

$f(y/x) = \frac{f(x,y)}{g(x)}$

$f(y=2/x=2) = \frac{f(2,2)}{g(x=2)} \Rightarrow \frac{5/30}{10/30} \Rightarrow \frac{1}{2}$

- - . -

Q.No.12

(i) Let X & Y have joint p.f

(ii) $f(x,y) = \frac{xy^2}{30}$, $x = 1,2,3$, $y = 1,2$

Find marginal p.f of X & Y.

$P(X) = \sum_y f(x,y)$

$= \sum_1^2 \frac{xy^2}{30}$

$= \frac{x}{30} + \frac{4x}{30} \Rightarrow \frac{5x}{30} \Rightarrow \frac{x}{6}$

$h(y) = \sum_x f(x,y) = \sum_1^3 \frac{xy^2}{30}$

$= \frac{y^2}{30} + \frac{2y^2}{30} + \frac{3y^2}{30} \Rightarrow \frac{6y^2}{30} \Rightarrow \frac{y^2}{5}$

(iii): Conditional p.f of Y given X.

$f(Y/X) = \frac{f(x,y)}{P(x)}$

$= \frac{xy^2/30_5}{x/6} \Rightarrow \frac{y^2}{5}$

Q.NO.15

(b): Two r.v.'s X and Y have joint p.d.f

$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & , 0<x<1, \; 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$

find marginal distributions of X and Y and their conditional distributions. Also find $P(X<\frac{1}{2} \mid Y=\frac{1}{2})$

(i) $g(x) = \int_y f(x,y)\,dy$

$= \int_{-\infty}^{0} f(x,y)\,dy + \int_0^1 f(x,y)\,dy + \int_1^{\infty} f(x,y)\,dy$

$= 0 + \int_0^1 \frac{2}{3}(x+2y)\,dy + 0$

$= \frac{2}{3}\left\{ xy\Big|_0^1 + \frac{2y^2}{2}\Big|_0^1 \right\} \Rightarrow \frac{2}{3}\{x+1\}$

$g(x) = \frac{2}{3}(x+1)$

$h(y) = \int_x f(x,y)\,dx$

$= \int_{-\infty}^{0} f(x,y)dx + \int_0^1 f(x,y)dx + \int_1^{\infty} f(x,y)dx$

$= 0 + \int_0^1 \frac{2}{3}(x+2y)dx + 0$

$= \frac{2}{3}\left\{ \frac{x^2}{2}\Big|_0^1 + 2xy\Big|_0^1 \right\} \Rightarrow \frac{2}{3}\left\{\frac{1}{2} + 2y\right\} \Rightarrow$

$h(y) = \frac{1}{3}(1+4y)$

$P(X|Y) = \frac{f(x,y)}{h(y)} \Rightarrow \frac{\frac{2}{3}(x+2y)}{\frac{1}{3}(1+4y)} \Rightarrow$

$= \frac{2(x+2y)}{1+4y}$

$P(Y|X) = \frac{f(x,y)}{P(X)} = \frac{\frac{2}{3}(x+2y)}{\frac{2}{3}(x+1)} \Rightarrow \frac{x+2y}{x+1}$

$P(X<\frac{1}{2} \mid Y=\frac{1}{2}) = ? \to 0$

$P(x<\frac{1}{2} \mid y=\frac{1}{2}) = \frac{P[x<\frac{1}{2} \text{ and } y=\frac{1}{2}]}{P[y=\frac{1}{2}]}$

$= \frac{\int_0^{1/2}\int_{1/2}^{1/2} f(x,y)\,dy\,dx}{\int_{1/2}^{1/2} h(y)\,dy} \Rightarrow \frac{0}{0}$

So undefined.

Note:-
If x and y are independent then
$f(x,y) = g(x)\,h(y)$
where $g(x)$ is marginal Prob. fun. of x
and $h(y)$ " " of y

Example (Assignment) 5.

Q:1:

Red	Black	White	Total	Select
5	3	4	12	3

X: no. of Red balls.
Y: no. of black balls
(i): Find Prob. dist. of X.
(ii): Find Prob. dist of Y
(iii): joint dist of (X,Y)
(iv): marginal Prob. dist of X and Y
From joint Prob. dist of (X,Y)
(v): Conditional Prob dist of X|Y and Y|X

Example 2:

Q:2 A r.v X has P.f

$$f(x) = \begin{cases} 0 & x<0 \\ A(x-2)^2 & 0 \le x < 2 \\ 0 & x \ge 2 \end{cases}$$

(1) Find $P[X=1]$, $P[X\ge 1]$, $P[X \le 1]$

Q:3 Example:-
If X and Y have joint P.f

$$f(x,y) = \begin{cases} A(x+2y), & 0\le x \le 1,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

(2) $P[\frac{1}{3} \le x \le \frac{1}{2} \text{ and } \frac{1}{4} \le y \le \frac{1}{3}]$
(1) $g(x), h(y), f(x/y), f(y/x)$
(3): $P[X \le 1/2 \mid Y \le 1/3]$
(4): $P[X \le 4/3] =$
(5): $P(Y \ge \frac{1}{2})$

$P[x^2 \le 2] = \int f$

$f(x,y)$

$\int\int \cancel{f(x,y)}\,dx$

Expectation of Random Variable:-

A r.v 'x' has Prob. fun. $f(x)$

$f(x) = \ldots\ldots$

Find $E(X^r)$; $r \geq 0, 1, 2, 3, \ldots$

$$E(X^r) = \begin{cases} \sum_x X^r f(x) & , \text{ if } X \text{ is d.r.v} \\ \int_x X^r f(x)\,dx & , \text{ if } X \text{ is c.r.v} \end{cases}$$

$$E(X) = \begin{cases} \sum_x X f(x) & , X \text{ is d.r.v} \\ \int_x X f(x) & , X \text{ is c.r.v} \end{cases}$$

$$E(X^2) = \begin{cases} \sum_x X^2 f(x) & , X \text{ is d.r.v} \\ \int_x X^2 f(x)\,dx & , X \text{ is c.r.v} \end{cases}$$

$$E(X^3) = \begin{cases} \sum x^3 f(x) & , X \text{ is d.r.v} \\ \int_x X^3 f(x)\,dx & , X \text{ is c.r.v} \end{cases}$$

$$E(3x^2+2) = \begin{cases} \sum_x (3x^2+2) f(x) & , x \text{ is d.r.v} \\ \int_x (3x^2+2) f(x)\,dx & , x \text{ is c.r.v.} \end{cases}$$

For joint

$$E(X^r Y^r) = \begin{cases} \sum_X \sum_Y (X^r Y^r) f(x, y) \\ \int_x \int_y (X^r Y^r) f(x, y)\,dy\,dx \end{cases}$$

$$E(XY) = \begin{cases} \sum_X \sum_Y (XY) f(x, y) \\ \int_x \int_y (XY) f(x, y)\,dy\,dx \end{cases}$$

- Mean of r.v is called expected value of X.

Example:-

let a r.v has Prob. dist—

X	P(X)	X P(X)	X^2	$X^2 P(X)$
−1	.125	−.125	1	.125
0	.50	0	0	0
1	.20	.20	1	.20
2	.05	.10	4	.20
3	.125	.375	9	1.125
		$\sum X P(X)$		$\sum X^2 P(X) = 1.65$

find $E(X)$ and $V(X)$ → (Variance of X)

Imp. Formulas:-

(1): Mean of r.v of $X = E(X)$

(2): Variance of r.v of $X = E(X^2) - [E(X)]^2$

(3): Covariance $(X, Y) = E(XY) - E(X).E(Y)$

Sol: $E(x) = \sum X P(X)$

$\boxed{E(X) = 0.55}$

$E(X^2) = \sum X^2 P(X) = 1.65$

$V(x) = E(X^2) - [E(X)]^2$

$= 1.65 - (0.55)^2$

$\boxed{V(X) = 1.3475}$

- Square root of Variance, we give standard deviation.

Example:-

$$f(y) = \binom{2}{y}\left(\frac{1}{2}\right)^y\left(\frac{1}{2}\right)^{2-y}, \quad y = 0, 1, 2.$$

$E(y)$, $Var(Y) = ?$, $E(y+2)$

Y	f(y)	y f(y)	y + 2	(y+2) f(y)	y^2	$y^2 f(y)$
0	1/4	0	2	2/4	0	0
1	2/4	$\frac{2}{4}$	3	6/4	1	$\frac{2}{4}$
2	1/4	2/4	4	4/4	4	4/4
		$\sum y f(y) = 1$		$\sum(y+2) f(y) = 3$		$\sum y^2 f(y) = \frac{6}{4}$

$E(y) = \sum y f(y) = 1$; $E(y^2) = \sum y^2 f(y) = 6/4$

$Var(Y) = E(y^2) - [E(y)]^2 = \frac{6}{4} - (1)^2 = \frac{6-4}{4} \Rightarrow \frac{2}{4} \Rightarrow \frac{1}{2}$

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Example:-

$$f(x) = \begin{cases} \frac{3}{8}(x-2)^2 & , \; 0 < x < 2 \\ 0 & , \; \text{otherwise} \end{cases}$$

Find $E(X)$, $Var(X)$, $E(X+3)$.

$$E(X) = \int_0^2 x \cdot \frac{3}{8}(x-2)^2 \, dx = \frac{3}{8}\int_0^2 x(x^2+4-4x)\,dx \Rightarrow$$

$$E(x) = \frac{3}{8}\int_0^2 (x^3+4x-4x^2)\,dx \Rightarrow \frac{3}{8}\left\{ \left|\frac{x^4}{4}\right|_0^2 + 4\left|\frac{x^2}{2}\right|_0^2 - 4\left|\frac{x^3}{3}\right|_0^2 \right\} \Rightarrow$$

$$= \frac{3}{8}\left\{ \frac{1}{4}(4-0) + 2(4-0) - \frac{4}{3}(8-0) \right\} \Rightarrow \frac{3}{8}\left\{1+8-\frac{32}{3}\right\} \Rightarrow \frac{3}{8}\left\{\frac{27-32}{3}\right\} \Rightarrow \frac{-5}{8}$$

$$E(X+3) = \int_0^2 (X+3)\,\frac{3}{8}(X-2)^2\,dx$$

$$E(X+3) = \frac{3}{8}\int_0^3 (x+3)(x^2+4-4x)\,dx \Rightarrow \frac{3}{8}$$

$$= \frac{3}{8}\int_0^3 (x^3+4x-4x^2+3x^2+12-12x)\,dx$$

$$= \frac{3}{8}\int_0^3 (x^3-x^2-8x+12)\,dx$$

$$= \frac{3}{8}\left\{ \left|\frac{x^4}{4}\right|_0^3 - \left|\frac{x^3}{3}\right|_0^3 - 8\left|\frac{x^2}{2}\right|_0^3 + 12\left|x\right|_0^3 \right\}$$

$$= \frac{3}{8}\left\{ \frac{1}{4}(3^4-0) - \frac{1}{3}(3^3-0) - 4(9-0) + 12(3-0) \right\}$$

$$E(x+3) = \frac{3}{8}\left\{\frac{3^4}{4} - 3^2\right\} \Rightarrow \frac{3}{8}\left\{\frac{81-36}{4}\right\} \Rightarrow \frac{3}{8}\left\{\frac{45}{4}\right\} \Rightarrow \frac{135}{32}$$

As $Var(X) = E(x^2) - [E(x)]^2$

$$E(x^2) = \int_0^2 x^2 \cdot \frac{3}{8}(x-2)^2\,dx \Rightarrow \int_0^2 \frac{3}{8}x^2(x^2+4-4x)\,dx \Rightarrow$$

$$= \frac{3}{8}\int_0^2 (x^4+4x^2-4x^3)\,dx.$$

$$= \frac{3}{8}\left\{ \left|\frac{x^5}{5}\right|_0^2 + 4\left|\frac{x^3}{3}\right|_0^2 - 4\left|\frac{x^4}{4}\right|_0^2 \right\}$$

$$= \frac{3}{8}\left\{ \frac{1}{5}(32-0) + \frac{4}{3}(8-0) - (16-0) \right\}$$

$$= 3\left\{\frac{4}{5} + \frac{4}{3} - 2\right\} \Rightarrow 3\left\{\frac{12+20-30}{15}\right\} \Rightarrow \frac{6}{15}$$

$$Var(x) = \frac{6}{15} - \left(\frac{-5}{8}\right)^2 \Rightarrow \frac{6}{15} - \frac{25}{64} \Rightarrow \frac{384-375}{960} \Rightarrow \frac{9}{960} \Rightarrow \frac{3}{320}$$

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Example.

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \le x \le 1, \; 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{cov}(X,Y) = E(XY) - E(X)\,E(Y) \quad \star$$

$$E(XY) = \int_0^1\int_0^1 xy \cdot \frac{2}{3}(x+2y)\,dy\,dx$$

$$= \int_0^1\int_0^1 \frac{2}{3}(x^2y+2xy^2)\,dy\,dx \Rightarrow \int_0^1 \frac{2}{3}\left(x^2\left|\frac{y^2}{2}\right|_0^1 + 2x\left|\frac{y^3}{3}\right|_0^1\right)dx \Rightarrow$$

$$= \frac{2}{3}\int_0^1 \left(x^2(1-0) + \frac{2}{3}x(1-0)\right)dx \Rightarrow \frac{2}{3}\int_0^1\left(x^2+\frac{2}{3}x\right)dx \Rightarrow$$

$$= \frac{2}{3}\left\{ \left|\frac{x^3}{3}\right|_0^1 + \frac{2}{3}\left|\frac{x^2}{2}\right|_0^1 \right\} \Rightarrow \frac{2}{3}\left\{\frac{1}{3}(1-0) + \frac{1}{3}(1-0)\right\} \Rightarrow \frac{4}{9}$$

$$E(x) = \int x\, g(x)\,dx$$

$$g(x) = \int_y f(x,y)\,dy = \int_0^1 \frac{2}{3}(x+2y)\,dy \Rightarrow \frac{2}{3}\left\{ x\left|y\right|_0^1 + 2\left|\frac{y^2}{2}\right|_0^1 \right\}$$

$$g(x) = \frac{2}{3}\{x+1\}$$

$$E(x) = \int_0^1 x \cdot \frac{2}{3}(x+1)\,dx \Rightarrow \frac{2}{3}\int_0^1 (x^2+x)\,dx \Rightarrow \frac{2}{3}\left\{ \left|\frac{x^3}{3}\right|_0^1 + \left|\frac{x^2}{2}\right|_0^1 \right\}$$

{See above page

Binomial Example:- If 60% of students in math dept prefer teacher A. what is prob that in sample of 10 students

(i) Exactly 3 will prefer A.

(ii) Atleast 5 students will prefer A.

$n = 10$, $P = .60$, $q = 1-P = 1-.60 = .40$

x = no. of student that prefer 'A'

$X \sim b(10, 0.60)$

$$b(X, 10, .60) = \binom{10}{x}(.60)^x(.40)^{10-x}, \quad x = 0,1,2,\ldots,10$$

we find

$$P(X=3) = \binom{10}{3}(.60)^3(.40)^{10-3}$$

$$P(X \ge 5) = \sum_{x=5}^{10}\binom{10}{x}(.60)^x(.40)^{10-x}$$

Binomial.

Plob disc

↑

Random Valiable

↑

Random exp.

d.p.d , c.p.d

↑

d.r.v , c.r.v

↑

discrete or continuous.

Plob disc

Discrete Plob disc — Continuous Plob. disc

(i): Binomial Plob disc

(ii): Hypergeometric

(iii): Negative Bionomial

(iv): Geometric Plob

- Binomial P.d → B.r.v → B.r. experiment

Binomial Random Experiment:-

An experiment that satisfies the following 4 Property is known as Binomial random experiment

(I): Each trial has two possible outcomes classified as "success" or "Failure"

(ii). Probability of success, denoted by P. remains same throughout all trials

(iii): Success trials are independent

(IV): The number of trials are fixed, say n.

Binomial Random Variable:-

A variable x which denotes the number of success in binomial random experiment is called binomial random Variable.

Binomial Plob. Distribution:-

The plob. distribution of binomial random Variable is known as binomial plob. disc. The binomial plob func is denoted by $b(x; n, P)$ defined as

$$b(x; n, P) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots n$$

X = no. of success in n trials

where n = no. of trials

P = plob of success

$q = 1 - P$ = plob of failure

n and P are called parameters of binomial plob. disc.

Note:- $X \sim b(n, P)$ lead as

A r.v X follow binomial plob dist with parameter n & P.

Example:- Dept. of math won 10 matches out of 15 of badminton. What is plob. that match will win 3 match in series of 5 matches.

$P = \frac{10}{15} = \frac{2}{3} = .67 = 67\%$

$n = 5$, $P = .67$, $q = 1 - P = 1 - .67 = .34$

$P(X=3) = ?$

$X \sim b(5, .67)$

$P(X=3) = \binom{5}{3} (.67)^3 (.33)^2$

Assignment 5:

Q.No.1

R	B	w	total	Select
5	3	4	12	3

X: no. of red balls
Y: no of black balls

(i). Prob dist of X = ?
(ii). Prob dist of Y = ?

X = 0, 1, 2, 3

X	P(X)
0	$\binom{5}{0}\binom{7}{3} / \binom{12}{3} = \frac{(1)(35)}{220} = \frac{35}{220}$
1	$\binom{5}{1}\binom{7}{2} / \binom{12}{3} = \frac{(5)(21)}{220} = \frac{105}{220}$
2	$\binom{5}{2}\binom{7}{1} / \binom{12}{3} = \frac{(10)(7)}{220} = \frac{70}{220}$
3	$\binom{5}{3}\binom{7}{0} / \binom{12}{3} = \frac{(10)(1)}{220} = \frac{10}{220}$

Y: no. of black balls
Y = 0, 1, 2, 3

Y	P(Y)
0	$\binom{3}{0}\binom{9}{3} / \binom{12}{3} = \frac{(1)\,84}{220} = \frac{84}{220}$
1	$\binom{3}{1}\binom{9}{2} / \binom{12}{3} = \frac{(3)(36)}{220} = \frac{108}{220}$
2	$\binom{3}{2}\binom{9}{1} / \binom{12}{3} = \frac{3(9)}{220} = \frac{27}{220}$
3	$\binom{3}{3}\binom{9}{0} / \binom{12}{3} = \frac{(1)(1)}{220} = \frac{1}{220}$

(iii): Joint dist of (x, y)

X \ Y	0	1	2	3	g(x)
0	$\frac{4}{220}$	$\frac{18}{220}$	$\frac{12}{220}$	$\frac{1}{220}$	$\frac{35}{220}$
1	$\frac{30}{220}$	$\frac{60}{220}$	$\frac{15}{220}$	0	$\frac{105}{220}$
2	$\frac{40}{220}$	$\frac{30}{220}$	0	0	$\frac{70}{220}$
3	$\frac{10}{220}$	0	0	0	$\frac{10}{220}$
h(y)	$\frac{84}{220}$	$\frac{108}{220}$	$\frac{27}{220}$	$\frac{1}{220}$	1

✓ $P(x=0 \text{ and } y=0) = \binom{5}{0}\binom{3}{0}\binom{4}{3} / \binom{12}{3} = \frac{4}{220}$

$P(x=0 \text{ and } y=1) = \binom{5}{0}\binom{3}{1}\binom{4}{2} / \binom{12}{3} = \frac{(3)6}{220} \Rightarrow \frac{18}{220}$

$P(x=0 \text{ and } y=2) = \binom{5}{0}\binom{3}{2}\binom{4}{1} / \binom{12}{3} = \frac{12}{220}$

$P(x=0 \text{ and } y=3) = \binom{5}{0}\binom{3}{3}\binom{4}{0} / \binom{12}{3} = \frac{1}{220}$

✓ $P(x=1 \text{ and } y=0) = \binom{5}{1}\binom{3}{0}\binom{4}{2} / \binom{12}{3} = \frac{30}{220}$

$P(x=1 \text{ and } y=1) = \binom{5}{1}\binom{3}{1}\binom{4}{1} / \binom{12}{3} = \frac{60}{220}$

$P(x=1 \text{ and } y=2) = \binom{5}{1}\binom{3}{2}\binom{4}{0} / \binom{12}{3} = \frac{15}{220}$

$P(x=1 \text{ and } y=3) = 0$ (b/c it is impossible)

$P(x=2 \text{ and } y=0) = \binom{5}{2}\binom{3}{0}\binom{4}{1} / \binom{12}{3} = \frac{40}{220}$

✓ $P(x=2 \text{ and } y=1) = \binom{5}{2}\binom{3}{1}\binom{4}{0} / \binom{12}{3} = \frac{30}{220}$

$P(x=2 \text{ and } y=2) = 0 = P(x=2 \text{ and } y=3) \rightarrow$ (b/c impossible)

$P(x=3 \text{ and } y=0) = \binom{5}{3}\binom{3}{0}\binom{4}{0} / \binom{12}{3} = \frac{10}{220}$

$P(x=3 \text{ and } y=1) = 0 = P(x=3 \text{ and } y=2) = P(x=3 \text{ and } y=3)$

(v): Conditional prob dist of X|Y and Y|X

$f(x|y) = \frac{f(x,y)}{g(x)}$, $f(x=0 \mid y=0) = \frac{f(0,0)}{g(x=0)} = \frac{4/220}{35/220}$

$f(x=0 \mid y=1) = \frac{f(0,1)}{g(x=0)} = \frac{18}{220} / \frac{35}{220} \Rightarrow \frac{18}{35}$

$f(y|x) = \frac{f(x,y)}{h(y)}$, $f(y=0 \mid x=0) = \frac{f(0,0)}{h(y=0)} = \frac{4/220}{84/220} \Rightarrow \frac{4}{84}$ {find all other x|y and y|x from table

Q.No.2

$$f(x) = \begin{cases} 0 & x < 0 \\ A(x-2)^2 & 0 \le x < 2 \\ 0 & x \ge 2 \end{cases}$$

(i) $P(X=1)$, $P(X \ge 1]$, $P(X \le 1] = ?$

$\int_0^2 f(x)dx = 1$

$\int_0^2 A(x-2)^2 = 1 \Rightarrow A\int_0^2 (x^2+4-4x)dx = 1 \Rightarrow$

$A\left\{ \frac{x^3}{3}\Big|_0^2 + 4x\Big|_0^2 - 4\left|\frac{x^2}{2}\right|_0^2 \right\} = 1$

$A\left\{ \frac{1}{3}(8-0) + 4(2-0) - 2(4-0) \right\} = 1$

$A\left\{ \frac{8}{3} + 8 - 8 \right\} = 1 \Rightarrow A\left\{ \frac{8}{3} \right\} = 1 \Rightarrow$

$A = \frac{3}{8}$ so

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{8}(x-2)^2 & 0 \le x < 2 \\ 0 & x \ge 2 \end{cases}$$

(i) $P(X=1) = \int_1^1 f(x)\,dx = 0$

(ii) $P(X \ge 1) = \int_1^\infty f(x)\,dx$

$= \int_1^2 f(x)dx + \int_2^\infty f(x)dx$

$= \int_1^2 \frac{3}{8}(x-2)^2 dx + 0$

$= \frac{3}{8}\left|\frac{(x-2)^3}{3}\right|_1^2 \Rightarrow \frac{3}{8}\frac{1}{3}\left\{(2-2)^3 - (1-2)^3\right\} \Rightarrow$

$= \frac{1}{8}\left\{(0)^3 - (-1)^3\right\} \Rightarrow \frac{1}{8}\left\{+1\right\} \Rightarrow$

$P(X \ge 1) = \frac{1}{8}$

$P(X \le 1) = \int_{-\infty}^1 f(x)\,dx$

$= \int_{-\infty}^0 f(x)dx + \int_0^1 f(x)\,dx$

$= 0 + \int_0^1 \frac{3}{8}(x-2)^2\,dx$

$= \frac{3}{8}\left|\frac{(x-2)^3}{3}\right|_0^1 \Rightarrow \frac{3}{8}\frac{1}{3}\left\{(1-2)^3 - (0-2)^3\right\}$

$= \frac{1}{8}\left\{-1+8\right\} \Rightarrow$

$P(X \le 1) = \frac{7}{8}$ Ans!

Q.No.3

$$f(x,y) = \begin{cases} A(x+2y) & 0 \le x \le 1,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

$\int_0^1\int_0^1 A(x+2y)\,dx\,dy = 1$

$\int_0^1 \left(A\left(x|y|_0^1 + 2\left|\frac{y^2}{2}\right|_0^1\right)\right)dx = 1$

$A\int_0^1 (x+1)dx = 1 \Rightarrow A\left\{\frac{x^2}{2}\Big|_0^1 + |x|_0^1\right\} = 1 \Rightarrow$

$A\left\{\left(\frac{1}{2}-0\right) + (1-0)\right\} = 1 \Rightarrow A\left(\frac{1}{2}+1\right) = 1 \Rightarrow$

$A\left(\frac{3}{2}\right) = 1 \Rightarrow \boxed{A = \frac{2}{3}}$

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0 \le x \le 1,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(I) (i): marginal p.f of X

$P(x) = \int_y f(x,y)\,dy \Rightarrow \int_0^1 \frac{2}{3}(x+2y)\,dy \Rightarrow$

$P(x) = \frac{2}{3}\left\{x|y|_0^1 + 2\frac{y^2}{2}\Big|_0^1\right\} \Rightarrow \frac{2}{3}\left\{x+1\right\}$

$$= \frac{\frac{2}{3}\int_0^{1/2}\left(\frac{x}{3}+\frac{1}{9}-0\right)dx}{\frac{1}{3}\left\{ y\Big|_0^{1/3} + 4\frac{y^2}{2}\Big|_0^{1/3}\right\}} \Rightarrow \frac{2\left\{\frac{x^2}{2(3)}\Big|_0^{1/2} + \frac{1}{9}\,x\Big|_0^{1/2}\right\}}{\left(\frac{1}{3}-0\right)+2\left(\frac{1}{9}-0\right)} \Rightarrow$$

$$= \frac{2\left\{\frac{1}{2(3)}\left(\frac{1}{4}-0\right)+\frac{1}{9}\left(\frac{1}{2}-0\right)\right\}}{\frac{1}{3}+\frac{2}{9}} \Rightarrow \frac{\frac{1}{12}+\frac{1}{9}}{\frac{3+2}{9}} \Rightarrow \frac{\frac{3+4}{36_4}}{\frac{5}{9}} \Rightarrow$$

$$= \frac{7}{4}\cdot\frac{1}{5} \Rightarrow \frac{7}{20}$$

1): $P[x \le 4/3] = ?$

$$P(x \le 4/3) = \int_{-\infty}^{4/3} g(x)\,dx \Rightarrow \int_{-\infty}^{0} g(x)\,dx + \int_0^1 g(x)\,dx + \int_1^{4/3} g(x)\,dx$$

$$= 0 + \int_0^1 \frac{2}{3}(x+1)\,dx + 0 \Rightarrow \frac{2}{3}\left|\frac{(x+1)^2}{2}\right|_0^1 \Rightarrow$$

$$= \frac{1}{3}\left\{(1+1)^2-(0+1)^2\right\} \Rightarrow \frac{1}{3}\{4-1\} \Rightarrow \frac{3}{3} \Rightarrow$$

$$= 1$$

$$P[Y \ge 1/2] = \int_{1/2}^{\infty} h(y)\,dy$$

$$= \int_{1/2}^{1}\frac{1}{3}(1+4y)\,dy + \int_1^{\infty} h(y)\,dy$$

$$= \frac{1}{3}\left\{ y\Big|_{1/2}^{1} + 4\left|\frac{y^2}{2}\right|_{1/2}^{1}\right\}$$

$$= \frac{1}{3}\left\{1-\frac{1}{2}+2\left(1-\frac{1}{4}\right)\right\} \Rightarrow \frac{1}{3}\left\{\frac{1}{2}+2\left(\frac{4-1}{4}\right)\right\} \Rightarrow$$

$$= \frac{1}{3}\left\{\frac{1}{2}+\frac{3}{2}\right\} \Rightarrow \frac{1}{3}\left\{\frac{4^2}{2}\right\} \Rightarrow \frac{2}{3} \text{ Ans!}$$

marginal P.f of Y

(ii) $$h(y) = \int_x f(x,y)\,dx \Rightarrow \int_0^1 \frac{2}{3}(x+2y)\,dx \Rightarrow$$

$$= \frac{2}{3}\left\{\frac{x^2}{2}\Big|_0^1 + 2y\,x\Big|_0^1\right\} \Rightarrow \frac{2}{3}\left\{\frac{1}{2}+2y\right\}$$

$$h(y) = \frac{1}{3}(1+4y)$$

(iii) $$f(x|y) = \frac{f(x,y)}{h(y)} \Rightarrow \frac{\frac{2}{3}(x+2y)}{\frac{1}{3}(1+4y)} \Rightarrow \frac{2(x+2y)}{1+4y}$$

(iv) $$f(y|x) = \frac{f(x,y)}{g(x)} \Rightarrow \frac{\frac{2}{3}(x+2y)}{\frac{2}{3}(x+1)} \Rightarrow \frac{x+2y}{x+1}$$

II) $P\left(\frac{1}{3} \le x \le 1/2 \text{ and } 1/4 \le y \le 1/3\right) = ?$

$$= \int_{1/3}^{1/2}\int_{1/4}^{1/3}\frac{2}{3}(x+2y)\,dy\,dx =$$

$$= \frac{2}{3}\int_{1/3}^{1/2}\left(x\,y\Big|_{1/4}^{1/3} + \frac{2y^2}{2}\Big|_{1/4}^{1/3}\right)dx \Rightarrow \frac{2}{3}\int_{1/3}^{1/2}\left\{x\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{9}-\frac{1}{16}\right)\right\}dx \Rightarrow$$

$$= \frac{2}{3}\int_{1/3}^{1/2}\left(\frac{x}{12}+\frac{7}{144}\right)dx \Rightarrow \frac{2}{3}\left\{\frac{1}{12}\left|\frac{x^2}{2}\right|_{1/3}^{1/2} + \frac{7}{144}\,x\Big|_{1/3}^{1/2}\right\} \Rightarrow$$

$$= \frac{2}{3}\left\{\frac{1}{12(2)}\left(\frac{1}{4}-\frac{1}{9}\right)+\frac{7}{144}\left(\frac{1}{2}-\frac{1}{3}\right)\right\}$$

$$= \frac{2}{3}\left\{\frac{1}{12}\cdot\frac{1}{2}\cdot\frac{5}{36}+\frac{7}{144}\cdot\frac{1}{6}\right\} \Rightarrow \frac{1}{3}\left\{\frac{5}{12\cdot 36}+\frac{7}{144(3)}\right\}$$

$$= \frac{5}{1296}+\frac{7}{1296} \Rightarrow \frac{12}{1296} \Rightarrow \frac{1}{108} \text{ Ans!}$$

III): $P[x \le 1/2 \mid y \le 1/3] = ?$

$$P[x \le \tfrac{1}{2} \mid y \le 1/3] = \frac{P[x \le 1/2 \text{ and } y \le 1/3]}{h(y \le 1/3)}$$

$$= \frac{\int_0^{1/2}\int_0^{1/3}\frac{2}{3}(x+2y)\,dy\,dx}{\int_0^{1/3} h(y)\,dy} \Rightarrow \frac{\frac{2}{3}\int_0^{1/2}\left\{x\left(\frac{1}{3}-0\right)+\frac{2y^2}{2}\Big|_0^{1/3}\right\}dx}{\int_0^{1/3}\frac{1}{3}(1+4y)\,dy}$$

See above Page.

Hyper Geometric Prob. Distribution:-

1). 1 possible outcome → not r.v.
2 „ „ → Binomial
3 „ „ → Trinomial
4 „ → multinomial

Hyper Geometric prob. dist:(Properties):

(1),(4) Same as binomial
(2): Prob. of success changes from trial to trial.
(3) Trials are dependent.

X = no. of success from n

Prob. fun:-

$h(x; n, k, N) = \binom{k}{x}\binom{N-k}{n-x} \Big/ \binom{N}{n}$, $x = 0,1,2,3, \ldots \min[n,k]$

$n \le N$
$x \le k$
$(n-x) \le (N-k)$

Parameters:-

n = sample size
N = Pop. size
K = Total no. success unit in pop.

Example:- A bag contains 4 red and 6 black balls. A sample of 3 balls is selected. what is prob. that (i) 2 red balls will be selected.

Sol:-

R	B	Total(N)	Select(n)
4	6	10	3

$\binom{4}{2}\binom{6}{1} / \binom{10}{3}$

$N = 10$, $n = 3$, $k = 4$

X = no. of red balls

$x = 0, 1, 2, 3$

$$P(X=x) = \binom{4}{x}\binom{6}{n-x} \Big/ \binom{10}{3} \quad , x = 0,1,2,3$$

$$\left\{ P(X=3) = \binom{4}{3}\binom{6}{3-3} \Big/ \binom{10}{3} \rightarrow 3 \text{ red balls (extra)} \right\}$$

(i) $P(X=2) = \binom{4}{2}\binom{6}{3-2} \Big/ \binom{10}{3}$

- When "selection" is used then we use hyper geometric. r.v.

Note ⟺ Selection.

Negative Binomial Prob. Distribution:-

(1) → (3) are same as binomial

(4): no. of trials are vary to achieve fixed no. of success (r)

~~Success fixed and trial vary~~

r.v = x = no. of trials to achieve r^{th} success.

Prob. of success

$$b^*(x; r, P) = \binom{x-1}{r-1} p^r q^{x-r}$$

Parameters. r = no. of success
P = Prob. of success

Exercise on Hyper geometric and Negative binomial (10 + 10 Problems)

$\boxed{\text{Ex of Binomial prob. dist}}$

Q.No. 41

P[defective]

$P = .01 \Rightarrow q = 1 - P = .99$

$n = 10$

X = no. of screws will be defective

$X \sim b(10, .01)$

$b(x, 10, .01) = \binom{10}{x}(.01)^x(.99)^{10-x}$

we find

Prob. that atleast two screws will be defective

$$P(x \ge 2) = \sum_{x=2}^{10} \binom{10}{x}(.01)^x(.99)^{10-x}$$

$$= \binom{10}{0}(.01)^0(.99)^{10-0} + \binom{10}{1}(.01)^1(.99)^9 + \ldots$$

or

$$P(x \ge 2) = 1 - P[x \le 1]$$
$$= 1 - \{P(x=0) + P(x=1)\}$$
$$=$$

Q.No.44

$P = 10\% = .1$, $q = 1-P = .9$

(a): $n = 6$

x = cosmetic flaws and classified as second

$P(x; n, p) = P(x; 6, .1) = \binom{6}{x}(.1)^x(.9)^{6-x}$

find only one is second

$P(x; 6, .1) = \binom{6}{1}(.1)^1(.9)^{6-1}$

(b): $n = 6$

find atleast two are second

$P(x; 6, .1) = \sum_{x=2}^{6}\binom{6}{x}(.1)^x(.9)^{6-x} = 1 - \sum_{x=0}^{1}\binom{6}{x}(.1)^x(.9)$

(c): $P = .9$, $q = .1$, $n = 5$

$x = 4$

Q.No.45.

$P = 25\% = .25$, $q = 1-P = .75$

$n = 20$

x = no. of drivers come to a complete stop

$P(x; 20, .25) = \binom{20}{x}(.25)^x(.75)^{20-x} = \ldots$

(a): $P(x \leq 6) = \sum_{x=0}^{6}\binom{20}{x}(.25)^x(.75)^{20-x}$

(b): $P(x=6) = \binom{20}{6}(.25)^6(.75)^{20-6}$

(c): $P(x \geq 6) = \sum_{x=6}^{20}\binom{20}{x}(.25)^x(.75)^{20-x} = \ldots$

$P(x \geq 6) = 1 - P(x \leq 5) = \sum_{x=0}^{5}\binom{20}{x}(.25)^x(.75)^{20-x}$

Q.No.46

x = customers want oversize version

$P = 60\% = .6$, $q = 1-P = 1-.6 = .4$

(a): $n = 10$

$P(x; 10, .6) = \binom{10}{x}(.6)^x(.4)^{10-x} \ldots$

find prob that atleast 6 want oversize version.

$P(x \geq 6) = \sum_{x=6}^{10}\binom{10}{x}(.6)^x(.4)^{10-x} \ldots$

Q.No.47

$P = 23\% = .23$, $q = 1-P = .77$

x = respondents stick with their tour group

(b): $n = 6$

$P(x; 6, .23) = \binom{6}{x}(.23)^x(.77)^{6-x}$

find prob that atleast two will stick with their tour group.

$P(x \geq 2) = \sum_{x=2}^{6}\binom{6}{x}(.23)^x(.77)^{6-x} = \ldots$

(a) $n = 6$

find prob that two will stick with their tour group.

$P(x=2) = \binom{6}{2}(.23)^2(.77)^{6-2} = \ldots$

(c): $n = 10$

find prob that none will stick with their t g

$P(x; 10, .23) = \binom{10}{x}(.23)^x(.77)^{10-x}$

$P(x=0) = \binom{10}{0}(.23)^0(.77)^{10-0} = .$

Q.No.48.

$P = 30\% = .3$, $q = 1-P = .7$

(a): $n = 10$

x = workers take public transportation daily

$P(x; 10, .3) = \binom{10}{x}(.3)^x(.7)^{10-x}$

find prob. that exactly 3 worker take...

$P(x=3) = \binom{10}{3}(.3)^3(.7)^{10-3}$

(b): $n=10$

find prob that atleast 3 workers take...

$P(x \geq 3) = \sum_{x=3}^{10} \binom{10}{x}(.3)^x(.7)^{10-x} = 1 - P(x \leq 2)$

Q.NO.51 (B)

$p = 5\% = .5$, $q = 1 - p = .5$

$n = 20$

x = no. of people believed country was in recession

$P(x; 20, .5) = \binom{20}{x}(.5)^x(.5)^{20-x}$

(a) $P(x=12) = \binom{20}{12}(.5)^{12}(.5)^{20-12}$

(b): $P(x \leq 5) = \sum_{x=0}^{5} \binom{20}{x}(.5)^x(.5)^{20-x} = \cdots$

Q.No.52

$p = 28\% = .28$, $q = 1 - p = .72$

$n = 15$

x = completed four years of college

$P(x; 15, .28) = \binom{15}{x}(.28)^x(.72)^{15-x}$

(a). $P(x=4) = \binom{15}{4}(.28)^4(.72)^{15-4}$

(b) $P(x \geq 3) = \sum_{x=3}^{15} \binom{15}{x}(.28)^x(.72)^{15-x} = \cdots$

$P(x \geq 3) = 1 - P(x \leq 2)$

$= 1 - \sum_{x=0}^{2} \binom{15}{x}(.28)^x(.72)^{15-x}$

Ex. of Hyper Geometric Prob. Distribution:-

Q.No.54

1st	2nd section	Total	Select
20	30	50	15

X = no. of students

$X = 0, 1, 2, 3, 4, 5, 6, \ldots, 15$

$h(x; 15, k, 50) = \binom{k}{x}\binom{50-k}{15-x} / \binom{50}{15}$

(a): Exactly 10 from 2nd section $\Rightarrow k = 30$

$h(x=10) = \binom{30}{10}\binom{50-30}{15-10} / \binom{50}{15}$

$= \binom{30}{10}\binom{20}{5} / \binom{50}{15}$

(b): at least 10 from 2nd section $\Rightarrow k = 30$

$x \geq 10 \Rightarrow x = 10, 11, 12, 13, 14, 15$

$h(x; 15, 30, 50) = \sum_{x=10}^{15} \binom{30}{x}\binom{50-30}{15-x} / \binom{50}{15}$ $x = 10, 11, 12, 13, \ldots 15$

Q.No.55(A)

1st	2nd	Total(N)	Select(n)
6	9	15	5.

X = no. of first printings among selected copies

$x = 0, 1, 2, 3, 4, 5$. $k = 6$

$h(x; 5, 6, 15) = \binom{6}{x}\binom{15-6}{5-x} / \binom{15}{5}$, $x = 0, 1, \ldots, 5$

(b). $P(x=2) = \binom{6}{2}\binom{15-6}{5-2} / \binom{15}{5}$

$P(x \leq 2) = \sum_{x=0}^{2} \binom{6}{x}\binom{15-6}{5-x} / \binom{15}{5}$. $x = 0, 1, 2$

$P(x \geq 2) = \sum_{x=2}^{5} \binom{6}{x}\binom{5-6}{5-x} / \binom{15}{5}$. $x = 2, 3, 5$

Ex. of Negative Binomial Distribution

Q: 62: A geological study indicates that an exploratory oil well drilled in a particular region should strike oil with Prob. 0.2.
(a): Find Prob. that third oil strike comes on fifth well drilled.

$P = .2$ $q = 1 - p = .8$

$r = 3$, $x = 5$

$b^*(x; r, P) = \binom{x-1}{r-1} p^r q^{x-r}$

x = drill strike oil

$b^*(x; 3, .2) = \binom{5-1}{3-1} (.2)^3 (.8)^{5-3}$

$= \binom{4}{2} (.008)(.8)^2$

$= .0307$

Q:2: 10 % of engines are defective. If engines randomly selected one at a time and tested, what is prob. that 1st nondefective engine will be found on second trial?

P[defective] $P = 10\% = .1$, $q = 1 - P = .9$ ($P = .9$

P[not defective]

$x = 2$, $r = 1$ $q = .1$)

$b^*(2; 1, .1) = \binom{2-1}{1-1} (.9)^1 (0.1)^{2-1}$

$= \binom{1}{0} (.9)(.1)$

Q:59: If 40% of employees have +ve indication of asbestos in their lungs, find prob. that 10 employees must be tested in order to find three positivities?

$P = 40\% = .4$, $q = 1 - p = .6$

$x = 10$, $r = 3$

$b^*(10; 3, .4) = \binom{10-1}{3-1} (.4)^3 (.6)^{10-3}$

$=$

Q: 4: Refer to Q.2. what is Prob. that the third nondefective engine will be found (a): on the 5th trial.
(b): on or before 5th trial.

$p = .9$, $q = .1$, $r = 3$

(a): $x = 5$

$b^*(5; 3, .1) = \binom{5-1}{3-1} (.9)^1 (.1)^{5-1}$

$= 1 (.9)(.1)^4 =$

(b) $x \le 5$

$b^*(x \le 5; 3, .9) = \sum_{x=0}^{5} \binom{x-1}{3-1} (.9)^1 (.1)^{x-1}$

Ex. of Hypergeometric

Question: 55 (B)

Q: 8: From a group of 20 Ph.D. engineers, 10 are randomly selected for employment. What is prob that the 10 selected include all 5 best engineers in group of 20?

$N = 20$, $n = 10$, $K = 5$ → i.e. there are only 5 in set of 5 best engineers

$x = x$ denotes the no. of best engineers among 10 selected i.e $x = 5$

$h(5; 10, 5, 20) = \binom{5}{5}\binom{20-5}{10-5} \Big/ \binom{20}{10}$ i.e $x = 5$

Question: 55 (D)

Q: A warehouse contains 10 printing machines, four of which are defective. A company selects five of the machines at random, thinking all are in working condition. What prob. that all five of machines are non-defective?

$N = 10$

defective	Not-def	Total	Select
4	6	10	5

x = all five machines are not-def. $\Rightarrow K = 6$

$$h(x; 5, 6, 10) = \binom{6}{x}\binom{10-6}{5-x} \Big/ \binom{10}{5}$$

$x = 0, 1, 2, 3, 4, 5$

So $h(x; 5, 6, 10) = \sum_{x=0}^{5} \binom{6}{x}\binom{10-6}{5-x} \Big/ \binom{10}{5}$

Q. No. 55 (F).

Texas	Hawaii	Total (N)	Select (n)
40	20	60	10

$$h(x; 10, K, 60) = \binom{K}{x}\binom{60-K}{10-x} \Big/ \binom{60}{10}$$

(a): Find Prob. that none of employees in Hawaii

$\Rightarrow K = 20$ and $x = 0$

$$h(0; 10, 20, 60) = \binom{20}{0}\binom{60-20}{10-0} \Big/ \binom{60}{10} \quad \text{— (i)}$$

(b): $K = 20$ and $x = 1$

$$h(1; 10, 20, 60) = \binom{20}{1}\binom{60-20}{10-1} \Big/ \binom{60}{10} \quad \text{— (ii)}$$

(C): $K = 20$ and $x \geq 2$

$$h(x \geq 2; 10, 20, 60) = \sum_{x=2}^{10} \binom{20}{x}\binom{60-20}{10-x} \Big/ \binom{60}{10}$$

$P(x \geq 2) = 1 - P(x \leq 1) = 1 - \{P(x=0) + P(x=1)\}$

see page.

$b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots n$

$\sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x} = 1 \qquad \because p + q = 1$

- $E(x) = \sum x \binom{n}{x} p^x q^{n-x}$ ‖ Ex. of -ve Binomial

Q: 56. Suppose that 30% of applicants for a certain industrial job possess advanced training in comp. programming.

(a) Find Prob 3rd applicant with advanced training is found on 8th interview.

(b): Find Prob. 1st applicant is found on 5th interview.

(a): $b^*(x; r, p) = \binom{x-1}{r-1} p^r q^{x-r}$

$p = 30\% = .3, \quad r = 3, \quad x = 8$

$q = 1 - p = .7$

$$b^*(8; 3, .3) = \binom{8-1}{3-1} (.3)^3 (.7)^{8-3}$$

$$= \binom{7}{2} (.3)^3 (.7)^5$$

(b): $r = 1, \quad x = 5$

$$b^*(5; 1, .3) = \binom{5-1}{1-1} (.3)^1 (.7)^{5-1}$$

Q. No. 57.

$p = 0.2, \quad q = 1 - p = 1 - .2 = .8$

(a): $r = 3, \quad x = 5$

$$b^*(5; 3, .2) = \binom{5-1}{3-1} (.2)^3 (.8)^{5-3}$$

(b): $r = 1, \quad x = 3$

$$b^*(3; 1, .2) = \binom{3-1}{1-1} (.2)^1 (.8)^{3-1}$$

- $E(X) = \frac{kq}{p}$
- $Var(X) = \frac{kq}{p^2}$

Geometric:-

when $r = 1$ then -ve binomial is called geometric

Q.NO.58.

$P = \frac{9}{10} = .9$, $q = 1-p = 1-.9 = .1$

$r = 1$

(a): $x = 3$

$b^*(3; 1, .9) = \binom{3-1}{1-1}(.9)^1(.1)^{3-1}$

(b): $x \geq 3$

$b^*(x \geq 3) = 1 - b^*(x \leq 2)$

$= 1 - \sum_{x=0}^{2} \binom{x-1}{1-1}(.9)^1(.1)^{x-1}$

Q.NO.60.

P[defective] = 10% = .1

P[Not defective] = 1 - .1 = .9

$r = 1$. $x = 2$ and

$P = .9$, $q = .1$

$b^*(2; 1, .9) = \binom{2-1}{1-1}(.9)^1(.1)^{2-1}$

$= \binom{1}{0}(.9)(.1) = .09$. Ans!

Q.NO.61.

P[Tel.phone lines are busy] = 60% = .6.

P[call meet] = 1 - .6 = .4

$r = 1$, $P = .4$, and $q = .6$

(i) $x = 1$

$X \sim b^*(x; r, p) = \binom{x-1}{r-1} p^r q^{x-r}$

$X \sim g(x; p) = p q^{x-1}$

$b^*(1; 1, .4) = \binom{1-1}{1-1}(.4)^1(.6)^{1-1} = .4$

(ii) $x = 2$

$b^*(2; 1, .4) = \binom{2-1}{1-1}(.4)^1(.6)^{2-1}$

(iii) $x = 3$

$b^*(3; 1, .4) = \binom{3-1}{1-1}(.4)^1(.6)^{3-1}$

Q:

In a group of 20 students, 4 of them dislike new exam pattern.

(i) what is prob. that a student selected at random will dislike new exam

(ii) what is prob that 5 out of 10 student will dislike new exam

(iii) 5th student will be 3rd student who like new exame

(iv) 3rd student will be 1st student who will dislike new exam pattern

Dislike	like	N
4	16	20

(i) $x = 1$ $k = 4$

Poisson Probability Distribution:

$X \sim P(\mu)$

$$P(x;\mu) = \frac{e^{-\mu}(\mu)^x}{x!}, \quad x = 0,1,2,3\ldots$$

μ = average rate of occurance of event in particular interval (i.e time / area / length)

x : no. of event that occure in interval

e.g Instructor recieve 10 calls on interval 3:00 – 4:30. Find Prob. Pick 5 calls

$\mu = 10$

x : no of calls b/w 3:00 – 4:30

$P[x=5]$

$$P(x=5) = \frac{e^{-10}(10)^5}{5!}$$

- Interval is fixed then event is poisson

	Discrete	continuous
Poisson →	Variable	fixed
exponential ↔	Fixed	Variable

Q:1: Let X denotes the no. of creatures of a particular type captured in a trap during a given time period. suppose that X has a poisson distribution with $\lambda = 4.5$, so on average traps will contain 4.5 creatures. Find the prob. that

(i): trap contains exactly 5 creatures

(ii): trap has at most five creatures.

(i) $x = 5$

$$P(x=5) = \frac{e^{-4.5}(4.5)^5}{5!} = .1708$$

(ii): $x \le 5$

$$P(x\le 5) = \sum_{x=0}^{5} \frac{e^{-4.5}(4.5)^x}{x!}$$

$$= e^{-4.5}\left[4.5 + \frac{(4.5)^2}{2!} + \frac{(4.5)^3}{3!} + \frac{(4.5)^4}{4!} + \frac{(4.5)^5}{5!}\right]$$

$= .7029$

Q:2 The mean no. of customers who arrive per minute at the bank during the noon-to-1 p.m hour is equal to 3.0. what is prob. that in given minute (i) exactly two customers will arrive. (ii) more than 2 customers will arrive.

$\mu = 3.0$

(i) $x = 2$

$$P(x=2) = \frac{e^{-3}(3)^2}{2!} = \frac{9}{2(2.71828)^3} = .2240$$

(ii) $x > 2$

$$P(x>2) = 1 - P(x \le 2)$$

$$= 1 - \sum_{x=0}^{2} \frac{e^{-3}(3)^x}{x!}$$

$$= 1 - \left\{\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!}\right\} \quad 1 - \left[\frac{e^{-3}(1)}{1} + \frac{3e^{-3}}{1} + \frac{9e^{-3}}{2}\right]$$

$$= 1 - \{.0498 + .1494 + .2240\}$$

$$= 1 - .4232 = .5768$$

Q:3: The number of work-related injuries per month in a manufacturing plant is known to follow a poisson distribution

with a mean of 2.5 work-related injuries a month. what is Prob. that in a given month no work related injuries occure?
(ii): That atleast one work-related injury occure?

$\mu = 2.5$

(i) $x = 0$

$$P(x=0) = \frac{e^{-2.5}(2.5)^0}{0!} = \frac{1}{(2.71828)^{2.5}} = .0821$$

(ii): $x \geq 1$

$$P(x \geq 1) = 1 - P(x \leq 0) \Rightarrow 1 - P(x=0) \Rightarrow$$
$$= 1 - .0821 \Rightarrow .9179.$$

Q.4 If production process is in control, the mean no. of chip parts per cookie is 6. what is prob. that in any particular cookie being inspected.

(a): less than five chip parts will be found?
(b): Exactly five chip parts ,, ,, ,, ?
(c): Five or more chip parts ,, ,, ,, ?
(d): Either four or five chip parts ,, ,, ,, ?

$\mu = 6$

(a): $x < 5$

$$P(x<5) = P(x \leq 4)$$
$$= \sum_{x=0}^{4} \frac{e^{-6}(6)^x}{x!}$$

(b): $x = 5$

$$P(x=5) = \frac{e^{-6}(6)^5}{5!}$$

(c): $x \geq 5$

$$P(x \geq 5) = 1 - P(x \leq 4)$$
$$= 1 - \sum_{x=0}^{4} \frac{e^{-6}(6)^x}{x!}$$

(d):

Q.5: Reffer to Q.4. How many cookies in a batch of 100 should the manager expect to discard if company policy requires that all chocolate-chip cookies sold have atleast four chocolate chip parts?

$\mu = 6 \times 100$

$x \geq 4$

Q.6 The U.S Department of Transportation maintains statistics for mishandled bags per 1000 airline passengers. In 2007, airlines had mishandled 7 bags per 1000 passengers. what is prob. that in the next 1000 passengers, airlines will have

(a): no mishandled bags?
(b): at least one mishandled bag?
(c): at least two mishandled bags?

$? \leftarrow \mu = \frac{7}{1000} = .007$ x: no. of mishandled bags

(a): $x = 0$

$$P(x=0) = \frac{e^{-.007}(.007)^0}{0!} = .0009$$

(b): $x \geq 1$

$$P(x \geq 1) = 1 - P(x \leq 0) = 1 - P(x=0) \Rightarrow$$
$$= 1 - .0009 \Rightarrow .9991$$

(c): $x \geq 2$

$$P(x \geq 2) = 1 - P(x \leq 1)$$
$$= 1 - \sum_{x=0}^{1} \frac{e^{-.7} (.7)^x}{x!}$$

Q. 7: A toll-free phone no. is available from 9 a.m to 9 p.m for your customers to register complaints about a product purchased from your company. Past history indicates that an average 0.4 calls are received per minute.

(a) Zero phone calls will be received?

(b): Three or more phone calls " " " ?

$\mu = .4$

(a): $x = 0$

$$P(x=0) = \frac{e^{-.4} (.4)^0}{0!}$$

(b): $x \geq 3$

$$P(x \geq 3) = 1 - P(x \leq 2)$$
$$= 1 - \sum_{x=0}^{2} \frac{e^{-.4} (.4)^x}{x!}$$

Example:- A student ask 4-Question on the average in the class 3–4:30 (P.m). what is Prob. that he/she will ask

(i): exactly 2 questions.

(ii): at least 2 Questions.

(iii): At most two quenstion.

In the next class (3–4:30.)

$\mu = 4$

(i) $x = 2$

(ii) $x \geq 2$

(iii) $x \leq 2$

Sol: average no. of ques asked 3–4:30 = μ

$\mu = 4$.

X: no. of ques that will asked in next time 3–4:30

(i) $P(x=2) = \frac{e^{-4}(4)^2}{2!}$

(ii) $P(x \geq 2) = 1 - P(x \leq 1) = 1 - P(x=0) - P(x=1)$

(iii) $P(x \leq 2) = P(x=2) + P(x=1) + P(x=0)$.

Q: A radar system installed at particular place to detect the noise of plan. The working capacity of the system is that it detect 10 noise on the average in 30 second time interval. what is prob. that it will detect 5 noise in next 30 time interval?

μ = average no. of noise detected in 30 S

$\mu = 10$.

X: no. of noise in 30 S

$x = 5$

$$P(x=5) = \frac{e^{-10}(10)^5}{5!}$$

• Time vary

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Exponential Prob. Distribution.

$$e(x;\lambda) = \lambda e^{-\lambda x}, \quad x > 0$$

λ: average rate of occurance of event

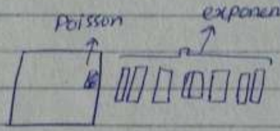
e.g. In previous example

$\lambda = 5$ second per noise

X: time to detect a noise

$P(x=2) =$

e.g



$\lambda = 10$ mint to serve single person

• Event is waiting for time to occure.

$\mu = 10$ Person in 60 mint

Q: The duration of a long call with a mean 3 minute is found to be exponential distributed. what is prob. that a call will ~~atleast more than~~ end after 2 minute

$\lambda = 3$

$P(x \geq 2) =$

$e(x;\lambda) = \lambda e^{-\lambda x}, \quad x > 0$

$$P(x>2) = \int_2^\infty 3e^{-3x}\,dx$$

$$= \frac{3e^{-3x}}{-3}\Big|_2^\infty \Rightarrow -\left(e^{-3(\infty)} - e^{-3(2)}\right) \Rightarrow$$

$$= e^{-6}$$

Weibul distribution → Presentation

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Normal Prob. Distripution:- (not included)

(i) Introduction
(ii) Prob function
(iii) Properties
(iv) Application

→ Presentation consists following steps

. Beta, Gamma dist.

. Transformation of r.v.

e.g $x \sim Exp(\lambda) \rightarrow e(x;\lambda) = \frac{e^{-\lambda}(\lambda)^x}{x!}$

$x^3 \sim ?$

$Y = U(a,b)$

$Y + Y$?

Deffinition:- A continuous r.v X is said to have a normal distribution with parameters μ and δ (or μ and δ^2). where $-\infty < \mu < \infty$ and $0 < \delta$. if prob density fun of X is

$$f(x;\mu,\delta) = \frac{1}{\sqrt{2\pi}\delta} e^{-\frac{\left(\frac{x-\mu}{\delta}\right)^2}{2}}, \quad -\infty < x < \infty$$

where $e \rightarrow 2.71828$

$\pi \rightarrow 3.14159$

μ = mean

δ = Standard deviation

X = Any value of continuous variable

Properties:-

(i) It is symmetrical and thus its mean and median are equal.
(ii) It is bell shaped in its appearance
(iii) It interquartile rang is equal to 1.33 standard deviation.
(iv) It has infinite range

steps for interquartile:-

e.g. (1) orderd data
median
find median of first half Q_1
find median 2nd half Q_3
$IQR = Q_3 - Q_1$

Transformation Formula:-
The z value is equal to difference b/w X and mean, μ, divided by standard deviation σ.
i.e $Z = \frac{X-\mu}{\sigma}$

In normal distribution, we can calculate Prob. that various values occure within certain ranges or intervals. However the exact Prob. of a particular value from a continuous distribution such as normal distribution is zero.

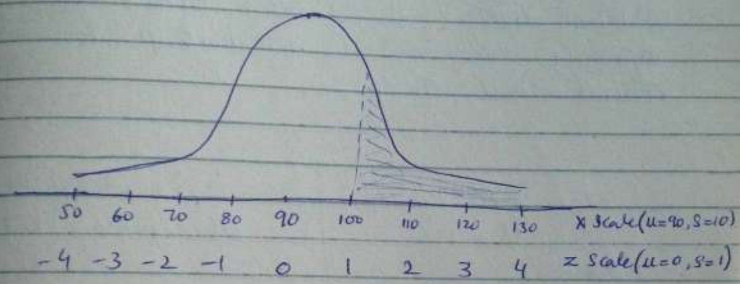
Example:- A radar unit is used to measure speeds of cars on a motor way. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. what is Prob. that a car picked at random is travelling at more than 100 km/h?

$\mu = 90$
$\sigma = 10$
$P(x > 100) = ?$
$x = 100$ Then $Z = \frac{100-90}{10} = 1$

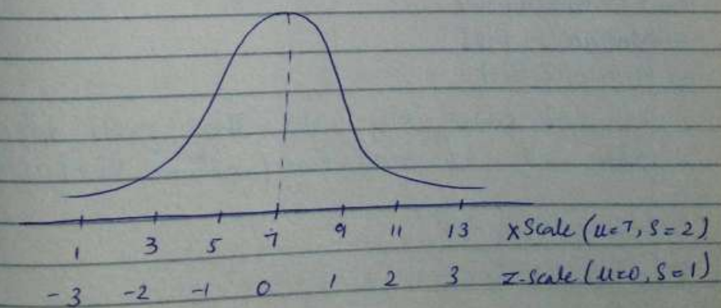
$P(x > 100) = P(z > 1)$
$= \int_1^\infty \frac{1}{\sqrt{2\pi(10)}} e^{-\frac{1}{2}z^2} dz$

or

$= [\text{total area}] - [\text{area to left of } z=1]$



e.g 2:
Download web page is normally distributed with $\mu = 7$ seconds and standard deviation $\sigma = 2$ secs. download time for 1 second?



$Z = \frac{1-7}{2} = -3$

Applications:-

Normal distribution (Gaussian distribution).

1) Numerous continuous variables common in business have distributions that closely resemble the normal distribution.
2) Can be used to approximate various discrete prob. dist—.
3) Provides basis for classical statistical inference b/c of its relationship to central limit Thm.

$$\mu = \frac{\Sigma x}{n}$$

go Data Analysis → Discriptive Statistics

Coloum:

	A	B
1	marks	Y/x

column name, Row no.

40 | = Sum(A2:A38)

41 | = Count(A2:A38)

42 | = A2

43 | = Average(A2:A38)

44 | = MEDIAN (A2:A38)

45 | = MODE(A2:A38)

If not know key word Then first select data and go to "More Functions" → Statistical → —

46 | = HarMean(A2:A38)

47 | = Skew (A2:A38) → −0.14

• only mean is valid for all observation.

48 | = Var (A2:A3)

Syllabus:- (final)

Presentation of data
Measures of central tendency

Measure of Dispersion

Prob.
Random Variable
Discrete Prob Dist
Continuous ″ ″ (exponential)

Rayleigh - Ritz method:-

Consider b.v.p

$$y'' + Q(x)\,y = F(x)$$

where

$y(a) = y_0$, $y(b) = y_n$

we construct the functional

$$I(u) = \int_a^b \left(\left(\frac{du}{dx}\right)^2 - Qu^2 + 2Fu\right)dx \quad \text{—}(2)$$

where

$$u(x) = C_0V_0(x) + C_1V_1(x) + C_2V_2(x) + \cdots + C_nV_n(x)$$
$$= \sum_{i=0}^{n} C_iV_i(x)$$

$$I[C_1, C_2 - C_n] = \int_a^b \left[\left(\frac{d}{dx}\sum C_iV_i(x)\right)^2 - Q\left(\sum C_iV_i\right)^2 + 2F\sum C_iV_i\right]dx \quad (3)$$

To minimize I, we take partial derivatives w.r.t each unknown in the C's we can solve. This will define $u(x)$

$$\frac{\partial I}{\partial C_i} = \int_a^b \left(2\left(\frac{du}{dx}\right)\frac{\partial}{\partial C_i}\left(\frac{du}{dx}\right)\right)dx - \int_a^b 2Qu\left(\frac{\partial u}{\partial C_i}\right)dx + 2\int_a^b \left(F\frac{\partial u}{\partial C_i}\right)dx \quad (4)$$

The no of miles that a particular car can run before its battery wears out is exponentially distributed with an average of 10,000 miles. The owner of car needs to take a 5000 mile trip. what is prob. that he will be able to complete trip without having to replace the car battery?

X: no of miles that car can run before its battery wear.

If jobs arrive every 15 seconds on average $\lambda = 4$ per minute, what is prob. of waiting less than or equal to 30 s i.e .5 mints.

$$P[T \le .5] = \int_0^{.5} 4e^{-4t}\,dt$$
$$= -e^{-4t}\Big|_0^{.5}$$
$$= 1 - e^{-2}$$
$$= .86$$

The average rate of job submission in a busy comp. centre is 4 per minute. If it can be assumed that the no. of submissions per minute interval is poisson distributed. Calculate prob. that

(a): At least 15 s will elapse b/w any two jobs.

$\lambda = 4$ per minute

$$P(t > 15\ s) = P(T > .25\ min)$$

$$= \int_{.25}^{\infty} \lambda e^{-\lambda T}\, dT$$

$$= -e^{-\lambda T}\Big|_{.25}^{\infty}$$

$$= e^{-1}$$

$$= .37$$