

Question # 2

Make dendrogram of clusters by applying the Hierarchical Agglomerative Single Linkage Clustering technique. How many clusters are there in the dendrogram for threshold (distance) = 3

The dataset is given by:

Data Point Features →	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	D ₈	D ₉	D ₁₀
x	1	2	3	4	5	1	2	3	4	5
y	3	4	1	2	1	4	1	2	5	4

	x	y
D ₁	1	3
D ₂	2	4
D ₃	3	1
D ₄	4	2
D ₅	5	1
D ₆	1	4
D ₇	2	1
D ₈	3	2
D ₉	4	5
D ₁₀	5	4

	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	D ₈	D ₉	D ₁₀
D ₁	0									
D ₂	1.4	0								
D ₃	2.8	3.1	0							
D ₄	3.1	2.8	1.4	0						
D ₅	4.4	4.2	2	1.4	0					
D ₆	1	1	3.6	3.6	5	0				
D ₇	2.2	3	1	2.2	3	3.1	0			
D ₈	2.2	2.2	1	1	2.2	2.8	1.4	0		
D ₉	3.6	2.2	4.1	3	4.1	3.1	4.4	3.1	0	
D ₁₀	4.1	3	3.6	2.2	3	4	4.2	2.8	1.4	0

$$(D_1, D_2) = (1, 3)(2, 4)$$

$$= \sqrt{(2-1)^2 + (4-3)^2}$$

$$= \sqrt{(1)^2 + (1)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$= 1.4$$

$$(D_1, D_3) = (1, 3)(3, 1)$$

$$= \sqrt{(3-1)^2 + (1-3)^2}$$

$$= \sqrt{(2)^2 + (-2)^2}$$

$$\sqrt{4+4}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

$$(D_2, D_3) = (2, 4), (3, 1)$$

$$= \sqrt{(3-2)^2 + (1-4)^2}$$

$$= \sqrt{(1)^2 + (-3)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$= 3\cdot 1$$

$$(D_2, D_4) = (1, 3) (4, 2)$$

$$= \sqrt{(4-1)^2 + (2-3)^2}$$

$$= \sqrt{(3)^2 + (-1)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10}$$

$$= 3\cdot 1$$

$$(D_2, D_4) = (2, 4) (4, 2)$$

$$= \sqrt{(4-2)^2 + (2-4)^2}$$

$$= \sqrt{(2)^2 + (-2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

$$(D_3, D_4) = (3, 1)(4, 2)$$

$$= \sqrt{(4-3)^2 + (2-1)^2}$$

$$= \sqrt{(1)^2 + (1)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$= 1\cdot\sqrt{2}$$

$$(D_1, D_5) = (1, 3)(5, 1)$$

$$= \sqrt{(5-1)^2 + (1-3)^2}$$

$$= \sqrt{(4)^2 + (-2)^2}$$

$$= \sqrt{16+4}$$

$$= \sqrt{20}$$

$$= 4\cdot\sqrt{5}$$

$$(D_2, D_5) = (2, 4)(5, 1)$$

$$= \sqrt{(5-2)^2 + (1-4)^2}$$

$$= \sqrt{(3)^2 + (-3)^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 4\cdot2$$

$$(D_3, D_5) = (3, 1)(5, 1)$$

$$= \sqrt{(5-3)^2 + (1-1)^2}$$

$$= \sqrt{(2)^2 + (0)}$$

$$= \sqrt{4}$$

$$= 2$$

$$D_4, D_5 = (4, 2)(5, 1)$$

$$= \sqrt{(5-4)^2 + (1-2)^2}$$

$$= \sqrt{(1)^2 + (-1)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$= 1\cdot4$$

$$(D_1, D_6) = (1, 3)(1, 4)$$

$$= \sqrt{(1-1)^2 + (4-3)^2}$$

$$= \sqrt{0 + (1)^2}$$

$$= \sqrt{1}$$

$$= 1$$

$$\begin{aligned}
 (D_2, D_6) &= (2, 4)(1, 4) \\
 &= \sqrt{(1-2)^2 + (4-4)^2} \\
 &= \sqrt{(-1)^2 + (0)^2} \\
 &= \sqrt{1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (D_3, D_6) &= (3, 1)(1, 4) \\
 &= \sqrt{(1-3)^2 + (4-1)^2} \\
 &= \sqrt{(-2)^2 + (3)^2} \\
 &= \sqrt{4+9} \\
 &= \sqrt{13} \\
 &= 3.6
 \end{aligned}$$

$$\begin{aligned}
 (D_4, D_6) &= (4, 2)(1, 4) \\
 &= \sqrt{(1-4)^2 + (4-2)^2} \\
 &= \sqrt{(-3)^2 + (2)^2} \\
 &= \sqrt{9+4} \\
 &= \sqrt{13} \\
 &= 3.6
 \end{aligned}$$

$$\begin{aligned}
 (D_5, D_6) &= (5, 1)(1, 4) \\
 &= \sqrt{(1-5)^2 + (4-1)^2} \\
 &= \sqrt{(-4)^2 + (3)^2} \\
 &= \sqrt{16+9} \\
 &= \sqrt{25} \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 (D_1, D_7) &= (1, 3)(2, 1) \\
 &= \sqrt{(2-1)^2 + (1-3)^2} \\
 &= \sqrt{(1)^2 + (-2)^2} \\
 &= \sqrt{1+4} \\
 &= \sqrt{5} \\
 &= 2\cdot 2
 \end{aligned}$$

$$\begin{aligned}
 (D_2, D_7) &= (2, 4)(2, 1) \\
 &= \sqrt{(2-2)^2 + (1-4)^2} \\
 &= \sqrt{(0)^2 + (-3)^2} \\
 &= \sqrt{9} \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 (D_3, D_7) &= (3, 1)(2, 1) \\
 &= \sqrt{(2-3)^2 + (1-1)^2} \\
 &= \sqrt{(-1)^2 + 0} \\
 &= \sqrt{1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (D_4, D_7) &= (4, 2)(2, 1) \\
 &= \sqrt{(2-4)^2 + (1-2)^2} \\
 &= \sqrt{(-2)^2 + (-1)^2} \\
 &= \sqrt{4+1} \\
 &= \sqrt{5} \\
 &= 2\cdot\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 (D_5, D_7) &= (5, 1)(2, 1) \\
 &= \sqrt{(2-5)^2 + (1-1)^2} \\
 &= \sqrt{(-3)^2 + 0} \\
 &= \sqrt{9} \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 (D_6, D_7) &= (1, 4)(2, 1) \\
 &= \sqrt{(2-1)^2 + (1-4)^2} \\
 &= \sqrt{(1)^2 + (-3)^2} \\
 &= \sqrt{1+9} \\
 &= \sqrt{10}
 \end{aligned}$$

$$\begin{aligned}
 &= 3 \cdot 1 \\
 (D_1, D_8) &= (1, 3)(3, 2) \\
 &= \sqrt{(3-1)^2 + (2-3)^2} \\
 &= \sqrt{(2)^2 + (-1)^2} \\
 &= \sqrt{4+1} \\
 &= \sqrt{5} \\
 &= 2 \cdot 2
 \end{aligned}$$

$$\begin{aligned}
 (D_2, D_8) &= (2, 4)(3, 2) \\
 &= \sqrt{(3-2)^2 + (2-4)^2} \\
 &= \sqrt{(1)^2 + (-2)^2} \\
 &= \sqrt{1+4} \\
 &= \sqrt{5} \\
 &= 2 \cdot 2
 \end{aligned}$$

$$\begin{aligned}
 (D_3, D_8) &= (3, 1)(3, 2) \\
 &= \sqrt{(3-3)^2 + (2-1)^2} \\
 &= \sqrt{0^2 + 1^2} \\
 &= \sqrt{1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (D_4, D_8) &= (4, 2)(3, 2) \\
 &= \sqrt{(3-4)^2 + (2-2)^2} \\
 &= \sqrt{(-1)^2 + 0^2} \\
 &= \sqrt{1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (D_5, D_8) &= (5, 1)(3, 2) \\
 &= \sqrt{(3-5)^2 + (2-1)^2} \\
 &= \sqrt{(-2)^2 + 1^2} \\
 &= \sqrt{4+1} \\
 &= \sqrt{5} \\
 &= 2\cdot\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 (D_6, D_8) &= (1, 4)(3, 2) \\
 &= \sqrt{(3-1)^2 + (2-4)^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{(2)^2 + (-2)^2} \\
 &= \sqrt{4+4} \\
 &= \sqrt{8} \\
 &= 2\cdot8
 \end{aligned}$$

$$\begin{aligned}
 (D_7, D_8) &= (2, 1)(3, 2) \\
 &= \sqrt{(3-2)^2 + (2-1)^2} \\
 &= \sqrt{(1)^2 + (1)^2} \\
 &= \sqrt{1+1} \\
 &= \sqrt{2} \\
 &= 1\cdot4
 \end{aligned}$$

$$\begin{aligned}
 (D_1, D_9) &= (1, 3)(4, 5) \\
 &= \sqrt{(4-1)^2 + (5-3)^2} \\
 &= \sqrt{(3)^2 + (2)^2} \\
 &= \sqrt{9+4} \\
 &= \sqrt{13}
 \end{aligned}$$

$$\begin{aligned}
 (D_2, D_9) &= (2, 4)(4, 5) \\
 &= \sqrt{(4-2)^2 + (5-4)^2} \\
 &= \sqrt{(2)^2 + (1)^2}
 \end{aligned}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

$$= 2\cdot 2$$

$$(D_3, D_9) = (3, 1)(4, 5)$$

$$= \sqrt{(4-3)^2 + (5-1)^2}$$

$$= \sqrt{1^2 + 4^2}$$

$$= \sqrt{1+16}$$

$$= \sqrt{17}$$

$$= 4\cdot 1$$

$$(D_4, D_9) = (4, 2)(4, 5)$$

$$= \sqrt{(4-4)^2 + (5-2)^2}$$

$$= \sqrt{0^2 + 3^2}$$

$$= \sqrt{9}$$

$$= 3$$

$$(D_5, D_9) = (5, 1)(4, 5)$$

$$= \sqrt{(4-5)^2 + (5-1)^2}$$

$$= \sqrt{(-1)^2 + 4^2}$$

$$= \sqrt{1+16}$$

$$= \sqrt{17}$$

$$= 4\cdot 1$$

$$(D_6, D_9) = (1, 4)(4, 5)$$

$$= \sqrt{(4-1)^2 + (5-4)^2}$$

$$= \sqrt{3^2 + 1^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10}$$

$$= 3\cdot 1$$

$$(D_7, D_9) = (2, 1)(4, 5)$$

$$= \sqrt{(4-2)^2 + (5-1)^2}$$

$$= \sqrt{2^2 + 4^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20}$$

$$= 4\cdot 4$$

$$(D_8, D_9) = (3, 2)(4, 5)$$

$$= \sqrt{(4-3)^2 + (5-2)^2}$$

$$= \sqrt{1^2 + 3^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$= 3\cdot 1$$

$$\begin{aligned}
 (D_{1,10}) &= (1,3)(5,4) \\
 &= \sqrt{(5-1)^2 + (4-3)^2} \\
 &= \sqrt{(4)^2 + (-1)^2} \\
 &= \sqrt{16+1} \\
 &= \sqrt{17} \\
 &= 4.1
 \end{aligned}$$

$$\begin{aligned}
 (D_{2,10}) &= (2,4)(5,4) \\
 &= \sqrt{(5-2)^2 + (4-4)^2} \\
 &= \sqrt{(3)^2 + (0)^2} \\
 &= \sqrt{9} \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 (D_{3,10}) &= (3,1)(5,4) \\
 &= \sqrt{(5-3)^2 + (4-1)^2} \\
 &= \sqrt{(2)^2 + (3)^2} \\
 &= \sqrt{4+9} \\
 &= \sqrt{13} \\
 &= 3.6
 \end{aligned}$$

$$\begin{aligned}
 (D_4, D_{10}) &= (4, 2)(5, 4) \\
 &= \sqrt{(5-4)^2 + (4-2)^2} \\
 &= \sqrt{(1)^2 + (2)^2} \\
 &= \sqrt{1+4} \\
 &= \sqrt{5} \\
 &= 2 \cdot 2
 \end{aligned}$$

$$\begin{aligned}
 (D_5, D_{10}) &= (5, 1)(5, 4) \\
 &= \sqrt{(5-5)^2 + (4-1)^2} \\
 &= \sqrt{0 + (3)^2} \\
 &= \sqrt{9}
 \end{aligned}$$

$$\begin{aligned}
 (D_6, D_{10}) &= (1, 4)(5, 4) \\
 &= \sqrt{(5-1)^2 + (4-4)^2} \\
 &= \sqrt{(4)^2 + (0)^2} \\
 &= \sqrt{16} \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 (D_7, D_{10}) &= (2, 1)(5, 4) \\
 &= \sqrt{(5-2)^2 + (4-1)^2} \\
 &= \sqrt{(3)^2 + (3)^2}
 \end{aligned}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 4.2$$

$$(D_8, D_{10}) = (3, 2)(5, 4)$$

$$= \sqrt{(5-3)^2 + (4-2)^2}$$

$$= \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

$$= 2.8$$

$$(D_9, D_{10}) = (4, 5)(5, 4)$$

$$= \sqrt{(5-4)^2 + (4-5)^2}$$

$$= \sqrt{(1)^2 + (-1)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$= 1.4$$

New Cluster 1

$$D_1, D_6 = 1 \rightarrow (\text{Minimum Value})$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_6)D_2]$

$$\text{MIN}(\text{dist}(D_1, D_2), (D_6, D_2))$$

$$= \min(1.4, 1)$$

$$= 1$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_6)D_3]$.

$$\text{MIN}(\text{dist}(D_1, D_3), (D_6, D_3))$$

$$= \min(2.8, 3.6)$$

$$= 2.8$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_6)D_4]$

$$\text{MIN}(\text{dist}(D_1, D_4), (D_6, D_4))$$

$$= \min(3.1, 3.6)$$

$$= 3.1$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_6)D_5]$

$$\text{MIN}(\text{dist}(D_1, D_5), (D_6, D_5))$$

$$= \min(4.4, 5)$$

$$= 4.4$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_6) D_7]$
 $\text{MIN}(\text{dist}(D_1, D_7), (D_6, D_7))$
= $\min(2.2, 3.1)$
= 2.2

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_8) D_8]$
 $\text{MIN}(\text{dist}(D_1, D_8), (D_6, D_8))$
= $(2.3, 2.8)$
= 2.3

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_9) D_9]$
 $\text{MIN}(\text{dist}(D_1, D_9), (D_6, D_9))$
= $\min(3.6, 3.1)$
= 3.1

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_{10}) D_{10}]$
 $\text{MIN}(\text{dist}(D_1, D_{10}), (D_6, D_{10}))$
= $(4.1, 4)$
= 4

	D ₁	D ₆	D ₂	D ₃	D ₄	D ₅	D ₇	D ₈	D ₉	D ₁₀
D ₁ D ₆	0									
D ₂	1	0								
D ₃	2.8	3.1	0							
D ₄	3.1	2.8	1.4	0						
D ₅	4.4	4.2	2	1.4	0					
D ₇	2.2	3	1	2.2	3	0				
D ₈	2.2	2.2	1	1	2.2	1.4	0			
D ₉	3.1	2.2	4.1	3	4.1	4.4	3.1	0		
D ₁₀	4	3	3.6	2.2	3	4.2	2.8	1.4	0	

New Cluster 2

$$(D_1, D_6, D_2) = 1 \rightarrow (\text{Minimum Value})$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_6, D_2), D_3]$

$$\text{MIN}(\text{dist}(D_1, D_6, D_3), (D_2, D_3))$$

$$= \min(2.8, 3.1)$$

$$= 2.8$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1, D_6, D_2), D_4]$

$$\text{MIN}(\text{dist}(D_1, D_6, D_4), (D_2, D_4))$$

$$= \min(3.1, 2.8)$$

$$= 2.8$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1 D_6, D_2) D_5]$

$$\text{MIN}(\text{dist}(D_1 D_6, D_5), (D_2, D_5))$$

$$= \min(4.4, 4.2)$$

$$= 4.2$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1 D_6, D_2) D_7]$

$$\text{MIN}(\text{dist}(D_1 D_6, D_7), (D_2, D_7))$$

$$= \min(2.2, 3)$$

$$= 2.2$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1 D_6, D_2) D_8]$

$$\text{MIN}(\text{dist}(D_1 D_6, D_8), (D_2, D_8))$$

$$= \min(2.2, 2.2)$$

$$= 2.2$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1 D_6, D_2) D_9]$

$$\text{MIN}(\text{dist}(D_1 D_6, D_9), (D_2, D_9))$$

$$= \min(3.1, 2.2)$$

$$= 2.2$$

To update the distance matrix $\text{MIN}[\text{dist}(D_1 D_6, D_2) D_{10}]$

$$\text{MIN}(\text{dist}(D_1 D_6, D_{10}), (D_2, D_{10}))$$

$$= \min(4, 3)$$

$$= 3$$

	D ₁ , D ₆ , D ₂	D ₃	D ₄	D ₅	D ₇	D ₈	D ₉	D ₁₀
D ₁ , D ₆ , D ₂	0							
D ₃	2.8	0						
D ₄	2.8	1.4	0					
D ₅	4.2	2	1.4	0				
D ₇	2.2	1	2.2	3	0			
D ₈	2.2	1	1	2.2	1.4	0		
D ₉	2.2	4.1	3	4.1	4.4	3.1	0	
D ₁₀	3	3.6	2.2	3	4.2	2.8	1.4	0

New Cluster 3

$$D_3, D_7 = 1 \rightarrow (\text{Minimum value})$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3, D_7), (D_1, D_6, D_2)]$

$$\text{MIN}(\text{dist}(D_3, D_1, D_6, D_2), (D_7, D_1, D_6, D_2))$$

$$= \min(2.8, 2.2)$$

$$= 2.2$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3, D_7), D_4]$

$$\text{MIN}(\text{dist}(D_3, D_4), (D_7, D_4))$$

$$= \min(1.4, 2.2)$$

$$= 1.4$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3, D_7), D_5]$
 $\text{MIN}[\text{dist}(D_3, D_5), (D_7, D_5)]$
= $\min(2, 3)$
= 2

To update the distance matrix $\text{MIN}[\text{dist}(D_3, D_7), D_8]$
 $\text{MIN}[\text{dist}(D_3, D_8), (D_7, D_8)]$
= $\min(1, 1.4)$
= 1

To update the distance matrix $\text{MIN}[\text{dist}(D_3, D_7), D_9]$
 $\text{MIN}[\text{dist}(D_3, D_9), (D_7, D_9)]$
= $\min(4.1, 4.4)$
= 4.1

To update the distance matrix $\text{MIN}[\text{dist}(D_3, D_7), D_{10}]$
 $\text{MIN}[\text{dist}(D_3, D_{10}), (D_7, D_{10})]$
= $\min(3.6, 4.2)$
= 3.6

	D ₁ D ₆ D ₂	D ₃ D ₇	D ₄	D ₅	D ₈	D ₉	D ₁₀
D ₁ D ₆ D ₂	0						
D ₃ D ₇	2.2	0					
D ₄	2.8	1.4	0				
D ₅	4.2	2	1.4	0			
D ₈	2.2	1	1	2.2	0		
D ₉	2.2	4.1	3	4.1	3.1	0	
D ₁₀	3	3.6	2	3	2.8	1.4	0

New Cluster 4

$$D_3 D_7, D_8 = 1 \rightarrow (\text{Minimum Value})$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7, D_8), D_1 D_6 D_2]$

$$\text{MIN}[\text{dist}(D_3 D_7, D_1 D_6 D_2), (D_8, D_1 D_6 D_2)]$$

$$= \min(2.2, 2.2)$$

$$= 2.2$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7, D_8), D_4]$

$$\text{MIN}[\text{dist}(D_3 D_7, D_4), (D_8, D_4)]$$

$$= \min(1.4, 1)$$

$$= 1$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3D_7, D_8), D_5]$

$$\text{MIN}[\text{dist}(D_3D_7, D_5), (D_8, D_5)]$$

$$= \min(2, 2.2)$$

$$= 2$$

To update the distance matrix $[\text{dist}(D_3D_7, D_8), D_9]$

$$\text{MIN}[\text{dist}(D_3D_7, D_9), (D_8, D_9)]$$

$$= \min(4.1, 3.1)$$

$$= 3.1$$

To update the distance matrix $[\text{dist}(D_3D_7, D_8), D_{10}]$

$$\text{MIN}[\text{dist}(D_3D_7, D_{10}), (D_8, D_{10})]$$

$$= \min(3.6, 2.8)$$

$$= 2.8$$

	D ₁ D ₆ D ₂	D ₃ D ₇ D ₈	D ₄	D ₅	D ₉	D ₁₀
D ₁ D ₆ D ₂	0					
D ₃ D ₇ D ₈	2.2	0				
D ₄	2.8	1	0			
D ₅	4.2	2	1.4	0		
D ₉	2.2	3.1	3	4.1	0	
D ₁₀	3	2.8	2	3	1.4	0

New clusters 5

$$(D_3 D_7 D_8, D_4) = 1 \rightarrow (\text{Minimum value})$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7 D_8, D_1), D_1 D_6 D_2)$

$$\text{MIN}[\text{dist}(D_3 D_7 D_8, D_1 D_6 D_2), (D_4, D_1 D_6 D_2)]$$

$$= \min(2.2, 2.8)$$

$$= 2.2$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7 D_8, D_4), D_5]$

$$\text{MIN}[\text{dist}(D_3 D_7 D_8, D_5), (D_4, D_5)]$$

$$= \min(2, 1.4)$$

$$= 1.4$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7 D_8, D_4), D_9]$

$$\text{MIN}[\text{dist}(D_3 D_7 D_8, D_9), (D_4, D_9)]$$

$$= \min(3.1, 3)$$

$$= 3$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7 D_8, D_4), D_{10}]$

$$\text{MIN}[\text{dist}(D_3 D_7 D_8, D_{10}), (D_4, D_{10})]$$

$$= \min(2.8, 2)$$

$$= 2$$

	D ₁ D ₆ D ₂	D ₃ D ₇ D ₈ D ₄	D ₅	D ₉	D ₁₀
D ₁ D ₆ D ₂	0				
D ₃ D ₇ D ₈ D ₄	2.2	0			
D ₅	4.2	1.4	0		
D ₉	2.2	3	4.1	0	
D ₁₀	3	2	3	1.4	0

New Cluster 6

(D₃D₇D₈D₄, D₁₀) = 2 → (Minimum value).

To update the distance matrix MIN[dist(D₃D₇D₈D₄,
D₁₀), D₁D₆D₂)]

$$\text{MIN}[\text{dist}(D_3D_7D_8D_4, D_{10}), (D_{10}, D_1D_6D_2)]$$

$$= \min(2.2, 3)$$

$$= 2.2$$

To update the distance matrix MIN[dist(D₃D₇D₈D₄,
D₁₀), D₅]

$$\text{MIN}[\text{dist}(D_3D_7D_8D_4, D_5), (D_{10}, D_5)]$$

$$= \min(1.4, 3)$$

$$= 1.4$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7 D_8 D_4, D_9), D_9]$

$$\text{MIN}(\text{dist}(D_3 D_7 D_8 D_4, D_9), (D_9, D_9))$$

$$= \min(3, 1.4)$$

$$= 1.4$$

	D ₁ D ₆ D ₂	D ₃ D ₇ D ₈ D ₄ D ₁₀	D ₅	D ₉
D ₁ D ₆ D ₂	0			
D ₃ D ₇ D ₈ D ₄ D ₁₀	2.2	0		
D ₅	4.2	1.4	0	
D ₉	2.2	1.4	4.1	0

New cluster 7

$$(D_3 D_7 D_8 D_4 D_{10}, D_5) = 1.4 \rightarrow (\text{Minimum Value})$$

To update the distance matrix $\text{MIN}[\text{dist}(D_3 D_7 D_8 D_4 D_{10}, D_5), D_1 D_6 D_2]$

$$\text{MIN}[\text{dist}(D_3 D_7 D_8 D_4 D_{10}, D_1 D_6 D_2), (D_5, D_1 D_6 D_2)]$$

$$= \min(2.2, 4.2)$$

$$= 2.2$$

To update the distance matrix $\text{MIN}(\text{dist}(D_3 D_7 D_8 D_4 D_{10}, D_5), D_9)$

$$\begin{aligned} & \text{MIN}(\text{dist}(D_3 D_7 D_8 D_4 D_{10}, D_9), (D_5, D_9)) \\ &= \min(1.4, 4.1) \\ &= 1.4 \end{aligned}$$

	$D_1 D_6 D_2$	$D_3 D_7 D_8 D_4 D_{10} D_5$	D_9
$D_1 D_6 D_2$	0		
$D_3 D_7 D_8 D_4 D_{10} D_5$	2.2	0	
D_9	2.2	1.4	0

New clusters 8

$(D_3 D_7 D_8 D_4 D_{10} D_5, D_9) = 1.4 \rightarrow (\text{minimum value})$

To update the distance matrix $\text{MIN}(\text{dist}(D_3 D_7 D_8 D_4 D_{10} D_5, D_1 D_6 D_2), D_9)$

$(D_3 D_7 D_8 D_4 D_{10} D_5, D_1 D_6 D_2) = 2.2$

$\text{MIN}(\text{dist}(D_3 D_7 D_8 D_4 D_{10} D_5, D_1 D_6 D_2), (D_9, D_1 D_6 D_2))$

$$= \min(2.2, 2.2)$$

$$= 2.2$$

	D ₁ D ₆ D ₂	D ₃ D ₇ D ₈ D ₄ D ₁₀ D ₅ D ₉
D ₁ D ₆ D ₂	O	
D ₃ D ₇ D ₈ D ₄ D ₁₀ D ₅ D ₉	2.2	O

D₁ D₆ D₂, D₃ D₇ D₈ D₄ D₁₀ D₅ D₉,