Regression Loss Functions:

Mean Squared Error Loss:

Iteration: 1

Actual → $y = [3, 6, 9, 12]$.

predicted $= [2, 5, 8, 11]$

$$MSE = \frac{1}{n} \times \Sigma(y - y\text{-predict})^2$$

$$MSE = \frac{1}{4} \times \left[(3-2)^2 + (6-5)^2 + (9-8)^2 + (12-11)^2\right].$$

$$MSE = \frac{1}{4}\left[(1)^2 + (1)^2 + (1)^2 + (1)^2\right].$$

$$MSE = \frac{1}{4}\left[1+1+1+1\right]$$

$$MSE = \frac{\cancel{4}}{\cancel{4}}$$

$$\boxed{MSE = 1}$$

Iteration: 2

Predicted values closer to actual.

Actual values → $y = [3, 6, 9, 12]$.

predicted values = $[2.5, 5.5, 8.5, 11.5]$

$$MSE = \frac{1}{n} \times \Sigma (y - y.predict)^2.$$

$$MSE = \frac{1}{4} \times \left[(3-2.5)^2 + (6-5.5)^2 + (9-8.5)^2 + (12-11.5)^2 \right]$$

$$MSE = \frac{1}{4} \times \left[(0.5)^2 + (0.5)^2 + (0.5)^2 + (0.5)^2 \right].$$

$$MSE = \frac{1}{4} \times \left[0.25 + 0.25 + 0.25 + 0.25 \right]$$

$$MSE = \frac{1}{4}$$

$$\boxed{MSE = 0.25}$$

Mean Squared Logarithmic Error Loss:

Actual value → $y = [3, 6, 9, 12]$

predicted values $= [2, 5, 8, 11]$

$$MSLE = \frac{1}{n} \times \Sigma\left[\ln(y\,\text{actual}+1) - \ln(y.\text{predict}+1)\right]^2$$

$$MSLE = \frac{1}{4} \times \Big[(\ln(3+1) - \ln(2+1))^2 +$$

$$(\ln(6+1) - \ln(5+1))^2 + (\ln(9+1) - \ln(8+1))^2 +$$

$$(\ln(12+1) - \ln(11+1))^2 \Big].$$

$$MSLE = \frac{1}{4} \times \Big[(\ln 4 - \ln 3)^2 + (\ln 7 - \ln 6)^2 +$$

$$(\ln 10 - \ln 9)^2 + (\ln 13 - \ln 12)^2 \Big].$$

$$MSLE = \frac{1}{4} \Big[(1.386 - 1.098)^2 + (1.945 - 1.791)^2 +$$

$$(2.302 - 2.197)^2 + (2.564 - 2.484)^2 \Big]$$

$$MSLE = \frac{1}{4} \times \Big[(0.29)^2 + (0.154)^2 + (0.105)^2 + (0.08)^2 \Big]$$

$$MSLE = \frac{1}{4} \times \Big[0.0841 + 0.0237 + 0.011 + 0.0064 \Big]$$

$$MSLE = \frac{0.1254}{4}$$

$$MSLE = 0.0313$$

Iteration : 2

Predicted values closer to actual.

Actual values → $y = [3, 6, 9, 12]$.

Predicted values = $[2.5, 5.5, 8.5, 11.5]$.

$$MSLE = \frac{1}{n} \times \Sigma\left[\ln(y+1) - \ln(y.predict+1)\right]^2$$

$$MSLE = \frac{1}{4} \times \left[(\ln(3+1) - \ln(2.5+1))^2 + (\ln(6+1) - \ln(5.5+1))^2 + (\ln(9+1) - \ln(8.5+1))^2 + (\ln(12+1) - \ln(11.5+1))^2\right].$$

$$MSLE = \frac{1}{4}\left[(\ln 4 - \ln 3.5)^2 + (\ln 7 - \ln 6.5)^2 + (\ln 10 - \ln 9.5)^2 + (\ln 13 - \ln 12.5)^2\right].$$

$$MSLE = \frac{1}{4} \times \left[(1.386 - 1.252)^2 + (1.945 - 1.871)^2 + (2.302 - 2.251)^2 + (2.564 - 2.525)^2\right].$$

$$MSLE = \frac{1}{4} \times \left[(0.134)^2 + (0.074)^2 + (0.051)^2 + (0.039)^2\right]$$

$$MSLE = \frac{1}{4} \times \left[0.0179 + 0.0054 + 0.0026 + 0.0015\right].$$

$$MSLE = \frac{0.0274}{4}$$

$$\boxed{MSLE = 0.006}$$

(c) Mean Absolute Error Loss:

Iteration : 1

$$MAE = \frac{1}{n} \times \Sigma(y_{actual} - y_{predicted})$$

Actual values → $y = [3, 6, 9, 12]$

predicted values $= [2, 5, 8, 11]$

$$MAE = \frac{1}{4} \times [(3-2) + (6-5) + (9-8) + (12-11)]$$

$$MAE = \frac{1}{4} \times (1+1+1+1)$$

$$MAE = \frac{\cancel{4}^{1}}{\cancel{4}}$$

$$\boxed{MAE = 1}$$

Iteration: 2

Predicted values closer to actual

Actual values → $y = [3, 6, 9, 12]$

predicted values $= [2.5, 5.5, 8.5, 11.5]$

$$MAE = \frac{1}{n} \times \Sigma(y_{actual} - y_{predicted})$$

$$MAE = \frac{1}{4} \times [(3-2.5)+(6-5.5)+(9-8.5)+(12-11.5)]$$

$$MAE = \frac{1}{4} \times [0.5+0.5+0.5+0.5].$$

$$MAE = \frac{2}{4}$$

$$\boxed{MAE = 0.5}$$

Binary Classification Loss functions:

Binary Cross-Entropy:

Iteration: 1

$$BCE = -\Sigma[y \times \log(y.predict) + (1-y) \times \log(1-y.predict)].$$

Actual values → $y = [1, 0, 0, 1]$

predicted values = $[1, 0, 1, 0]$.

$$= -([1 \times \log(1) + (1-1) \times \log(1-1)] +$$
$$[0 \times \log(0) + (1-0) \times \log(1-0)] +$$
$$[0 \times \log(1) + (1-0) \times \log(1-1)] +$$
$$[1 \times \log(0) + (1-1) \times \log(1-0)]).$$

$$\boxed{BCE = 0}$$

Iteration: 2

Predicted values closer to actual.

Actual values → $y = [1, 0, 0, 1]$.

Predicted values = $[0.9, 0.1, 0.9, 0.1]$.

$$BCE = -\Sigma[y \times \log(y.predict) + (1-y) \times \log(1-y.predict)].$$

$$= -[(1 \times \log(0.9) + (1-1) \times \log(1-0.9)) +$$
$$(0 \times \log(0.1) + (1-0) \times \log(1-0.1)) +$$
$$(0 \times \log(0.9) + (1-0) \times \log(1-0.9)) +$$
$$(1 \times \log(0.1) + (1-1) \times \log(1-0.1))].$$

$$BCE = -[(1 \times -0.045) + 0 + 0 + 0 + (1 \times -1)$$
$$= (-0.045 - 1)$$
$$\boxed{BCE = 1.0}$$

(b) **Hinge Loss:**

Iteration: 1

$$HL = \max(0, 1 - (y \times y.predict)).$$

Actual values → $y = [1, -1, 1, -1]$

predicted values = $[0.8, -0.2, 0.7, -0.3]$.

$$HL = \left[\max\left[0, 1-(1 \times 0.8)\right] + \max(0, 1-(-1 \times -0.2)) + \max(0, 1-(1 \times 0.7)) + \max(0, 1-(-1 \times -0.3))\right].$$

$$HL = \left[\max(0, 0.2) + \max(0, 0.8) + \max(0, 0.3) + \max(0, 0.7)\right].$$

$$HL = 0.2 + 0.8 + 0.3 + 0.7$$

$$\boxed{HL = 2}$$

Squared Hinge Loss.

Iteration: 1

$$HL = \max(0, 1-(y \times y.\text{predict}))^2$$

Actual values $\rightarrow y = [1, -1, -1, 1]$.

predicted values $= [1, -1, 1, -1]$.

$$HL = \left[\max(0, 1-(1 \times 1))^2 + \max(0, 1.-(-1 \times -1))^2 + \max(0, 1-(-1 \times 1).)^2 + (\max(0, 1-(1 \times -1))^2\right]$$

$$HL = \left[\max(0, 1-1)^2 + \max(0, 1-1)^2 + \max(0, 1+1)^2 + \max(0, 1+1)^2\right]$$

$$HL = \max(0, 0) + \max(0, 0) + \max(0, 4) + \max(0, 4)$$

$$HL = 0 + 0 + 4 + 4$$

$$\boxed{HL = 4}$$

Iteration : 2

Actualvalues $\rightarrow y = [1, -1, -1, 1]$

Predicted values closer to actual values.

predicted values $= [0.9, -0.9, 0.9, -0.9]$

$HL = \max(0, 1 - (y \times y \cdot predict))^2$

$HL = [\max(0, 1 - (1 \times 0.9))^2 + \max(0, 1 - (-1 \times -0.9))^2 +$
$\max(0, 1 - (-1 \times 0.9))^2 + \max(0, 1 - (1 \times -0.9))^2]$

$HL = [\max(0, 1 - 0.9)^2 + \max(0, 1 - 0.9)^2 +$
$\max(0, 1 + 0.9)^2 + \max(0, 1 + 0.9)^2]$

$HL = [\max(0, 0.1)^2 + \max(0, 0.1)^2 +$
$\max(0, 1.9)^2 + \max(0, 1.9)^2]$.

$HL = [\max(0, 0.01) + \max(0, 0.01) +$
$\max(0, 3.61) + \max(0, 3.61)]$.

$HL = \max(0.01 + 0.01 + 3.61 + 3.61)$

$\boxed{HL = 7.24}$

3). Multi-class Classification Loss functions:

(a). Multi-class Cross Entropy loss:

Iteration: 1

$$MCE = -\sum(\text{actual} \times \log(\text{predict}))$$

Actual values → $y = [0, 1, 0, 0]$.

Predicted values $= [0.1, 0.6, 0.2, 0.1]$.

$$MCE = -(0 \times \log(0.1) + 1 \times \log(0.6) + 0 \times \log(0.2) + 0 \times \log(0.1))$$

$$MCE = -(0 \times -1 + 1 \times -0.22 + 0 \times -0.698 + 0 \times -1)$$

$$MCE = -(0 - 0.22 + 0 + 0)$$

$$\boxed{MCE = 0.22}$$

Iteration: 2

Predicted values closer to actual

Actual value → $y = [0, 1, 0, 0]$.

Predicted values → $y = [\cdot 1, 0.9, \cdot 1, \cdot 1]$

$$MCE = -\sum(\text{actual} \times \log(\text{predict}))$$

$$MCE = -\left(0 \times \log(.1) + 1 \times \log(0.9) + 0 \times \log(.1) + 0 \times \log(.1)\right)$$

$$MCE = -\left(0 + (-0.045) + 0 + 0\right)$$

$$\boxed{MCE = 0.045}$$

Sparse Multiclass Cross-Entropy Loss:-

Iteration: 1

$$SMCE = -\sum(\log(\text{predicted}))$$

Actual values $\rightarrow$ $y = [3, 6, 9, 12]$.

Predicted values = $[2, 5, 8, 11]$

$$SMCE = -\left(\log(2) + \log(5) + \log(8) + \log(11)\right)$$

$$SMCE = -\left(0.30 + 0.69 + 0.9 + 1.04\right)$$

$$\boxed{SMCE = -2.93}$$

Iteration: 2

Predicted closer to actual. $y = [3, 6, 9, 12]$

Predicted values = $[2.5, 5.5, 8.5, 11.5]$.

$$SMCE = -\sum(\log(\text{predicted})).$$

$$SMCE = -\left(\log(2.5) + \log(5.5) + \log(8.5) + \log(11.5)\right)$$

$$SMCE = -(0.397 + 0.74 + 0.929 + 1.06)$$

$$\boxed{SMCE = -3.126}$$

c) **Kullback Leibler Divergence Loss:-**

Iteration: 1

$$\text{KL Divergence} = \Sigma\left(\text{actual} \times \log\left(\frac{\text{actual}}{\text{predicted}}\right)\right)$$

Actual values $\rightarrow y = [3, 6, 9, 12]$.

Predicted values $= [2, 5, 8, 11]$.

$$KLD = (3 \times \log(3/2)) + (6 \times \log(6/5)) + (9 \times \log(9/8)) + (12 \times \log(12/11))$$

$$KLD = (3 \times 0.17) + (6 \times 0.07) + (9 \times 0.05) + (12 \times 0.03)$$

$$KLD = 0.51 + 0.42 + 0.45 + 0.36$$

$$\boxed{KLD = 1.74}$$

Iteration: 2

Predicted closer to actual values.

Actual values $\rightarrow y = [3, 6, 9, 12]$.

Predicted values $= [2.5, 5.5, 8.5, 11.5]$.

$$KLD = \Sigma\left(\text{actual} \times \log\left(\frac{\text{actual}}{\text{predicted}}\right)\right)$$

$$KLD = (3 \times \log(3/2.5)) + (6 \times \log(6/5.5)) + (9 \times \log(9/8.5)) + (12 \times \log(12/11.5))$$

$$KLD = (3 \times 0.079) + (6 \times 0.037) + (9 \times 0.248) + (12 \times 0.184)$$

$$KLD = (0.237 + 0.222 + 2.232 + 2.208)$$

$$\boxed{KLD = 4.899}$$