Show how to multiply this matrix chain optimally.

$A_1 = 30 \times 35$

$A_2 = 35 \times 15$

$A_3 = 15 \times 5$

$A_4 = 5 \times 10$

$A_5 = 10 \times 20$

$A_6 = 20 \times 25$

m

	1	2	3	4	5	6
1	0	15750	7875	9375	11875	15125
2		0	2625	4375	7125	10500
3			0	750	2500	5375
4				0	1000	3500
5					0	5000
6						0

S

0	1	1	3	3	3
	0	2	3	3	3
		0	3	3	3
			0	4	5
				0	5
					0

$A_1 \cdot A_2 = m[1,2]$

$30 \times 35 \cdot 35 \times 15 = 30 \times 35 \times 15 = 15750$

$A_2 \cdot A_3 = m[2,3]$

$35 \times 15 \cdot 15 \times 5 = 35 \times 15 \times 5 = 2625$

$A_3 \cdot A_4 = m[3,4]$

$15 \times 5 \cdot 5 \times 10 = 15 \times 5 \times 10 = 750$

$A_4 \cdot A_5 = m[4,5]$

$5 \times 10 \cdot 10 \times 20 = 5 \times 10 \times 20 = 1000$

$A_5 . A_6 = m[5,6]$

$10 \times 20 . 20 \times 25 = 10 \times 20 \times 25 = 5000$

$m[i,j] = \min\{ m[i,k] + m[k+1,j] + P_{i-1} P_k P_j\}$

j	i					
30	35	15	5	10	20	25
0	1	2	3	4	5	6

$(A_1 . A_2) A_3$

$$m[\overset{i}{1},\overset{k}{2}] + m[\overset{k+1}{3},\overset{j}{3}] + \overset{i-1}{P_0} \overset{j}{P_3} \overset{k}{P_2}$$

$$15750 + 0 + 30 \times 5 \times 15$$

$$= 18000$$

$A_1 (A_2 . A_3)$

$$m[\overset{i}{1},\overset{k}{1}] + m[\overset{k+1}{2},\overset{j}{3}] + \overset{i-1}{P_0} \overset{k}{P_1} \overset{j}{P_3}$$

$$0 + 2625 + 30 \times 35 \times 5$$

$$\boxed{= 7875}$$

$(A_2 . A_3) A_4$

$$m[\overset{i}{2},\overset{k}{3}] + m[\overset{k+1}{4},\overset{j}{4}] + \overset{i-1}{P_1} \overset{k}{P_3} \overset{j}{P_4}$$

$$2625 + 0 + 35 \times 5 \times 10$$

$$= \boxed{4375}$$

$A_2 (A_3 . A_4)$

$$m[\overset{i}{2},\overset{k}{2}] + m[\overset{k+1}{3},\overset{j}{4}] + \overset{i-1}{P_1} \overset{k}{P_2} \overset{j}{P_4}$$

$$0 + 750 + 35 \times 15 \times 10$$

$$= 6000$$

Left column:

$(A_3 \cdot A_4) A_5$

$$m[3,4] + m[5,5] + P_2 P_4 P_5$$

$$750 + 0 + 15 \times 10 \times 20$$

$$= 3750$$

$A_3(A_4 \cdot A_5)$

$$m[3,3] + m[4,5] + P_2 P_3 P_5$$

$$0 + 600 + 15 \times 5 \times 20$$

$$\boxed{= 2500}$$

Right column:

$(A_4 \cdot A_5) A_6$

$$m[4,5] + m[6,6] + P_3 P_5 P_6$$

$$1000 + 0 + 5 \times 20 \times 25$$

$$\boxed{= 3500}$$

$A_4(A_5 \cdot A_6)$

$$m[4,4] + m[5,6] + P_3 P_4 P_6$$

$$0 + 5000 + 5 \times 10 \times 25$$

$$= 6250$$

$$\rightarrow \quad A_1(A_2 \cdot A_3 \cdot A_4)$$
$$\rightarrow \quad (A_1 A_2)(A_3 A_4) \quad \Big\} \quad 3 \text{ ways}$$
$$\rightarrow \quad (A_1 \cdot A_2 A_3) A_4$$

$$\rightarrow A_1(A_2 \cdot A_3 \cdot A_4)$$

$$\therefore \quad (A_2 \cdot A_3 \cdot A_4) = 4375 .$$

$$4375 + 30 \times 35 \times 10$$

$$= 14875$$

$$\rightarrow (A_1 \cdot A_2)(A_3 \cdot A_4)$$

$$15750 + 750 + 30 \times 15 \times 10$$

$$= 21000$$

$$\rightarrow (A_1 \cdot A_2 \cdot A_3) A_4$$

$$\therefore (A_1 A_2 A_3) = 7875$$

$$7875 + 30 \times 5 \times 10$$

$$\boxed{= 9375}$$

Right side note:

$A_3 \cdot A_4$

$15 \times 5 \, . \, 5 \times 10$

$\overline{15 \times 10}$

$A_2 \cdot (15 \times 10)$

$35 \times 15 \quad 15 \times 10$

(35×10)

$A_1 \cdot (35 \times 10)$

$30 \times 35 \, . \, 35 \times 10$

$\rightarrow$ $A_2(A_3 . A_4 . A_5)$
$\rightarrow$ $(A_2 A_3)(A_4 . A_5)$ $\Bigg\}$ 3 ways
$\rightarrow$ $(A_2 . A_3 . A_4) A_5$

$\rightarrow$ $A_2(A_3 . A_4 . A_5)$

$\therefore (A_3 . A_4 . A_5) = 2500$

$2500 + 35 \times 15 \times 20$

$= 13000$

$\rightarrow$ $(A_2 . A_3)(A_4 . A_5)$

$2625 + 1000 + 35 \times 5 \times 20$

$\boxed{= 7125}$

$\rightarrow$ $(A_2 . A_3 . A_4) A_5$

$\therefore (A_2 . A_3 . A_4) = 4375$

$4375 + 35 \times 10 \times 20$

$= 11375$

$\rightarrow$ $A_3(A_4 . A_5 . A_6)$
$\rightarrow$ $(A_3 . A_4)(A_5 . A_6)$
$\rightarrow$ $(A_3 . A_4 . A_5) A_6$

$\rightarrow$ $A_3(A_4 . A_5 . A_6)$

$\therefore (A_4 . A_5 . A_6) = 3500$

$3500 + 15 \times 5 \times 25$

$\boxed{= 5375}$

→ $(A_3 \cdot A_4)(A_5 \cdot A_6)$

$\qquad$ $750 + 5000 + 15 \times 10 \times 25$

$\qquad\qquad = 9500$

→ $(A_3 \cdot A_4 \cdot A_5) A_6$

$\qquad\qquad$ ∴ $(A_3 \cdot A_4 \cdot A_5) = 2500$

$\qquad 2500 + 15 \times 20 \times 25$

$\qquad\qquad = 10000$

→ $A_1(A_2 \cdot A_3 \cdot A_4 \cdot A_5)$
→ $(A_1 \cdot A_2 \cdot A_3)(A_4 \cdot A_5)$ $\Big\}$ 4 ways
→ $(A_1 \cdot A_2 \cdot A_3 \cdot A_4) \cdot A_5$.
→ $(A_1 \cdot A_2)(A_3 \cdot A_4 \cdot A_5)$

→ $A_1(A_2 \cdot A_3 \cdot A_4 \cdot A_5)$

$\qquad\qquad$ ∴ $(A_2 A_3 A_4 A_5) = 7125$

$\qquad 7125 + 30 \times 35 \times 20$

$\qquad\qquad = 28125$

→ $(A_1 \cdot A_2 \cdot A_3)(A_4 \cdot A_5)$

$\qquad 7875 + 1000 + 30 \times 5 \times 20$

$\qquad\qquad \boxed{= 11875}$

→ $(A_1 \cdot A_2 \cdot A_3 \cdot A_4) A_5$ $\qquad$ ∴ $(A_1 \cdot A_2 \cdot A_3 \cdot A_4) = 9375$.

$\qquad = 9375 + 30 \times 10 \times 20$

$\qquad\qquad = 15375$

→ $(A_1 \cdot A_2)(A_3 \cdot A_4 \cdot A_5)$

$\qquad 15750 + 2500 + 30 \times 15 \times 20$

$\qquad\qquad = 27250$

→ $A_2 (A_3 . A_4 . A_5 A_6)$
→ $(A_2 . A_3 . A_4) (A_5 . A_6)$ } 4 ways
→ $(A_2 . A_3 . A_4 . A_5) A_6$
→ $(A_2 . A_3)(A_4 . A_5 . A_6)$
→ $A_2 (A_3 . A_4 . A_5 . A_6)$

∴ $(A_3 A_4 A_5 A_6) = 5375$

$5375 + 35 \times 15 \times 25$

$= 18500$

→ $(A_2 . A_3 . A_4) (A_5 . A_6)$

$4375 + 5000 + 35 \times 10 \times 25$

$= 18125$

→ $(A_2 . A_3 . A_4 . A_5) A_6$

∴ $(A_2 A_3 A_4 A_5) = 7125$

$7125 + 35 \times 20 \times 25$

$= 24625$

→ $(A_2 . A_3) (A_4 . A_5 . A_6)$

$2625 + 3500 + 35 \times 5 \times 25$

$= 10500$

→ $A_1(A_2 . A_3 . A_4 . A_5 . A_6)$
→ $(A_1 . A_2 . A_3)(A_4 . A_5 . A_6)$ } 3 ways.
→ $(A_1 . A_2 . A_3 . A_4 . A_5) A_6$

→ $A_1 (A_2 . A_3 . A_4 . A_5 . A_6)$

$18500 + 30 \times 35 \times 25$

$= 44750$

→ $(A_1 A_2 A_3 A_4 A_5) A_6$

$11875 + 30 \times 20 \times 25$

$= 26875$

→ $(A_1 A_2 A_3) (A_4 A_5 A_6)$

$7875 + 3500 + 30 \times 5 \times 25$

$\boxed{= 15125}$