KNAPSACK PROBLEM

item	weight	Value
1	2	12
2	1	10
3	3	20
4	2	15

Capacity
($W = 5$)

Solution:-

TABLE

Capacity

items	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	12	12	12	12
2	0	10	12	22	22	22
3	0	10	12	22	30	32
4	0	10	15	25	30	37

F1:- $V[i,j] = \max\{V[i-1,j],\ v_i + V[i-1, j-w_i]\} \Rightarrow \text{if}(j - w_i \geq 0)$

F2:- $V[i,j] = V[i-1, j] \Rightarrow \text{if}(j - w_i < 0)$

where, i = item, j = capacity, v_i = value, w_i = weight

initially $V[0, j]$ & $V[i, 0] = 0$

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$i = 1$, $j = 1$, $w_1 = 2$

check $j - w_i$ $\therefore$ weight of selected item $= w_1 \Rightarrow 2$

$\Rightarrow j - w_i = 1 - 2 = \boxed{-1 < 0}$

F2 :-

$V[i, j] = V[i-1, j]$

$V[1, 1] = V[0, 1]$

$V[1, 1] = 0$

$i = 1$, $j = 2$, $w_1 = 2$

check $j - w_i \Rightarrow 2 - 2 = \boxed{0 = 0}$

F1 :-

$V[i, j] = \max\{V[i-1, j], v_i + V[i-1, j-w_i]\}$ $\therefore v_1 = 12$

$V[1, 2] = \max\{V[0, 2], 12 + V[0, 0]\}$ $V[0, 2] = 0$

$V[1, 2] = \max\{0, 12\}$ $V[0, 0] = 0$

$V[1, 2] = 12$

$i = 1$, $j = 3$, $w_i = 2$

check $j - w_i \Rightarrow 3 - 2 = \boxed{1 > 0}$

F1 :-

$V[1, 3] = \max\{V[0, 3], 12 + V[0, 1]\}$ $\therefore V[0, 3] = 0$

$V[1, 3] = 12$ $V[0, 1] = 0$

$i = 1$, $j = 4$, $w_1 = 2$

check $j - w_i \Rightarrow 4 - 2 = \boxed{2 > 0}$

F1 :-

DATE / /   M T W T F S S

$V[1,4] = \max\{V[0,4], 12 + V[0,2]\}$ $\therefore V[0,4] = 0$

$V[1,4] = 12$ $V[0,2] = 0$

$i = 1$, $j = 5$, $w_1 = 2$

check $j - w_i \Rightarrow$ $5 - 2 = \boxed{3 > 0}$

F1:-

$V[1,5] = \max\{V[0,5], 12 + V[0,3]\}$ $\therefore V[0,5] = 0$

$V[1,5] = 12$ $V[0,3] = 0$

$i = 2$, $j = 1$, $w_2 = 1$

check $j - w_i \Rightarrow$ $1 - 1 = \boxed{0 = 0}$

F1:-

$V[2,1] = \max\{V[1,1], 10 + V[1,0]\}$ $\therefore V[1,1] = 0$

$V[2,1] = 10$ $V[1,0] = 0$

$10 + 0 = 10$

$i = 2$, $j = 2$, $w_2 = 1$

check $j - w_i \Rightarrow$ $2 - 1 = \boxed{1 > 0}$

F1:-

$V[2,2] = \max\{V[1,2], 10 + V[1,1]\}$ $\therefore V[1,2] = 12$

$V[2,2] = \max\{12, 10\}$ $V[1,1] = 0$

$10 + 0 = 10$

$V[2,2] = 12$

$i = 2$, $j = 3$, $w_2 = 1$

check $j - w_i \Rightarrow$ $3 - 1 = \boxed{2 > 0}$

F1:-

$V[2,3] = \max\{V[1,3], 10 + V[1,2]\}$ $\therefore V[1,3] = 12$

$V[1,2] = 12$

DATE / /

$V[2,3] = \max\{12, 22\}$ $\therefore 10+12=22$

$V[2,3] = 22$

$i=2$, $j=4$, $w_2 = 1$

check $= j - w_i \Rightarrow 4-1 = \boxed{3 > 0}$

F1 :-

$V[2,4] = \max\{V[1,4], 10 + V[1,3]\}$ $\therefore V[1,4] = 12$

$V[1,3] = 12$

$V[2,4] = 22$

$10+12=22$

$i=2$, $j=5$, $w_2 = 1$

check $j - w_i \Rightarrow 5-1 = \boxed{4 > 0}$

F1 :-

$V[2,5] = \max\{V[1,5], 10 + V[1,4]\}$ $\therefore V[1,5] = 12$

$V[1,4] = 12$

$V[2,5] = 22$

$10+12=22$

$i=3$, $j=1$, $w_3 = 3$

check $j - w_i \Rightarrow 1-3 = \boxed{-2 < 0}$

F2 :-

$V[3,1] = V[3-1, 1]$

$V[3,1] = V[2,1]$

$V[3,1] = 10$

$i=3$, $j=2$, $w_3 = 3$

check $j - w_i \Rightarrow 2-3 = \boxed{-1 < 0}$

F2 :-

$V[3,2] = V[2,2]$

$V[3,2] = 12$

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$i = 3$, $j = 3$, $w_3 = 3$

$j - w_i \Rightarrow 3 - 3 = \boxed{0 = 0}$

F1:-

$V[3,3] = \max\{V[2,3], 20 + V[2,0]\}$ $\therefore V[2,3] = 22$

$V\{3,3\} = \max\{22, 20\}$ $V[2,0] = 0$

$V[3,3] = 22$

$i = 3$, $j = 4$, $w_3 = 3$

$j - w_i \Rightarrow 4 - 3 = \boxed{1 > 0}$

F1:-

$V[3,4] = \max\{V[2,4], 20 + V[2,1]\}$ $\therefore V[2,4] = 22$

$V(3,4) = \max\{22, 30\}$ $V[2,1] = 10$

$20 + 10 = 30$

$V[3,4] = 30$

$i = 3$, $j = 5$, $w_3 = 3$

$j - w_i \Rightarrow 5 - 3 = \boxed{2 > 0}$

F1:-

$V[3,5] = \max\{V[2,5], 20 + V[2,2]\}$ $\therefore V[2,5] = 22$

$V[3,5] = \max\{22, 32\}$ $V[2,2] = 12$

$20 + 12 = 32$

$V[3,5] = 32$

$i = 4$, $j = 1$, $w_4 = 2$

$j - w_i \Rightarrow 1 - 2 = \boxed{-1 < 0}$

F2:-

$V[4,1] = V[3,1]$

$V[4,1] =$

DATE / /   M T W T F S S

$i=4$, $j=2$, $w_4=2$

$j-w_i \Rightarrow 2-2 = \boxed{0 = 0}$

F1:-

$V[4,2] = \max\{V[3,2], 15+V[3,0]\}$ $\therefore V[3,2]=12$

$V[4,2] = 15$ $V[3,0]=0$

$15+0=15$

$i=4$, $j=3$, $w_4=2$

$j-w_i \Rightarrow 3-2 = \boxed{1 > 0}$

F1:-

$V[4,3] = \max\{V[3,3], 15+V[3,1]\}$ $\therefore V[3,3]=22$

$V[4,3] = \cancel{22}\ 25$ $V[3,1]=10$

$15+10=25$

$i=4$, $j=4$, $w_4=2$

$j-w_i \Rightarrow 4-2 = \boxed{2 > 0}$

F1:-

$V[4,4] = \max\{V[3,4], 15+V[3,2]\}$ $\therefore V[3,4]=30$

$V[4,4] = 30$ $V[3,2]=12$

$15+12=27$

$i=4$, $j=5$, $w_4=2$

$j-w_i \Rightarrow 5-2 = \boxed{3 > 0}$

F1:-

$V[4,5] = \max\{V[3,5], 15+V[3,3]\}$ $\therefore V[3,5]=32$

$V[4,5] = 37$ $V[3,3]=22$

$15+22=37$

(BackTracking)

	0	1	2	3	4	5
0	0	$\boxed{0}$	0	0	0	0
1	0	12	$\boxed{12}$	12	12	12
2	0	10	22	$\boxed{22}$	22	22
3	0	10	12	22	30	32
4	0	10	15	25	30	$\boxed{37}$

1) Start from max profit which is 37
2) Backtrack from Last cell
3) Find the Max Value
4) Include that item

Max 37 item 4 selected

item 4 original proifit = 15

37 - 15 = 22

$$\begin{array}{r} 37 \\ -15 \\ \hline 22 \end{array}$$

Find profit 22 moving upwards diagonally.

22 found ~~item~~ in row 3 but was generated by row 2, so we selet row 2 / item 2

item 2 original profit = 10

22 - 10 = 12

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Find profit 12 moving upwards diagonally

12 found in row 1

item 1 select

item 1 original profit = 12

12 - 12 = 0

Moving upwards diagonally until we find profit 0.

Optimal Solution includes items [4, 2, 1]