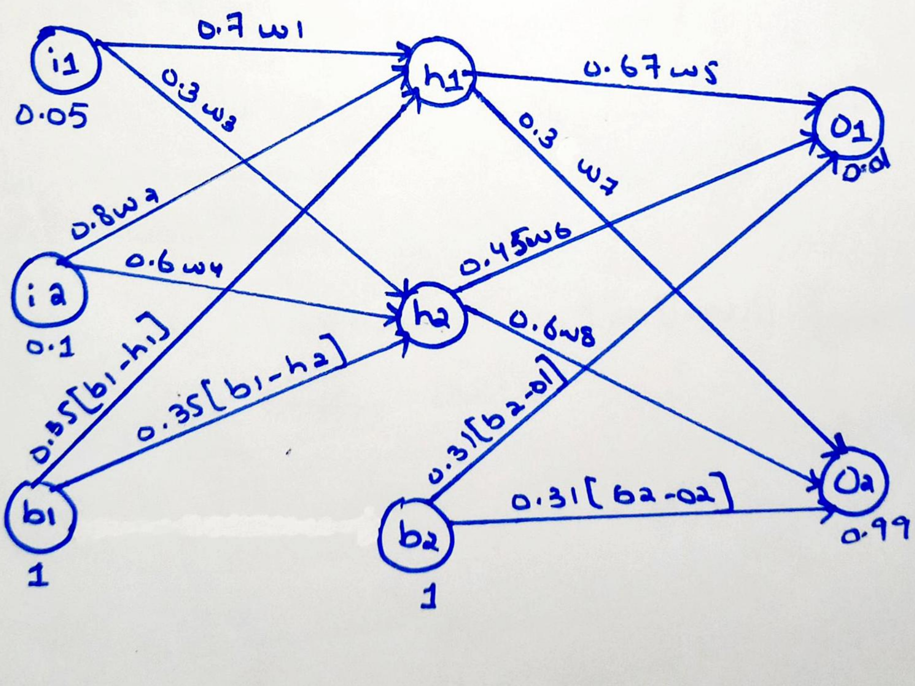
Question No # 1

Please use the following network to update all the weights except the bias weights using backward propagation for MLP. Input is given as (i1 = 0.05) and (i2 = 0.1) and target values for (o1 = 0.01 and (o2 = 0.99) and learning rate = 1. Execute for only one iteration. All neurons have sigmoid activation function.



Forward Propagation:-

-> Weighted sum for the hidden layer.
Find H_1, H_2.

$$H_1 = (w_1 \times i1) + (w_2 \times i2) + (b1 \times 0.35)$$
$$= (0.7 \times 0.05) + (0.8 \times 0.1) + (1 \times 0.35)$$
$$= 0.035 + 0.08 + 0.35.$$

$$\boxed{H_1 = 0.465}$$

$$H_2 = (w_3 \times i1) + (w_4 \times i2) + (b_1 \times 0.35)$$
$$= (0.3 \times 0.05) + (0.6 \times 0.1) + (1 \times 0.35)$$
$$= 0.015 + 0.06 + 0.35$$

$$\boxed{H_2 = 0.425}$$

-> Apply the sigmoid activation function to the hidden layer units.

$$h_1 = \frac{1}{1 + e^{-0.465}}$$

$$h_1 = \frac{1}{1 + 0.628}$$

$$h_2 = \frac{1}{1+e^{-0.425}}$$

$$h_2 = \frac{1}{1+0.653}$$

$$\boxed{h_2 = 0.6046}$$

→ Weighted sum for the output layer units:

Find O_1, O_2.

$$O_1 = (w_5 \times h_1) + (w_6 \times h_2) + (b_2 \times 0.31)$$

$$= (0.67 \times 0.614) + (0.45 \times 0.604) + (1 \times 0.31)$$

$$= 0.411 + 0.2718 + 0.31$$

$$\boxed{O_1 = 0.9928}$$

$$O_2 = (w_7 \times h_1) + (w_8 \times h_2) + (b_2 \times 0.31)$$

$$= (0.3 \times 0.614) + (0.6 \times 0.604) + (1 \times 0.31)$$

$$= 0.1842 + 0.3624 + 0.31$$

$$\boxed{O_2 = 0.8584}$$

→ Apply the sigmoid activation function to the output layer units.

$$O_1 = \frac{1}{1+e^{-0.9928}}$$

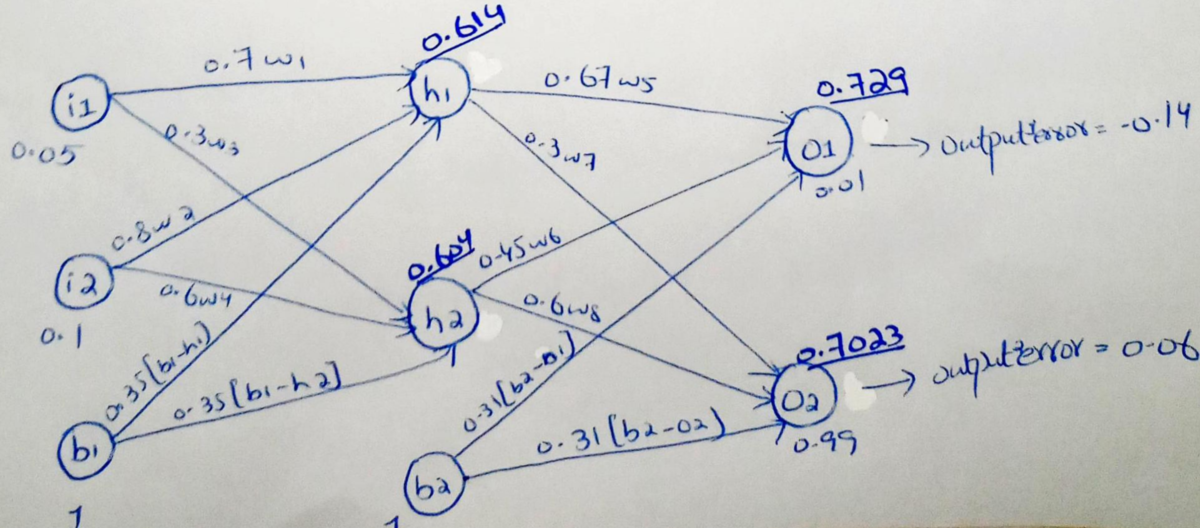
$$O_1 = \frac{1}{1+0.3705}$$

$$\boxed{O_1 = 0.729}$$

$$O_2 = \frac{1}{1+e^{-0.8584}}$$

$$O_2 = \frac{1}{1+0.4238}$$

$$\boxed{O_2 = 0.7023}$$



→ Calculate error:-

• For output units:-

Target ($o_1 = 0.01$)

$$\text{output}_1 = o_1(1-o_1)(\text{Target} - o_1)$$

$$= 0.729(1-0.729)(0.01-0.729)$$

$$= 0.729(0.0271)(-0.719)$$

$$\boxed{\text{output}_1 = -0.14}$$

Target ($o_2 = 0.99$)

$$\text{output}_2 = o_2(1-o_2)(\text{Target} - o_2)$$

$$= 0.7023(1-0.7023)(0.99-0.7023)$$

$$= 0.7023(0.2977)(0.2877)$$

$$\boxed{\text{output}_2 = 0.06}$$

• Total error:-

$$\text{Total error} = 0.5 \times \left((\text{Target}\, o_1 - \text{sigmoid}\, o_1)^2 + (\text{Target}\, o_2 - \text{sigmoid}\, o_2)^2 \right)$$

$$= 0.5 \times \left((0.01 - 0.729)^2 + (0.99 - 0.7023)^2 \right)$$

$= 0.5\left((-0.719)^2 + (0.287)^2\right)$

$= 0.5\,(0.5169 + 0.0823)$

$= 0.5\,(0.5992)$

$\boxed{\text{Total error} = 0.2996}$

Backward Propagation:-

→ Updating the weights between the hidden layer units.

$$\Delta w_{ij} = \eta \, \delta_j \, O_i$$

η is a learning rate which is equal to $= 1$.

$$\Delta w_5 = \eta \, \overset{\text{error}}{O_1} \, h_1$$

$$= 1(-0.14)(0.614)$$

$$\Delta w_5 = -0.085$$

$$w_5(\text{new}) = \Delta w_5 + w_5(\text{old})$$

$$w_5(\text{new}) = -0.085 + 0.67$$

$$\boxed{w_5(\text{new}) = 0.585}$$

$$\Delta w_7 = \eta \, o_2^{error} \, h_1$$

$$= 1 \times 0.06 \times 0.614$$

$$\Delta w_7 = 0.036$$

$$w_7(new) = \Delta w_7 + w_7(old)$$

$$= 0.036 + 0.3$$

$$\boxed{w_7(new) = 0.336}$$

$$\Delta w_6 = \eta \, o_1^{error} \, h_2$$

$$= 1 \times -0.14 \times 0.604$$

$$\Delta w_6 = -0.084$$

$$w_6(new) = \Delta w_6 + w_6(old)$$

$$= -0.084 + 0.45$$

$$\boxed{w_6(new) = 0.366}$$

$$\Delta w_8 = \eta \, o_2^{error} \, h_2$$

$$= 1 \times 0.06 \times 0.604$$

$$\Delta w_8 = 0.036$$

$$w_8(new) = \Delta w_8 + w_8(old)$$

$$= 0.036 + 0.6$$

$$\boxed{w_8(new) = 0.636}$$

$\Delta w_1 = \eta h_1 i_1$

$= 1 \times 0.614 \times 0.05$

$\Delta w_1 = 0.0307$

$w_1(new) = \Delta w_1 + w_1(old)$

$= 0.0307 + 0.7$

$\boxed{w_1(new) = 0.73}$

$\Delta w_3 = \eta h_2 i_1$

$= 1 \times 0.604 \times 0.05$

$\Delta w_3 = 0.0302$

$w_3(new) = \Delta w_3 + w_3(old)$

$= 0.0302 + 0.3$

$\boxed{w_3(new) = 0.3302}$

$\Delta w_2 = \eta h_1 i_2$

$= 1 \times 0.614 \times 0.1$

$\Delta w_2 = 0.0614$

$w_2(new) = \Delta w_2 + w_2(old)$

$= 0.0614 + 0.8$

$$\Delta w_4 = \eta\, h_2\, i_2$$

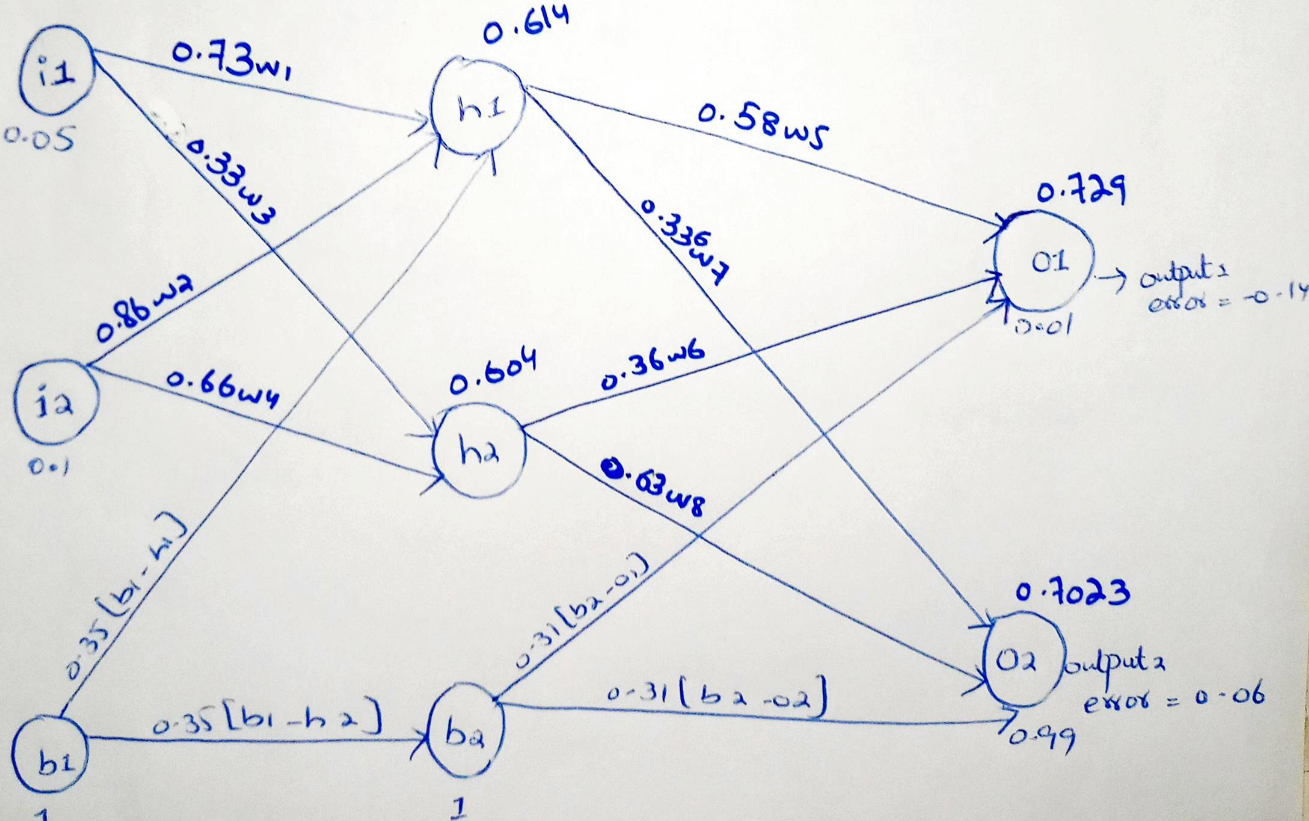
$$= 1 \times 0.604 \times 0.1$$

$$\Delta w_4 = 0.0604$$

$$w_4(\text{new}) = \Delta w_4 + w_4(\text{old})$$

$$= 0.0604 + 0.6$$

$$\boxed{w_4(\text{new}) = 0.6604}$$



Forward Propagation With Updated Weights:-

→ Calculate the weighted sum for the hidden layer units:-

$$h_1 = (w_1 \times i_1) + (w_2 \times i_2) + (b_1 \times 0.35)$$

$$= (0.73 \times 0.05) + (0.86 \times 0.1) + (1 \times 0.35)$$

$$= 0.0365 + 0.086 + 0.35$$

$$\boxed{h_1 = 0.4725}$$

$$h_2 = (w_3 \times i_1) + (w_4 \times i_2) + (b_1 \times 0.35)$$

$$= (0.33 \times 0.05) + (0.66 \times 0.1) + (1 \times 0.35)$$

$$= 0.0165 + 0.066 + 0.35$$

$$\boxed{h_2 = 0.4325}$$

→ Apply the sigmoid activation function to the hidden layer units:-

$$h_1 = \frac{1}{1 + e^{-0.4725}}$$

$$h_1 = \frac{1}{1 + 0.623}$$

$$\boxed{h_1 = 0.6159}$$

$$h_2 = \frac{1}{1+e^{-0.4325}}$$

$$h_2 = \frac{1}{1+0.64}$$

$$\boxed{h_2 = 0.606}$$

→ Weighted sum for the output layer units

$$O_1 = (w_5 * h_1^{output}) + (w_6 \times h_2^{output}) + (b_2 \times 0.31)$$

$$O_1 = (0.58 * 0.614) + (0.38 \times 0.604) + (1 \times 0.31)$$

$$O_1 = (0.356 + 0.229 + 0.31)$$

$$\boxed{O_1 = 0.895}$$

$$O_2 = (w_7 \times h_1) + (w_8 \times h_2) + (b_2 \times 0.31)$$

$$= (0.336 \times 0.614) + (0.63 \times 0.604) + (1 \times 0.31)$$

$$= 0.206 + 0.3805 + 0.31$$

$$\boxed{O_2 = 0.896}$$

→ Apply the sigmoid activation function to the output layer units:-

$$O_1 = \frac{1}{1+e^{-0.895}}$$

$$\boxed{O_1 = 0.709}$$

$$O_2 = \frac{1}{1+e^{-0.896}}$$

$$\boxed{O_2 = 0.71}$$