

Digital Logic Design:

- Introduction
 - Number System
 - Digital logic Gates (Symbol, Definition, Truth table)
- AND :-

It's an electronic gate which is true when all the inputs are true.

- OR :-

True when one of the input is true.

- Boolean Algebra.
- Combinational Circuit (Adder / Subtractor) /
(when you changed the input, the Max / demum, output is changed) Coder / Decoder
- Sequential Circuit (Flipflops
(Changes output based on Registers → Memory input and previous outputs) Counters).

Number System:-

Weighted | Not weighted

- Decimal
- Binary
- HexaDecimal
- Octal

Symbols & Digits.

Total digits are 10.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

	(2)	(8)	(5)	(16)
Decimal	Binary	Octal	Pedal	HexaDecimal
0	0	0	0	0
1	1	1	1	1
2	10	2	2	2
3	11	3	3	3
4	100	4	4	4
5	101	5	10	5
6	110	6	11	6
7	111	7	12	7
8	1000	10	13	8
9	1001	11	14	9
10	1010	12	20	A
11	1011	13	21	B
12	1100	14	22	C
13	1101	15	23	D
14	1110	16	24	E
15	1111	17	30	F
16	100000	20	31	1A
17	10001	21	32	1B
18	10010	22	33	1C
19	10011	23	34	1D
20	10100	24	40	1E
21	10101	25	41	1F
22	10110	26	42	2A
23	10111	27	43	2B
24	11000	30	44	2C
25	11001	31	50	2D
26	11010	32	51	2E
27	11011	33	52	2F

Decimal	Binary	Octal	Pedal	Hexa Decimal
28	11100	34	53	3A
29	11101	35	54	3B
30	11110	36	60	3C
31	11111	37	61	3D
32	100000	40	62	3E
33	100001	41	63	3F
34	10010	42	64	4A
35	100011	43	70	4B
36	100100	44	71	4C
37	100101	45	72	4D
38	100110	46	73	4E
39	100111	47	74	4F
40	101000	50	80	5A

$$\begin{array}{r}
 2 | 5473 \\
 2 | 2736 - 1 \\
 2 | 1368 - 0 \\
 2 | 684 - 0 \\
 2 | 342 - 0 \\
 2 | 171 - 0 \\
 2 | 85 - 1 \\
 2 | 42 - 1 \\
 2 | 21 - 0 \\
 2 | 10 - 1 \\
 2 | 5 - 0 \\
 2 | 2 - 1 \\
 \hline
 & 1 - 0
 \end{array}$$

$$(1010101100001)_2$$

$$\begin{aligned}
 (5473)_{10} &= 4096 + 1024 + 256 + 64 + 32 + 1 \\
 &= 2^{12} + 2^{10} + 2^8 + 2^6 + 2^5 + 2^0 \\
 &= 1010101100001
 \end{aligned}$$

$$\begin{array}{ccccccccccccccccc}
 2^0 & 2^1 & 2^2 & 2^3 & 2^4 & 2^5 & 2^6 & 2^7 & 2^8 & 2^9 & 2^10 & 2^11 & 2^12 & 2^13 & 2^14 & 2^15 & 2^16 & 2^17 & 2^18 & 2^19 & 2^20 \\
 1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 & 512 & 1024 & 2048 & 4096 & 8192 & 16384 & 32768 & 65536 & 131072 & 262144 & 524288 & 1048576
 \end{array}$$

$$\begin{array}{ccccccccccccc}
 2^0 & 2^1 & 2^2 & 2^3 & 2^4 & 2^5 & 2^6 & 2^7 & 2^8 & 2^9 & 2^10 & 2^11 & 2^12 & 2^13 & 2^14 & 2^15 \\
 1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 & 512 & 1024 & 2048 & 4096 & 8192 & 16384 & 32768
 \end{array}$$

(2)	(5)	(8)	(16)
Binary	Pedal	Octal	Hexa Decimal
1	1	1	1
2	5	8	16
4	25	64	256
8	125	512	4096
16	625	4096	65536
32	3125	32768	1048576
64	15625	262144	16777216
128	78125	2097152	268435456
256	390625	16777216	4294967296
512	1953125	134217728	
1024	9765625	1073741824	
2048	48828125	8589934592	
4096	244140625		
8192	1220703125		
16384	6103515625		
32768			
65536			
131072			
262144			
524288			
1048576			
2097152			
4194304			
8388608			
16777216			
33554432			

$$\rightarrow (1234)_{10} = ()_2$$

Date: 04/04/2023

$$512 + 256 + 128 + 64 + 16 + 2$$

$$2^9 + 2^7 + 2^6 + 2^4 + 2^3$$

$$(1001101001)_2$$

$$\rightarrow (1234)_{10} = ()_5$$

$$625 + 125 + 25 + 5 + 1$$

$$? \times 5^4 + ? \times 5^3 + ? \times 5^2 + ? \times 5^1 + ? \times 5^0$$

$$\rightarrow (1234)_{10} = ()_8$$

$$512 + 64 + 8 + 1$$

$$? \times 8^3 + ? \times 8^2 + ? \times 8^1 + ? \times 8^0$$

$$2 \times 8^3 + 3 \times 8^2 + 2 \times 8^1 + 2 \times 8^0$$

$$= (2322)_8$$

$$\rightarrow (1234)_{10} = ()_{16}$$

$$256 + 16 + 1$$

$$? \times 16^2 + ? \times 16^1 + ? \times 16^0$$

$$4 \times 16^2 + 1 \times 16^1 + 2 \times 16^0$$

$$(4D2)_{16}$$

BCD (Binary Coded Decimal) :-

Binary Coded Digits.

25863.

0010 0101 1000 0110 0011

When answer is invalid
or carry is generated.

$$\begin{array}{r} 11001 \\ 0111 \\ \hline 10000 \\ 0110 \\ \hline 0001, 0110 \\ 16 \end{array}$$

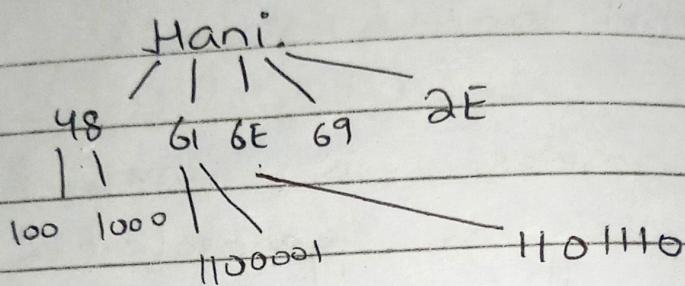
→ GRAY CODE :-

0000	→	0
0001	→	1
0011	→	2
0010	→	3
0110	→	4
0111	→	5
0101	→	6
0100	→	7
1100	→	8

→ ASCII :-

(American Standard Code for Information Interchange)

Hanzala.



Assignment # 1 .

→ What is BCD? How to convert
BCD to Binary and
Binary to BCD .

→ What is gray code? How to
convert BCD to gray and
gray to BCD .

→ Write down your name and
registration number in ASCII?

MSB

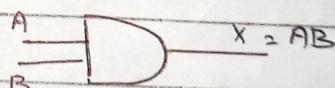
(Most Significant Bit)

LSB

(Least Significant Bit)

$$\begin{array}{r}
 & \overset{1}{\cancel{1}} \overset{1}{\cancel{1}} \overset{1}{\cancel{1}} \overset{1}{\cancel{1}} \\
 & \overset{1}{1} \overset{1}{1} 0 \overset{1}{1} & \overset{1}{1} \overset{1}{1} 0 \overset{1}{1} \\
 + & 0 0 1 \overset{1}{1} & 0 0 1 \overset{1}{1} \\
 \hline
 1 0 0 0 \overset{1}{1} & 0 0 0 0
 \end{array}$$

$$\begin{array}{r}
 \overset{+3}{\cancel{1}} \overset{+3}{\cancel{0}} \overset{+3}{\cancel{0}} \overset{+3}{\cancel{0}} \\
 \hline
 -11 \\
 \hline
 0111
 \end{array}
 \quad
 \begin{array}{r}
 8 \\
 -3 \\
 \hline
 5
 \end{array}$$

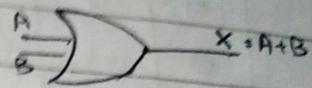
AND :-

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

OR :-

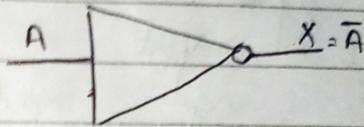
A	B	OR
0	0	0
0	1	1
1	0	1
1	1	1

D



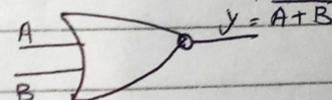
NOT :-

Input	Output
0	1
1	0



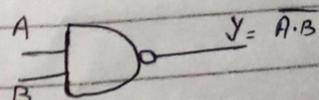
NOR :-

A	B	NOR
0	0	1
0	1	0
1	0	0
1	1	0



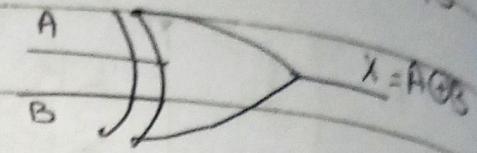
NAND :-

A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0



XOR:-

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

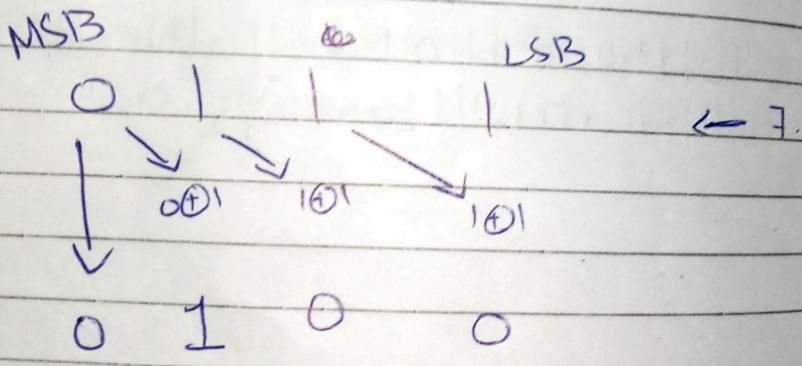


OR gate is an electronic circuit
the output of which is high
if any input is high

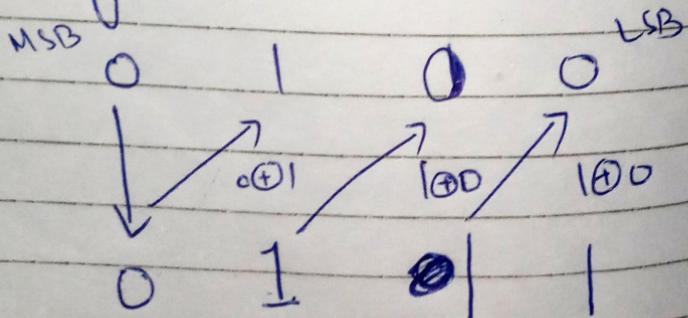
XOR ~~Table~~

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

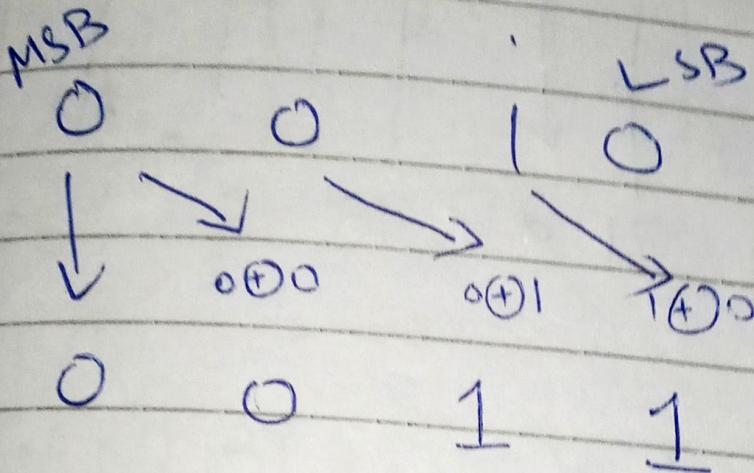
BCD to Gray



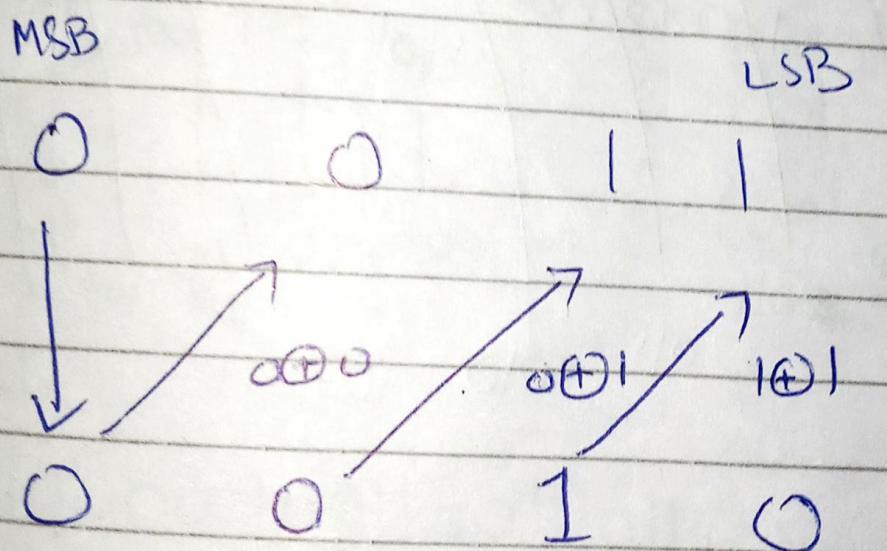
Gray to BCD



BCD to Gray



Gray to BCD



Expressions

DLD
Tutorials?

Not :-

$$X = \bar{A}$$

OR :-

$$X = \sum_{i=1}^N A_i = A_1 + A_2 + A_3 + \dots$$

$$X = A + B$$

AND :-

$$X = \prod_{i=1}^N A_i = A_1 \cdot A_2 \cdot A_3 \cdot \dots$$

XOR :-

$$X = A \oplus B$$

$$A\bar{B} + \bar{A}B$$

NOR :-

NAND:-

$$A\bar{B} + \bar{A}B.$$

A	B	\bar{A}	\bar{B}	$A\bar{B}$	$\bar{A}B$	$A\bar{B} + \bar{A}B$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

→ Boolean Algebra :-

Logical Expressions

$$A + A = A$$

$$A + 0 = A$$

$$A \cdot A = A$$

$$A \cdot 1 = A$$

$$A \cdot 0 = 0$$

$$A + B = B + A$$

$$AB = BA$$

$$A(B+C) = AB+AC$$

$$(A+B)+C = A+(B+C)$$

$$A + | A \quad X = A+A=A$$

$$0 + | 0 \quad X = 0$$

$$1 + | 1 \quad X = 1$$

$$A | X = A+0 = A$$

$$0 | 0+0 = 0$$

$$1 | 1+0 = 1$$

A	A	$X = A \cdot A = A$
0	0	0
1	1	1

A	1	$X = A \cdot 1 = A$
0	1	0
1	1	1

Logic Gates:-

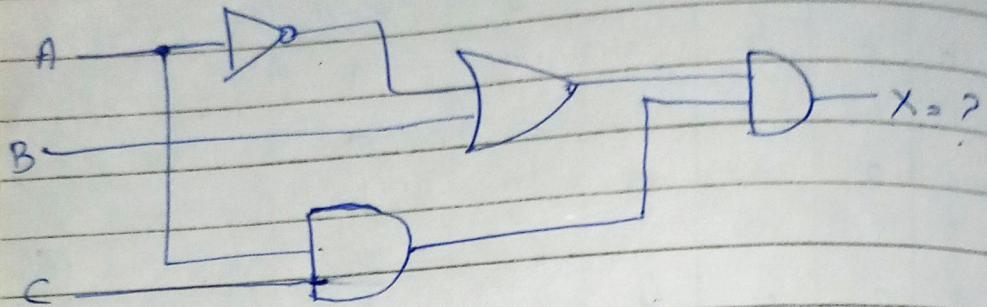
Logic Gates is an electronic circuit whose output is true/high when some conditions at the input are satisfied.

A	0	$X = A \cdot 0 = 0$
0	0	0
1	0	0

A	B	C	$B+C$	$X = A(B+C)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

DLD Lecture 1 - 9

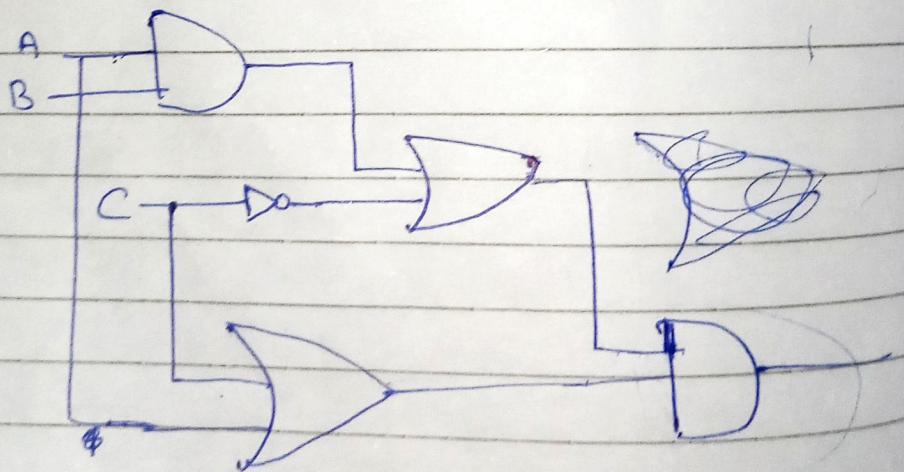
OR = +
And = *



$$X = (\bar{A} \vee B) \wedge (\bar{A} \wedge C)$$

$$X = (\bar{A} + B) \cdot A \cdot C$$

$$\rightarrow (AB + \bar{C}) (A + C)$$



$$\Rightarrow (\bar{A} + B)AC = AC(\bar{A} + B)$$

$$AC\bar{A} + ACB = CA\bar{A} + ACB$$

$$A\bar{A}C + ACB = 0 + ACB$$

$$0 + ACB = 0 + ACB$$

$$\Rightarrow X = (AB + \bar{C})(A + C)$$

BBQ

$$ABA + ABC + \bar{C}A + \bar{C}C$$

$$BA\bar{A} + ABC + \bar{C}A + \bar{C}C$$

$$B(A) + ABC + \bar{C}A + 0$$

$$BA + ABC + \bar{C}A$$

$$AB + ABC + \bar{C}A$$

$$AB(1 + C) + A\bar{C}$$

$$AB(1) + A\bar{C}$$

$$AB + A\bar{C}$$

$$A(B + \bar{C})$$

$$1 + C = 1$$

$$AB \cdot 1 = AB$$

Theorems

$$A \cdot A = A$$

$$B \cdot B = B$$

$$\rightarrow n + n = \underline{\underline{n}}$$

$$n + n = (n + n) \cdot 1$$

$$= (n + n) \cdot (n + \bar{n})$$

$$= \begin{matrix} A & B \\ A & B \end{matrix} \\ = nn + n\bar{n} + n\bar{n} + n\bar{\bar{n}}$$

$$\therefore n + \bar{n} = 1$$

$$= \frac{nn + n\bar{n}}{B}$$

$$A + A = A$$

$$A \cdot \bar{A} = 0$$

$$= n + 0$$

$$= n$$

$$\rightarrow n(n + y) =$$

$$\begin{aligned} & nx + ny \\ &= n + ny \\ &= n(1 + y) \\ &= n(1) \\ &= n \end{aligned}$$

$$\rightarrow n + ny = n(1 + y)$$

$$= n \cdot 1$$

$$= n$$

$$\rightarrow n(n' + y) =$$

$$n \cdot n' = 0$$

$$nx' + ny$$

$$0 + ny$$

$$= ny$$

$$\Rightarrow ny + n' \underbrace{(z + yz)}_{\cdot}.$$

$$\cdot ny + yz + n' z.$$

$$y \cdot (n+yz) + n' z$$

↔

$$\Rightarrow ny + n' z + yz \cdot)$$

$$= ny + n' z + yz(n + \bar{n})$$

$$= ny + n' z + \underbrace{nyz}_{\rightarrow} + \underbrace{n' yz}_{\rightarrow}.$$

$$= ny + nyz + n' z + n' yz.$$

$$= ny(1+z) + n' z(1+y)$$

$$= ny + n' z,$$

→ Standard form :-

$$= y + u'z + yz.$$

$$= uy(z+z') + u'(y+y')z + (u+u')yz$$

$$= \cancel{uyz} + \cancel{uyz'} + \cancel{u'yz} + \cancel{u'y'z} + \cancel{u'yz} + \cancel{u'y'z}.$$

$$x = uyz + u'y'z + uyz' + u'y'z'$$

$$x = \underset{1}{y'z} + \underset{1}{u'y'z} + \underset{1}{uyz'} + \underset{1}{u'y'z'}$$

if $u=1$	if $u=0$	if $u=1$	if $u=0$
$y=1$	$y=1$	$y=1$	$y=0$
$z=1$	$z=1$	$z=0$	$z=1$

x	y	z	x
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$\rightarrow F = A + B'C$$

$$= A(B+B')(C+C') + (A+A')B'C$$

$$= A(BC + BC' + B'C + B'C') + AB'C + A'B'C$$

$$= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$$

$$= \begin{matrix} ABC \\ 1 \end{matrix} + \begin{matrix} ABC' \\ 1 \end{matrix} + \begin{matrix} AB'C \\ 1 \end{matrix} + \begin{matrix} AB'C' \\ 1 \end{matrix} + \begin{matrix} A'B'C \\ 1 \end{matrix}$$

if $A=1$	if $A=1$	if $A=1$	if $A=1$	if $A=0$
$B=1$	$B=1$	$B=0$	$B=0$	$B=0$
$C=1$	$C=0$	$C=1$	$C=0$	$C=1$

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$F = x + y'z.$$

$$x(y+z')(z+z') + (x+n')(y'z)$$

$$x(yz+yz'+y'z+y'z') + ny'z + n'y'z.$$

$$xyz + nyz' + xy'z + ny'z' + xy'z + n'y'z.$$

$$nyz + nyz' + xz' + ny'z' + n'y'z.$$

if $n=1$ if $n=1$ if $n=1$ if $n=1$ if $n=0$
 $y=1$ $y=1$ $y=0$ $y=0$ $y=0$
 $z=1$ $z=0$ $z=1$ $z=0$ $z=1$

x	y	z	x
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

→ Minterm:-

Two binary variables x and y combined with an AND operation.

Four possible combinations:

$x'y'$, $x'y$, xy' , xy .

Each of these four AND terms is called a minterm.

X	Y	F
0	0	0
0	1	0
1	0	0
1	1	1

Minterm
of product
sum

Maxterm:-

Each variable are performed in OR operation

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	1

Maxterm

Product of sum

Minterm / Maxterm

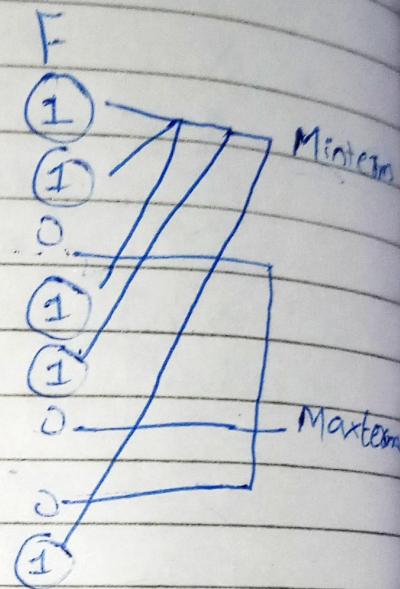
$$x'y'z' + xy'z + xyz' + xyz$$

X	Y	Z	F.
0	0	0	1
0	(0)	1	2
0	1	0	3
0	1	1	4
1	0	0	5
1	0	1	6
1	1	0	7
1	1	1	8

$$= (x'y'z') \cdot (x+y'+z) \cdot (x+y'+z') \cdot \\ (x'+y+z')$$

$$\Rightarrow F = x'y'z' + x'y'z + x'yz + xy'z' + xyz$$

	x	y	z	F
✓	0	0	0	1
✓	0	0	1	1
	0	1	0	0
✓	0	1	1	1
✓	1	0	0	1
	1	0	1	0
	1	1	0	0
✓	1	1	1	1



$$F = (x + y' + z) \cdot (x' + y + z) \cdot (x' + y' + z)$$

Each minterm is obtained from the AND term of the n variables, with each variable being primed if the corresponding bit of the primary number is a zero (0) and unprimed if a 1.

Each minterm is obtained from OR Term of the n variables with each variable being unprimed if the corresponding bit is a (0) and primed if a (1)

DLD Lecture

Minterms

Maxterms

x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'y'z'$	m_2	$x+y'+z$	M_2
0	1	1	$x'y'z$	m_3	$x+y'+z'$	M_3
1	0	0	$x'y'z'$	m_4	$x'+y+z$	M_4
1	0	1	$x'y'z$	m_5	$x'+y+z'$	M_5
1	1	0	$x'y'z'$	m_6	$x'+y'+z$	M_6
1	1	1	$x'y'z$	m_7	$x'+y'+z'$	M_7

$$M_5 = \bar{x} + \bar{y} + \bar{z} = 111$$

$$2^4 = 16$$

$$F = x'j'z + nyz' + nyz$$

$$F = n'j'z + ny(z+z')$$

$$F = n'j'z + ny(1)$$

$$F = n'j'z + nyj$$

$$F = n'j'z + ny(zz')$$

$$F = ny'z + nyz + nyz'$$

$$F = x'j'z + nyz' + nyz$$

$$= m_1 + m_6 + m_7 = \Sigma(1, 6, 7)$$

DD lecture

$$F = \Sigma (3, 7, 9, 12, 14, 15)$$

$$F_{\text{exp}} = m_3 + m_7 + m_9 + m_{12} + m_{14} + m_5 \quad (\text{Minterm})$$

w, z, y, x. (4 variables)

$$F_{\text{pos}} = \Pi (0, 1, 2, 4, 5, 6, 8, 10, 11, 13)$$

$$F_{\text{pos}} = M_0, M_1, M_2, M_4, M_5, M_6, M_8, M_{10}, M_{11}, M_3 \quad (\text{Maxterm})$$

→ Mapping :-

Making a table of cells

No. of ~~variables~~ ^{Cells} in a table, 2^n

n is the number of variables.

		n	0	1
		*	00	01
n=2		0	00	01
00, 01, 10, 11	0			
	1	10	11	

$$\text{Cells: } 2^3 = 8$$

x	y	00	01	10	11
0	000	001	010	011	
1	100	101	110	111	

Followed by Gray Code

One bit changed at a time.

0 → 000	000 → 0
1 → 001	001 → 1
2 → 010	011 → 3
3 → 011	010 → 2
4 → 100	110 → 5
5 → 101	100 → 4
6 → 110	101 → 5
7 → 111	111 → 7

$$F, \Sigma(0, 3, 4, 5, 7)$$

m_0, m_3, m_4, m_5, m_7

(Minterm)

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

yz'w'	0	1
00	1	0
01	0	1
11	0	1
10	1	1

$$y'z' + yz + zw =$$

$$F = n'y'z'w' + n'y'zw + ny'z'w + ny'zw + nyzw$$

$$F_1 \in \{0, 3, 4, 5, 7\}$$

$\wedge \vee z^2$

	00	01	11	10
0	1	0	1	0
1	1	1	1	0

2, 4,

$$F = y'z' + yz + nz.$$

$\wedge \vee z^2$

	00	01	11	10
0	1	1	1	1
1	0	1	1	0

$$F_2 = (n' + z)$$

000
001
010

$$①. F = \Sigma(0, 2, 4, 5, 6, 7)$$

$$F = m_0, m_2, m_4, m_5, m_6, m_7$$

x	y	z	F	
0	0	0	1	m_0
0	0	1	0	m_2
0	1	0	1	m_4
0	1	1	0	m_5
1	0	0	1	m_6
1	0	1	1	m_7
1	1	0	1	m_6
1	1	1	1	m_7

Boolean Expression:-

$$= x'y'z + x'y'z' + xy'z' + xy'z + xyz' + xyz$$

$$= x'z'(y+y') + xy'(z+z') + xy(z+z')$$

$$= x'z'(1) + xy'(1) + xy(1)$$

$$= x'z' + xy' + xy$$

$$= x'z' + x(y+y')$$

$$= x'z' + x(1)$$

$$= x'z' + x$$

K-Mapping :-

	\bar{z}	z	
\bar{x}	1	0	
x	1	0	
\bar{y}	1	1	
y	1	1	

$$= \bar{z}' + x$$

x	y	z	\bar{x}	\bar{z}	$\bar{x}\bar{z}$	$\bar{x}z'$
0	0	0	1	1	1	1
0	0	1	1	0	0	0
0	1	0	1	1	1	1
0	1	1	1	0	0	0
1	0	0	0	1	0	1
1	0	1	0	0	0	1
1	1	0	0	1	0	1
1	1	1	0	0	0	1

$$\bar{z} + x$$

-1

0

1

0

1

1

1