First Order Logic (FOL):

First order logic is also known as predicate logic or first-order predicate logic. First order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.

Objects
Relations
Quantifies

Two main parts:
Syntax
Semantics

→ More expensive than propositional logic
→ $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\sim$
→ $\forall$, $\exists$
→ $P(x)$; p is a predicate
x is a subject
Predicate (subject).

Example:

John is tall.
Tall (John)
Chinky is a cat.
Cat (Chinky)

Quantifiers in FOL:

A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.

Two types of Quantifiers:

- Universal Quantifier $(\rightarrow)$ $(\forall)$
- Existential Quantifier $(\wedge)$ $(\exists)$

→ Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.

- → For all
- → Everyone
- → Every thing

For example:

All man drink coffee

$\forall x \; man(x) \rightarrow drink(x, coffee)$

→ Existential Quantifier are the type of quantifiers which express that the statement within its scope is true for at least one instance of something.

- → for some
- → At least one

For example:

Some boys are intelligent.

$\exists x: boys(x) \wedge intelligent(x)$

Some red cats don't like tofu

$\exists y((Red(y) \wedge Cat(y)) \wedge \sim likesTofu(y))$

Examples:

All happy people smile

$\forall x: people(x) \wedge happy(x) \rightarrow smile(x)$

All boys like cricket.

$\forall x: boys(x) \rightarrow like(x, cricket)$

Some boys like football

$\exists x: boys(x) \wedge like(x, football)$

All birds fly

$\forall x. birds(x) \rightarrow fly(x)$

Every man respects his parent.
↓
All

$\forall x. man(x) \rightarrow respects(x, parent)$

Some boys play cricket.

$\exists x\ boys(x) \wedge play(x, cricket)$

Not all students like both maths and science.

$\sim \forall x[student(x) \rightarrow likes(x, maths) \wedge like(x, science)]$

Free Variable:

A variable is said to be a free variable in a formula if it occurs outside the scope of the quantifier.

e.g;

$\forall x \exists y [P(x, y, z)]$, where z is a free variable.

Bound Variables

A variable is said to be a bound variable in a formula if it occurs within the scope of the quantifier.

e.g.:

$\forall x [A(x) B(y)]$, here x and y are the bound variable.

Predicate: is a function that returns a truth value.

e.g.:

Cat(x) →	x is a cat
Prime(x)	x is prime
has taken(x, y)	x has taken y
less than (x, y)	$x < y$
Sum(x, y, z)	$x + y = z$
greater than 5(x)	$x > 5$
has N chars(s, n)	x has length n

Predicates can have varying numbers of arguments and input types.

Statements with Quantifiers:

$\exists x$ Even(x)	T	2, 4, 6
$\forall x$ Odd(x)	F	2, 4, 6
$\forall x$ (Even(x) $\vee$ odd(x))	T	every integer is either even or odd
$\exists x$ (Even(x) $\wedge$ odd(x))	F	no integer is both even and odd.
$\forall x$ Greater(x+1, x)	T	adding 1 makes a bigger number
$\exists x$ (Even(x) + Prime(x))	T	Even(2) is true and Prime(2) is true

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For example:

→ $\forall x \, \exists y \; Greater(y, x)$

For every positive integer, there is some larger positive integer.

→ $\exists y \, \forall x \; Greater(y, x)$

There is a positive integer that is larger than every other positive integer.

→ $\forall x \, \exists y \, (Greater(y, x) \wedge Prime(y))$

For every positive integer, there is a prime that is larger.