Probability 20-Dec-2023

→ Poisson Distribution:-

$$P(X;\mu) = \frac{e^{-\mu}e^{x}}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

A random variable (x) follow a poisson distribution if its probability function is:-

	Descrete	Continous
Poisson Dist.	Vary	fixed
Exponential Dist	fixed	Time Vary (Waiting Time)

where,

X: # of outcomes occur in particular time, area, Volume, Length

μ: average # of outcomes occur in particular time, area, volume, Length.

Example:- Past record shows that an instructor asked 5 Questions on average in final exam. What percent of chance that he will ask.

i) At most 4 Questions.

ii) Exactly 6 Questions.

SOLUTION:-

Given,

μ = average # of Questions in final exam.

$\mu = 5$

X: # of Questions in final exam.

i) $P[x > 4]$

$$P[x > 4] = 1 - P[x \leq 4]$$

$$P[X \leq a] + P[X > a] = 1$$

$$P[X > 4] = \sum_{X \geq 5}^{\infty} \frac{e^{5}(5)^{x}}{x!}$$

ii) $P[X = 6]$

$$P[X = 6] = \frac{e^{-5}(5)^{6}}{6!}$$

→ Exponential Prob Dist:-

$$e(x;\lambda) = \lambda e^{-\lambda x}, \quad x>0$$

A r.v (x) follow an exponential Dist if its prob function is defined as.

where,

X: Waiting time to occur in an event

λ: Average waiting time to occur in an event.

Example:- An instructor observed that the student of MTH 262 solved a problem within 30 minutes on average. Find the probability that the student will solve a problem b/w 25 to 35 minutes in final exam.

Given,

λ = average to solve a problem

$\lambda = 30$

X = time to solve a problem

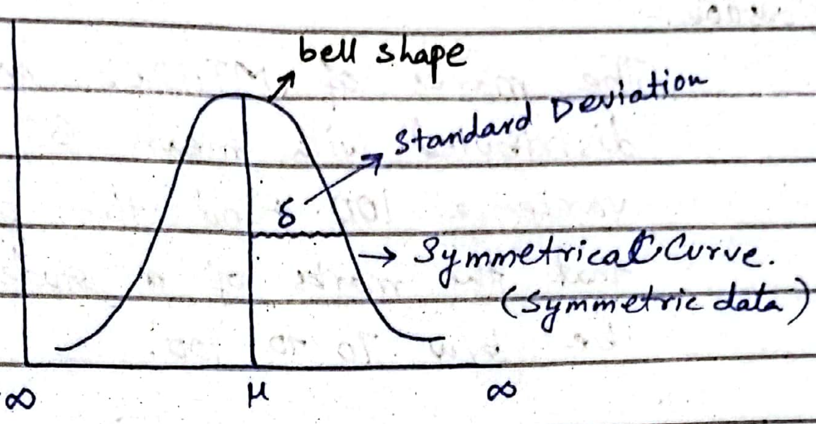
$P[25 \leq X \leq 35]$

$P[X;30] = 30e^{-30x}, \quad x>0$

$$P[25 \leq X \leq 35] = \int_{25}^{35} 30e^{-30x}\,dx$$

MOST IMPORTANT!!!

→ Normal Distribution (Most Important!!!!)



where,

Mean = μ = ?
Variance = δ^2 = ?
} Given

Skewness = $\gamma_1 = 0$

Kurtosis = $\gamma_2 = 0$

$$f(x;\mu;\delta^2) = \frac{1}{\sqrt{2\pi}\,\delta} e^{-\frac{1}{2\delta^2}(x-\mu)^2}, \quad -\infty < x < \infty$$

Assignmen 4:-

→ Normal Dist Introduction + Applications.

→ Hypothesis testing Introduction + Applications.

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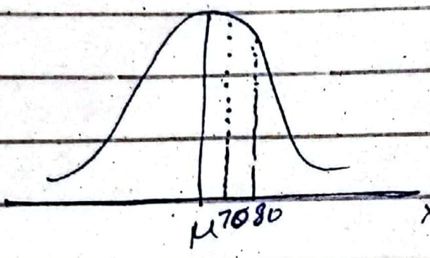
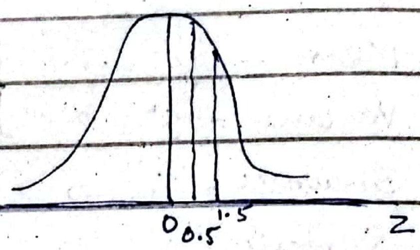
22-Dec-2023

Question:-

The marks of MTH262 are normally distributed with mean 65 and varience 100. Find the probability that the marks of a student will be b/w 70 to 100.





$X \sim N(\mu, \sigma^2)$

$E(X) = \mu$

$Var(X) = \sigma^2$

$$Z = \frac{X - \mu}{\sigma}$$

$$E(z) = \frac{E(x) - \mu}{\sigma}$$

$$= \frac{\mu - \mu}{\sigma}$$

$$= \frac{0}{\sigma} = 0$$

$$V(z) = \frac{V(x) - \mu}{\sigma^2}$$

$$\therefore V(ax+b) = a^2 var(x) + 0$$

$$V(z)\ \sigma^2/\sigma^2 = 1$$

$Z \sim N(0, 1)$

Standard Normal Variable

X : Marks of student

$X \sim N(65, 100)$

$P[70 \le X \le 80] = P[0.5 \le Z \le 1.5]$

$P[Z < 1.5] - P[Z < 0.5]$

$\therefore Z = \frac{X - \mu}{\sigma}$

$\Rightarrow Z = \frac{X - 65}{10}$

At $X = 70$

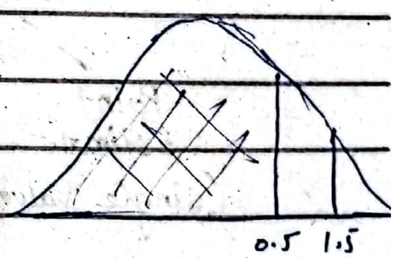
$\Rightarrow Z = \frac{70 - 65}{10}$

$= 0.5$

At $X = 80$

$\Rightarrow Z = \frac{80 - 65}{10}$

$= 1.5$



Statistical Infrence:-

Sampling theory

Sampling

Population $\longrightarrow$ Sample

Population: parameter — Unknown { μ, δ^2, $P \rightarrow$ proportion }

Sample: Statistic — $\bar{x}$, s^2, p

Inference

Estimator

- Point Estimate (single Value)
- Interval $\rightarrow$ [70 to 85)

$\rightarrow$ $\bar{x}$ is a point estimate of μ

$\rightarrow$ s^2 is a point estimate of δ^2

$\rightarrow$ p is a point estimate of P

Interval Estimation:-

$\rightarrow$ Point Estimate $\pm$ Margin of Error

$\rightarrow$ Point Estimate $\pm$ (reliability Coeff) SE of Point Estimate

reliability Coeff: How much confident you are for the result.

SE: Standard Error

Population is Normal

Confidence Error

$100(1-\alpha)\%$ C.I for μ

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Hypothesis Testing:-

The procedure in which we prove if the claim is right or not.

valid

Two Hypothesis,

One against the claim

Other in favour of claim.