Longest Common Subsequence (LCS)

$$C[i,j] \begin{cases} F1 \leftarrow \quad c[i-1, j-1] + 1 \qquad \Rightarrow \quad \text{if } x[i] = y[j] \\ F2 \leftarrow \quad \max\{ C[i, j-1], C[i-1, j]\} \Rightarrow \text{otherwise} \end{cases}$$

$$\therefore \text{if } x[i] \mathrel{!=} y[j]$$

$$X = ABCB \rightarrow i = 4$$

$$Y = BDCAB \rightarrow j = 5$$

intially $C[i, 0]$ & $C[0, j] = 0$

		D	1	2	3	4	5
		Yi	B	D	C	A	B
0	Xi	0	0	0	0	0	0
1	A	0	0	0	0	1	1
2	B	0	1	1	1	1	2
3	C	0	1	1	2	2	2
4	B	0	1	1	2	2	3

$X_i = 1$, $Y_j = 1$

$X_1 = A$ $Y_1 = B$

$\Rightarrow \text{if}(X_i == Y_j) \Rightarrow X_1 = Y_1 \Rightarrow A == B$ // False

F2 :-

$$C[1,1] = \max\{C[1,0], C[0,1]\} \quad \therefore C[1,0] = 0$$
$$C[0,1] = 0$$
$$C[1,1] = 0$$

$X_i = 1 \quad , \quad Y_j = 2$

$X_2 = A \quad , \quad Y_2 = D$

$if(X_2 == Y_2) \implies A == D \;//\; False$

F2:-

$$C[1,2] = \max\{C[1,1], C[0,2]\} \quad \therefore C[1,1] = 0$$
$$C[0,2] = 0$$
$$C[1,2] = 0$$

$X_i = 1 \quad , \quad Y_j = 3$

$X_1 = A \quad , \quad Y_3 = C$

$if(X_1 == Y_3) \implies A == C \;//\; False$

F2:-

$$C[1,3] = \max\{C[1,2], C[0,3]\} \quad \therefore C[1,2] = 0$$
$$C[0,3] = 0$$
$$C[1,3] = 0$$

$X_i = 1 \quad , \quad Y_j = 4$

$X_1 = A \quad , \quad Y_4 = A$

$if(X_1 == Y_4) \implies A == A \;//\; True$

F1:-

$$C[1,4] = C[0,3] + 1 \qquad \therefore C[0,3] = 0$$
$$C[1,4] = 1$$

$X_i = 1$, $Y_j = 5$

$X_1 = A$, $Y_5 = B$

if $(X_1 == Y_5)$ => $A == B$ // False

F2 :-

$C[1,5] = \max\{C[0,4], C[0,5]\}$ ∴ $C[1,4] = 1$

$C[1,5] = \max\{1, 0\}$ $C[0,5] = 0$

$C[1,5] = 1$

$X_i = 2$, $Y_j = 1$

$X_2 = B$, $Y_1 = B$

if $(X_2 == Y_2)$ => $B == B$ // True

F1 :-

$C[2,1] = C[1,0] + 1$ ∴ $C[1,0] = 0$

$C[2,1] = 1$

$X_i = 2$, $Y_j = 2$

$X_2 = B$, $Y_2 = D$

if $(X_2 == Y_2)$ => $B == D$ // False

F2 :-

$C[2,2] = \max\{C[2,1], C[1,2]\}$ ∴ $C[2,1] = 1$

$C[2,2] = 1$ $C[1,2] = 0$

$X_i = 2$, $Y_j = 3$

$X_2 = B$, $Y_3 = C$

if $(X_2 == Y_3)$ => $B == C$ // False

F2:-

$C[2,3] = \max\{C[2,2], C[1,3]\}$ $\therefore C[2,2] = 1$

$C[2,3] = 1$ $C[1,3] = 0$

$X_i = 2$, $Y_j = 4$

$X_2 = B$, $Y_4 = A$

if $(X_2 == Y_4) \Rightarrow B == A$ // False

F2:-

$C[2,4] = \max\{C[2,3], C[1,4]\}$ $\therefore C[2,3] = 1$

$C[2,4] = 1$ $C[1,4] = 1$

$X_i = 2$, $Y_j = 5$

$X_2 = B$, $Y_5 = B$

if $(X_2 == Y_5) \Rightarrow B == B$ // True

F1:-

$C[2,5] = C[1,4] + 1$ $\therefore C[1,4] = 1$

$C[2,5] = 2$ $1+1 = 2$

$X_i = 3$, $Y_j = 1$

$X_3 = C$, $Y_1 = B$

if $(X_3 == Y_1) \Rightarrow C == B$ // False

F2:-

$C[3,1] = \overset{max}{\{}C[3,0], C[2,1]\}$ $\therefore C[3,0] = 0$

$C[3,1] = 1$ $C[2,1] = 1$

$X_i = 3$, $Y_j = 2$

$X_3 = C$, $Y_2 = D$

if $(X_3 == Y_2) \Rightarrow C == D$ // False

F2:-

$C[3,2] = \max\{C[3,1], C[2,2]\}$ $\therefore C[3,1] = 1$

$C[3,2] = 1$ $C[2,2] = 1$

$X_i = 3$, $Y_j = 3$

$X_3 = C$, $Y_3 = C$

$if(X_3 == Y_3) \Rightarrow C == C$ // True

F1:-

$C[3,3] = C[2,2] + 1$ $\therefore C[2,2] = 1$

$C[3,3] = 2$ $1 + 1 = 2$

$X_i = 3$, $Y_j = 4$

$X_3 = C$, $Y_4 = A$

$if(X_3 == Y_4) \Rightarrow C == A$ // False

F2:-

$C[3,4] = \& \max\{C[3,3], C[2,4]\}$ $\therefore C[3,3] = 2$

$C[3,4] = \max\{2, 1\}$ $C[2,4] = 1$

$C[3,4] = 2$

$X_i = 3$, $Y_j = 5$

$X_3 = C$, $Y_5 = B$

$if(X_3 == Y_5) \Rightarrow C == B$ // False

F2:-

$C[3,5] = \max\{C[3,4], C[2,5]\}$ $\therefore C[3,4] = 2$

$C[3,5] = 2$ $C[2,5] = 2$

$X_i = 4$, $Y_j = 1$

$X_4 = B$, $Y_1 = B$

$if(X_4 == Y_1) \Rightarrow B == B$ // True

F1:-

$C[4,1] = C[3,0] + 1$ $\therefore C[3,0] = 0$

$C[4,1] = 1$ $0 + 1 = 1$

$X_i = 4$, $Y_j = 2$

$X_4 = B$, $Y_2 = D$

$if (X_4 == Y_2) \implies B == D$ // False

F2:-

$$C[4,2] \overset{max}{=} \{C[4,1], C[3,2]\} \quad \therefore C[4,1] = 0$$
$$C[3,2] = 1$$

$C[4,2] = 1$

$X_i = 4$, $Y_j = 3$

$X_4 = B$, $Y_3 = C$

$if (X_4 == Y_3) \implies B == C$ // False

F2:-

$$C[4,3] = max\{C[4,2], C[3,3]\} \quad \therefore C[4,2] = 1$$
$$C[3,3] = 2$$

$C[4,3] = max\{1, 2\}$

$C[4,3] = 2$

$X_i = 4$, $Y_j = 4$

$X_4 = B$, $Y_4 = A$

$if (X_4 == Y_4) \implies B == A$ // False

F2:-

$$C[4,4] = [max\{C[4,3], C[3,4]\} \quad \therefore C[4,3] = 2$$
$$C[3,4] = 2$$

$C[4,4] = 2$

$X_i = 4$, $Y_j = 5$

$X_4 = B$, $Y_5 = B$

$if (X_4 == Y_4) \implies B == B$ // True

F1:-

$$C[4,5] = C[3,4] + 1 \quad \therefore C[3,4] = 2$$
$$2 + 1 = 3$$

$C[4,5] = 3$

(BACKTracking)

		0	1	2	3	4	5
	Y_j	B	D	C	A	B	
0	X_i	0	0	0	0	0	0
1	A	0	0	0	0	1	1
2	B	0	①←	1	1	1	2
3	C	0	1	1	②←	2	2
4	B	0	1	1	2	2	③

Reverse Order= BCB

Straight Order= BCB

1) LCS = 3   Select LCS

2) Backtrack from Last cell ~~diag~~

3) Find the matching Strings from X_i & Y_j & Select them

4) Go to the top.

At $C[4,5]$ = B == B   ; Select the subsequence

$$B$$

Move upwards diagonally and find maching values.

At $C[3,4]$ = C != A   ; Move to the left

At $C[3,3]$ = C == C   ; Select subsequence

$$BC$$

Move upwards diagonally

At $C[2,2]$ = B != D   ; Move to the left

At $C[2,1]$ = B == B   ; select the subsequence

$$BCB$$

Move upwards untill 0

Since LCS = 3 , the LCS would be BCB