

# STATS

Lecture 1  
8. Sep.

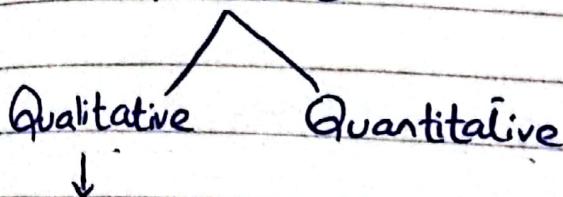
## Variable:

- Any characteristic whose values can change from one element to another element
- Any characteristic that can assume different value

## Data:

Set of information about variables.

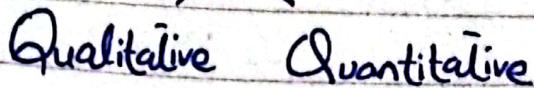
## Variable



Those variables  
which can't be  
numerically  
measured

Those variables  
which can be  
numerically  
measured

## Data



Data about  
qualitative  
variables.

e.g. Blood Group

Data about  
quantitative  
variables

e.g. CGPA, Weight

## Quantitative Variables

Discrete



Countable variable  
(How many?)

Continuous



Measurable Variable  
(How much?)

(n) **Population:** → (Statistical Population)  
Set of objects under study.

(n) **Sample:**

Subset of population is called  
sample.

**Sampling:**

Process of sample selection

## Sampling

Random / Non-Random

Probability



Each sample has  
chance to be  
selected.

Can be generalization  
for the population

A sample selected  
in random sampling.

Non-Probability



Sample is selected on  
personal judgement

Each sample does not  
have the chance to  
be selected.

Result of non-random  
samples is only valid  
for themselves.

# Population

Finite      Infinite



Known number      Unknown number  
of elements      of elements

Finite → Real / Hypothetical  
Infinite → Real / Hypothetical.

Real

Elements exist  
physically.

Hypothetical

Elements does not  
exist physically.

Target

## Target Population:

→ Under the study / whole

## Sampled Population:

- Selected sample / access
- from which we can draw the conclusion.

## Types of Random Sampling:

- Simple random sampling (Homogenous).
- Systematic sampling (Sequence, etc)
- Stratified sampling (Heterogeneous).
- Cluster sampling.

## Simple Random Sampling:

- Selected sample from the population using random method.
- Best for homogenous / same population.

## Stratified Sampling:

- Breaking heterogeneous population into homogenous population

## Systematic Sampling:

→ Select interval

$$\frac{N}{n} = k$$

Population      Sample

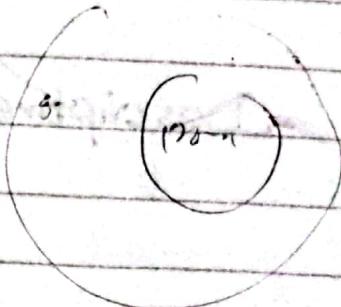
interval

$$k = \frac{N}{n}$$

$$k = \frac{25}{3}$$

$$k = 8.33$$

$$K = 8$$



→ List of unit is known as sampling frame

## Cluster Sampling:

→ When we do not have the data / info about the samples, we select the samples as clusters.

→ Natural selection - Random selection

## Statistic:

→ Any numerical value that is calculated from sample

## Parameter:

→ Any numerical value that is

calculated from population.

## Statistics:

→ Statistics is a science of conducting studies to collect, organize, summarize, analyze and draw conclusion from data

## Statistics

Descriptive                    Inferential

↓  
Info about  
population or  
sample

↓  
Conclusion drawn  
from the selected  
sample

# STATS

## ASSIGNMENT 1

Lecture 3

15. Sep

### Data visualization

What → Introduction

Why → Objects

How → Methods

Which → Best

Application? Dataset

Report → Summary

### Presentation of Data:

Data presentation is the method by which people summarize, organize and communicate information using a variety of tools, such as diagrams, distribution charts, histogram and graphs.

### Objective:

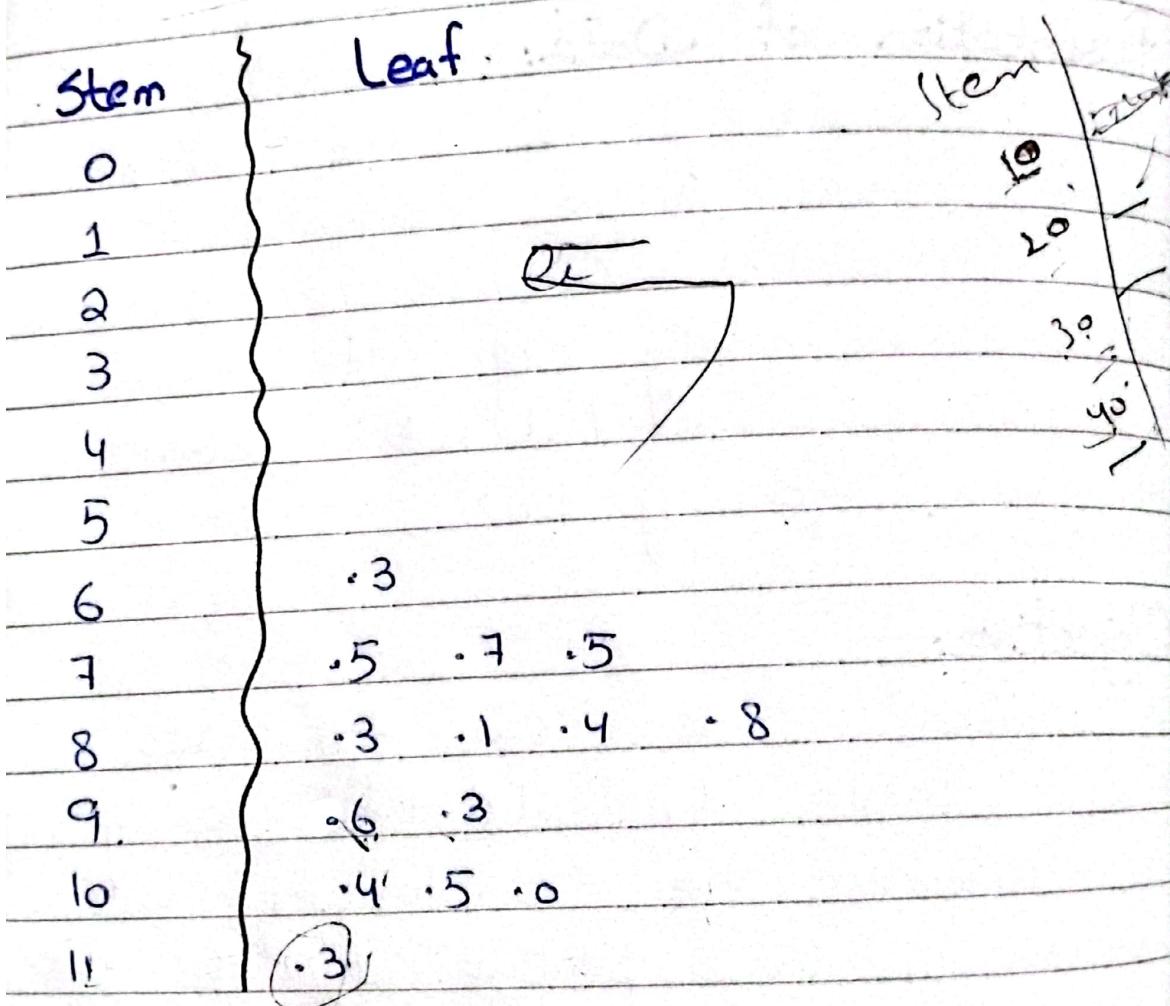
- It condense the raw data into a form suitable for statistical analysis.
- It remove the complexities and highlights the feature of data.
- It facilitates comparison and drawing conclusion from data.
- It helps in statistical analysis by separating elements of data set into homogeneous groups.

Method:

Textual Method:

- Rearrangement from lowest to highest
- Stem and leaf plot.

11.3, 9.6, 10.4, 7.5, 8.3, 10.5, 10.0, 9.3, 8.1,  
7.7, 7.5, 8.4, 6.3, 8.8.



Tabular Method:

Presenting data in the form  
of table.

## Types of Tabular Method:

### Frequency Distribution:

A frequency distribution is a table that distributes a data set into a suitable number of categories (classes). Data presentation in the form of frequency table called grouped data.

$$K = 1 + 3.22 \log(N)$$

$$W = \frac{\text{max} - \text{min}}{K}$$

2, 4, 8, 4, 8, 1, 2, 32, 12, 10, 5, 7, 5, 5, 3, 4,  
24, 19, 4, 14

$$K = 1 + 3.22 \log(\frac{N}{20})$$

$$K = 5.18$$

$$\boxed{K = 5}$$

$$W = \frac{\text{max} - \text{min}}{K}$$

$$W = \frac{32 - 1}{5}$$

$$W = \frac{31}{5} \Rightarrow W = 6.2 \Rightarrow \boxed{W = 6}$$

Classes } Tallies } frequency.

1 - 6 IIIII II 12

7 - 12 IIII 4

13 - 18 I 1

19 - 24 II 2

25 - 30 - 0

31 - 36. I 1

THIS MUL

MUL

RECORDS OF CLASS MEETING

11/11/11

RECORDS OF CLASS MEETING

11/11/11

RECORDS OF CLASS MEETING

11/11/11

# STATS

Lecture 4  
20 Sep.

## Presentation of Data:

- Stem and Leaf
- Frequency Distribution (F.D)
  - Cumulative F.D
  - Percentage F.D
- Graphs
  - Histogram
  - Frequency Polygon
  - Frequency Curve
  - Time series plot

Tabular Method

## Frequency Distribution:

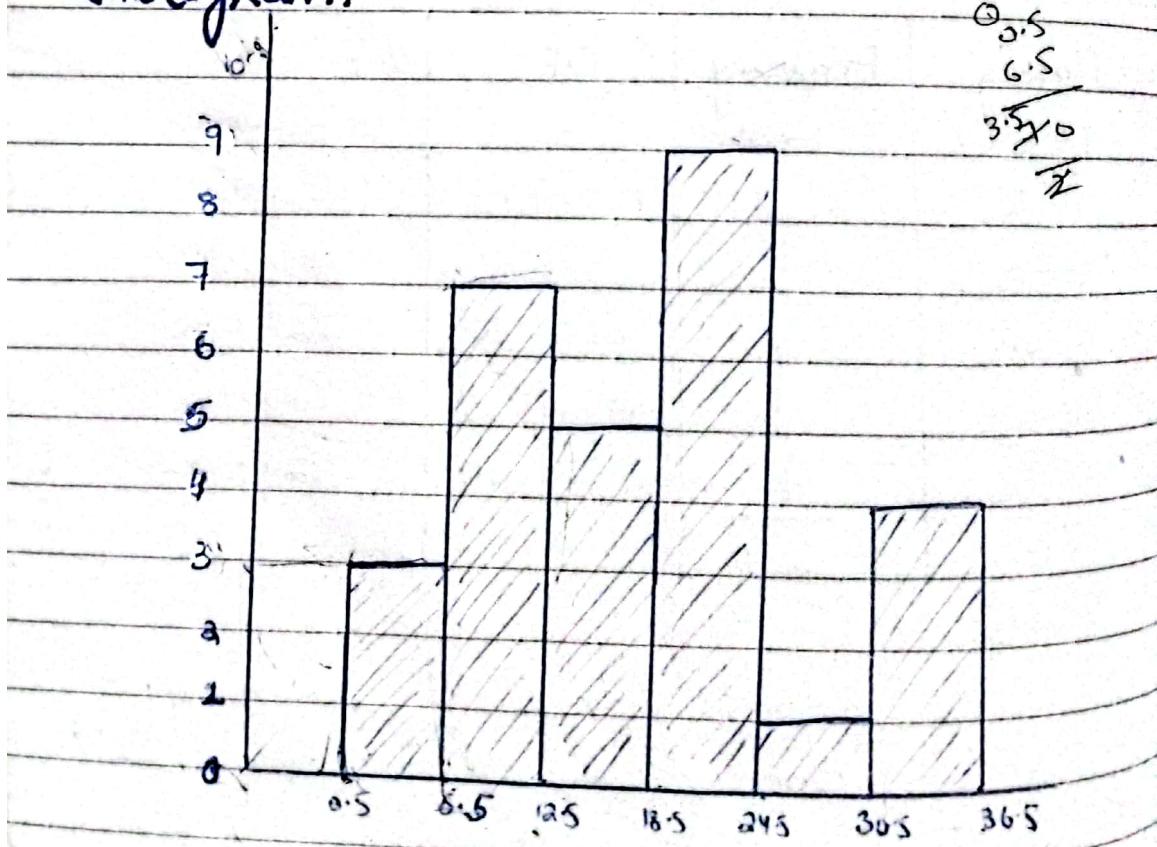
| Classes | Frequency      | C.F       | % F                       |
|---------|----------------|-----------|---------------------------|
| 1-6     | 3              | 3         | $\frac{3}{29} \times 100$ |
| 7-12    | 7              | $3+7=10$  | $\frac{7}{29} \times 100$ |
| 13-18   | 5              | $10+5=15$ | $\frac{5}{29} \times 100$ |
| 19-24   | 9              | $15+9=24$ | $\frac{9}{29} \times 100$ |
| 25-30   | 1              | $24+1=25$ | $\frac{1}{29} \times 100$ |
| 31-36   | $\frac{4}{29}$ | $25+4=29$ | $\frac{4}{29} \times 100$ |

# Graphs:

Histogram  $\rightarrow$  { F.D with class boundaries  
Plot frequency against C.B }

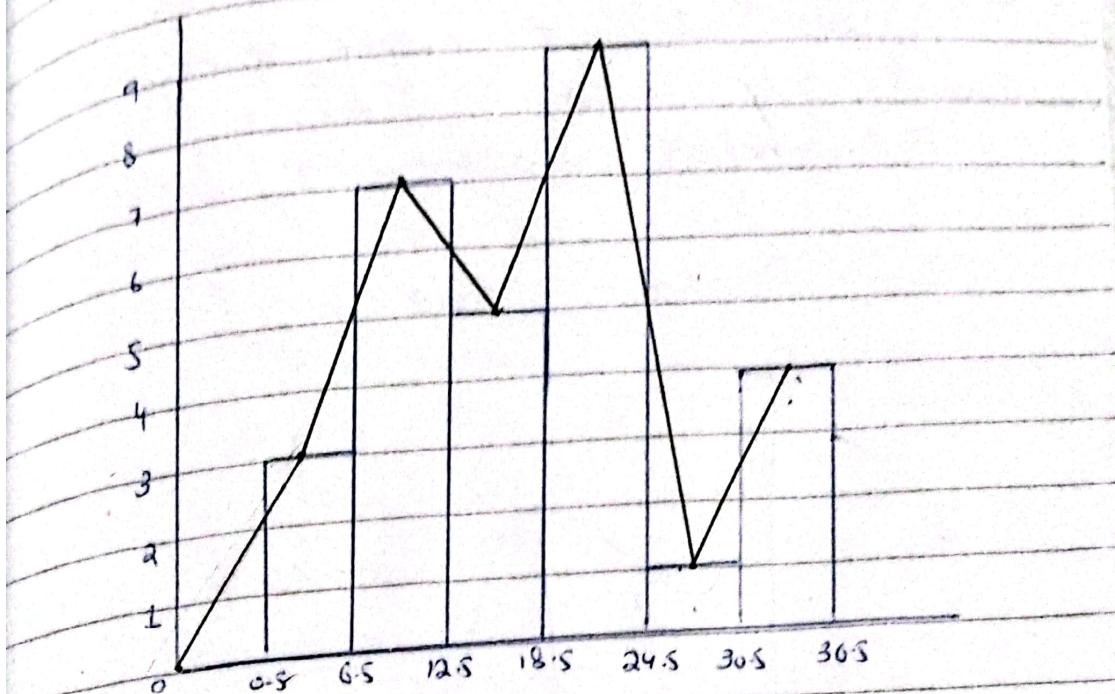
| Classes | Frequency | C.B         | X (Mid Point) |
|---------|-----------|-------------|---------------|
| 1-6     | 3         | 0.5 - 6.5   | 3.5           |
| 7-12    | 7         | 6.5 - 12.5  | 9.5           |
| 13-18   | 5         | 12.5 - 18.5 | 15.5          |
| 19-24   | 9         | 18.5 - 24.5 | 21.5          |
| 25-30   | 1         | 24.5 - 30.5 | 27.5          |
| 31-36   | 4         | 30.5 - 36.5 | 33.5          |

## Histogram:



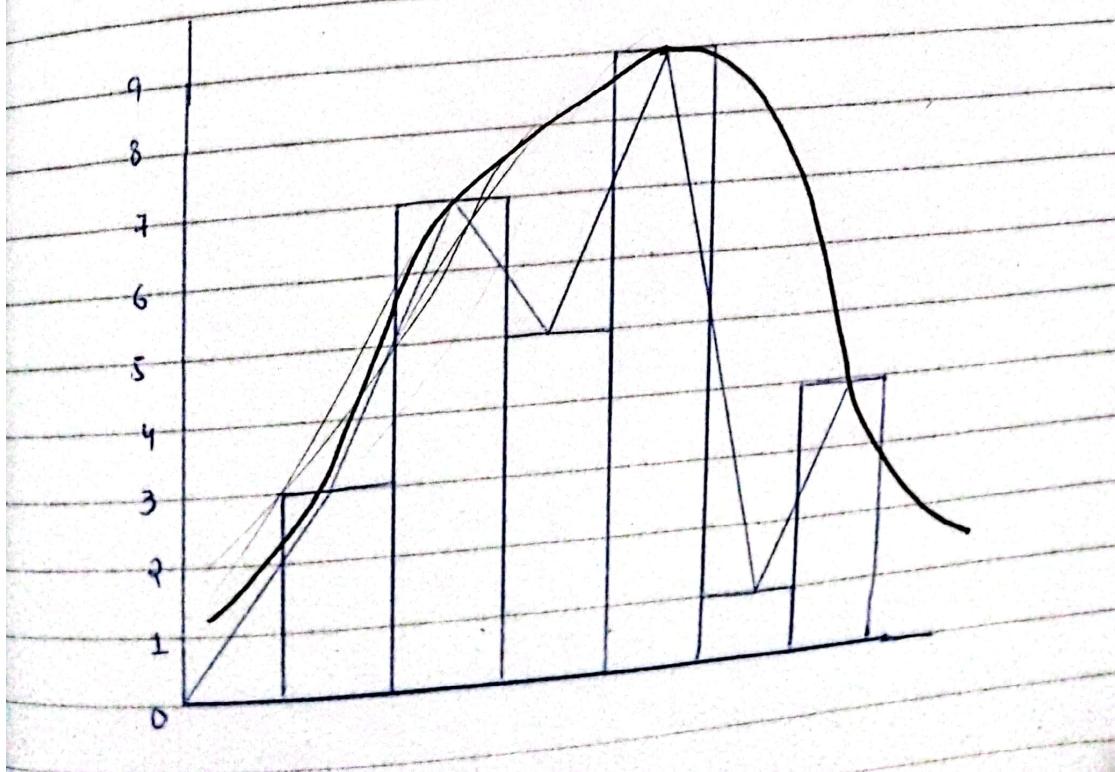
Frequency Polygon:

→ Histogram or midpoint (x)



Frequency Curve:

→ Frequency polygon or histogram

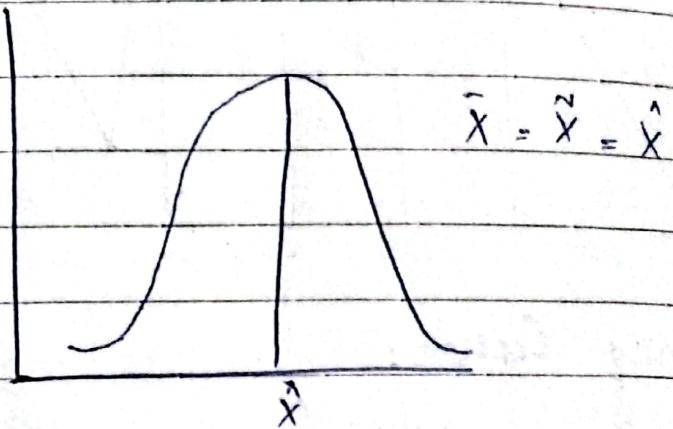


# Skewed And Symmetric Distribution

## Symmetric Data:

Data set are evenly spread around the center  
Bell shaped curve

$$\text{Mean} = \text{Median} = \text{Mode}$$



## Skewed Data:

# Measure of Central Tendency

Lecture 16  
4. Oct

## Arithmetic Mean:-

Arithmetic mean is a value which is obtained by dividing the sum of all values by their number.

[Population mean]

$$\mu = \frac{x_1 + x_2 + \dots + x_N}{N} \rightarrow \text{parameter}$$

Greek Latin

Population size = N

Sample size = n

[Sample mean]

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \rightarrow \text{statistic}$$

n,

Variable

$\bar{x}, \bar{y}, s^2, p$

## Geometric Mean:-

Geometric mean is a value which is obtained by  $N^{\text{th}}$  under-root of product of all N values.

$$\rightarrow G.M = (x_1 \cdot x_2 \cdot x_3 \cdots x_N)^{\frac{1}{N}}$$

OR

$$\rightarrow G.M = \left[ \prod_{i=1}^N x_i \right]^{\frac{1}{N}}$$

OR

$$\rightarrow G.M = \text{Antilog} \left[ \frac{\sum_{i=1}^N \log x_i}{N} \right]$$

OR

$$\rightarrow G.M = \text{Antilog} \left[ \frac{\log x_1 + \log x_2 + \dots + \log N}{N} \right]$$

### Harmonic Mean:-

Harmonic mean is the reciprocal of arithmetic mean of reciprocal of values.

$$\rightarrow H.M = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_N}}$$

OR

$$\rightarrow H.M = \frac{N}{\sum_{i=1}^N \left( \frac{1}{x_i} \right)}$$

→ Gives more importance to smaller value and less importance to greater value

Example: 70, 37, 55, 63, 59, 68, 56, 54

$$\text{Mean} = \frac{70 + 37 + 55 + 63 + 59 + 68 + 56 + 54}{8}$$

$$= \frac{462}{8}$$

$$\text{Mean} = 57.75$$

$$G.M = \left( 70 \times 37 \times 55 \times 63 \times 59 \times 68 \times 56 \times 54 \right)^{\frac{1}{8}}$$

$$G.M = (108879398812800)^{\frac{1}{8}}$$

$$G.M =$$

Mode:- Most repeated value.

Median:- Mid value sorting the dataset into ascending order.

# Measure of Dispersion:-

→ Absolute Measure

$$\rightarrow \text{Range} = X_{\max} - X_{\min}$$

$$\rightarrow \text{Mean Deviation (M.D.)}$$



Population:-

$$M.D. = \frac{\sum_{i=1}^N |X_i - \mu|}{N} \quad \therefore \mu = \text{Mean}$$

Sample:-

$$M.D. = \frac{\sum_{i=1}^n |X_i - \bar{x}|}{n} \quad \therefore |X_i - \bar{x}| = \text{Deviation}$$

→ Variance

Population:-

$$\sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N}$$

Sample:-

$$\sigma^2 = \frac{\sum_{i=1}^n (X_i - \bar{x})^2}{n}$$

→ Standard Deviation

$$S.D. = \sqrt{\text{Variance}}$$

Population:-

$$S = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}}$$

Sample:-

$$S = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{x})^2}{n}}$$

Relative Measure :-

Coefficient of Range =  $\frac{X_{\max} - X_{\min}}{X_{\max} + X_{\min}}$

Coefficient of Mean Deviation:  $\frac{\text{Mean Deviation}}{\text{Mean}} \times 100$

Coefficient of Variance =  $\frac{\text{Standard Deviation}}{\text{Mean}} \times 100$

Example :-

Mean data shows the marks of MTH605.  
Find all measures of dispersion.

| X  | $X - \bar{u}$ | $ X - \bar{u} $ | $X^2 =  X - \bar{u} ^2$ |
|----|---------------|-----------------|-------------------------|
| 70 | -3.83         | 3.83            | 14.66                   |
| 82 | 8.17          | 8.17            | 66.74                   |
| 70 | -3.83         | 3.83            | 14.66                   |
| 71 | -2.83         | 2.83            | 8.008                   |
| 75 | 1.17          | 1.17            | 1.36                    |
| 75 | 1.17          | 1.17            | 1.36                    |
|    |               | 21              | 106.78                  |

$$\bar{u} = \frac{\sum x}{N} = \frac{70+82+70+71+75+75}{6} \Rightarrow \frac{443}{6} \Rightarrow 73.83$$

Range.  $X_{\max} - X_{\min}$

$$= 82 - 70 \Rightarrow 12$$

## → Absolute Measure:

Mean Deviation (Population) :-

$$M.D = \frac{|\bar{x}_i - \mu|}{N}$$

$$M.D = \frac{21}{6}$$

$$\boxed{M.D = 3.5}$$

Variance (Population) :-

$$\sigma^2 = \frac{(x_i - \mu)^2}{N}$$

$$\sigma^2 = \frac{106.78}{6}$$

$$\boxed{\sigma^2 = 17.79}$$

Standard Deviation

$$S.D = \sqrt{\text{Variance}}$$

$$S.D = \sqrt{17.79}$$

$$\boxed{S.D = 4.217}$$

## → Relative Measures:

$$\text{Coeff Range} = \frac{X_{\max} - X_{\min}}{X_{\max} + X_{\min}}$$

$$\text{Coeff Range} = \frac{82 - 70}{82 + 70}$$

$$\text{Coeff - Range} = \frac{12}{152}$$

$$\boxed{\text{Coeff - Range} = 0.078}$$

$$\text{Coeff M.D} = \frac{\text{M.D}}{\text{Mean} \rightarrow (\mu)} \times 100$$

$$= \frac{3.5}{73.83} \times 100$$

$$= 0.047 \times 100$$

$$\boxed{\text{Coeff M.D} = 4.74}$$

$$\text{Coeff Variance} = \frac{\text{S.D}}{\text{Mean} \rightarrow (\mu)} \times 100$$

$$= \frac{4.217}{73.83} \times 100$$

$$= 0.057 \times 100$$

$$\boxed{\text{Coeff Variance} = 5.71}$$

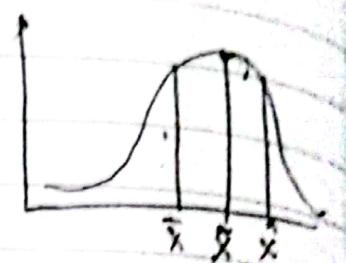
## Absolute Measure of Skewness:

Symmetric:-

$$\text{Mean} = \text{Median} = \text{Mode}$$

$$\text{Mean} - \text{Mode} = 0$$

$$\text{Mean} - \text{Median} = 0$$

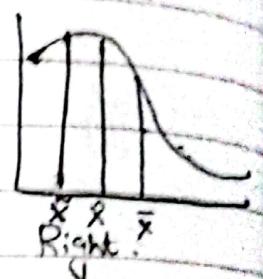


Right Skewed: -ively skewed

$$\text{Mean} > \text{Median} > \text{Mode}$$

$$\text{Mean} - \text{Mode} > 0$$

$$\text{Mean} - \text{Median} > 0$$

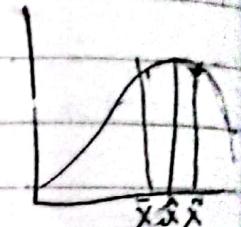


Left Skewed: -ively skewed

$$\text{Mean} < \text{Median} < \text{Mode}$$

$$\text{Mean} - \text{Mode} < 0$$

$$\text{Mean} - \text{Median} < 0$$



## Pearson Coefficient of Skewness: (Relative)

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D}} \quad OR$$

$$Sk_p = \frac{3(\text{Mean} - \text{Mode})}{\text{S.D}}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^2}$$

$$\alpha = \frac{\text{Mean} - \text{Median}}{\text{S.D}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\text{Coef of Skewness} = \gamma_1 = \sqrt{\beta_1} = \sqrt{\frac{\mu_3^2}{\mu_2^2}} = \frac{\mu_3}{\mu_2^{3/2}}$$

## Moments:

rth moment about mean :-

$$u_r = \frac{\sum_{i=1}^N (x_i - u)^r}{N}$$

$$u_1 = \frac{\sum_{i=1}^N (x_i - u)^1}{N} = 0$$

$$u_2 = \frac{\sum_{i=1}^N (x_i - u)^2}{N} = \sigma^2$$

$$u_3 = \frac{\sum_{i=1}^N (x_i - u)^3}{N} = \text{Not specified} = \text{Skewness}$$

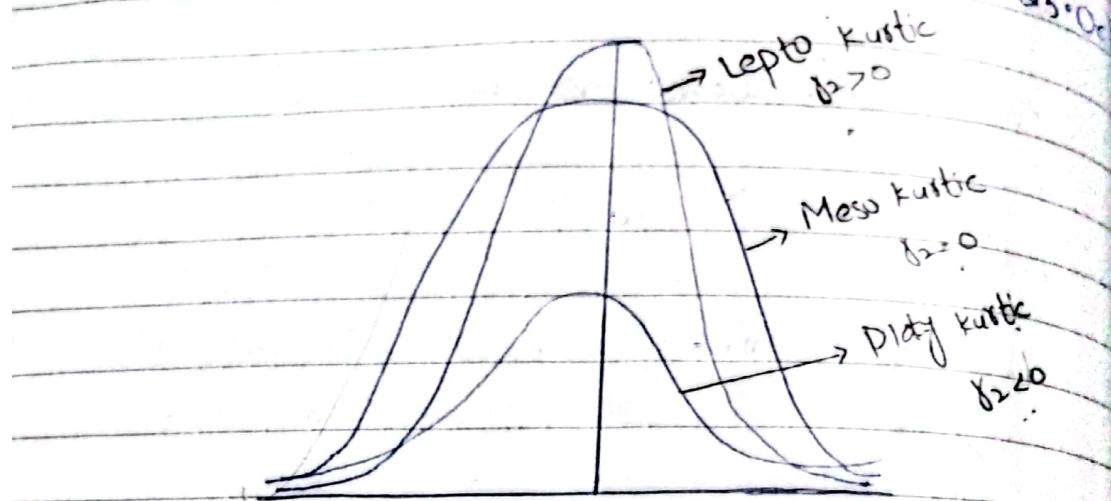
$$u_4 = \frac{\sum_{i=1}^N (x_i - u)^4}{N} = \text{Not specified} = \text{Kurtosis}$$

∴ Parameter is a constant value.

Mean is a

## Measure of Kurtosis :-

Lecture  
25.01.



## Coefficient of Kurtosis:

$$\gamma_2 = \beta_2 - 3$$

$$\gamma_2 = \frac{m_4}{m_2^2} - 3$$

$$\gamma_2 = \frac{m_4}{(\sigma^2)^2} - 3$$

$$\gamma_2 = \frac{m_4}{\sigma^4} - 3$$

→ What will be the degree of skewness in case of Kurtosis.

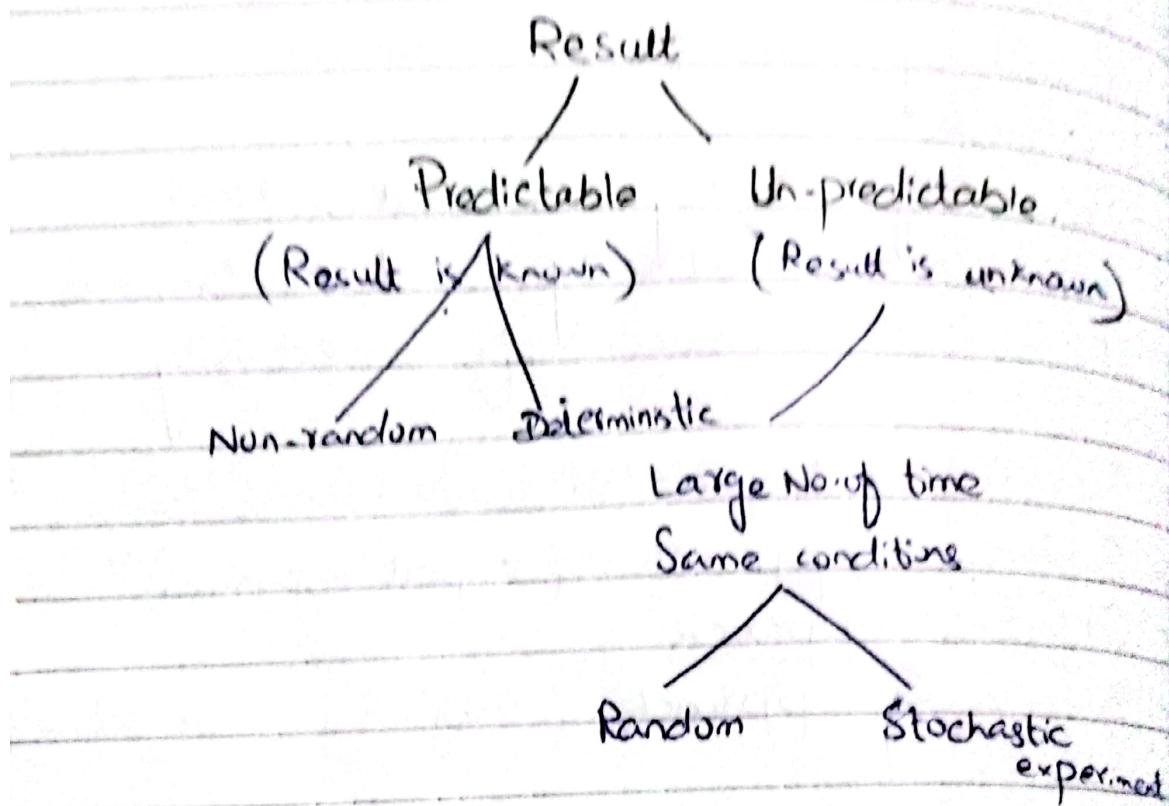
Degree of  $\gamma_1$  will be zero  
 $\gamma_1$  will be symmetric

## Probability:

- Uncertainty:
  - ↳ Caused by incomplete information
  - or Variation
- Probability theory is mathematical tool for uncertainty and randomness.
- Numerical measure of uncertainty.
- Two Approaches to Measure Uncertainty:
  - Subjective approach (expert opinion)
  - Objective approach
  - Bayesian approach (Subjective + Objective).
- Subjective Approach:-  
Probability of occurrence of any event on the basis of personal judgement.
- Objective Approach- (Mathematical Model).  
On the basis of previous data, we find the probability for any event to occur.

## Experiment:

- Activity that produces ~~obtained result~~ result
- It has two outcomes.



## Sample Space:

Set of all possible outcomes.

## Event:

Subset of sample space.

$$S = \{ \text{win, loss, Draw} \}$$

$$2^n = 2^3 = 2 \times 2 \times 2 = 8.$$

|                           |                                      |
|---------------------------|--------------------------------------|
| $A_1 = \{ \}$             | $A_5 = \{ \text{win, loss} \}$       |
| $A_2 = \{ \text{win} \}$  | $A_6 = \{ \text{win, draw} \}$       |
| $A_3 = \{ \text{loss} \}$ | $A_7 = \{ \text{loss, draw} \}$      |
| $A_4 = \{ \text{Draw} \}$ | $A_8 = \{ \text{win, loss, draw} \}$ |

$$B = \{ A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8 \} \rightarrow (\text{event space})$$

- $B$  is a power set or even space
- Set of all possible subsets is called event space.