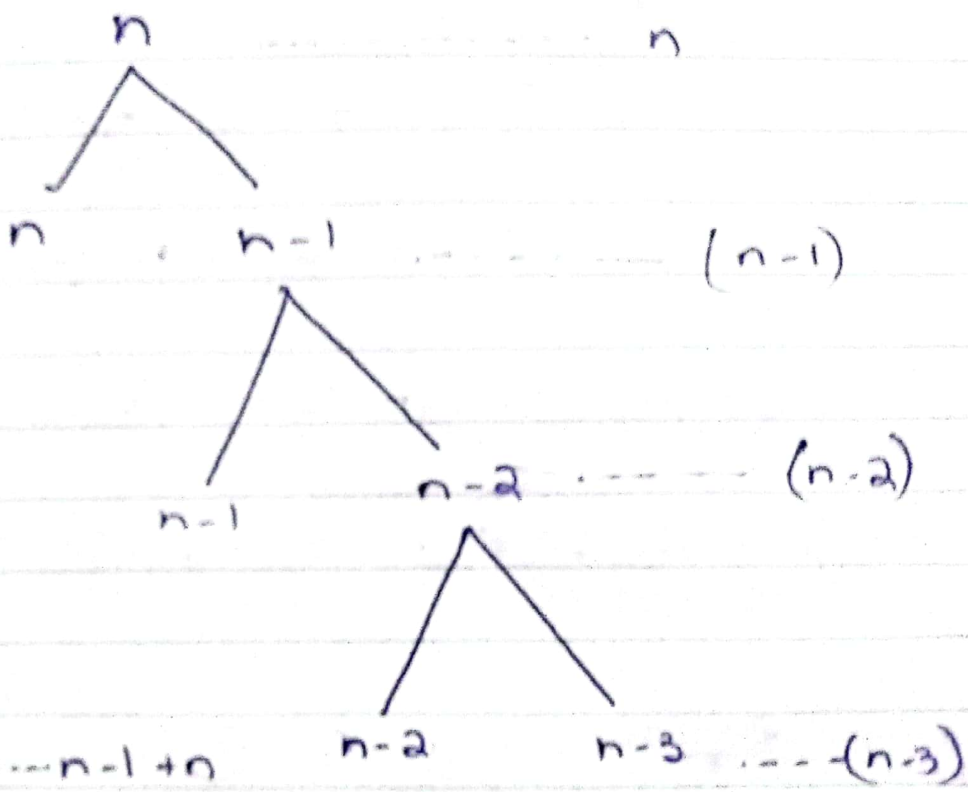
Paper Question #5

Recursion Tree.

$T(n) = T(n-1) + n$.



$0+1+2+3 \cdots n-1+n$

Test(0)

$\frac{n(n+1)}{2}$, sum of first

n natural numbers

$O(n^2)$.

$$T(n) = T(n-1) + n \rightarrow (1)$$

→ Iteration Method.

$$T(n-1) = T((n-1)-1) + n$$

$$= T(n-1-1) + (n-1)$$

$$T(n-1) = T(n-2) + (n-1) \rightarrow (2)$$

$$T(n-2) = T((n-2)-1) + (n-2)$$

$$T(n-2) = T(n-3) + (n-2) \rightarrow (3)$$

eq (2) put in (1).

$$T(n) = T(n-2) + (n-1) + n \rightarrow (4)$$

eq (3) put in (4)

$$T(n) = T(n-3) + (n-2) + (n-1) + n$$

$$\vdots$$

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$$T(n) = T(n-k) + (n-(k-1)) + (n-(k-2)) + (n \ldots$$

$$\therefore n-k = 1$$

$T(n) = T(1) + (n-1) + (n-2) + \cdots n+1$

$= 1 + n(n+1)/2$

$= O(n^2).$

Question # 4.

Master theorem.

$\rightarrow T(n) = 8T\left(\frac{n}{2}\right) + n$

$a = 8, \; b = 2, \; f(n) = n$

$T(n) = n^{\log_b^a} \, u(n)$

$T(n) = n^{\log_2^8} \, u(n)$

$T(n) = n^3 \, u(n) \rightarrow n^3 \, O(1)$

$= O(n^3)$ Ans.

$u(n)$ depends of $h(n)$

$h(n) = \dfrac{f(n)}{n^{\log_b^a}}$

$= \dfrac{n}{n^{\log_2^8}}$

$h(n) = \dfrac{\cancel{n}^1}{n^{\cancel{3}2}} \Rightarrow h(n) = \dfrac{1}{n^2} \Rightarrow h(n) = n^{-2}$

$\rightarrow$ $T(n) = T\left(\frac{n}{2}\right) + n^2$

$a = 1$, $b = 2$, $f(n) = n^2$

$T(n) = n^{\log_b^a} \, u(n)$

$= n^{\log_2^1} \, u(n)$

$= n^0 \, u(n)$

$= 1 \cdot u(n)$

$= u(n) \Rightarrow O(n^2 \log n)$.

$u(n)$ depends on $h(n)$

$h(n) = \frac{f(n)}{n^{\log_b^a}}$

$= \frac{n^2}{n^{\log_2^1}}$

$= \frac{n^2}{n^0}$

$= \frac{n^2}{1} \Rightarrow h^2$.