PROBABILITY :-

27-09-2

Uncertainity:-

=> Caused by incomplete information

=> OR Variation

=> Probability theory is mathematical tool for uncertanity & randomness.

=> Numerical measure of uncertainity

=> Two approaches to measure uncertainity.

→ Subjective approach (expert Opinion)

→ Objective approach

→ Baysian approach (Subjective + Objective)

Subjective approach:-

→ Probability of occurance of any event on the basis of personal judgement

Objective approach:-

→ Mathematical model

→ ~~On the basis of previous data, we find the probability for any event to occur.~~

→ Probability of an event is determined using well defined mathmetical Procedure or method.

Experiment:-

Activity that produces some results.

→ It has two outcomes.

Result

Predictable
Result is known
(Non-Random / Deterministic)

Unpredictable
Result is Unknown.

Ideal Condition
Repeated large No. of time

Under Same Conditions
(Random / Stochastic)

⇒ Two types:- Experiment

→ Predictable :-

Non-Random
Deterministic

→ Unpredictable:-

Random
Stochastic (Experiment)

→ The distribution of random variable is called probability distribution

$2^3 = 2\times2\times2$
$= 8$

Experiment → Random exp → Sample space → event

⇒ Sample Spaces:-

Set of all possible outcomes

→ Event:-

Subset of sample space

$S = \{Win, Loss, Draw\}$ → Sample Space

Possible subsets $= 2^n = 2^3 = 2\times2\times2 = 8$

Possible outcomes	No. of outcomes	Possible outcomes	No. of outcomes
$A_1 = \phi$	$n(A_1) = 0$	$A_5 = \{W, L\}$	$n(A_5) = 2$
$A_2 = \{W\}$	$n(A_2) = 1$	$A_6 = \{W, D\}$	$n(A_6) = 2$
$A_3 = \{L\}$	$n(A_3) = 1$	$A_7 = \{L, D\}$	$n(A_7) = 2$
$A_4 = \{D\}$	$n(A_4) = 1$	$A_8 = \{W, L, D\}$	$n(A_8) = 3$

Event space → $B = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$

→ B is called Power set or Event Space

→ Set of all possible subsets is called Event Space

$A_1 = \phi$ → Impossible event

Event which has no outcome

Q.

$(7\times7\times32)$

$(3.\times3+1)32$

(PROBABILITY)

(15-NOV-2023)

→ Types of Events.

1- Null Event / Impossible Event

→ $n(A) = 0$

No. of Outcomes = None

2- Simple Event

→ $n(A) = 1$ → 1 possible outcome

=> Rolling a dice

$S = \{1, 2, 3, 4, 5, 6\}$ → Sample Space

6 possible outcomes.

$n(S) = 6$

$A_1 = \phi$, $A_2 = \{1\}$, $A_3 = \{1, 2\}$ .... → Events

3- Compound Event

→ $n(A) \geq 2$

No. of possible outcomes ≥ 2

4- Sure Event

→ $n(A)$ = Sample Space

All possible outcomes.

A - Mutually Exclusive Events. / Disjoint Event

Let A & B be any two events defined in a sample space (S).
A & B will be mutually exclusive iff (if and only if) $A \cap B = \phi$

When two events have no common possible outcomes.
ie,
$A_1 = \{1, 2, 3\}$, $A_2 = \{4, 5, 6\}$

B. Equally likely Events.

Let A & B be any two events defined in a sample space (S).
A & B will be known as Equally Likely if A is as likely to occur as event B.

C. Collectively Exhaustive Events.
Let A & B be any two events defined in a sample space (S).
A & B will be Collectively Exhaustive iff $A \cup B = S$ $(A \cap B \neq \phi)$
$A \cup B$ is equal to sample space.

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Probability:-

Numerical measure of uncertainty or occurance of an event.

Approaches of Probability.

A- SUBJECTIVE APPROACH

The Probability of an event is determined on basis of personal judgment

→ Expert knowledge.

B. OBJECTIVE APPROACH

The Probability of an event is determined using well defined mathematical procedure or method.

1- Classical or prior defination of Prob of event:

Let A be any event defined in a sample Space 'S'.

Prob of event A is denoted by P(A), defined as.

$$P(A) = \frac{n(A)}{n(S)} = \frac{\text{No. of outcomes in event } A}{\text{No. of outcomes in Sample Space}}$$

2- Relative frequency or posterior defination of Prob of event:

17th-NOV-202

if an experiment is repeated large number of time say n.

$$P(A) = \lim_{n\to\infty} \frac{n(A)}{n(S)}$$

Example:- Find probability of one head in tossing:-

(i) one coin

(ii) two coins

(iii) Three coins

SOL:- (i) One coin

$S = \{H, T\}$, $n(S) = 2$

P(one Head) = ?

Let event A: one Head

$A = \{H\}$, $n(A) = 1$

$$P[\text{one Head}] = P(A) = \frac{n(A)}{n(S)} = \frac{1}{2} = 0.5 = 50\%$$

(ii) Two coins.

$S = \{HH, HT, TH, TT\}$, $n(S) = 4$

$P(\text{one Head}) = ?$

$A = \{HT, TH\}$ $n(A) = 2$

$P[\text{one Head}] = P(A)$

$= \frac{n(A)}{n(S)}$

$= \frac{2}{4}$

$= 0.5$

$= 50\%$

A) At least One Head

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{3}{4}$

$= 0.75$

$= 75\%$

(iii) Three coins.

$n(S) = 8$

$S = \{HHH, HHT, HTH, THH, THT, TTH, TTT, HTT\}$

$P(\text{one Head}) = ?$

$A = \{HTT, THT, TTH\}$ $n(A) = 3$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{3}{8}$

$= 0.37$

$= 37\%$

B) At least one head

$n(A) = 7$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{7}{8}$

$= 0.87$

$= 87\%$

Example:- A pair of balance diece are rolled. Find the probability that
i) The sum of two numbers is 8
ii) Same numbers on both diece
iii) Product of two numbers is 8.

SOL:- $n(S) = 6 \times 6 = 36$

1st diece

S =

2nd diece	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

i) The sum of two numbers is 8

$n(S) = 36$

$A = \{(2,6), (6,2), (4,4), (5,3), (3,5)\}$

$n(A) = 5$

$P(A) = \frac{5}{36} = 0.13 = 13\%$

ii) Same Numbers on both diece

$n(S) = 36$

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(A) = 6$

$P(A) = \frac{6}{36} = 0.16 = 16\%$

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iii) Product of two numbers is 8

$n(S) = 36$

$A = \{(2,4), (4,2)\}$

$n(A) = 2$

$P(A) = \frac{2}{36} = 0.05 = 5\%$

Example:- A card is drawn at random from 52 cards. Find the probability that the selected card is

i) a king

ii) King of black

iii) Ace or Queen card.

SOL

$n(S) = 52$

i) a King. (4 Kings)

Let A = King → $n(A) = 4$

$P(A) = \frac{4}{52} = 0.07 = 7\%$

ii) King of black (2 Black King)

Let A_b : King of black → $n(A_b) = 2$

$P(A_b) = \frac{2}{52} = 0.03 = 3\%$

iii) Ace or Queen card (4 Ace, 4 Queens)

Let A_c : Ace or Queen → $n(A) = 8$

$P(A) = \frac{8}{52} = 0.15 = 15\%$

1) Counting Rule
2) Addition Law
3) Multiplication Law
4) Bayes theorem

Random Variable → Prob distribution → (Mean, Varience, Kurtosis)

↓

Types (Binomial

Selected = 3

5 - men 3 women

→ Selet 1 man & women

Let event A be ~~the probability of~~ 1 man and women

$$P(A) = \frac{\binom{5}{1}\binom{3}{2}}{\binom{8}{3}} = \frac{5 \times 3}{56} = \frac{15}{56}$$

$= 0.26$

$= 26\%$

Examples-

1) a king.

$$= \frac{\binom{4}{1}\binom{48}{0}}{\binom{52}{1}} = \frac{4 \times 1}{52}$$

$= 0.07$

2) King of black

$$= \frac{\binom{2}{1}\binom{50}{0}}{\binom{52}{1}} = \frac{2 \times 1}{52}$$

$= 0.03$

3) Ace or Queen Card

$$\frac{\binom{8}{1}\binom{44}{0}}{\binom{52}{1}} = \frac{8 \times 1}{52} = 0.15$$

Boxcar contains 6 electronic system
Two are randomly selected.
a) Two are defective. (Find the probability that at least one is defective). (Find the probability that both are defective)
b) ~~Find~~ ~~f~~our are defective, find the probabilities in part a

at least 1 let event A be at least 1 defective

Given = ~~n(~~ S = 6

$n(S) = \binom{6}{2}$

$n(A) =$

a)

$$p(A) = \frac{\binom{2}{1}\binom{4}{1} + \binom{2}{2}\binom{4}{0}}{\binom{6}{2}}$$

$$= \frac{2 \times 4 + 1 * 1}{15}$$

$$= \frac{8+1}{15} = \frac{9}{15} = 0.6$$

both defective

let event B be both defective

$$p(B) = \frac{\binom{2}{2}\binom{4}{0}}{\binom{6}{2}} = \frac{1 \times 1}{15} = \frac{1}{15} = 0.06.$$

b) at least 1

let A be at least 1 defective

$$n(A) = \frac{\binom{4}{1}\binom{2}{1} + \binom{4}{2}\binom{2}{0}}{\binom{6}{2}} = \frac{4 \times 2 + 6 * 1}{15} = \frac{14}{15} = 0.9$$

both defective

let B be both defective

$$P(B) = \frac{\binom{4}{2}\binom{2}{0}}{\binom{6}{2}} = \frac{6 \times 1}{15} = \frac{6}{15} = 0.4$$

A class has 20 male & 5 female students. 5 male & 2 female failed. One student is selected at random.

A) A male

B) A female & failed.

C) A female or a failed.

Given:-

	Male	Female	Total
	20	5 F	25
Failed	5	2	7 (5 M, 2 F)

A) $S = 25$

Let A be : A male

$$p(A) = \frac{\binom{20}{1}\binom{5}{0}}{\binom{25}{1}} = \frac{20 \times 1}{25} = 0.8$$

B) $S = 25$

let B be : A Femal & failed.

$$p(B) = \frac{\binom{2}{1}\binom{23}{0}}{\binom{25}{1}} = \frac{2 \times 1}{25} = 0.08$$

$C = A \cup B = A + B - A \cap B$

let event B.

C) $S = 25$ let A

let C be : A female or failed.

$$p(C) = \frac{\binom{5}{1}\binom{20}{0} + \binom{7}{1}\binom{18}{0} - \binom{2}{1}}{\binom{25}{01}}$$

~~P(A∩B)~~

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{\binom{5}{1}\binom{20}{0}}{\binom{25}{1}} + \frac{\binom{7}{1}\binom{18}{0}}{\binom{25}{1}} - \frac{\binom{2}{1}\binom{23}{0}}{\binom{25}{1}}$

$= \frac{5\times 1}{25} + \frac{7\times 1}{25} - \frac{2\times 1}{25}$

$= \frac{5}{25} + \frac{7}{25} - \frac{2}{25}$

$= 0.2 + 0.28 - 0.08$

$= 0.48 - 0.08$

$= 0.4$

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1-DEC-2023.

Random Variable (R.V)

Probability Function:-

$$P(X = x) = \binom{2}{x}\left(\frac{1}{2}\right)^{2-x}\left(\frac{1}{2}\right)^{x}, \; x = 0, 1, 2$$

$$P[x = 0] = \binom{2}{0}\left(\frac{1}{2}\right)^{2-0}\left(\frac{1}{2}\right)^{0} = \frac{1}{4} = 0.25$$

$$P[x = 1] = \binom{2}{1}\left(\frac{2}{4}\right)^{2-1}\left(\frac{1}{2}\right)^{1} = 0.5$$

Probability Distribution

X	P(x)
0	1/4
1	2/4
2	1/4

$\sum P(x) = 1$

Probability Mass Function. $\left(\sum_x f(x) = 1\right)$ if $f(x) \geq 0$

If Random variable is Descrete

Probability Density Function $\left(\int f(x)\,dx = 1\right)$ if $f(x) \geq 0$

If Random variable is Continous.

if $f(x) \geq 0$; $\forall x$

& $\sum_x f(x) = 1$

if $f(x) \geq 0$

$\int f(x)\,dx = 1$

6-Dec-2023

Random Variable:- (R.V) A variable whose value is unknown.

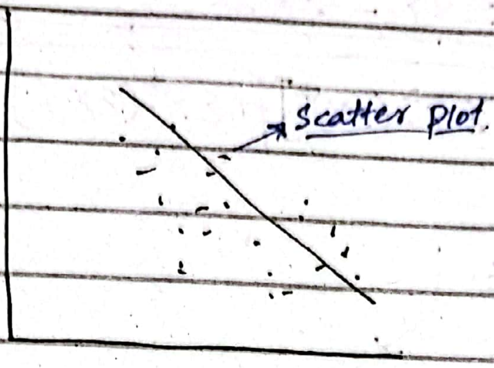
Random Experiment:- Result is unknown.

Sample Space:- Is a set of all possible outcomes.

Probability Distribution:- The distribution of random variable is Prob dist.

Co-relation :-

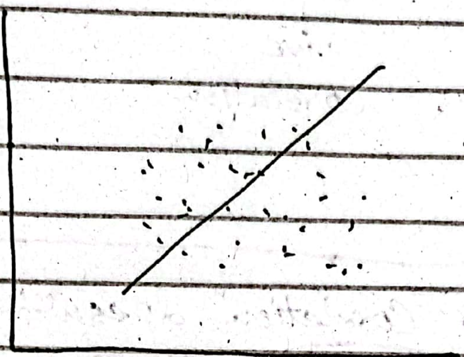
Negative Slope



→ Negative Slope

→ Shows -ive relationship

Positive Slope



→ Positive Slope

→ Shows +ive relationship

Co-Relation

+ive Relationship	-ive Relationship
100%	-100%

Corelation :- is study of linear relationship b/w two variable.

→ We don't know about dependent & independent variable.

Corelation Coefficient :- Measure the stregth of a linear relationship b/w two variables

→ Sample Corelation = r

→ Population Corelation = ρ(rho)

→ Greek letter rho

Sample Corelation:-

$$r = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{\left[n\Sigma Y^2 - (\Sigma Y)^2\right]\left[n\Sigma X^2 - (\Sigma X)^2\right]}}$$

$$-1 \le r \le 1$$

Perfect -ive corelation

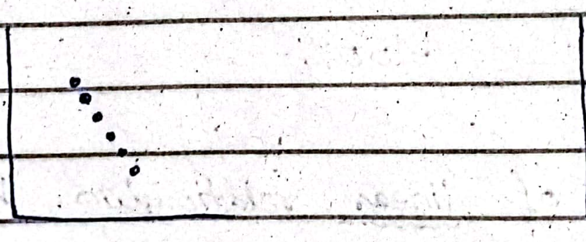
Perfect +ive corelation

$r = 0$

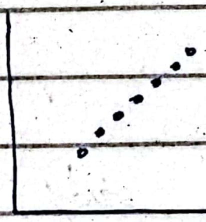
$\Rightarrow$ linear Corelation doesn't exist

Population Corelation:-

$$\rho = \frac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{\left[N\Sigma Y^2 - (\Sigma Y)^2\right]\left[N\Sigma X^2 - (\Sigma X)^2\right]}}$$



$r = -1.00$

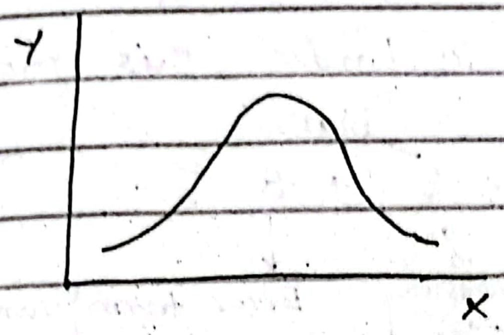


$r = 1.00$

$\Rightarrow$ When we know about dependent & independent variable then we go for regression not corelation.

8-Dec-2023

Regression:-



→ $Y = 0$

→ Non-Linear relation

→ Dependent / Independent variables known.

→ Check dependence of dependent variable on independent variable

→ In regression, we study dependent variable.

⇒ Regression is the study of dependence of a variable on one or more variables.

Types of Regression:-

Regression → Simple, Multiple

Simple: independent variable $= 1$

Multiple: independent variable ≥ 2

Regression → Linear, Non-Linear

Simple Linear Regression Model.

Population:- $Y = \alpha + \beta X$ → Deterministic Model.

if x is study hours & Y is marks. by putting the value of X, we will find the exact value of Y. This is ideal situation & this doesn't happen in real life. To handle this problem

Population:- $$Y = \underbrace{\alpha}_{(Y \text{ estimate})} + \underbrace{\beta}_{[\text{regression coeff}]} x + \underbrace{\varepsilon}_{\text{Error term } \{\text{random variable}\}}$$

bias

Sample:- $Y = a + bx + e_i$ → Complete

$\hat{Y} = a + bx$ → Estimated

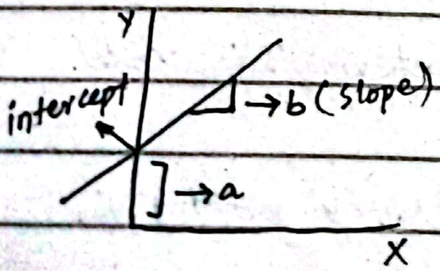
1st formula $$b = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(X-\bar{X})^2}$$ → Estimater (for β)

2nd formula $$b = \frac{\text{Covarience}(X,Y)}{\text{Varience}(X)}$$

3rd formula $$b = \frac{n\Sigma XY - (\Sigma Y)(\Sigma X)}{n\Sigma X^2 - (\Sigma X)^2}$$

$$a = \bar{Y} - b\bar{X}$$

$$\underbrace{e = Y - \hat{Y}}_{\text{Error}} \rightarrow \underbrace{e^2}_{\text{Square}} \rightarrow \underbrace{\frac{\sum_{i=1}^{N} e^2}{N}}_{\text{Mean [Estimater]}}$$



at X = 0, change in the value of Y

$$MSE = Var(\bar{X}) + (bias(\bar{x}))^2$$

if bias = 0 then. $MSE = Var(\bar{X})$

X_i	$\bar{X}$
1	m_1
2	m_2
3	m_3
⋮	⋮

$Mean(\bar{X})$

$\downarrow$

$E(\bar{X})$

$Bias\left(E(\bar{x}) - \mu = 0\right)$

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13-Dec-2023

Random Variable ⟶ Prob distribution ⟶ Mass func, Density func.

Probability Distribution:-

Example:- A box contains 4 Red balls, 3 white balls, & 6 green balls. Two balls are selected at random. Find the probability

1) No white ball

$p[\text{no white ball}] = \binom{3}{0}\binom{10}{2} \Big/ \binom{13}{2}$

2) 1 white ball

$p[1 \text{ white ball}] = \dfrac{\binom{3}{1}\binom{10}{1}}{\binom{13}{2}}$

3) 2 white balls

$p[2 \text{ white balls}] = \dfrac{\binom{2}{2}\binom{10}{0}}{\binom{13}{0}}$

4) 3 white balls

Impossible event bcz we can only select 2 white balls.

Example:- A box contains 4 Red balls ------ Two balls are selected at random. Let X denotes the number of white balls. Find prob distribution of X.

Prob Distribution

X	P(X = X)	X P(X)	X^2 P(X)
0	0.5769	0	0
1	0.3846	0.3846	0.3846
2	0.0385	0.0770	0.3080
	$\Sigma P = 1$	$\Sigma = 0.4616$	$\Sigma = 0.6926$

A.r.v X has p.m.f $f(x) = \frac{\binom{3}{x}\binom{10}{2-x}}{\binom{13}{2}}$, $x = 0, 1, 2$

Find $P[x \le 1] = P[x=0] + P[x=1]$

$M_r = E(X^r)$ $\begin{cases} \sum x^r f(x) & X \text{ is descrete Random variable} \\ \int x^r f(x)\,dx & X \text{ is continous Random variable} \end{cases}$	Expectation of r.v
	rth raw moment of a.r.v X with prob fun $M_r = \frac{\sum x^r}{n}$

→ Mean (1 variable)

$Mean(X) = E(X)$ | Expected value of

→ Variance (1 variable) | $M_r = E(X^r)$

$Var(X) = E(X^2) - [E(X)]^2$

→ Mean (Whole data)

$Mean(X) = E(X) = \sum X\, P(X) = 0.4616$

→ Var (Whole data)

$Var(x) = \sum X^2 P(X) - [\sum X\, P(X)]^2$

$= 0.6926 - [0.4616]^2$

$Var(x) = 0.4795 \Rightarrow \mu^2$

$\mu_r = E(X - E(X))^r$ $\begin{cases} \sum (X - E(x))^r f(x) & X \text{ is descrete random var} \\ \int (X - E(x))^r f(x)\,dx & X \text{ is continous random var} \end{cases}$	rth mean moment $\mu_r = E(X - E(X))^r$
	$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$
	$\beta_2 = \frac{\mu_4}{\mu_2^2}$

Continous Random Variable :-

Distribution Function $(F(x))$

Integrate the value of x from $-\infty$ to x

→ $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$ (Prob func given)

→ $f(x) = \frac{d}{dx} F(x)$ (Distribution func given)

Derivate the distribution Func

Example:- A random variable X has p.d.f $f(x)$ defined as.

$$f(x) = C(x - x^3) \quad , \; 0 \le x \le 1$$

1) Find the value of C.
2) $P[\frac{1}{2} \le x \le \frac{3}{4}]$, $P[x = \frac{3}{4}]$
3) $E(X)$; $Var(x)$

Sol:-

1) $\int_{X} f(x)\,dx = 1$

$$\int_0^1 c(x - x^3)\,dx = 1$$

$$c\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 1$$

$$c\left[\frac{1}{4}\right] = 1$$

$$\Rightarrow c = 4$$

2) $P\left[\frac{1}{2} \le x \le \frac{3}{4}\right] = \int_{1/2}^{3/4} 4(x - x^3)\,dx$

$$= 4\left[\frac{x^2}{2} - \frac{x^4}{4}\right]\Bigg|_{1/2}^{3/4}$$

3) $$E(X) = \int_0^1 x f(x)\,dx$$

$$= \int_0^1 x\, 4(x - x^3)\,dx$$

$$= \int_0^1 4(x^2 - x^4)\,dx$$

$$= \left[4\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Bigg|_0^1\right]$$

$$= 4\left[\frac{5-3}{15}\right]$$

$$= \frac{8}{15}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \int_0^1 x^2 f(x)\,dx$$

$$= \int_0^1 x^2\, \cancel{f}\, 4(x - x^3)\,dx$$

$$= \int_0^1 4(x^3 - x^5)\,dx$$

$$= \left[4\left(\frac{x^4}{4} - \frac{x^6}{6}\right)\Bigg|_0^1\right]$$

$$= 4\left[\frac{3-2}{12}\right]$$

$$= \frac{4}{12} - \left[\frac{8}{15}\right]^2$$

= 0.049

$\frac{d}{dx} e^{-5x} = e^{-5x}(-$

$\int \frac{e^{-5x}(-5)}{(-5)} = \frac{e^{-5x}}{-5}$

Example:- A r.v. has p.d.f

$$f(x) = A e^{-5x}, \quad x > 0$$

1) Find A, $P[X \geq 5]$, $P[X \leq 5]$
2) $E(X)$, $Var(X)$

Example:- A r.v. X has p.m.f

$$f(x) = \frac{e^{-5} 5^x}{x!} ; \quad x = 0, 1, 2, 3, \ldots$$

1) Find $P[X \geq 5]$, $P[X \leq 5]$
2) $P\{2 \leq X \leq 7\}$
3) ~~E(x~~ $Var(X)$

15-DEC-2023

Binomial Distribution:-

A random variable x has binomial distribution if it has the probability function.

$$b(\underset{(r.v)}{X} : \underbrace{n, P}_{(parameters)}) = \binom{n}{x} P^x q^{n-x} \; ; \; \underset{(value\ of\ r.v)}{x} = 0, 1, 2, 3, \ldots, n$$

where,

X : Number of successful outcomes
n : Number of trail (Sample size)
P : Probability of successful outcomes in a single trail.
$q = 1 - P$: Probability of failed outcomes in a single trail.

Example:- The past feedback of an instructor shows that 80% student are in favour of teaching methodology.

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Example: The past record of a student shows that he got 4 A grades out of 20. The student will apear in 6 exams. Find the prob that he will get at most 2 A grades.

$n = 6$

X : number of A Grades

$P[\text{A Grade}] = \frac{4}{20} = \frac{1}{5} = 0.20$

$P[X \leq 2]$

$\therefore 1-P = 1-0.20$

$1-P = 0.80$

$X \sim b(n, P)$

$b(x: 6, 0.20) = \binom{6}{x}(0.20)^{x}(0.80)^{6-x}$

$P[X \leq 2] = \sum_{0}^{2}\binom{6}{x}(0.20)^{x}(0.80)^{6-x}$

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