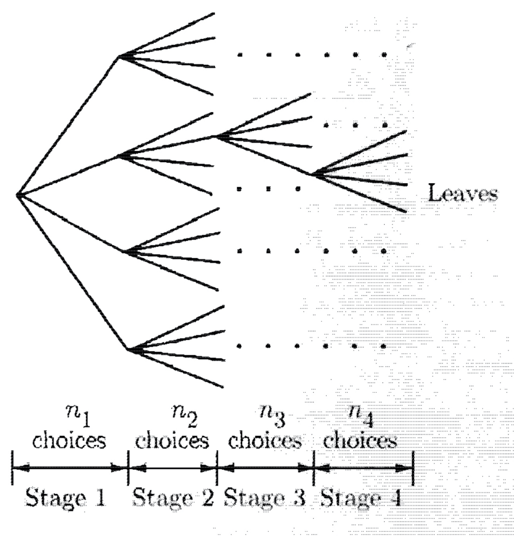
EE5110: Probability Foundations for Electrical Engineers

July - Nov 2024

1 Counting

1. Counting is fundamental to probability theory
   - to enumerate elementary outcomes and to identify events of interest
   - to assign probability to elementary outcomes and events
   - useful in uniform and general probability measures

2. Principle of counting:



   - The total number of possible outcomes is $n_1 \times n_2 \times n_3 \times n_4$
   - Example: A vehicle registration number is a 2 digit sequence. How many distinct registration numbers are there? (Hint: 90 or 100)
   - Example: Consider an n-element set. How many subsets does it have (including itself and the empty subset)? (Hint: 2^n)

3. Permutation: Count the different ways in which we can pick r out of n elements and arrange them in a sequence.
   - Answer: $n \times (n-1) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$

- Example: Count the different ways in which you can pick 2 players out of 10 players and rank them in a sequence? (Hint: 90)
- Example:Count the different ways in which you can rank order 10 players in a sequence? (Hint: 10!)

4. Combination: Count the different ways in which we can pick k out of n elements. (In combination, there is no ordering of the selected elements.)
   - Answer: $\frac{n!}{(n-k)!k!}$
   - Example: Count the different ways in which you can pick 2 players out of 10 players? (Hint: 45)
   - Example: Count the different ways in which you pick 10 out of 10 players? (Hint: 1)
   - Binomial formula: $2^n = \sum_{k=0}^{n} \binom{n}{k}$

5. Partition: We are given an n-element set and nonnegative integers $n_1, \cdots, n_r$ whose sum is equal to n. We consider partitions of the set into r disjoint subsets of size $n_1, \cdots, n_r$. (Combination is a partition into 2 sets.)
   - Answer: $\frac{n!}{n_1!n_2!\cdots n_r!}$
   - Example: Count the number of ways in which you can distribute a deck of cards among four players?
   - Example (Anagrams): How many different words (letter sequences) can be obtained by rearranging the letters in the word TATTOO?

6. Miscellaneous
   - Example: Suppose we throw n distinguishable balls into one of the k distinguishable bins. Then, the number of possible outcomes is _____
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