1. Bending a flexure under torque

The bending moment is given by

$$M'(s) = \frac{\mathrm{d}M}{\mathrm{d}s} = F_w \sin\theta \approx F_w \theta \tag{1}$$

The derivative of the bending angle is given by

$$\theta'(s) = \frac{1}{EI} M(s) \longrightarrow \theta''(s) = \frac{1}{EI} M'(s) = \frac{F_w}{EI}\theta(s) \tag{2}$$

where we assume E and I to be constant along the flexure axis s. Using the ansatz $\theta(s) = A\exp(-s\,\nu) + B\exp(s\,\nu)$, we see $\theta''(s) = \nu^2\theta(s)$, which leads to $\nu^2 = F_w/(EI)$. The first boundary condition is that $\theta = 0$, hence

$$\theta(0) = A + B = 0 \longrightarrow A = -B. \tag{3}$$

Hence

$$\theta(s) = A^* \sinh(s\,\nu) \text{ with } \nu = \left(\frac{F_w}{EI}\right)^{1/2}. \tag{4}$$

From (2) we obtain

$$M(s) = EI\,\theta(s)' = A^*\,\nu\cosh(s\,\nu). \tag{5}$$

Our second boundary condition is that $M(L) = \tau$, hence

$$M(L) = A^*\,\nu\cosh(L\,\nu) = \tau \longrightarrow A^* = \frac{\tau}{\nu\cosh(L\,\nu)}. \tag{6}$$

With that, we can calculate the torque that the flexure exerts on the fixed end. It is

$$M(0) = \frac{\tau}{\nu\cosh(L\,\nu)} \tag{7}$$

So, the complete solution is

$$\theta(s) = \frac{\tau}{\nu\cosh(L\,\nu)}\sinh(s\,\nu). \tag{8}$$

The x and y coordinates can be obtained by integrating out

$$x(s) = \int_0^s \cos\big(\theta(s^*)\big)\mathrm{d}s^* \text{ and } y(s) = \int_0^s \sin\big(\theta(s^*)\big)\mathrm{d}s^* \tag{9}$$

In the small-angle approximation, we find

$$x(s) = s + c_o \text{ and } y(s) = \frac{\tau}{EI\,\nu^2\cosh(L\,\nu)}\big(\cosh(s\,\nu) - 1\big) + c_1 \tag{10}$$

The integration constant c_o is 0 because $x = 0$ corresponds to $s = 0$. The integration constant c_1 is obtained from $y(0) = 0$, which is true for $c_1 = 0$. Since $x = s$, the s in $y(s)$ can be replaced by x. Substituting ν^2 with $F_w/(EI)$ we obtain,

$$y(x) = \frac{\tau}{F_w\cosh(L\,\nu)}\big(\cosh(x\,\nu) - 1\big). \tag{11}$$

This equation is identical to the one that Clive obtained for $F = 0$.