
Bias-UQ-PCR

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GET DATA

`src.get_data.cycle_to_index(cycle: int) → int`

Parameters

cycle (*int*) – cycle number *i*, 0 to *n*

Returns

index in matrix or array

Return type

int

`src.get_data.file_to_numpy(file_abs_path: str) → ndarray`

Parameters

file_abs_path (*str*) – path to file name

Notes

All fluorescence values are divided by 10^6 before any analysis

Returns

n by *m* matrix of fluorescence, **F**

Return type

np.ndarray

`src.get_data.index_to_cycle(index: int) → int`

Parameters

index (*int*) – index of cycle (0 to *n*-1)

Returns

cycle number (1 to *n*)

Return type

int

WELLS

`src.wells.column_row_to_well(ix: int, iy: int) → str`

Convert indices to well

Parameters

- **ix** (*int*) – x (column) index
- **iy** (*int*) – y (row) index

Returns

name of well

Return type

str

`src.wells.number_to_column(number: int) → int`

Get column index associated with well number

Parameters

number (*int*) – well number (0 to 95)

Returns

column (0 to 11)

Return type

int

`src.wells.number_to_row(number: int) → int`

Get the row number associated with a well number

Parameters

number (*int*) – well number

Returns

row number (0 to 7)

Return type

int

`src.wells.number_to_well(number: int) → str`

Get well name from number

Parameters

number (*int*) – well number (0 to 95)

Returns

well name (A1 to H12)

Return type

str

src.wells.**well_to_column**(well: str) → int

Convert well name to column

Parameters

well (str) – name of well

Returns

ix – x-index for well (row)

Return type

int

src.wells.**well_to_column_row**(well: str) → Tuple[int]

Convert well name to column, row

Parameters

well (str) – name of well

Returns

column index, row index

Return type

tuple(int, int)

src.wells.**well_to_number**(well: str) → int

Returns number of well

Parameters

well (str) – name of well (e.g., "A1")

Returns

well number (0 to 95)

Return type

int

src.wells.**well_to_row**(well: str) → int

Well to y index

Parameters

well (str) – well name

Returns

iy – y-index of well (row)

Return type

int

AMPLIFICATION

```
class src.amplification.Amplification(R: float, pbar: float, E_U0: array, V_U0: ndarray)
```

Parameters

- **l1** (*float*) – Largest eigenvalue of **A**, λ_1
- **l2** (*float*) – Smallest eigenvalue of **A**, λ_2
- **x1** (*np.array*) – Right eigenvector of **A** corresponding to λ_1 , \mathbf{x}_1
- **x2** (*np.array*) – Right eigenvector of **A** corresponding to λ_2 , \mathbf{x}_2
- **z1** (*np.array*) – Left eigenvector of **A** corresponding to λ_1 , \mathbf{z}_1
- **z2** (*np.array*) – Left eigenvector of **A** corresponding to λ_2 , \mathbf{z}_2
- **K1** (*np.ndarray*) – Matrix **K**₁ defined in (25)
- **K2** (*np.ndarray*) – Matrix **K**₂ defined in (25)

```
__init__(R: float, pbar: float, E_U0: array, V_U0: ndarray)
```

Parameters

- **R** (*float*) – ratio of amplification probabilities, R
- **pbar** (*float*) – geometric mean of amplification probabilities, \bar{p}
- **E_U0** (*np.array*) – initial expected values of both strands $\mathbb{E}[\mathbf{U}_0]$, 2 by 1 matrix
- **V_U0** (*np.ndarray*) – initial variances of both strands $\text{Var}[\mathbf{U}_0]$, 2 by 2 matrix

```
get_A() → ndarray
```

Returns

matrix **A** calculated by decomposition (13)

Return type

np.ndarray

```
get_Atoi(i: int) → ndarray
```

Parameters

i (*int*) – cycle number

Returns

matrix **A**^{*i*} calculated by decomposition (15)

Return type

np.ndarray

get_EXi_over_EYi (*cycles: array*) \rightarrow array

Calculate $\mathbb{E}[X_i] / \mathbb{E}[Y_i]$ for a variety of cycles i

Parameters

cycles (*np.array*) – cycles (integers) to calculate for each i

Returns

$\mathbb{E}[X_i] / \mathbb{E}[Y_i]$

Return type

np.array

get_E_Ui (*i: int*) \rightarrow array

Parameters

i (*int*) – cycle number

Returns

matrix $\mathbb{E}[\mathbf{U}_i] = \mathbf{A}^i \mathbb{E}[\mathbf{U}_0]$

Return type

np.array

get_V_Ui (*i: int*) \rightarrow ndarray

Parameters

i (*int*) – cycle number

Returns

Var $[\mathbf{U}_i]$ obtained by Equation (27)

Return type

np.ndarray

initialize()

Initialize \mathbf{K}_ℓ via (25), $\nu_{j,k}$ via (28a), and $\eta_{j,k}^\ell$ via (28b).

src.amplification.eval_R (*p_fr: float, p_rf: float*)

Calculate R from p_{rf} and p_{fr}

Parameters

- **p_fr** (*float*) – probability of forward to reverse amplification, p_{fr}
- **p_rf** (*float*) – probability of reverse to forward amplification, p_{rf}

Returns

$$R = \sqrt{\frac{p_{\text{rf}}}{p_{\text{fr}}}}$$

Return type

float

src.amplification.get_pfr_prf (*R: float, pbar: float*)

Get p_{fr} and p_{rf} from R and \bar{p}

Parameters

- **R** (*float*) – square root of ratio of probabilities, R
- **pbar** (*float*) – geometric mean of probabilities, \bar{p}

Return type

tuple(p_{fr} , p_{rf})

MOLAR FLUORESCENCE

```
class src.molar_fluorescence.MolarFluorescence(C: array, files: List[str], name: str)
```

Parameters

- **f** (*np.ndarray*) – molar fluorescences for each cycle/well **f**, *n* by number of wells matrix
Determined in units of fluorescence divided by (pmol/L)
- **df** (*np.ndarray*) – standard deviation in molar fluorescences σ , *n* by number of wells matrix
Determined in units of fluorescence divided by (pmol/L)
- **cv** (*np.ndarray*) – Coefficient of variation in molar fluorescences σ/f ,
- **q** (*int*) – number of plates (different concentrations)
- **n** (*int*) – number of cycles
- **m** (*int*) – number of wells
- **F** (*np.ndarray*) – raw fluorescence data (scaled by 10^6 as described in `get_data.py`). Tensor of dimensions $n \times m \times q$

```
__init__(C: array, files: List[str], name: str)
```

Parameters

- **C** (*np.array*) – concentrations of reporter in pmol/L
- **files** (*List[str]*) – list of file names (absolute paths)
- **name** (*str*) – name of reporter (e.g., FAM, Probe)

```
calculate()
```

Perform calculations of **f** and standard deviation σ

```
print_cv()
```

Print information relating to coefficient of variation

KINETIC PCR

```
class src.kinetic_PCR.HydrolysisProbes(C: float, Vol: float, f_plus: ndarray, f_minus: ndarray, R: float,
                                       pbar: float, E_U0: array, V_U0: ndarray, s_plus: ndarray,
                                       s_minus: ndarray, **kwargs)
```

Specific subclass for hydrolysis probes

Parameters

- **n** (*int*) – number of cycles
- **m** (*int*) – number of wells
- **d** (*np.ndarray*) – array of incremental increases in fluorescences, n by m
- **b** (*np.ndarray*) – array of background signals, n by m
- **dd** (*np.ndarray*) – array of standard deviation in incremental increases in fluorescences, n by m
- **db** (*np.ndarray*) – array of standard deviation in background signals, n by m
- **E_F** (*np.ndarray*) – expected value of fluorescence, n by m, $\mathbb{E}[\mathbf{F}]$
- **V_F** (*np.ndarray*) – variance of fluorescence, n by m, $\text{Var}[\mathbf{F}]$
- **E_DX** (*np.array*) – expected value of change in X, length n, $\mathbb{E}[\Delta X_i]$
- **V_DX** (*np.array*) – variance value of change in X, length n, $\text{Var}[\Delta X_i]$
- **cycles** (*np.array*) – cycle numbers, 1 to n

```
__init__(C: float, Vol: float, f_plus: ndarray, f_minus: ndarray, R: float, pbar: float, E_U0: array, V_U0:
         ndarray, s_plus: ndarray, s_minus: ndarray, **kwargs)  $\rightarrow$  None
```

Parameters

- **R** (*float*) – ratio of amplification probabilities, R
- **pbar** (*float*) – geometric mean of amplification probabilities, \bar{p}
- **E_U0** (*np.array*) – initial expected values of both strands $\mathbb{E}[\mathbf{U}_0]$, 2 array
- **V_U0** (*np.ndarray*) – initial variances of both strands $\text{Var}[\mathbf{U}_0]$, 2 by 2 matrix
- **f_plus** (*np.ndarray*) – molar fluorescences for each cycle/well of active reporter \mathbf{f}^+ , n by number of wells matrix Determined in units of fluorescence divided by (pmol/L)
- **f_minus** (*np.ndarray*) – molar fluorescences for each cycle/well of inactive reporter \mathbf{f}^- , n by number of wells matrix Determined in units of fluorescence divided by (pmol/L)
- **C** (*float*) – Total concentration of reporters in pmol/L, C

- **Vol** (*float*) – Total volume of solution in L, V
- **s_plus** (*np.ndarray*) – standard deviation in molar fluorescences for each cycle/well of active reporter σ^+ , n by number of wells matrix Determined in units of fluorescence divided by (pmol/L)
- **s_minus** (*np.ndarray*) – standard deviation molar fluorescences for each cycle/well of inactive reporter σ^- , n by number of wells matrix Determined in units of fluorescence divided by (pmol/L)
- **kwargs** (*dict*) – extra kwargs for Amplification class

calculate()

Performs calculations, filling in E_F, V_F, E_DX, and V_DX.

get_Cov_XiX0(i)

Parameters

i (*int*) – cycle number

Returns

$\text{Cov}[X_i, X_0]$, see (43)

Return type

float

get_E_DX(i) → float

Parameters

i (*int*) – cycle number

Returns

$\mathbb{E}[\Delta X_i]$

Return type

float

get_V_DX(i)

Parameters

i (*int*) – cycle number

Returns

$\text{Var}[\Delta X_i]$

Return type

float

src.kinetic_PCR.plot_fluorescence_curve(kls: [HydrolysisProbes](#), ax: *axis*, wml: *int*)

Parameters

- **kls** ([HydrolysisProbes](#)) – Contains all amplification data. Has already computed values
- **ax** (*matplotlib.axes*) – axes to plot on
- **wml** (*int*) – well number

PLOTTING FIGURES

`plot_figures.plot_figure2(R=0.9, pbar=0.85, max_cycle=4)`

Parameters

- **R** (*float, optional*) – choice for value of R , defaults to 0.9
- **pbar** (*float, optional*) – choice for value of \bar{p} , defaults to 0.85
- **max_cycle** (*int, optional*) – choice for maximum cycle to plot, defaults to 4

`plot_figures.plot_figure6(model: HydrolysisProbes, wml: int)`

Parameters

- **model** ([HydrolysisProbes](#)) – Contains all of the parameters for calculation of fluorescence curves
- **wml** (*int*) – well number (0 to 95)

Notes

The methods `model.E_U0` and `model.V_U0` are updated in place during plotting of each subplot

LIMIT OF DETECTION

class `src.lod.LOD`(*input_type: str*)

Parameters

input_type (*str*) – Type of nucleic acids input. Either "ds-DNA" (see (4)), "fs-RNA" (see (5)), or "rs-RNA" (see (6)).

L(*chi, kappa, R, pbar, r*)

Parameters

- **chi** (*float*) – parameter relating $\mathbb{E}[I]$ to $\text{Var}[I]$ (see (50))
- **kappa** (*float*) – number of standard deviations allowed to be above background, κ see (51)
- **R** (*float*) – square-root of amplification efficiency ratio (see (3))
- **pbar** (*float*) – geometric mean of amplification efficiencies (see (2))
- **r** (*float*) – probability of reverse-transcription

Returns

L as in (56)

Return type

int

M(*chi, L*)

Parameters

- **chi** (*float*) – relationship between expected value and variance
- **L** (*int*) – limit of detection from (56)

Returns

M as in (57)

Return type

float

alpha(*R*)

Parameters

R (*float*) – square-root of amplification efficiency ratio

Returns

α as in (35a)

Return type

float

beta($R, pbar, r$)**Parameters**

- **R** (*float*) – square-root of amplification efficiency ratio (see (3))
- **pbar** (*float*) – geometric mean of amplification efficiencies (see (2))
- **r** (*float*) – probability of reverse-transcription

Returns β as in (35b)**Return type**

float

`src.lod.main(chi=1, kappa=3, N=100)`

Estimate range of LOD due to different assays. Prints out results.

Parameters

- **chi** (*float, optional*) – Relationship between expected value and variance, defaults to 1 (Poisson distribution)
- **kappa** (*float, optional*) – Number of standard deviations for statistical significance, defaults to 3
- **N** (*int, optional*) – number of points in interval, defaults to 100

`src.lod.my_int(x)`**Parameters****x** (*float*) –**Returns** $\min \{y \in \mathbb{N} \mid y \geq x\}$ **Return type**

int

NotesThe set $\mathbb{N} := \{1, 2, \dots\}$.

```
>>> my_int(0.9)
1
>>> my_int(1.1)
2
>>> my_int(1.)
1
>>> my_int(2.)
2
>>> my_int(2.01)
3
>>> my_int(1.999)
2
```

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