The tikz package

This is a general purpose graphics package. To load it for this document, I used:

```
\usepackage{tikz}
\usetikzlibrary{matrix,arrows,decorations.pathmorphing}
```

There are now three ways to enter commutative diagrams using tikz: with the package tikz-cd, with matrix, and directly with tikz (listed roughly in order of decreasing ease but increasing flexibility).

$$\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "d"] & D
\end{tikzcd}$$

```
$\begin{CD}
A@>a>>B\\
@VVbV@VVcV\\
C@>d>>D
\end{CD}$
(amscd)
```

$$\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "d"] & D
\end{tikzcd}$$

```
\begin{tikzcd}
A \arrow{r}{a} \arrow{d}{b}
               &B \arrow{d}{c}\\
C \arrow{r}{d} &D
\end{tikzcd}
(tikz-cd)
```

$$\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "d"] & D
\end{tikzcd}$$

```
\begin{tikzpicture}
\matrix(m)[matrix of math nodes,
row sep=2.6em, column sep=2.8em,
text height=1.5ex, text depth=0.25ex]
{A&B\\
C&D\\};
\path[->,font=\scriptsize,>=angle 90]
(m-1-1) edge node[auto] {$a$} (m-1-2)
        edge node[auto] {$b$} (m-2-1)
(m-1-2) edge node[auto] {$c$} (m-2-2)
(m-2-1) edge node[auto] {$d$} (m-2-2);
\end{tikzpicture}
(matrix)
```

$$\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "d"] & D
\end{tikzcd}$$

```
\begin{tikzpicture}[scale=1.5]
\node (A) at (0,1) {$A$};
\node (B) at (1,1) {$B$};
\node (C) at (0,0) {$C$};
\node (D) at (1,0) {$D$};
\path[->,font=\scriptsize,>=angle 90]
(A) edge node[above]{$a$} (B)
(A) edge node[right]{$b$} (C)
(B) edge node[right]{$c$} (D)
(C) edge node[above]{$d$} (D);
\end{tikzpicture}
(tikz)
```

This is part of: Guide to Commutative Diagrams, www.jmilne.org/not/CDGuide.html
September 6, 2012

Using tikz-cd

Load[1] this with \usepackage{tikz-cd}. As the code on p.1 illustrates, the syntax for tikz-cd is similar to that of array. Note that tikz-cd handles large objects and tall labels better than amscd:

$$\begin{tikzcd}
A\times A\times A\times A\times A\times A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "d"] & D
\end{tikzcd}
\qquad
\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "\overline{A^{A^A}}"] & D
\end{tikzcd}$$

The next example illustrates the use of different arrows in a commutative diagram:

$$\begin{tikzcd}
A \arrow[r, hook] \arrow[rd, two heads] & B \arrow[d, dotted] \arrow[r, hookleftarrow] & C \arrow[ld, two heads] \\
& D &
\end{tikzcd}$$

```
\begin{tikzcd}
A \arrow[hook]{r}\arrow[two heads]{rd}
&B \arrow[dotted]{d}\arrow[hookleftarrow]{r}
&C \arrow[two heads]{ld}\\
&D
\end{tikzcd}
```

Now an example with labels on the arrows:

$$\begin{tikzcd}
A \arrow[r, hook, "u", "b"'] \arrow[rd, two heads, "u", "b"'] & B \arrow[d, dotted, "r", "l"'] \arrow[r, hookleftarrow, "u", "b"'] & C \arrow[ld, two heads, "b", "u"'] \\
& D &
\end{tikzcd}$$

```
\begin{tikzcd}
A \arrow[hook]{r}{u}[swap]{b}
  \arrow[two heads]{rd}{u}[swap]{b}
&B \arrow[dotted]{d}{r}[swap]{l}
   \arrow[hookleftarrow]{r}{u}[swap]{b}
&C \arrow[two heads]{ld}{b}[swap]{u}\\
&D
\end{tikzcd}
```

Long labels may cause problems:

$$\begin{tikzcd}
A \arrow[r] \arrow[d] & B \arrow[r, "\text{very long label}"] \arrow[d] & C \arrow[d] \\
D \arrow[r] & E \arrow[r] & F
\end{tikzcd}$$

```
\begin{tikzcd}
A\arrow{r}\arrow{d}
&B\arrow{r}{\text{very long label}}\arrow{d}
&C\arrow{d}\\
D\arrow{r}&E\arrow{r}&F
\end{tikzcd}
```

However, this can be fixed as follows:

$$\begin{tikzcd}[column sep=large]
A \arrow[r] \arrow[d] & B \arrow[r, "\text{very long label}"] \arrow[d] & C \arrow[d] \\
D \arrow[r] & E \arrow[r] & F
\end{tikzcd}$$

```
\begin{tikzcd}[column sep=large]
A\arrow{r}\arrow{d}
...
```

tikz-cd does not have a problem with objects of different heights.

$$\hat{A} \longrightarrow \prod_{n\in Z} A_n \longrightarrow \prod_{n\in Z} A_n.$$

[1] Before using tikz-cd, check that your TEX installation is using version 2.10 of pgf — you can do this by running TEX on a file containing \pgfversion.

Curving arrows is easy.

$$\begin{tikzcd}
A \arrow[r, bend left] \arrow[r, bend right] & B
\end{tikzcd}$$

```
\begin{tikzcd}
A\arrow[bend left]{r}\arrow[bend right]{r}&B
\end{tikzcd}
```

When a diagram is numbered, this is placed correctly:

$$\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "d"] & D
\end{tikzcd} \tag{1}$$

Two more examples:

$$\begin{tikzcd}
& A \arrow[ldd, "f"'] \arrow[rd, "c" description] \arrow[rrd, "d" description] \arrow[rrrd, "e" description] & & & \\
& B \arrow[ld] \arrow[r] & C \arrow[r] & D \arrow[r] & E \\
F & & & &
\end{tikzcd}$$

```
\begin{tikzcd}
&A\arrow{ldd}[swap]{f}\arrow{rd}[description]{c}
  \arrow{rrd}[description]{d}
  \arrow{rrrd}[description]{e}\\
&B\arrow{ld}\arrow{r}&C\arrow{r}&D\arrow{r}&E\\
F
\end{tikzcd}
```

$$\begin{tikzcd}
T \arrow[drr, bend left, "x"] \arrow[ddr, bend right, "y"'] \arrow[dr, dotted, "{(x,y)}" description] & & \\
& X \times_Z Y \arrow[r, "p"] \arrow[d, "q"] & X \arrow[d, "f"] \\
& Y \arrow[r, "g"] & Z
\end{tikzcd}$$

```
\begin{tikzcd}
T\arrow[bend left]{drr}{x}
\arrow[bend right]{ddr}[swap]{y}
\arrow[dotted]{dr}[description]{(x,y)} & & \\
& X \times_Z Y \arrow{r}{p} \arrow{d}{q} & X \arrow{d}{f} \\
& Y \arrow{r}{g} & Z
\end{tikzcd}
```

Using `matrix`

The code on p.1 sets up a matrix named `m` with some options, and then places *A*, *B*, *C*, and *D* at the four positions of a 2×2 matrix. The next line specifies normal arrows with labels in scriptsize and a nondefault arrow head, and the following line specifies an arrow from the (1,1) position of the matrix `m` to the (1,2) position with a label *a* in the default position.

Note that `tikz` handles large objects and tall labels better than amscd:

$$\begin{tikzcd}
A \times A \times A \times A \times A \times A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\
C \arrow[r, "d"] & D
\end{tikzcd}
\qquad
\begin{tikzcd}
A \arrow[r] \arrow[d] & B \arrow[d] \\
C \arrow[r, "\overline{A^{A^A}}"] & D
\end{tikzcd}$$

To my eyes, the arrow heads are too small.[2] This can be fixed by adding `>=angle 90`, as an option to the path or to the whole picture:

$\longrightarrow$ `\path[->](1,1) edge (2,1);`

$\longrightarrow$ `\path[->,>=angle 90](1,1) edge (2,1);`

Here is the code for some arrows.

$\longrightarrow$ `\path[->](1,1) edge (2,1);`

$\longmapsto$ `\path[|->](1,1) edge (2,1);`

$\overline{\qquad}$ `\path[-](1,1) edge (2,1);`

$\hookrightarrow$ `\path[right hook->](1,1) edge (2,1);`

$\twoheadrightarrow$ `\path[->>](1,1) edge (2,1);`

$\cdots\!\rightarrow$ `\path[dotted,->](1,1) edge (2,1);`

$\dashrightarrow$ `\path[dashed,->](1,1) edge (2,1);`

$\bullet\!\!\longrightarrow$ `\path[*->](1,1) edge (2,1);`

$=\!=$ `\draw[double distance = 1.5pt](1,1) -- (2,1);`

$\rightsquigarrow$ `http://tex.stackexchange.com/questions/12678/`

The next example illustrates the use of the different arrows in a commutative diagram

$$\begin{tikzcd}
A \arrow[r, hook] \arrow[dr, two heads] & B \arrow[d, dotted] & C \arrow[l, hook'] \arrow[dl, two heads] \\
 & D &
\end{tikzcd}$$

```
\begin{tikzpicture}[>=angle 90]
\matrix(a)[matrix of math nodes,
row sep=3em, column sep=2.5em,
text height=1.5ex, text depth=0.25ex]
{A&B&C\\
&D\\};
\path[right hook->](a-1-1) edge (a-1-2);
\path[->>](a-1-1) edge (a-2-2);
\path[dotted,->](a-1-2) edge (a-2-2);
\path[left hook->](a-1-3) edge (a-1-2);
\path[->>](a-1-3) edge (a-2-2);
\end{tikzpicture}
```

[2] See `http://tex.stackexchange.com/questions/37320/` for an erudite discussion of this problem, with solutions.

Now an example with labels on the arrows:

$$\begin{tikzcd}
A \arrow[r, hook, "u", "b"'] \arrow[dr, two heads, "u", "b"'] & B \arrow[d, dotted, "u"', "b"] & C \arrow[l, hook', "u"', "b"] \arrow[dl, two heads, "u"', "b"] \\
& D &
\end{tikzcd}$$

```
\path[right hook->,font=\scriptsize]
(a-1-1) edge node[above]{$u$}
             node[below]{$b$} (a-1-2);
\path[->>,font=\scriptsize]
(a-1-1) edge node[above]{$u$}
             node[below]{$b$} (a-2-2)
(a-1-3) edge node[above left]{$u$}
             node[below right]{$b$} (a-2-2);
\path[dotted,->,font=\scriptsize]
(a-1-2) edge node[left]{$u$}
             node[right]{$b$} (a-2-2);
\path[left hook->,font=\scriptsize]
(a-1-3) edge node[above]{$u$}
             node[below]{$b$} (a-1-2);
```

Long labels may cause a problem:

$$\begin{tikzcd}
A \arrow[r] \arrow[d] & B \arrow[r, "\text{very long label}"] \arrow[d] & C \arrow[d] \\
D \arrow[r] & E \arrow[r] & F
\end{tikzcd}$$

```
\begin{tikzpicture}
\matrix(m)[matrix of math nodes,
row sep=3em, column sep=2.5em,
text height=1.5ex, text depth=0.25ex]
{A&B&C\\
D&E&F\\};
\path[->,font=\scriptsize]
(m-1-1) edge (m-1-2)
        edge (m-2-1)
(m-1-2) edge node[auto] {very long label} (m-1-3)
        edge (m-2-2)
(m-1-3) edge (m-2-3)
(m-2-1) edge (m-2-2)
(m-2-2) edge (m-2-3);
\end{tikzpicture}
```

However, this can be fixed by setting `column sep=5.0em`.

$$\begin{tikzcd}[column sep=5.0em]
A \arrow[r] \arrow[d] & B \arrow[r, "\text{very long label}"] \arrow[d] & C \arrow[d] \\
D \arrow[r] & E \arrow[r] & F
\end{tikzcd}$$

`tikz` does not have a problem with objects of different heights.

$$\begin{tikzcd}
\hat{A} \arrow[r] & \prod_{n\in Z} A_n \arrow[r] & \prod_{n\in Z} A_n.
\end{tikzcd}$$

But that is because of the options `text height=1.5ex, text depth=0.25ex`. When you omit them, you get:

$$\begin{tikzcd}
\hat{A} \arrow[r] & \prod_{n\in Z} A_n \arrow[r] & \prod_{n\in Z} A_n.
\end{tikzcd}$$

Curving arrows is easy.

$$\begin{tikzcd}
A \arrow[r, bend left] \arrow[r, bend left=40] \arrow[r, bend left=60] \arrow[r, bend left=80] \arrow[r, bend right] & B
\end{tikzcd}$$

```
\begin{tikzpicture}
\matrix(m)[matrix of math nodes,
row sep=3em, column sep=2.8em,
text height=1.5ex, text depth=0.25ex]
{A&B\\};
\path[->]
(m-1-1) edge [bend left] (m-1-2)
        edge [bend left=40] (m-1-2)
        edge [bend left=60] (m-1-2)
        edge [bend left=80] (m-1-2)
        edge [bend right] (m-1-2);
\end{tikzpicture}
```

Arrows may not attach themselves correctly to the nodes:

$$\begin{tikzcd}
 & & & H^2(L/K) \\
0 \arrow[r] & \mathrm{Br}(L/K) \arrow[r] & \bigoplus_v \mathrm{Br}(L^v/K_v) \arrow[ur] \arrow[dr] & \\
 & & & \mathbb{Q}/\mathbb{Z}
\end{tikzcd}$$

To fix this, use

```
(m-2-3.north east) edge (m-1-4)
(m-2-3.south east) edge (m-3-4);
```

$$\begin{tikzcd}
 & & & H^2(L/K) \\
0 \arrow[r] & \mathrm{Br}(L/K) \arrow[r] & \bigoplus_v \mathrm{Br}(L^v/K_v) \arrow[ur] \arrow[dr] & \\
 & & & \mathbb{Q}/\mathbb{Z}
\end{tikzcd}$$

Two more examples:

$$\begin{tikzcd}
& A \arrow[ddl, "f"'] \arrow[dr, "c" description] \arrow[drr, "d" description] \arrow[drrr, "e" description] & & & \\
& B \arrow[dl] \arrow[r] & C \arrow[r] & D \arrow[r] & E \\
F & & & &
\end{tikzcd}$$

```
\[
\begin{tikzpicture}[descr/.style={fill=white}]
\matrix(m)[matrix of math nodes, row sep=3em, column sep=2.8em,
text height=1.5ex, text depth=0.25ex]
{&A\\&B&C&D&E\\F\\};
\path[->,font=\scriptsize]
(m-1-2) edge node[above left] {$f$} (m-3-1)
        edge node[descr] {$c$} (m-2-3)
        edge node[descr] {$d$} (m-2-4)
        edge node[descr] {$e$} (m-2-5);
\path[->]
(m-2-2) edge (m-3-1)
        edge (m-2-3);
\path[->]
(m-2-3) edge (m-2-4);
\path[->]
(m-2-4) edge (m-2-5);
\end{tikzpicture}
\]
```

$$\begin{tikzcd}
T \arrow[drr, bend left, "x"] \arrow[dr, dotted, "{(x,y)}" description] \arrow[ddr, bend right, "y"'] & & \\
& X \times_Z Y \arrow[r, "p"'] \arrow[d, "q"] & X \arrow[d, "f"] \\
& Y \arrow[r, "g"] & Z
\end{tikzcd}$$

```
\[
\begin{tikzpicture}[descr/.style={fill=white}]
\matrix(m)[matrix of math nodes, row sep=3em, column sep=2.8em,
text height=1.5ex, text depth=0.25ex]
{T\\&X\times_Z Y&X\\&Y&Z\\};
\path[->,font=\scriptsize]
(m-1-1) edge [bend left=10] node[above] {$x$} (m-2-3)
(m-1-1) edge [bend right=10] node[below] {$y$} (m-3-2);
\path[->,dotted,font=\scriptsize]
(m-1-1) edge node[descr] {$(x,y)$} (m-2-2);
\path[->,font=\scriptsize]
(m-2-2) edge node[below] {$p$} (m-2-3)
(m-2-2) edge node[right] {$q$} (m-3-2);
\path[->,font=\scriptsize]
(m-2-3) edge node[right] {$f$} (m-3-3);
\path[->,font=\scriptsize]
(m-3-2) edge node[above] {$g$} (m-3-3);
\end{tikzpicture}
\]
```

One final example: the extended snake lemma says that the exact commutative diagram in blue gives rise to the exact sequence in red.

$$\begin{tikzcd}
0 \arrow[r] & \operatorname{Ker} f \arrow[r] & \operatorname{Ker} a \arrow[r] \arrow[d] & \operatorname{Ker} b \arrow[r] \arrow[d] & \operatorname{Ker} c \arrow[d] \arrow[lldd, out=0, in=180, "d"'] & & \\
 & & A \arrow[r, "f"] \arrow[d, "a"] & B \arrow[r] \arrow[d, "b"] & C \arrow[r] \arrow[d, "c"] & 0 & \\
 & 0 \arrow[r] & A' \arrow[r] \arrow[d] & B' \arrow[r, "g'"] \arrow[d] & C' \arrow[d] & & \\
 & & \operatorname{Coker} a \arrow[r] & \operatorname{Coker} b \arrow[r] & \operatorname{Coker} c \arrow[r] & \operatorname{Coker} g' \arrow[r] & 0
\end{tikzcd}$$

```
\begin{tikzpicture}[>=angle 90]
\matrix[matrix of math nodes,row sep=3em, column sep=3em,
text height=1.5ex, text depth=0.25ex]
{|[name=00]|0&|[name=kf]|\Ker f&|[name=ka]| \Ker a &|[name=kb]|
\Ker b &|[name=kc]| \Ker c \\
&&|[name=A]| A &|[name=B]| B &|[name=C]| C &|[name=01]| 0 \\
&|[name=02]| 0 &|[name=A’]| A’ &|[name=B’]| B’ &|[name=C’]| C’ \\
&&|[name=ca]| \Coker a &|[name=cb]| \Coker b &|[name=cc]|
\Coker c &|[name=cg]|\Coker g’&|[name=04]|0\\};
\draw[->,font=\scriptsize]
          (ka) edge (A)
          (kb) edge (B)
          (kc) edge (C);
\draw[->,font=\scriptsize,blue,thick]
          (A) edge node[auto] {$f$} (B)
          (B) edge (C)
          (C) edge (01)
          (A) edge node[auto] {$a$} (A')
          (B) edge node[auto] {$b$} (B')
          (C) edge node[auto] {$c$} (C')
          (02) edge (A')
          (A') edge (B')
          (B') edge node[auto] {$g'$} (C');
\draw[->,font=\scriptsize]
          (A') edge (ca)
          (B') edge (cb)
          (C') edge (cc);
\draw[->,red]
          (00) edge (kf)
          (kf) edge (ka)
          (ka) edge (kb)
          (kb) edge (kc)
          (ca) edge (cb)
          (cb) edge (cc)
          (cb) edge (cc)
          (cc) edge (cg)
          (cg) edge (04)
          (kc) edge[out=0,in=180,red] node[above left] {$d$} (ca);
\end{tikzpicture}
```

For the last diagram, I added the following lines to the preamble

```
\usepackage{amsmath}
\DeclareMathOperator{\Coker}{Coker}
\DeclareMathOperator{\Ker}{Ker}
```

Using tikz directly

Instead of using a matrix grid, you can use tikzpicture directly to construct a diagram.

$$\begin{tikzcd} A \arrow[r, bend left, "\alpha"] \arrow[r, bend right, "\beta"'] \arrow[r, phantom, "\Downarrow"] & B \end{tikzcd}$$

```
\begin{tikzpicture}
\node (A) at (-1,0) {$A$};
\node (B) at (1,0) {$B$};
\node at (0,0) {\rotatebox{270}{$\Rightarrow$}};
\path[->,font=\scriptsize,>=angle 90]
 (A) edge [bend left] node[above] {$\alpha$} (B)
     edge [bend right] node[below] {$\beta$} (B);
\end{tikzpicture}
```

(rotatebox requires graphicx.)

$$\begin{tikzcd}
 & X\otimes(Y\otimes(Z\otimes T)) \arrow[dl, "1\otimes\phi"'] \arrow[dr, "\phi"] & \\
X\otimes((Y\otimes Z)\otimes T)) \arrow[d, "\phi"'] & & (X\otimes Y)\otimes(Z\otimes T) \arrow[d, "\phi"] \\
(X\otimes(Y\otimes Z))\otimes T \arrow[rr, "\phi\otimes 1"] & & ((X\otimes Y)\otimes Z)\otimes T
\end{tikzcd}$$

Pentagon Axiom

```
\begin{tikzpicture}
\node (P0) at (90:2.8cm) {$X\otimes (Y\otimes (Z\otimes T))$};
\node (P1) at (90+72:2.5cm) {$X\otimes ((Y\otimes Z)\otimes T))$} ;
\node (P2) at (90+2*72:2.5cm) {$\mathllap{(X\otimes (Y\otimes Z))}\otimes T$};
\node (P3) at (90+3*72:2.5cm) {$((X\otimes Y)\mathrlap{\otimes Z)\otimes T}$};
\node (P4) at (90+4*72:2.5cm) {$(X\otimes Y)\otimes (Z\otimes T)$};
\draw
(P0) edge[->,>=angle 90] node[left] {$1\otimes\phi$} (P1)
(P1) edge[->,>=angle 90] node[left] {$\phi$} (P2)
(P2) edge[->,>=angle 90] node[above] {$\phi\otimes 1$} (P3)
(P4) edge[->,>=angle 90] node[right] {$\phi$} (P3)
(P0) edge[->,>=angle 90] node[right] {$\phi$} (P4);
\end{tikzpicture}
```

Here I used \mathllap and \mathrlap to adjust the positions of the nodes. They require the package mathtools.

When you number a displayed commutative diagram

```
\begin{equation}
\begin{tikzpicture}
......
```

```
\end{tikzpicture}
\end{equation}
```

$$\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "c"] & B \arrow[r, "b"] \arrow[d, "d"] & C \arrow[d, "e"] \\
D \arrow[r, "f"] & E \arrow[r, "g"] & F
\end{tikzcd} \tag{2}$$

the number appears below the level of the diagram. To centre the number, use:

```
\begin{equation}
\begin{tikzpicture}[baseline=(current bounding box.center)]
......
\end{tikzpicture}
\end{equation}
```

$$\begin{tikzcd}
A \arrow[r, "a"] \arrow[d, "c"] & B \arrow[r, "b"] \arrow[d, "d"] & C \arrow[d, "e"] \\
D \arrow[r, "f"] & E \arrow[r, "g"] & F
\end{tikzcd} \tag{3}$$

Here's an example with multiple arrows:

$$\begin{tikzcd}
Y \times_X Y \arrow[r, shift left, "p_1"] \arrow[r, shift right, "p_2"'] & Y \arrow[r] & X
\end{tikzcd}$$

```
\begin{tikzpicture}
\node (a) at (0,0) {$Y\times_X Y$};
\node (b) at (2,0) {$Y$};
\node (c) at (3.5,0) {$X$};
\path[->,font=\scriptsize,>=angle 90]
([yshift= 2pt]a.east) edge node[above] {$p_1$} ([yshift= 2pt]b.west)
([yshift= -2pt]a.east) edge node[below] {$p_2$} ([yshift= -2pt]b.west)
(b) edge (c);
\end{tikzpicture}
```