$$C_e^{\mathrm{I}}\frac{\partial T_e^{\mathrm{I}}}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_e^{\mathrm{I}}\frac{\partial T_e^{\mathrm{I}}}{\partial x}\right) - G^{\mathrm{I}}\left(T_e^{\mathrm{I}} - T_l^{\mathrm{I}}\right) + S, \tag{1a}$$

$$C_l^{\mathrm{I}}\frac{\partial T_l^{\mathrm{I}}}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_l^{\mathrm{I}}\frac{\partial T_l^{\mathrm{I}}}{\partial x}\right) + G^{\mathrm{I}}\left(T_e^{\mathrm{I}} - T_l^{\mathrm{I}}\right), \tag{1b}$$

$$C_e^{\mathrm{II}}\frac{\partial T_e^{\mathrm{II}}}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_e^{\mathrm{II}}\frac{\partial T_e^{\mathrm{II}}}{\partial x}\right) - G^{\mathrm{II}}\left(T_e^{\mathrm{II}} - T_l^{\mathrm{II}}\right), \tag{1c}$$

$$C_l^{\mathrm{II}}\frac{\partial T_l^{\mathrm{II}}}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_l^{\mathrm{II}}\frac{\partial T_l^{\mathrm{II}}}{\partial x}\right) + G^{\mathrm{II}}\left(T_e^{\mathrm{II}} - T_l^{\mathrm{II}}\right). \tag{1d}$$

$$S = \sqrt{\frac{\beta}{\pi}}\frac{(1-R)I_0}{t_p\delta}\exp\left[-\frac{x}{\delta} - \beta\left(\frac{t-2t_p}{t_p}\right)^2\right]. \tag{2}$$

$$T_e^{\mathrm{I}}(x,0) = T_l^{\mathrm{I}}(x,0) = T_0, \tag{3a}$$

$$T_e^{\mathrm{II}}(x,0) = T_l^{\mathrm{II}}(x,0) = T_0, \tag{3b}$$

$$\frac{\partial T_e^{\mathrm{I}}(x=0,t)}{\partial x} = \frac{\partial T_e^{\mathrm{II}}(x=L,t)}{\partial x} = 0, \tag{4a}$$

$$\frac{\partial T_l^{\mathrm{I}}(x=0,t)}{\partial x} = \frac{\partial T_l^{\mathrm{II}}(x=L,t)}{\partial x} = 0. \tag{4b}$$

Further, at the interface, i.e., at $x = L_1$, the two layers are in perfect thermal contact, i.e.,

$$T_e^{\mathrm{I}}(x,t) = T_e^{\mathrm{II}}(x,t), \tag{5a}$$

$$T_l^{I}(x,t) = T_l^{\mathrm{II}}(x,t), \tag{5b}$$

$$\kappa_e^{\mathrm{I}}\frac{\partial T_e^{\mathrm{I}}(x,t)}{\partial x} = \kappa_e^{\mathrm{II}}\frac{\partial T_e^{\mathrm{II}}(x,t)}{\partial x}, \tag{5c}$$

$$\kappa_l^{\mathrm{I}}\frac{\partial T_l^{\mathrm{I}}(x,t)}{\partial x} = \kappa_l^{\mathrm{II}}\frac{\partial T_l^{\mathrm{II}}(x,t)}{\partial x}. \tag{5d}$$