

통계분석

Statistical Analysis

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Assignment 02. Bayes Theorem

$$P(A \cap B) = P(B)P(A|B) = P(A)P(B|A)$$

Q1. Drunken Drivers (50pts)

A group of police officers have breathalyzers displaying false drunkenness in 5% of the cases in which the driver is sober. However, the breathalyzers never fail to detect a truly drunk person. One in a thousand drivers is driving drunk. Suppose the police officers then stop a driver at random, and force the driver to take a breathalyzer test. It indicates that the driver is drunk. We assume you don't know anything else about him or her. How high is the probability he or she really is drunk?

A1 : 약 0.2%

Breathalyzer(B+, B-), Drunken(D+, D-)

- $P(D+)=0.001$, $P(D-)=0.999$
- $P(B+ | D-) = 0.05$, $P(B+ | D+)=1$

Want to calculate : $P(D+ | B+)$

	B+	B-	total
D+	0.001	0	0.001
D-	0.04995	0.94905	0.999
total	0.05095	0.94905	1

풀이과정

$$\begin{aligned}P(B+) &= P(B+ | D-)P(D-) + P(B+ | D+)P(D+) \\&= 0.05 \times 0.999 + 1 \times 0.001 = 0.05095\end{aligned}$$

$$\begin{aligned}P(D+ | B+) &= P(B+ | D+)P(D+) / P(B+) \\&= (1 \times 0.001) / 0.05095 = 0.00196271 \\&= 0.196271\%, \text{ 약 } \mathbf{0.2\%}\end{aligned}$$

Q2. Terrorist Identification (50pts)

In a city of 1 million inhabitants let there be 100 terrorists and 999,900 non-terrorists. To simplify the example, it is assumed that all people present in the city are inhabitants. Thus, the base rate probability of a randomly selected inhabitant of the city being a terrorist is 0.0001, and the base rate probability of that same inhabitant being a non-terrorist is 0.9999. In an attempt to catch the terrorists, the city installs an alarm system with a surveillance camera and automatic facial recognition software.

The software has two failure rates of 1%:

- The false negative rate: If the camera scans a terrorist, a bell will ring 99% of the time, and it will fail to ring 1% of the time.
- The false positive rate: If the camera scans a non-terrorist, a bell will not ring 99% of the time, but it will ring 1% of the time.

Suppose now that an inhabitant triggers the alarm. What is the chance that the person is a terrorist? In other words, what is $P(T|B)$, the probability that a terrorist has been detected given the ringing of the bell?

A2 : 약 1%

Bell(B+, B-), Terrorist(T+, T-)

- $P(T+)=0.0001$, $P(T-)=0.9999$
- $P(B- | T+) = 0.01$, $P(B+ | T-)=0.01$
- $P(B+ | T+) = 0.99$, $P(B- | T-)=0.99$

Want to calculate : $P(T+ | B+)$

	B+	B-	total
T+	0.000099	0.000001	0.0001
T-	0.009999	0.989901	0.9999
total	0.010098	0.989902	1

풀이과정

$$\begin{aligned}P(B+) &= P(B+ | T-)P(T-) + P(B+ | T+)P(T+) \\&= 0.01*0.9999 + 0.99*0.0001 = \\&0.010098\end{aligned}$$

$$\begin{aligned}P(T+ | B+) &= P(B+ | T+)P(T+) / P(B+) \\&= (0.99*0.0001) / 0.010098 = 0.00980392 \\&= 0.980392\%, \text{ 약 } 1\%\end{aligned}$$