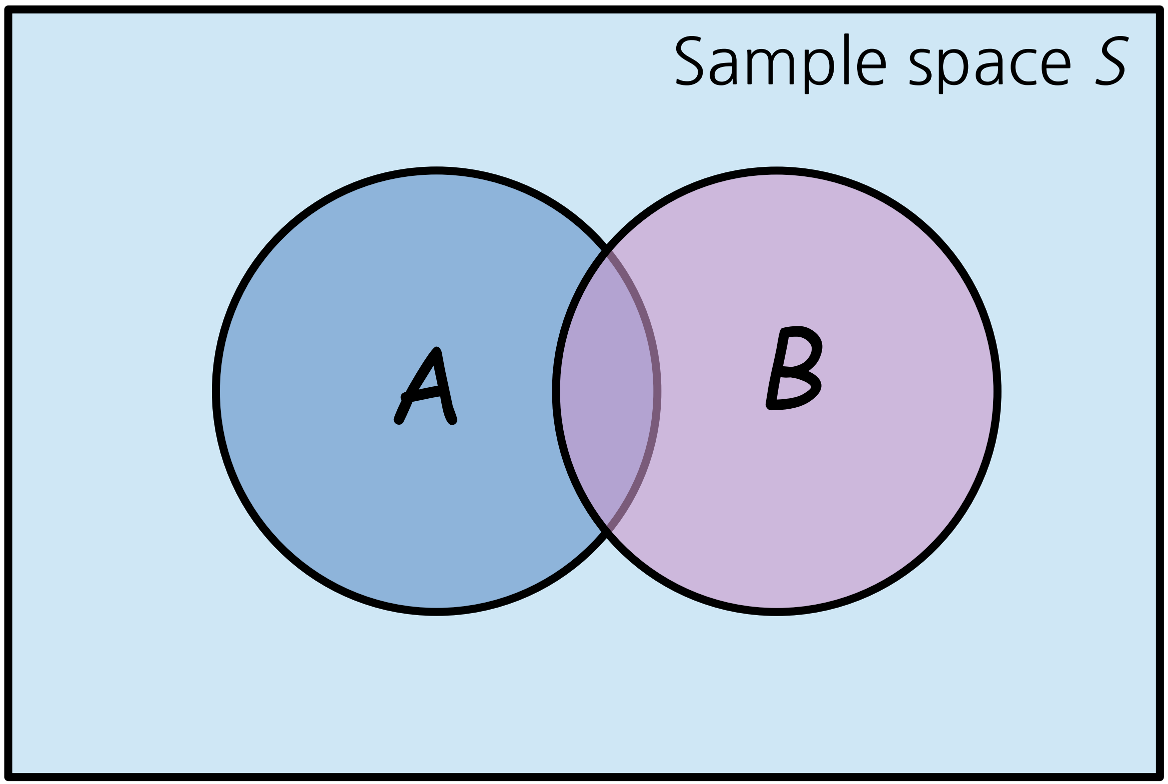
통계분석
Statistical Analysis

Probability



Relation between A and B?

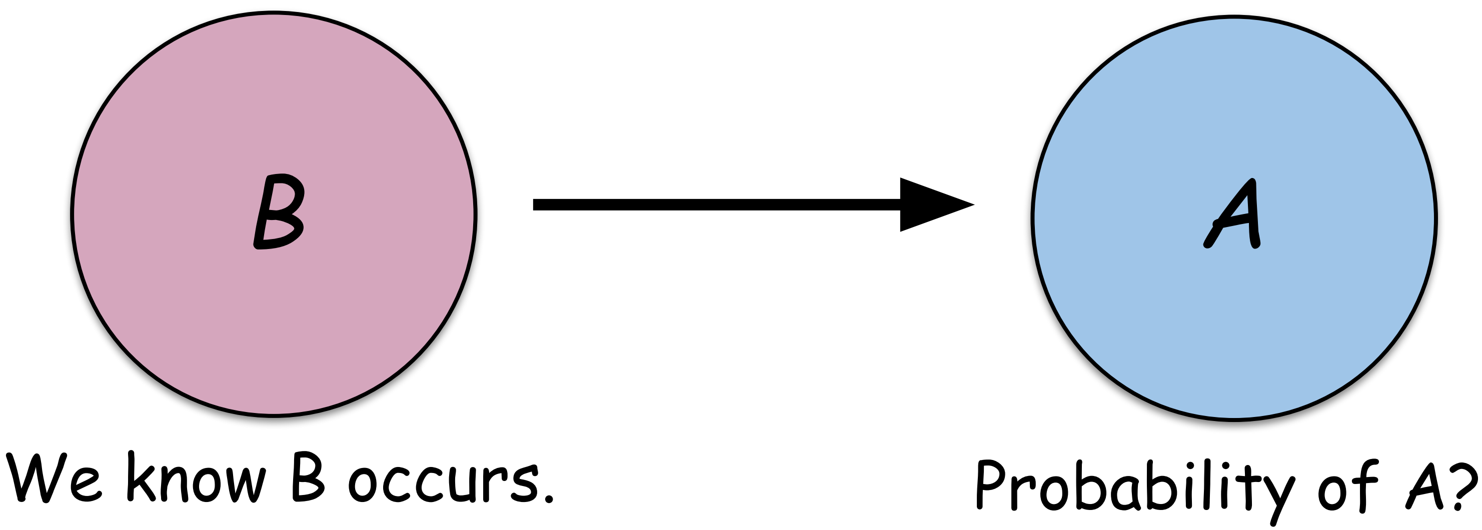
Probability

1. Conditional Probability

2. Independence

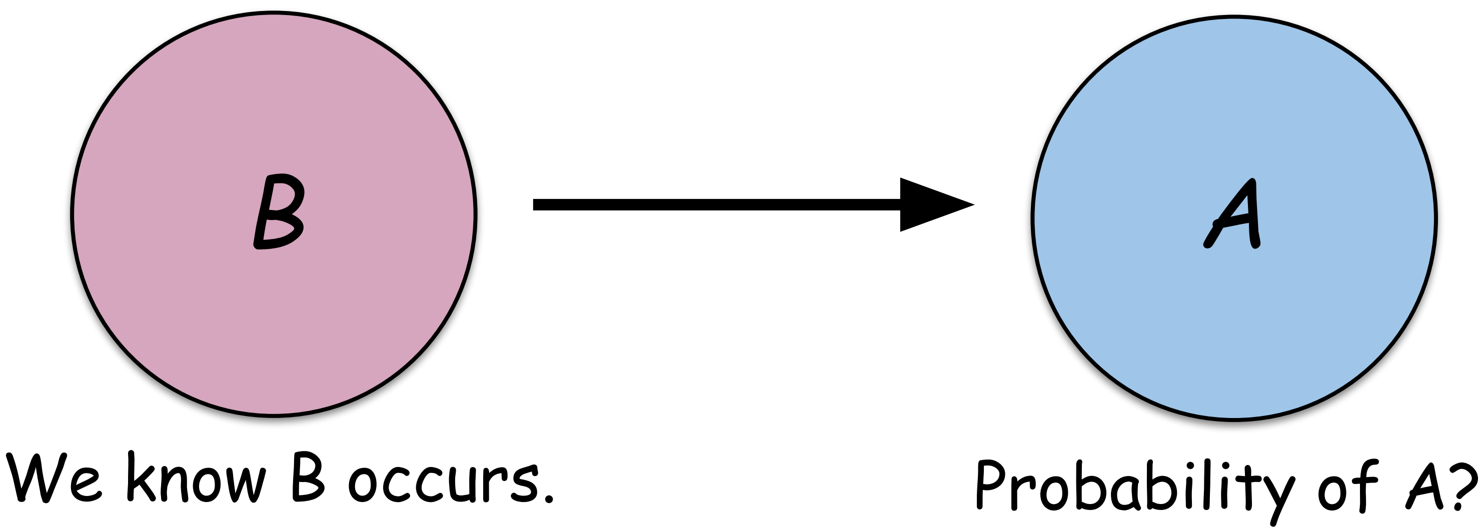
Conditional Probability
(조건부확률)

Question: What is the probability that the event A occurs provided the event B has occurred *(condition)*?



Conditional Probability

$$P(A|B) = P(A \cap B)/P(B) \quad [P(B) \neq 0]$$



Conditional Probability

$$P(A|B) = P(A \cap B)/P(B) \quad [P(B) \neq 0]$$

Be careful!

$$P(A|B) \neq P(A \cap B)$$

Conditional Probability

$$P(A|B) = P(A \cap B)/P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{N(A \cap B)/N}{N(B)/N} = \frac{N(A \cap B)}{N(B)}$$

$$P(A \cap B) = \frac{N(A \cap B)}{N}$$

N : Total number of trials

$N(A)$: The number of outcomes A among N experiments

$N(B)$: The number of outcomes B among N experiments

$\boxed{\text{The sample space changes from S to B.}}$

Conditional Probability

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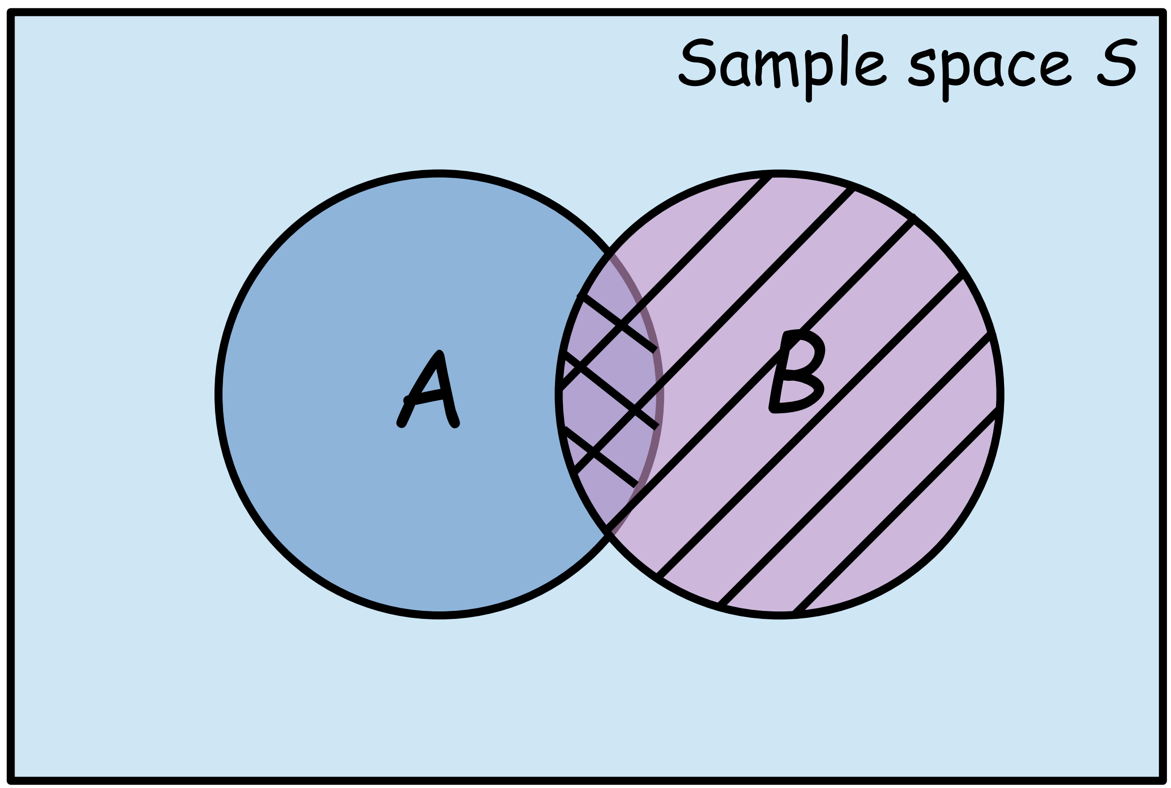

Conditional Probability: Examples

Tossing three times

A: at least two heads

B: first trial is a tail

<u>coins are fair : P(H) = P(T) = 0.5</u>

cf. unfair coin : P(H) ≠P(T)

HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

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HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

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Tossing three times

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HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

A <u>HHH</u> | <u>HH</u>T | <u>H</u>T<u>H</u> | T<u>HH</u> | HTT | THT | TTH | TTT

B HHH | HHT | HTH | <u>T</u>HH | HTT | <u>T</u>HT | <u>T</u>TH | <u>T</u>TT

A and B

Conditional Probability: Examples

Tossing three times

A: at least two heads

B: first trial is a tail

$$P(A) = 0.5$$
$$P(B) = 0.5$$
$$P(A \cap B) = 0.125$$
$$P(A|B) = 0.25 \neq P(A) = 0.5$$

$$P(A|B) \neq P(A \cap B)$$

Multiplication Rule

$$P(A|B) = P(A \cap B)/P(B)$$

$$\rightarrow P(A \cap B) = P(B)P(A|B)$$

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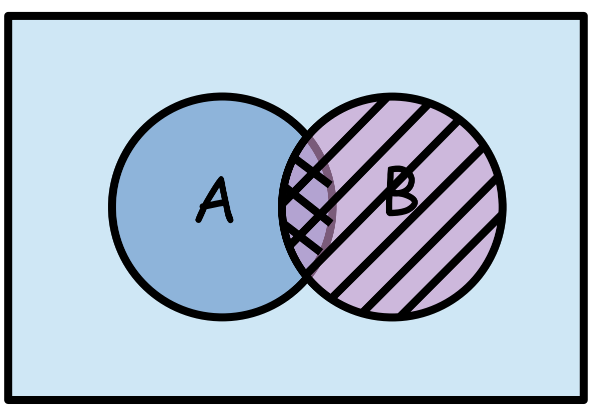
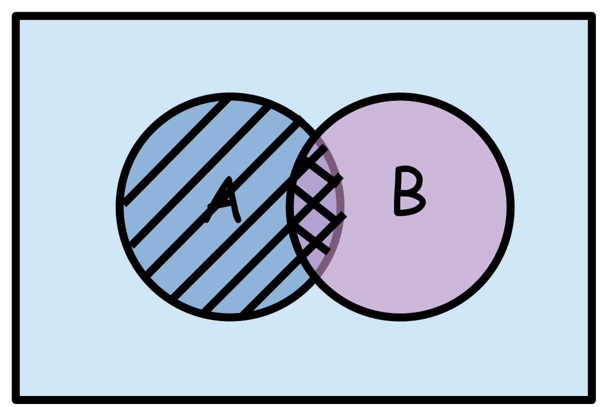


$P(A|B)$



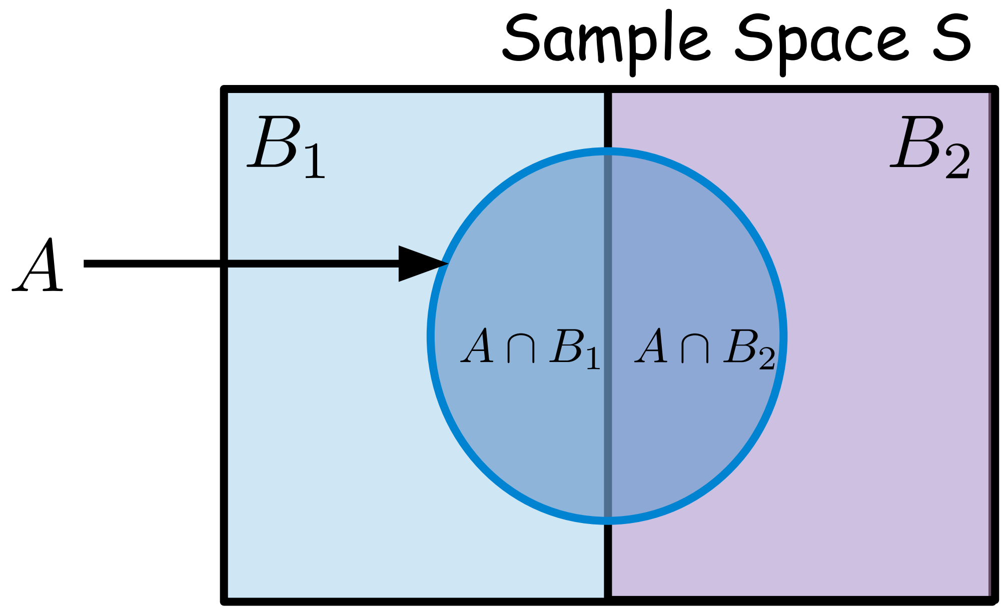
$P(B|A)$

Law of Total Probability

- Mutually exclusive (or disjoint) events

$$B_1 \cup B_2 = S \qquad B_1 \cap B_2 = \phi$$

$$\begin{aligned} \rightarrow P(A) &= P(A \cap B_1) + P(A \cap B_2) \\ &= P(A|B_1)P(B_1) + P(A|B_2)P(B_2) \end{aligned}$$



Independence (독립사건)

- Question: When can we say that events A and B are
  independent from a probability viewpoint?

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  independent from a probability viewpoint?
- If A and B are independent, the probability that A occurs is **NOT affected** by whether B takes place or not.
- Regardless of B, the probability of A remains the same.

Independence

- A and B are independent.

$$P(A) = P(A|B)$$

Independence (독립사건)

- Question: When can we say that events A and B are
  <u>independent</u> from a probability viewpoint?

- If A and B are independent, <u>the probability that A occurs</u> is <u>**NOT affected**</u> by <u>whether B takes place or not</u>.

- Regardless of B, the probability of A remains the same.

- A and B are independent.

$$P(A) = P(A|B) = P(A|B^c)$$

Independence

- A and B are independent.

$$P(A) = P(A|B) \leftrightarrow P(A \cap B) = P(A)P(B)$$

A is independent of B
Probability of A is not changed whether B occurs or not.

$$\leftrightarrow P(B) = P(B|A)$$

B is independent of A
Probability of B is not affected by the fact that A happens.

Conditional Probability: Examples

Tossing one fair coin three times

A: at least two heads

B: first trial is a tail

A and B are NOT independent.

HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

A HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

B HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

A and B

$$P(A|B) = 0.25 \neq P(A) = 0.5$$

Conditional Probability: Examples

Tossing one fair coin three times

A: at least two heads

B: ?????????

HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

A HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

B HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

Which B satisfying $P(A) = P(A|B)$??

Which B is independent of A?

Conditional Probability: Examples

Tossing three times

A: at least two heads

B: ?????????

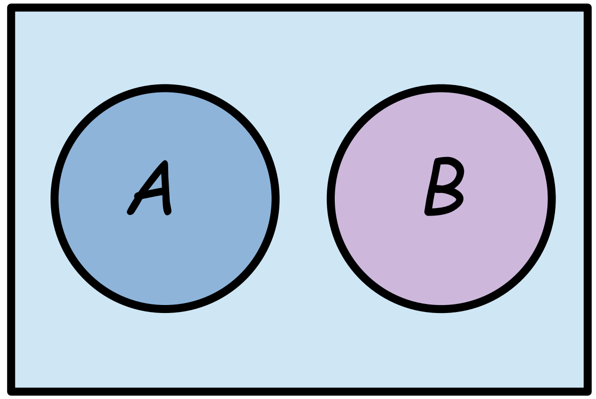
HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

A HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

B HHH | HHT | HTH | THH | HTT | THT | TTH | TTT

$$P(A) = P(A|B) = 0.5$$

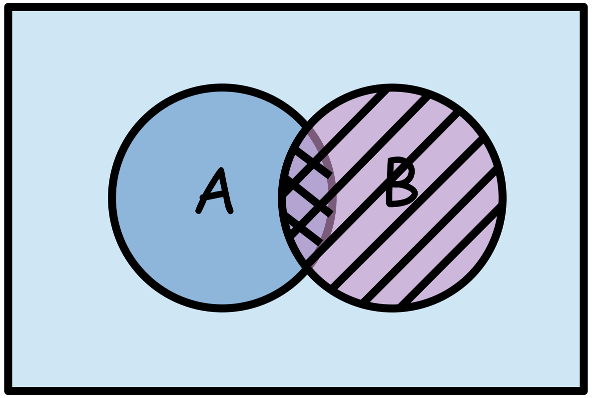
Don't be confused!



- It does not mean that A and B are independent.

$$P(A \cap B) = 0 \neq P(A)P(B)$$

Independence and Intersection



$P(A|B)$

We need a finite intersection of A and B for independence of A and B.