Q.

$A(2,1)$, $B(-1,1)$, $C(5,5)$, $D(x,y)$ → 좌표 연산.

$\vec{AB} = (-3,0)$, $\vec{AC} = (3,4)$, $\vec{AD} = (x-2, y-1)$

$|\vec{AB}| = 3$, $|\vec{AC}| = 5$, $|\vec{AD}| = 5$

$$\cos\theta = \frac{\vec{AB}\cdot\vec{AD}}{|\vec{AB}||\vec{AD}|} = \frac{\vec{AC}\cdot\vec{AD}}{|\vec{AC}||\vec{AD}|}$$

$$\frac{-3x+6}{3} = \frac{3x-6+4y-4}{5}$$

$$-x+2 = \frac{3dx+4dy-10}{5}$$

$$-5x+10 = 3dx+4y-10$$

$8x+4y-20=0$, $2x+y-5=0$ $\therefore y=-2x+5$

$$|\vec{AD}|^2 = 5^2 = (x-2)^2+(y-1)^2$$
$$= (x-2)^2+(-2x+4)^2$$

$$x^2-4x+4+4x^2-16x+16=25$$

$$5x^2-20x-5=0$$

$$x^2-4x-1=0$$

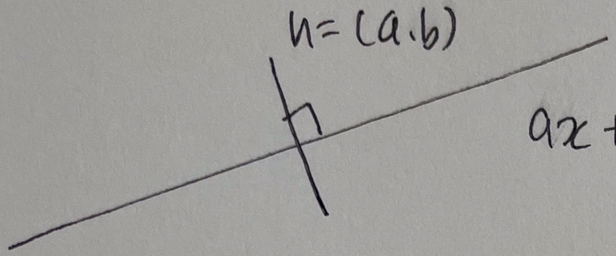
$$\frac{4\pm\sqrt{16+4}}{2} = 2\pm\sqrt{5}$$

but $x<2$ $\therefore x=2-\sqrt{5}$

$\therefore D=(x,y)$

$D=(2-\sqrt{5}, 1+2\sqrt{5})$

Q2



$ax+by+c=0$

수직임을 증명하라.

$ax+by+c=0$ 과 평행한 직선 中

$O(0.0)$을 지나는 직선

$$y = -\frac{a}{b}x$$

직선의 방향 Vector $= (b,-a)$

내적 값이 0이면, 두 벡터가 수직.

$$(a,b)\cdot(b-a) = ab-ab = 0$$

∴ 벡터와 직선은 수직임이 증명.

Q3

직선1 ⇒ $x-y+1=0$

직선2 ⇒ $2x+y-1=0$

직선1의 법선벡터 $V_1=(1,-1)$

직선2의 법선벡터 $V_2=(2,1)$

$$\cos\theta = \frac{V_1 \circ V_2}{|V_1||V_2|}$$

$$= \frac{1}{\sqrt{2}\sqrt{5}}$$

$$\cos\theta = \frac{1}{\sqrt{10}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$$

$$= 71.56505118$$

∴ 사잇각 $\theta = 71.6°$

Q4

$Q(1,1,0)$, $n(1,0,-2)$, $P(0,0,0)$

Q를 지나고 n에 수직인 평면과 P 사이 최단거리 ?

최단거리 d = projection PQ in n

$PQ = (1,1,0)$

$|\vec{PQ}| = 1$, $|n| = \sqrt{5}$

$$d = (PQ \cdot n) \times \frac{n}{|n|^2}$$

$$= 1 \times \frac{(1,0,-2)}{5}$$

$$= \frac{1}{5}(1,0,-2)$$

$$= \frac{1}{5} \times \sqrt{5} = \frac{1}{\sqrt{5}}$$

$\therefore$ 최단거리 $d = \frac{1}{\sqrt{5}}$

Q5

1) $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

오른손의 법칙을 이용. (+)

$\overrightarrow{AB} \times \overrightarrow{AC}$ 의 값이 음수라면, 시계방향

양수라면, 반시계방향

2) $A(5,1)$, $B(0,6)$, $C(0,0)$

$\overrightarrow{AB} = (-5, 5)$, $\overrightarrow{AC} = (-5, -1)$

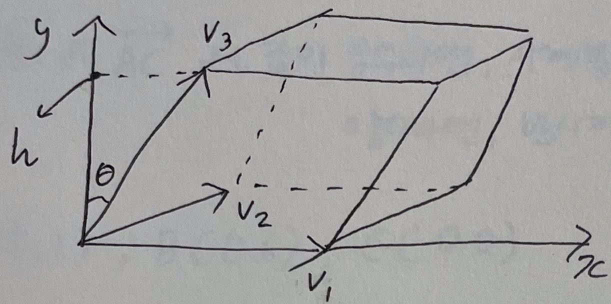
$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} i & j & k \\ -5 & 5 & 0 \\ -5 & -1 & 0 \end{pmatrix} = 5 - (-25) = 30$$

위와 동일하게 +, 즉 반시계방향의 순서.

Q6

$V_1 = (5, 1, -1)$, $V_2 = (0.4.1)$, $V_3 = (0.-1, 5)$

Volume $= |V_1 \times V_2| \times$ 높이 h



h = V_3을 $V_1 \times V_2$에 projection.

$h = V_3 \cos\theta$

$|V_1 \times V_2| \cdot |V_3| \cos\theta = (V_1 \times V_2) \cdot V_3$

$$V_1 \times V_2 = \begin{pmatrix} i & j & k \\ 5 & 1 & -1 \\ 0 & 4 & 1 \end{pmatrix}$$

$= (1+4), -(5), 20$

$= 5, -5, 20$

$(V_1 \times V_2) \cdot V_3 = (5, -5, 20) \cdot (0, -1, 5)$

$= 5 \times 0 + 5 \times 1 + 20 \times 5$

$= 0 + 5 + 100 = 105$

$\therefore$ Volume $= 105$.