통계분석
Statistical Analysis

Testing Hypotheses :
Continuous Random Variables

Testing Hypothesis : Normal Distribution

- Example: IQs of People

  IQ is normally distributed with standard deviation 15.

- Hypotheses

  $H_0 = $ The average IQ of People is 120. $\mu = 120$

  $H_a = $ The average IQ of People is greater than 120. $\mu > 120$
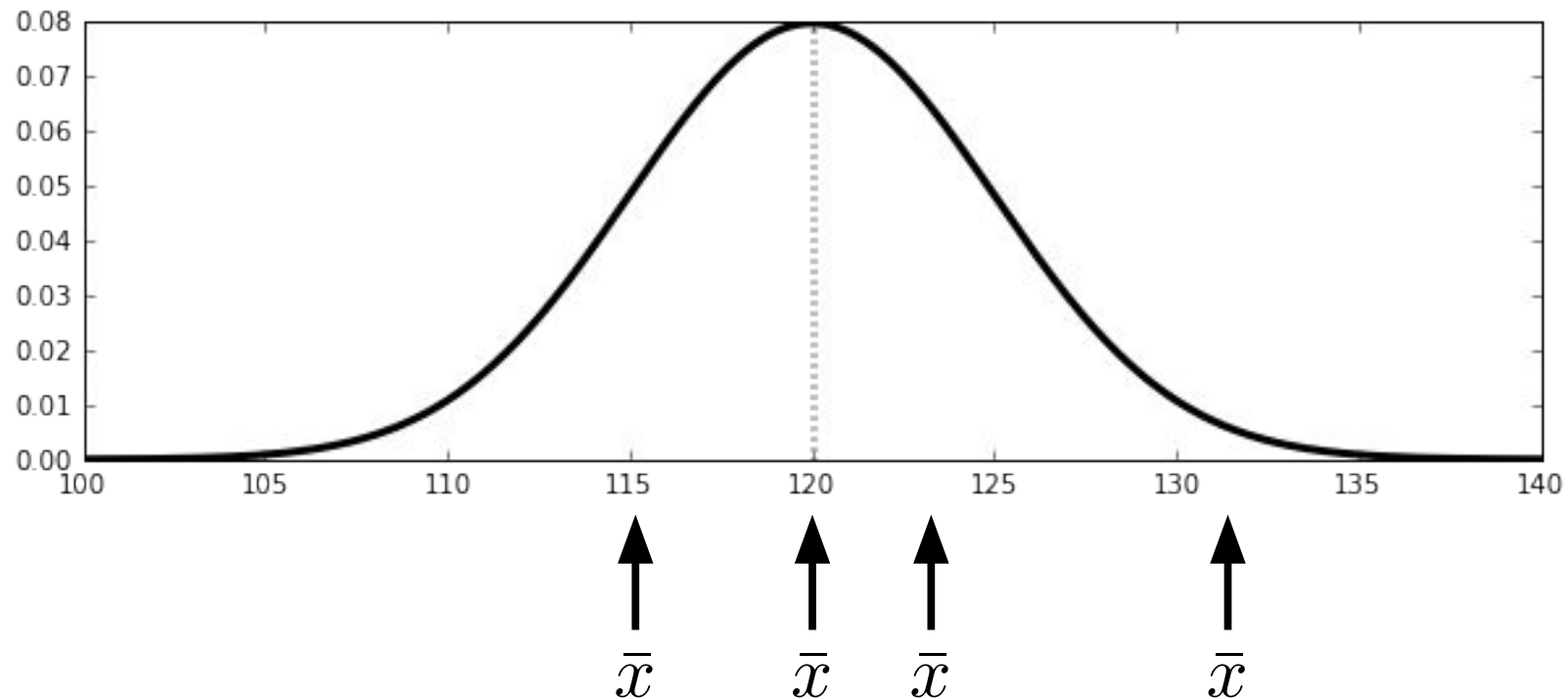
- Experiments and Test statistics

  IQ test for 9 people

  A test statistic $=$ Sample mean $= \bar{x}$

  $$\bar{x} \sim N(\mu, \sigma^2/n = 15^2/9)$$
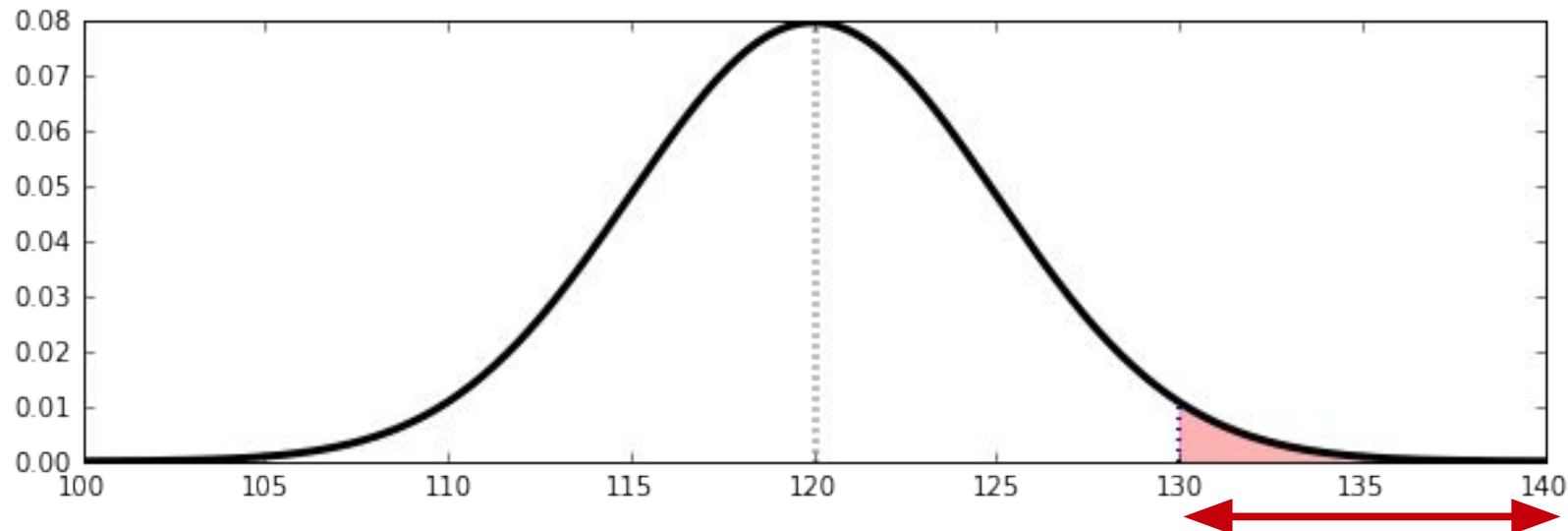
Hypothesis Testing with Normal Distribution

$$\bar{x} \sim N(\mu, \sigma^2/n = 15^2/9)$$

$\bar{x}$ can have any value in principle,

but each of $\bar{x}$ has a different probability.

Setting up Rejection Region

$$\bar{x} \sim N(\mu, \sigma^2/n = 15^2/9)$$

Rejection region: $\bar{x} \geq 130$

- It means that if the sample mean of IQ of 9 people is greater 130, then we conclude that the null hypothesis is false.

- Since the alternative hypothesis is that the population average is larger than 120, the rejection region is upper-sided, not two-sided.

Setting up Rejection Region

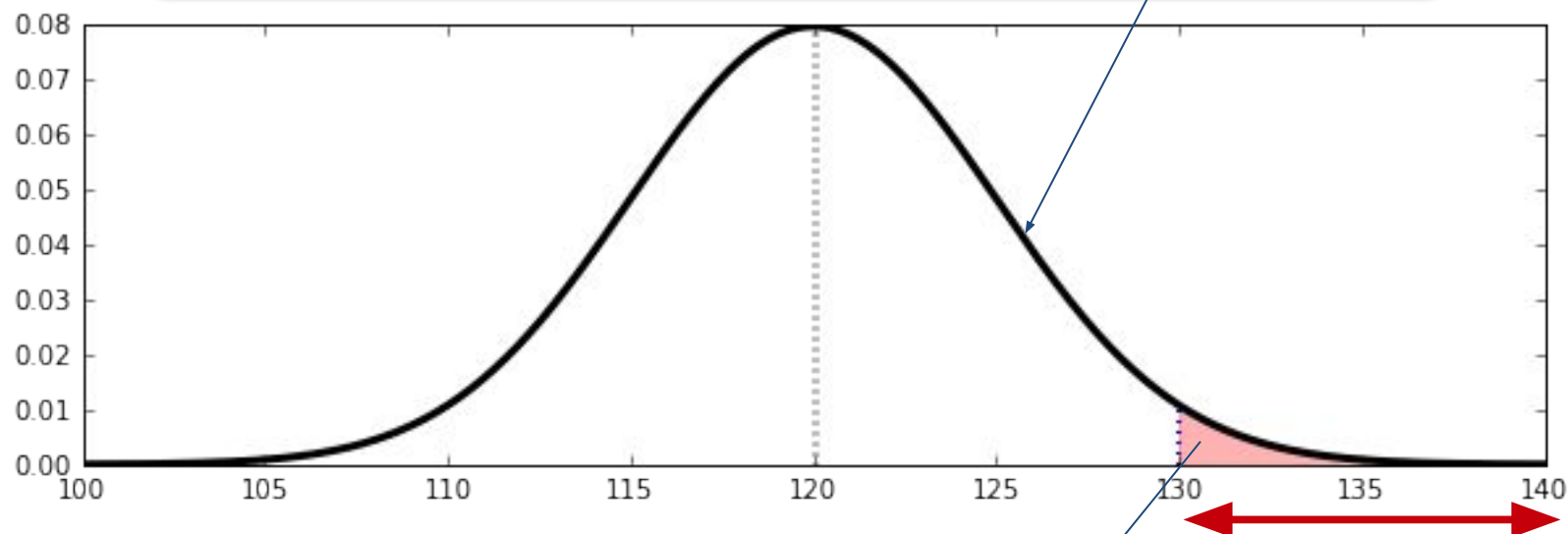$H_a =$ The average IQ of People is greater than 120. $\mu > 120$

Rejection Region is upper-sided

$\mu < 120$ ⟶ Rejection Region is lower-sided

$\mu \neq 120$ ⟶ Rejection Region is Two-sided (both-sided)

Type I Error

Type I Error $= P(\text{Reject } H_0 | H_0 \text{ is true})$
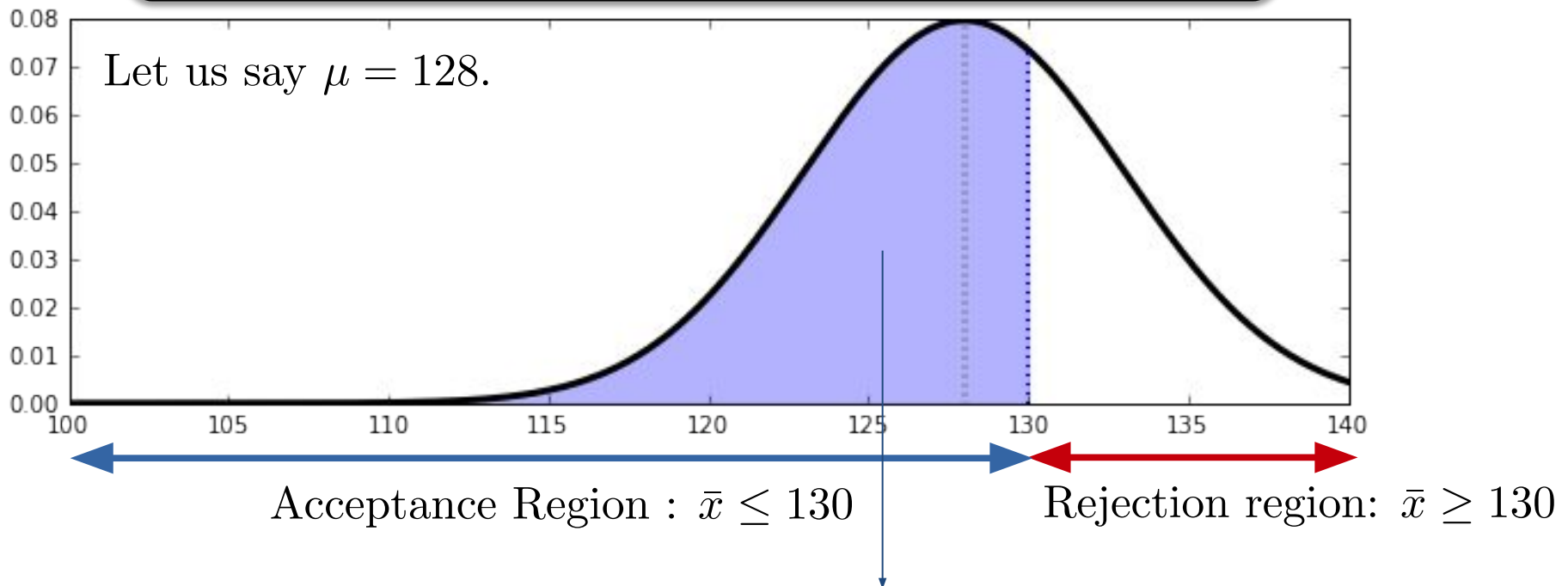

Rejection region: $\bar{x} \geq 130$

Even if the null hypothesis is true, it is possible that the sample mean is greater than 130.

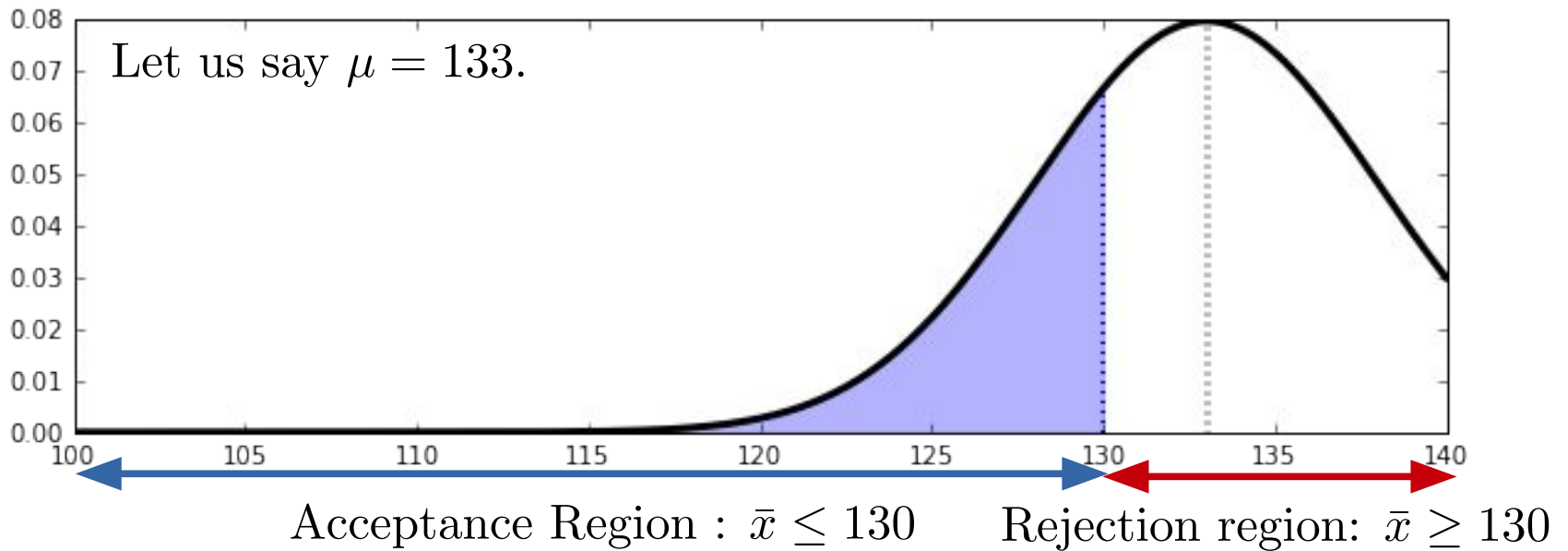$$\alpha = P(\bar{x} \geq 130 | \mu = 120) \approx 0.0228$$

Type II Error

Type II Error $= P(\text{Not reject } H_0 | H_0 \text{ is false})$

Let us say $\mu = 128$.

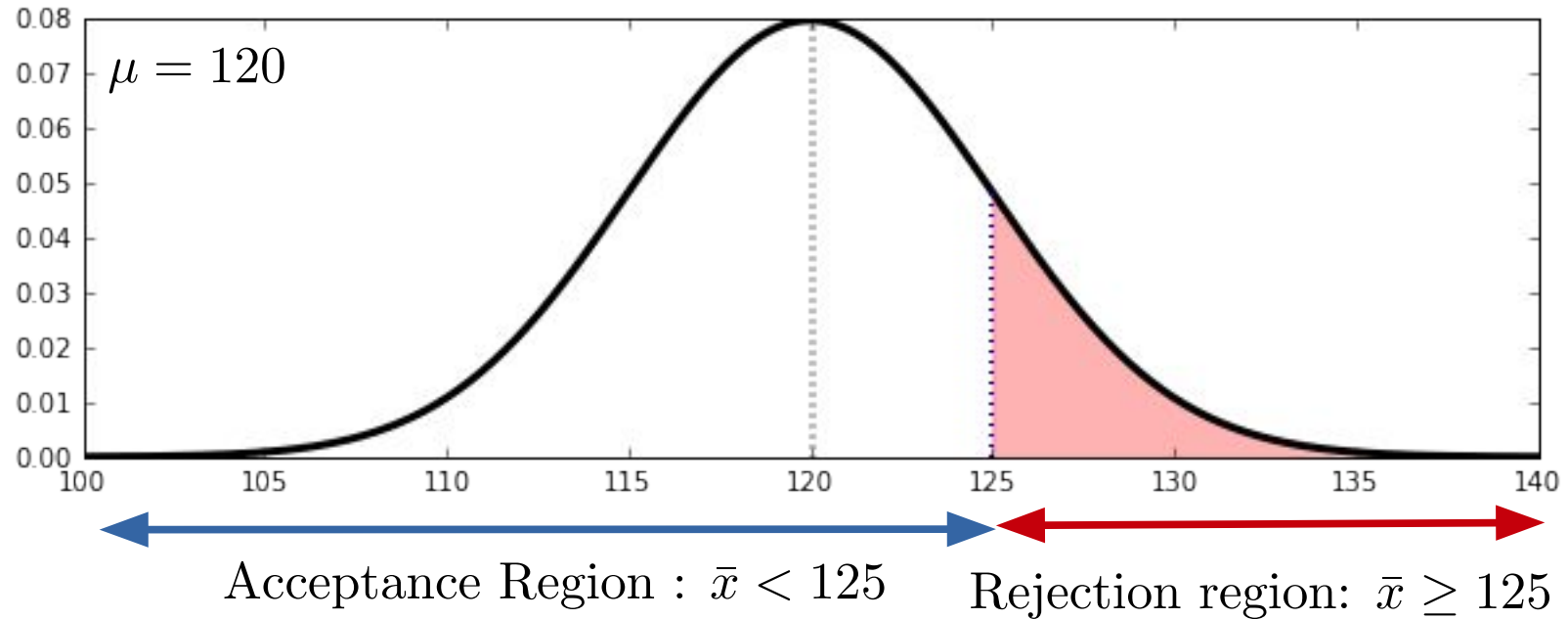$$\beta(128) = P(\bar{x} \leq 130 | \mu = 128) \approx 0.6554$$

Acceptance Region : $\bar{x} \leq 130$

Rejection region: $\bar{x} \geq 130$

Type II Error

Type II Error $= P(\text{Not reject } H_0 | H_0 \text{ is false})$

Let us say $\mu = 133$.

Acceptance Region : $\bar{x} \leq 130$ Rejection region: $\bar{x} \geq 130$

$$\beta(133) = P(\bar{x} \leq 130 | \mu = 133) \approx 0.2743$$

Changing Rejection Region

$\mu = 120$

Acceptance Region : $\bar{x} < 125$ Rejection region: $\bar{x} \geq 125$

Type I Error

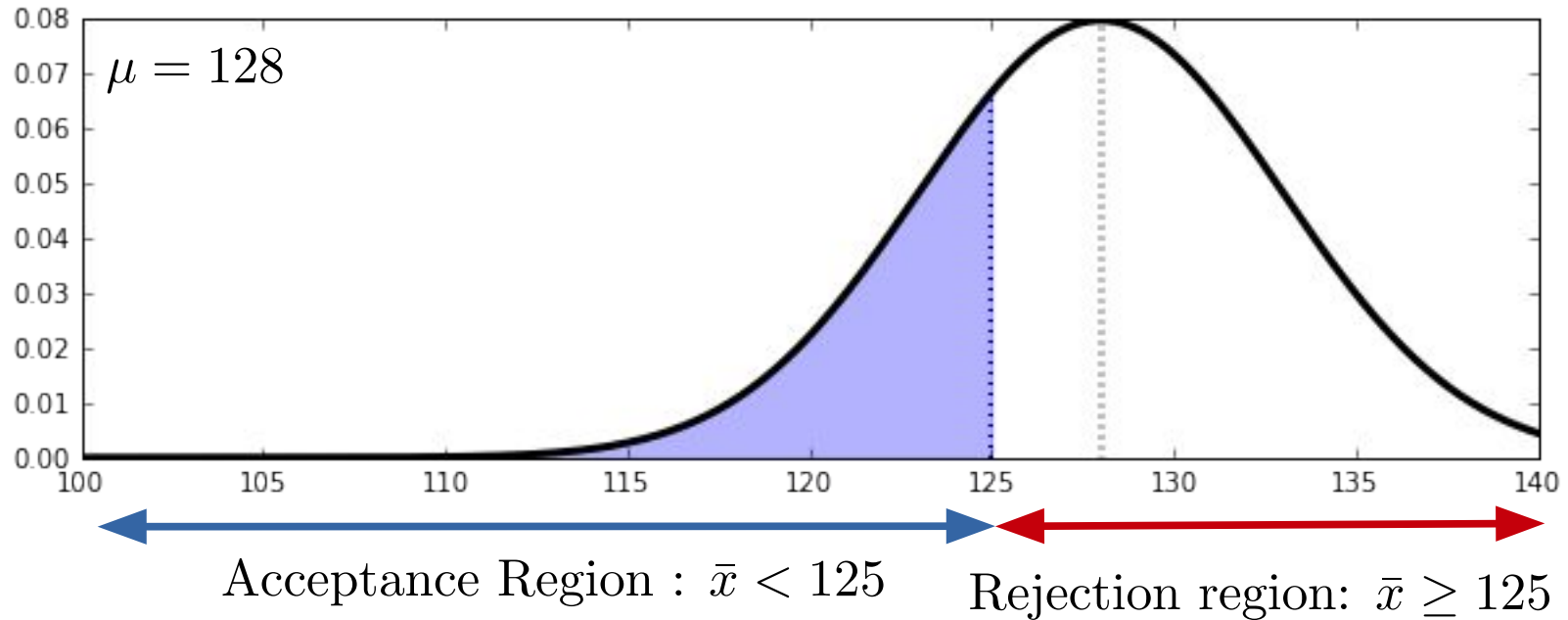Acceptance Region : $\bar{x} < 125$ Rejection region: $\bar{x} \geq 125$

Type I Error $= P(\text{Reject } H_0 | H_0 \text{ is true})$

$$\alpha = P(\bar{x} \geq 125 | \mu = 120) \approx 0.1587$$

Type II Error

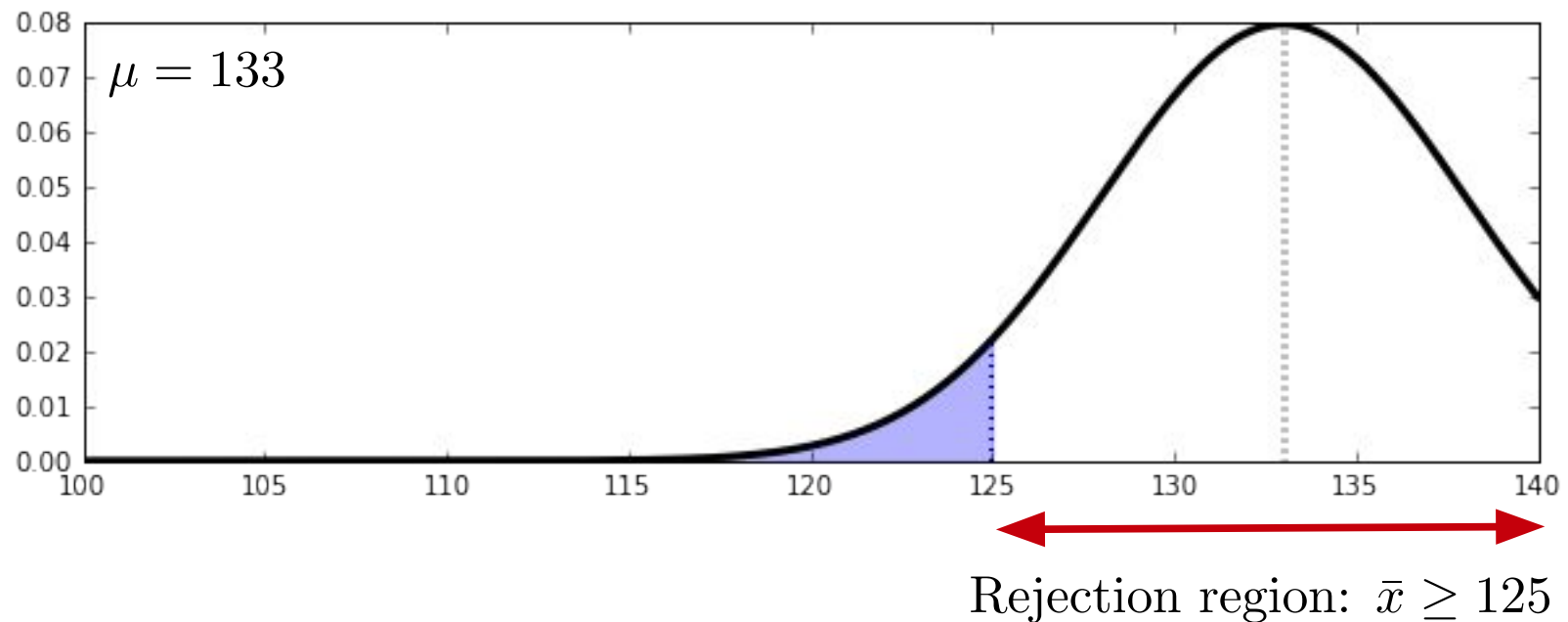Acceptance Region : $\bar{x} < 125$ Rejection region: $\bar{x} \geq 125$

Type II Error $= P(\text{Not reject } H_0 | H_0 \text{ is false})$

$$\beta(128) = P(\bar{x} \leq 125 | \mu = 128) \approx 0.2743$$

Type II Error

Rejection region: $\bar{x} \geq 125$
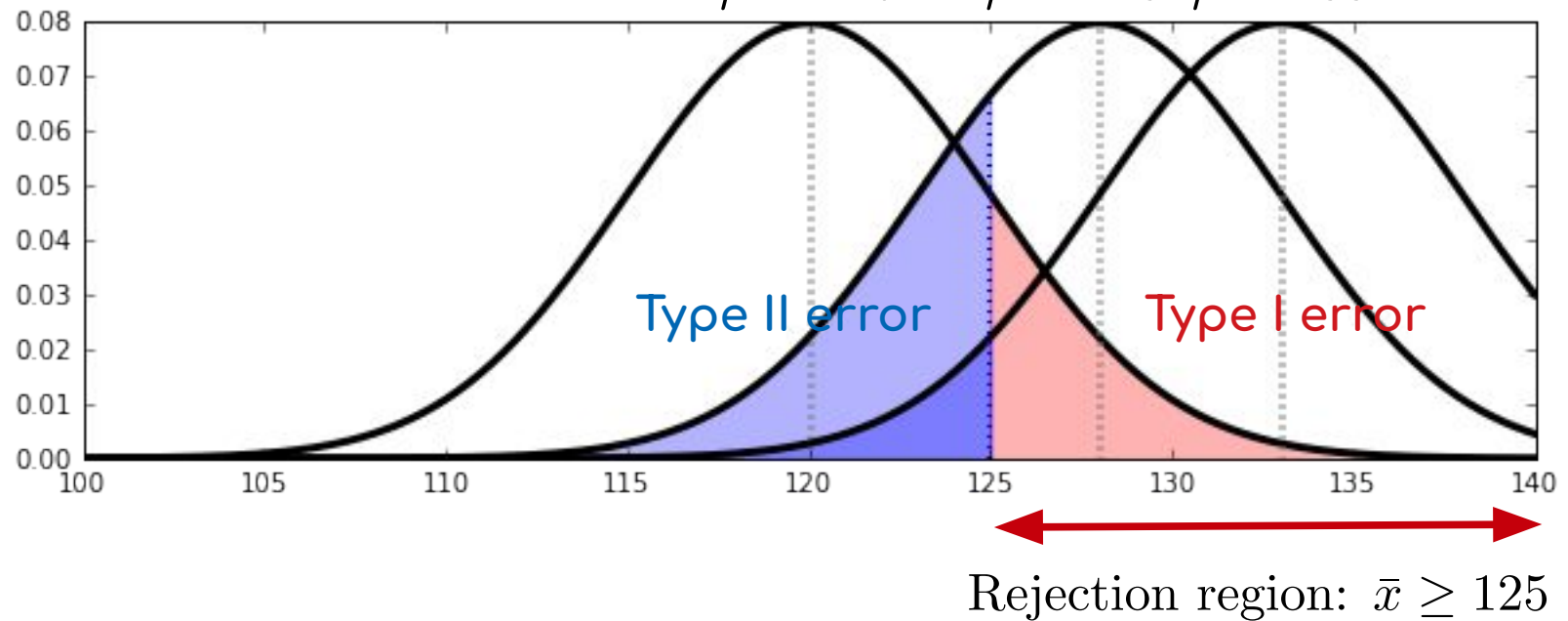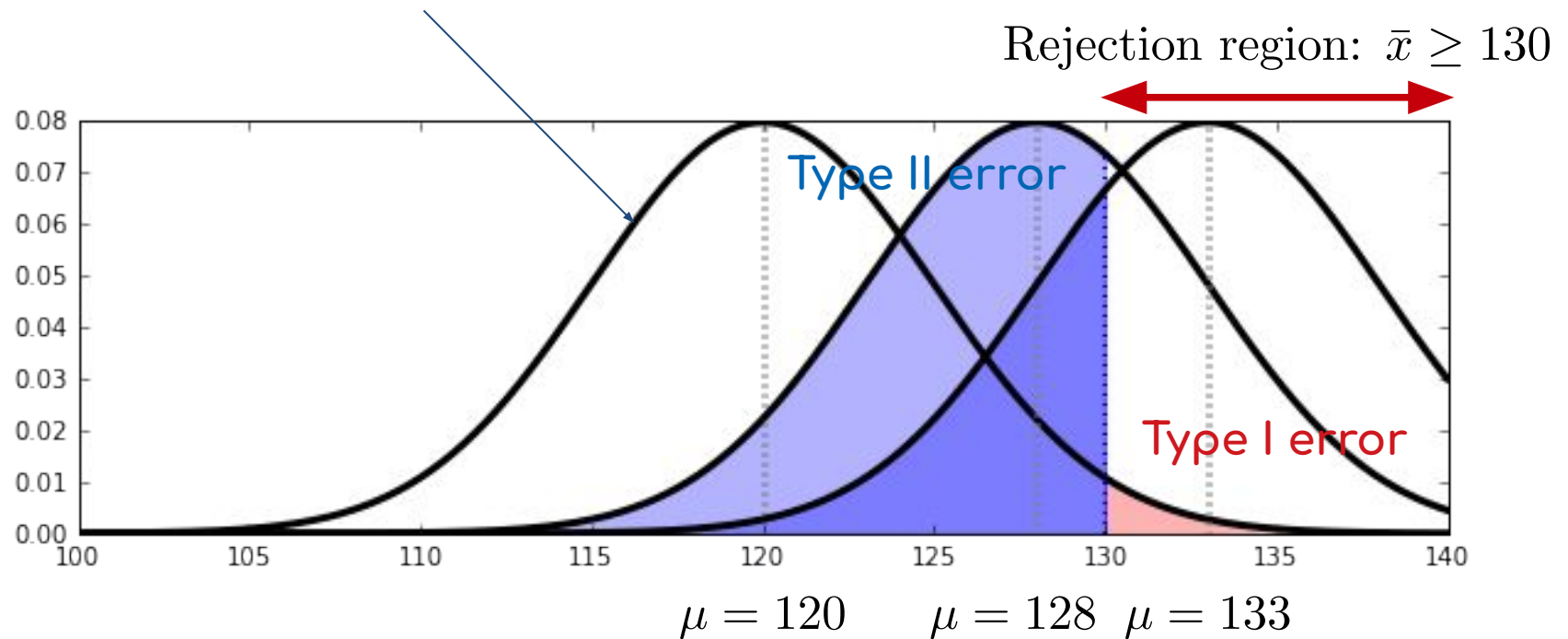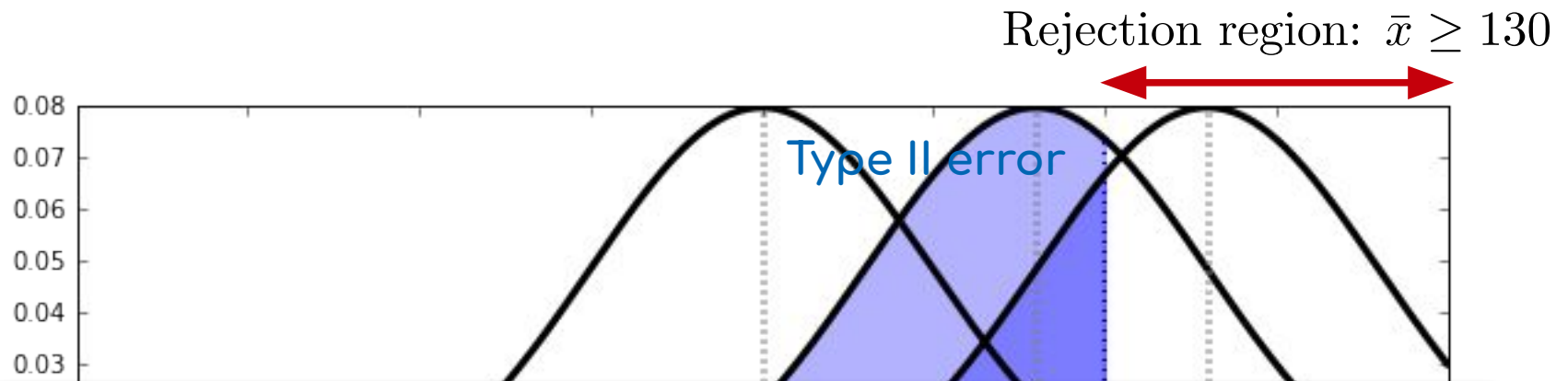
Type II Error $= P(\text{Not reject } H_0 | H_0 \text{ is false})$

$$\beta(133) = P(\bar{x} \leq 125 | \mu = 133) \approx 0.05480$$

Errors

Rejection region: $\bar{x} \geq 130$

Type II error

Type I error

$\mu = 120$ $\mu = 128$ $\mu = 133$

Type II error

Type I error

Rejection region: $\bar{x} \geq 125$
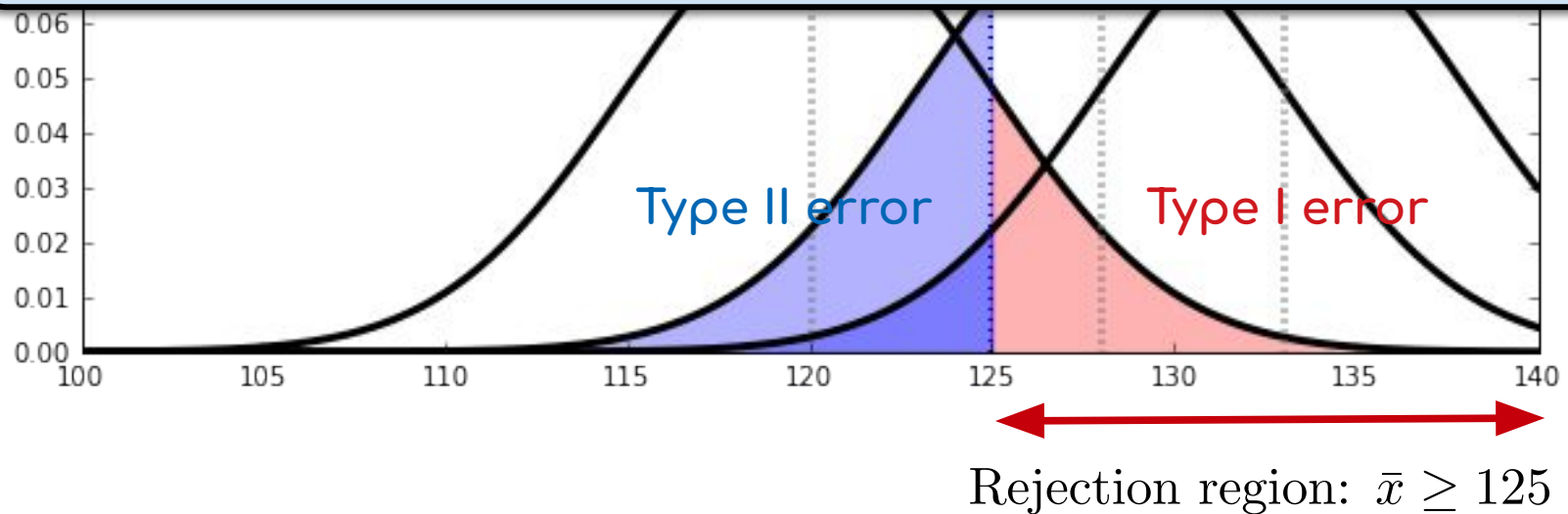
Errors

Rejection region: $\bar{x} \geq 130$

Type II error

Trade-off :
If we adjust the rejection region
in order to decrease (increase) the type I error,
the type II errors increase (decrease).

Type II error   Type I error

Rejection region: $\bar{x} \geq 125$

Significance Level : 유의 수준

- Previously, we specify the rejection region, and then calculate the type I error α.

- We can do this reversely.
  First we specify the type I error of the test.
  Then we decide the rejection region that gives the type I error that we previously set up.

- The type I error that we set up is called the significance level (유의 수준).

$$\text{significance level } \alpha = \text{type I error}$$
$$= P(\text{Reject } H_0 | H_0 \text{ is true})$$

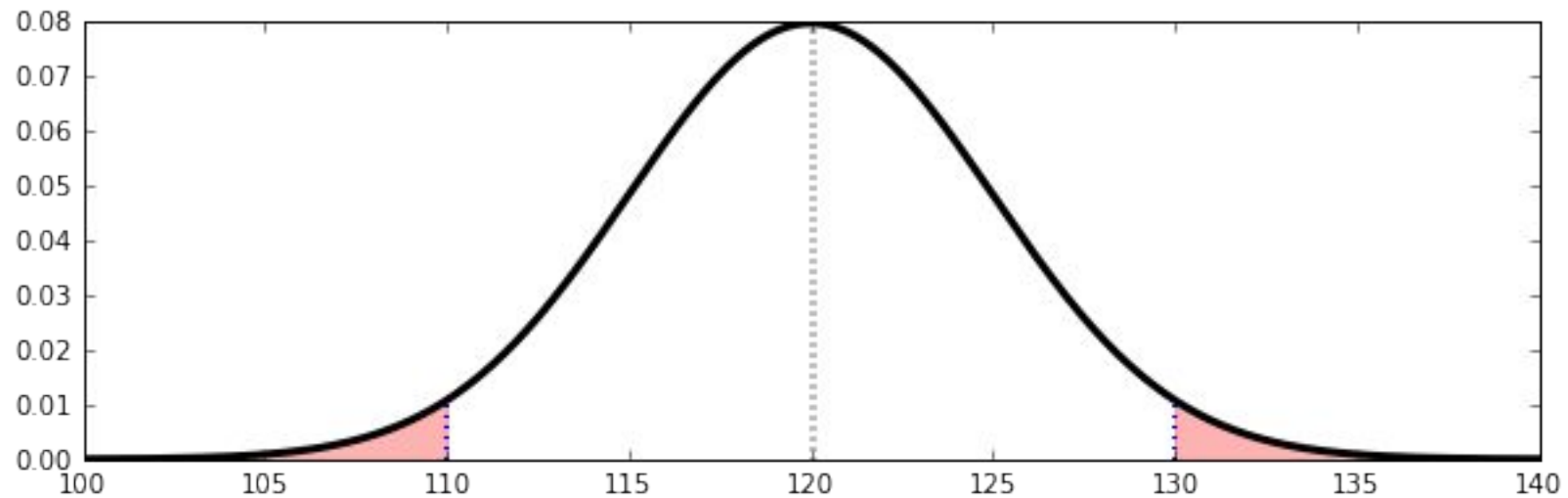$$(1 - \alpha) = \text{Probability that the null hypothesis is correct}$$

One-Sided and Two-Sided Tests : 단측검정과 양측검정

- Null Hypothesis: $\theta = \theta_0$

- Alternative Hypothesis: $\begin{cases} \theta \neq \theta_0 \\ \theta > \theta_0 \\ \theta < \theta_0 \end{cases}$

- Depending on the alternative hypothesis, the rejection region is different.

- 대립가설을 어떤 것을 선택하느냐에 따라 기각영역(rejection region)이 달라진다.
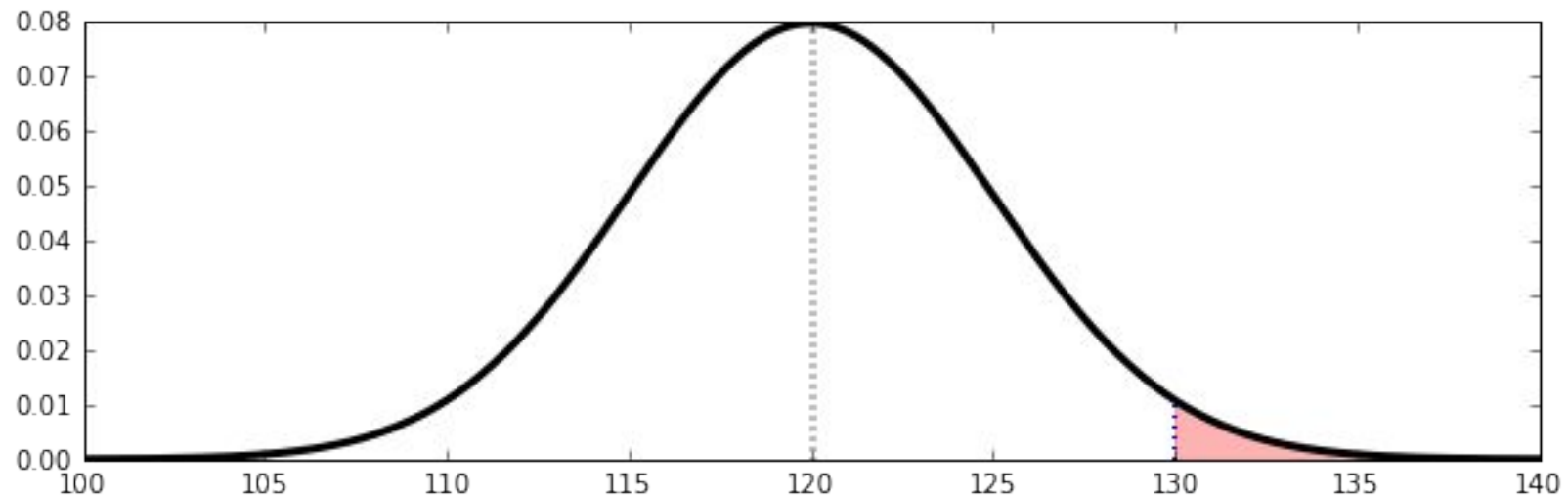
Two-Sided Rejection Regions

- Null Hypothesis: $\theta = \theta_0$
- Alternative Hypothesis: $\theta \neq \theta_0$

The rejection region is two-sided (or two-tailed).
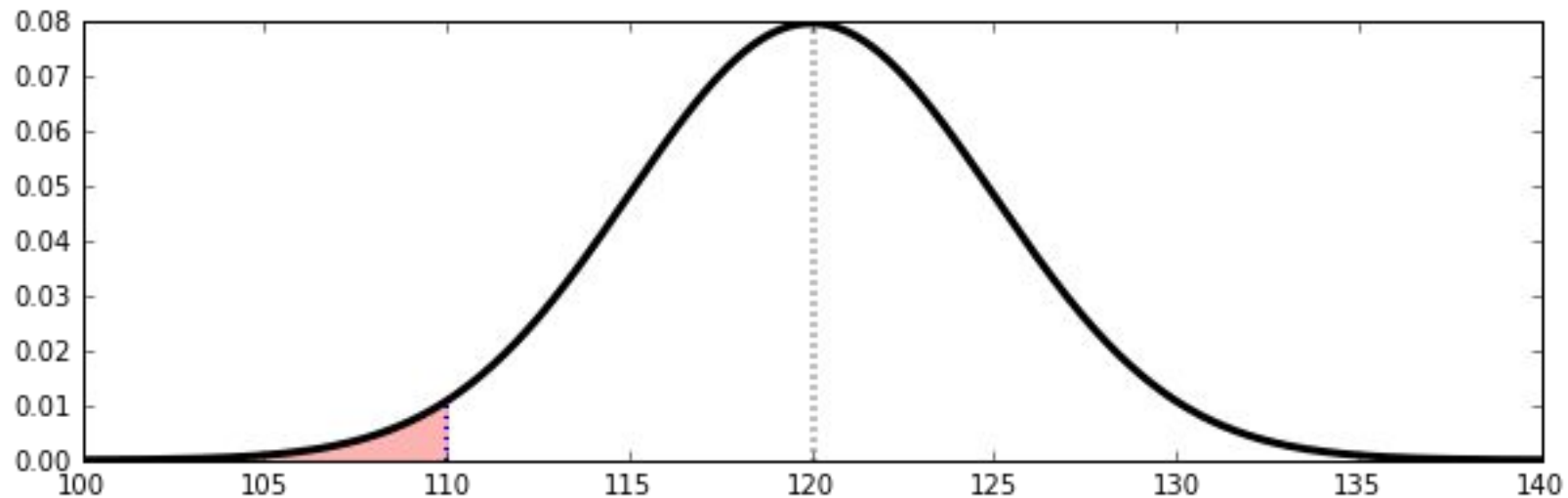
One-Sided Rejection Region : Upper-tailed

- Null Hypothesis: $\theta = \theta_0$
- Alternative Hypothesis: $\theta > \theta_0$

The rejection region is upper-tailed (우측검정).

One-Sided Rejection Region : Lower-tailed

- Null Hypothesis: $\theta = \theta_0$
- Alternative Hypothesis: $\theta < \theta_0$

The rejection region is lower-tailed (좌측검정).