통계분석
Statistical Analysis

Hypothesis Testing with Normal Distribution

Case I. z-test

Normal Distribution with known variance

population is normal, $\sim N(\mu, \sigma^2)$

Standard deviation σ is known.

$\bar{X}$ = Sample mean from random sample of n elements

- Setting up hypotheses

Null hypothesis: $\mu = \mu_0$

Alternative hypothesis: $\mu \neq \mu_0 \quad \mu > \mu_0 \quad \mu < \mu_0$

- Designing hypothesis test : test statistic and significance level

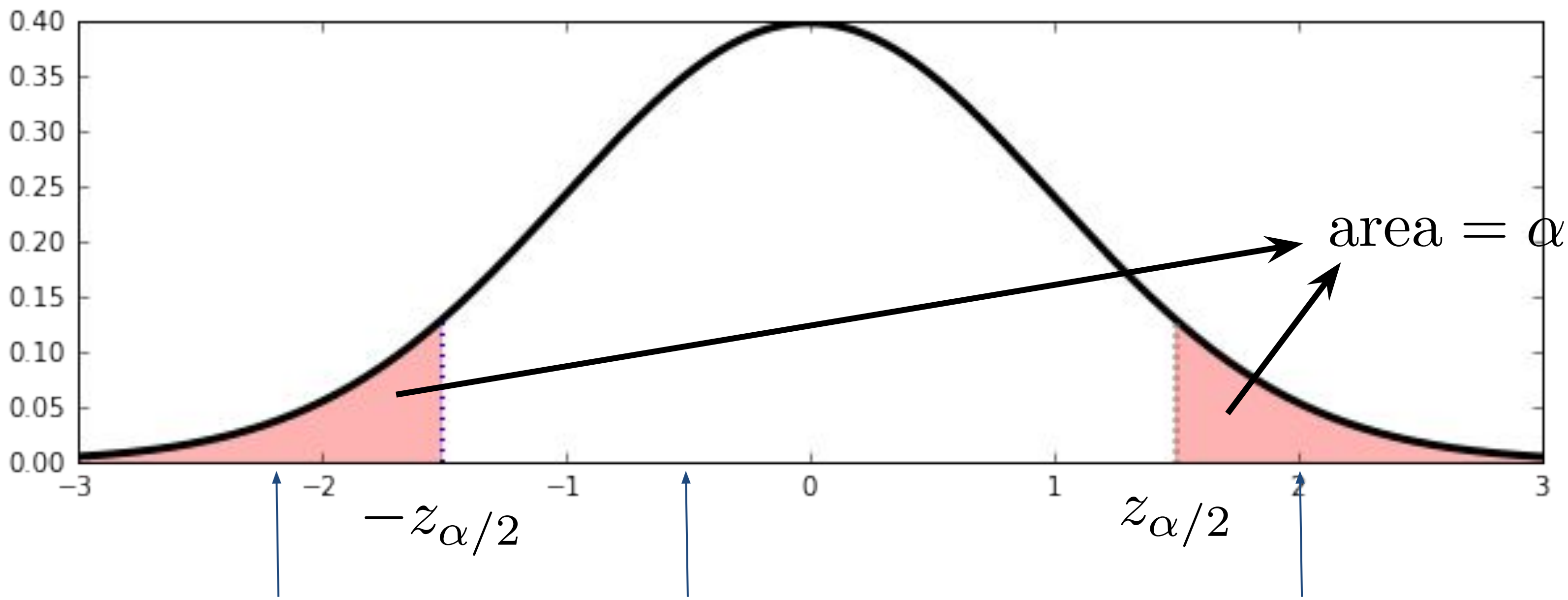
Standardized test statistic $= Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$

Significance Level $= \alpha$

This is called z-test.

Two-sided z-Test

- Alternative hypothesis: $\mu \neq \mu_0$

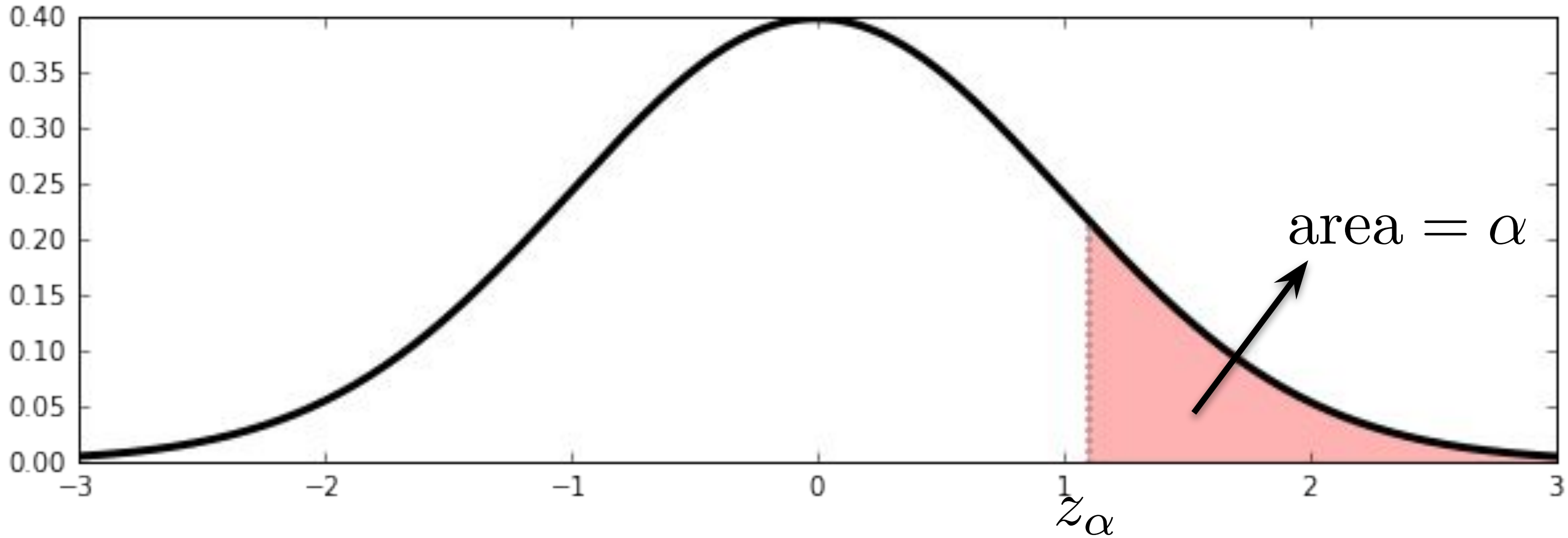


Rejection region for level α test (two-sided test)

$$z \leq -z_{\alpha/2} \text{ or } z \geq z_{\alpha/2}$$

Upper-tailed z-Test

Alternative hypothesis: $\mu > \mu_0$

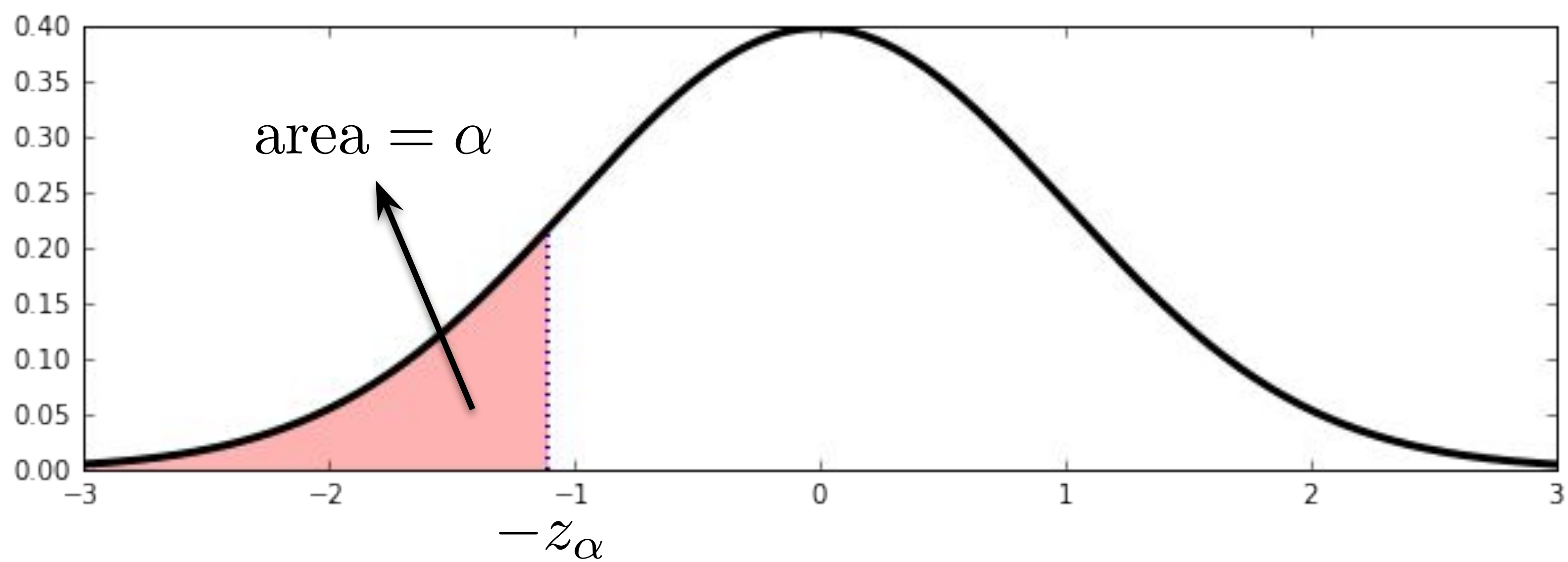


Rejection region for level α test (upper-tailed test)

$$z \geq z_\alpha$$

Lower-tailed z-Test

Alternative hypothesis: $\mu < \mu_0$



Rejection region for level α test (lower-tailed test)

$$z \leq -z_\alpha$$

Case II. z-Test for Large Sample

The sample size is ***large enough*** to apply ***Central limit theorem***.

It is not required that the population distribution is normal.

$\bar{X} = \text{Sample mean from n random samples}$
$S^2 = \text{Sample variance from n random samples}$

Null hypothesis: $\mu = \mu_0$

Alternative hypothesis: $\mu \neq \mu_0$ $\quad \mu > \mu_0 \quad \mu < \mu_0$

$$\text{Standardized test statistic} = Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim N(0,1)$$

$\text{Significance Level} = \alpha$

z-test for a large sample

Case III. t-Test

The sample size is ***not large enough*** for Central limit theorem.

The population distribution is normal $\sim N(\mu, \sigma^2)$

The population variance is unknown.

$\bar{X} = \text{Sample mean from n random samples}$
$S^2 = \text{Sample variance from n random samples}$

Null hypothesis: $\mu = \mu_0$

Alternative hypothesis: $\mu \neq \mu_0$ $\quad \mu > \mu_0 \quad \mu < \mu_0$

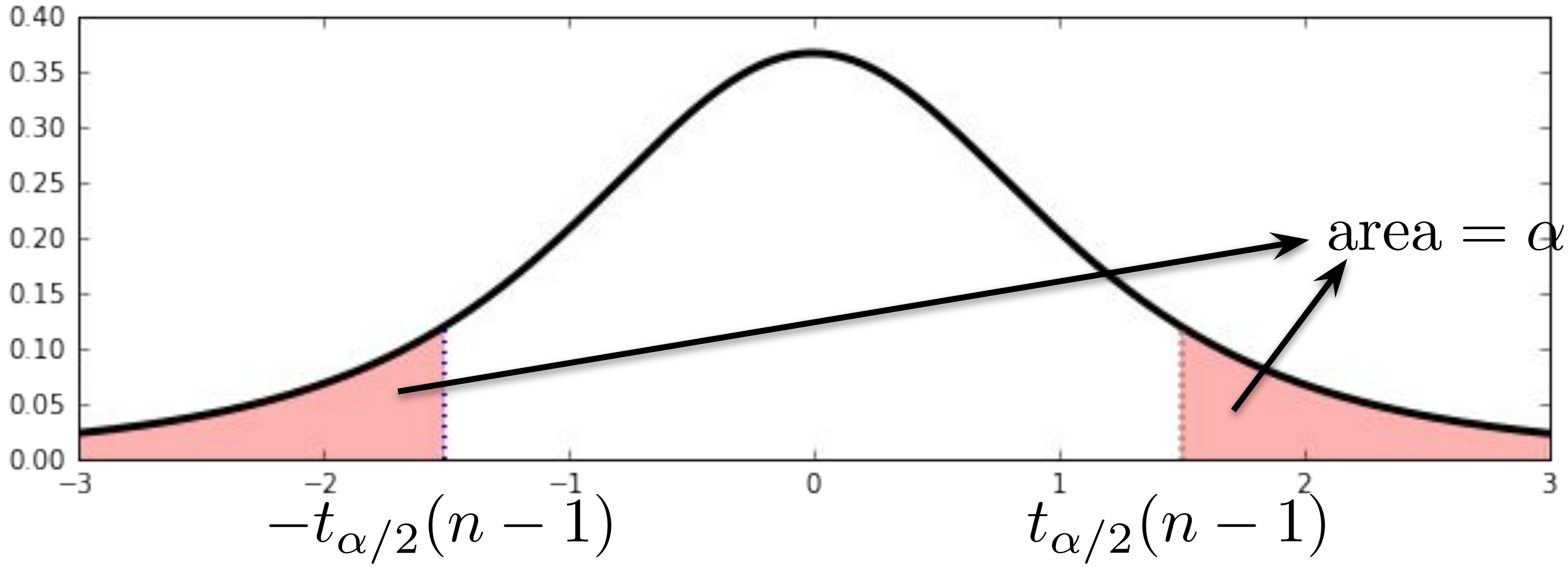
$$\text{Test statistic} = T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1)$$

$\text{Significance Level} = \alpha$

This is called t-test.

Two-tailed t-Test

Alternative hypothesis: $\mu \neq \mu_0$

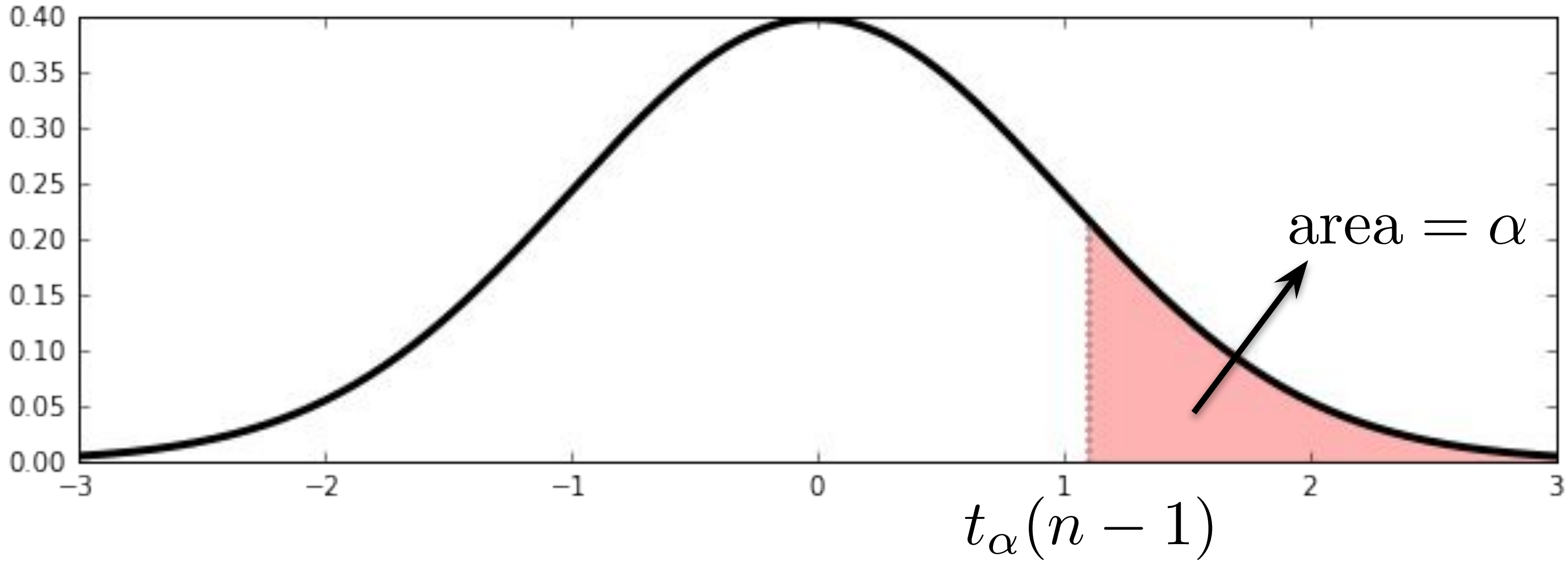


Rejection region for level α test (two-tailed test)

$$t \leq -t_{\alpha/2}(n-1) \text{ or } t \geq t_{\alpha/2}(n-1)$$

Upper-tailed t-Test

Alternative hypothesis: $\mu > \mu_0$

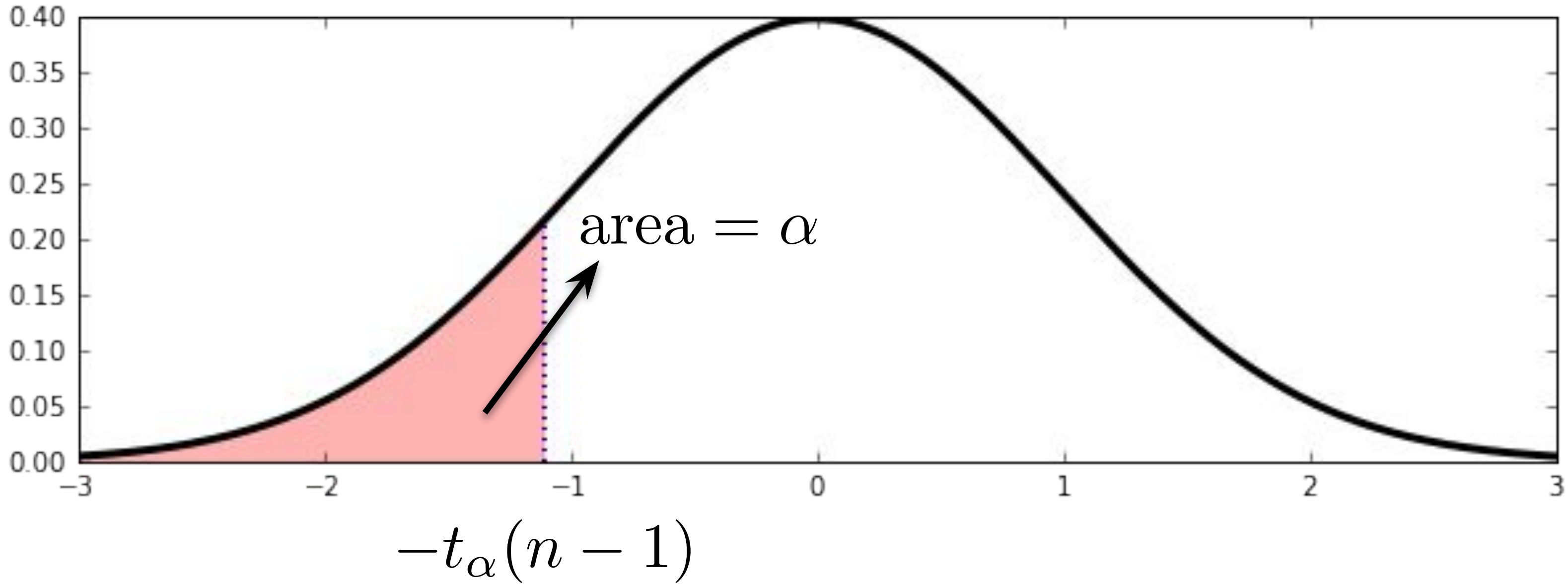


Rejection region for level α test (upper-tailed test)

$$t \geq t_\alpha(n-1)$$

Lower-tailed t-Test

Alternative hypothesis: $\mu < \mu_0$



Rejection region for level α test (lower-tailed test)

$$t \leq -t_\alpha(n-1)$$