Solving Sudoku using Network Analysis

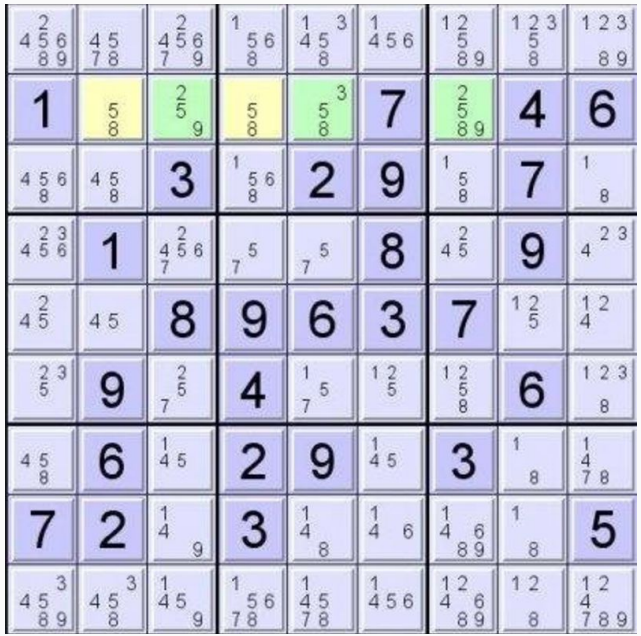
- Ulrik Soderstrom
- Thomas Varner
- Solomon Gaim

Problem Statement & Motivation

- Sudoku Puzzles are among a difficult and relevant class of problems
  - Nondeterministic puzzles
- Goals:
  - Implement the best known algorithm to solve Sudoku (Dancing Links)
  - Model a Sudoku puzzle as a network
  - Improve the algorithm using heuristics based on network analysis

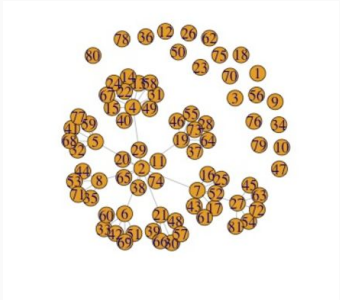
Dancing Links

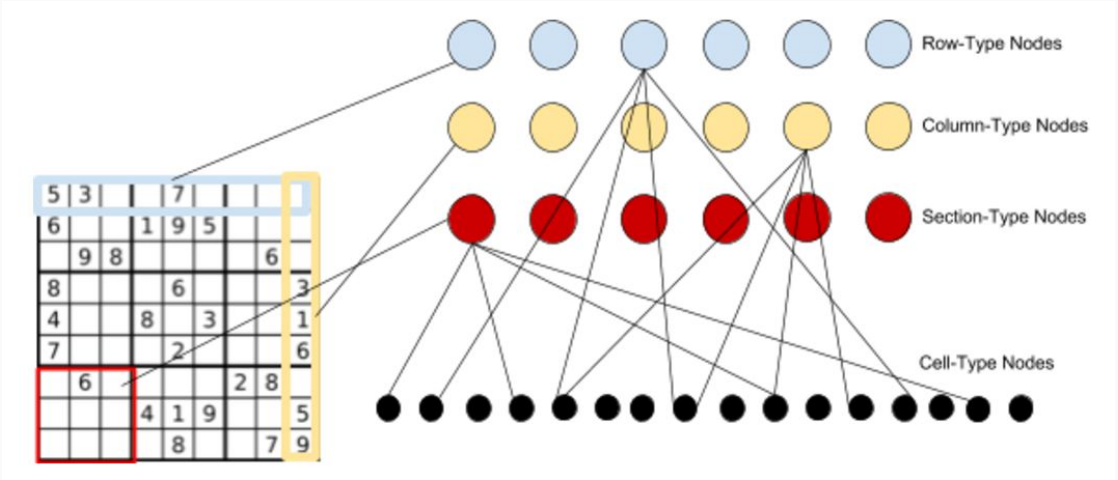
- Randomly choose cells, make guess
- Recursively exhaust solution space
    - Depth-first search, backtrack if board cannot be true



Graphical Representation

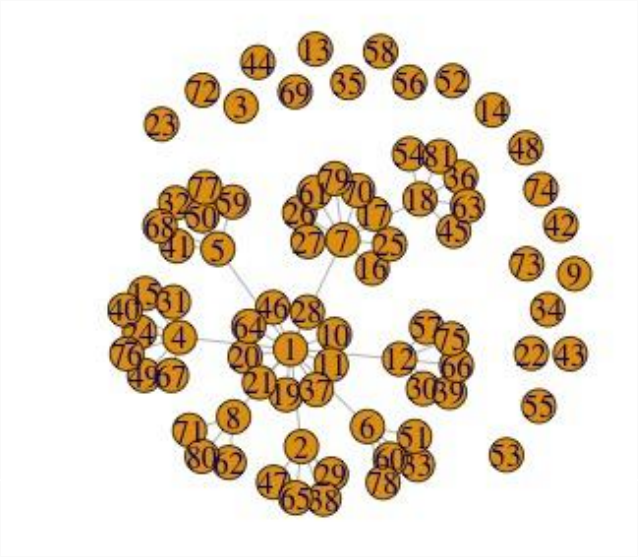
- MultiGraph
  - Shared Value Model
  - Node Representation
- igraph Representation



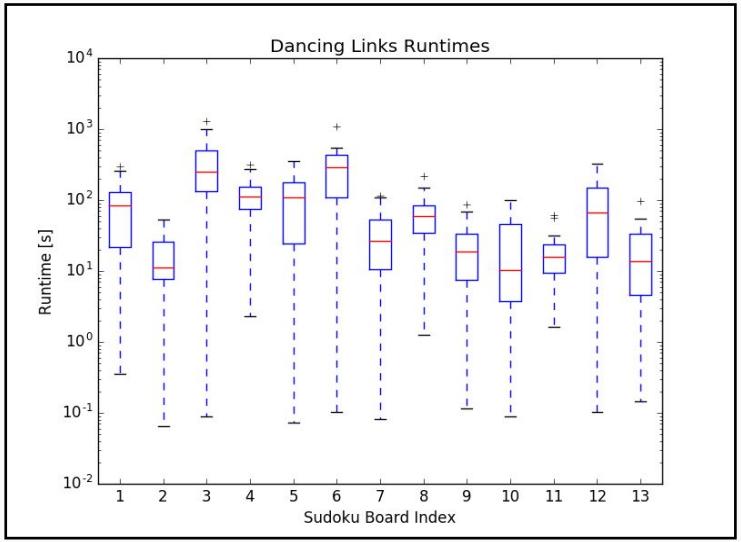


Hypothesis: Developed Heuristic

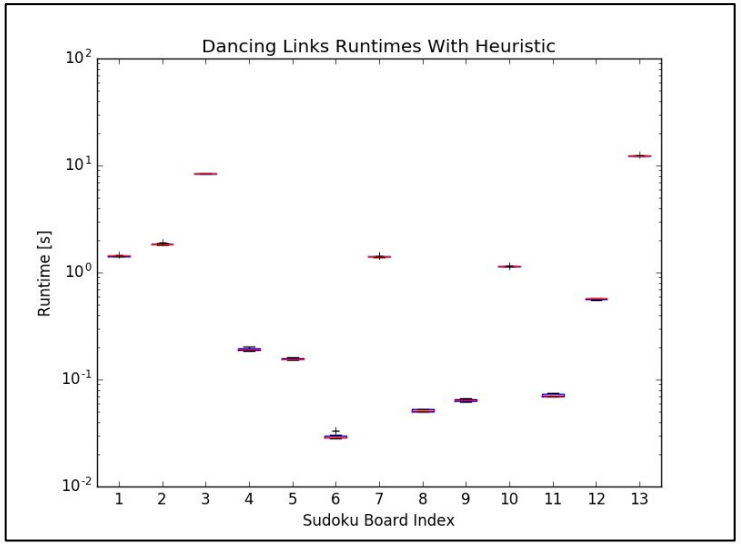
- Choose cells with the lowest degree centrality first
  - Exhaust Solution Space
  - Recursive
  - Decision Based



Runtime Comparison Between Boards



General Dancing Links



Network Analysis Based Heuristic

Comparative Result

- 13 Sudoku Boards, each with 17 givens and one unique solution
- Two-sided, paired hypothesis test:
  - H_o: mean runtime does not change once the heuristic is applied, H_A: otherwise
  - Confidence Level of 95%
  - 20 trials per board
- Results

Board #	1	2	3	4	5	6	7	8	9	10	11	12	13
T-stat	-4.9	-4.9	-4.6	-6.9	-5.0	-5.3	-4.5	-5.7	-4.7	-3.5	-5.5	-4.5	-1.8
Pvalue	9e-5	9e-5	1e-4	7e-5	4e-5	2e-4	1e-5	1e-4	2e-3	2e-5	2e-4	3e-5	0.08

Conclusions

- Best known Dancing Links heuristics significantly improved
  - Network Analysis applications alter stack feature
- 40 tests of 13 boards with Random Dancing Links and Network Analysis Dancing Links
  - Significant change in runtimes and efficiency
  - No board exceptions

Questions

Sources

- [1] Knuth, D (1975). “Estimating the efficiency of backtrack programs,” Mathematics of Computation 29, 121–136.
- [2] Knuth, D (2000). "Dancing links". Millennial Perspectives in Computer Science. P159. 187. arXiv:cs/0011047
- [3] Floyd, R (1967) “Nondeterministic algorithms,” Journal of the ACM 14, 636–644.
- [4] Golomb, S. Baumart, L (1965). “Backtrack programming,” Journal of the ACM 12, 516–524.
- [5] McGuire, Gary (2012). "There is no 16-Clue Sudoku: Solving the Sudoku Minimum Number of Clues Problem." Web.
- [6] Scott, D (1958) “Programming a combinatorial puzzle,” Technical Report No. 1 (Princeton, New Jersey: Princeton University Department of Electrical Engineering), ii + 14 + 5 pages.
- [7] Kjell Ericson. Generate and Solve Sudoku <https://kjell.haxx.se/sudoku/>
- [8] https://sourceforge.net/projects/dancinglinks/ (Image File)