Difficulty-Matching High Hit Factor Algorithm for 23 and 24 Series Classifiers

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Abstract

This article presents a novel approach for addressing difficulty discrepancies within the USPSA classifier system by applying a difficulty-matching algorithm to the high hit factors (HHF) of classifiers across the 23, 24, and 99-22 series. The proposed method uses the Weibull distribution to model the observed hit factors and applies maximum likelihood estimation (MLE) to derive the best-fit parameters for each classifier. By regressing these parameters against existing USPSA high hit factors, the algorithm generates difficulty-matched HHFs that better align with the overall classification system, minimizing misclassification risks due to statistical arbitrage. The results show that certain classifiers in the 23 and 24 series are too easy or too difficult, while the 99-22 series classifiers also exhibit significant variability in difficulty. The difficulty-matching high hit factor algorithm offers a more consistent and equitable way to assess shooter performance across all classifiers, with potential recommendations for adjusting current high hit factors to eliminate misclassification opportunities and enhance classification fairness.

1 Introduction

The high hit factors for the 23 and 24 series classifiers were poorly established, causing significant and irreversible damage to the classifier system and rankings. In particular, the hit factors for the 23 series were set too low, while those for the 24 series were set too high, resulting in erroneous classifications and widespread dissatisfaction within the shooting community. Recognizing these issues, the USPSA Board of Directors has suspended the 23 and 24 series from the classification system, effective October 30, 2024, pending new insights for establishing accurate high hit factors.

Fluctuations in classifier difficulty are not uncommon, primarily due to inconsistencies in the high hit factor determination method. These inconsistencies create opportunities for “statistical arbitrage,” giving rise to “sandbaggers” and “paper GMs”, shooters who significantly underperform or overperform relative to their classification.

The classifier committee and HitFactor.Info have already established a reasonable, data-driven method to determine high hit factors. This approach is designed based on the population of shooters across various classifications, such as Grandmaster (approximately 1%), Master (5%), A-class (15%), and so on. By using this method, the difficulty of the classifiers is normalized, ensuring that the distribution of shooters in each class remains consistent.

The HitFactor.Info method also requires the removal of B/C/D flags, which introduces the following changes:

- Classifier scores that are 5% lower than the current class threshold **will be included** in the calculations.
- Duplicated scores will be treated differently, and the higher score **will not be directly used** as-is.

By implementing these changes, we observed that classification results strongly correlate with match finishes (as measured by the ELO rating provided by Shooting Sports Analyst). This alignment is a desirable feature in any classification system.

Recognizing that the removal of B/C/D flags is not an easy task and may take time, we have also developed an alternative approach. This method matches the "difficulty" of the 23 and 24 series classifiers to that of the 99-22 series classifiers. By doing so, we can establish new high hit factors for the 23 and 24 series classifiers as a temporary solution, providing a stopgap measure before implementing the removal of the B/C/D flags.

2 Methodology

The proposed method leverages a statistical approach utilized by HitFactor.Info. In this section, we will briefly introduce the HitFactor.Info approach, followed by an explanation of the proposed method.

2.1 Hitfactor.Info Method

In essence, the HitFactor.Info method models hit-factor data using a statistical distribution. From this modeled distribution, a high hit factor is determined by aligning it with a target population. For instance, if the top 5% of shooters are classified as at least Master class, the method identifies the 95th percentile hit factor and divides it by 85% to establish the high hit factor. This ensures that only the top 5% of performers are awarded Master or Grandmaster classification.

2.1.1 The Weibull Distribution

It has been observed that the hit-factor data of a stage often conforms to a Weibull distribution [1, 2].

Probability density function

The probability density function (PDF) of a Weibull distribution is given by:

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}\,, \tag{1}$$

for $x \geq 0$, where x is the random variable (e.g. hit factor), λ is the scale parameter, k is the shape parameter, and $e \approx 2.718\ldots$ is the base of the natural logarithm.

Cumulative distribution function

The cumulative distribution function (CDF) is expressed as

$$F(x) = \int_{-\infty}^{x} f(x')\,dx' = 1 - e^{-\left(\frac{x}{\lambda}\right)^k}\,. \tag{2}$$

This function describes the probability of finding a hit factor less than x.

Percentile function

To determine the hit factor for the top $P\%$ of shooters, the percentile (quantile) function is used:

$$Q(p) = F^{-1}(p) = \lambda\,(-\ln(1-p))^{\frac{1}{k}} \tag{3}$$

where p represents the probability that the hit factor is less than or equal to $Q(p)$. To find the hit factor corresponding to the top $P\%$ of shooters, simply substitute $p = 1 - P\%$.

2.1.2 Finding the Shape and Scale Parameters

To use the percentile function, the shape parameter k and the scale parameter λ that best fit the observed distribution must be determined. For this purpose, we adopt the maximum likelihood estimation (MLE) method, which is the academic standard for estimating the parameters of a probability distribution based on observed data, such as hit factors.

Maximum likelilhood estimation

The likelihood function estimates how likely an observed data sample is to be drawn from a given statistical distribution. By maximizing the likelihood with respect to the parameters k and λ, the best-fit distribution can be determined.

To keep things simple and avoid delving too deeply into statistical jargon (apologies to my statistician friends), we express the likelihood function for the Weibull distribution as:

$$\mathcal{L}(\lambda, k) = \prod_{i=1}^{n} f(x_i; \lambda, k)\,, \tag{4}$$

where x_i represents the i^{th} observed hit factor of a stage, and $f(x_i; \lambda, k)$ is the probability density function of a Weibull distribution with parameters k and λ. The best-fit parameters, $\hat{k}$ and $\hat{\lambda}$ can then be obtained by maximizing the likelihood function:

$$\hat{\lambda}, \hat{k} = \arg\max \mathcal{L}(\lambda, k)\,. \tag{5}$$

Log-likelihood

Since $f(x_i; \lambda, k)$ is a number smaller than 1, the product of all $f(x_i; \lambda, k)$ becomes exceedingly small with a large sample size, making it impractical to handle numerically. To address this, it is common to use the natural logarithm of the likelihood function, known as the log-likelihood function:

$$l(\lambda, k) = \ln \mathcal{L}(\lambda, k)\,, \tag{6}$$

This transformation converts the product $\prod_{i=1}^{n} f(x_i; \lambda, k)$ into a sum $\sum_{i=1}^{n} \ln f(x_i; \lambda, k)$, yielding more manageable values.

Optimizing the parameters

Since commonly available optimization algorithms are designed to minimize a "cost function" (or "loss function"), the maximum likelihood estimation method can be adapted by minimizing the negative log-likelihood function. This approach effectively converts the MLE problem into an optimization problem. It relies on the assumption that maximizing the likelihood is equivalent to maximizing the log-likelihood function, which in turn is equivalent to minimizing the negative log-likelihood function.

To formally state the optimization problem, we aim to minimize the cost function:

$$J(\lambda, k) = -l(\lambda, k) = -\sum_{i=1}^{n} \ln f(x_i; \lambda, k)\,. \tag{7}$$

Minimizing this function yields the optimal values $\hat{\lambda}$ and $\hat{k}$ corresponding to the Weibull distribution that best fits the observed hit-factor data.

This function can be minimized using many available numerical optimization algorithms. For convenience, the Nelder-Mead [3] method is employed, with the initial parameters given by:

$$k_0 = 3.6\,, \tag{8}$$

and

$$\lambda_0 = \frac{\text{med}(x_i)}{(\ln 2)^{1/k_0}}\,, \tag{9}$$

where $\text{med}(x_i)$ represents the median of the observed hit factors. Here, k_0 is chosen in such a way it represents a distribution with close to 0 skewness. And, λ_0 is simply a rearrangement of the equation related the median and the scale parameter λ.

2.1.3 Determining a recommended high hit factor

(Note that this is **not** the proposed high hit factor method.)

With the best-fit Weibull distribution, we can now determine the high hit factor by matching a class population. Based on observation of shooter's current classfication percentage, roughly 5% of shooters are Masters and Grandmasters, meaning the top 5% of shooters currenlty have an 85% classification or above.

Using the percentile function (Eqn. (3)), we can calculate the hit factor attained by the top 5% of shooters (the 95$^{\text{th}}$ percentile). By dividing this value by the corresponding percentage, i.e., 85%, we obtain the high hit factor (HHF):

$$\begin{aligned} \mathrm{HHF} &= \frac{Q(p=0.95)}{85\%} \\ &= \frac{\lambda\left(-\ln(1-0.95)^{1/k}\right)}{85\%} \end{aligned} \tag{10}$$

Using the best-fit parameters for the classifier CM 23-01, $\lambda = 6.45$ and $k = 3.3$, we get

$$\mathrm{HHF} = 10.5812\,. \tag{11}$$

Thus, the reccomended high hit factor for classifier 23-01 is 10.5812.

Note that this approach **differs slightly** from the method used by the current version of HitFactor.Info, for reasons not discussed here. Also, note that the top 5% percentile and the 85% classification threshold are based on observations and may be subject to change.

2.2 Difficulty Matching

In Sec. 2.1.3, we established that high hit factors can be determined by matching the percentile p with a hit factor percentage HF% on the Weibull distribution. For instance, setting $p = 0.95$ and HF% = 85% implies that the high hit factor is designed such that only the top 5% of shooters achieve an 85% or higher classification. This effectively defines the "difficulty" of a classifier.

The percentile and percentage values can be adjusted for individual classifiers to make them relatively easier or harder. For example, a classifier where only the top 1% of shooters achieve an 85% or higher classification is more difficult than one where the top 5% of shooters achieve the same.

The concept of difficulty matching is straightforward. First, we treat the percentile p and hit factor percentage HF% as free variables and identify a pair that best corresponds to the difficulty level of the 99-22 series classifiers. Then, using the best-fit Weibull distribution for the 23 and 24 series classifiers, we can calculate the **difficulty-matched high hit factors** as follows:

$$\mathrm{HHF}(p, \mathrm{HF\%}) = \frac{Q(p)}{\mathrm{HF\%}}\,, \tag{12}$$

where p and HF% are the percentile and percentage values and $Q(p)$ is the percentile function from Eqn. (3). This approach is akin to reverse-engineering the USPSA's methodology and applying it to the 23 and 24 series.

2.2.1 Regression

To determine the best-fit percentile and percentages that describe the difficulty of the 99-22 series, we can regress these parameters against the existing high hit factors previously set by USPSA. This regression is achieved by minimizing an error function defined as:

$$J'(p, \mathrm{HF\%}) = \sum_{i=1}^{n}\left[\ln\left(\frac{\mathrm{HHF}_i(p, \mathrm{HF\%})}{\mathrm{HHF}_i}\right)\right]^2\,, \tag{13}$$

where HHF_i represents the current high hit factor of the i^{th} classifier stage set by USPSA, and $\mathrm{HHF}_i(p, \mathrm{HF\%})$ represents the high hit factor estimation for that stage using the Weibull method from Eqn. (12). This approach resembles a least-squares fit, with the cost function based on the sum of the squared logarithmic errors.

The reason for using a logarithmic error here is that hit factors are typically compared in terms of relative proportions rather than absolute differences. For instance, a hit factor increasing from 9 to 10 represents a 90% relative comparison, with an absolute difference of 1. Similarly, an increase from 90 to 100 also represents a 90% relative comparison but with an absolute difference of 10. Clearly, in both cases, the difference should be considered as 10%, not as 1 and 10.

The differential evolution algorithm [4] is chosen to minimize the cost function $J'(p, \mathrm{HF\%})$, as it efficiently optimizes parameters within specified boundaries. This is essential since the percentile p is bounded within (0, 1), and the percentage HF% is bounded within (0, 100%).

3 Results

In this section, the proposed method is applied to the 23 and 24 series classifiers to determine difficulty-matched high hit factors in the Carry Optics division.

3.1 Classifier CM 23-01

Here, we use the classifier CM 23-01 as a highlighted example.

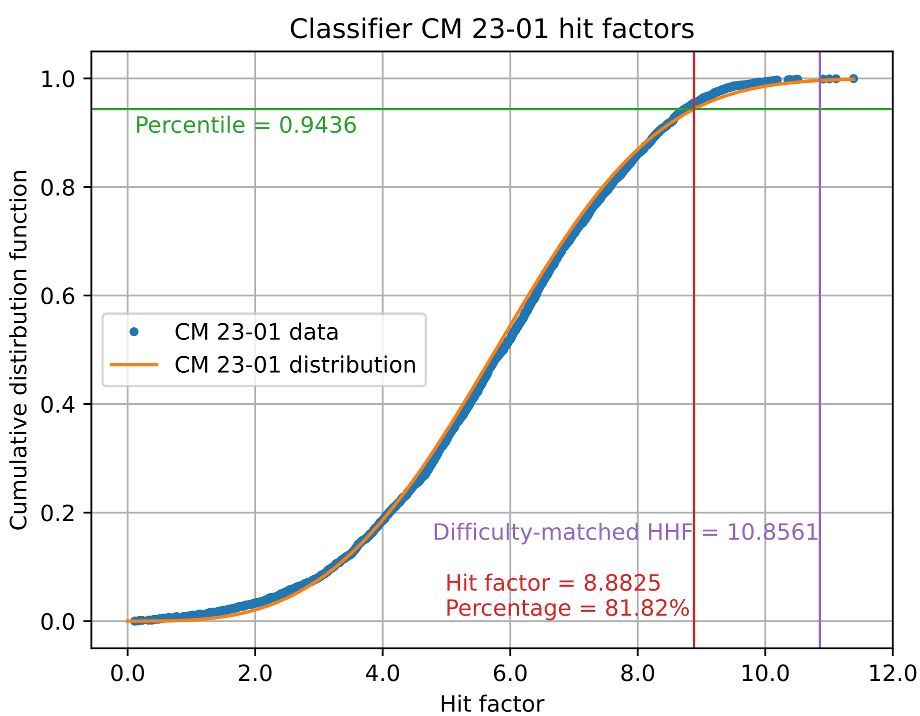

Figure 1: Cumulative distribution function of the CM 23-01 classifier hit-factor data.

Fig. 1 illustrates the best-fit Weibull cumulative distribution function for the CM 23-01 classifier hit-factor data. Through regression, the percentile and hit factor percentage that align with the difficulty of the 99-22 series were determined to be $p = 0.9436$ and $\mathrm{HF\%} = 81.82\%$, respectively

As shown in the figure, the hit factor corresponding to $p = 0.9436$ is **8.8825**, representing the **81.82% score** if the difficulty of CM 23-01 is equalized to that of the 99-22 series classifiers. This results in a difficulty-matched high hit factor of:

$$\text{HHF} = \frac{8.8825}{81.82\%} = 10.8561\,. \tag{14}$$

Note that this difficulty-matched high hit factor is slightly higher than the recommended high hit factor from HitFactor.Info. This difference arises because the recommended HHFs from HitFactor.Info are calculated without accounting for B/C/D flags, which slightly increases the overall difficulty of the classification system.

According to the difficulty-matched high hit factor, the percentages of shooters achieving scores above 95% (GM), 85% (M), and 75% (A) of the high hit factor are 0.284%, 2.50%, and 12.23%, respectively. This population distribution aligns more closely with those of the 99-22 series.

For example, the current populations achieving GM, M, and A scores for the CM 22-04 classifier are 0.49%, 3.41%, and 11.26%, respectively. In contrast, the CM 23-01 classifier currently has much higher percentages, 0.71%, 4.75%, and 16.56%, which indicates a less consistent level of difficulty compared to the 99-22 series

3.2 23 and 24 Series Classifiers

The difficulty-matching high hit factor algorithm was applied to all 23 and 24 series classifiers, as well as the 99-22 series classifiers. The resulting difficulty-matched high hit factors were then compared to the current high hit factors set by USPSA.

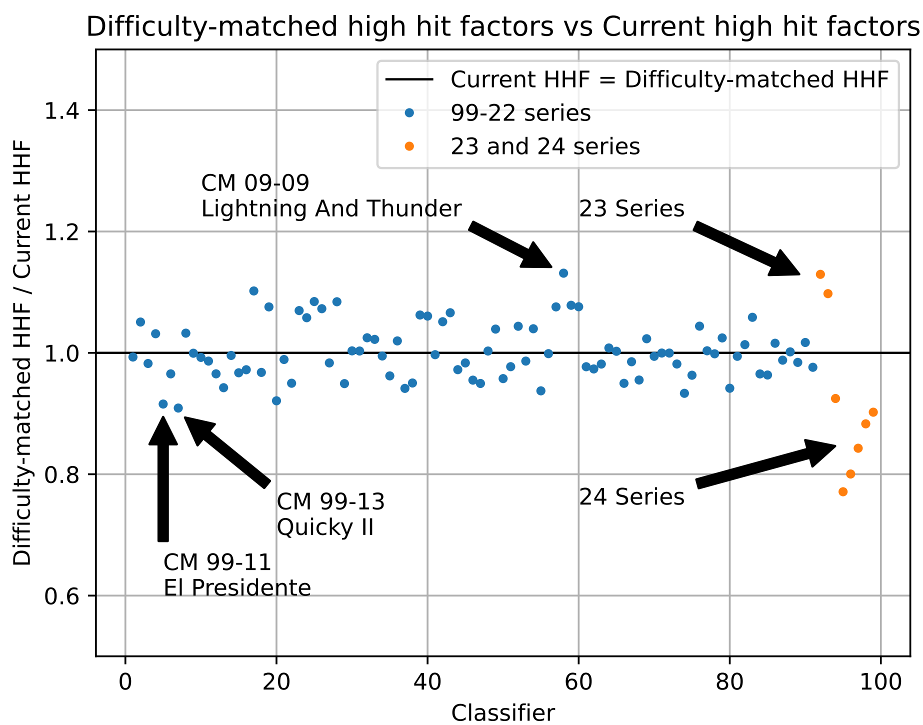

Figure 2: The ratio between difficulty-matched high hit factors and the current high hit factors set by USPSA

Figure 2 illustrates the ratio of difficulty-matched high hit factors to the current high hit factors. Blue markers represent the 99-22 series classifiers, while yellow markers denote the 23 and 24 series classifiers. Points above the $y = 1$ line indicate relatively easy classifiers, where the current high hit factor (HHF) is lower than the difficulty-matched HHF. Conversely, points below the $y = 1$ line signify relatively difficult classifiers, where the current HHF is higher than the difficulty-matched HHF.

As evident from the comparison, the two 23 series classifiers and the six 24 series classifiers are clear outliers within the classification system. Specifically, the 23 series classifiers lie well above the $y = 1$ line, suggesting that they are overly easy, whereas the 24 series classifiers fall significantly below the $y = 1$ line, indicating they are disproportionately difficult.

Classifier	Current HHF	Difficulty-matched HHF
CM 23-01	9.6123	10.8561
CM 23-02	10.2284	11.2267
CM 24-01	12.25	11.3288
CM 24-02	13.089	10.0932
CM 24-04	12.2	9.7647
CM 24-06	13.4883	11.3684
CM 24-08	12.4033	10.9527
CM 24-09	12.1488	10.9610

Table 1: Difficulty-matched high hit factors for the 23 and 24 series classifiers in the Carry Optics division.

Table 1 provides the suggested difficulty-matched high hit factors for the 23 and 24 series classifiers. These values aim to align the difficulty of these classifiers with those of the 99-22 series. We recommend updating the high hit factors accordingly, particularly if the removal of B/C/D flags cannot be implemented successfully.

4 Discussions

4.1 Statistical Arbitrage

Figure 2 illustrates not only the difficulty-matched high hit factors for the 23 and 24 series but also those of the 99-22 series classifiers. The figure highlights that some 99-22 series classifiers, such as CM 09-09 Lightning And Thunder, exhibit difficulty levels comparable to the relatively easy 23 series. Conversely, the method also identifies classifiers widely regarded as challenging, such as CM 99-11 El Presidente and CM 99-13 Quicky II. This underscores the variability in difficulty across classifiers within the 99-22 series and emphasizes the potential value of applying the difficulty-matching methodology to the entire classification system.

The fluctuation in difficulty among classifiers introduces opportunities for shooters to be misclassified through a form of **"statistical arbitrage."** Imagine Fig. 2 as analogous to a price chart in trading. Shooters who perform on the relatively easy classifiers achieve scores that exceed their true classification level, similar to selling or shorting a stock priced above its mean.

In this context, the B and D flags function as protective mechanisms against the impact of relatively difficult classifiers, akin to halting sales at certain price thresholds However, the interplay between fluctuating difficulty levels and the protective B/D flags creates a system where shooters can manipulate classifications through statistical arbitrage.

While the difficulty-matched high hit factor algorithm can address inconsistencies for the 23 and 24 series, **we recommend also updating the high hit factors of the 99-22 series** to eliminate this arbitrage opportunity and create a more consistent and fair classification system.

5 Conclusions

In conclusion, the proposed difficulty-matching high hit factor algorithm provides a robust method for addressing the inconsistency in classifier difficulty within the USPSA system. By using Weibull distribution-based regression, we have derived difficulty-matched high hit factors that align more accurately with the intended classification levels, minimizing the potential for misclassification through statistical arbitrage. The analysis revealed significant discrepancies in the difficulty of classifiers across the 23, 24, and 99-22 series, with some classifiers being either too easy or too difficult. The difficulty-matching algorithm offers a solution to these issues, ensuring a more consistent and equitable classification system. We recommend updating the high hit factors of both the 23 and 24 series classifiers, as well as revisiting the 99-22 series, to further reduce classification inconsistencies and eliminate opportunities for misclassification, ultimately enhancing fairness in the shooting sports classification system.

References

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