

```

method GCD1(a: int, b: int) returns (r: int)
  requires a > 0 && b > 0 && a != b
  ensures r == gcd(a, b)
  decreases a % b
{
  {a > 0 && b > 0 && a != b}
  {(0 < a < b) || (0 < b < a)}
  {(0 < a < b) || false || false || 0 < b < a || false}
  {(0 < a < b) || (0 < a < b && b == gcd(a, b)) || false || 0 < b < a || (b ==
gcd(a, b) && 0 < b < a)}
  {(0 < a < b && a % b != 0) || (0 < a < b && b == gcd(a, b)) || (0 < a < b && 0
< b < a) || (a % b != 0 && 0 < b < a) || (b == gcd(a, b) && 0 < b < a)}
  {(0 < a < b) && (a % b != 0 || b == gcd(a, b)) && (0 < b < a)}
  {(a > b || b > 0 && a > 0) && true && a < b || a % b == 0 || (b > 0 && a % b >
0)}
  {(a < b ==> b > 0 && a > 0) && (a % b == 0 ==> b == gcd(a, b)) && ((a > b && a
% b != 0) ==> b > 0 && a % b > 0)}
  if a < b {
    {b > 0 && a > 0}
    {b > 0 && a > 0 && true}
    {b > 0 && a > 0 && gcd(b, a) == gcd(a, b)}
    {b > 0 && a > 0 && forall r' :: r' == gcd(b, a) ==> r' == gcd(a, b)}
    r := GCD1(b, a);
    {r == gcd(a, b)}
  } else if (a % b == 0) {
    {b == gcd(a, b)}
    r := b;
    {r == gcd(a, b)}
  } else {
    {b > 0 && a % b > 0}
    {b > 0 && a % b > 0 && true}
    {b > 0 && a % b > 0 && gcd(b, a % b) == gcd(a, b)}
    {b > 0 && a % b > 0 && forall r' :: r' == gcd(b, a % b) ==> r' == gcd(a,
b)}}
    {true && forall r' :: r' == gcd(b, a % b) ==> r' == gcd(a, b)}
    r := GCD1(b, a % b);
    {r == gcd(a, b)}
  }
  {r == gcd(a, b)}
}
{r == gcd(a, b)}

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method GCD2(a: int, b: int) returns (r: int)
  requires a >= 0 && b >= 0
  ensures r == gcd(a, b)
  decreases b + 1
{
  {a >= 0 && b >= 0}                                (from gcd rules assumption)
  {true}
  {true && true}                                       (used rule i and tautology
condition)
  {b == 0 ==> a == gcd(a, b) && b != 0 ==> true}
  if b == 0 {
    {a == gcd(a, b)}
    r := a;
    {r == gcd(a, b)}
  } else {
    {true}
    {gcd(b, a % b) == gcd(a, b)}
    {true && forall r' :: r' == gcd(b, a % b) ==> r' == gcd(a, b)}
    r := GCD2(b, a % b);
    {r == gcd(a, b)}
  }
  {r == gcd(a, b)}
}
{r == gcd(a, b)}

```