Homework 5

2021 年 11 月 19、26 日布置

2021 年 12 月 3 日交

1

对于球面

$$ds^2 = a^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

计算其 Gauss 曲率：

$$
\begin{aligned}
K = {} & \frac{1}{2g}\left[2\frac{\partial^2 g_{12}}{\partial x_1 \partial x_2} - \frac{\partial^2 g_{11}}{\partial x_2^2} - \frac{\partial^2 g_{22}}{\partial x_1^2}\right] \\
& - \frac{g_{22}}{4g^2}\left[\left(\frac{\partial g_{11}}{\partial x_1}\right)\left(2\frac{\partial g_{12}}{\partial x_2} - \frac{\partial g_{22}}{\partial x_1}\right) - \left(\frac{\partial g_{11}}{\partial x_2}\right)^2\right] \\
& + \frac{g_{12}}{4g^2}\left[\left(\frac{\partial g_{11}}{\partial x_1}\right)\left(\frac{\partial g_{22}}{\partial x_2}\right) - 2\left(\frac{\partial g_{11}}{\partial x_2}\right)\left(\frac{\partial g_{22}}{\partial x_1}\right) + \left(2\frac{\partial g_{12}}{\partial x_1} - \frac{\partial g_{11}}{\partial x_2}\right)\left(2\frac{\partial g_{12}}{\partial x_2} - \frac{\partial g_{22}}{\partial x_1}\right)\right] \\
& - \frac{g_{11}}{4g^2}\left[\left(\frac{\partial g_{22}}{\partial x_2}\right)\left(2\frac{\partial g_{12}}{\partial x_1} - \frac{\partial g_{11}}{\partial x_2}\right) - \left(\frac{\partial g_{22}}{\partial x_1}\right)^2\right]
\end{aligned}
$$

答案为 $K = \frac{1}{a^2}$。解答时请把每一项写出来

2

局域 Minkowski 坐标系（自由落体坐标系）$\{\xi^\mu\}$ 中 Christoffel symbol ($\Gamma^\lambda_{\ \mu\nu}$) 为 0，在另外一任意坐标系 $\{x^\mu\}$ 中, 证明 $\{x^\mu\}$ 系中的克氏符为

$$\Gamma^\lambda_{\ \mu\nu} = \frac{\partial x^\lambda}{\partial \xi^\alpha}\frac{\partial^2 \xi^\alpha}{\partial x^\mu \partial x^\nu}$$