Homework 13

2022 年 12 月 5 日，9 日布置

2022 年 12 月 12 日交

1 轨道进动

对于度规

$$d\tau^2 = \left(1 - \frac{2GM}{r} + 2(\beta - \gamma)\frac{G^2M^2}{r^2}\right)dt^2 - \left(1 + 2\gamma\frac{GM}{r}\right)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2$$

请证明：

$$\varphi = \frac{2 - \beta + 2\gamma}{3}\text{rad/circle}$$

2 光线偏折

对于度规

$$d\tau^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \left(1 + 2\gamma\frac{GM}{r}\right)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2$$

求出偏折角为

$$\delta = \frac{4GM}{r_0}\left(\frac{1+\gamma}{2}\right)$$

3 雷达回波延迟

对于度规

$$B(r) = 1 - \frac{2GM}{r}$$
$$A(r) = 1 + \gamma\frac{2GM}{r}$$

计算对应时间为

$$t(r, r_0) = \sqrt{r^2 - r_0^2} + (1+\gamma)GM\ln\left(\frac{r + \sqrt{r^2 - r_0^2}}{r_0}\right) + GM\left(\frac{r - r_0}{r + r_0}\right)^{1/2}$$

$$\Delta t = 4GM\left\{1 + \frac{1+\gamma}{2}\ln(\frac{r_E, r_M}{R_\odot^2})\right\} = 5.9\text{km}\left\{1 + 11.2\frac{1+\gamma}{2}\right\}$$