Background flow for Eady model

- f-plane geometry (β = 0).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}, g = 9.81\ ms^{-2}, \Theta_0 = 300\ K, \Delta\Theta = 30, or\ 40;$
- The basic zonal wind has a constant vertical shear: $\frac{d\bar{u}}{dZ} = \Lambda = constant$ or $\bar{u} = \Lambda Z$
  - Thermal wind relation:

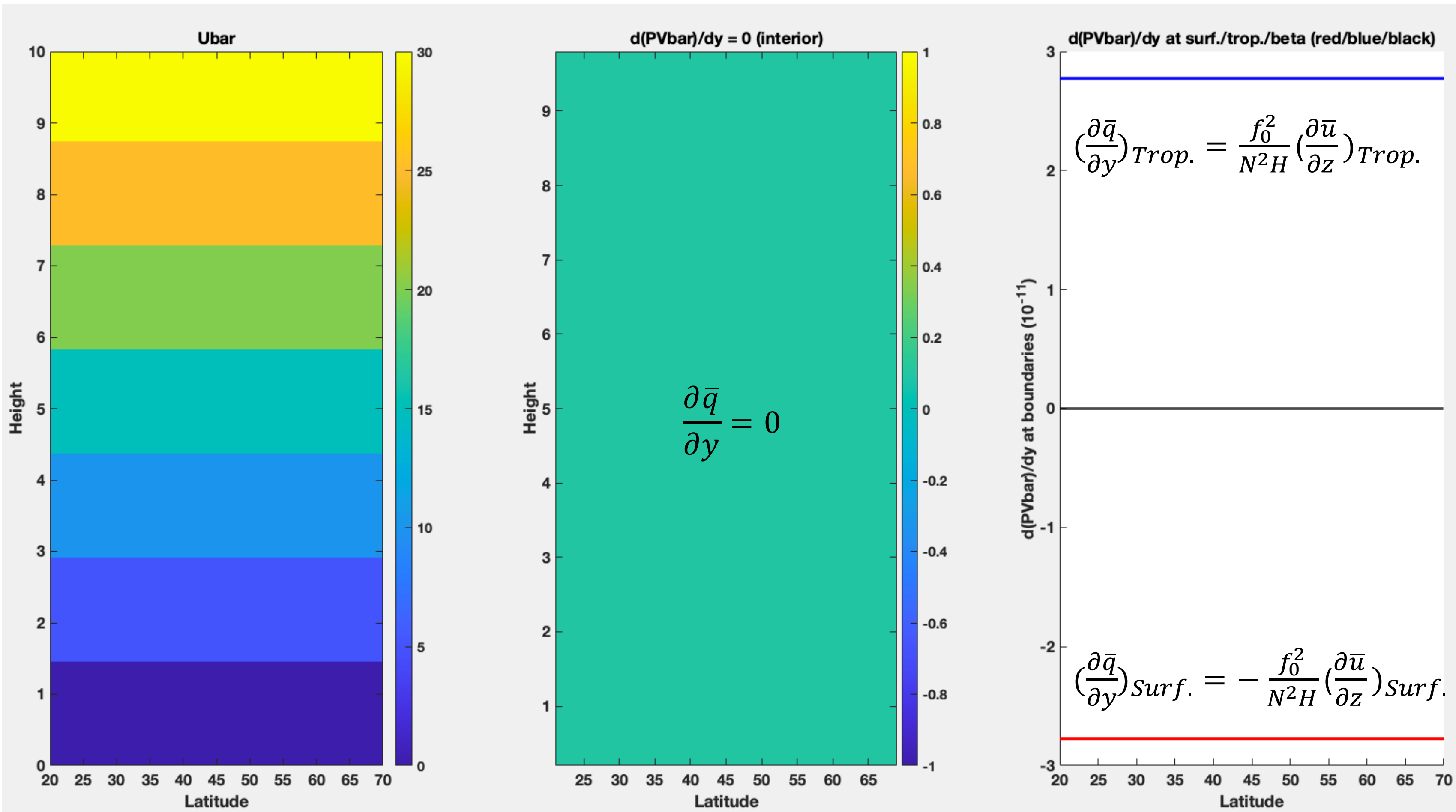
$$\frac{d\bar{u}}{dZ} = \Lambda = -\frac{g}{f_0\Theta_0}\frac{\partial\bar{T}}{\partial y} = \frac{g}{f_0\Theta_0}\frac{\bar{T}_{south} - \bar{T}_{north}}{L_y};$$

$$\frac{g}{f_0\Theta_0}\frac{\bar{T}_{south} - \bar{T}_{north}}{L_y} = \frac{g}{f_0\Theta_0}\frac{\Delta\bar{T}}{L_y};$$

$$\bar{u}_{j,k} = (\frac{g}{f_0\Theta_0}\frac{\Delta\bar{T}}{L_y})Z_k - U_0\ \ for\ all\ j\ (j = 1, 2, \dots, J + 1)$$

Eady Model's background flow

$\Delta\bar{T} = 60; U_0 = 0$



Mid-Latitude Synoptic Meteorology: Dynamics, Analysis, and Forecasting, by Gary Lackmann

Eady Model's Solution

$\Delta\bar{T} = 50; m = 8$

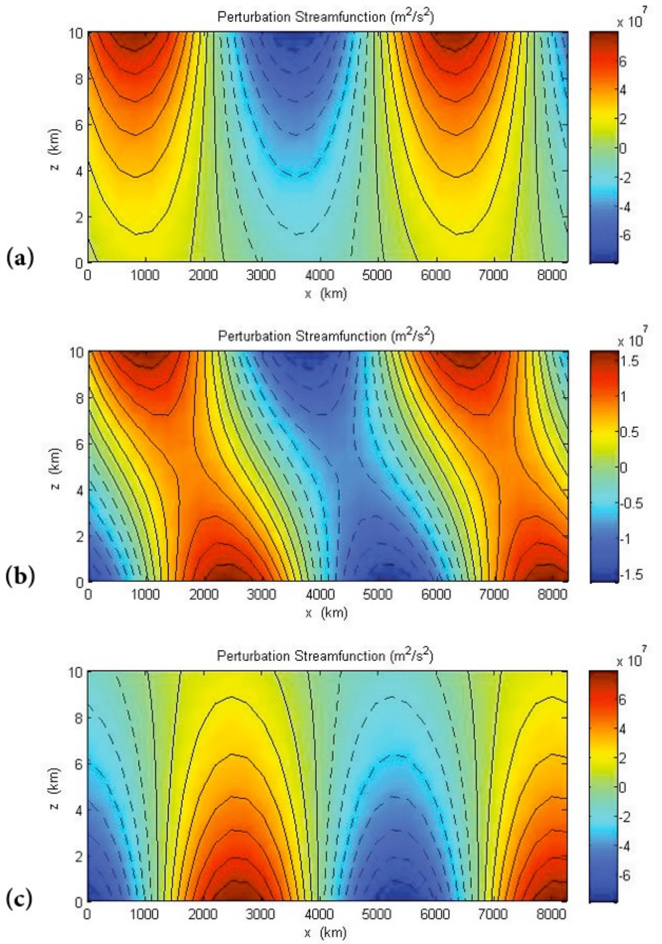


Figure 7.6. Perturbation streamfunction for most unstable Eady mode, contoured and shaded: (a) top disturbance only; (b) full solution; and (c) lower disturbance only.

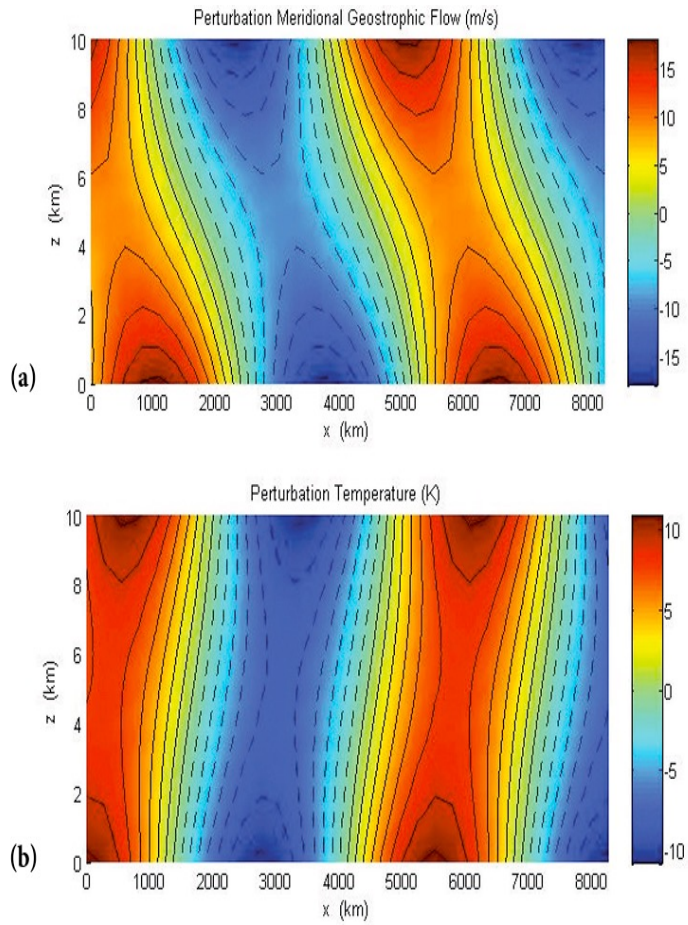
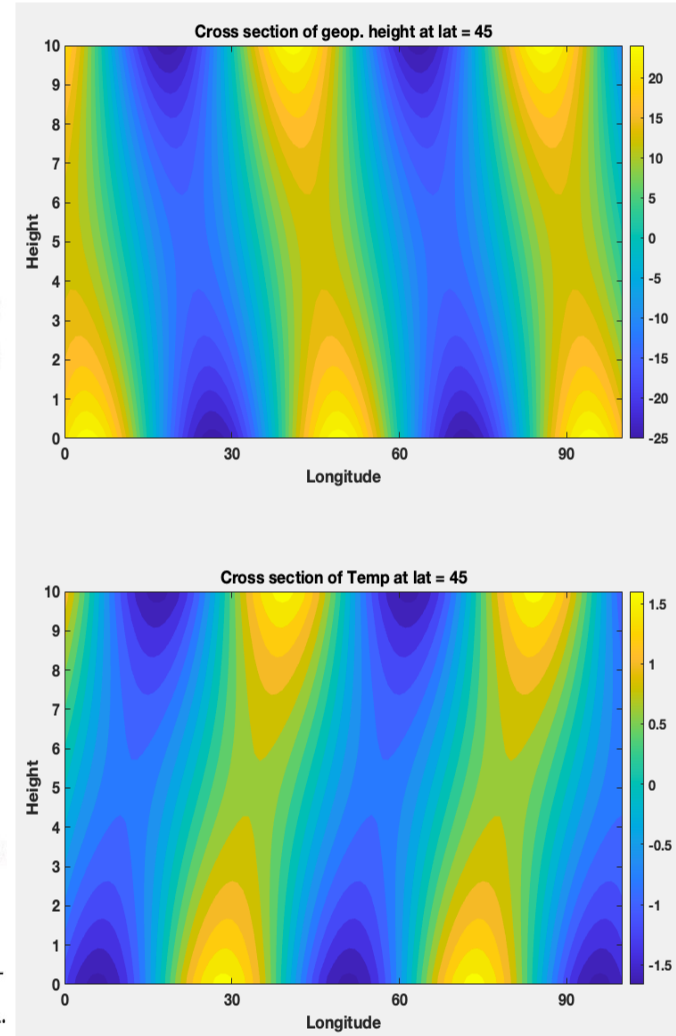


Figure 7.7. Perturbation (a) meridional geostrophic wind and (b) temperature components, shaded and contoured with negative values dashed.



Eady Model's Solution

$\Delta\bar{T} = 60; m = 8$

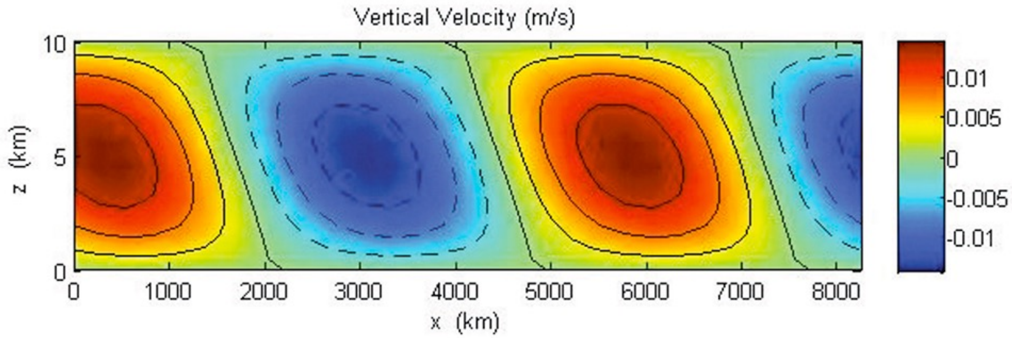
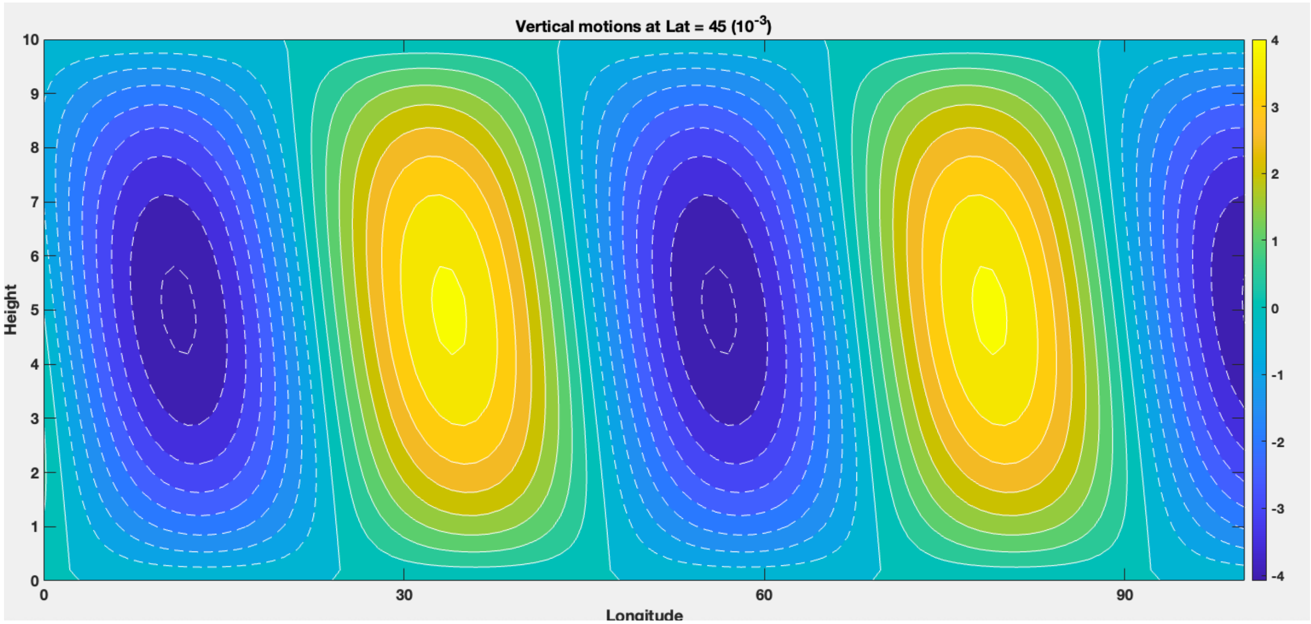


Figure 7.8. Vertical velocity for most unstable Eady mode; warm colors and solid contours correspond to ascent, and cold colors and dashed contours correspond to descent.



Eady Model’s Solution

$\Delta\bar{T} = 50; m = 8$

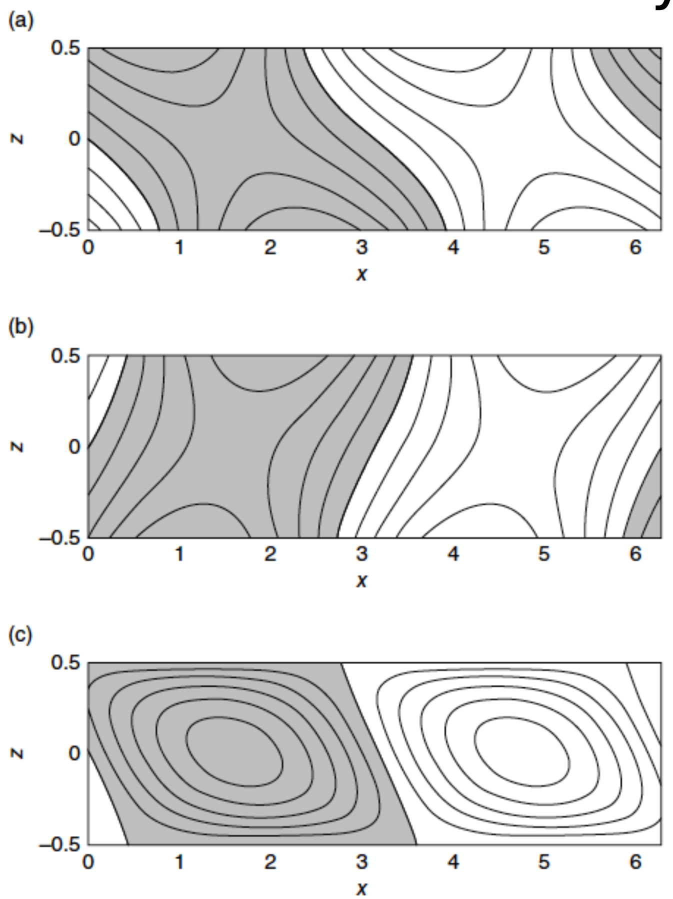
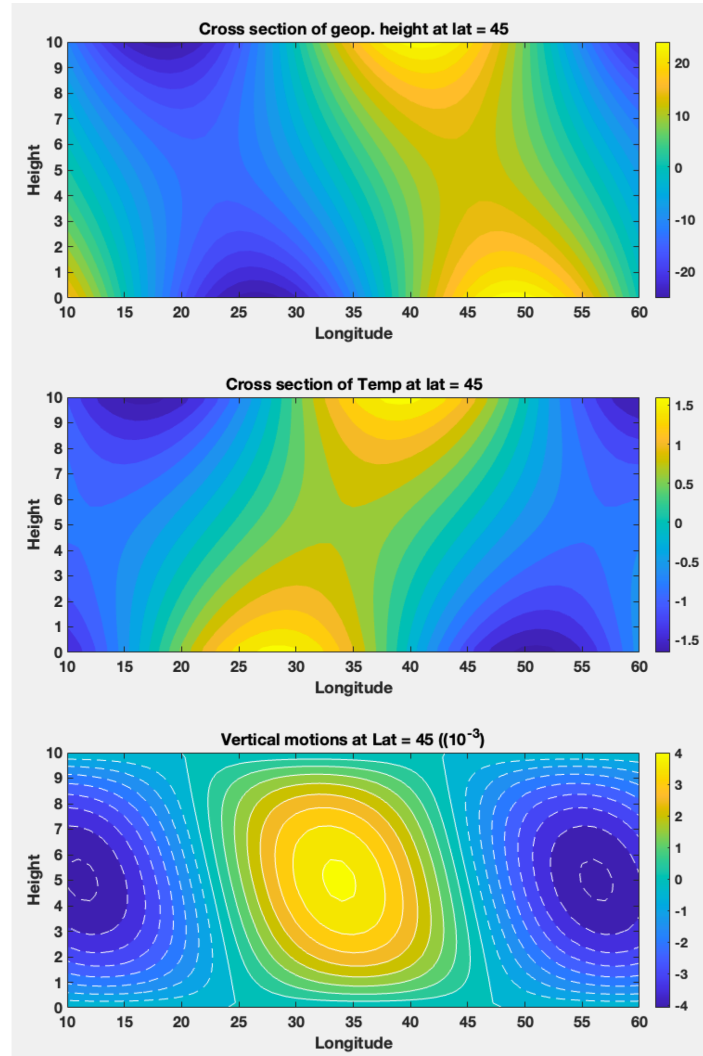
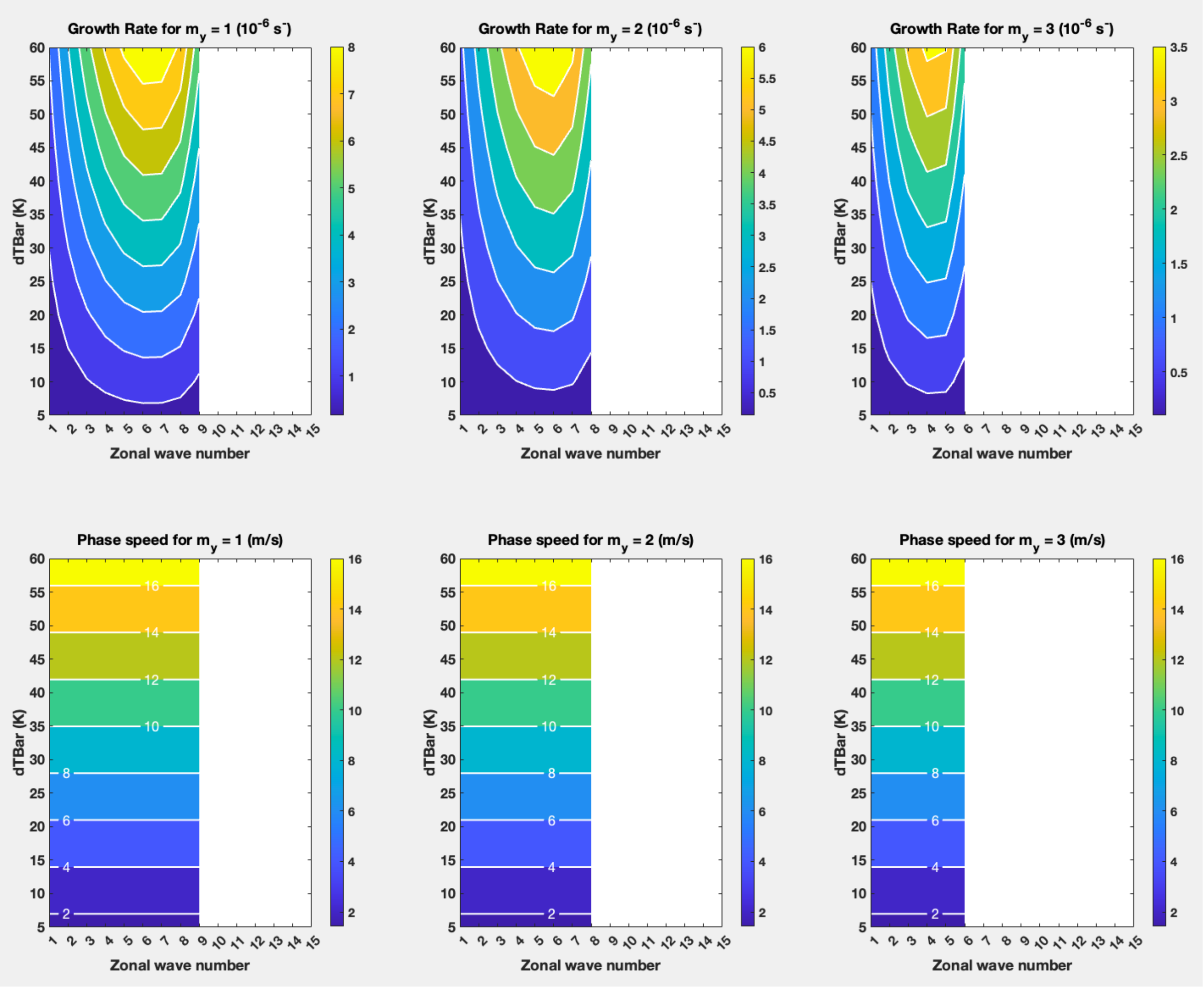


Figure 14.11 The structure of the fastest growing Eady wave. (a) Poleward velocity component, (b) buoyancy perturbation and (c) vertical velocity. In (a), the low and high in streamfunction and pressure are located on the zero contours of the poleward wind. In each case, the field has been arbitrarily normalized so that its maximum value is 1; contour interval 0.2; negative values shaded



Eady Model's Solution

$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = from\ 5\ to\ 60K$



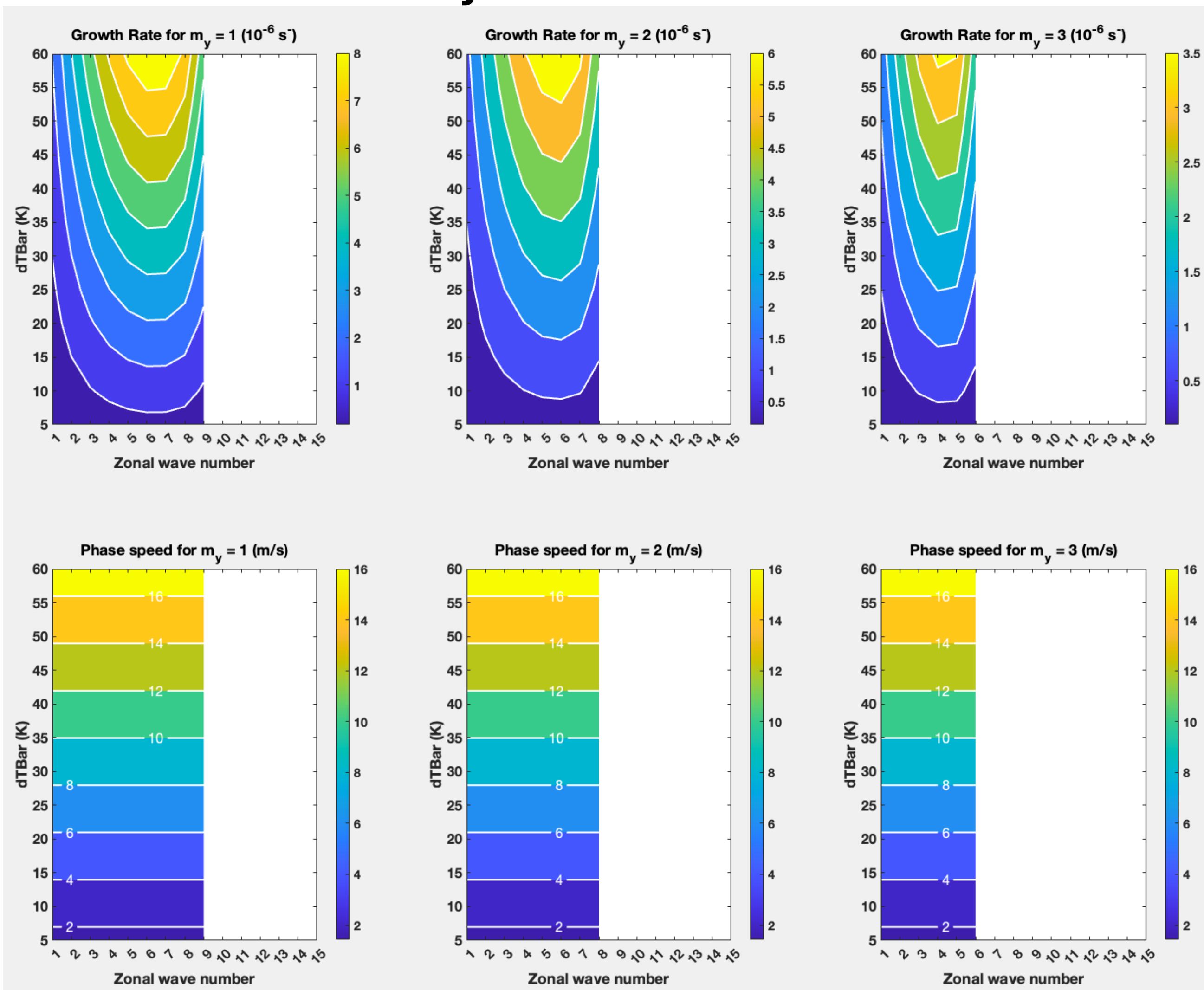
Eady Model's Solution

How to obtain solutions for $\Delta\bar{T}\ from\ 5\ to\ 60K\ and\ m_0\ from\ 1\ to\ 15\ in\ a\ single\ run$?

```
model parameters (like NN2, Lx, jj, kk, ll etc.)

Evec_output=zeros(ll,ll,15, 12);
Eval_output=zeros(ll,ll,15,12);
B_output=zeros(ll,ll,15,12);
For slevel = 1:12
    dTbar = 5*slevel
    for m0 = 1:15

        your original model code here

        % do not save data for individual solutions for each slevel and m0, but do the following
        Evec_output(:,:,m0,slevel)=eigVec2(:,:);
        Eval_output(:,m0,slevel)=eigVal2(:);
        B_output(:,:,m0,slevel)=B(:,:);
    end
end

save filename.mat
```

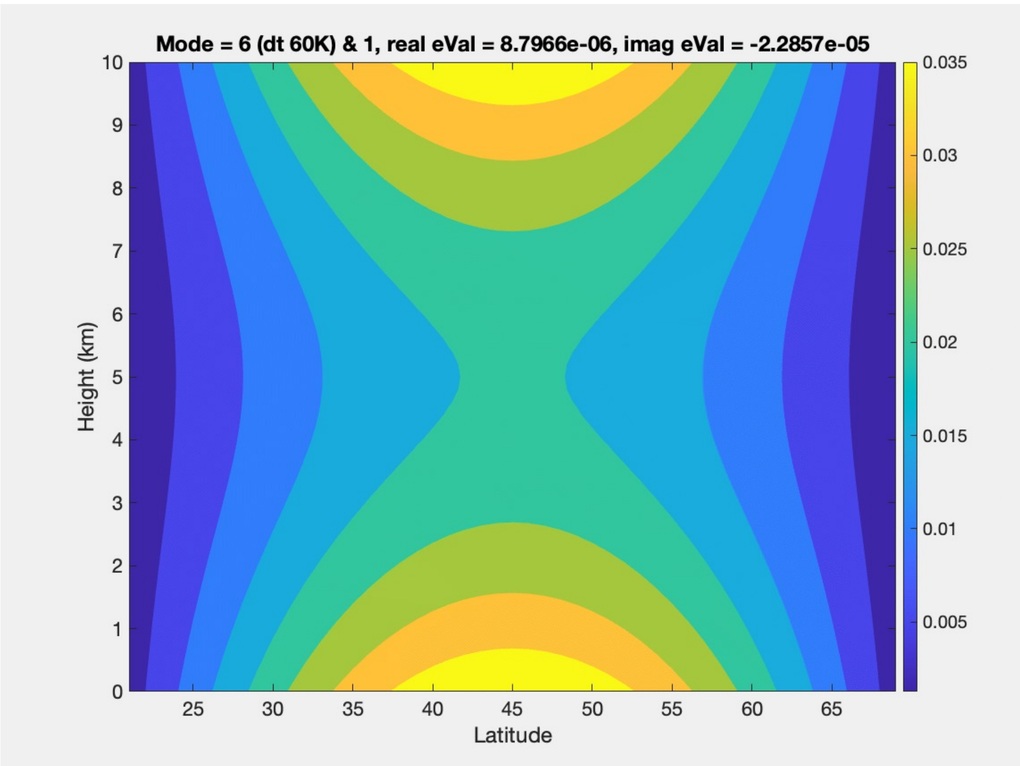
Eady Model's Solution

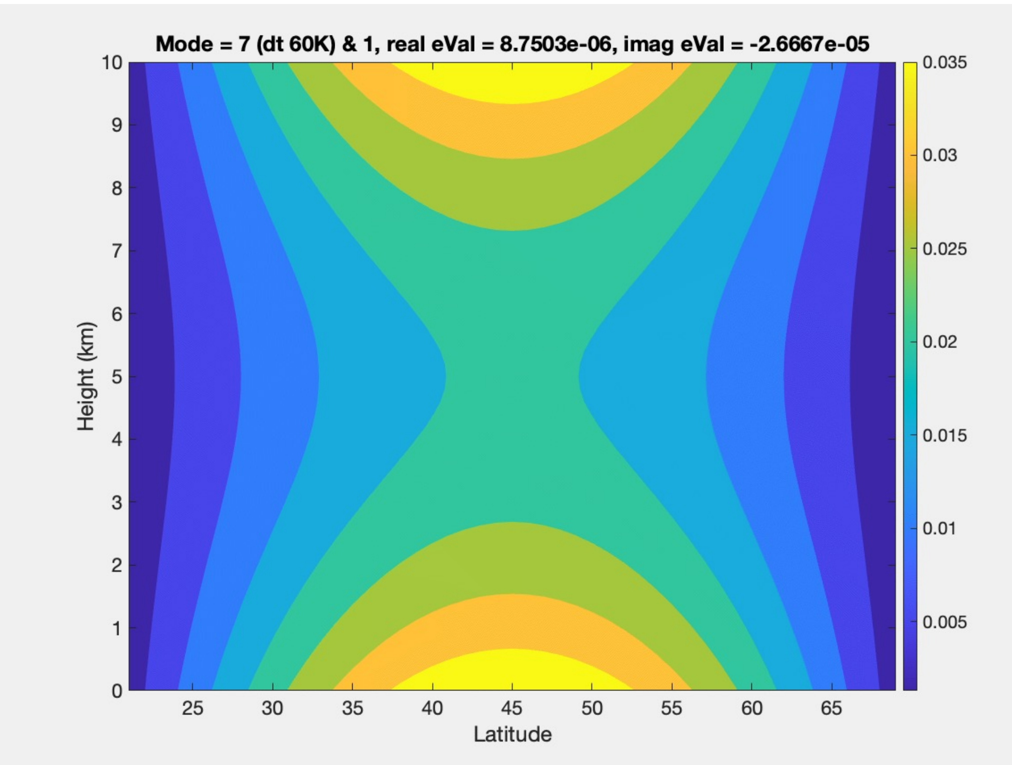
$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = from\ 5\ to\ 60K$



Eady Model's Solution

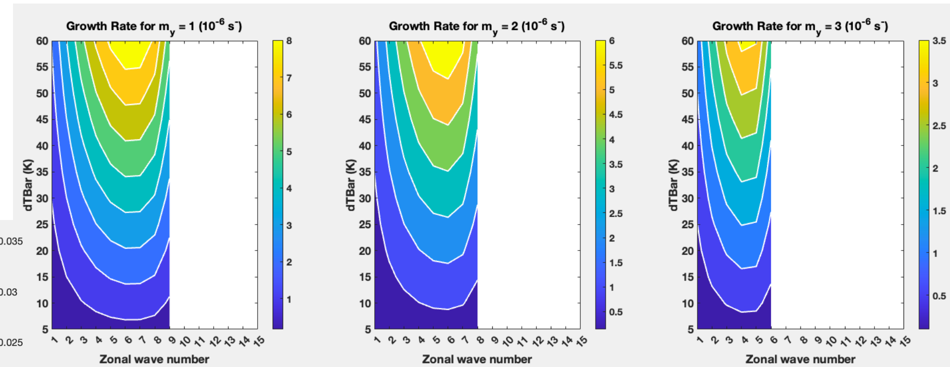
$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = from\ 5\ to\ 60K$

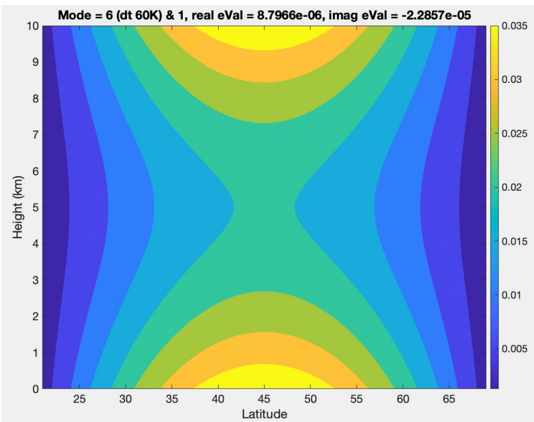


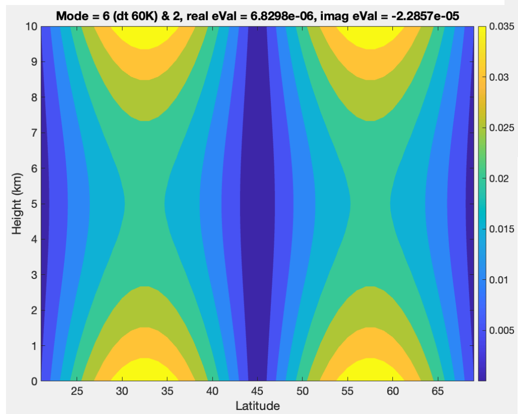


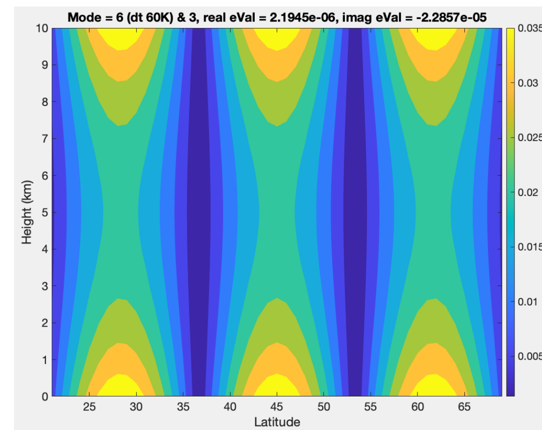
Eady Model’s Solution

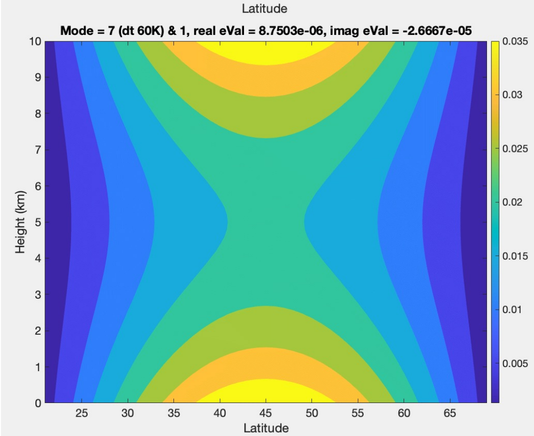
$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = from\ 5\ to\ 60K$

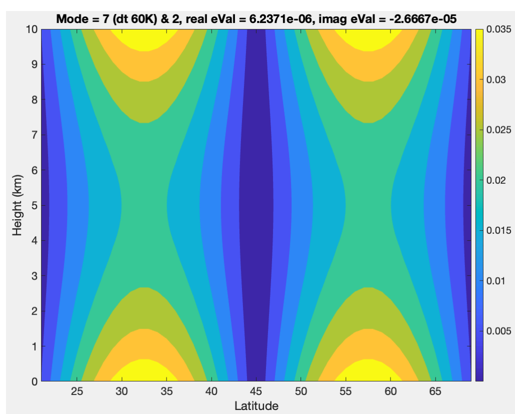






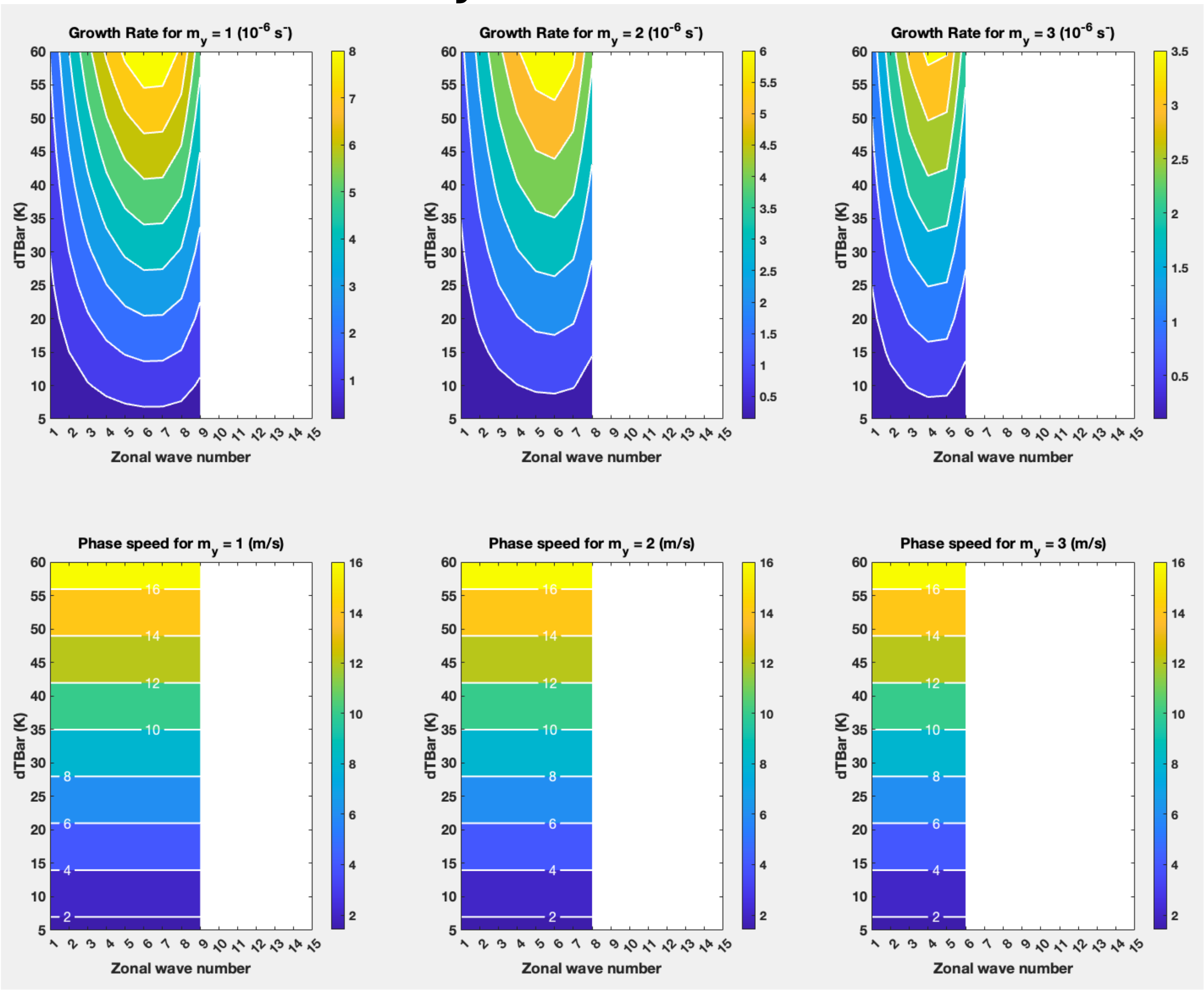






Eady Model's Solution

$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = from\ 5\ to\ 60K$



Eady Model's Solution

$\Delta\bar{\Theta} = 40K; \Delta\bar{T} = 60K$

$k: non - dimensional\ zonal\ wave\ number$

$$k = \frac{NH}{f_0}\frac{2\pi m}{L_X}; \quad \mathrm{m} = \frac{f_0 L_X}{2\pi NH}k$$

```
m = (f0*Lx/(2*pi*sqrt(NN2)))*[0.5:0.5:3];
```

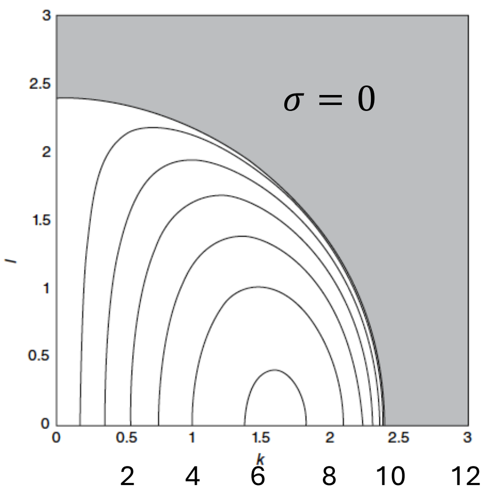
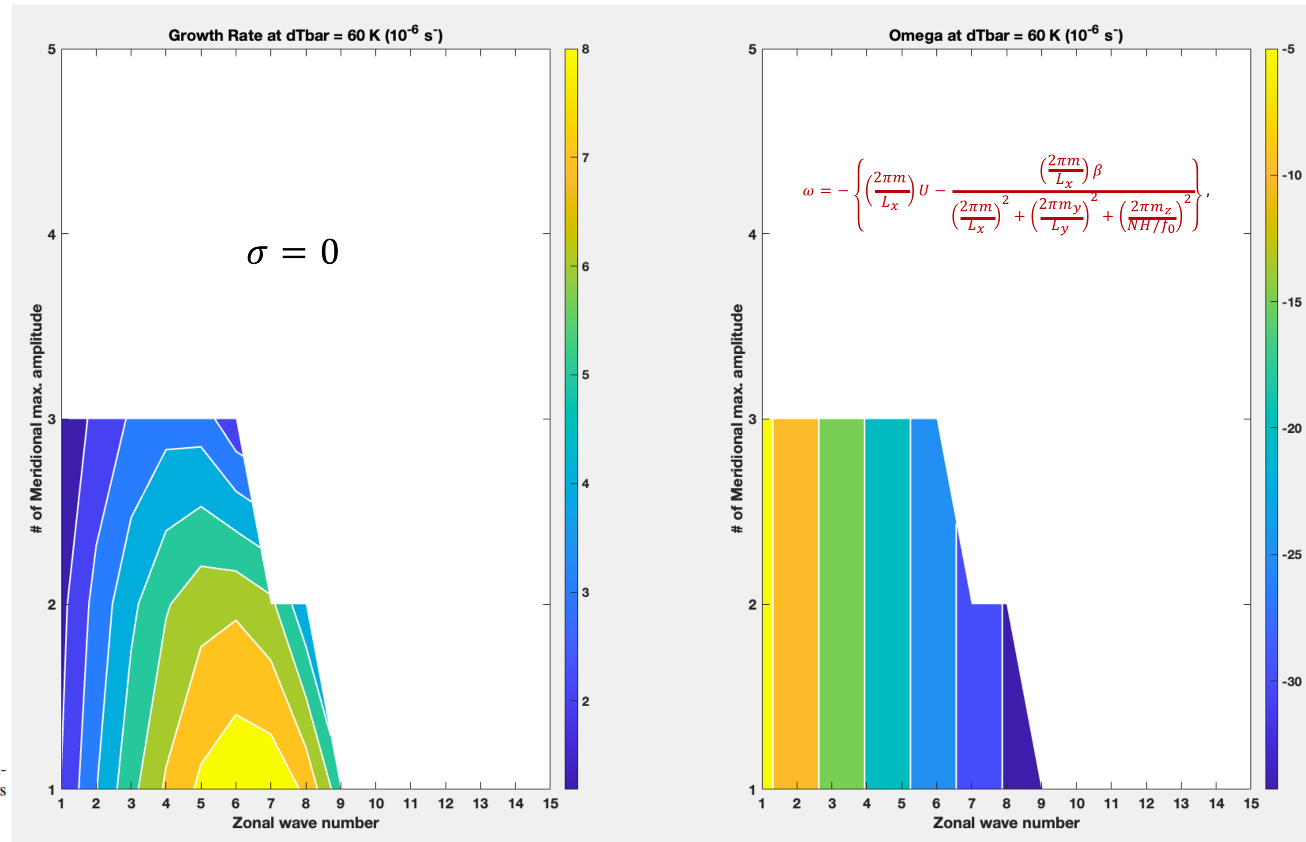


Figure 14.10 Growth rate for the Eady model as a function of zonal wavenumber k and meridional wavenumber l. Contour interval $0.05(f_0 / N)\bar{u}_z$. Shading indicates regions where there is no instability



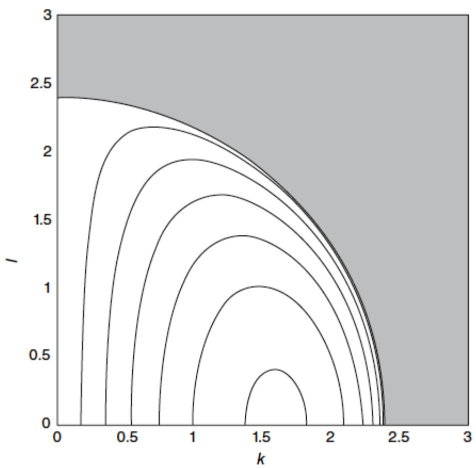
Eady Model’s Solution

$\Delta\overline{\Theta} = 30K;\ \Delta\overline{T} = 60K$

$k: non-dimensional\ zonal\ wave\ number$

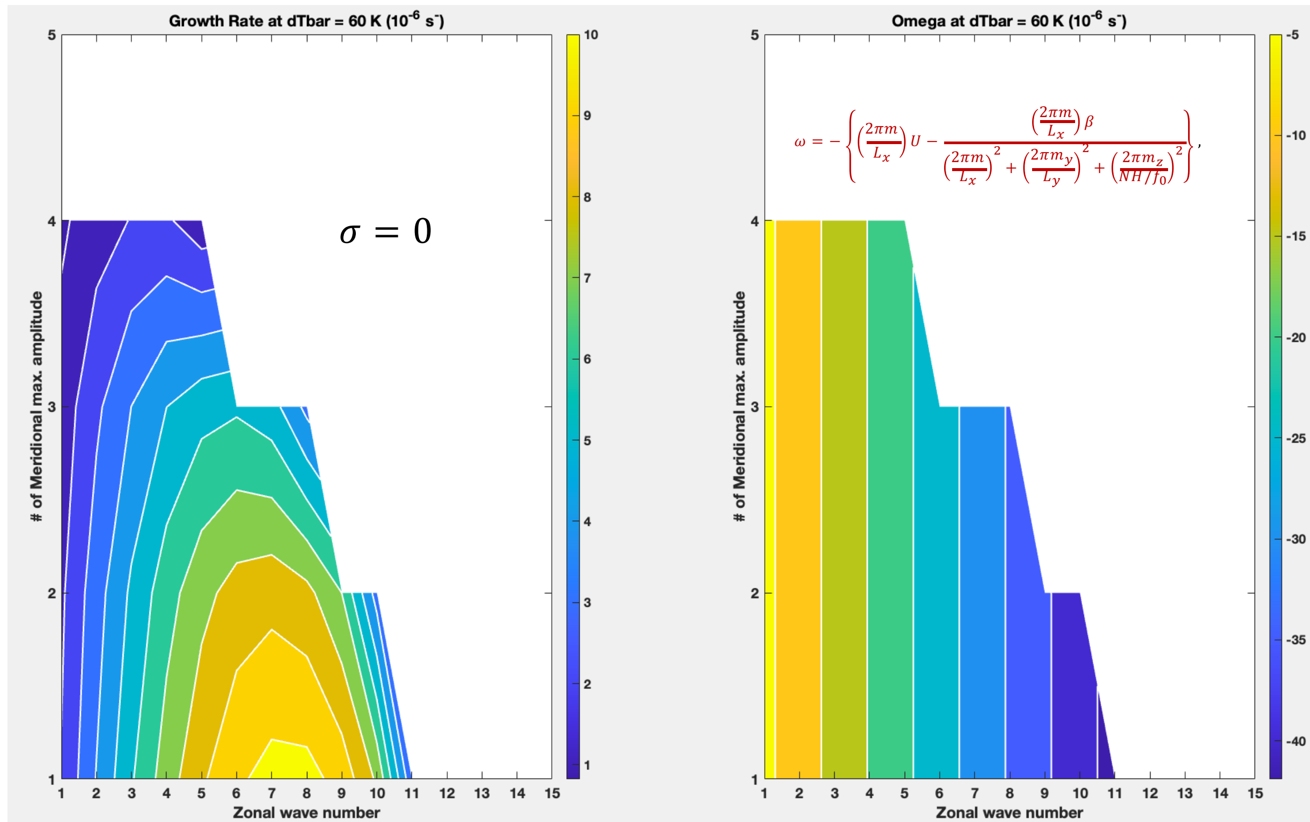
$$k = \frac{NH}{f_0}\frac{2\pi m}{L_X};\quad \mathrm{m} = \frac{f_0 L_X}{2\pi NH}k$$

$\sigma = 0$



2.3 4.7 7.0 9.4 11.7 14.

Figure 14.10 Growth rate for the Eady model as a function of zonal wavenumber k and meridional wavenumber l. Contour interval $0.05(f_0/N)\overline{u}_z$. Shading indicates regions where there is no instability

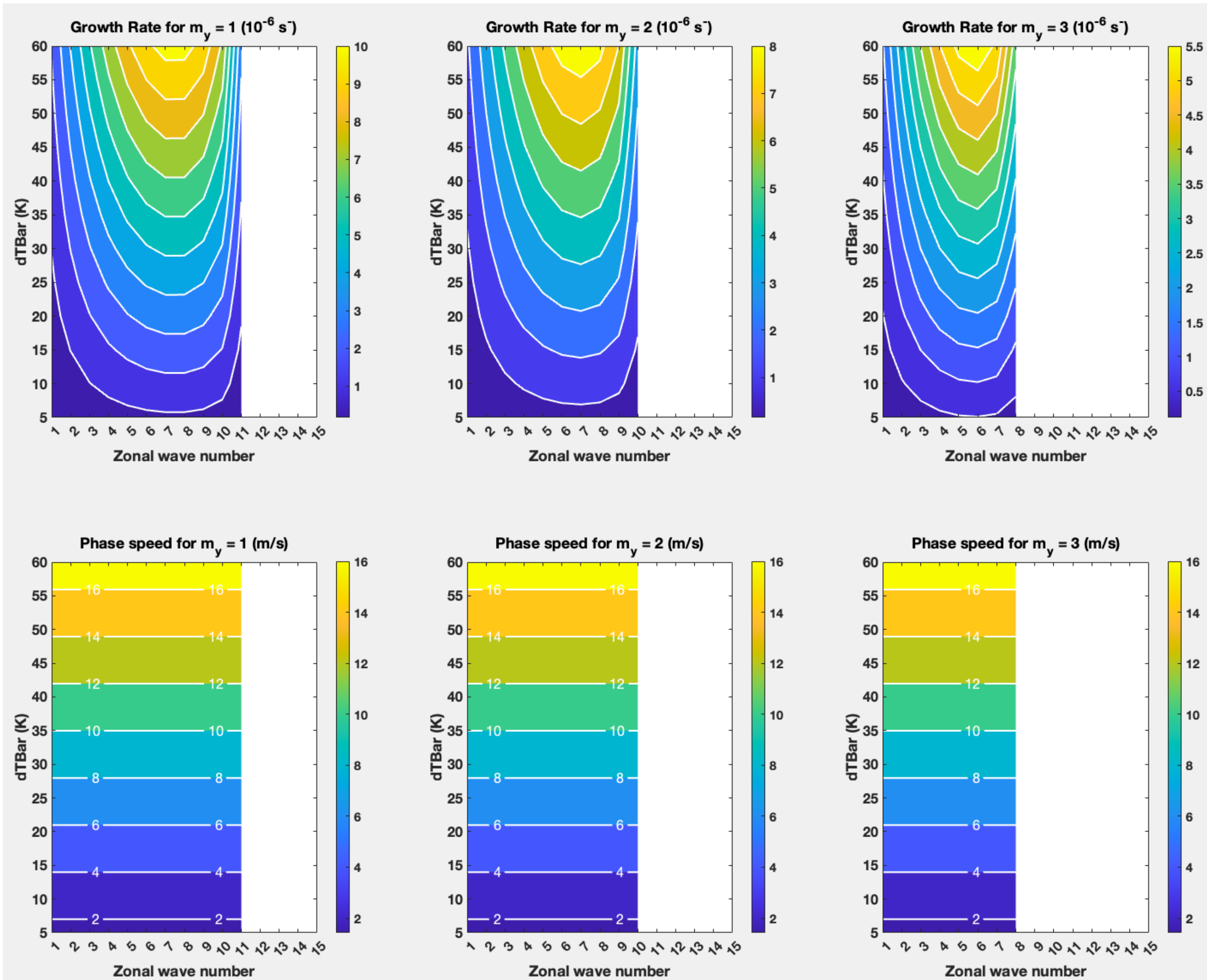


m = (f0*Lx/(2*pi*sqrt(NN2)))*[0.5:0.5:3];

2.33851984025044 4.67703968050089 7.01555952075133 9.35407936100177 11.6925992012522 14.0311190415027

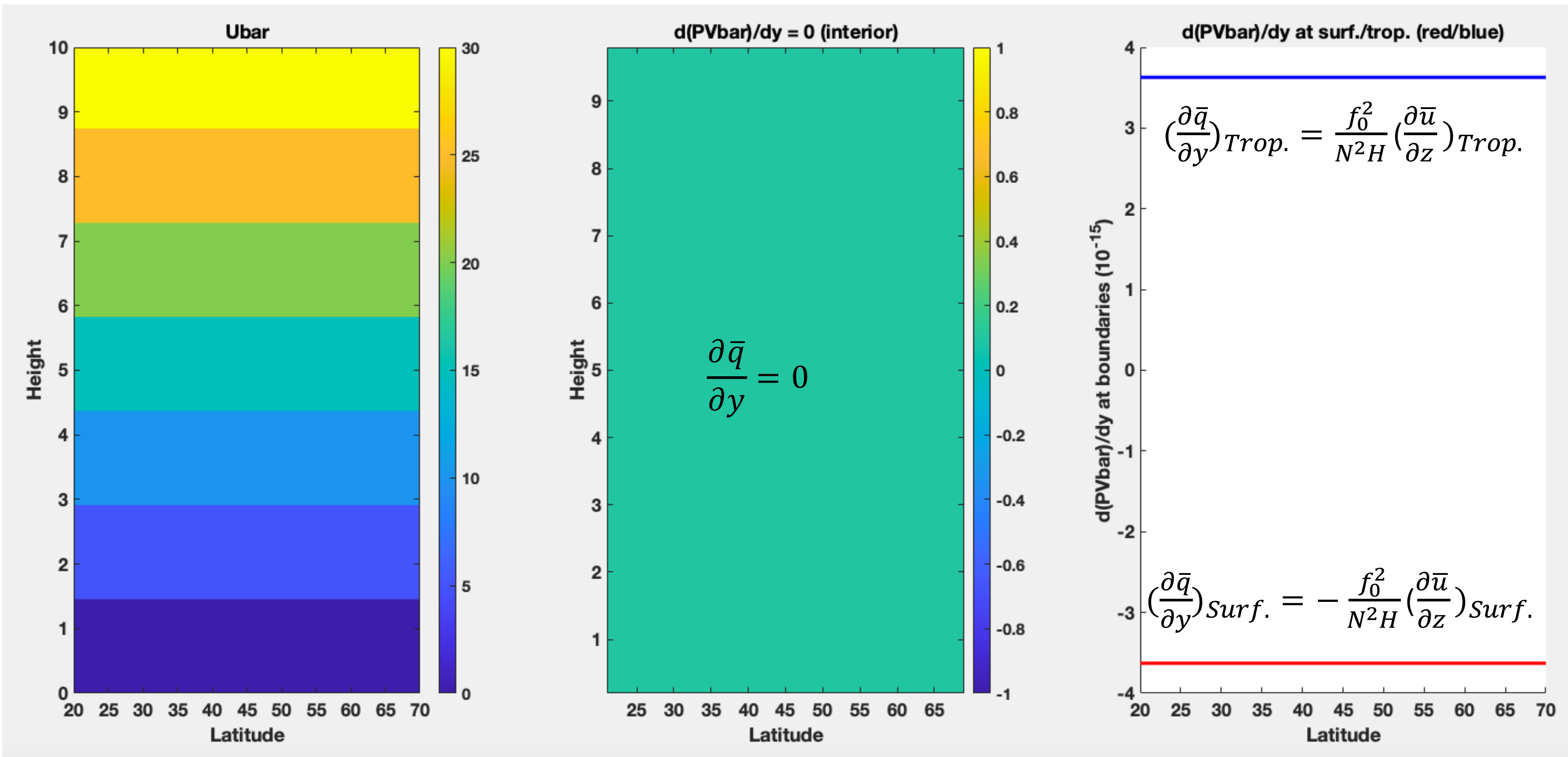
Eady Model's Solution

$\Delta\bar{\Theta} = 30K; \Delta\bar{T} = from\ 5\ to\ 60K$



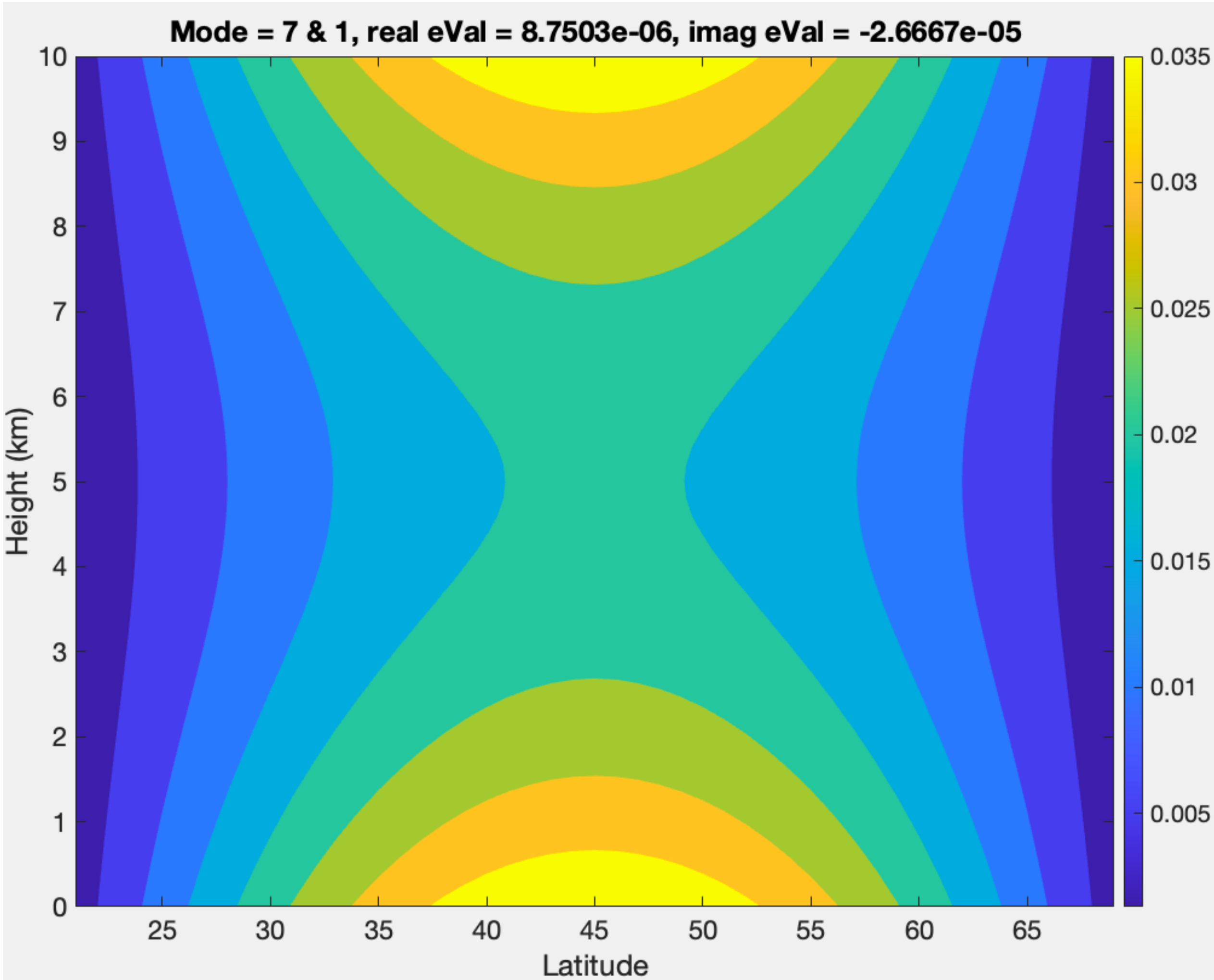
Eady Model's background flow

$\Delta\bar{T} = 60; U_0 = 0$

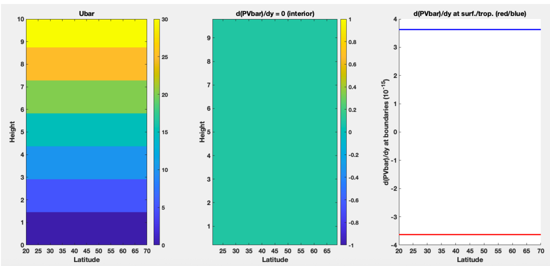


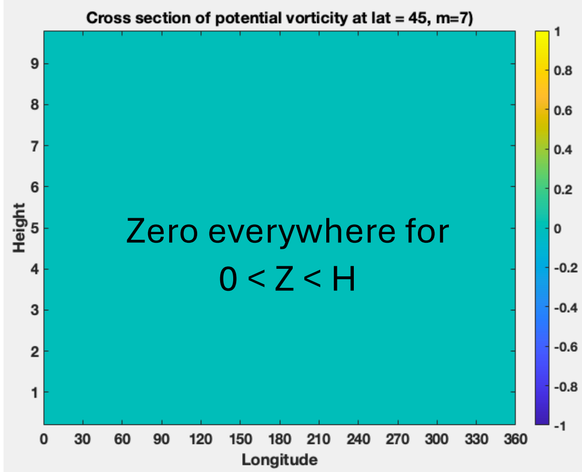
Eady Model's Solution

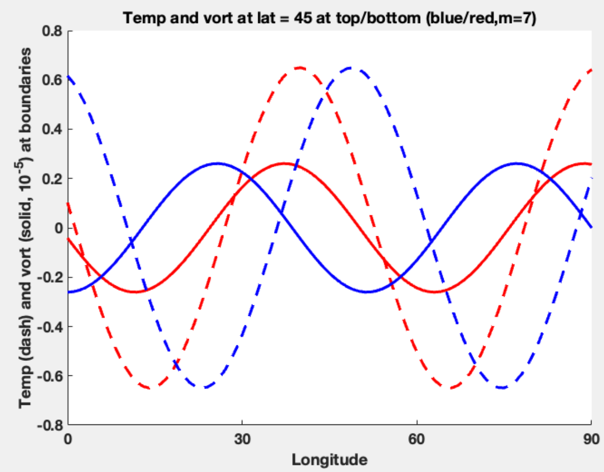
$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = 60K; m = 7$

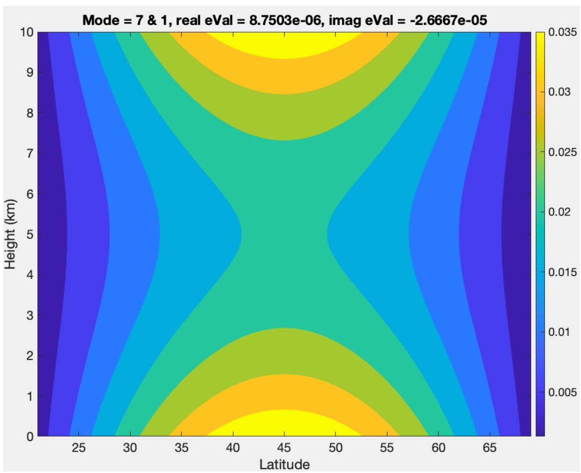


Do unstable waves in Eady model have potential vorticity?

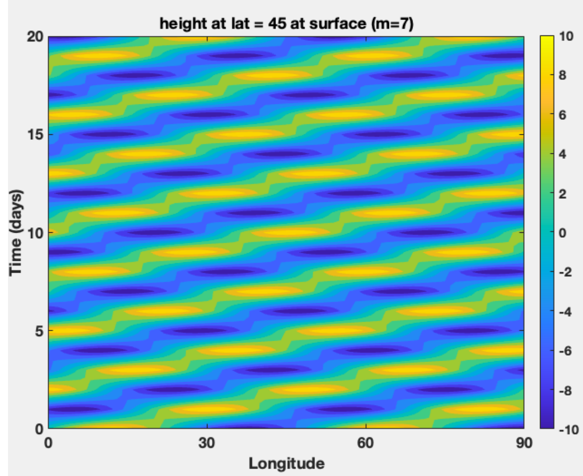


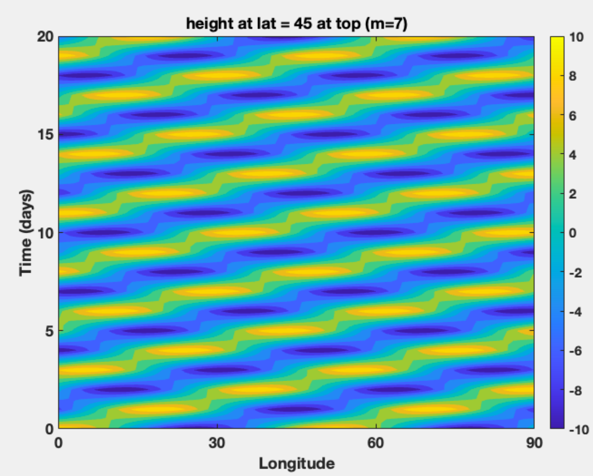






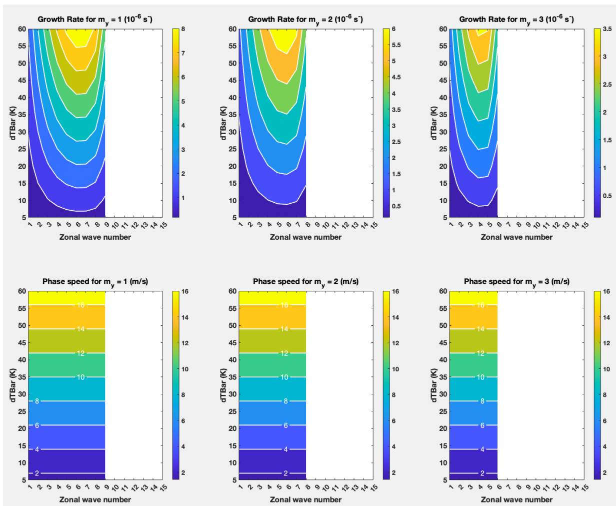
Phase speed=(Lx/4)/(5day*86400sec/day)=16.4 m/s



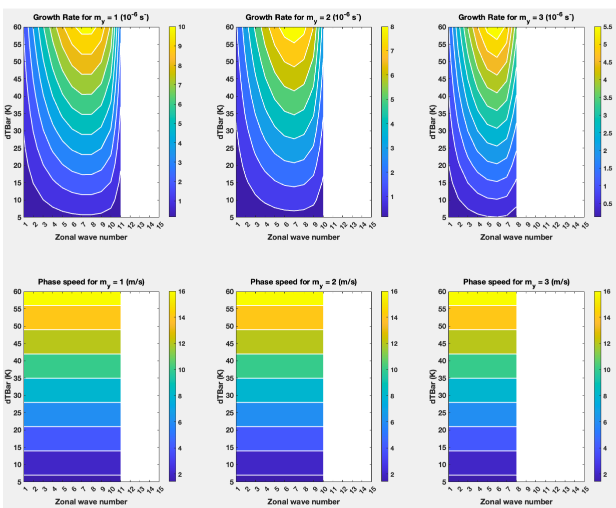


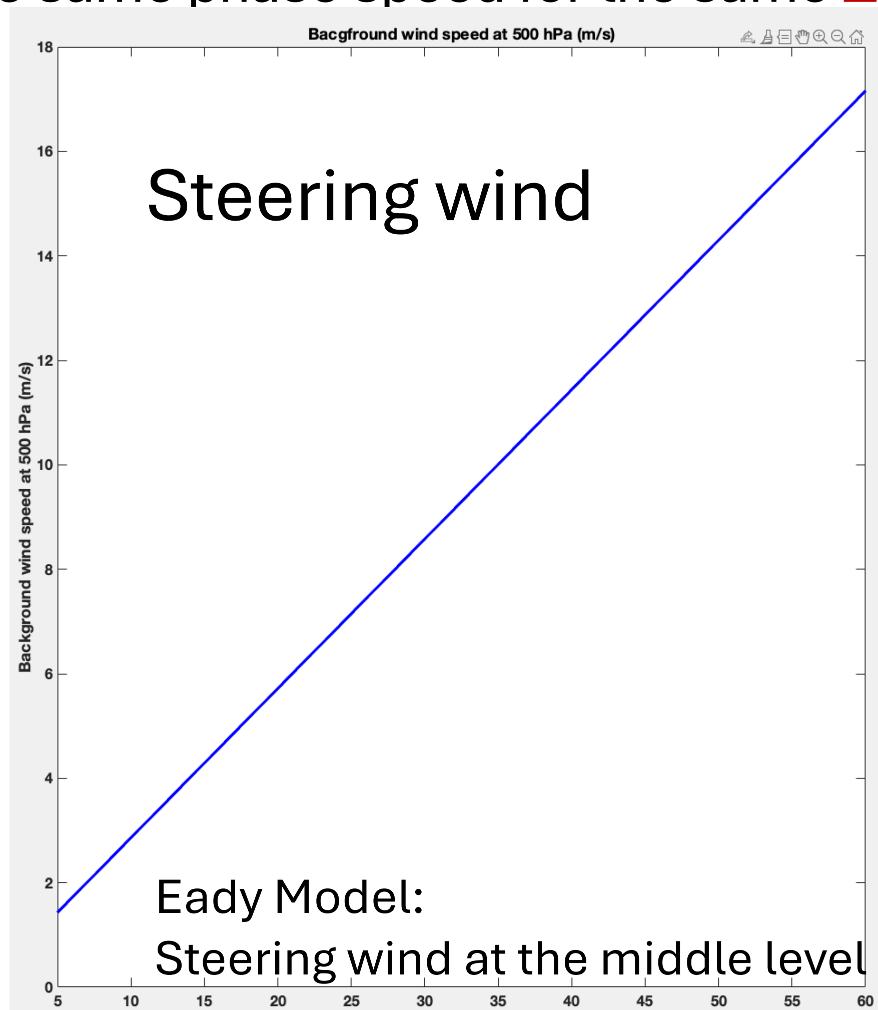
Why do all unstable waves travel at the same phase speed for the same ΔT_{bar}?

$\Delta\overline{\Theta} = 40K$



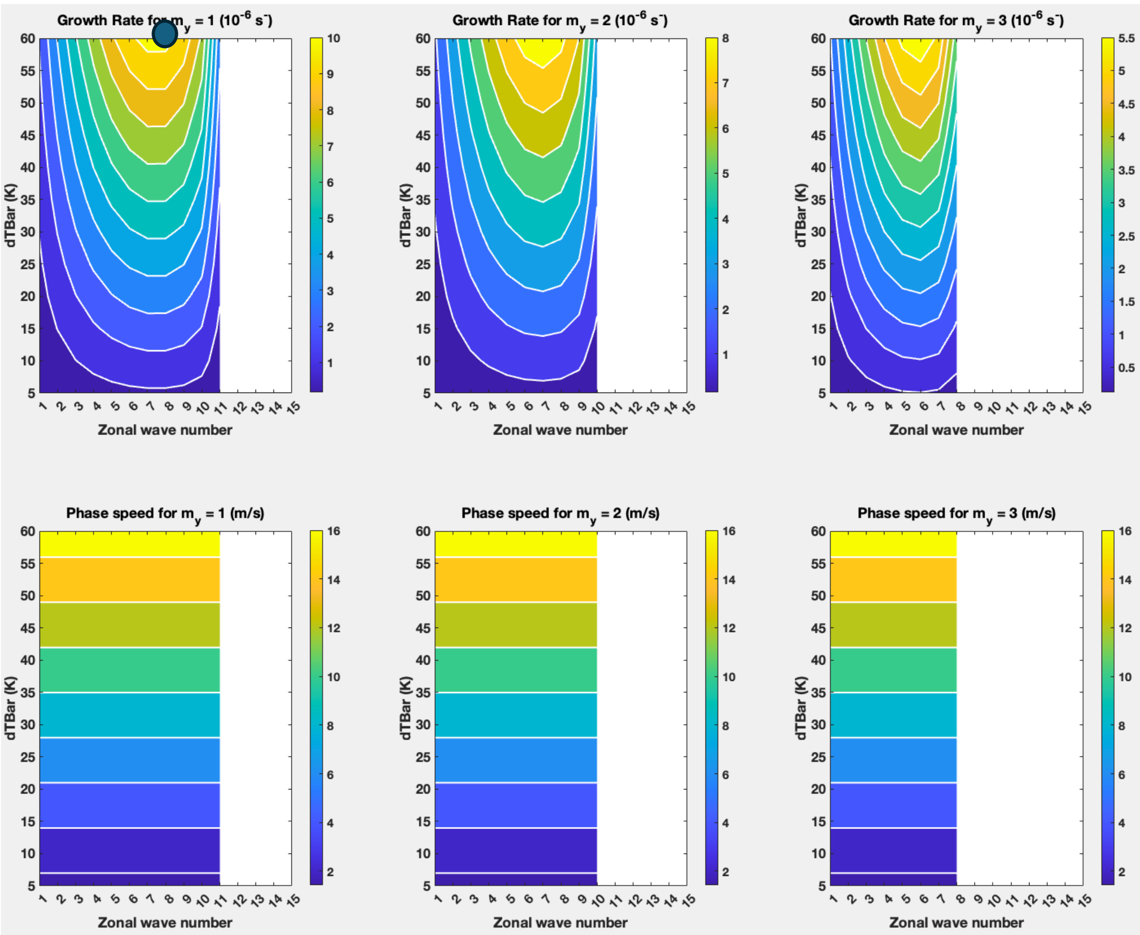
$\Delta\overline{\Theta} = 30K$



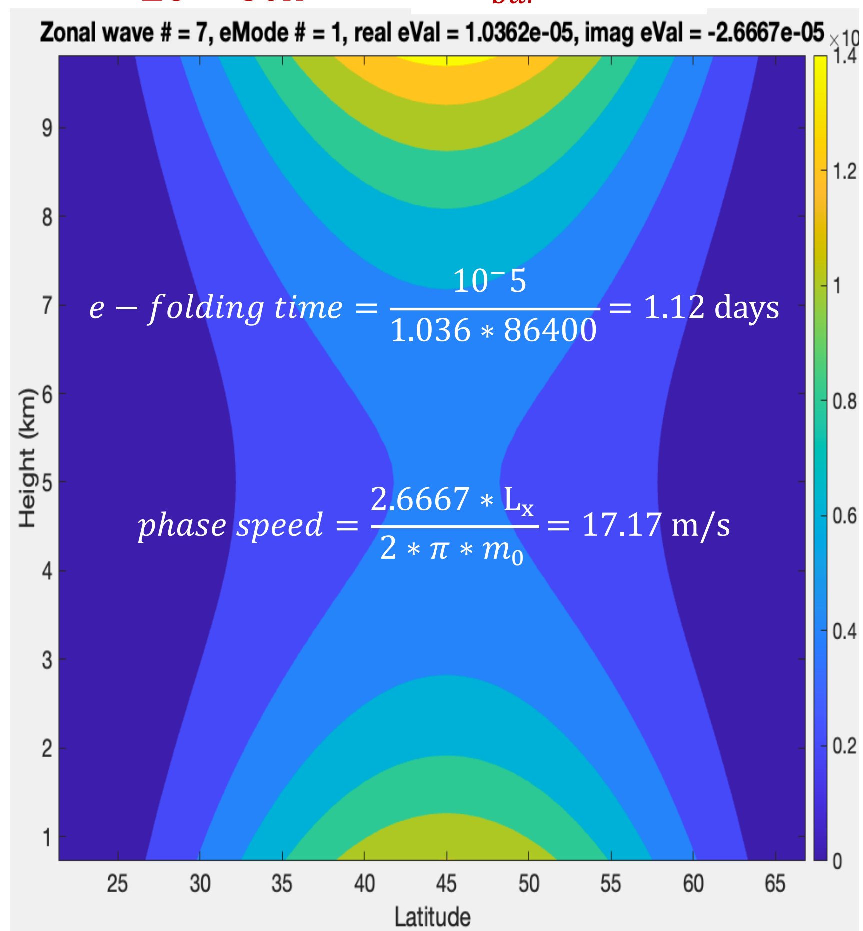


$\Delta\overline{T}_{bar}$

The most unstable mode



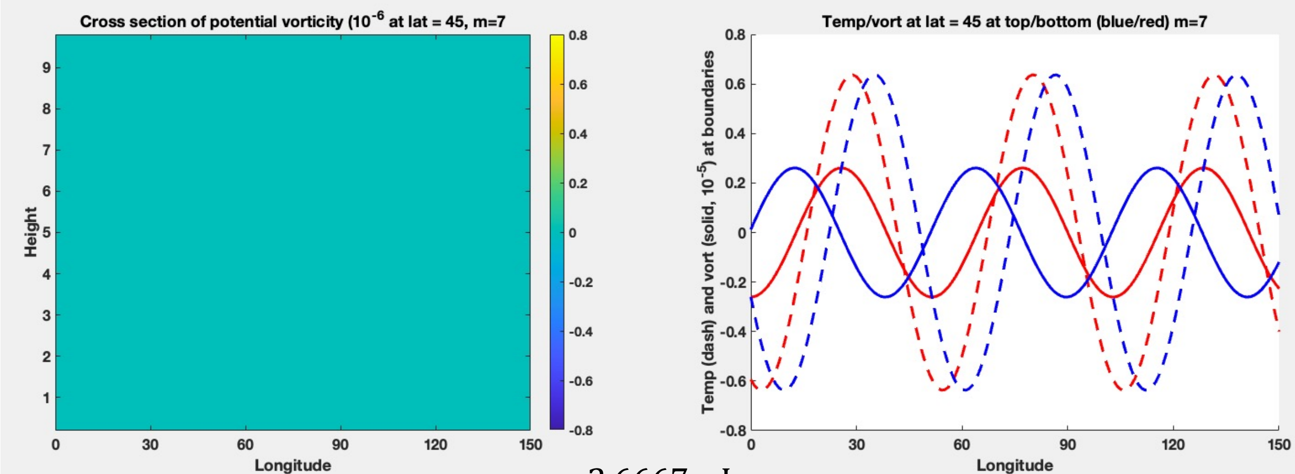
$\Delta\bar{\Theta} = 30K$ $\quad$ $\Delta\bar{T}_{bar} = 60\ \mathrm{K}$



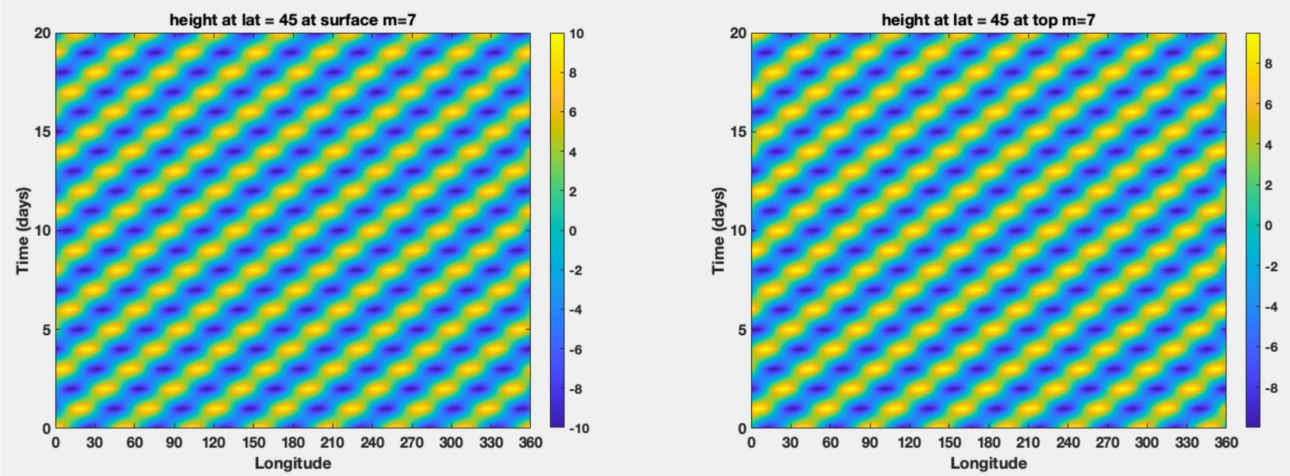
The most unstable mode

$$\Delta\bar{\Theta} = 30K$$

$$\Delta\bar{T}_{bar} = 60\text{ K};$$
$$\mathrm{m}_0 = 7;$$
$$\mathrm{n}_{\mathrm{mode}} = 1$$



$$phase\ speed = \frac{2.6667 * \mathrm{L_x}}{2 * \pi * m_0} = 17.17\text{ m/s}$$



$$\mathrm{L_x} = 2.8301\mathrm{e} + 07;$$

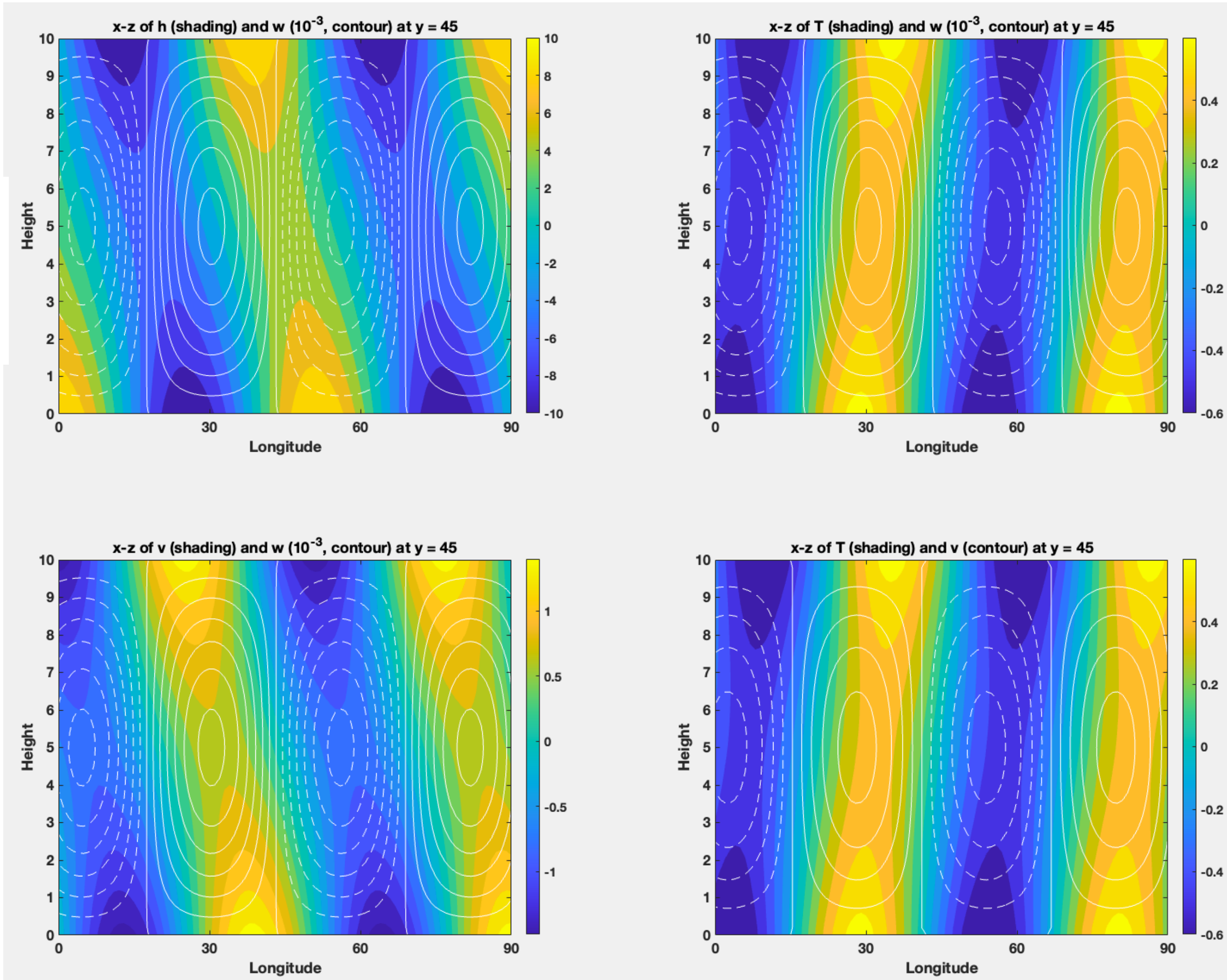
$$\text{Time} \approx 18.5\text{ days}$$

$$\frac{\mathrm{L_x}}{time} = 17.7\text{ m/s}$$

Relationships among variables

$\Delta\overline{\Theta} = 30K$

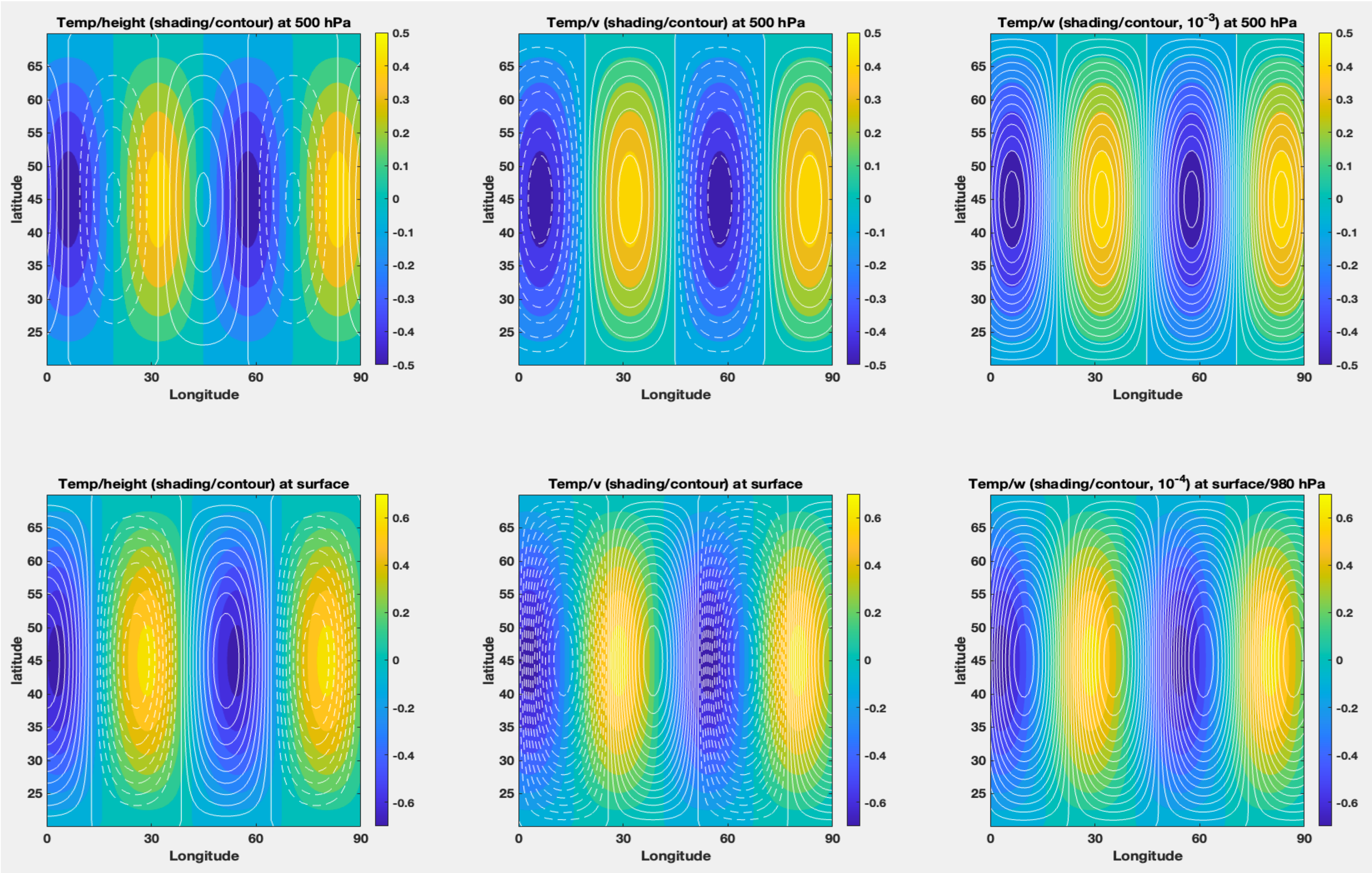
$\Delta\bar{\bar{T}}_{bar} = 60$ K;
$\mathrm{m}_0 = 7$;
$\mathrm{n}_{\mathrm{mode}} = 1$



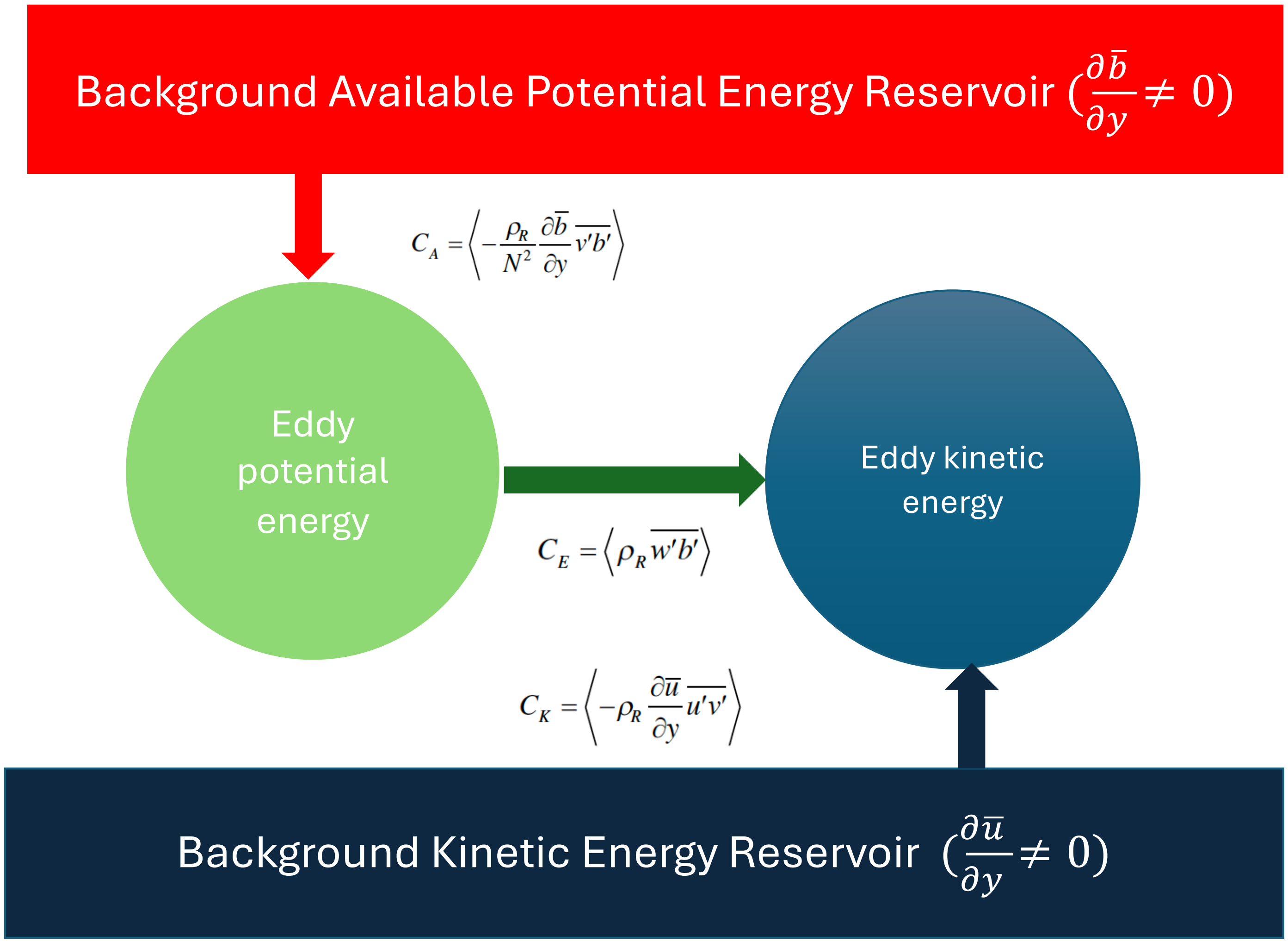
Relationships among variables

$$\Delta\overline{\Theta} = 30K$$

$$\Delta\bar{T}_{bar} = 60\ \mathrm{K};$$
$$\mathrm{m}_0 = 7;$$
$$\mathrm{n}_{\mathrm{mode}} = 1$$

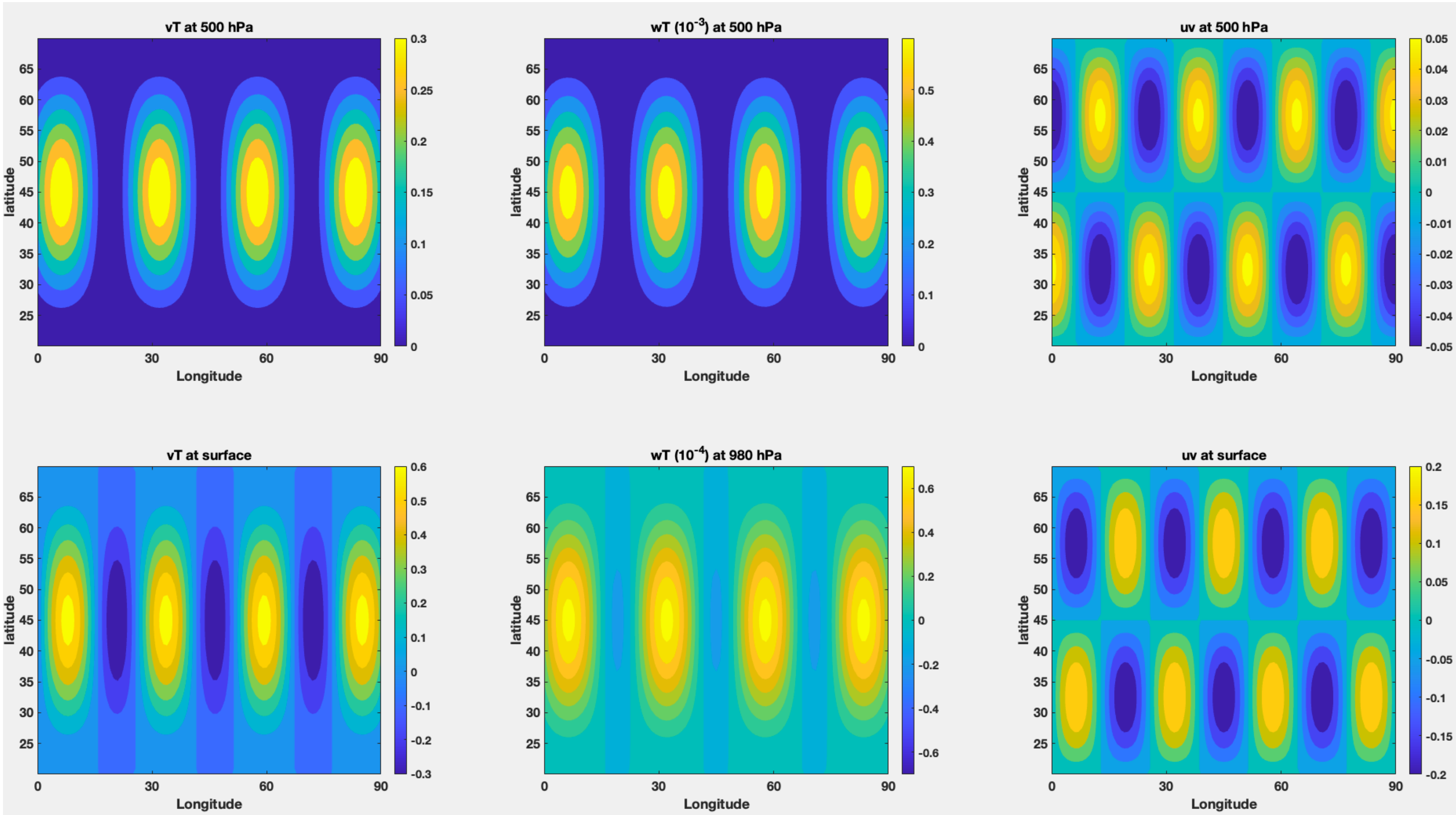


4.7 Energetics of baroclinic waves (H16.5)



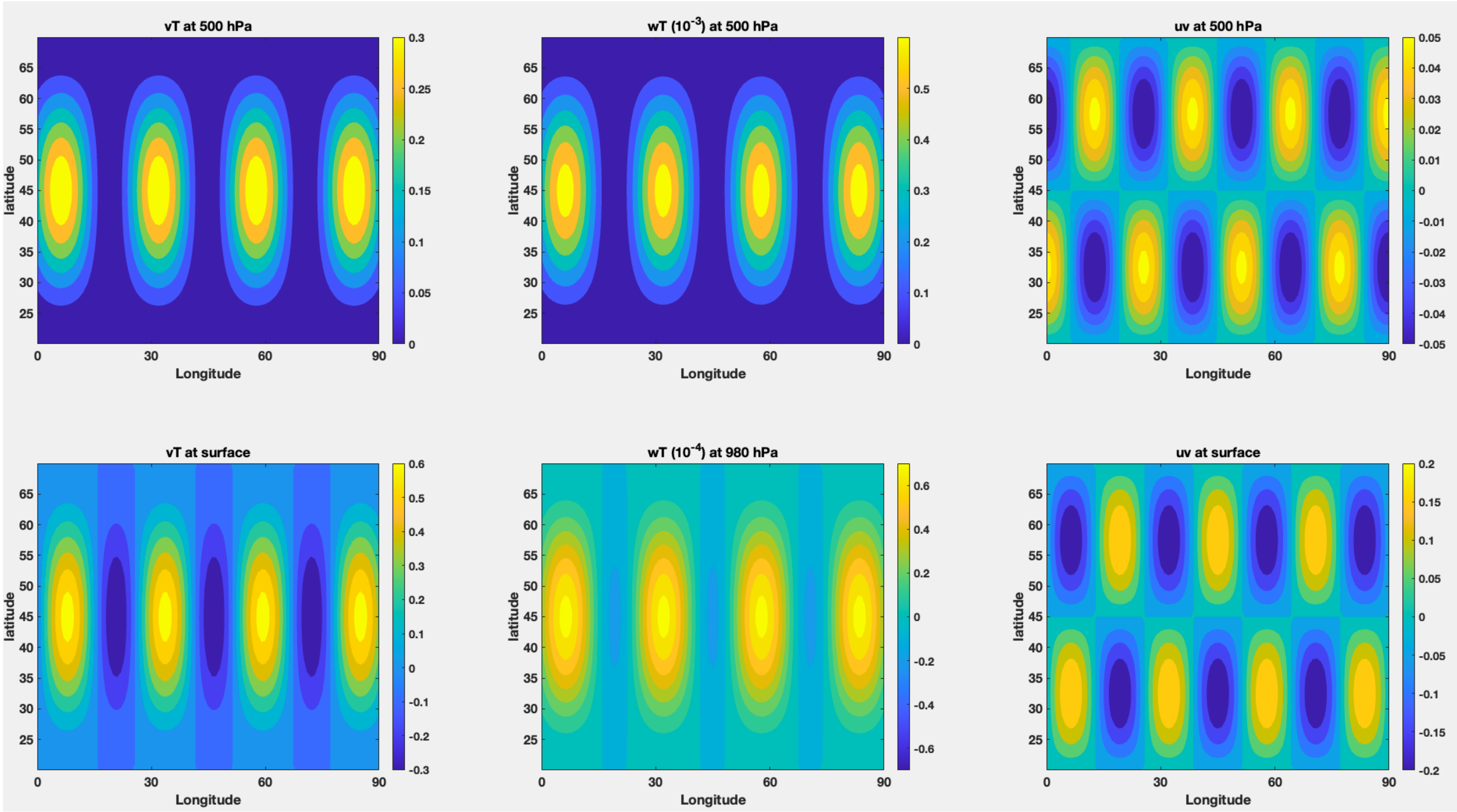
Fluxes of unstable modes

$\Delta\bar{\Theta} = 30K$ $\Delta\bar{T}_{bar} = 60\ \mathrm{K}; \mathrm{m}_0 = 7; \mathrm{n}_{\mathrm{mode}} = 1$



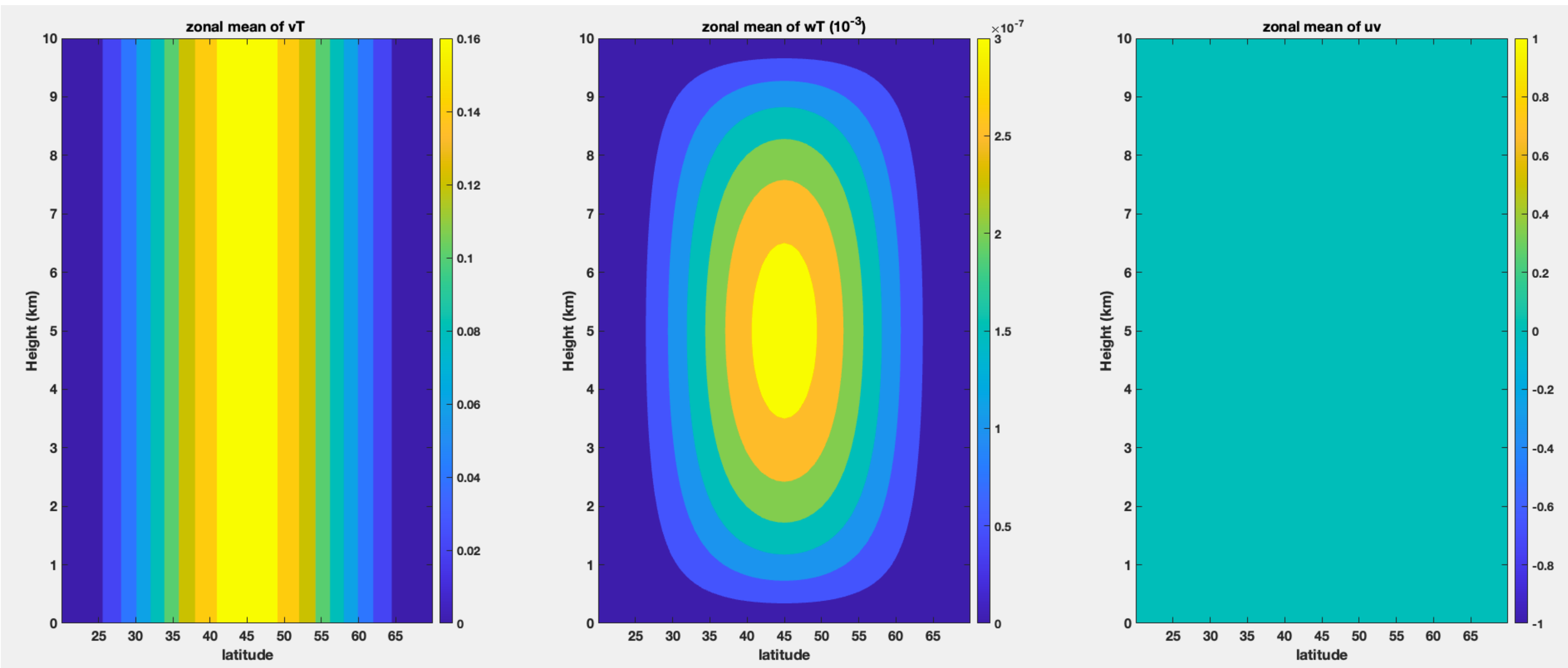
Maps of fluxes of unstable modes

$\Delta\bar{\Theta} = 30K$ $\Delta\bar{T}_{bar} = 60\ \mathrm{K}; \mathrm{m}_0 = 7; \mathrm{n}_{\mathrm{mode}} = 1$

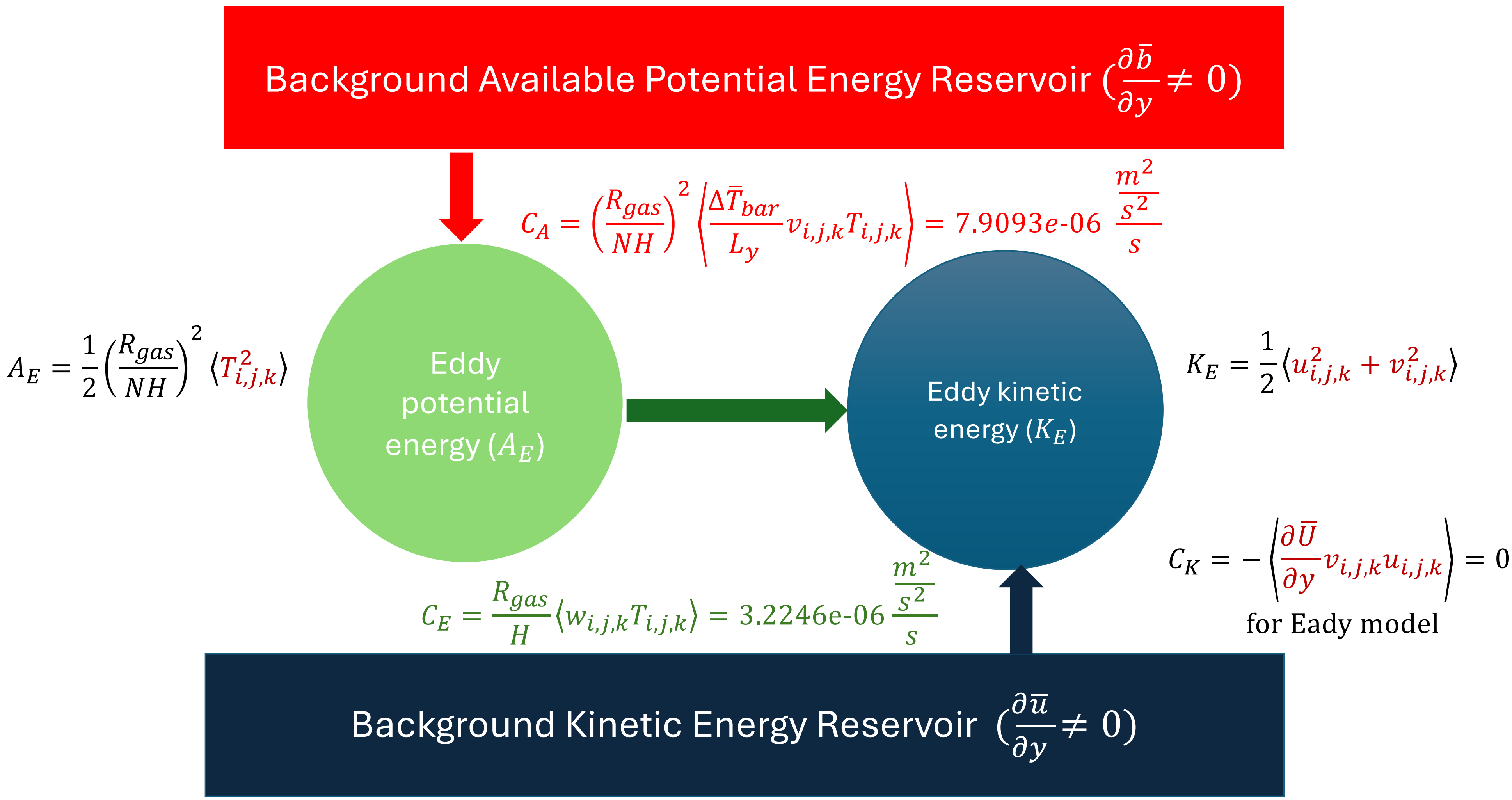


Zonal mean of fluxes

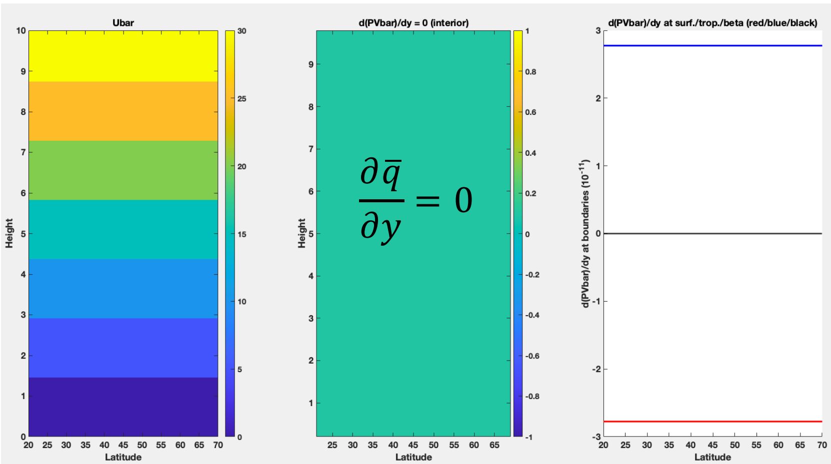
$\Delta\bar{\Theta} = 30K$ $\Delta\bar{T}_{bar} = 60\ \mathrm{K}; \mathrm{m}_0 = 7; \mathrm{n}_{\mathrm{mode}} = 1$



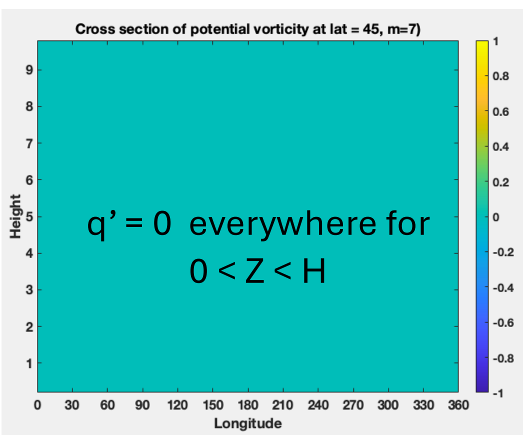
4.7 Energetics of baroclinic waves (H16.5)



What determines the height of baroclinic waves in Eady model?



$$\frac{\partial q'}{\partial t} = -v' \frac{\partial \bar{q}}{\partial y} = 0 \rightarrow q' = 0$$



$$Assuming\ uniform\ in\ y,$$

$$q' = -\left(\frac{2\pi m}{L_x}\right)^2 \Psi + \frac{f_0^2}{N^2}\frac{\partial^2 \Psi}{\partial z^2} = 0$$

$$q' = -\left(\frac{2\pi m}{L_x}\right)^2 \Psi + \frac{f_0^2}{N^2}\frac{\Psi}{H_R^2} = 0$$

$$Rossby\ height: H_R = \frac{f_0 L_X}{2\pi m N}$$

- Small m (longer waves), larger value of H_R $(deeper)$;
- Larger m (shorter waves), smaller value of H_R $(shallower)$;

What makes shorter waves shallower and longwave deeper?

$$Assuming\ uniform\ in\ y,$$

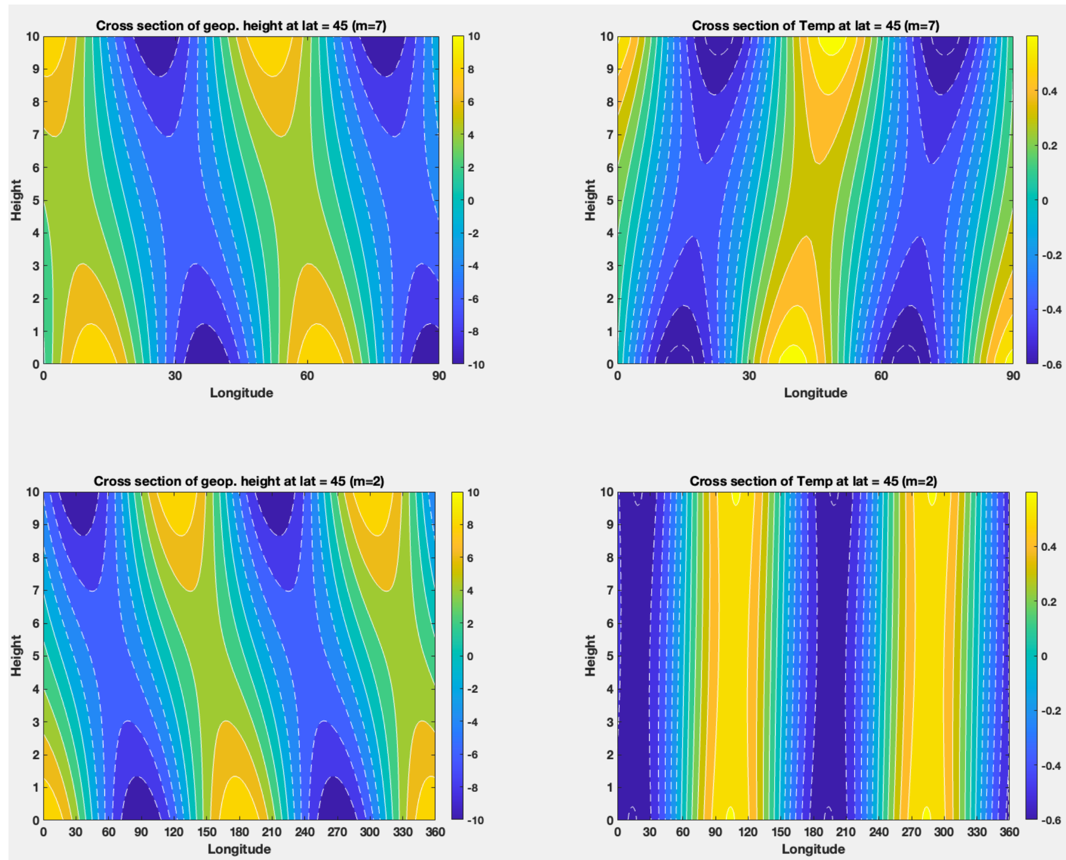
$$q' = -\left(\frac{2\pi m}{L_x}\right)^2 \Psi + \frac{f_0^2}{N^2}\frac{\partial^2 \Psi}{\partial z^2} = 0$$

$$\frac{f_0^2}{N^2}\frac{\partial}{\partial z}\frac{\partial \Psi}{\partial z} = \left(\frac{2\pi m}{L_x}\right)^2 \Psi$$

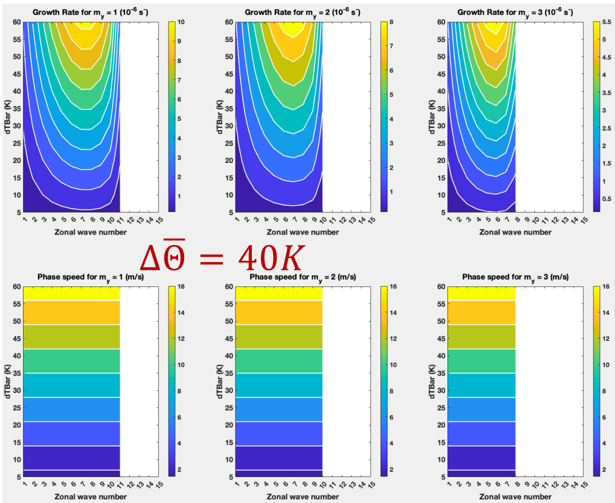
Given same $|\Psi|$, to achieve

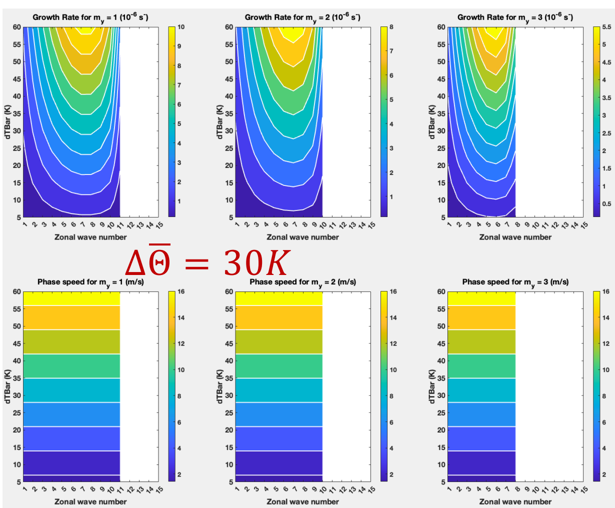
- Small m (longer waves), larger value of H_R $(deeper)$;
- Larger m (shorter waves), smaller value of H_R $(shallower)$;

✓ For longer (shorter) waves, amplitude of temperature anomalies decreases with height slower (faster), implying longer (shorter) waves have weaker (stronger) tilting of temperature fields with height.

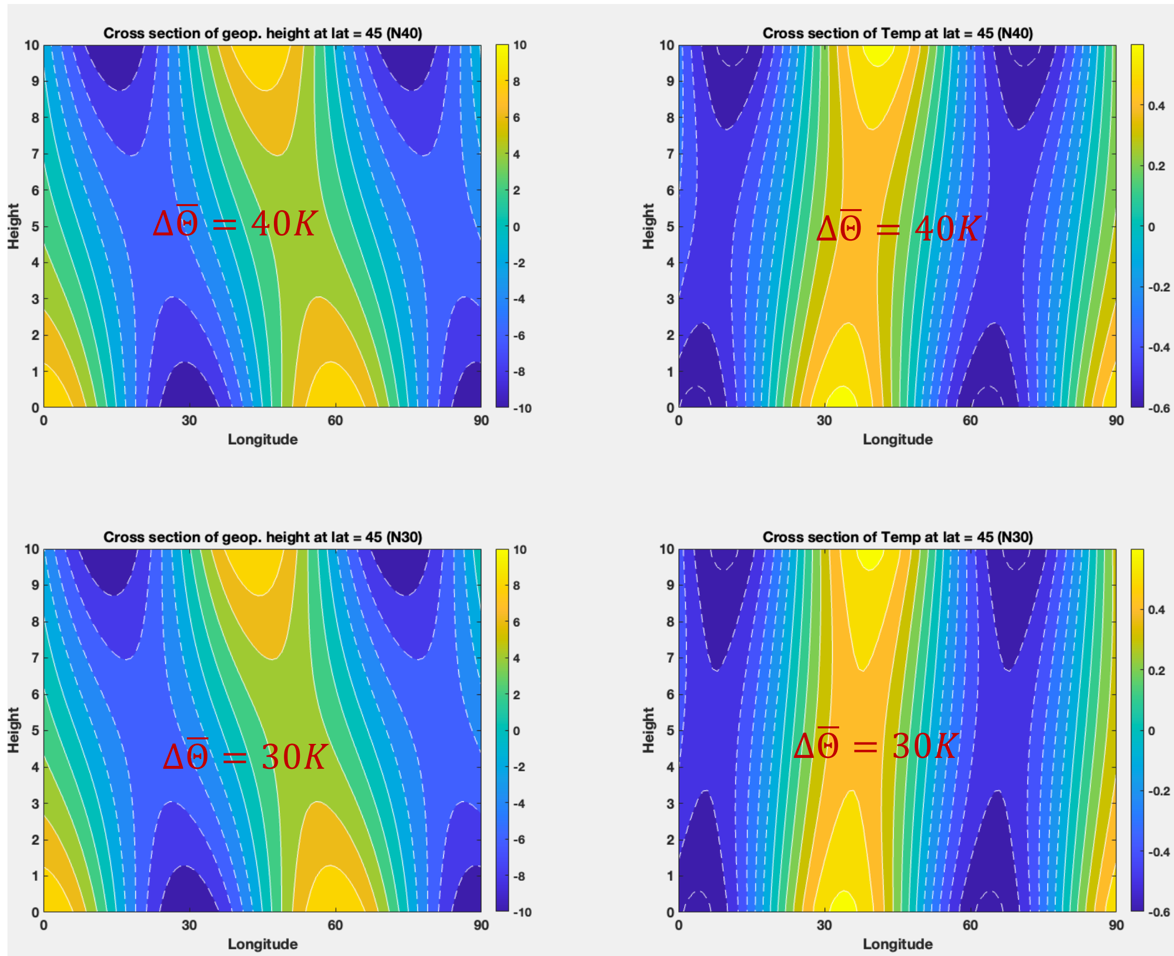


Why do waves become less unstable (less # waves/slow growing) for more stably stratified atmosphere ?

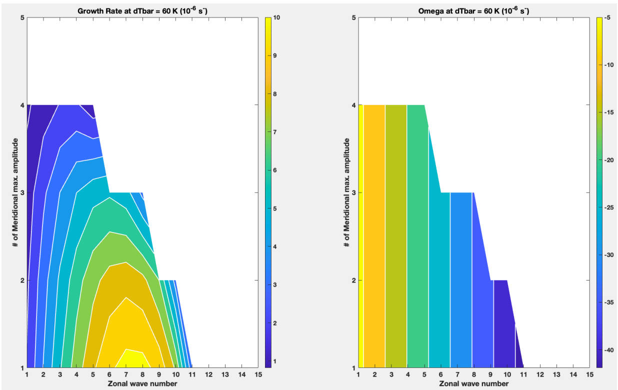


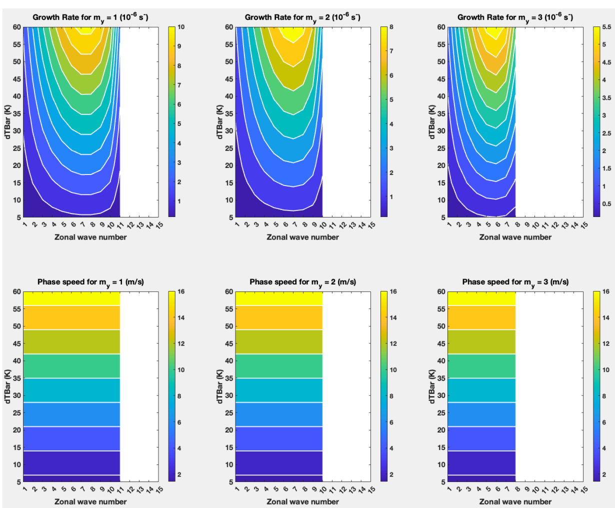


$$\textit{Rossby height}: H_R = \frac{f_0 L_X}{2\pi m N}$$

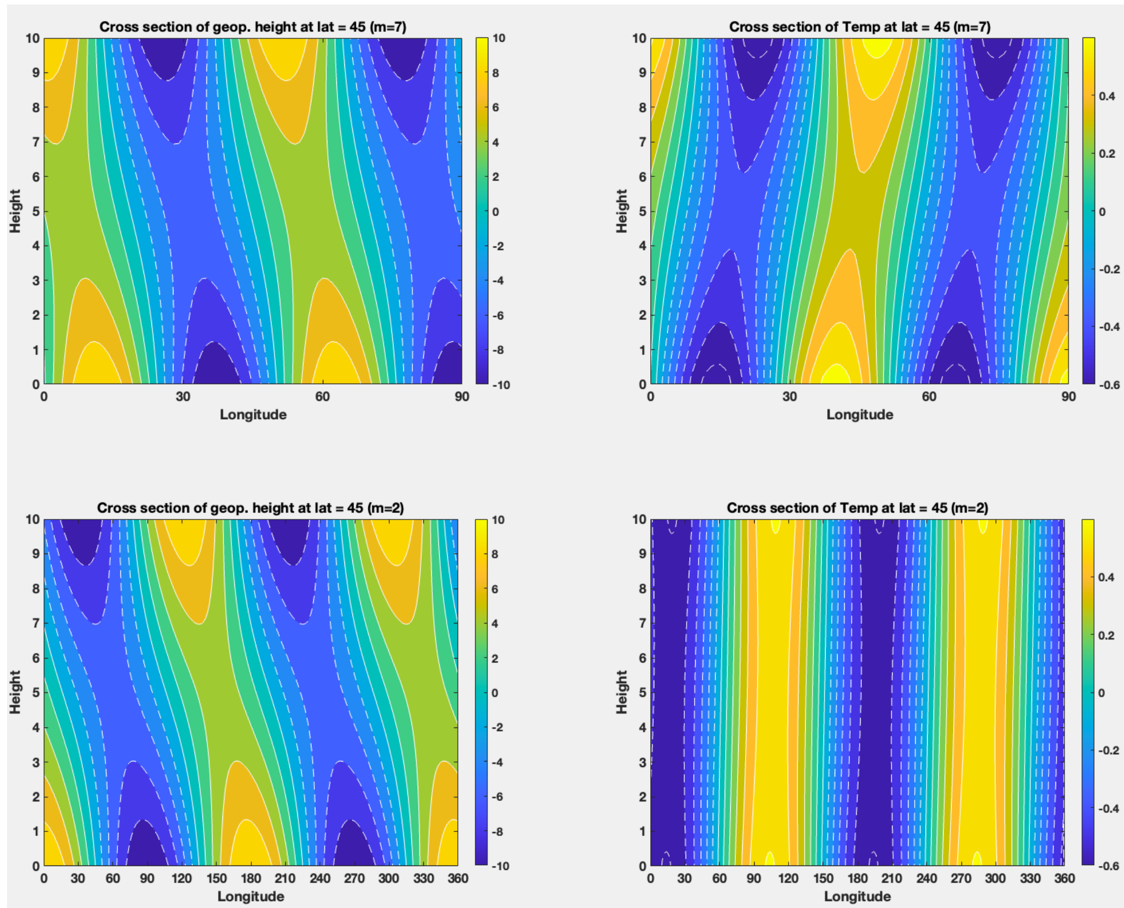


Why can’t shorter zonal waves become unstable in the Eady model?

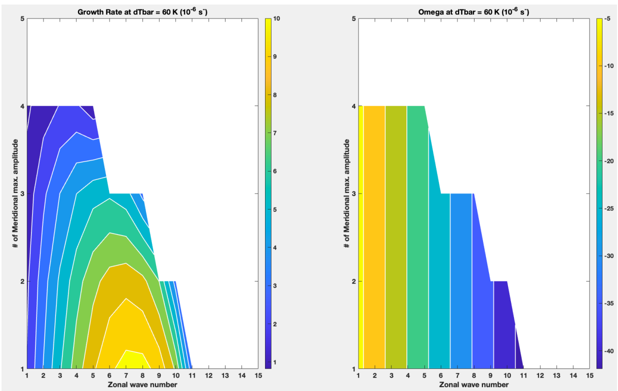


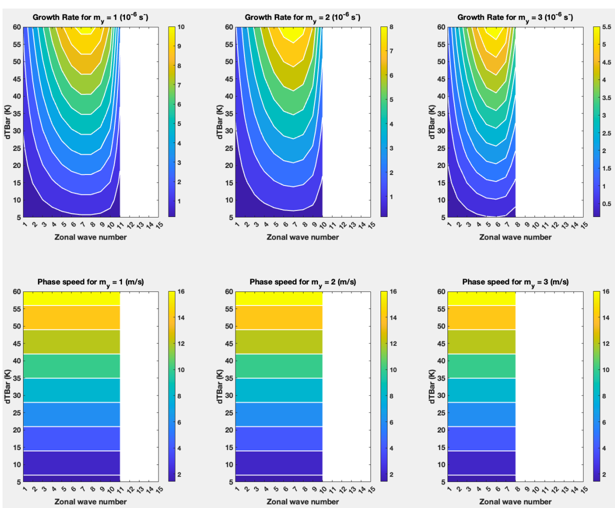


$$\text{Rossby height: } H_R = \frac{f_0}{N\sqrt{[\left(\frac{2\pi m}{Lx}\right)^2 + \left(\frac{2\pi n}{Ly}\right)^2]}}$$

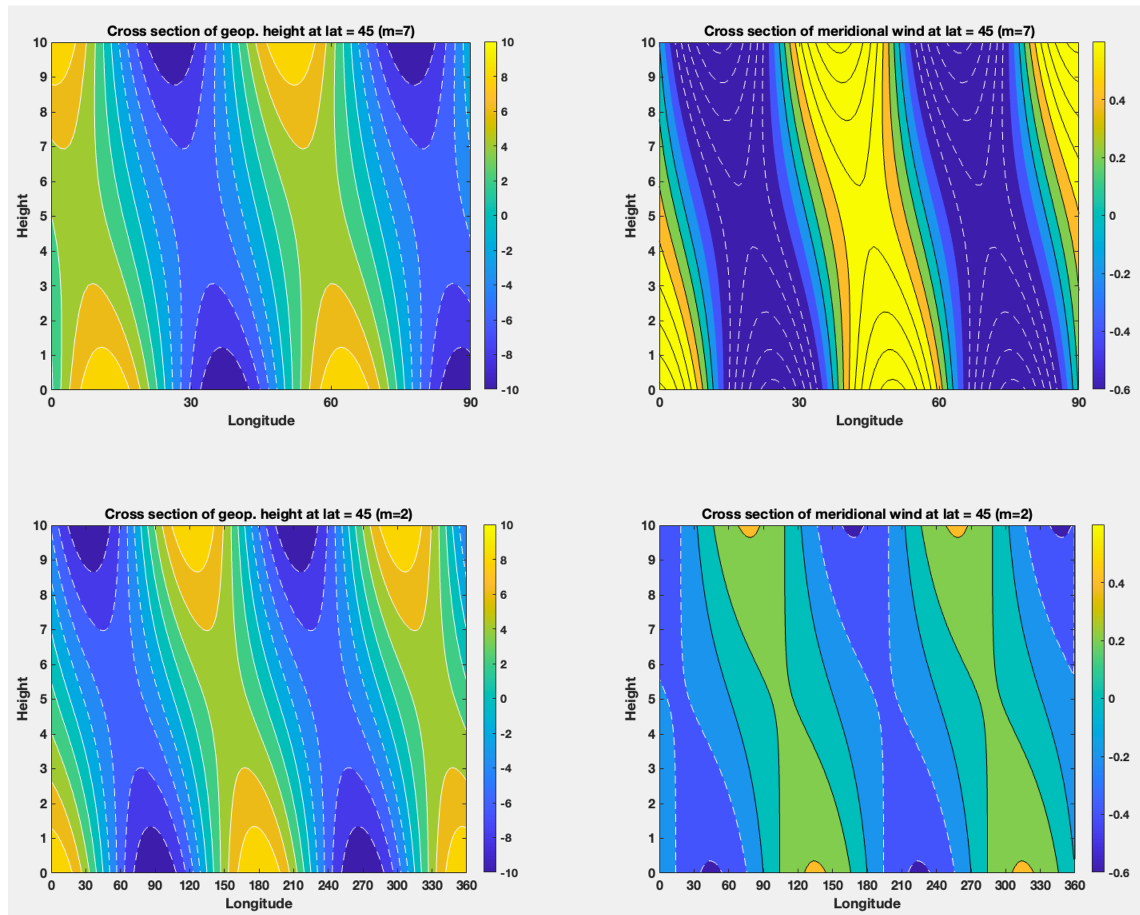


Why are planetary waves less unstable despite they are deeper than synoptic waves?





$$meridional\ wind: v' = \frac{g}{f_0}\frac{\partial h'}{\partial x} \propto \mathrm{m}\,\frac{g}{f_0}h'$$



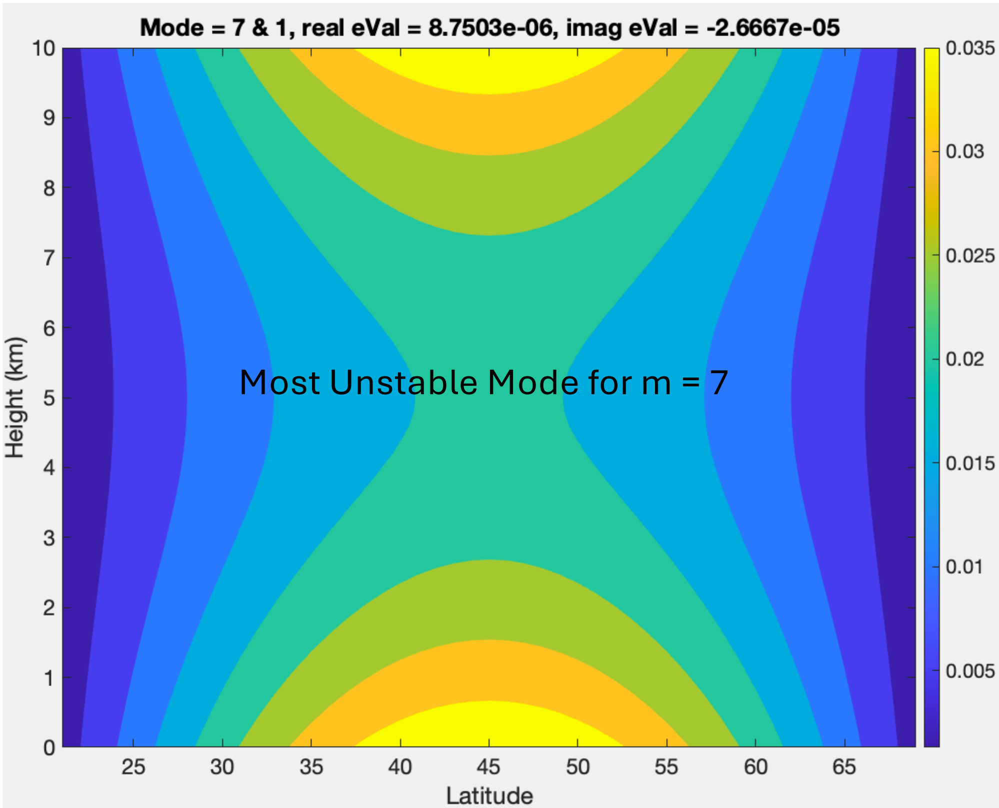
Eady Model's Solution

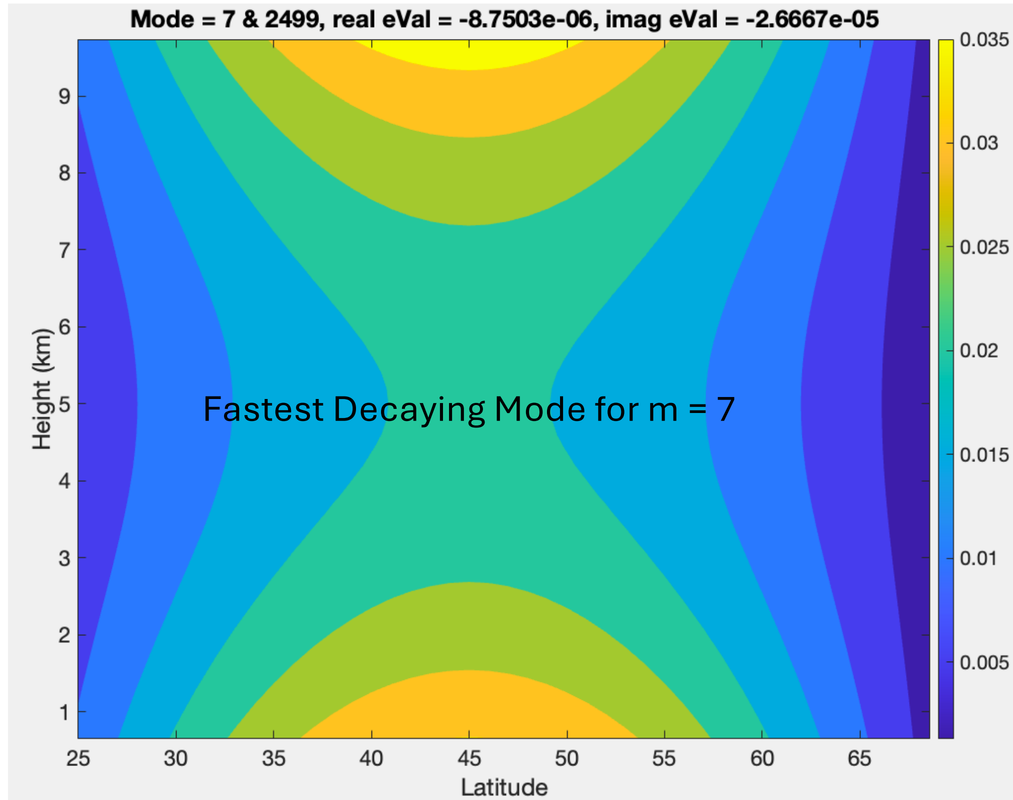
$$\frac{\Psi}{\bar{u}-c}=\frac{\Psi^*}{\bar{u}-c^*}: if\ (\Psi,c)\ corresponds\ to\ a\ solution\ of\ a\ linear\ system,$$
$$so\ does\ (\Psi^*,c^*)$$

In our model: $\frac{\Psi}{\bar{u}-\sqrt{-1}\lambda/m}=\frac{\Psi^*}{\bar{u}-\sqrt{-1}\lambda^*/m}$

Eady Model's Solution

$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = 60K; m = 7$





Eady Model's Solution

$\Delta\overline{\Theta} = 40K; \Delta\overline{T} = 60K; m = 7$

Most Unstable Mode for m = 7

Fastest Decaying Mode for m = 7