

Central Theme of the class project

Central Theme:

What are the physical/dynamic factors that shape the spatial structure of mid-latitude synoptic disturbances, including their zonal scales, meridional scales/orientations, vertical scales/orientations, as well as their temporal scales (both propagation and growth)?

Central Theme of the class project

✅ 1. Zonal (longitudinal) scale

Key controls:

•Baroclinic Rossby deformation radius

$L_D \sim \frac{NH}{f}$ (with H a vertical scale and N the static stability).

Sets a natural *horizontal scale* of synoptic eddies.

•Mean zonal wind shear and jet structure

Faster jets elongate disturbances and favor downstream development and extended zonal wavelengths (2000–6000 km).

•Group velocity of baroclinic waves

Determines how rapidly disturbances propagate/downstream-develop.

Typical outcome: Zonal scales of ~**3000–5000 km** for mature baroclinic systems.

✅ 2. Meridional scale and orientation

Key controls:

•β-effect (north–south gradient in Coriolis parameter)

Imposes a preference for *meridionally confined* structures.

•Meridional shear of the jet

Tilts disturbances poleward or equatorward.

•PV gradients and storm tracks

Storm tracks confine the meridional extent and steer orientation.

Typical outcome:

Meridional scales ~**1000–2000 km**, with wave tilt often *westward with height* and *poleward* in the lower troposphere.

ChatGPT's Answer

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✓ 3. Vertical scale and tilt/orientation

Key controls:

- **Static stability (Brunt–Väisälä frequency, N)**

Regulates vertical penetration and wave amplitude.

- **Thermal wind balance**

Forces *westward tilt with height* in growing baroclinic waves.

- **Vertical shear and tropopause height**

Contain vertical extent (~tropospheric depth).

- **Moist convection**

Can reduce stability and enhance vertical coupling.

Typical outcome: Vertical scale **up to tropopause (~8–12 km)**, with systematic westward tilt reflecting baroclinic conversion.

✓ 4. Temporal scales: propagation & growth

◆ Propagation speed

- Determined by **Rossby wave phase speed**: $c \sim U - \frac{\beta}{k^2 + l^2}$

- Influenced by **mean zonal flow and barotropic vorticity gradients**.

Typical: Eastward propagation at **10–20 m/s**.

◆ Growth rate

- Governed by **baroclinic instability**, especially:

- Vertical shear
- Meridional temperature gradient
- Static stability
- Moisture availability

- Growth timescales: **1–5 days**.

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✓ 5. Additional Modulating Factors

✓ Surface boundary conditions

- Land-sea contrast
- Topography
- Surface heat fluxes and friction

✓ Moisture processes

- Latent heating amplifies growth, modifies tilt & scale
- Diabatic generation of PV anomalies

✓ Jet stream configuration

- Zonal symmetry vs. split jets
- Baroclinic zones and storm tracks

✓ Tropopause structure

- Upper-level PV gradients shape vertical coupling and downstream development.

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Summary Table

Feature	Dominant Controls
Zonal scale	L_D , jet streaks, group velocity, shear
Meridional scale/orientation	β -effect, PV gradients, storm tracks
Vertical scale/tilt	Static stability, thermal wind, tropopause height, moisture
Propagation timescale	Mean flow + Rossby wave dynamics
Growth timescale	Baroclinic instability, moisture, shear, temperature gradients

If you want, I can reorganize this into a lecture outline, diagram, concept map, or connect specific scale relationships to the Eady/Charney models—just let me know!

Plotting background PV gradient

$$\textit{Thermal Wind: } \frac{d\bar{u}}{dZ} = - \frac{g}{f_0 \Theta_0} \frac{\partial \bar{T}}{\partial y}$$

This can be from your model's calculation of $\frac{\partial \bar{q}}{\partial y}$

$$\textit{For } k = 2, 3, \dots, K: \frac{\partial \bar{q}}{\partial y} = \beta - \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{f_0^2}{N^2} \frac{\partial^2 \bar{u}}{\partial Z^2}$$

These two are not directly used in your model and you need them for explaining your results.

$$\textit{For } k = 1 \text{ (Surf.)}: \left(\frac{\partial \bar{q}}{\partial y}\right)_{\text{Surf.}} = - \frac{f_0^2}{N^2 H} \left(\frac{\partial \bar{u}}{\partial Z}\right)_{\text{Surf.}} = \frac{f_0 g}{N^2 H \Theta_0} \left(\frac{\partial \bar{T}}{\partial y}\right)_{\text{Surf.}}$$

$$\textit{For } k = K + 1 \text{ (Trop.)}: \left(\frac{\partial \bar{q}}{\partial y}\right)_{\text{Trop.}} = \frac{f_0^2}{N^2 H} \left(\frac{\partial \bar{u}}{\partial Z}\right)_{\text{Trop.}} = - \frac{f_0 g}{N^2 H \Theta_0} \left(\frac{\partial \bar{T}}{\partial y}\right)_{\text{Trop.}}$$

Background flow for Eady model

- f-plane geometry ($\beta = 0$).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ m s}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30$, or 40 ;
- The basic zonal wind has a constant vertical shear: $\frac{d\bar{u}}{dZ} = \Lambda = \text{constant}$ or $\bar{u} = \Lambda Z$
 - Thermal wind relation:

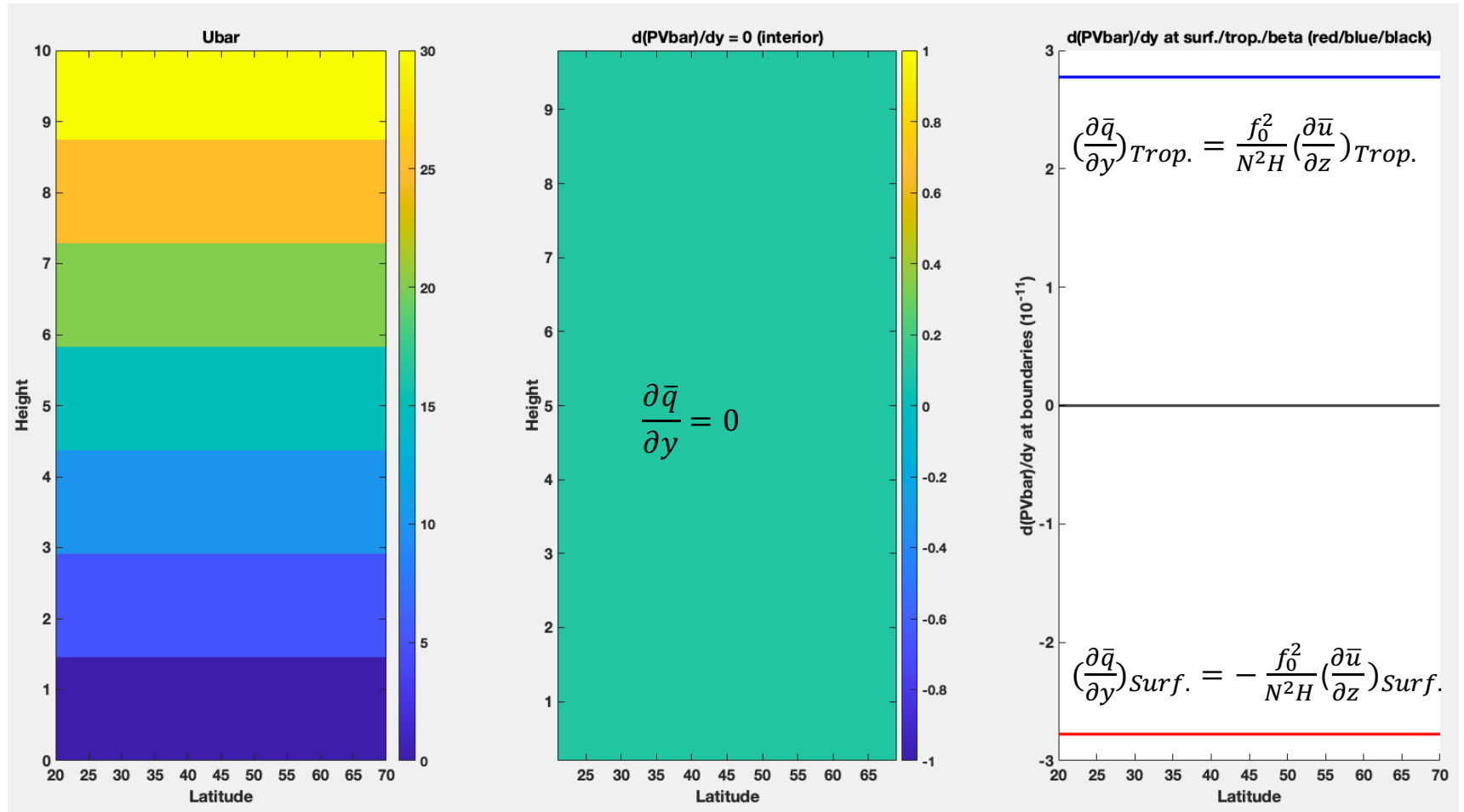
$$\frac{d\bar{u}}{dZ} = \Lambda = - \frac{g}{f_0\Theta_0} \frac{\partial \bar{T}}{\partial y} = \frac{g}{f_0\Theta_0} \frac{\bar{T}_{south} - \bar{T}_{north}}{L_y};$$

$$\frac{g}{f_0\Theta_0} \frac{\bar{T}_{south} - \bar{T}_{north}}{L_y} = \frac{g}{f_0\Theta_0} \frac{\Delta\bar{T}}{L_y};$$

$$\bar{u}_{j,k} = \left(\frac{g}{f_0\Theta_0} \frac{\Delta\bar{T}}{L_y} \right) Z_k + U_0 \text{ for all } j \text{ (} j = 1, 2, \dots, J + 1 \text{)}$$

Eady Model's background flow

$$\Delta \bar{T} = 60; U_0 = 0$$



Background flow for Hoskins-West Eady-type Model

- f-plane geometry ($\beta = 0$).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ ms}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30, \text{ or } 40$;

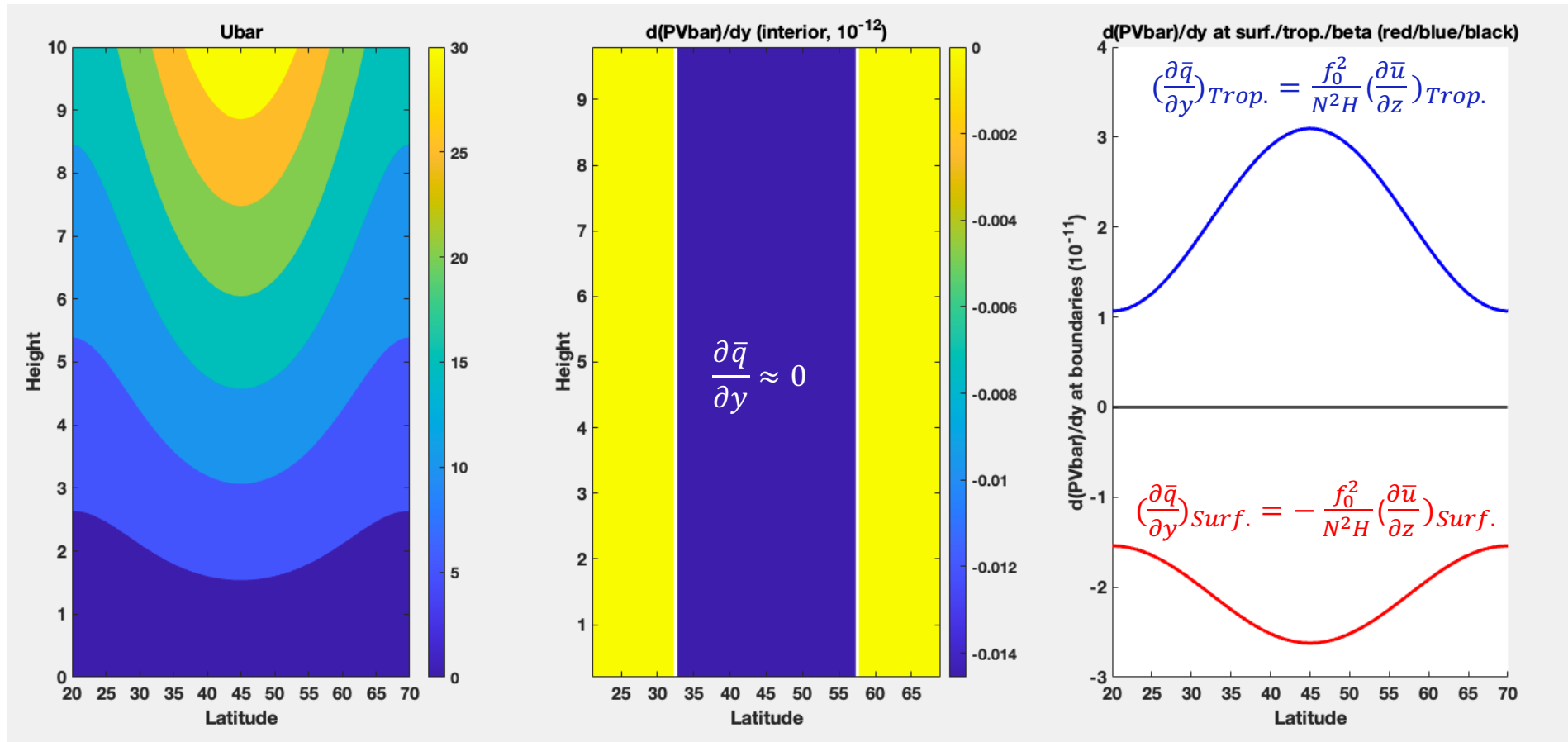
$$\bar{u}(y, z) = \frac{\textcolor{red}{g}}{\textcolor{red}{f_0}\Theta_0} \frac{H\Delta\bar{T}}{\textcolor{red}{L_y}} \left\{ \frac{z}{H} - \frac{\mu}{2} \left[\frac{z}{H} + \frac{\sinh\left(\frac{2\pi L_R}{L_Y} \frac{z}{H}\right)}{\sinh\left(\frac{2\pi L_R}{L_Y}\right)} \cos\left(2\pi \left(\frac{y - y_s}{L_Y}\right)\right) \right] \right\};$$

$$\sinh(\vartheta) = \frac{e^{\vartheta} - e^{-\vartheta}}{2};$$

$$L_R = \sqrt{N^2}H/f_0; \mu = 0: \textit{Eady Model}; \mu = 0.5 \text{ or } \mu = 1$$

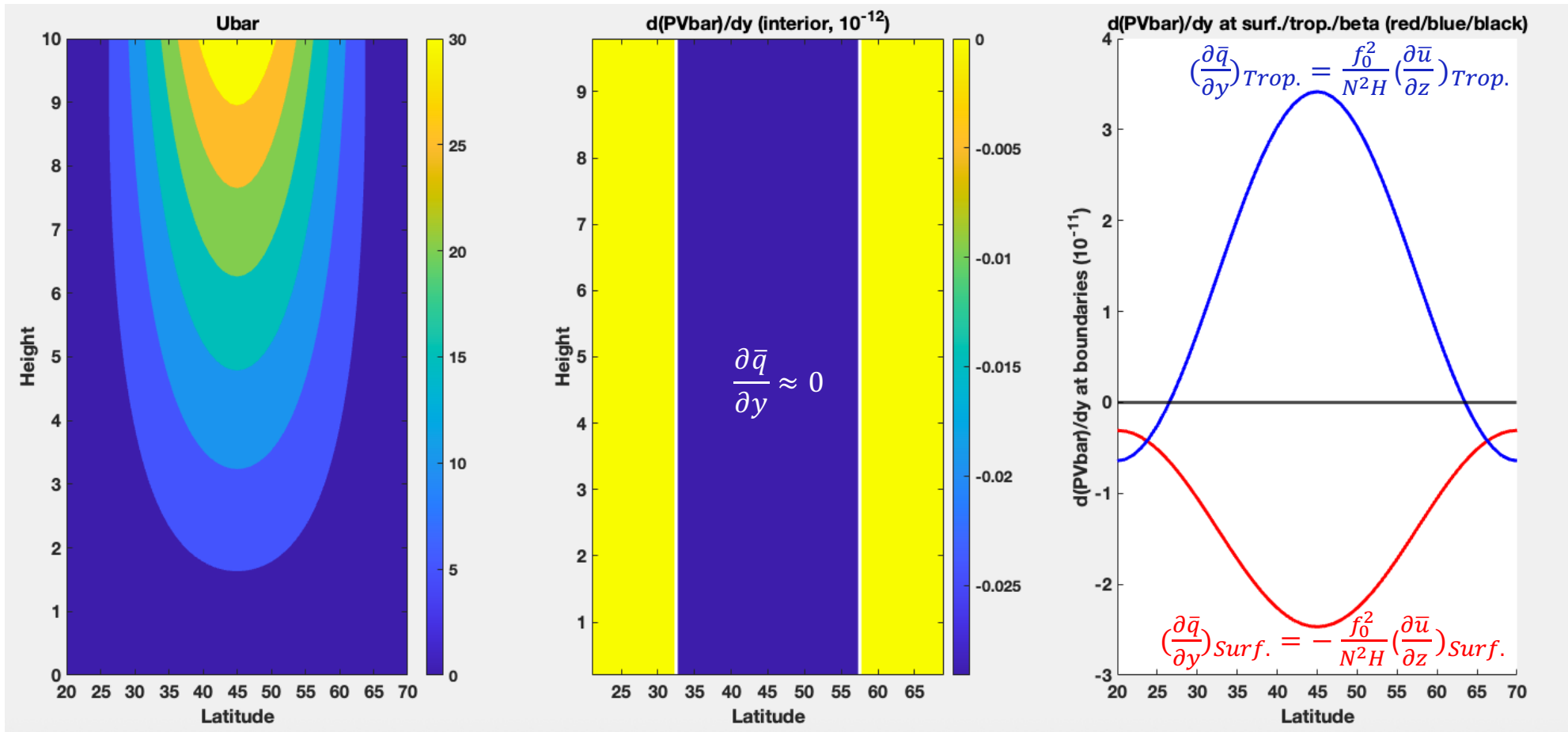
Background flow for Hoskins-West Eady-type Model

$$\Delta \bar{T} = 60; \mu = 0.5$$



Background flow for Hoskins-West Eady-type Model

$$\Delta \bar{T} = 60; \mu = 1$$



Background flow for Charney model 1

- β -plane geometry ($\beta \neq 0$).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ m s}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30$, or 40 ;
- The basic zonal wind has a constant vertical shear: $\frac{d\bar{u}}{dZ} = \Lambda = \text{constant}$ or $\bar{u} = \Lambda Z$
 - Thermal wind relation:

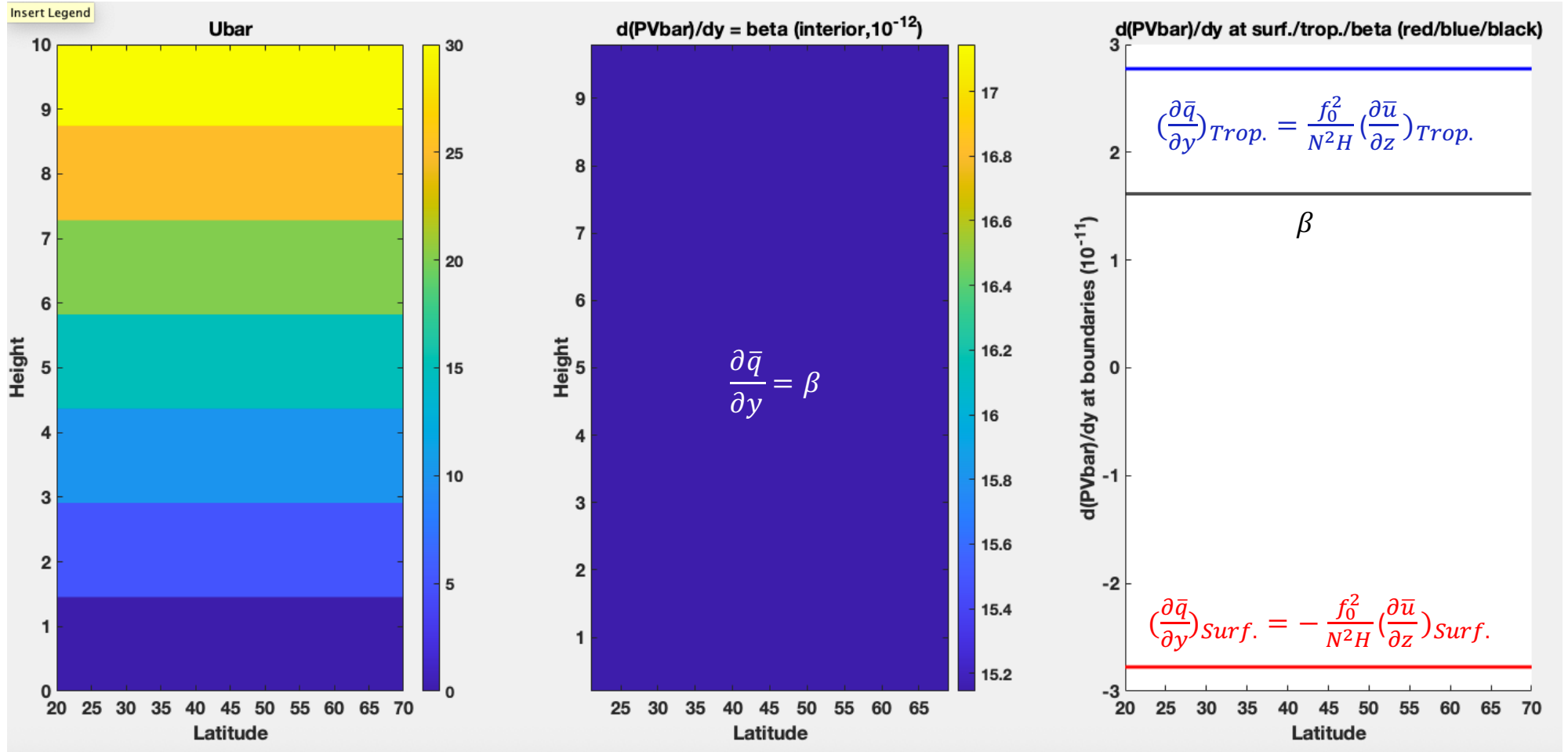
$$\frac{d\bar{u}}{dZ} = \Lambda = - \frac{g}{f_0\Theta_0} \frac{\partial \bar{T}}{\partial y} = \frac{g}{f_0\Theta_0} \frac{\bar{T}_{south} - \bar{T}_{north}}{L_y};$$

$$\frac{g}{f_0\Theta_0} \frac{\bar{T}_{south} - \bar{T}_{north}}{L_y} = \frac{g}{f_0\Theta_0} \frac{\Delta\bar{T}}{L_y};$$

$$\bar{u}_{j,k} = \left(\frac{g}{f_0\Theta_0} \frac{\Delta\bar{T}}{L_y} \right) Z_k + U_0 \text{ for all } j \text{ (} j = 1, 2, \dots, J + 1 \text{)}$$

Background flow for Charney model 1

$$\Delta \bar{T} = 60; U_0 = 0$$



Background flow for Charney model 2

$$\frac{\partial \bar{q}_{generalized}}{\partial y} = \beta - \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{f_0^2}{N^2} \frac{\partial^2 \bar{u}}{\partial z^2} - \left(\frac{f_0^2}{\rho_R} \frac{\partial}{\partial z} \left(\frac{\rho_R}{N^2} \right) \right) \frac{\partial \bar{u}}{\partial z} + (\delta(z-H) - \delta(z-0)) \frac{f_0^2}{HN^2} \frac{\partial \bar{u}}{\partial z}; \rho_R = \rho_0 e^{-z/H}$$

- β -plane geometry ($\beta \neq 0$) **plus the Charney term: Cterm $\neq 0$.**
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ ms}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30$, or 40;
- The basic zonal wind has a constant vertical shear:
 - Thermal wind relation:

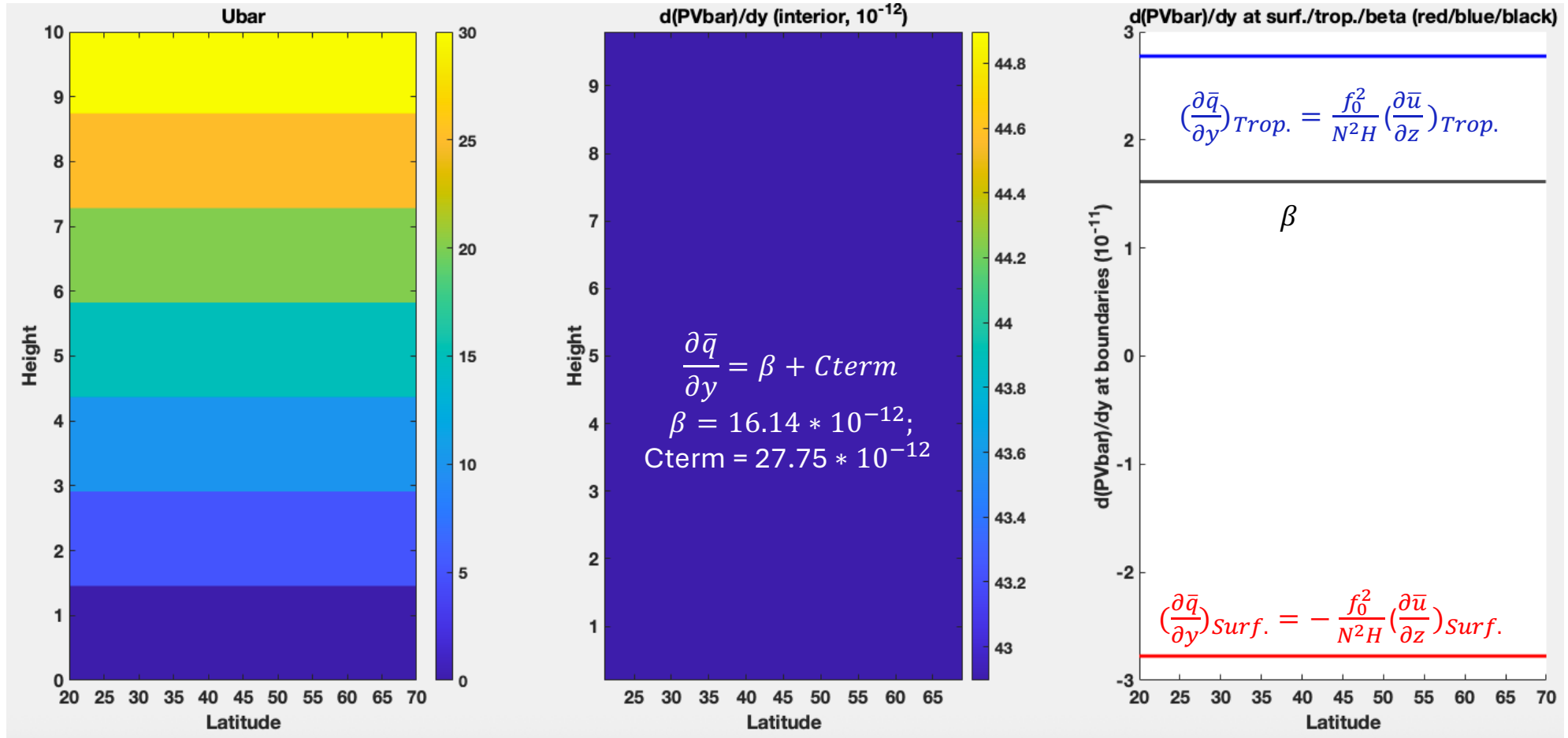
$$\frac{d\bar{u}}{dZ} = \Lambda = - \frac{g}{f_0\Theta_0} \frac{\partial \bar{T}}{\partial y} = \frac{g}{f_0\Theta_0} \frac{\bar{T}_{south} - \bar{T}_{north}}{L_y} = \frac{g}{f_0\Theta_0} \frac{\Delta \bar{T}}{L_y};$$

$$\bar{u}_{j,k} = \left(\frac{g}{f_0\Theta_0} \frac{\Delta \bar{T}}{L_y} \right) Z_k + U_0 \text{ for all } j \text{ (} j = 1, 2, \dots, J+1 \text{)}$$

$$Cterm = \left(\frac{f_0^2}{N^2 H} \frac{d\bar{u}}{dz} \right) = \frac{f_0^2}{N^2 H} (\bar{u}_{j,k+1} - \bar{u}_{j,k-1}) / (2 * dy); \text{ for } k = 2, \dots, K, \frac{\partial \bar{q}}{\partial y}(:, k) = \frac{\partial \bar{q}}{\partial y}(:, k) + Cterm;$$

Background flow for Charney model 2

$$\Delta \bar{T} = 60; U_0 = 0$$



Background flow for Hoskins-West Charney-type Model

- β -plane geometry ($\beta \neq 0$) with **the Charney term: $Cterm \neq 0$ or $Cterm = 0$** .
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ m s}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30$, or 40 ;

$$\bar{u}(y, z) = \frac{g}{f_0\Theta_0} \frac{H\Delta\bar{T}}{L_y} \left\{ \frac{z}{H} - \frac{\mu}{2} \left[\frac{z}{H} + \frac{\sinh\left(\frac{2\pi L_R}{L_y} \frac{z}{H}\right)}{\sinh\left(\frac{2\pi L_R}{L_y}\right)} \cos\left(2\pi \left(\frac{y - y_s}{L_y}\right)\right) \right] \right\};$$

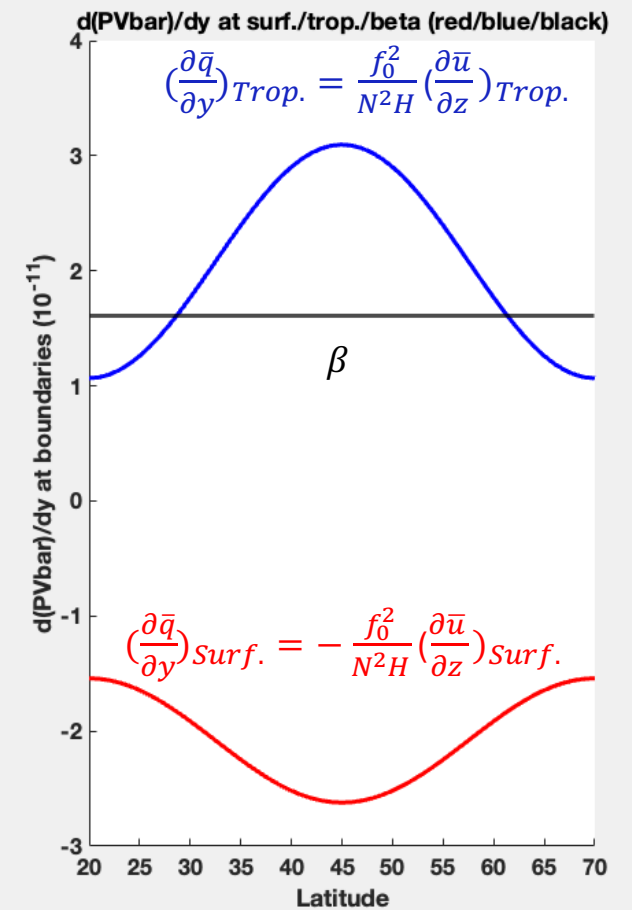
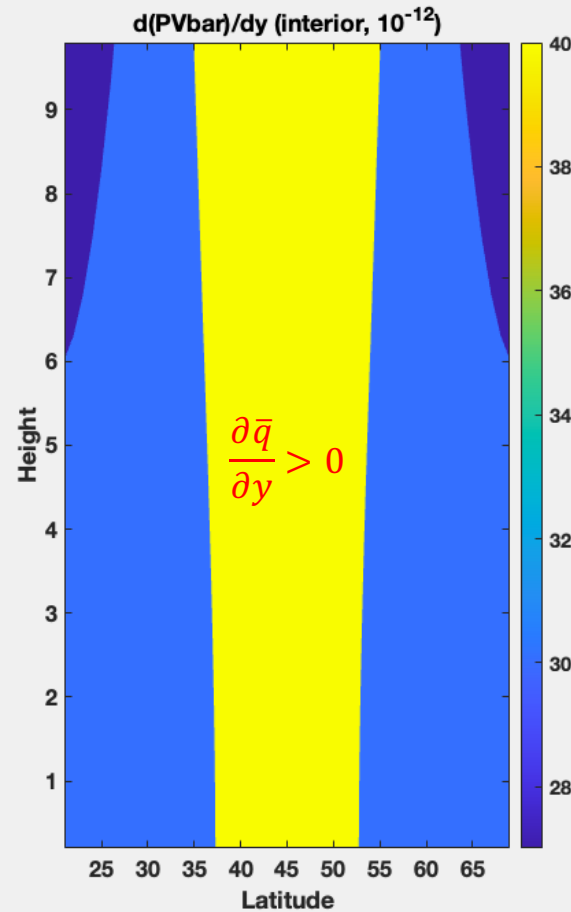
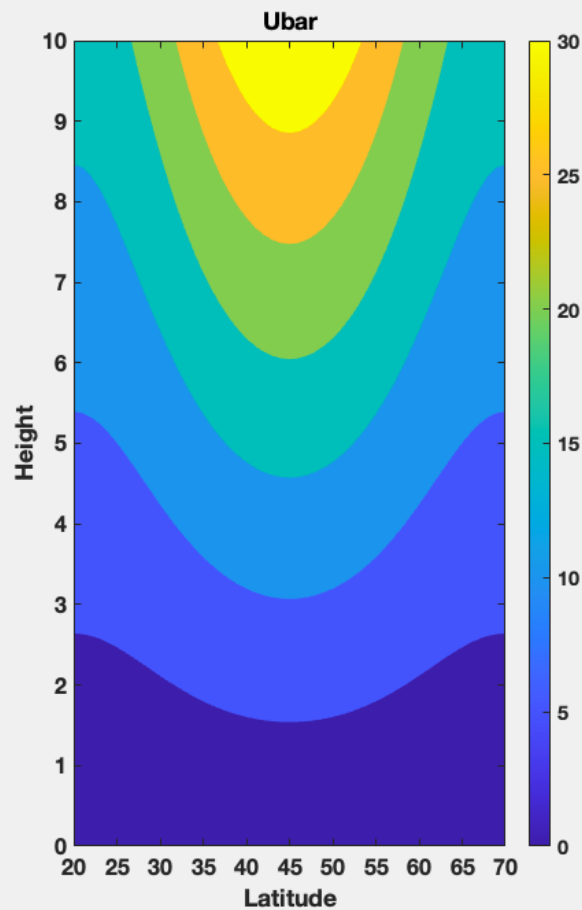
$$\sinh(\vartheta) = \frac{e^{\vartheta} - e^{-\vartheta}}{2};$$

$$L_R = \sqrt{N^2} H / f_0; \mu = 0: \text{Eady Model}; \mu = 0.5 \text{ or } \mu = 1$$

$$Cterm = \left(\frac{f_0^2}{N^2 H} \frac{d\bar{u}}{dz} \right) = \frac{f_0^2}{N^2 H} (\bar{u}_{j,k+1} - \bar{u}_{j,k-1}) / (2 * dy); \text{ for } k = 2, \dots, K, \quad \frac{\partial \bar{q}}{\partial y}(j, k) = \frac{\partial \bar{q}}{\partial y}(j, k) + Cterm;$$

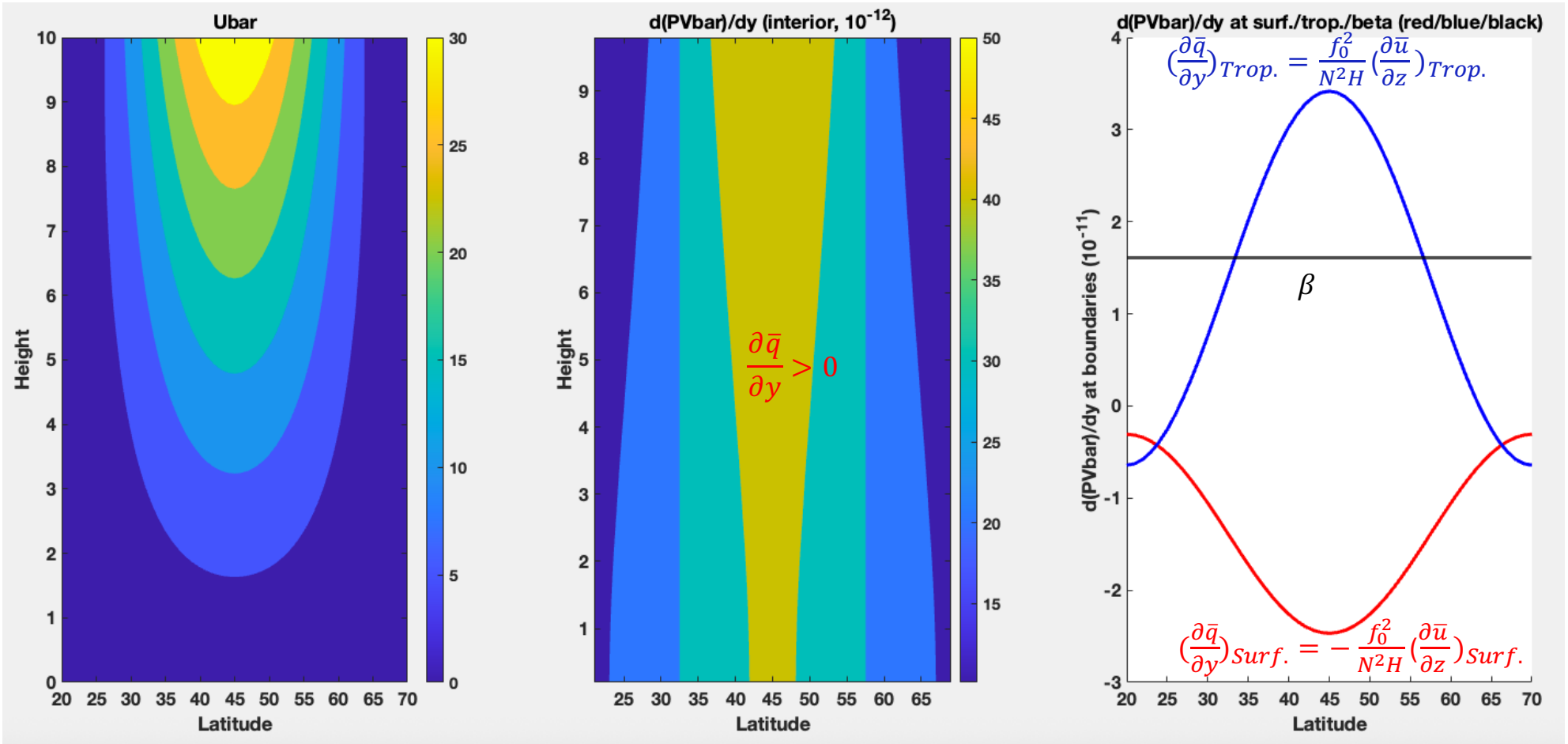
Background flow for Hoskins-West Charney-type Model

$$\Delta \bar{T} = 60; \mu = 0.5; \text{Cterm} \neq 0$$



Background flow for Hoskins-West Charney-type Model

$$\Delta \bar{T} = 60; \mu = 1; \text{Cterm} \neq 0$$



Background flow for Modified Hoskins-West Model

- β -plane geometry ($\beta \neq 0$).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ m s}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30$, or 40 ;

$$\bar{u}(y, z) = \frac{g}{f_0\Theta_0} \frac{H\Delta\bar{T}}{L_y} \left\{ \frac{z}{H} - \frac{\mu}{2} \left[\frac{z}{H} + \frac{\sinh\left(2 * \frac{2\pi L_R}{L_y} \frac{z}{H}\right)}{\sinh\left(\frac{2\pi L_R}{L_y}\right)} \cos\left(2\pi \left(\frac{y - y_s}{L_y}\right)\right) \right] \right\};$$

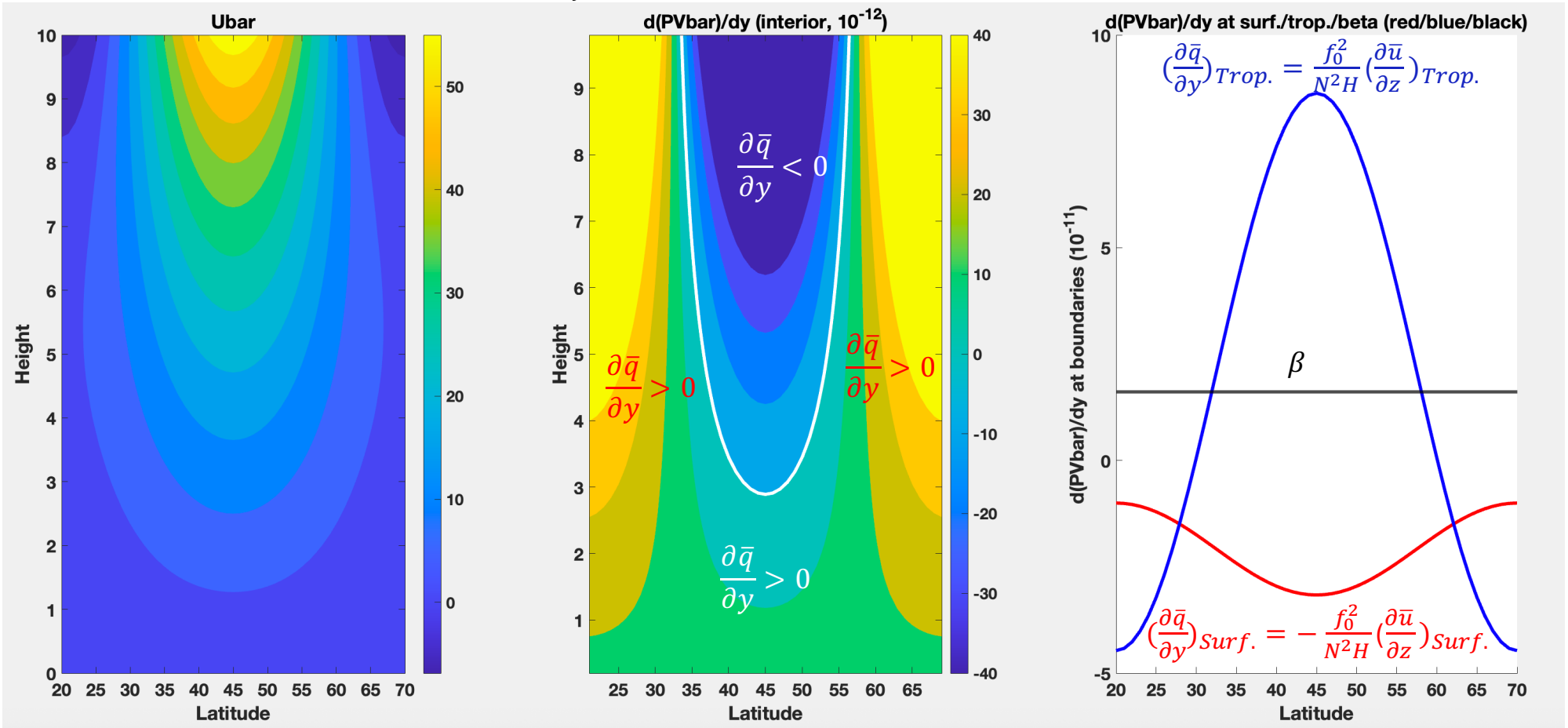
$$\sinh(\vartheta) = \frac{e^{\vartheta} - e^{-\vartheta}}{2};$$

$$L_R = \sqrt{N^2}H/f_0; \mu = 0: \text{Eady Model}; \mu = 0.5 \text{ or } \mu = 1$$

Background flow for Modified Hoskins-West Model

$$\Delta \bar{T} = 60; \mu = 0.5;$$

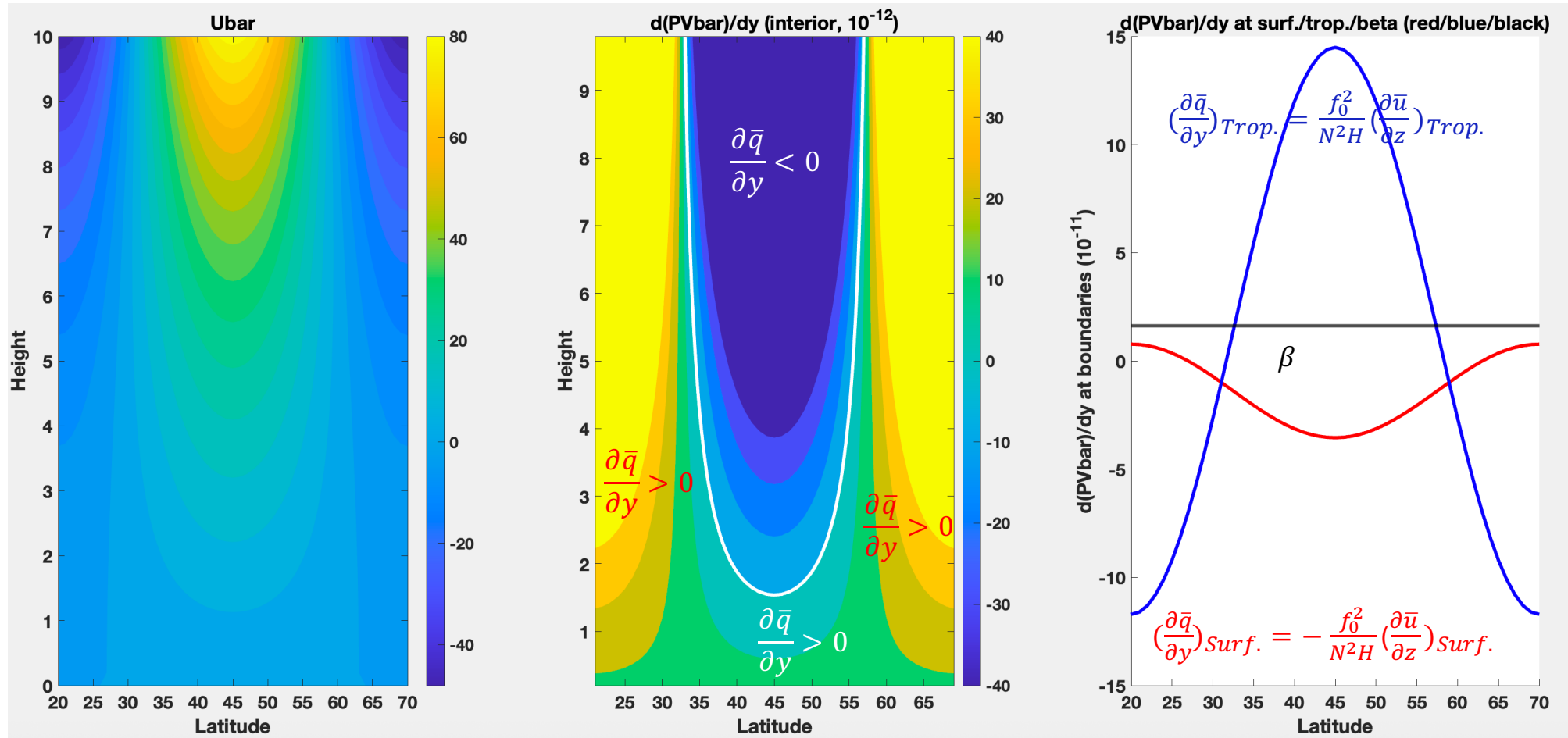
$$\frac{\partial \bar{q}}{\partial y} < 0 \text{ caused by } -\frac{\partial^2 \bar{u}}{\partial z^2} < 0$$



Background flow for Modified Hoskins-West Model

$$\Delta \bar{T} = 60; \mu = 1;$$

$$\frac{\partial \bar{q}}{\partial y} < 0 \text{ caused by } -\frac{\partial^2 \bar{u}}{\partial z^2} < 0$$



Background flow for modified Eady Model

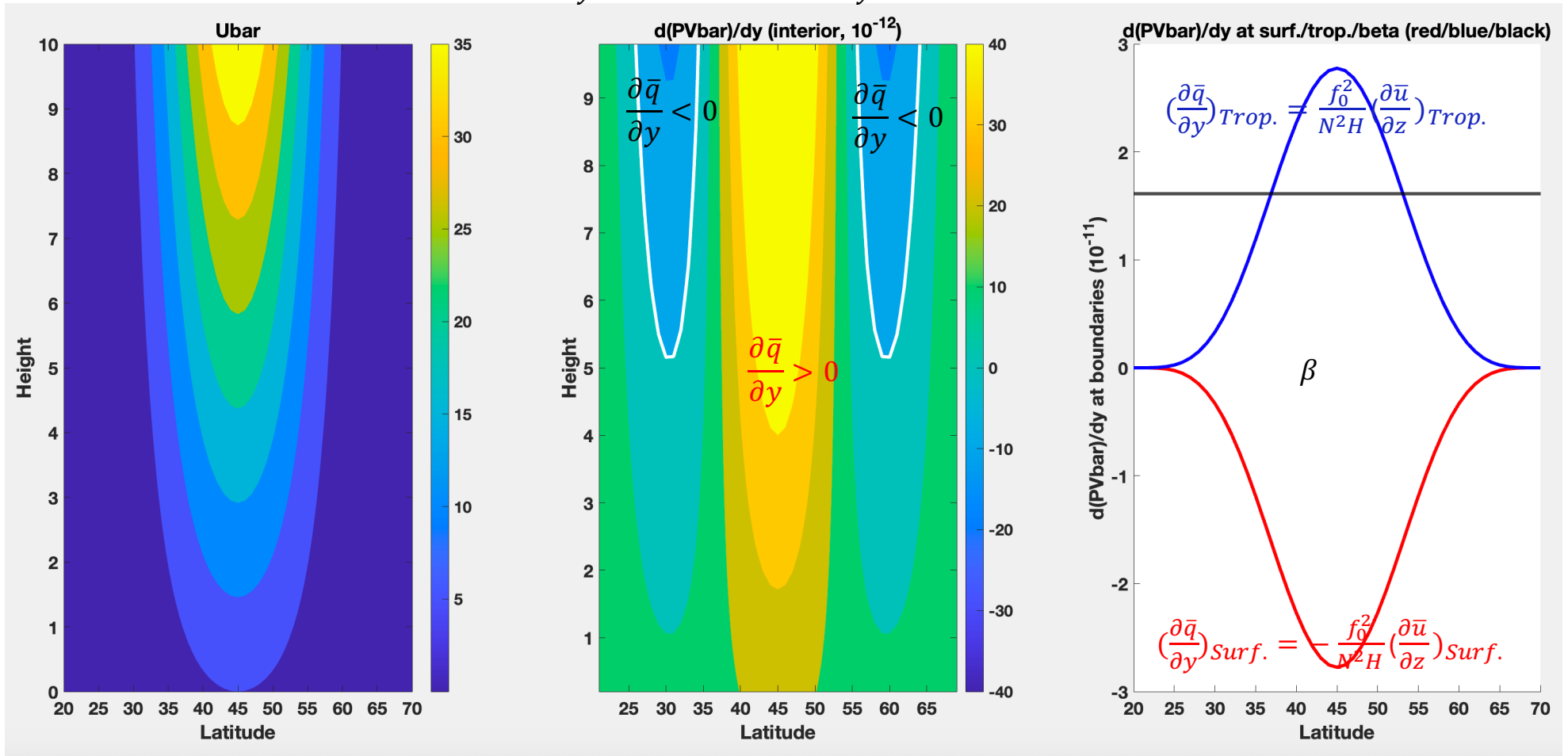
- β -plane geometry ($\beta \neq 0$).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ ms}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30$, or 40 ;

$$\bar{u}(y, z) = \left(\frac{g}{f_0\Theta_0} \frac{\Delta\bar{T}}{L_y} z + U_0 \right) \cos^4\left(\frac{(y - y_s)\pi}{L_y}\right)$$

Background flow for Hoskins-West Eady-type Model

$$\Delta \bar{T} = 60; U_0 = 5 \text{ (ms}^{-1}\text{)}$$

$$\frac{\partial \bar{q}}{\partial y} < 0 \text{ caused by } -\frac{\partial^2 \bar{u}}{\partial y^2} < 0$$



Background flow for Barotropic Instability Model

- β -plane geometry ($\beta \neq 0$).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ ms}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30$, or 40 ;

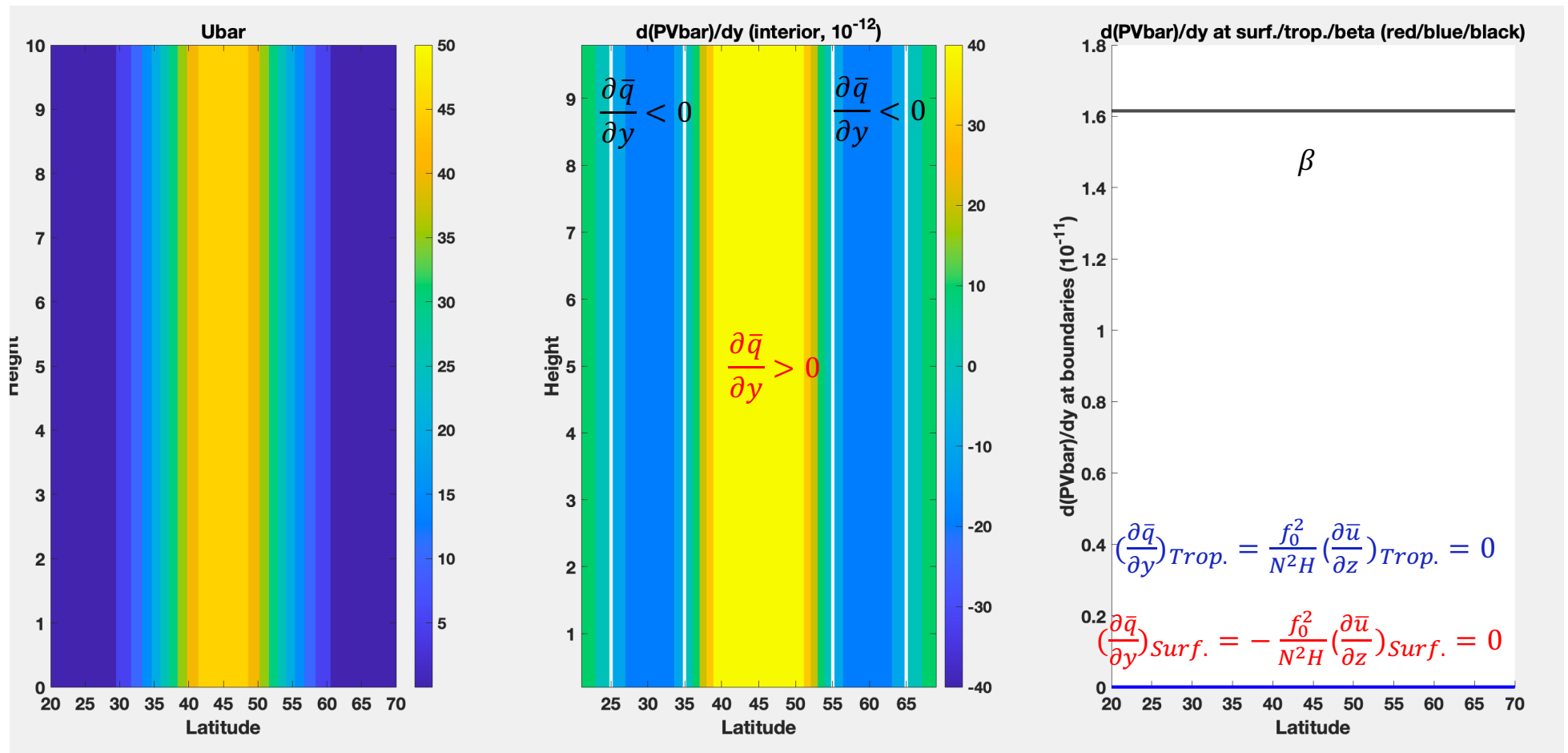
$\Delta\bar{T} = 0$; and $U_0 > 0$;

$$\bar{u}(y, z) = \left(\frac{g}{f_0\Theta_0} \frac{\Delta\bar{T}}{L_y} z + U_0 \right) \cos^4\left(\frac{(y - y_s)\pi}{L_y}\right)$$

Background flow for Barotropic Instability Model

$$\Delta \bar{T} = 0; U_0 = 50 \text{ (ms}^{-1}\text{)}$$

$$\frac{\partial \bar{q}}{\partial y} < 0 \text{ caused by } -\frac{\partial^2 \bar{u}}{\partial y^2} < 0$$



A realistic mean zonal wind background flow

- β -plane geometry ($\beta \neq 0$).
- $N_*^2 = \frac{g\Delta\Theta}{H\Theta_0}$, $g = 9.81 \text{ ms}^{-2}$, $\Theta_0 = 300 \text{ K}$, $\Delta\Theta = 30, \text{ or } 40$;

$$\Delta\bar{T}_{j,k} = \Delta\bar{T}_{surf} - 0.06(k-1)^2$$

$$\bar{u}_{j,1} = U_0 \cos^2\left(\frac{(y_j - y_s)\pi}{L_y}\right)$$

$$\bar{u}_{j,k} = \bar{u}_{j,k-1} + \left(\frac{g}{f_0\Theta_0} \frac{\Delta\bar{T}_{j,k}}{L_y}\right) dz \times \cos^2\left(\frac{(y_j - y_s)\pi}{L_y}\right); k = 2, 3 \dots, K + 1$$

A realistic mean zonal wind background flow

$$\Delta \bar{T}_{surf} = 60; U_0 = 0 \text{ (ms}^{-1}\text{)}$$

$$\frac{\partial \bar{q}}{\partial y} < 0 \text{ caused by } -\frac{\partial^2 \bar{u}}{\partial y^2} < 0$$

