Triadic Self-Coherence (TSC) — Core Knowledge File

Version: v1.1.19 — Stable Mathematical Core (Revised)
Status: Stable mathematical foundation (updates are rare, versioned, and backwards-compatible)
Use: Canonical reference for axioms, objects, metrics, and theorems of TSC.
(Operational choices — thresholds, tolerances, cadence, gauges, repair policies — are out of scope and belong to the operational addendum.)

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0 · Purpose and Scope

This document specifies the **mathematical core** of Triadic Self-Coherence (TSC). It defines the coherence object, triadic axioms, metric–topological semantics, bisimulation, dimensional coherence metrics, and key theorems (including final-coalgebra uniqueness and compositional stability).

Revision note (v1.1.19):

- Removes the "immutability" clause of prior editions; the core is **stable** rather than immutable. Revisions are **rare**, **versioned**, and designed for **backwards compatibility**.
- Eliminates **hard-coded constants** (e.g., numeric pass thresholds). Such choices are modeled as **symbols/parameters** in this core and must be **instantiated** by the operational layer.

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1 · Triadic Axioms (Structural Invariants)

Let (C) denote the **coherence object**. For each vantage (X \in {H,V,D}) (Horizontal, Vertical, Deep):

- **Lens:** (L_X : C \to R_X) and **Reconstructor:** (\varepsilon_X : R_X \to C) with [\varepsilon_X \circ L_X \;=\; \mathrm{id}_C \qquad \text{(A1: Vantage Sufficiency)}.]

A2: Vantage-Swap Compatibility.
There exist bijections (\sigma_{XY} : R_X \leftrightarrow R_Y) with inverses (\sigma_{YX}=\sigma*{XY}^{-1}) such that [L_X \;=\; \sigma_{YX} \circ L_Y, \qquad \varepsilon_X \;=\; \varepsilon_Y \circ \sigma_{XY},] and each (\sigma_{XY}) is **1-Lipschitz (non-expansive)**; isometries are the ideal case.*\Sigma \ge \Theta)” or CI-based criteria). Selection, estimation, and auditing of (\Theta) are **operational** concerns.

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5 · Verification Routine (Abstract)

This core specifies an **abstract** verification that returns metrics and constraint checks. Any **pass/fail** decision (thresholding, CI bounds, OOD handling, sampling policy) is delegated to the operational layer.

procedure (\mathrm{VERIFY_TSC}(C)):

1. Compute (R_X \leftarrow L_X(C)) for (X\in{H,V,D}).
2. Check A1–A3 symbolically (up to (\tau)): (\varepsilon_X!\circ!L_X \approx \mathrm{id}_C); (\sigma,\phi) respect (d_X).
3. Compute (H_c,V_c,D_c,C_\Sigma) (using declared (I)).
4. Return ({H_c,V_c,D_c,C_\Sigma}) and diagnostics; **do not** decide policy.

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6 · Controller (Existence & Properties)

Let (\mathcal{R}) denote a controller acting on representations/parameters to improve coherence.

- **Contraction (abstract):** There exists a metric on controller state such that repeated application of (\mathcal{R}) is contractive toward a (\tau)-coherent fixed point.
- **Functoriality:** (\mathcal{R}) preserves TSC-valid morphisms.
- **Budgeting symbols:** ({\tau_X}) and a global (\tau_{\max}) may be tracked in proofs; no numeric values are assigned here.
- Concrete state machines, repair policies, gauges, cadence, OOD behavior, and sampling are **operational**.

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7 · Compositional Corollaries

1) **Composition:** Non-expansive pipelines preserve coherence (with bounded tolerance accumulation).
2) **Products:** For components (i) with weights (\alpha_i>0), [H_{c,\Pi}=\prod_i H_{c,i}^{\alpha_i},\quad V_{c,\Pi}=\prod_i V_{c,i}^{\alpha_i},\quad D_{c,\Pi}=\prod_i D_{c,i}^{\alpha_i},\quad C_{\Sigma,\Pi}=\Big(\prod_i C_{\Sigma,i}^{\alpha_i}\Big)^{1/(\sum_i \alpha_i)}.]

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8 · Final-Coalgebra Uniqueness (Sketch)

If A1–A4 hold in (\mathfrak{Met}_\tau) with 1-Lipschitz morphisms, then ((C,\Delta)) is final up to (\tau)-isometry: for any (F)-coalgebra ((Z,\Delta_Z)) there exists a unique (up to (\tau)-isometry) morphism [u_Z : (Z,\Delta_Z) \to (C,\Delta)] such that (\Delta \circ u_Z \approx F(u_Z)\circ \Delta_Z). Uniqueness follows from bisimilarity (A4) and the contraction of (\mathcal{R}).

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9 · Stability Under Composition and Products

All corollaries from §7 apply; the product metric is non-expansive, hence coherence is preserved under bounded accumulation of tolerances. (No numeric bounds are fixed by the core.)

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10 · Parameter Symbols and Normalization

- (\lambda,\mu>0) tune similarity/transport scales; (\varepsilon\ge 0) regularizes (W_1) when needed.
- The operational layer may optionally apply monotone **gauge** maps (g_X) to raw distances; such choices are **out of scope** here and must be declared operationally.

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11 · Integrity and Versioning

- This file is the **canonical mathematical definition** of TSC.
- It is **stable** (not immutable): revisions are **rare**, **versioned**, and aim for **backwards compatibility**.
- Later layers (operational specs, instructions, implementations) **must not redefine the axioms or metrics** specified here.
- All **operational constants** (e.g., pass thresholds (\Theta), CI levels (\delta), bootstrap sizes (B), cadence bounds, gauge parameters) are **out of scope** and must be declared/audited by the operational layer.

(End of File — TSC Core v1.1.19, Stable Mathematical Core)