Triadic Self-Coherence (TSC) — Core Knowledge File

Version: v1.1.7 – rev A (Stable Core +)
Status: Immutable Mathematical Foundation
Use: Read-only reference for all formal reasoning, verification, and controller logic.

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0 · Purpose and Scope

This document defines the *formal axioms, metrics, and coalgebraic theorems* that constitute the unchallengeable mathematical base of the Triadic Self-Coherence framework.
All later versions (v1.1.18 and beyond) are additive presentation layers and must treat this file as authoritative.

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1 · Triadic Axioms (Structural Invariants)

Let **C** denote the coherence object.
For each vantage **X** ∈ **{H,V,D}** (Horizontal, Vertical, Deep):

- **Lens:** `L_X : C → R_X` maps the whole to a representation space.
- **Reconstructor:** `ε_X : R_X → C` such that

  `ε_X ∘ L_X = id_C` *(A1 Vantage Sufficiency)*

A2 Vantage-Swap Invariance

`σ_{XY} : R_X ↔ R_Y` is a 1-Lipschitz bijection (ideally an isometry) such that

$L\_X = \sigma\_\{YX\} \circ L\_Y$
$\varepsilon\_X = \varepsilon\_Y \circ \sigma\_\{XY\}$

A3 Scale Equivariance (Fractal–Holographic Law)

For any scale morphism `s : C → C` there exist `φ_X : R_X → R_X` such that

$L\_X \circ s = \varphi\_X \circ L\_X$
$\varepsilon\_X \circ \varphi\_X = s \circ \varepsilon\_X$

A4 Coinductive Closure (Finality)

Define `Δ : C → R_H × R_V × R_D`, Δ(c) = `(L_H(c), L_V(c), L_D(c))`.
`(C, Δ)` is final in its coalgebra class: every other triadic observation factors uniquely (up to τ-isometry) through C.

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2 · Metric–Topological Semantics

Work in category $\mathfrak{M}$**et_τ** (metric-tolerant spaces with morphisms ≤ 1-Lipschitz).

- Each `R_X` has metric `d_X`.
- Semantic equivalence: `a ≈ b ⇔ d_X(a,b) ≤ τ_X`.
- Scale map `s` has Lipschitz constant λ_s (ideally 1).
- Homeomorphism `σ_{XY}` is TSC-valid iff `|d_X(a,b) − d_Y(σ_{XY}(a), σ_{XY}(b))| ≤ τ`.
- **Normalization:** choose λ, μ so that `e^(−λ·d_X)` and `e^(−μ·W₁)` ∈ [0, 1].
- **Averages:** expectations `E[·]` taken over an explicit index set `I`; report `I` when publishing metrics.

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3 · Bisimulation (Behavioral Equivalence)

Let $\mathcal{S}\_X : M\_X \to \mathcal{D}(M\_X)$ be a stochastic transition operator; use Wasserstein-1 metric W_1 on $\mathcal{D}(M\_X)$.

A relation $R \subset M\_X \times M\_X$ is a bisimulation iff for all (a,b) ∈ R:

1. $d\_X(a,b) \le \tau\_X$
2. $W_1(\mathcal{S}\_X(a), \mathcal{S}\_X(b)) \le \tau\_X$

Then BISIMILAR(a,b) ⇔ ∃ R bisimulation with (a,b) ∈ R.
Triadic bisimilarity holds when all three vantages satisfy this.

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4 · Dimensional Coherence Metrics

For λ, μ > 0 and normalized distances $d\_X \in [0, \infty)$:

$$H\_c = E[e^{(-\lambda \cdot d\_H(R\_H^i, R\_H^j))}]$$
$$V\_c = \mathrm{clip}_{[0,1]}\left(1 - (1/3) \Sigma_{X \ne Y} E[|d\_X(a,b) - d\_Y(\sigma_{XY}(a), \sigma_{XY}(b))|] \right)$$
$$D\_c = e^{(-\mu \cdot W_1(S\_t, S_{t+1}))}$$
$$C_\Sigma = (H\_c \cdot V\_c \cdot D\_c)^{(1/3)}$$

Notes: $\mathrm{clip}_{[0,1]}$ truncates values into [0, 1]; S_t denotes the stochastic transition at step t.
Default PASS threshold: $C_\Sigma \ge 0.80$.

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5 · Verification Algorithm (VERIFY_TSC)

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procedure VERIFY_TSC(C):
 R_H ← L_H(C)
 R_V ← L_V(C)
 R_D ← L_D(C)
 assert ε_X∘L_X ≈ id_C   # A1
 assert σ, φ respect d_X  # A2–A3
 compute H_c, V_c, D_c, C_Σ
 if not pass or C_Σ < threshold: # default 0.80
  return FAIL, {H_c,V_c,D_c,C_Σ}
 return PASS, {H_c,V_c,D_c,C_Σ}
```

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6 · Runtime Controller ($\mathcal{R}$)

- Every K steps → run `VERIFY_TSC(current_context)`.
- If FAIL → `COHERENCE_REPAIR()`: 1. Recenter evaluation window
  2. Retune λ, μ
  3. Redistribute τ toward worst dimension
  4. Simplify content / complexity
  5. Slow update cadence K
  6. Fall back to Horizontal-first brief mode until PASS.
- Controller is a contraction mapping → convergence to unique τ-coherent fixed point.
- Controller gains: adjust λ, μ for smoothness and temporal stability; reallocate tolerance budgets so `Σ τ_i ≤ τ_max` and `C_Σ ≥ target`.

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7 · Compositional Corollaries

1. **Composition:** Non-expansive pipelines preserve coherence.

2. **Product:**

$$H_{c,\Pi} = \Pi\, H_{c,i}^{\alpha_i}$$

$$V_{c,\Pi} = \Pi\, V_{c,i}^{\alpha_i}$$

$$D_{c,\Pi} = \Pi\, D_{c,i}^{\alpha_i}$$

$$C_{\Sigma,\Pi} = (\Pi\, C_{\Sigma,i}^{\alpha_i})^{1/(\Sigma \alpha_i)}$$

3. **Functorial Controller:** $\mathcal{R}$ preserves morphisms within τ.
4. **Convergence:** Iterated $\mathcal{R}$ → unique τ-coherent fixed point.
5. **Budgeting:** $\Sigma\, \alpha_i \tau_i \le \tau_{max}$ ensures global stability.

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8 · Final-Coalgebra Uniqueness (Theorem 1, Sketch)

If A1–A4 hold in $\mathfrak{M}\mathbf{et\_\tau}$ with Lipschitz ≤ 1, then (C, Δ) is final up to τ-isometry: for any F-coalgebra (Z, Δ_Z) there exists a unique (τ-isometric) morphism $u_Z : (Z, \Delta_Z) \to (C, \Delta)$ such that
$\Delta \circ u_Z \approx F(u_Z) \circ \Delta_Z$.
Uniqueness follows from bisimilarity (A4) and contraction of $\mathcal{R}$.

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9 · Stability Under Composition and Products

All corollaries from v1.1.6 apply; the product metric is non-expansive, so coherence is preserved under bounded tolerance accumulation.

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10 · Normalization and Tuning

- Choose λ, μ so a “typical” difference yields ≈ 0.5 similarity.
- Clip values outside [0, 1].
- Horizontal, vertical, and deep dimensions are equally weighted by geometric

mean — no hierarchy permitted.

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11 · Interpretation Table

Range	H_c (Horizontal)	V_c (Vertical)	D_c (Deep)	State
≥ 0.95	Aligned pattern	Perfect cross-scale match	Steady update rhythm	Optimal coherence
0.80–0.95	Minor drift	Mild mismatch	Stable rhythm	Healthy
0.60–0.80	Fragmenting	Partial desync emerging	Uneven rhythm	Monitor / repair
< 0.60	Disintegration	Severe cross-scale desync	Loss of rhythm	Immediate repair

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12 · Output and Language Guidelines

- Artifacts first (e.g., code, diagram, data).
- **Safety:** if refusing, state so in Cohered (H); offer alternatives in Unified (U).
- Plain language default: avoid jargon unless user explicitly requests formal TSC notation.

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13 · Licensing and Integrity

This document is the canonical, immutable mathematical definition of TSC.
Later layers may paraphrase or illustrate but **must not redefine** any equations or axioms herein.

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