Triadic Self-Coherence (TSC) — Core Knowledge File
Version: v1.1.7 – rev A (Stable Core +)   **Status:** Immutable Mathematical Foundation
Use: Read-only reference for all formal reasoning, verification, and controller logic.

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0 · Purpose and Scope
This document defines the *formal axioms, metrics, and coalgebraic theorems* that constitute the unchallengeable mathematical base of the Triadic Self-Coherence framework.
All later versions (v1.1.18 and beyond) are additive presentation layers and must treat this file as authoritative.

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1 · Triadic Axioms (Structural Invariants)

Let $begin:math:text$C$end:math:text$ denote the coherence object.
For each vantage $begin:math:text$X∈\\{H,V,D\\}$end:math:text$ (Horizontal, Vertical, Deep):

- **Lens:** $begin:math:text$L_X:C→R_X$end:math:text$ maps the whole to a representation space.
- **Reconstructor:** $begin:math:text$\\varepsilon_X:R_X→C$end:math:text$ such that
  $begin:math:display$
  \\varepsilon_X ∘ L_X = \\mathrm{id}_C .
  \\tag{A1 Vantage Sufficiency}
  $end:math:display$

A2 Vantage-Swap Invariance
$begin:math:text$σ_{XY}:R_X↔R_Y$end:math:text$ is a 1-Lipschitz bijection (ideally an isometry) such that
$begin:math:display$
L_X = σ_{YX} ∘ L_Y,\\qquad
\\varepsilon_X = \\varepsilon_Y ∘ σ_{XY}.
$end:math:display$

A3 Scale Equivariance (Fractal–Holographic Law)
For any scale morphism $begin:math:text$s:C→C$end:math:text$ there exist
$begin:math:text$ϕ_X:R_X→R_X$end:math:text$ such that

$$
L_X \circ s = \phi_X \circ L_X,\qquad
\varepsilon_X \circ \phi_X = s \circ \varepsilon_X.
$$

A4 Coinductive Closure (Finality)
Define Δ :$C\to R_H\times R_V\times R_D$, Δ(c) = $(L_H(c),L_V(c),L_D(c))$.
(C,Δ) is final in its coalgebra class: every other triadic observation factors uniquely (up to τ-isometry) through C.

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2 · Metric–Topological Semantics

Work in category $\mathbf{Met}_{\tau}$ (metric-tolerant spaces with morphisms ≤ 1-Lipschitz).

- Each R_X has metric d_X.
- Semantic equivalence: $a\approx b \Leftrightarrow d_X(a,b)\le\tau_X$.
- Scale map s has Lipschitz constant λ_s (ideally 1).
- Homeomorphism σ_{XY} is TSC-valid iff $|d_X(a,b)-d_Y(\sigma_{XY}(a),\sigma_{XY}(b))|\le\tau|$.
- **Normalization:** choose λ, μ and distance scales so that $e^{-\lambda\cdot d_X}$ and $e^{-\mu\cdot W_1}$ ∈ [0, 1].
- **Averages:** expectations $E[\cdot]$ are taken over an explicit index set I (e.g., pairs within a time window); report I when publishing metrics.

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3 · Bisimulation (Behavioral Equivalence)

Let $\mathcal{S}_X:M_X\to\mathcal{D}(M_X)$ be a stochastic transition operator;
use Wasserstein-1 metric W_1 on $\mathcal{D}(M_X)$.

A relation R⊂M_X×M_X is a bisimulation iff for all (a,b)∈R:
1. $begin:math:text$d_X(a,b)≤τ_X$end:math:text$
2. $begin:math:text$W₁(\\mathcal{S}_X(a),\\mathcal{S}_X(b))≤τ_X$end:math:text$

Then BISIMILAR(a,b) ⇔ ∃ R bisimulation with (a,b)∈R.
Triadic bisimilarity holds when all three vantages satisfy this.

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4 · Dimensional Coherence Metrics

For λ, μ > 0 and normalized distances d_X ∈ [0, ∞):

$begin:math:display$
\\begin{aligned}
H_c &= E_{(i,j)∈I}\\!\\left[e^{−λ\\,d_H(R_H^i,R_H^j)} \\right],\\\\[2mm]
V_c &= \\mathrm{clip}_{[0,1]}\\!\\Big(1 − \\tfrac13 \\!\\sum_{X≠Y}\\! E_{(a,b)∈I}\\,| d_X(a,b) − d_Y(σ_{XY}(a),σ_{XY}(b)) | \\Big),\\\\[2mm]
D_c &= e^{−μ\\,W₁(S_t,S_{t+1})},\\\\[2mm]
C_Σ &= (H_c V_c D_c)^{1/3}.
\\end{aligned}
$end:math:display$

Notes: `clip_[0,1]` truncates values into [0, 1]; $begin:math:text$S_t$end:math:text$ denotes the stochastic transition at step t.
Default PASS threshold: $begin:math:text$C_Σ ≥ 0.80$end:math:text$.

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5 · Verification Algorithm (VERIFY_TSC)

```
procedure VERIFY_TSC(C):
   R_H ← L_H(C); R_V ← L_V(C); R_D ← L_D(C)
   assert ε_X∘L_X ≈ id_C          # A1
   assert σ, ϕ respect d_X         # A2–A3
   compute H_c, V_c, D_c, C_Σ
   if not pass or C_Σ < threshold:  # default threshold 0.80 (see Interpretation Table)
```

```
        return FAIL, {H_c,V_c,D_c,C_Σ}
    return PASS, {H_c,V_c,D_c,C_Σ}
```

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6 · Runtime Controller ($\mathcal{R}$)

- Every K steps → run VERIFY_TSC(current_context).
- If FAIL → COHERENCE_REPAIR():
  (1) Recenter evaluation window
  (2) Retune λ, μ
  (3) Redistribute τ toward worst dimension
  (4) Simplify content / complexity
  (5) Slow update cadence K
  (6) Fall back to Horizontal-first brief mode until PASS.
- Controller is a contraction mapping → convergence to unique τ-coherent fixed point.
- Controller gains: adjust λ, μ for smoothness and temporal stability; reallocate tolerance budgets so $\Sigma \tau_i \le \tau_{max}$ and $C_\Sigma \ge$ target.

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7 · Compositional Corollaries

1. **Composition:** Non-expansive pipelines preserve coherence.
2. **Product:** $$H_{c,\Pi}=\prod H_{c,i}^{\alpha_i},\\ V_{c,\Pi}=\prod V_{c,i}^{\alpha_i},\\ D_{c,\Pi}=\prod D_{c,i}^{\alpha_i},\\ C_{\Sigma,\Pi}=\left(\prod C_{\Sigma,i}^{\alpha_i}\right)^{1/(\Sigma \alpha_i)}$$.
3. **Functorial Controller:** $\mathcal{R}$ preserves morphisms within τ.
4. **Convergence:** Iterated $\mathcal{R}$ → unique τ-coherent fixed point.
5. **Budgeting:** $\Sigma \alpha_i \tau_i \le \tau_{max}$ ensures global stability.

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8 · Final-Coalgebra Uniqueness (Theorem 1, Sketch)

If A1–A4 hold in $\mathbf{Met}_{\tau}$ with Lipschitz ≤ 1, then (C,Δ) is final up to τ-isometry: for any F-coalgebra (Z,Δ_Z) there exists a unique (τ-isometric) morphism $u_Z:(Z,\Delta_Z)\to(C,\Delta)$ such that $\Delta\circ u_Z \approx F(u_Z)\circ\Delta_Z$. Uniqueness follows from bisimilarity (A4) and contraction of $\mathcal{R}$.

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9 · Stability Under Composition and Products

All corollaries from v1.1.6 apply; the product metric is non-expansive, so coherence is preserved under bounded tolerance accumulation.

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10 · Normalization and Tuning

- Choose λ, μ so a "typical" difference yields ≈ 0.5 similarity.
- Clip values outside [0, 1].
- Horizontal, vertical, and deep dimensions are equally weighted by geometric mean—no hierarchy permitted.

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11 · Interpretation Table

Range	H_c (Horizontal)	V_c (Vertical)	D_c (Deep)	State
≥ 0.95	Aligned pattern	Perfect cross-scale match	Steady update rhythm	Optimal coherence
0.80–0.95	Minor drift	Mild mismatch	Stable rhythm	Healthy
0.60–0.80	Fragmenting	Partial desync emerging	Uneven rhythm	Monitor / repair
< 0.60	Disintegration	Severe cross-scale desync	Loss of rhythm	Immediate repair

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12 · Output and Language Guidelines

- Artifacts first (e.g., code, diagram, data).
- Safety: if refusing, state so in Cohered (H); offer alternatives in Unified (U).
- Plain language default: avoid jargon unless user explicitly requests formal TSC notation.

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13 · Licensing and Integrity

This document is the canonical, immutable mathematical definition of TSC.
Later layers may paraphrase or illustrate but **must not redefine** any equations or axioms herein.

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