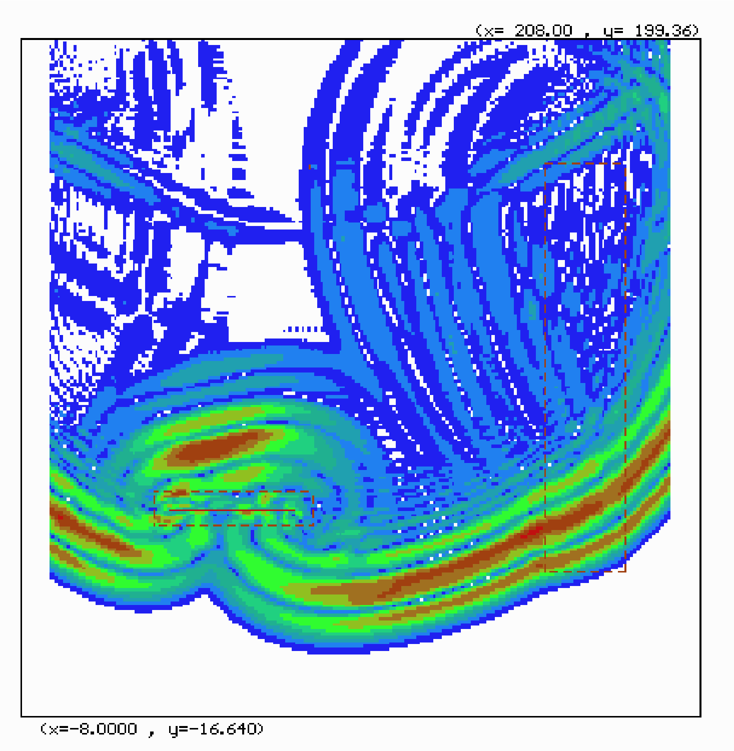
WAVE
User Manual



Includes:

Preface to WAVE Manual and changes in ver 3.51 by Mark Hildyard (2007)

WAVE ver 2.0 Manual by Mark Hildyard (1992)

Updates on WVPLOT by Lindsay Linzer (2007)

Tutorials

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History and Acknowledgements

Prof. Peter Cundall wrote the first 2D version of WAVE around 1990. This included the basic grid scheme, absorbing boundaries and the ubiquitous “stope” construct. He extended the code to 3D the following year and later (amongst other things) introduced fault slip logic and applied stress boundaries for static solutions.

Mark Hildyard wrote the first version of WVPLOT around 1990. He then took over the development of WAVE, converting it to fourth order accuracy and introducing aspects such as irregular mining layouts, crack opening/closing behaviour, crack intersections which also allow voids to be modelled, and source rupture models.

Steve Donovan worked on a version of WAVE suitable for parallel solution in 2006.

The developments of WAVE formed part of research projects envisioned by Dr John Napier, aimed at understanding rock mass behaviour and fracture zone behaviour in the vicinity of mine excavations. These projects were performed from the 1990s to the mid-2000s, at Chamber of Mines Research Organisation (funded by the Chamber of Mines of South Africa) and later at the Council for Scientific and Industrial Research (CSIR) Division of Mining Technology (funded by Safety in Mining Research Advisory Committee (SIMRAC) under the auspices of the Mine Health and Safety Council, South Africa). WAVE’s development is largely due to Dr Napier’s continued vision and the sponsorship behind these projects.

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Preface to the 1992 WAVE manual

by Mark W. Hildyard, 2007

A WAVE user manual was written in 1992. Since then there have been numerous additions of features and some changes to the command structure. Nevertheless, much of the basic structure remains unchanged, so that the manual is a useful aid as an introduction to using WAVE.

Probably the most fundamental change has been due to computing power. While the examples shown are primarily two-dimensional examples, WAVE's principal usage since then has almost exclusively been in three-dimensional modelling. Projections of model sizes and run-times were made in 1992 based appropriately on a 50 MHz 80486 processor.

Current projections (2007) for a 2 GHz Pentium 4, are shown in Table 1, and illustrate that models with more than 50 million elements can now be solved on a single processor desktop computer. An MPI version of WAVE is in development and will allow much larger models to be solved on a computing cluster.

Table 1 Memory and Runtime requirements on a 2 GHz Pentium 4

Elements along 1 dimension	Total Elements in 3D (mil. elems)	Memory required	Estimated Runtime (hours)
100	1	40 MB	0.1
150	3	140 MB	0.5
200	8	325 MB	1.4
300	27	1.1 GB	7
400	64	2.6 GB	23

Probably WAVE's most powerful feature is the "stope" construct. It was named a "stope" as it was originally introduced for modelling tabular mining excavations. However this construct has also been used to successfully model cracks, faults, interfaces, free surface boundaries and three dimensional voids.

WAVE (Extensions since 1992)

Since the 1992 manual, the following features have been added to WAVE:

1. Fourth and higher order solution for grid equations.

2. Frictional sliding on cracks (Mohr-coulomb failure with cohesion weakening)

3. Opening and closing behaviour of cracks (tensile failure and closure restrictions)

4. Stope (or crack) outlines in three-dimensions. The stope/ crack is still planar, but can have an irregular outline. This has primarily been used for representing mining layouts but has wider application.

5. Crack intersection. Orthogonal cracks can intersect. This has been limited to the case where cracks are open at the intersection point. This feature has been used to allow voids in three-dimensions such as mining tunnels.

6. Source rupture models.

7. Various smaller enhancements, and also some developments which have never been extended to three dimensions.

The above features will be written into an up to date version of the WAVE manual.

WAVE Applications

WAVE was originally written for mining research but has also been used in a number of other applications, including nuclear waste disposal, vibrations due to pile-driving, and detecting cracks in wave-guides. Examples of these applications can be found in the following references:

- dynamic fault slip due to stope advance (Hildyard et al., 1995; Napier et al., 1997)
- the effects of mining layouts on recorded seismic waveforms (Hildyard, Napier and Young, 2001; Milev et al., 2005)
- modelling seismograms from explosions (Hildyard and Milev, 2001a,b)
- the effects of cracks on seismic waveforms (Hildyard, 2001; Hildyard, Napier and Young, 2001; Hildyard and Young, 2002).
- determining crack density and from ultrasonic measurements (Hildyard, 2001; Pettitt et al., 2004).

WAVE Theory

The most detailed reviews of the WAVE theory and implementation are found in unpublished material – Cundall (1992) and SIMRAC reports from 1990-2003. Significant detail is also given in Hildyard (2001). Some detail of the theory is published in Hildyard et al. (1995), and the implementation of cracks is shown in Hildyard and Young (2002).

Contents of 1992 WAVE manual

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1. Introduction

The program WAVE was written to model wave propagation in a 2-D or 3-D elastic medium. This wave propagation can be followed from its initiation at a source, through its reflection and refraction from structures in the stope vicinity.

Interlaced finite difference equations are used on an orthogonal grid with uniform grid spacing, with an emphasis on optimal speed, accuracy and memory usage.

For example, a machine with 4 MBytes of memory would support a 2-D model with over 300 by 300 elements. On a 50 MHz 486, a model with this grid size should take less than half an hour to complete. A large 2-D model of 800 by 800 elements can be run with 16 Mbytes of memory[1].

A variety of dynamic sources can be simulated in WAVE. It provides assorted 'smoothed-step' and cosine wave-shapes, which can be applied to shear or dilational sources of any size and placed at any point in the grid. Absorbing boundaries are provided which simulate an infinite rock mass and reduce reflections from the edges of the numerical grid. Material properties may vary throughout the grid, allowing features such as dykes and softened material to be represented. The orthogonality of the grid means that the relative orientations of features must be approximated in the grid. This orthogonality gives WAVE its simplicity and efficiency of solution.

Although openings may be constructed by setting the elastic constants to zero over part of the grid, a special discontinuity - called a *stope* - is provided, which has zero thickness, and can exhibit non-linear stress-displacement behaviour. This feature is intended to represent a thin, mined-out seam, with yielding or hardening support (hydraulic supports, or backfill). Passive faults are provided as a similar construct to that of the stope.

Plotting and printing of any quantity is provided. Plots involve 'snapshots' of the grid (or part of the grid) or time histories. Snapshots show grid variable values at some instant in time. Time histories show how a particular parameter at a point or line of points in the grid evolves with time (these are essentially seismograms). Plot data is saved to file and can be analysed later with the 'WVPLOT' post-processing package, where various representations are supported.

2. Operation

WAVE has a command driven interface. On start-up, WAVE expects input from the keyboard. The program can be run interactively in this manner, or from a prepared data file of commands, using the CALL command.

Prior to setting up a WAVE model, a user should have a clear insight into its finite difference mesh or grid. The program has a finite difference formulation, which means that the equations for stresses and velocities are solved for at discrete points called the grid. It is a two-step method, where velocities are calculated from known stresses applied to equations of motion, while stresses are calculated from known velocities applied to linear elastic constitutive equations. The stresses and velocities are thus known at different points in time.

[1] Refer to updated table of memory and runtime requirements in the Preface

The grid is interlaced for numerical purposes. 'Interlacing' means that the points in space and time at which the equations are solved for, differs for the different stresses and velocities. They are in fact separated in space by one half a grid length. A cell or element is the collection of stress and velocity variables conceptually associated with one point (i.e. sharing the same indexing in the grid). Figure 2.1 shows the arrangement of a unit cell in the grid, with the positions at which velocity and stress components are calculated.

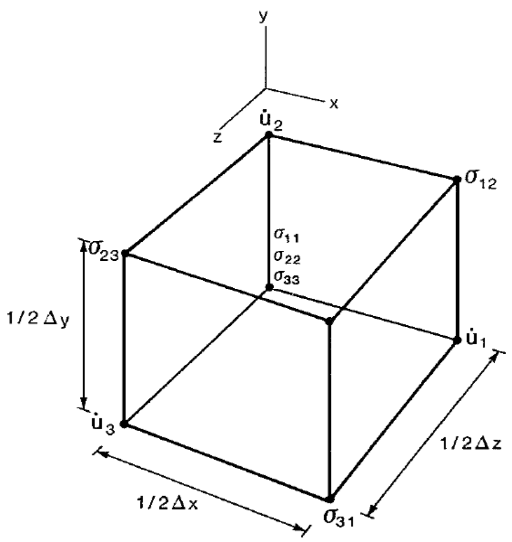


Figure 2.1 Location of variables on a unit cell

A number of steps or phases are normally required in a WAVE run. These are outlined in the following sections.

2.1 Grid definition

WAVE's grid is orthogonal and the grid spacing along any direction is uniform. To define the grid, the number of grid points and the spacing between grid points must be specified for the x- and y-directions (and the z-direction if the model is three-dimensional). The density of grid points in the mesh affects the accuracy and resolution of the model, while the total number of grid points affects the run-time.

On a 16 MByte 386 machine, the maximum 2-D problem size would be 800x800 elements (a WAVE grid need not be square), and the approximate run-time 3 hours. Appendix A shows the relationship between memory size, maximum number of elements, maximum square-grid size and run-time. The run-time increases in 2-D with $N^{3/2}$ and in 3-D with $N^{4/3}$, where N is the total number of elements. However, with N_1 the number of elements in one dimension of a square grid, the order is N_1^3 and N_1^4 respectively. A 2-D problem using a 200x200 element grid would take one eighth of the time if run on a 100x100 grid.

A time-saving practice is to run a first approximation of a model on a reduced grid spacing, and later to refine the mesh to satisfy the frequency considerations (*cf.* source specifications), while giving sufficient resolution.

2.2 Property definitions

Material properties can vary throughout the grid. These properties cannot vary arbitrarily from zone to zone (as they can in FLAC). Instead, a maximum of twenty different materials can be defined, each with its set of properties. Each zone in the mesh is then associated with one of these material types. The properties to be specified are the Bulk Modulus (K), the Shear Modulus (G) and the Density (P). Appendix B gives formulae for conversions between these parameters and other elastic parameters i.e. Young's Modulus (E), Poisson's Ratio (v), Shear-wave velocity (c_s) and Compressional-wave velocity (C_p).

2.3 Layout of geology

The geology to be modelled must be laid out by specifying the distribution of the various materials, and the location of any special constructs such as stopes and passive faults (not available in WAVE ver 2.0). Different materials can be used to represent dykes, discontinuities of rock type and softened areas of rock.

With the ‘stope’ construct one can represent a number of different features that are not only mining voids where the reef is mined (i.e. stopes). These features are: stopes with or without support, parting planes and discontinuities, free surfaces and reflecting boundaries. (The WAVE grid has absorbing boundaries which simulate an infinite rockmass, but a stope can simulate a free surface and hence a reflecting boundary). Features are specified in grid elements and the layout is conceptually simple due to the orthogonality of the grid. Note that the location of material properties is somewhat 'fuzzy' due to the interlaced grid - there is a half zone width of indeterminacy in the location of the material boundary.

2.4 Point and line history positions

A large amount of data is available during a dynamic run, and due to space limitations only selected data can be stored. It is thus important to consider well the data required for useful analysis, before the model is solved. Point and line histories indicate positions and variables in the mesh for which WAVE must accumulate data at different time steps, for time histories.

2.5 Initial boundary stresses

Initial boundary stresses are required to solve for the initial static stress state. If not specified WAVE solves only the dynamic problem, without the superimposed virgin stress state.

A preexisting stress state can be imposed. In this case, the boundary stresses may be set and then cycled to a stable solution, prior to an event.

e.g. **Bound s11=-35e6 s22=-70e6 s12=0e6**

Note that compressive stress is negative.

The **Initial** command can be used in conjunction with the **Boundary** command to give the grid a predefined stress state.

2.6 Source representation

Seismic sources are implemented in WAVE by forcing nodes in the grid to take on particular stress values. In representing a source then, consideration must be given as to which stresses to perturb. The examples in Chapter 5 implement two different sources. An explosion was simulated with a hydrostatic pulse in the stresses s_{11} and s_{22}, while slip on a fault was simulated with a drop in shear stress s_{12} along a line in the mesh.

In addition to selecting a suitable representation of the source, the source through its introduction of frequencies of disturbance into the mesh, has implications for the size and number of grid elements. The finite difference scheme suffers from the dispersion of high frequencies. Dispersion implies that a disturbance is not propagated through the mesh at a constant velocity, but the different frequencies in its composition are propagated at different velocities (higher frequencies at slower velocities). As a result the frequencies separate out with propagation from the source. The specification of the source therefore has far-reaching consequences on the validity of a WAVE simulation.

Three considerations are important when defining the source: the shape, pulse duration and the area it spans. These are chosen to ensure that the frequencies present in the mesh satisfy a criterion, such that frequencies below this criterion do not suffer significant dispersion within the propagation distances under consideration.

A good rule is that

$$\lambda > 10\,\Delta x$$

where λ is the shortest wavelength to be propagated and Δx is the largest grid spacing.

Pulse duration T, is understood to be the duration of the transient portion of the Pulse. This duration implies a dominant wavelength, which as a first approximation we will take as the critical wavelength by:

$$\lambda = c\,T$$

where c is the propagation speed of the wave.

Noting that the time-step used by WAVE based on sampling considerations is:

$$\Delta t = \frac{\Delta x}{2c_p}$$

where the c_p used is the fastest propagation speed of all materials, and applying the rule that $\lambda > 10\,\Delta x$, the duration of the source must always be <u>at least</u> 20 times greater than the time step Δt, which is reported when cycling the model in WAVE.

However, there may be widely differing material properties in a model. More strictly applying the criterion throughout the grid, we see that the shortest wavelength must over at least 10 grid lengths in all materials, and hence the slowest propagation speed should be used:

$$\lambda = c_s\, T$$

where $c_s = \left(\frac{K^{-2/3} G}{\rho} \right)^{1/2}$ is the slowest shear wave propagation speed of all the materials in the model, and applying the criterion, we clearly require $c_{\min} T > 10\, \Delta x$.

Therefore the grid element size and source frequency are coupled parameters: choice of one restricts the choice of the other. In a model one may need to choose a particular frequency range; alternatively, memory and run-time considerations may enforce a minimum grid element size. To calculate the corresponding parameter in either case, we have the following guideline:

Element size from a given frequency: $\Delta x < \frac{1}{10} \left(\frac{c_{\min}}{f} \right)$

Pulse duration for a given element size: $T > \frac{1}{c_{\min}} (10\, \Delta x)$

A pulse will contain higher frequencies with shorter wavelengths than that of the dominant, yet with significant contribution to the energy in the pulse. The duration of the pulse may need to be around three times the above approximation, depending on the **pulse shape**. Various pulse shapes are provided. These differ in smoothness and in the smoothness of their derivatives, and hence in their frequency content. Appendix C shows these waveforms together with their first and second derivatives. These vary from the worst case as follows:

- Step Pulse - discontinuous
- Step Pulse - discontinuous
- Cos Pulse - discontinuous in 1st derivative
- S-Shaped Step - discontinuous in 2nd derivative
- Ricker Wavelet - smooth in all derivatives.

The **source application area** or span is relevant in terms of the wave shape decay with distance from the source. A point source is the worst case and requires a continuous second derivative. A planar source would not apply the same constraint on the choice of original source pulse shape. In practice, the best approach is to first select a source based on the physical problem. Analysis afterwards of frequencies in the waveforms can show the presence of significant magnitudes of unreliable frequencies. The post-processing package allows a waveform to be filtered to the above 10% frequency criterion. Substantial differences between this waveform and the unfiltered waveform would indicate respecifying the problem, either by:

- Increasing the number of grid zones (i.e. decreasing zone size)
- Increasing the duration (of the transient part) of the pulse
- Choosing an alternative, smoother source shape.

2.7 Generating a solution

This is done by issuing the Cycle command and the number of time steps to be cycled. The program calculates the critical time step Δt from the given material properties and grid element sizes. It then steps through successive time increments solving the mesh for the specified conditions. If initial boundary stresses are specified, they should be cycled through to give a stable virgin condition, before the source functions are applied. On applying the source, the mesh should be cycled until the transient effects have died out. The time step is calculated according to the propagation speed in the fastest material - it takes approximately two time steps to propagate across one zone. Hence the time for a source to complete its influence on the mesh can be estimated.

2.8 Plotting

This involves requesting relevant data from the solution to be saved to file, for analysis later. Five types of plots can be requested. Snapshots, Grid Dumps and Line Snapshots can be dumped to file at various stages of cycling – these are based on the grid condition at an instant in time. Requested Point and Line Histories are accumulated with cycling and can be written to file at the end of cycling.

3. Commands for WAVE

The commands listed below may be given in any order that is physically meaningful; normally the first command should be **Grid**. The following conventions are used in describing these commands.

- When parameters are arranged horizontally across a line, they must all be given, except for optional parameters, denoted by brackets **< >**.

- Parameters arranged vertically after a command denote the selection of options that may be given: any or all of these parameters may be given in any order. For example: **Print M S22 XV** will display the percentage of the available memory used, the σ_{22} stresses and the x-velocities, respectively.

- Parameters that start with a lower-case letter stand for numbers. For example, **n** may be given as 100 and s22 may be given as 1.2e- 7.

- Commands and parameter keywords may be typed in full, or truncated to a few letters. It is recommended that at least the first three letters of the keyword are used to avoid ambiguity. For example, the command Print Memory may be given as **Print Memory**, **PRI MEM**, **pri mem**, **print mem**, etc. "**p m**", will in this case produce the same result, but such shortcuts are not recommended.

- Parameters may be separated by any number of spaces, commas, brackets () or slashes /.

- Many commands refer to the grid variables of stress and velocity .To avoid having to repeat the same list many times, the symbol *gvar* will denote a variable name from the following list:

S11	stress σ_{11}

S22	stress σ_{22}
S33	stress σ_{33}
S12	stress σ_{12}
S23	stress σ_{23}
S31	stress σ_{31}
XVel	velocity u_1
YVel	velocity u_2
ZVel	velocity u_3
Dil	Dilational stress $\left(\sigma_{11}+\sigma_{22} \ <\sigma_{33}>\right)$

In 2-D mode (see the **Grid** command), the variables S33, S23, S31 and ZVel should not be given as parameters; if they are, an error message will be produced.

Grid locations are referred to by their indices: for example, index **i** refers to the ith zone in the x-direction. Indices **j** and **k** refer to the y and z directions, respectively. Note that the finite difference grid is *interlaced*: each variable is stored at a different point within a zone (see Figure 2.1 in Section 1)

Where the symbol *range* occurs in a command specification, ranges for one or more of the grid indices may be given: e.g. i**= 2,3 j = 2,6 k = 1**. If an index (**i**, **j** or **k**) is omitted, its associated range is assumed to be the maximum (the whole grid width). Indices may be given in any order.

Call **filename**

Input lines are taken from the file **filename** rather than the keyboard. Control is returned to the keyboard when either (a) the end of the file is reached, or (b) a **Return** statement is encountered in the file, or (c) an error is detected on an input line. The **Call** command will also set the root filename (less an extension) which is then used for output files created by commands such as **Save** and **Plot**. The **Name** command can alter this root filename.

Cycle **n**

n calculation cycles are performed. Each calculation cycle corresponds to an increment in time equal to one timestep. The final time is printed out after the **n** calculation cycles have been executed. The time step is calculated from the formula:

$$\Delta t = \frac{\min\left(\Delta x, \Delta y, \Delta z\right)}{2c_{p-\max}}$$

where $c_{p-\max}$ is the fastest p-wave speed considering all materials in the model.

Damp **dfrac dfreq**

There is provision in the code for mass-proportional viscous damping. **dfrac** is the fraction of critical damping at a frequency of **dfreq**.

Grid **nx ny nz dx dy < dz >**

The number of zones in the finite difference grid is specified: **nx** is in the x-direction, **ny** in the y-direction and **nz** in the z-direction. If **nz** is given as 1, then WAVE solves the *plane-strain, two dimensional* problem, in the plane of the x and y axes. The grid spacings must also be given on the **Grid** command: **dx**, **dy** and **dz** in the x, y and z directions, respectively. The parameter **dz** is optional if 2-D mode is requested (i.e. **nz = 1**); if it is given as in this example, it is ignored. The **Grid** command must be given before any other commands that refer to a grid (e.g. **Initial**, **History**).

History ***gvar*** **i, j, k <Dt = n> <*gvar* i, j, k >**

Point history request. The command instructs WAVE to track the specified grid variable *gvar* at grid location **i, j, k**. Several requests can be placed on one line. Histories need not be started at zero time, but all histories must be started at the same time. Using **Dt = n**, the user can set the sample rate at which histories are recorded i.e. 1 sample every **n** cycles. If not specified, this defaults to 1 sample every cycle. The sample rate can only be specified in the very first history request, and becomes a global setting for all point histories:

e.g. **his s12 (1,2,3) dt=10 xvel (3,2,1) s33 (4,4,4).**

Note that point histories are accumulated in a temporary file 'hst.tmp' in the current directory. The file is automatically deleted when closed.

Initial ***gvar*** **val** ***range***

The grid variable *gvar* is given the value **val** over the specified range of zones. Note that there is no point in specifying initial values for the stress components, since the program works in terms of stress increments.

Lhist ***gvar range*** **< Dx = n > < Dt = n >**

Line history request. Logically equivalent to the **History** request, except that the history is maintained for a series of points along a line. The command instructs WAVE to track the specified grid variable *gvar* over the given *range*. Only one of **i, j** or **k** may vary in this range, yielding a line parallel to the x, y or z axis. **Dx = n** sets the sample spacing in space, and refers to this line history only (i.e. one in every n points along the line is tracked by the history). Line histories need not be started at zero time, but all must be started at the same time. **Dt = n** sets the sample rate at which line histories are recorded i.e. 1 sample every **n** cycles. If not specified, this defaults to 1 sample every cycle. The sample rate can only be specified in the **first** line history request, and becomes a global setting for all line histories:

e.g. **Lhis s12 i=1,100 j=50 k=1 dx=5 dt = 10.**

Note that line histories are accumulated in a temporary file called ‘Lhst.tmp’ in the current directory. The file is automatically deleted when closed.

Material **n range**

The material number is set to n for the range of zones, range. **n** must be between 1 and 20. The default material number (when starting WAVE) is 1. The **Property** command associates physical properties with given material numbers.

Name **filename**

This sets the root file name to be used for output i.e. files generated with the **Plot** and **Save** commands. **Filename** should be the full path, but without any file-name extension. If not specified, the root file-name used for output is that of the remote file (if a **Call** command has been issued). Otherwise the name used defaults to 'wave', and files such as 'wave.sav' or 'wave.snp' will be generated in the current directory. (Refer to the **Plot** and **Save** commands).

New

All data concerning the current run is erased, and WAVE re-starts from scratch.

Plot **Snap** ***gvar*** **<Rep=n>** **< I** **<DI=n> <DJ=n> <DK=n>**
J
K >
LSnap *gvar range*
Dump
Hist n1 < , n2, n3 ...>
LHist n1 < , n2, n3 ...>

PSnap *gvar*
PHist n1 < , n2, n3 ...>

Keywords **Snap**, **Lsnap**, **Hist**, **Lhist** and **Dump** generate output in a form suitable for the program WVPLOT – the WAVE post-processing package.

Snap, **Lsnap** and **Dump** are snapshot type output, i.e. they record the state of the grid at an instant in time. **Snap** indicates a grid snapshot, and causes the values of the specified grid variable (*gvar*) at an instant in time and over a section of the grid in the x-y plane, to be saved to file. The section is that previously selected by the **Select** command, defaulting to an entire plane. The optional **Repeat** parameter indicates that the snapshot must be added to a repeat list. If it is the first in the list it must also specify the repeat frequency (in cycles). These snapshots are then produced every **n** cycles. If the **Repeat** parameter is not given, a single snapshot is produced of the current grid. If WAVE is in 3-D mode, the **i, j** or **k** parameter is required, indicating the slice in the x, y or z dimension at which the plane snapshot will be made. The optional **di**, **dj** and **dk** parameters set the sample spacing in space along the x, y and z directions.

Lsnap indicates a line snapshot, and saves the values of the specified variable (*gvar*) at the given instant, but over a line rather than an area of the grid. The *range* parameter must vary in one direction only. Keyword **Dump** generates a snapshot of *all* grid variables at the given instant and over the selected grid area.

The keyword **Hist** causes one or more point histories to be plotted. The numbers **n1, n2** ... refer to the sequential history numbers as they were requested with the **History** command. Similarly **Lhist** causes one or more line histories to be plotted, as requested in the **Lhistory** command.

Snapshots and Line histories are saved to a file ‘fname.SNP’, histories and line snaps to ‘fname.HST’ and dumps to ‘fname.DMP’, where *fname* is the root filename as set by the **Name** and **Call** commands. In addition a file ‘fname.MAP’ is created containing information on the format of each of the plots. The data can be viewed in a variety of ways with WVPLOT. Data in ‘fname.SNP’ (i.e. Snapshots and Line histories) can be displayed as multi-line graphs, colour-shaded plots (with shading proportional to the value of *gvar* at a grid point), or contours. Data in ‘fname.RST’ follows a line in time or in space, and are displayed as line graphs/seismograms. Functions can be performed on the data allowing combinations of graphs, integration, frequency analysis and filtering. Data in ‘fname.DMP’ can be displayed as snapshots of stress tensors, velocity or displacement vectors.

The keywords **Psnap** and **Phist** provide direct postscript output to an ASCII file ‘fname.PSC’. The file can be printed on a postscript compatible device to give a hard copy. Each postscript plot request produces a page of plotted material. The keyword **Psnap** causes a section through the grid to be plotted, with shading intensity proportional to the magnitude of the specified variable *gvar*. The keyword **Phist** causes one of more histories to be plotted. Only four histories can be placed on a page; if more than four are specified, the remainder will be placed on subsequent pages.

Print ***gvar***
History
Lhist
Memory
Sources
STopes

The command **Print** *gvar* prints out the selected grid variable over a range selected by a previously given **Select** command. Keywords **History** and **Lhist** produce a list of those histories and line histories that have been requested, with their maximum and minimum values. Keyword **Memory** reports the amount of WAVE's main memory currently in use. Keywords **Sources** and **Stopes** produce summaries of the sources and stopes currently defined.

PROp **< Mat n >** **Dens d**
Bulk b
Shear s

The keywords **Dens**, **Bulk** and **Shear** cause the density, bulk modulus and shear modulus to be set to **d**, **b** and **s**, respectively, for material number **n**. If the **Mat** keyword is not given, the material number defaults to 1. Appendix B has useful conversion formulae between these and other elastic parameters.

Quit

The program stops. The command **Save** should be used prior to **QUIT** if the current run will need to be continued in the future.

Restore **filename**

The previously saved state on file **filename** is restored to memory, overwriting the current state. WAVE reports the grid size, elapsed time and cycle count of the restored state. If no filename is given, the default filename of 'save.SAV' is used.

Return

This command, when placed in a **Call**ed data file, will cause control to be returned to the user. **Return** does nothing if entered from the keyboard.

Save **filename**

The current state (including all grid variables, histories, options, selections, etc.) is saved to file **filename**. The command has no effect on the current state. If the file name is omitted, the current root filename, as set by the **Name** or **Call** commands, is used with an extension '.SAV'. If no root filename has been set, a default file name of 'wave.SAV' is used.

Select ***range***

The given range is stored in memory, and is used to restrict the grid range on subsequent **Print** *gvar* or **Plot Snap** commands. If **Select** is given without parameters, the whole grid is selected.

SET **Log** **ON**
OFF
Append **ON**
OFF

The keywords **Set Log ON** cause the file 'wave.LOG' to be opened, and all subsequent screen output to be recorded on the file. The command **Set Log OFF** inhibits subsequent file recording, but the file remains open; a future **Set Log ON** command starts the recording process again. Note that an existing 'wave.LOG' file will be overwritten when **Set Log ON** is given for the first time in a WAVE run. An existing file can be preserved by renaming it before starting WAVE.

The **Append** attribute determines whether plot files from a previous run are appended to or overwritten. If the **Set Log ON** command has been given, then when the first plot command is given, and the plot files of the name already exist, they will be opened and advanced to the end of the file for appending. If not set, the old files of the same name will first be deleted.

SOurce **< Ampl** a **>** ***range***
< Begin $t1$ **>**
<End $t2$ **>**
< Period T **>**
<Type ***gvar*** **>**
<Waveform **Cos**
Step
SS
S_Shape
SSH2
Rck
CCos >

A dynamic source is created by this command. The specified grid variable (given by the **Type** keyword) will be controlled over the given range. If **Type Dil** is given as the type, then $(\sigma_{11}+\sigma_{22})/2$ will be controlled in 2-D mode, and $(\sigma_{11}+\sigma_{22}+\sigma_{33})/3$ will be controlled in 3-D mode. The control will be applied at time $t1$, given by the BEGIN keyword, and will end at time $t2$, given by the END keyword. Outside of this time range, normal calculations will be done for the specified grid range. The maximum amplitude of the dynamic waveform is given by the AMPL keyword and the duration of the pulse given by the PERIOD keyword. If the time $(t2-t1)$ is greater than the specified period then the controlled variable will be *held at its final value* for the remaining time after the pulse is finished.

Various wave shapes are supported by the **Wave** keyword. Taking a as the amplitude, T as the period, t as the time from the start of the pulse (i.e. $t = time - t1$) and $a(t)$ the amplitude at time t, the wave shapes have the following definitions:

Wave Cos produces a single cosine pulse: $a(t) = a/2\left(1-\cos\left(2\pi t/T\right)\right)$

Wave Step produces a discontinuous or step change in the given grid variable. The period T need not be specified, since there is no transient duration. Ultimately, it is a bad choice, because of the high frequencies it introduces into the mesh. However, it is useful as a first implementation of the source, since there are no period/frequency considerations (as with the other sources), and it provides a worst case frequency content. This helps in choosing a 'better behaved' source.

Waveforms **SS, S_Shape** and **SSH2** are various alternatives yielding a smooth step change over the given transient duration:

SS: $a(t) = a\left(t/T\right)^2\left(3-2t/T\right)$

S_Shape: $a(t) = a/2\left(1-\cos\pi t/T\right)$

SSH2: $a(t) = a/T\left(t - T/2\pi\sin\left(2\pi t/T\right)\right)$

Wave Rck selects a Ricker Wavelet. The Ricker Wavelet as an analytic function is infinitely differentiable (i.e. all derivatives are smooth). It is band limited in frequency but infinite in duration. The function as implemented in WAVE as a time-limited approximation. Defining $t_s = T/2$, $t_0 = T/6$ and $\tau = \left(t-t_s\right)/t_0$, the value at time t is: $a(t) = a\left(1-2\tau^2\right)\exp\left(-\tau^2\right)$

Wave Ccos produces a continuous train of cosine pulses. Here T is the period of the cos wave, which is excited for the full duration of $t1$ to $t2$. After some time the source excitation is a single frequency: $a(t) = a/2\left(1-\cos\left(2\pi t/T\right)\right)$. Appendix C shows each waveform together with its first and second derivative.

Stope	**< Num**	**n >**	***range***
	< Type	**0,1,2,3 >**	
	< KN	**v >**	
	< KS	**v >**	
	< Ratio	**r >**	
	< Slope	**v >**	
	< i0	**n >**	
	< j0	**n >**	
	< k0	**n >**	

A zero-thickness slit is created in the grid. Rectilinear stopes are allowed in either the x-z, the y-z, and for three-dimensional models, the x-y plane orientations. At least one of the indices in the specified range must be constant. This defines the orientation of the stope:

e.g. **i= 50,80 j =20 k=20,60** defines a stope in the x-z plane.

For a stope in the x-z plane, the boundary conditions on the slit are: $\sigma_{22} = 0$ and $\sigma_{12} = 0$ if unsupported. Both u_1 and σ_{11} are double-valued on the slit – the value used depends on the direction of approach to the slit.

Keyword **Num** addresses a particular stope allowing parameters of an existing stope to be altered. However the *range* must be selected in the first definition of a stope. Other parameters refer to the stiffnesses of support/backfill in the stope.

Four types of support are possible:

Type 0: No support -other parameters are ignored.

Type 1: **KN** the normal stiffness and **KS** the shear stiffness are fixed over the entire stope.

Type 2: **i0**, **j0** and **k0** specify positions in the stope from where support is to be applied - a maximum of two need be specified, depending on the stope's orientation. Support is applied in the direction of increasing **i**, **j** or **k** through to the end of the stope. **KN** is the maximum normal stiffness. Slope is then used to vary normal stiffness from zero at **i0** / **j0** / **k0** to the maximum **KN**, at some point in the stope - it is held at this value until the end of the stope. The normal stiffness is varied according to the formula:

$$KN(i) = \min\left(KN, Slope*(i - i0)\right) \quad \text{for} \quad i \geq i0$$

The *shear stiffness* is calculated from the given **KN** multiplied by a shear **Ratio**. Shear stiffness also varies along the stope, but the slope used is **Slope * Ratio**.

Type 3: Sliding interface (fault). This stope type assumes a uniform shear and normal stiffness. It allows sliding to occur if the shear stress satisfies the following criteria:

$$|Tau| > COH - \tan(FRIC)(Sn + SIG\_N)$$

where COH is cohesion, $FRIC$ is friction angle, Sn the current normal stress and SIG_N a static normal stress given by the user. SIG_N is available to specify a static normal stress, if boundary stresses have not been used to give a pre-existing stress state.

Slip weakening is implemented according to the formula:

$$Cohesion = \max\left(0, COH * \left(1 - \frac{Tot\_Slip}{SW\_Disp}\right)\right)$$

Tot_Slip is the total of all slip increments, SW_disp is the slip weakening displacement. A high default is used for this parameter.

A stoping width can be given with the $SWID$ parameter. No normal or shear stress is allowed on the surface if the crack is open. Shear and normal bonds are broken.

$TSTR$ is the tensile strength. Once exceeded, shear and normal bonds are broken.

e.g. Stope n=2 type=3 i=20,30 j=10
Stope n=2 kn=1e12 ks=1e10 coh=1e6 fric=30 sw_disp=.01 swid=0 tstr=0

4. WVPLOT

WVPLOT is the WAVE post-processor. To run WVPLOT, open a DOS box and type '**WVPLOT'**. When one runs WVPLOT for the first time, a settings file containing default settings is written to the directory containing the data files. The settings file is called: 'WVPL407.SET'. The settings file contains the default drive and directory, the video mode, and display. <u>Once the settings have been selected, it is advisable not to delete this file, and rather copy it into new directory containing WAVE output files</u>.

4.1 Introduction to WVPLOT

The *display option* is the core of the WVPLOT program. It first lists the WAVE generated files found in the currently selected directory – all files with a '.MAP' extension. The user then selects a file to be plotted. If an error occurs, or the file is not found, the user is prompted to respecify the default drive and directory. Once a file is chosen, the user decides whether to display snapshot, history or dump data sets. In each case a summary listing of the available data sets is given with the relevant data, such as positions and maximums. The graph types and options available to each of these data set types are given in detail later in this section.

Retrieving images refers to previously displayed plots which were saved as images. It allows two options: either to restore a particular image, or sequence a slide show. In either case all files in the default directory with extension '.Inn' where 'nn' is the image number, are listed. The user must then specify a file and the image number with which to start (a slide show will always start with image 1). In *Restore mode*, after the chosen image number has been displayed, pushing the carriage return button will display each image in sequence. The slide show automatically advances through the sequence of images. The time delay between slides can be increased/decreased using the '+' or '-' keys. Sequencing can be paused with the spacebar and resumed with any other key. When the end of the sequence is reached, it wraps around to display image 1. Option *Quit* returns to the main menu. Hard copies of saved images may be created by restoring the image to screen and using a screen capture program (or using the print screen key).

The option *Settings* presents a menu, allowing the global settings to be altered. These settings include the default drive and directory and the video mode. Other refer to a specific graph type. Many of the settings can also be changed while displaying graphs, and therefore their functions are displayed later in this manual. Settings are saved to a file called 'SETS.WAV'. The following features can be altered in the settings menu, listed her according to the graph types they affect:

- Snapshot settings:
  - Interpolation
  - Fast mode
  - Colour distribution method (log/ linear/ quadratic…)
  - Extent of distribution. This is used for log distributions only. If it is set to 256 (default) then the smallest value differentiated is 1/256, the maximum value.
  - VGA256 mode. This toggles a VGA monitor between standard mode and a 256 colour mode. This should only be used with the colour-shaded plots, where it gives greater colour resolution at the expense of a reduced pixel resolution.

- o Quantity of colours. This defaults to 12 colours in standard VGA/EGA mode, and 48 colours in the VGA256 mode. The palettes are redefined to give a rainbow distribution of colour.
- o Quantity of levels. The number of levels displayed in the colour map and contour plots. This defaults to 12 or 48 depending on the display mode, but need not be the same as the quantity of colours.

- History settings:
  - o Fourier low and high pass frequencies.
  - o The number of times the data set is extended in length (e.g. if this is set to 1 and a given data set has 200 points, then this is rounded to 256 (the nearest power of 2) and the extended to 512 points. The waveform is centralised with zero-padding on either side.

- Dump settings:
  - o The vector scale can be fixed at a definite value (the vector value represented by 1cm of the plotted) or allowed to be calculated for each graph.

4.2 Running WVPLOT for the first time

If WVPLOT has not been run from a directory before, the ‘WVPL407.SET’ file will not exist, and the first screen will appear as shown in Figure 4.1.

Figure 4.1 Default WVPLOT display

Press the ‘S’ or ‘s’ key on the keyboard to access the settings menu. The following screen should appear.

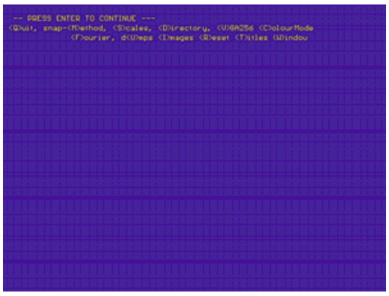


Figure 4.2 Settings menu

Press the ‘C’ key for colour mode and ‘W’ if a white background is preferred to the default black screen (this is preferable if one wishes to make screen dumps for a documented that will be printed). The screen will only change to the new colour once some images have been displayed (either snapshots or histories).

The first display screen should now appear as follows:

Figure 4.3 White background of WVPLOT display screen

4.3 Loading data files / images into WVPLOT

To load output files created by WAVE, press the ‘D’ key of the *Display Graphs* option (first menu item of Figure 4.1 and Figure 4.3). This will list all WAVE generated files found in the currently selected directory (which is saved in the settings file) with a ‘.MAP’ extension.

The user selects the file to be plotted by typing in the name of the ‘.MAP’ file, <u>without the extension</u>. Be careful not to make a typo in the first letter of the filename, because the first letter cannot be deleted. If an error occurs, or the file is not found, the user is prompted to re-specify the default drive and directory (a path may be specified). This can be exceedingly tricky. The screen will appear as shown in Figure 4.4.

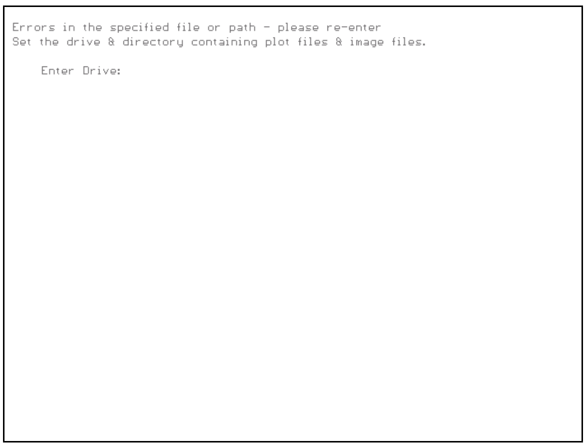


Figure 4.4 Error selecting the file to be viewed in WVPLOT

Type the letter of the drive name. You will then be prompted to enter the directory. Here lies the secret: type '.' and then press the 'Enter' button. The screen should look something like Figure 4.5.

Select the file to be plotted by typing in the name of the '.MAP' file, <u>without the extension</u>.

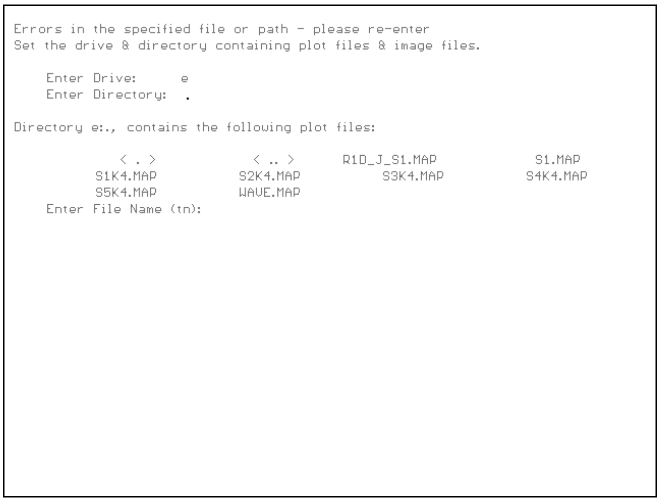


Figure 4.5 Error selecting the file to be viewed in WVPLOT

Once a file is chosen, select whether snapshots or histories are to be displayed, or whether data sets are to be dumped (Figure 4.6).

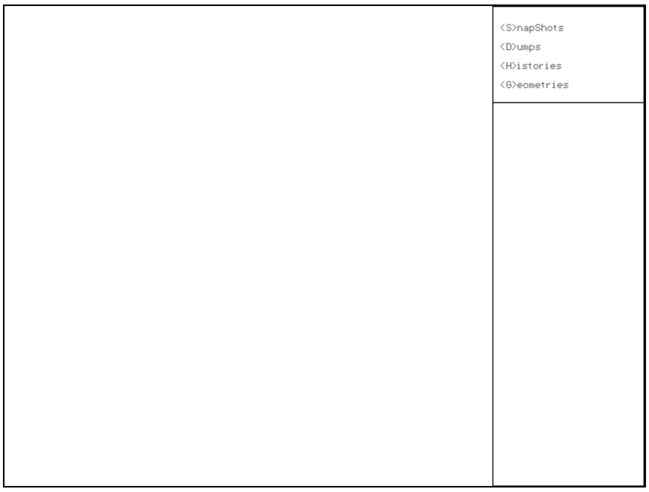


Figure 4.6 Options in WVPLOT

Select the option by typing the relevant letter. A summary of the available data sets will then be listed. This summary also includes summary information for each data set, such as recording positions, maxima and minima. In Figure 4.7, a listing of the available snapshots is displayed.

Pressing any button will allow the user to navigate through the listing. Pressing the 'Escape' button will skip to the end of the data set listing (Figure 4.8).

To select a snapshot to be viewed, type the snapshot number, space, followed by a zero. The syntax is shown in **Error! Reference source not found.**. In this example, snapshot 1 is selected. To skip through the snapshots available, press the 'Enter' button or the space bar. Figure 4.10 shows snapshot #7.

SUMMARY OF PLOTS AVAILABLE

Snap	Time	Var	Xqty	Yqty	Max	Min	Geom
1	3.84E-04	Xdisp	21	21	7.38E-08	-6.27E-08	1
2	3.84E-04	Ydisp	21	21	7.32E-08	-6.27E-08	1
3	3.84E-04	Zdisp	21	21	-1.70E-06	-3.43E-06	1
4	3.84E-04	Xdisp	21	21	1.33E-07	-1.43E-07	1
5	3.84E-04	Ydisp	21	21	1.33E-07	-1.43E-07	1
6	3.84E-04	Zdisp	21	21	0.00	-3.52E-06	1
7	3.84E-04	Xdisp	21	21	1.91E-07	-2.02E-07	1
8	3.84E-04	Ydisp	21	21	1.91E-07	-2.02E-07	1
9	3.84E-04	Zdisp	21	21	0.00	-3.58E-06	1
10	3.84E-04	Xdisp	21	21	1.27E-07	-1.35E-07	1
11	3.84E-04	Ydisp	21	21	1.27E-07	-1.35E-07	1
12	3.84E-04	Zdisp	21	21	0.00	-3.70E-06	1
13	3.84E-04	Xdisp	21	21	3.82E-07	-3.90E-07	1
14	3.84E-04	Ydisp	21	21	3.82E-07	-3.90E-07	1
15	3.84E-04	Zdisp	21	21	0.00	-3.69E-06	1
16	3.84E-04	Xdisp	21	21	5.49E-07	-5.49E-07	1
17	3.84E-04	Ydisp	21	21	5.49E-07	-5.49E-07	1
18	3.84E-04	Zdisp	21	21	0.00	-4.02E-06	1

.... Any Key to Continue

Figure 4.7 Options in WVPLOT

SUMMARY OF PLOTS AVAILABLE

Snap	Time	Var	Xqty	Yqty	Max	Min	Geom
19	3.84E-04	Xdisp	21	21	7.17E-07	-7.26E-07	1
20	3.84E-04	Ydisp	21	21	7.17E-07	-7.26E-07	1
21	3.84E-04	Zdisp	21	21	0.00	-3.71E-06	1

Enter Snapshot number with which to start:

Figure 4.8 End of list of available snapshots

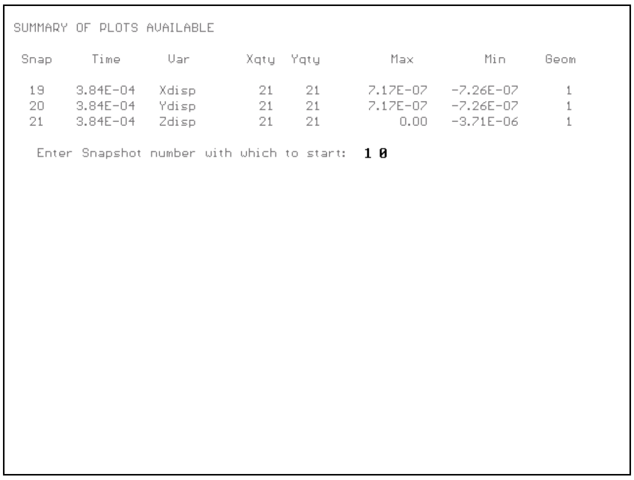


Figure 4.9 Syntax for selecting a snapshot to be displayed.

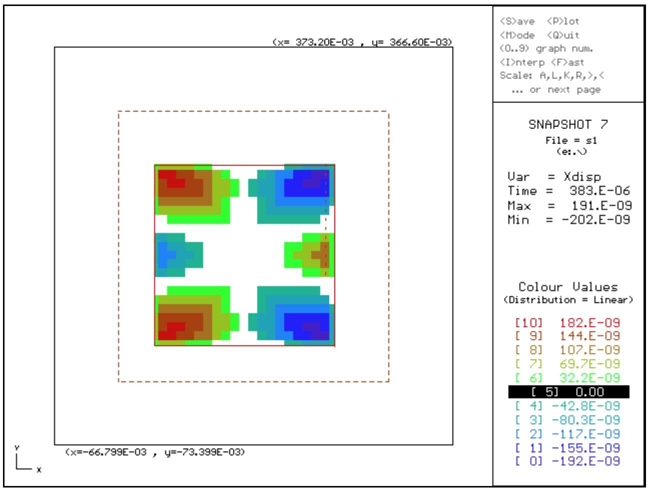


Figure 4.10 Snapshot #7

4.4 Snapshot commands

The data provided by snapshots are values of a grid variable sampled over a 2-D ‘area’. The area may be two dimensions in space (as generated in WAVE by the **Plot Snap** command) or one dimension in space, and the other in time (as in Line History data, and generated in WAVE by the **Plot Lhist** command). Snapshot data can be displayed in two ways:

- **Colour shaded graphs.** The colour shading at any position corresponds to the value of the variable at that point in the grid. Various methods are provided by which a colour is associated with a value, for example: log, linear or quadratic distribution.

- **Contour plot.** A standard contour plot is produced, with the colour levels chosen for the colour-shaded plot.

The following list of commands is available to the user. Note that many of these commands are toggles, some of which are case sensitive, i.e. the lower case letter is the inverse of the upper case letter. The commands include:

Q, q	Quits to the previous menu. Escape also works.
0…9 CR	Snapshot number. Enter the number of the snapshot, and follow this by a carriage return CR.
I, i	Interpolate on/off (Figure 4.11)
z, Z	Zoom / unzoom snapshot
←, →	Move image right / left
↑ , ↓	Move image down / up
Ctrl L	Restore scale, and move image to centre of display area
D, d	Toggle between linear scale, logarithmic scale and quadratic (look at header of legend)
K,k	Keeps the scale of the current image. Applies the kept, stored scale to other graphs if the Link option is used.
L, l	Link on/off. Useful if one wants to use the same scale for all the graphs (must Keep the scale before one can Link).
F1	Help
< , >	Increase/decrease the number of divisions of the scale i.e. colour levels

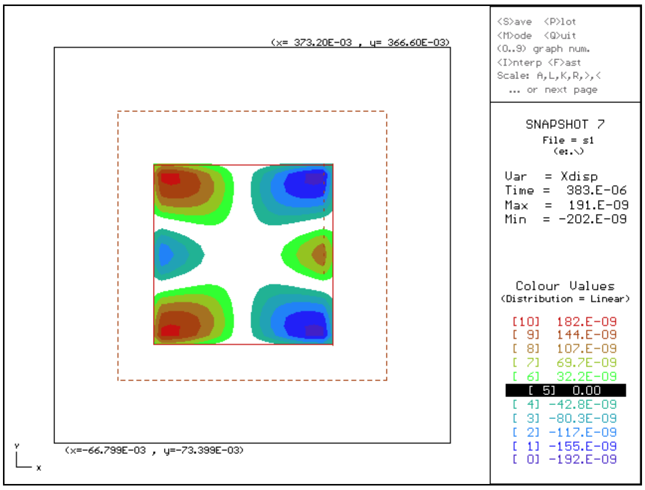


Figure 4.11 Example of interpolation

4.5 History commands

Histories contain data recordings of the variation of a grid variable over a line. This may be a line in either space (as generated by WAVE by the **Plot Lsnap** command), or in time (as generated by WAVE by the **Plot Hist** command).

To display histories, press the ‘H’ or ‘h’ key in Figure 4.6. The screen shown by Figure 4.12 will then be displayed. To navigate to the end of the list (useful if there are many hundreds of histories), press the ‘Escape’ key. To step through the list, press the ‘Enter’ key.

SUMMARY OF PLOTS AVAILABLE

Hist	Steps	T-st	T-fin	Var	X	Y	Z	Max	Min
1	1201	4.0E-02	1.2E+01	V-absMX	15	5	5	1.77E-01	1.66E-02
2	1201	4.0E-02	1.2E+01	V-absMX	15	7	5	2.02E-01	3.35E-02
3	1201	4.0E-02	1.2E+01	V-absMX	15	9	5	1.92E-01	3.36E-02
4	1201	4.0E-02	1.2E+01	V-absMX	15	11	5	1.91E-01	3.40E-02
5	1201	4.0E-02	1.2E+01	V-absMX	15	13	5	1.90E-01	3.41E-02
6	1201	4.0E-02	1.2E+01	V-absMX	15	15	5	2.22E-01	3.45E-02
7	1201	4.0E-02	1.2E+01	V-absMX	15	17	5	2.07E-01	3.42E-02
8	1201	4.0E-02	1.2E+01	V-absMX	15	19	5	1.87E-01	3.40E-02
9	1201	4.0E-02	1.2E+01	V-absMX	15	21	5	1.37E-01	3.38E-02
10	1201	4.0E-02	1.2E+01	V-absMX	15	23	5	1.59E-01	3.35E-02
11	4001	0.0	6.1E-03	aXVEL	15	15	301	1.76E-02	-1.03E-02
12	4001	0.0	6.1E-03	YVEL	15	15	301	1.88E-02	-9.39E-03
13	4001	0.0	6.1E-03	aZVEL	15	15	301	9.57E-03	-1.88E-01
14	4001	0.0	6.1E-03	V-abs	15	15	301	1.88E-01	0.00
15	4001	0.0	6.1E-03	aXVEL	15	15	301	1.76E-02	-1.03E-02
16	4001	0.0	6.1E-03	aYVEL	15	15	301	1.76E-02	-1.07E-02
17	4001	0.0	6.1E-03	aZVEL	15	15	301	9.57E-03	-1.88E-01
18	4001	0.0	6.1E-03	V-abs	15	15	301	1.88E-01	0.00

.... Any Key to Continue

Figure 4.12 List of histories available to be viewed

To display a single history, enter the history number (leftmost column of Figure 4.12) followed by a zero. Refer to Figure 4.13. To view the selected history, press the ‘Enter’ key. The screen shown in Figure 4.14 should now be displayed.

73	4001	0.0	6.1E-03	aZVEL	15	15	1101	3.47E-02	-4.15E-02
74	4001	0.0	6.1E-03	V-abs	15	15	1101	4.22E-02	0.00
75	4001	0.0	6.1E-03	aXVEL	15	17	1101	1.28E-02	-1.01E-02
76	4001	0.0	6.1E-03	YVEL	15	17	1101	1.42E-02	-1.38E-02
77	4001	0.0	6.1E-03	aZVEL	15	17	1101	2.91E-02	-3.55E-02
78	4001	0.0	6.1E-03	V-abs	15	17	1101	3.55E-02	0.00
79	4001	0.0	6.1E-03	aXVEL	15	19	1101	1.09E-02	-9.47E-03
80	4001	0.0	6.1E-03	YVEL	15	19	1101	2.01E-02	-1.81E-02
81	4001	0.0	6.1E-03	aZVEL	15	19	1101	3.06E-02	-3.54E-02
82	4001	0.0	6.1E-03	V-abs	15	19	1101	3.54E-02	0.00
83	4001	0.0	6.1E-03	aXVEL	15	21	1101	9.63E-03	-9.58E-03
84	4001	0.0	6.1E-03	YVEL	15	21	1101	2.47E-02	-3.06E-02
85	4001	0.0	6.1E-03	aZVEL	15	21	1101	2.78E-02	-3.51E-02
86	4001	0.0	6.1E-03	V-abs	15	21	1101	3.51E-02	0.00
87	4001	0.0	6.1E-03	aXVEL	15	23	1101	9.82E-03	-1.12E-02
88	4001	0.0	6.1E-03	YVEL	15	23	1101	3.07E-02	-3.94E-02
89	4001	0.0	6.1E-03	aZVEL	15	23	1101	2.18E-02	-3.46E-02
90	4001	0.0	6.1E-03	V-abs	15	23	1101	4.02E-02	0.00

Enter Histories to display (max 8, 0 to end) **21 0**

Figure 4.13 Selection of history #21.

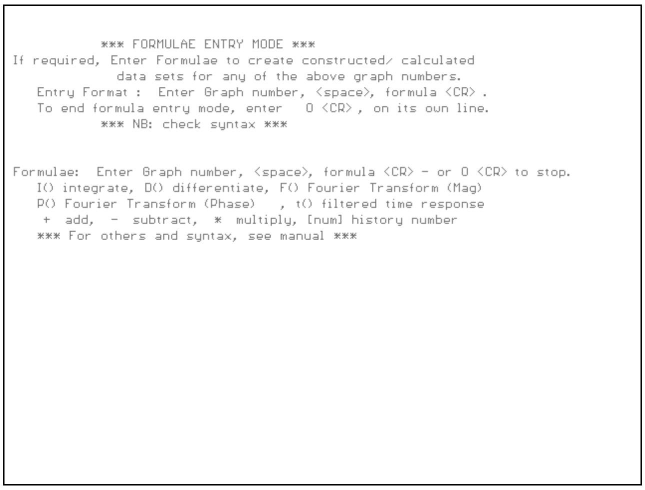


Figure 4.14 Formula entry screen

The screen in Figure 4.14 gives the user the opportunity to apply a formula to the history about to be displayed. Enter a zero for now - more on formula mode later and in the tutorial in Section 5.2. <u>Note that entering an incorrect formula can cause a dramatic crash</u>.

An example of a history plot is shown in Figure 4.15.

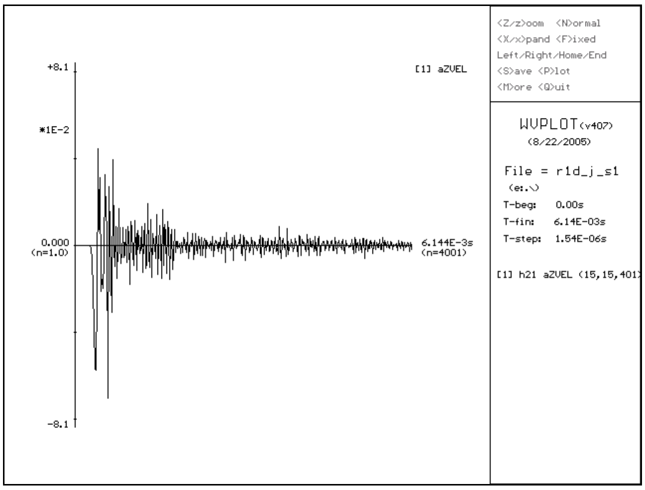
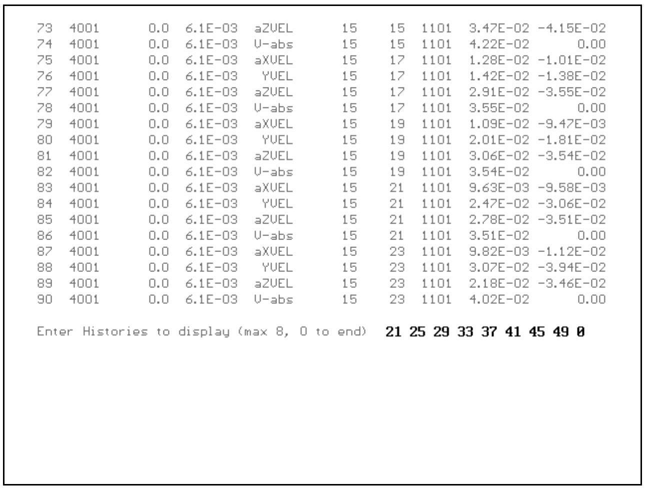


Figure 4.15 History plot

To select another history, enter ‘M’ for more, and the list of available histories will be listed.

To display multiple histories for comparison purposes, enter the list of required history numbers (maximum of 8) ending with a zero. For an example of the syntax, refer to Figure 4.16.



73	4001	0.0	6.1E-03	aZVEL	15	15	1101	3.47E-02	-4.15E-02
74	4001	0.0	6.1E-03	V-abs	15	15	1101	4.22E-02	0.00
75	4001	0.0	6.1E-03	aXVEL	15	17	1101	1.28E-02	-1.01E-02
76	4001	0.0	6.1E-03	YVEL	15	17	1101	1.42E-02	-1.38E-02
77	4001	0.0	6.1E-03	aZVEL	15	17	1101	2.91E-02	-3.55E-02
78	4001	0.0	6.1E-03	V-abs	15	17	1101	3.55E-02	0.00
79	4001	0.0	6.1E-03	aXVEL	15	19	1101	1.09E-02	-9.47E-03
80	4001	0.0	6.1E-03	YVEL	15	19	1101	2.01E-02	-1.81E-02
81	4001	0.0	6.1E-03	aZVEL	15	19	1101	3.06E-02	-3.54E-02
82	4001	0.0	6.1E-03	V-abs	15	19	1101	3.54E-02	0.00
83	4001	0.0	6.1E-03	aXVEL	15	21	1101	9.63E-03	-9.58E-03
84	4001	0.0	6.1E-03	YVEL	15	21	1101	2.47E-02	-3.06E-02
85	4001	0.0	6.1E-03	aZVEL	15	21	1101	2.78E-02	-3.51E-02
86	4001	0.0	6.1E-03	V-abs	15	21	1101	3.51E-02	0.00
87	4001	0.0	6.1E-03	aXVEL	15	23	1101	9.82E-03	-1.12E-02
88	4001	0.0	6.1E-03	YVEL	15	23	1101	3.07E-02	-3.94E-02
89	4001	0.0	6.1E-03	aZVEL	15	23	1101	2.18E-02	-3.46E-02
90	4001	0.0	6.1E-03	V-abs	15	23	1101	4.02E-02	0.00

Enter Histories to display (max 8, 0 to end) **21 25 29 33 37 41 45 49 0**

Figure 4.16 Selection of multiple histories to be plotted

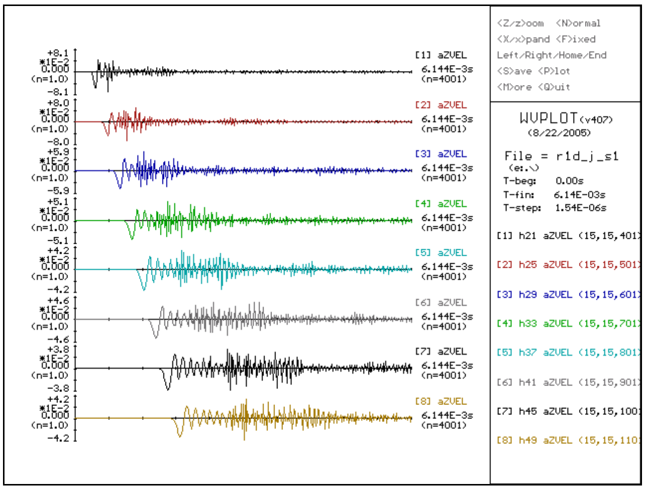


Figure 4.17 Display of multiple histories

The following commands are available:

W / w	Display all graphs on one axis (Figure 4.18)
1,2,3,…8	To change the scale of the graphs, one at a time. The scale of the graphs can be changed from linear-linear, absolute linear, linear-log, and log-log.
Z / z	Zoom/unzoom horizontal axis
X / x	Expand/contract vertical axis
F1	Help
- / +	Decrease / increase number of significant figures
← / →	Move graph right / left along horizontal axis
A / a	Saves an ASCII file of the graph. Only hit this once, otherwise multiple versions of the same data will be appended to the same file. A '*.ASC' file will be created. Rename this file if multiple seismograms are to be saved in ASCII format.

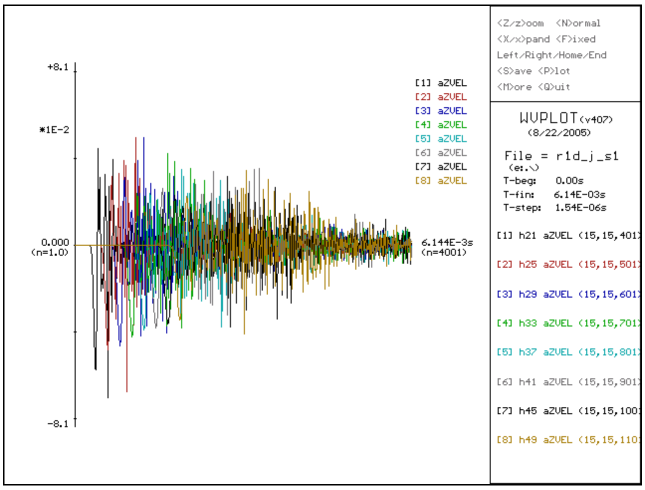


Figure 4.18 Use of the ‘W’ or ‘w’ key to display multiple graphs on the same axis

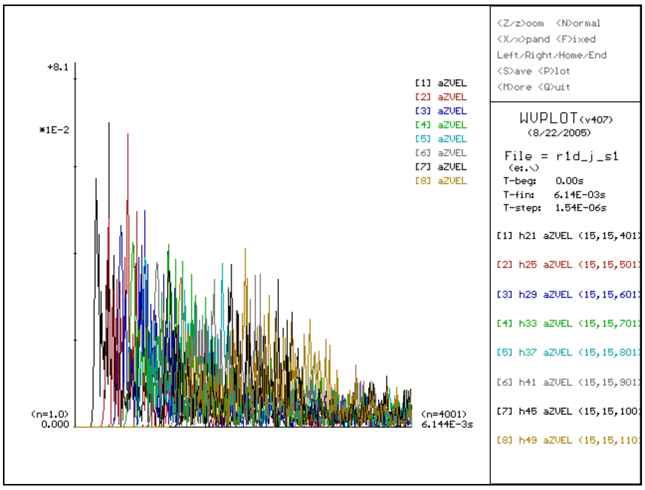


Figure 4.19 Using the ‘1’, ‘2’ … ‘8’ keys to display all the graphs on an absolute linear scale

4.6 Formula entry mode

Refer to Figure 4.14. <u>Note that this mode is very, very fussy about the syntax used to describe the formula, and incorrect entry of a formula can cause a serious crash</u>. The operation of Formula entry mode is best illustrated by means of example. A slightly more complicated example is given in the tutorial in Section 5.2.

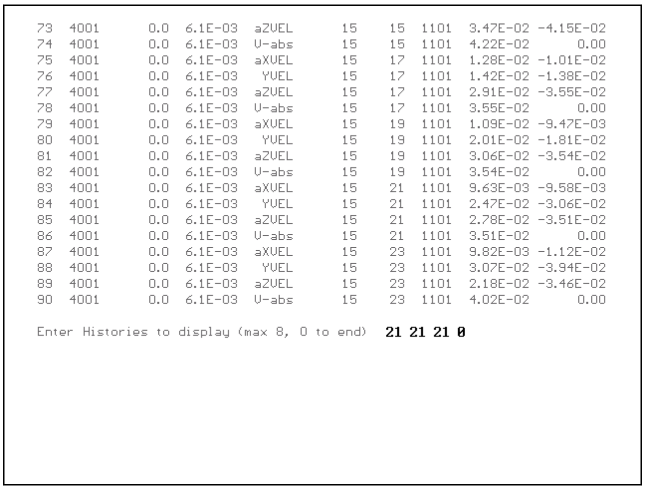
Example: Plot graph 21 in the time domain, integrate graph 21, differentiate graph 21. Show all graphs at once.

1. First one will need to select history #21 three times in the history selection mode, as shown in Figure 4.20.

Figure 4.20

2. Next, apply the formulae to each graph, one at time, hitting the ‘Enter’ key between each one, as shown in Figure 4.21. The graphs now are labelled 1, 2

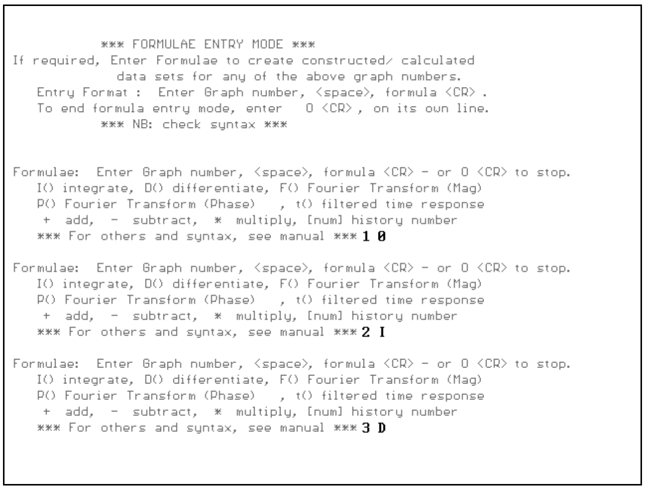
and 3. To exit the Formula Entry Mode, enter zero ‘0’. The result of applying these formulae is shown in Figure 4.22.



```
73  4001     0.0  6.1E-03  aZVEL     15    15  1101  3.47E-02 -4.15E-02
74  4001     0.0  6.1E-03  V-abs     15    15  1101  4.22E-02      0.00
75  4001     0.0  6.1E-03  aXVEL     15    17  1101  1.28E-02 -1.01E-02
76  4001     0.0  6.1E-03   YVEL     15    17  1101  1.42E-02 -1.38E-02
77  4001     0.0  6.1E-03  aZVEL     15    17  1101  2.91E-02 -3.55E-02
78  4001     0.0  6.1E-03  V-abs     15    17  1101  3.55E-02      0.00
79  4001     0.0  6.1E-03  aXVEL     15    19  1101  1.09E-02 -9.47E-03
80  4001     0.0  6.1E-03   YVEL     15    19  1101  2.01E-02 -1.81E-02
81  4001     0.0  6.1E-03  aZVEL     15    19  1101  3.06E-02 -3.54E-02
82  4001     0.0  6.1E-03  V-abs     15    19  1101  3.54E-02      0.00
83  4001     0.0  6.1E-03  aXVEL     15    21  1101  9.63E-03 -9.58E-03
84  4001     0.0  6.1E-03   YVEL     15    21  1101  2.47E-02 -3.06E-02
85  4001     0.0  6.1E-03  aZVEL     15    21  1101  2.78E-02 -3.51E-02
86  4001     0.0  6.1E-03  V-abs     15    21  1101  3.51E-02      0.00
87  4001     0.0  6.1E-03  aXVEL     15    23  1101  9.82E-03 -1.12E-02
88  4001     0.0  6.1E-03   YVEL     15    23  1101  3.07E-02 -3.94E-02
89  4001     0.0  6.1E-03  aZVEL     15    23  1101  2.18E-02 -3.46E-02
90  4001     0.0  6.1E-03  V-abs     15    23  1101  4.02E-02      0.00

Enter Histories to display (max 8, 0 to end)  21 21 21 0
```

Figure 4.20 Selecting history # 21 three times, for three different displays



```
          *** FORMULAE ENTRY MODE ***
If required, Enter Formulae to create constructed/ calculated
            data sets for any of the above graph numbers.
   Entry Format :  Enter Graph number, <space>, formula <CR> .
   To end formula entry mode, enter   0 <CR> , on its own line.
          *** NB: check syntax ***


Formulae:  Enter Graph number, <space>, formula <CR> - or 0 <CR> to stop.
   I() integrate, D() differentiate, F() Fourier Transform (Mag)
   P() Fourier Transform (Phase)   , t() filtered time response
    +  add,  -  subtract,  *  multiply, [num] history number
   *** For others and syntax, see manual *** 1 0

Formulae:  Enter Graph number, <space>, formula <CR> - or 0 <CR> to stop.
   I() integrate, D() differentiate, F() Fourier Transform (Mag)
   P() Fourier Transform (Phase)   , t() filtered time response
    +  add,  -  subtract,  *  multiply, [num] history number
   *** For others and syntax, see manual *** 2 I

Formulae:  Enter Graph number, <space>, formula <CR> - or 0 <CR> to stop.
   I() integrate, D() differentiate, F() Fourier Transform (Mag)
   P() Fourier Transform (Phase)   , t() filtered time response
    +  add,  -  subtract,  *  multiply, [num] history number
   *** For others and syntax, see manual *** 3 D
```

Figure 4.21 Example of formulae to be applied to history # 21. No formula is applied to graph 1, graph 2 is integrated, and graph 3 is differentiated. Result shown in Figure 22. To end, type zero ‘0’

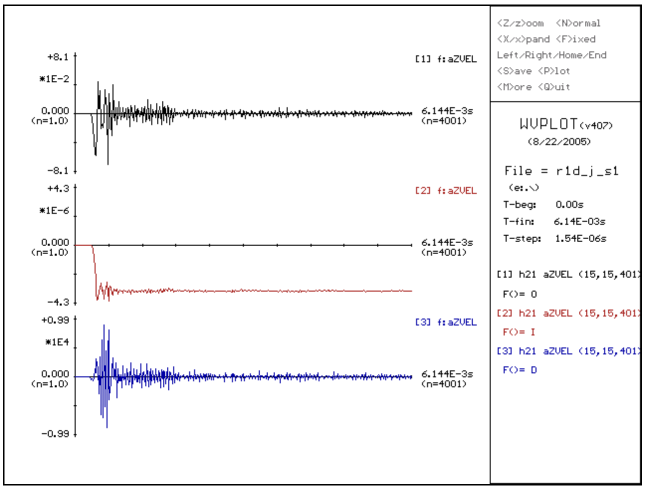


Figure 4.22 History # 21 has been integrated (middle) and differentiated (bottom)

There are a number of functions available in Formula Mode, for example:

I	Integrate
D	Differentiate
F	Fourier transform (amplitude spectrum)
P	Fourier transform (phase spectrum)

Formulae can also be specified from ASCII files located in the same directory as the WAVE output. These files must be named 'form.1', 'form.2' etc. If this method is being used to input formulae, do not enter a formula in Formula Mode (Figure 4.21).

An example of the content of an ASCII formula file, giving the formula for windowing the P-phase between 25% and 32% of the range of frequencies present in the signal, and then calculating the amplitude spectrum, is given as:

F,100,100,25,32,10

The first value after the 'F' denotes the low frequency cut-off; the second value gives the high frequency cut-off; the next two values indicate the frequency band to be passed, and the last value specifies the taper length. These values are all specified as percentages of the highest frequency present in the signal, where 100% = highest frequency. The commas indicate that the values are parameters.

4.7 Grid dumps

Data supplied here are the values of all grid variables over an area in the grid (as generated by the WAVE command PLOT DUMP). Due to the interlaced mesh, the grid variables of an element (as held by WAVE) are not all at the same point in the grid. However, the values provided have been interpolated so that all stress variables associated with the grid element correspond to the same position as the interlaced mesh. Similarly all velocity and displacement variables refer to a common position. This allows the stresses to be combined into principal stress vectors at a point, and velocities into vectors.

The data is loaded into memory, with a maximum of 50x50 elements (larger data sets will be truncated) and can then displayed as stress or velocity plots. The data sets can be accessed sequentially or selected individually by specifying the number of the data dump. The vector scale is chosen so that vectors do not overlap, but can be expanded or reduced. The image can be saved to file for alter recapture on screen in, for example, a slide show. The graph can be plotted either directly to a plotted or saved to a HPGL file.

5. Tutorials

Two simple examples have been chosen to demonstrate features available in WAVE, typical data file structure and some of the plot and output types available. Example 1 shows the use of the property and stope command to place mining features, source frequency considerations and variations of the snapshot type outputs. Example 2 demonstrates backfill support in stopes, point history and line history type outputs, and combining and applying formulae to history outputs to obtain (for example) relative movements.

5.1 Example 1: Propagation from an explosive source

This example traces wave propagation from an explosion in the rock mass through reflections from various features such as stopes and material interfaces such as dykes. The layout in Figure 5.1 is a schematic of the geometry modelled. The listing in Figure 5.2 shows the WAVE data file (named ‘expl’) interleaved with comments. This would be executed with the **Call** command.

For first-time users, the following steps are helpful:

1. To run WAVE, type the name of WAVE’s executable at the command line (in the example shown in Figure 5.3, the name of the executable is ‘wv_512’):
   `> WAVE`
2. The following text should be returned to the screen:
   `WAVE … version3.51`
3. Type:
   `CALL expl_eg1`

The job should then run, and produce the output shown in Figure 5.3. WAVE is now ready to either run a new/different job or the user can exit by typing:
   `QUIT`

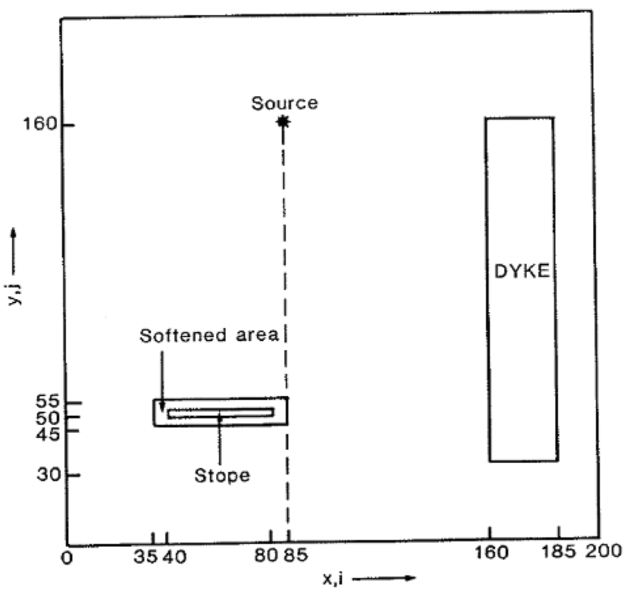


Figure 5.1 Basic model geometry for Example 1 (‘expl’)

```
** (a) Specify grid size **
*       2-D grid of 200x200 elements of size 1m x 1m
g 200 200 1 1 1

** (b) Set material properties for rock-mass, dyke and stope area **
pro mat=1 d=2700 s=3.5e10 b=4.5e10
pro mat=2 d=2800 s=5.0e10 b=6.5e10
pro mat=3 d=2600 s=1.4e10 b=2.4e10

** (c) Define positions of dyke and stope **
mat 2 i=160,185 j=30,160
st n=1 i=40,80 j=50

mat 3 i=35,85 j=45,55
** (d) Specify source **
so typ=dil amp=10e6 wave=cos per=4e-3 beg=0 end=500 i=85 j=160

** (e) Set plot types and th repeat frequency for these plot types **
pl snap vabs di=1 dj=1 rep=10

** (f) Cycle for period of interest **
cy 480

** (g) Return from remote file **
ret
```

Figure 5.2 Data file for Example 1 (from file 'expl')

The **Property** command defines 'typical' dynamic values for modulii in the rockmass, dyke and stope area. The **Material** and the **Stope** commands then define the positions of features, i.e. stope and dyke. The **Source** used is dilatational, where a **COS** pulse is applied to the stresses σ_{11} and σ_{22} at the source position. In this instance we first choose an element length of 1 metre, and from this calculate a suitable source duration – if we were interested in a particular frequency range, we would rather calculate a suitable element length from some maximum frequency. Using the material properties, we can construct Table 5.1 which shows P and S-velocities for the rockmass, dyke and softened materials.

Table 5.1 Frequency calculations for the materials in Example 1

Material	**Bulk (K)**	**Shear (G)**	**Density (ρ)**	**Cp**	**Cs**	**Dx**	**Fp**	**Fs**
Rockmass	4.5e+10	3.5e+10	2700	5827	3600	1	583	360
Dyke	6.5e+10	5.0e+10	2700	6983	4303	1	698	430
Softened	2.4e+10	1.4e+10	2700	3975	2277	1	398	228

```
[sdonovan@localhost Manual]$ wv_512
           WAVE:  ... version3.51
 wave:call expl_eg1
 * * setting root output file name to expl_eg1 * *
 >** (a) Specify grid size **
 >*         2-D grid of 200x200 elements of size 1m x 1m
 >g 200 200 1 1 1
  Note: 2-D mode is set: 1-2 plane is active
 bounds: 0 1 1 1 201 201 1
 sizes: 40401 201 201 1 242406 210000000
 >** (b) Set material properties for rock-mass, dyke and stope area **
 >pro mat=1 d=2700 s=3.5e10 b=4.5e10
 >pro mat=2 d=2800 s=5.0e10 b=6.5e10
 >pro mat=3 d=2600 s=1.4e10 b=2.4e10
 >** (c) Define positions of dyke and stope **
 >mat 2 i=160,185 j=30,160
 >st n=1 i=40,80 j=50
 >mat 3 i=35,85 j=45,55
 >** (d) Specify source **
 >so typ=dil amp=10e6 wave=cos per=4e-3 beg=0 end=500 i=85 j=160
 >** (e) Set plot types and th repeat frequency for these plot types **
 >pl snap vabs di=1 dj=1 rep=10
 ** snapshot repeat frequency =   10 cycles **
 >** (f) Cycle for period of interest **
 >cy 480
         (Min.) timestep =  7.29140E-05

        Cycle     20  of    480
        Cycle     40  of    480
        Cycle     60  of    480
        Cycle     80  of    480
        Cycle    100  of    480
        Cycle    120  of    480
        Cycle    140  of    480
        Cycle    160  of    480
        Cycle    180  of    480
        Cycle    200  of    480
        Cycle    220  of    480
        Cycle    240  of    480
        Cycle    260  of    480
        Cycle    280  of    480
        Cycle    300  of    480
        Cycle    320  of    480
        Cycle    340  of    480
        Cycle    360  of    480
        Cycle    380  of    480
        Cycle    400  of    480
        Cycle    420  of    480
        Cycle    440  of    480
        Cycle    460  of    480
        Cycle    480  of    480
   ** Cycle    480, time = 3.49986E-02 **
 >** (g) Return from remote file **
 >ret
 wave:
```

Figure 5.3 Screendump after running Example 1 'expl'

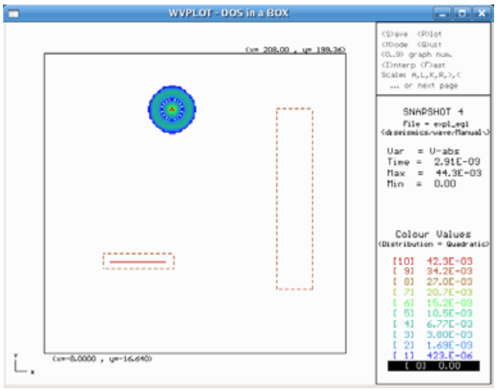
Strictly applying the 10% frequency criterion of Section 2, we would have $f_{\max} = \frac{c_{\min}}{10\Delta x}$ where $c_{\min}$ would be the shear wave velocity in the softened material, giving $f_{\max}$ = 225Hz. However, in this study we are mainly interested in P-wave propagation; also the softened material covers a very small area. We therefore base the critical frequency on c_p for the rockmass, which gives $f_{\max}$ = 600Hz, or a minimum period $T_{\min}$ = 1.7x10^{-3}s. A good rule for the **COS** pulse source shape is that the period should be at least 2.5 times this minimum period. Hence a duration of 4x10^{-3}s was used.

The plot command indicates to plot a grid snapshop of the dilational stress ($\sigma_{11} + \sigma_{22}$) every 10 cycles. The model is then cycled for 480 cycles, giving 48 snapshots.

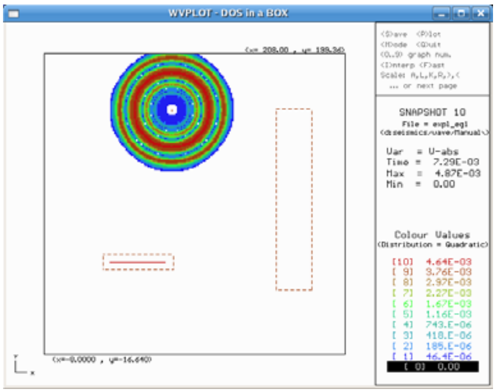
Figure 5.4 (a) to (g) shows a selection of the sequence of snapshots taken from the above model. These were displayed using the colour map option, 256 colour mode, 48 colours, a quadratic distribution and a range of 256 from the maximum to the minimum value.

The sequence shows the wavefront expanding from the source point, deforming due to increased velocity in the dyke, reflection from the dyke interface, wraparound from the footwall of the stope, reflection from the stope and finally interactions in the rockmass of reflections from the dyke and stope.

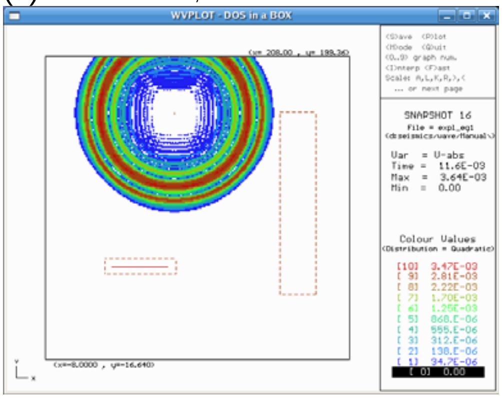
Snapshots were captured doing screendumps of the screen (using the ‘Print Screen’ key on the keyboard). An example of a horizontal direction and dual direction line plot for snapshots 29 and 36 respectively. See Figure 5.5 (a) and (b).



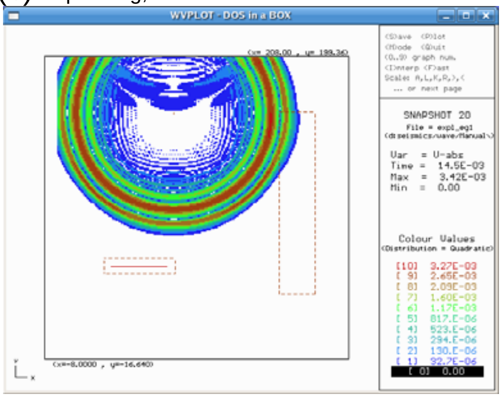
(a) Source location, t=2.8ms



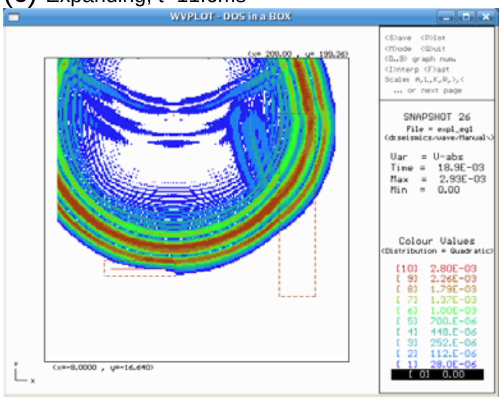
(b) Expanding, t=7.2ms



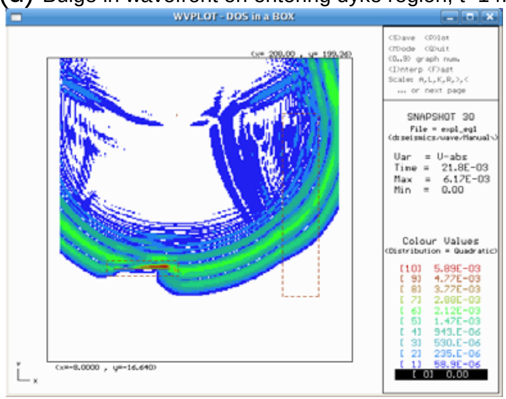
(c) Expanding, t=11.6ms



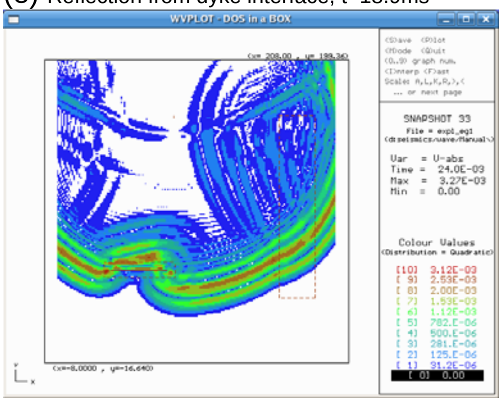
(d) Bulge in wavefront on entering dyke region, t=14.5ms



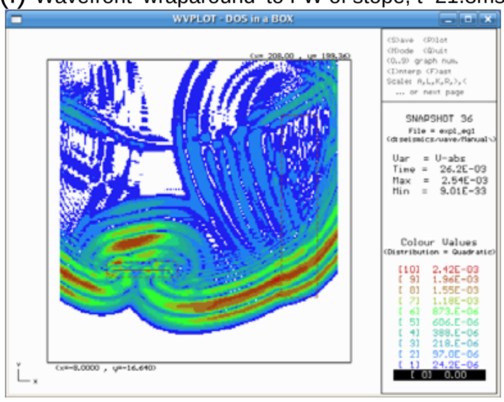
(e) Reflection from dyke interface, t=18.9ms



(f) Wavefront ‘wraparound’ to FW of stope, t=21.8ms

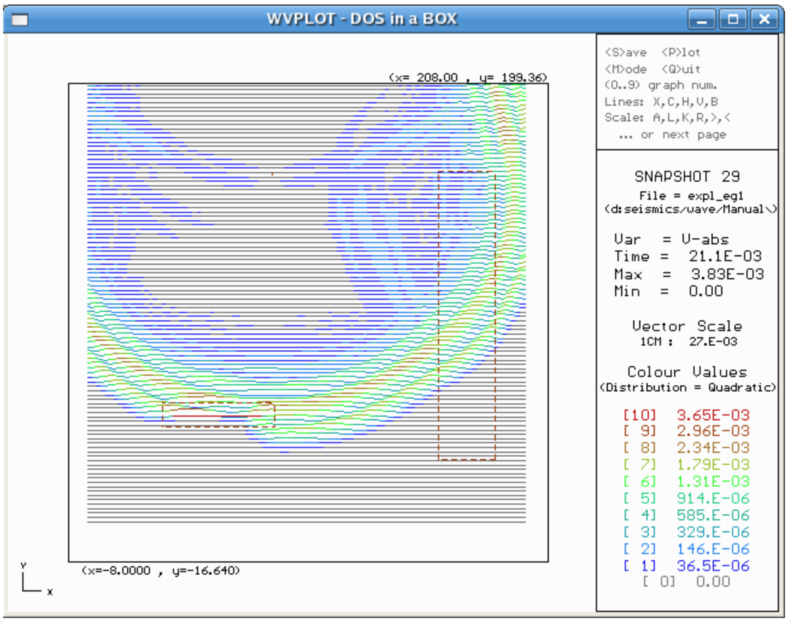


(g) Reflection from stope, t=24ms

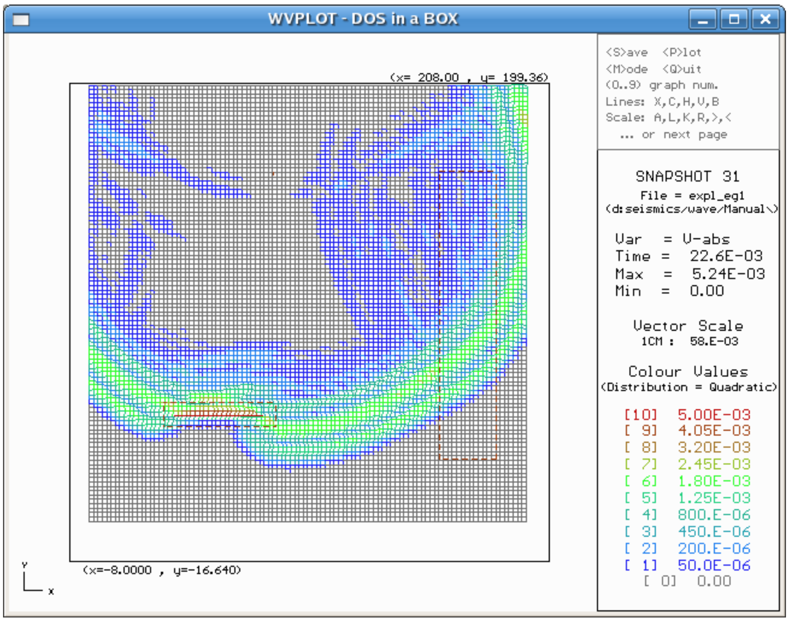


(h) Interaction of reflections from dyke and stope, t=26ms

Figure 5.4 Selected snapshots showing propagation from an explosion (Example 1, ‘expl’)



(a) Horizontal line representation



(b) Dual line representation

Figure 5.5 Colour plots of line representations of snapshot outputs for Example 1 (‘expl’)

5.2 Example 2: Ground motion in a filled stope

This example considers the motions induced by a shearing source and wave propagation along the stope hangingwall. Cases are compared for a filled and unfilled stope. (The WAVE models named ‘sh_bkf’ and ‘sh_nobkf’, respectively.) The layout shown in Figure 5.6 shows the modeled geometry and the listing shown in Figure 5.7 give the WAVE data file, for the case of the filled stope.

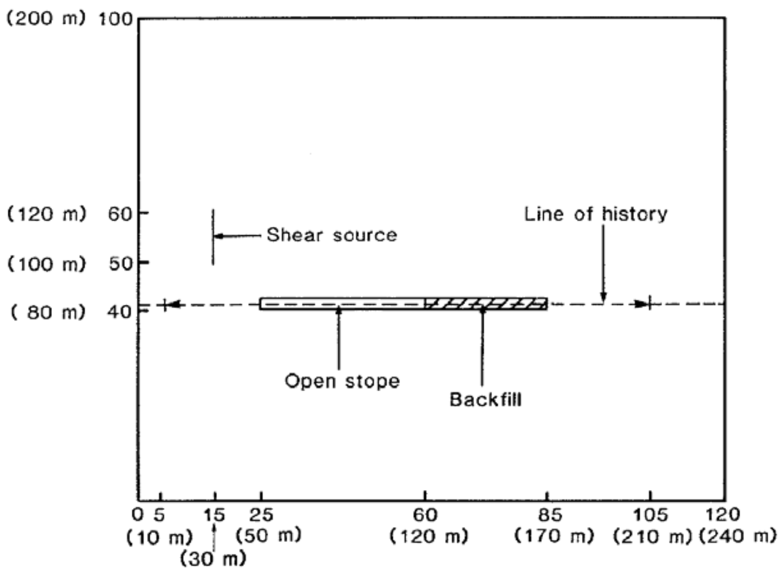


Figure 5.6 Basic model geometry for Example 2 (‘sh_bkf’)

```
g 120 100 1 2 2 1
pro mat=1 d=2700 s=3.9e10 b=4.52e10
source type=s12 wave=ss amp=1e6 per=10e-3 beg=0 end=1000 i=15 j=50,60
stope n=1 i=25,85 j=40
stope n=1 type=2 kn=15e9 slope=5e9 i0=60 rat=0
his yvel(25,40,1) dt=5
his yvel(25,39,1)
his yvel(27,40,1) yvel(27,39,1)
his yvel(30,40,1) yvel(30,39,1)
his yvel(40,40,1) yvel(40,39,1)

lhis yvel dt=8 dx=1 i=5,105 j=42 k=1
lhis yvel dx=1 i=5,105 j=40 k=1
lhis yvel dx=1 i=5,105 j=39 k=1

select i=1,100 j=1,100 k=1
cy 800
pl his 1 2 3 4 5 6 7 8
pl lhis 1 2 3
ret
```

Figure 5.7 Data file for Example 2 (‘sh_bkf’)

A 2-D grid of 120x100 elements each of 2m x 2m was used. The first stope command locates the position and extent of the stope. The second specifies the backfill characteristics (support type 2, cf. Section 2). The normal stiffness is varied over three elements to a maximum of 15 GPa. No shear stiffness is specified (ratio = 0). "Typical" dynamic values for modulii were used for the rockmass, yielding Table 5.2, showing P and S-wave velocities, and the resulting maximum frequencies.

Table 5.2 Frequency calculations for the materials in Example 2

Material	**Bulk (K)**	**Shear (G)**	**Density (ρ)**	**Cp**	**Cs**	**Dx**	**Fp**	**Fs**
Rockmass	4.52e+10	3.9e+10	2700	6000	3800	2	300	190

Using the shear wave velocity and applying the 10% frequency criterion gives $f_{\max}$ = 190 Hz, or a minimum period $T_{\min}$ = 5.2x10^{-3} s. A shearing source was used, with an 's-shaped' step pulse applied to the stress σ_{12} and a transient duration of 10^{-2} s.

Point histories were specified at various positions along the stope hanging and footwalls. Line histories were requested along line parallel to the stope. The point histories were sampled every 5 seconds, and the line histories every 8 seconds.

The line history along y = 40 (the stope hangingwall) is plotted in Figure 5.8 – both for the filled stope listed above, as well as for the case of an unfilled stope. To interpret the graph, note that time progression is along the horizontal axis, while the line of the stope is along the vertical axis – running forward from the stope face and source end to the remote and filled end (cf. the layout in Figure 5.6). The perturbation in the lines and the colour scale represent the vertical velocity of the hangingwall.

What can be seen is that in the unfilled stope Figure 5.8(a), the wave propagates to the remote end of the stope where some energy is reflected and some transmitted. Figure 5.8 (b) shows the case with backfill. Here some of the energy is first reflected from the start of the backfill. The energy reaching the remote end of the stope, and thus the reflection from the remote end, is reduced.

To generate the line plots shown in Figure 5.8:

1. Run WVPLOT by typing 'wvplot' at the cursor.

2. Select 'D' using the keyboard to select the WAVE output files to be displayed.

3. Type in the name of the *.MAP file, without the extension.

4. Type 'S" for snapshots. To display snapshot 2, type '2'

5. Type 'M" for map colours. If no image is displayed, press 'Ctrl L'.

6. For a line plot, type 'Ctrl G' to cycle between the different plot types until a line plot is displayed.

7. Zoom / unzoom using the 'X' or 'x' key.

Figure 5.9 is an analysis of the point histories defined at x = 30, very close to the stope face. Figure 5.9 (a) (i) and (ii) are the vertical velocities in the hanging- and footwall, respectively. These are the specified histories 5 and 6. The waveforms show clearly the initial pulse followed by lesser reflections from the backfill and remote end of the stope. Graph (iii) shows the relative vertical velocity, obtained by a formula subtracting history 6 from history 5. Graph (iv) then integrates the difference to give relative vertical displacement.

Figure 5.9 (b) gives the frequency compositions of the above waveforms – also obtained using formulae. It is important to note that since the histories were sampled only once every 5 cycles, the maximum frequency reflected here is incorrect; in fact, the maximum is 120 Hz, not 600 Hz. From our previous discussion, we require frequencies above 190 Hz to have an insignificant contribution. The frequencies around 120 Hz are below 1% of the dominant, so we have probably satisfied this criterion satisfactorily.

The following steps on using formulae should be useful for first time users.

To generate the display shown in Figure 5.9(a):

1. Run WVPLOT by typing the following at the cursor:
   ```
   > WVPLOT
   ```

2. Select ‘D’ using the keyboard to select the WAVE output files to be displayed.

3. Type in the name of the *.MAP file, without the extension.

4. Select ‘H’ for histories using the keyboard.

5. Select the histories to be viewed, up to a maximum of eight, separated by spaces and ending in zero:
   ```
   > 5 6 5 5 0
   ```
   Each history will be viewed as a separate graph. In this example, four graphs will be plotted, using histories 5 and 6.

6. After selecting the histories, WVPLOT will enter ** FORMULAE ENTRY MODE**. While in this mode, to subtract history 6 from history 5, and plot the result in graph 3, type the following:
   ```
   > 3 [5]-[6]
   ```

   To subtract the integrated history 6 from integrated history 5, and plot the result in graph 4, type the following:
   ```
   > 4 i([5]-[6])
   ```

To generate the display shown in Figure 5.9(b):

6. In ** FORMULAE ENTRY MODE**, to plot the Fourier transform of history 5 in graph 1 type:
   ```
   > 1 F
   ```

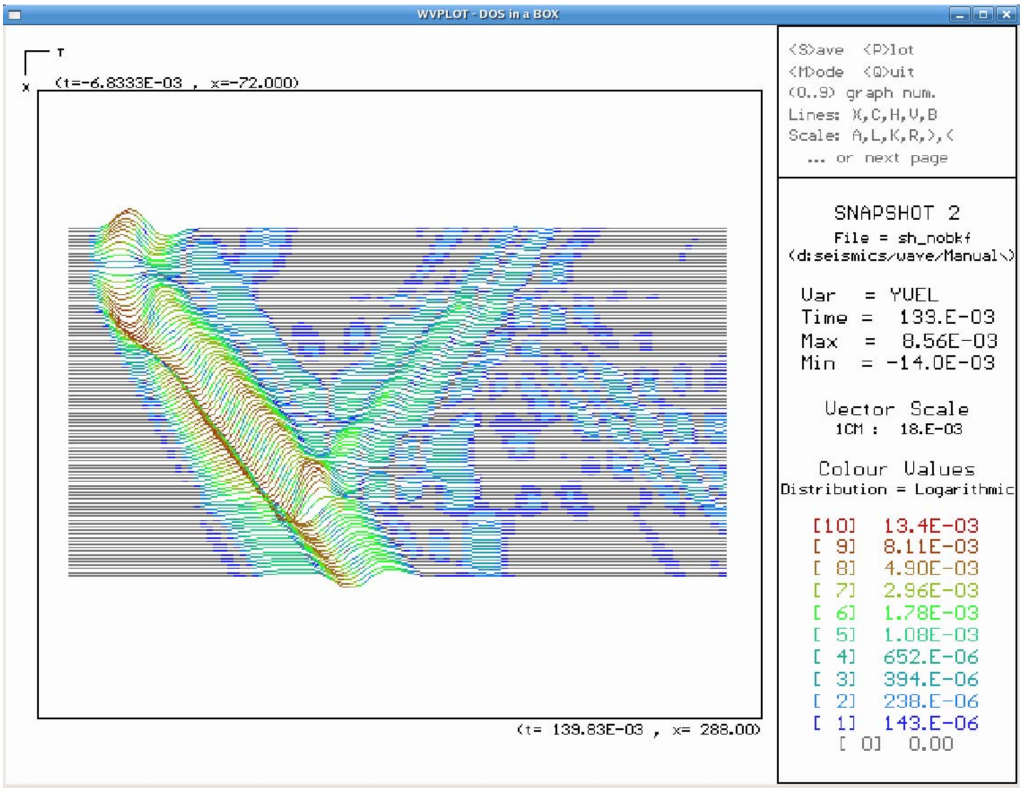
   To plot the Fourier transform of history 6 in graph 2:
   ```
   > 2 F
   ```

To calculate and plot the difference between the Fourier transforms of history 5 and history 6 in graph 3:

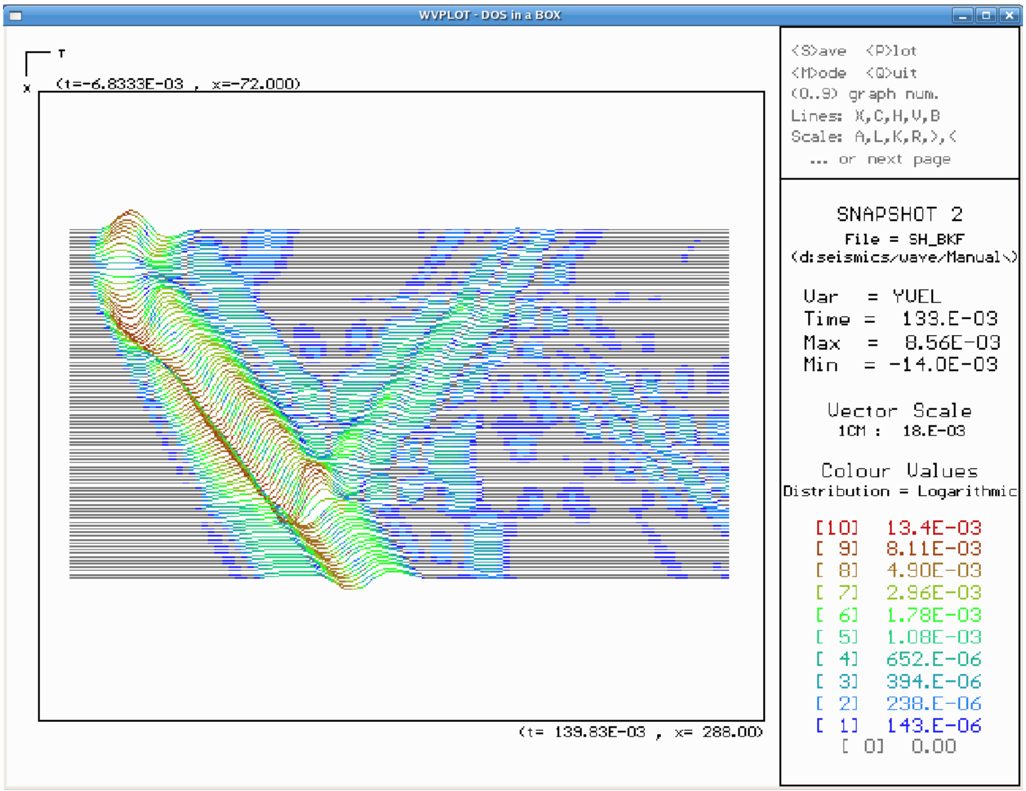
```
> 3 F([5]-[6])
```

To plot the Fourier transform of the difference between the integrals of histories 5 and 6 in graph 4 type:

```
> 4 F(i([5]-[6]))
```

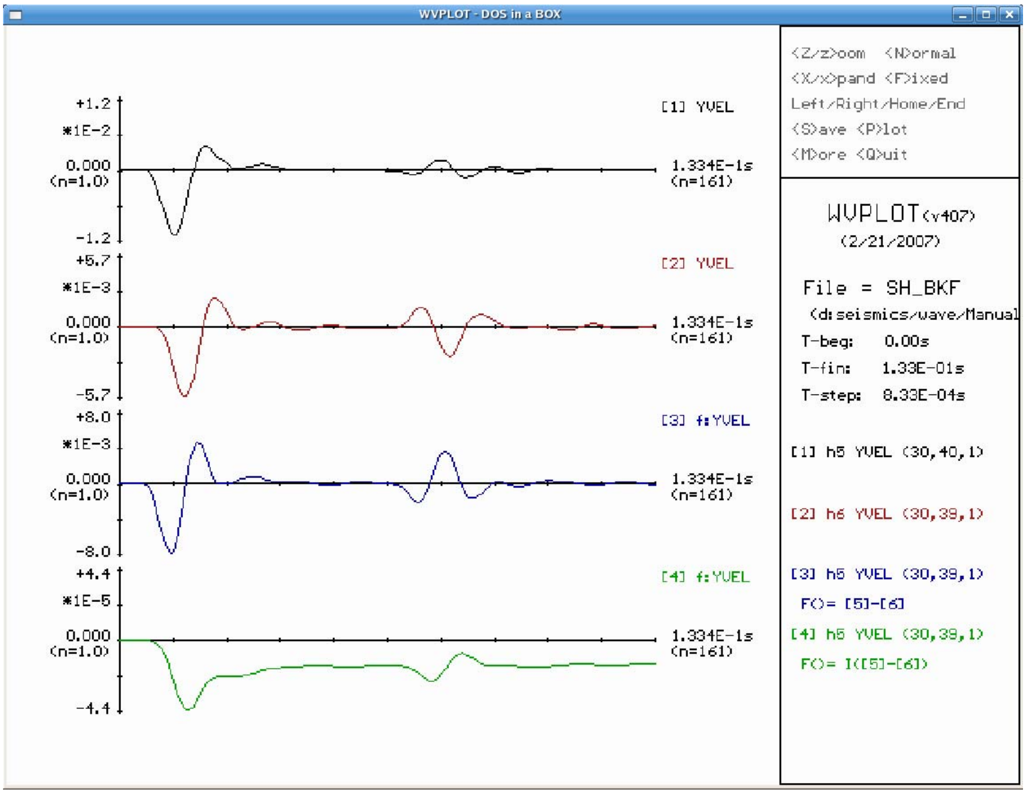


(a) Unfilled stopes



(b) Stope with backfill

Figure 5.8 Line histories along the line of the stope of the hangingwall



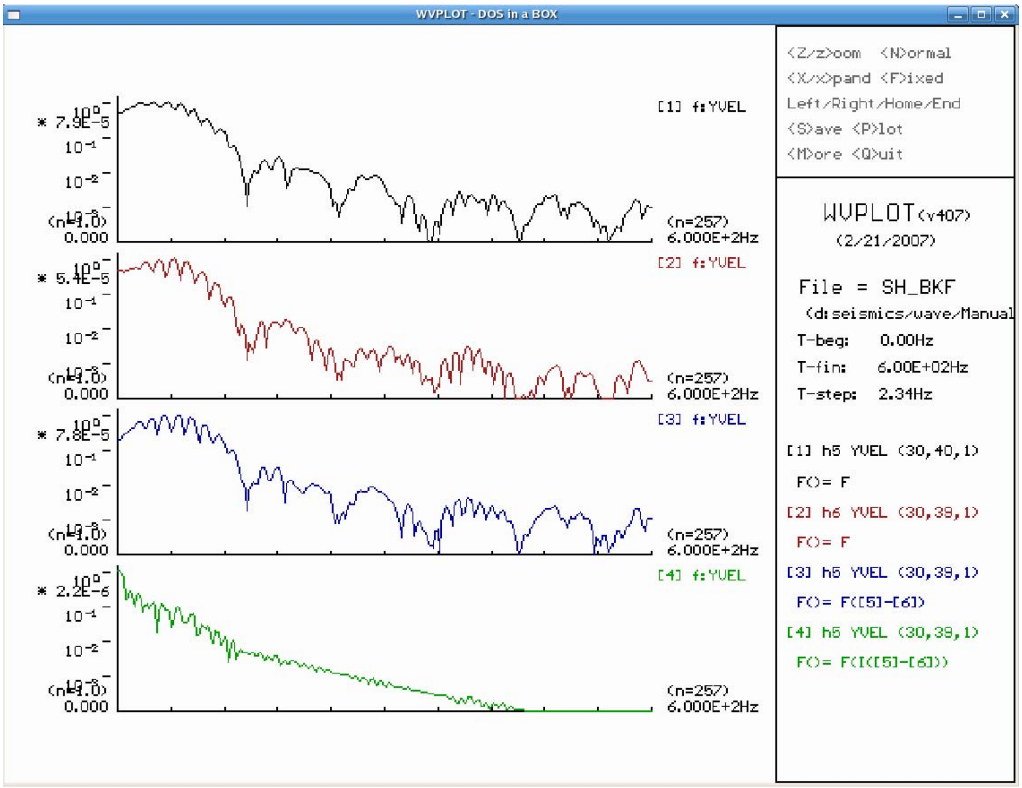


Figure 5.9 Plots of point histories, calculated histories and their frequency composition

6. References

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7. Appendices

Appendix A: Memory, grid-size and runtime applications

The following runtime predictions are based on a 50 MHz 486 and a simple model with a square grid, single material and the timesteps to completion equal to three times the elements along one direction. In the tables, N is the total number of elements in the model, while N_1 is the number of elements along one dimension of the square grid.

Table 7.1 Relationship between available memory, largest model size, and predicted runtime**

	2 Dimensions				3 Dimensions			
	Total Elements (N)	**Elements 1-D (N_1)**	**Runtime (sec)**	**Runtime (H:M:S)**	**Total Elements (N)**	**Elements 1-D (N_1)**	**Runtime (sec)**	**Runtime (H:M:S)**
1 MB	10 333	100	40	00:00:40	6 200	17	9	00:00:09
2 MB	53 666	230	487	00:08:06	32 300	30	87	00:01:27
3 MB	97 000	310	1192	00:19:51	58 200	37	201	00:03:21
4 MB	140 333	373	2076	00:34:35	84 200	42	334	00:05:34
8 MB	313 666	559	6987	01:56:27	188 200	56	1056	00:17:36
16 MB	660 333	811	21 336	05:55:00	396 200	72	2886	00:48:06
64 MB	2.7 mil	1654	181 000	50 hrs	1.6 mil	117	20 100	05:35:00
256 MB	11 mil	3324	1.5 mil	408 hrs	6.6 mil	186	129 000	35 hrs
1GB	44 mil	6657	11.8 mil	3277 hrs	26.6 mil	297	836 000	232 hrs

** This illustration has been updated in Table 7.1b to show more up to date (2007) memory and run-time requirements, based on a 2GHz Pentium 4 processor.

Table 7.1b Memory and Runtime requirements on a 2 GHz Pentium 4

Elements along 1 dimension	Total Elements in 3D (mil. elems)	Memory required	Estimated Runtime (hours)
100	1	40 MB	0.1
150	3	140 MB	0.5
200	8	325 MB	1.4
300	27	1.1 GB	7
400	64	2.6 GB	23

The following two tables illustrate the order of the runtimes relative to the size of the model. The subscript *a* in each table refer to entries in the first row of the table.

For a 2-D model: the runtime is O(3/2) relative to the total number of elements (N) and, O(3), relative to the number of elements in one direction only (N_1).

Table 7.2 Relationship between number of elements in a 2-D model and runtime

Total elements (N)	**Elements in 1-D (N_1)**	**Runtime (sec) (t)**	$\frac{t}{t_a}$	$\left(\frac{N}{N_a}\right)^{3/2}$	$\left(\frac{N_1}{N_{1a}}\right)^3$
10 000	100	40	1.0	1.0	1.0
20 000	141	112	2.8	2.8	2.8
30 000	173	207	5.2	5.2	5.2
40 000	200	320	8.0	8.0	8.0
80 000	282	897	22.4	22.4	22.4

For a 3-D model: the runtime is O(4/3) relative to the total number of elements (N) and O(4) relative to the number of elements in one direction only (N_1).

Table 7.3 Relationship between number of elements in a 3-D model and runtime

Total elements (N)	**Elements in 1-D (N_1)**	**Runtime (sec) (t)**	$\frac{t}{t_a}$	$\left(\frac{N}{N_a}\right)^{3/2}$	$\left(\frac{N_1}{N_{1a}}\right)^3$
8 000	20	17	1.0	1.0	1.0
16 000	25	43	2.5	2.5	2.5
24 000	29	74	4.3	4.3	4.3
32 000	32	109	6.4	6.4	6.4
64 000	40	275	16.0	16.0	16.0

Appendix B: Conversion formulae for elastic parameters

WAVE's material properties are entered in terms of the Bulk Modulus (K), Shear Modulus (G) and density (ρ). These properties are sometimes better known in terms of other parameters. The following are useful conventions:

$$C_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \qquad C_s = \sqrt{\frac{G}{\rho}}$$

$$K = \left(C_p{}^2 - \tfrac{4}{3}C_s{}^2\right)\rho \qquad G = C_s{}^2\rho$$

$$E = \frac{9KG}{3K+G} \qquad \nu = \frac{3K-2G}{2\left(3K+G\right)}$$

$$K = \frac{E}{3\left(1-2\nu\right)} \qquad G = \frac{E}{2\left(1+\nu\right)}$$

$$K = \lambda + \tfrac{2}{3}G$$

C_p and C_s are the P-wave and S-wave velocities, respectively. E is known as the Young's modulus, ν is Poisson's ratio and λ is one of the Lamé parameters.