

Solving the Boltzmann equation for electron kinetics using Petrov-Galerkin approach

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- **Importance:** Distribution function of electrons defines transport and reaction properties
→ Need to couple plasma model with electron kinetics (Y3)
- Evolution of $f = f(\mathbf{x}, \mathbf{v}, t)$ obeys the **Boltzmann equation**

$$\partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f - \mathbf{E} \cdot \nabla_{\mathbf{v}} f = \sum_a C_a(f)$$

where, for example, in case of $a = \text{elastic collisions}$

$$C_a(f) = n_0 \int_{S^2} v \underbrace{\sigma_a(v, \omega)}_{\text{scat. cross sec.}} (f(v') - f(v)) d\omega$$

- **Main challenge:** 6+1 dimensions
- **Idea:** FEM in \mathbf{x} , spectral in \mathbf{v}
- **Currently (Y1):** spatially homogeneous case $f = f(\mathbf{v}, t)$

$$\partial_t f = \sum_a C_a(f)$$

to investigate efficient velocity-space discretizations

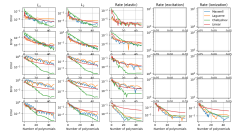
- **Approach:** Petrov-Galerkin

– Nearly isotropic and symmetric $f \rightarrow$ employ spherical harmonics

$$f(\mathbf{v}) = \sum_{k,l,m} h_{k,l,m}(t) \Phi_k(v) \underbrace{Y_{lm}(v_\theta, v_\phi)}_{\text{sph. harm.}} \rightarrow \text{ODEs for } h_{k,l,m}(t)$$

Results:

- Investigating choice of $\Phi_k(v)$:
 - Tested Laguerre, Maxwell, Chebyshev, Linear polys using Bolsig+ data
- Solver implementation:
 - Python
 - Tensorized 5D integrations: 0.0093s vs 5.2256s loop-based
 - Arbitrary choice of $\Phi_k(v)$
 - Reactions implemented: Elastic, Excitation, ionization
 - Testing and verification are underway



$z=0, t = 1.960000E-08s$

