Solving the Boltzmann equation for electron kinetics using Petrov-Galerkin approach

Milinda Fernando, Daniil Bochkov, Todd Oliver, George Biros

- **Importance:** Distribution function of electrons defines transport and reaction properties
  $\rightarrow$ Need to couple plasma model with electron kinetics (Y3)
- Evolution of $f = f(\boldsymbol{x}, \boldsymbol{v}, t)$ obeys the **Boltzmann equation**

$$\partial_t f + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f - \boldsymbol{E} \cdot \nabla_{\boldsymbol{v}} f = \sum_a C_a(f)$$

where, for example, in case of $a =$ elastic collisions

$$C_a(f) = n_0 \int_{S^2} v \underbrace{\sigma_a(v, \omega)}_{\text{scat. cross sec.}} \left(f(v') - f(v)\right) \, \mathrm{d}\omega$$

- **Main challenge**: 6+1 dimensions
- **Idea**: FEM in $\boldsymbol{x}$, spectral in $\boldsymbol{v}$
- **Currently (Y1)**: spatially homogeneous case $f = f(\boldsymbol{v}, t)$

$$\partial_t f = \sum_a C_a(f)$$

to investigate efficient velocity-space discretizations

- **Approach:** Petrov-Galerkin
  - Nearly isotropic and symmetric $f \rightarrow$ employ spherical harmonics

$$f(\boldsymbol{v}) = \sum_{k,l,m} h_{k,l,m}(t) \Phi_k(v) \underbrace{Y_{lm}(v_\theta, v_\phi)}_{\text{sph. harm.}} \quad \rightarrow \quad \text{ODEs for } h_{k,l,m}(t)$$

Results:

- Investigating choice of $\Phi_k(v)$:
  - Tested Laguerre, Maxwell, Chebyshev, Linear polys using Bolsig+ data
- Solver implementation:
  - Python
  - Tenzorized 5D integrations: 0.0093s vs 5.2256s loop-based
  - Arbitrary choice of $\Phi_k(v)$
  - Reactions implemented: Elastic, Excitation, Ionization
  - Testing and verification are underway