

EXERCISE :The Redshift, Hubble's Law, and the Age of the Universe

Aims: To recreate the steps Hubble took, to learn basic techniques such as approximating linear data, and to understand how the age of the Universe is derived.

In our galaxy, the Doppler effect has been very useful in determining the rotation rates of planets, of the sun, and even the motions of stars around the galactic center. The use of spectroscopy in determining velocities toward or away from us (radial velocities) has been extended to external galaxies as well. As a consequence, a very important relationship between a galaxy's distance and its velocity was discovered by Edwin Hubble in 1929. Hubble was able to determine distances to and velocities of 24 galaxies using the then largest telescope in the world—the 2.5-m (100") reflector on Mt. Wilson. The distances were primarily determined by using the Cepheid variable techniques.

Table I. Hubble-1929

Object Name	Dist. (Mpc)	Vd. (km/s)	Object Name	Dist. (Mpc)	Vd. (km/s)	Object Name	Dist. (Mpc)	Vd. (km/s)
SMC	0.032	+170	5194	0.5	+270	1055	1.1	+450
LMC	0.034	+290	4449	0.63	+200	7331	1.1	+500
6822	0.214	-130	4214	0.8	+300	4258	1.4	+500
598	0.263	-70	3031	0.9	-30	4151	1.7	+960
221	0.275	-185	3627	0.9	+650	4382	2.0	+500
224	0.275	-220	4826	0.9	+150	4472	2.0	+850
5357	0.45	+200	5236	0.9	+500	4486	2.0	+800
4736	0.5	+290	1068	1.0	+920	4649	2.0	+1090

SMC = Small Magellenic Cloud; LMC = Large Magellenic Cloud; All object numbers are preceded by "NGC." 1 parsec = 3.26 light years; 1 Mpc = megaparsec = 10 parsecs.

Procedure: Step 1

Plot velocity versus distance for Hubble's data in Table I. Use graph sheet #1 that is included in this write-up. Then, starting at the origin, draw the best straight line to fit the points. Note that the line goes through the graph's origin since we are at velocity = 0 and distance = 0, and all velocities are determined with respect to us.

From these data, Hubble was able to conclude that there exists a linear relationship between a galaxy's velocity and its distance from us. This relationship is now called Hubble's Law and it can be written as

$$v = H_0 d, \quad (1)$$

where d is the object's distance (in megaparsecs = Mpc = millions of parsecs), v is its velocity in km/s, and H_0 is called the Hubble constant. Equation (1) is the equation of the line you have drawn through Hubble's data and H_0 is the slope of the line.

Step 2

Determine the slope of the line you have drawn through Hubble's data, and, thus, the value of Hubble's constant. (Do this by taking the value of velocity and dividing it by the corresponding distance, that is, slope = change in y / change in x , or Dy/Dx)

$$H_0 = \text{slope} = \frac{\text{velocity}}{\text{distance}} = \text{-----} \text{ km / sec / Mpc}$$

(Note: This value is almost 10 times the current value as we shall see below.)

Equation 1 says that if the velocity of a galaxy can be determined (using the Doppler effect), then its distance can be inferred. On the data given in Table I, all velocities except five are positive values. This means that the majority of these galaxies are moving away (receding) from us. In fact, the first six galaxies in Table I are members of what is called the Local Group of galaxies. This Local Group is a collection of about 20 galaxies, including the Andromeda Galaxy, the Large and Small Magellenic Clouds, and our own galaxy. All of these members are gravitationally bound to one another and are moving through space as a cluster. (Some of these nearest galaxies are the negative velocity values given in Table I.)

One might try to deduce from Hubble's data that not only is the universe expanding but also that our galaxy is located at the center of the expansion. However, we now realize that every galaxy appears to be moving away from every other galaxy and that it is only an illusion that the center of the expansion is our own Milky Way.

Step 3

After Hubble's announcement of a distance-velocity correlation of galaxies, astronomers attempted to observe the redshift of even more distant galaxies to verify Hubble's relationship and to extend it. Such observations were reported in 1931 by Milton Humason, who worked with Hubble at the Mt. Wilson Observatory. His data covered 46 galaxies with redshifts as large as +19,700 km/sec, which was then thought to correspond to a distance of 35 million parsecs. Later, in 1936, Humason reported velocities as high as 40,000 km/sec, or 72 million parsecs. With the introduction of the 5-meter Hale telescope on Palomar Mountain in 1949, Humason measured the redshift of a cluster of galaxies in the constellation Hydra and found it to be 60,500 km/sec, or almost 110 million parsecs. Table II lists a summary of the data through 1936.

Table II. Humason - 1936

Cluster	Vd. (km/s)	Dist. (Mpc)	Cluster	Vd. (km/s)	Dist. (Mpc)	Cluster	•Vel. (km/s)	Dist. (Mpc)
Virgo	890	1.6	Leo	19,600	35.1	Anon 5	19,000	34.0
Pegasus	3,810	6.8	7814	1,000	1.8	Anon 6	42,000	75.3
Pisces	4,630	8.3	7868	5,700	10.2	Anon 7	15,400	27.6
Cancer	4,820	8.6	7869	6,700	12.0	Anon 9	39,000	69.9
Perseus	5,230	9.4	7872	7,000	12.5	Anon 10	21,000	37.6
Coma	7,500	13.4	Gem	24,000	43.0	Anon 11	9,200	16.5
U.Maj.	11,800	21.1	Gem	23,000	41.2	Anon 12	12,400	22.2

"NGC" precedes the 7814, 7868, 7869, and 7872.

Plot the data give in Table II onto graph sheet #2, also included in this write-up. Note that Hubble's original data from Table I occupies a very small area in this new plot- Illustrate this fact by drawing a box in the lower left comer that indicates the velocity and distance ranges covered by Hubble. Compute the value of the Hubble constant from these data.

$$H_0 = \text{slope} = \frac{\text{velocity}}{\text{distance}} = \text{-----} \text{ km / sec / Mpc}$$

Step 4

If the expansion of the universe has been constant, and, therefore, the value of Hubble's constant is indeed constant, then $1/H_0$ (in appropriate units) is the age of the universe (more precisely, a value called the Hubble Time), Therefore, using the fact that 1 Mpc = 3.08×10^{19} km and 1 year = 3.15×10^7 sec, we can calculate the age of the Universe, or the time since the expansion began. Note that if the expansion is slowing, the universe is actually younger than the Hubble Time. In what follows, substitute the value of Hubble's constant (including its units) from the results of graph 1 (given in step 2) and multiply out the numbers and units, cancelling units where possible.

$$\text{Age of the Universe} = \left(\frac{1}{H_0} \right) \left(\frac{3.08 \times 10^{19} \text{ km}}{1 \text{ Mpc}} \right) \left(\frac{1 \text{ year}}{3.15 \times 10^7 \text{ sec}} \right) = \text{-----} \text{ years.}$$

Step 5

If the expansion of the universe has been constant, and, therefore, the value of Hubble's constant is indeed constant, then $1/H_0$ (in appropriate units) is the age of the universe (more precisely, a value called the Hubble Time). Therefore, using the facts that 1 Mpc = 3.08×10^{19} km and 1 year = 3.15×10^7 sec, we can calculate the age of the Universe, or the time since the expansion began. Note that if the expansion is slowing, the universe is actually younger than the Hubble Time. In what follows, substitute the value of Hubble's constant (including its units) from the results of graph 2 (given in step 4) and multiply out the numbers and units, cancelling units where possible.

$$\text{Age of the Universe} = \left(\frac{1}{H_0} \right) \left(\frac{3.08 \times 10^{19} \text{ km}}{1 \text{ Mpc}} \right) \left(\frac{1 \text{ year}}{3.15 \times 10^7 \text{ sec}} \right) = \text{-----} \text{ years.}$$

Step 6

The formula $v = c$ is only valid for velocities much less than the speed of light. For larger velocities, we must use the relativistic formula;

$$z = \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1.$$

The largest redshift measured for a quasar (as of 1997) is $z = 4.89$. Solving the above equation for v , we can compute the recessional velocity for a quasar.

$$\text{Velocity} = c \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \text{_____ km/sec.}$$

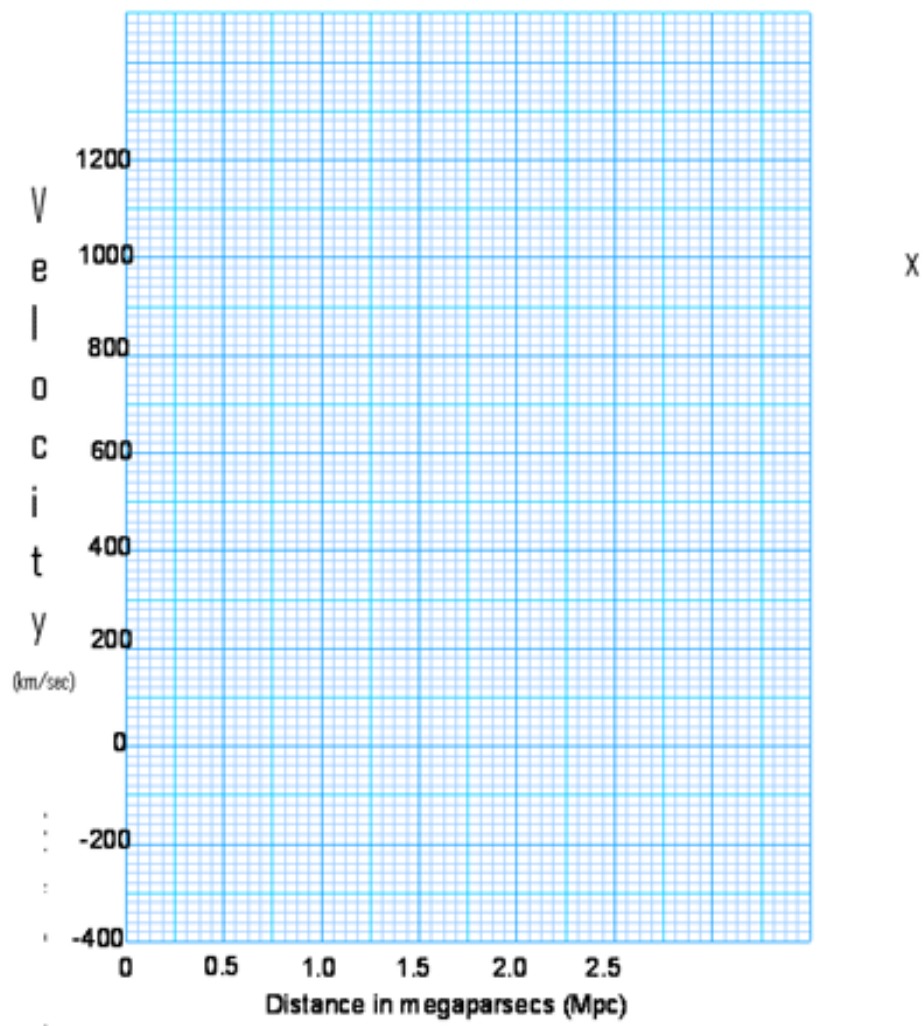
Calculate how far away the quasar must be.

$$\text{Distance} = \frac{\text{velocity}}{H_0} = \text{_____ Mpc} = \text{_____ light years.}$$

(Remember, 1 parsec = 3.26 light years.)

Compare this answer to the age of the Universe in step 5.

Based on a lab exercise developed at the Astronomy Program, University of Maryland, and rewritten by Jay M. Pasachoff.



Graph 1 Plot for Step 1

Recessional velocity vs distance

