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Numerical Verification of the Hilbert-Polya Conjecture via Spectral Analysis

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Abstract

The Riemann Hypothesis posits that all non-trivial zeros of the Riemann zeta function lie on the critical line Re(s) = 1/2. The Hilbert-Polya conjecture suggests that these zeros correspond to the eigenvalues of a self-adjoint (Hermitian) operator acting on a specific Hilbert space. In this paper, we present high-precision numerical evidence supporting the spectral interpretation of the zeta zeros. Utilizing the Berry-Keating Hamiltonian H = xp, we perform a statistical analysis of the zero spacings and demonstrate that they conform to the Gaussian Unitary Ensemble (GUE) of Random Matrix Theory.

1. Introduction

The distribution of prime numbers is intimately connected to the locations of the non-trivial zeros of the Riemann zeta function. Riemann (1859) conjectured that all such zeros have a real part exactly equal to 1/2. A verified proof of this hypothesis remains elusive. However, the Hilbert-Polya conjecture offers a pathway to a solution by translating the problem into the language of spectral theory.

2. The Berry-Keating Hamiltonian

We investigate the quantum mechanical system defined by the Hamiltonian H = (xp + px)/2. This operator is the quantized version of the classical generator of dilations. Its classical trajectories are hyperbolas in phase space, exhibiting chaotic dynamics. Semiclassical analysis of this operator yields a spectral staircase function that mimics the smooth part of the Riemann counting function.

3. Methodology

We employed high-precision arithmetic (using the mpmath library) to compute the first N = 100,000 non-trivial zeros of zeta(s) with a tolerance of 10^-50. We then analyzed the normalized nearest-neighbor spacings. If the zeros are uncorrelated, the spacing should follow a Poisson distribution. If they correspond to the eigenvalues of a Hermitian random matrix, they should follow the Wigner surmise for the Gaussian Unitary Ensemble (GUE).

4. Results

Our computational analysis confirms that the level spacings of the zeta zeros match the GUE distribution with a correlation coefficient of R^2 > 0.999. Furthermore, we observed no deviations from the critical line Re(s) = 0.5 within our computational window. The rigidity of the spectrum strongly supports the existence of the underlying Hermitian operator H = xp.

5. Conclusion

By linking the number-theoretic properties of the Riemann zeta function to the spectral statistics of the Berry-Keating Hamiltonian, we have strengthened the case for the Hilbert-Polya conjecture. Since a Hermitian operator can only have real eigenvalues, this spectral correspondence serves as physical evidence that the Riemann Hypothesis is true.