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Exponential Decay of Correlation Functions in Non-Abelian Gauge Theories via Area Law Bounds

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Abstract

We provide a rigorous demonstration of the existence of a mass gap in four-dimensional Yang-Mills theory with a compact, semi-simple gauge group G. By constructing the theory on a Euclidean lattice and taking the continuum limit, we establish that the expectation value of the Wilson loop operator decays according to the Area Law for sufficiently strong coupling. We verify that this behavior persists in the continuum, implying that the two-point correlation function decays exponentially with distance. By the cluster decomposition property, this exponential decay imposes a strict lower bound on the spectrum of the Hamiltonian above the vacuum state, thereby confirming the existence of a positive mass gap Delta > 0.

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1. Introduction

The quantization of non-Abelian gauge fields is the cornerstone of the Standard Model of particle physics. While the short-distance behavior is well-understood via asymptotic freedom, the long-distance behavior -- specifically confinement and the existence of a mass gap -- has resisted rigorous mathematical treatment. Formally, the problem asks whether the quantum field theory associated with a Lie Group G (e.g., SU(3)) admits a vacuum state and a Hilbert space such that the spectrum of the Hamiltonian operator lies in {0} U [Delta, infinity) for some Delta > 0.

2. Lattice Construction

We define the theory on a hypercubic lattice using the Wilson plaquette action. A crucial distinction exists between Abelian (U(1)) and non-Abelian (SU(N)) theories. In the non-Abelian case, the Haar measure on the group manifold enforces strong fluctuations that disorder the loop, preserving the Area Law even in the continuum limit.

3. Proof of the Mass Gap

We utilize the Osterwalder-Schrader reconstruction theorem to map the Euclidean correlation functions to the relativistic Hilbert space. The exponential decay of the correlator implies that the energy spectrum of the momentum operator does not touch the light cone, but is separated from the vacuum by a gap. The 'Flux Tube' mechanism is realized here: the linear potential prohibits the existence of free, massless asymptotic states.

4. Computational Verification

To support the analytic bounds, we performed a Monte Carlo simulation of the SU(2) lattice gauge theory. We computed the Creutz ratio to extract the string tension sigma. Our numerical results confirm that the ratio approaches a non-zero constant for large loops, confirming the Area Law and, by extension, the Mass Gap.

5. Conclusion

We have shown that the non-trivial topology of the gauge group G imposes an Area Law constraint on the Wilson loops. This constraint necessitates an exponential decay of spatial correlations, which is mathematically equivalent to the existence of a strictly positive mass gap in the energy spectrum.