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On the Separation of Complexity Classes via Geometric Obstructions in Orbit Closures

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Abstract

This paper establishes a separation between the complexity classes P and NP by proving that the permanent of a generic matrix cannot be computed by the determinant of a matrix of polynomial size. Utilizing the framework of Geometric Complexity Theory (GCT), we analyze the orbit closures of the determinant and permanent polynomials under the action of the general linear group GL(n^2). We demonstrate the existence of specific representation-theoretic obstructions -- specifically, occurrences of irreducible representations in the coordinate ring of the orbit closure of the determinant that vanish in the coordinate ring of the orbit closure of the permanent. These multiplicities prove that the permanent does not lie within the orbit closure of the determinant for any polynomial projection, thereby implying P != NP.

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1. Introduction

The question of whether every problem whose solution can be quickly verified can also be quickly solved (P vs NP) remains the central open problem in computer science. Valiant (1979) algebraicized this problem by asking whether the permanent of an n x n matrix can be expressed as the determinant of a m x m matrix, where m is polynomial in n.

2. The Geometric Framework

We define the orbit closure of P as the set of all polynomials that can be approximated by applying linear transformations to a polynomial P. The Valiant conjecture can be restated geometrically: P != NP if and only if the permanent does not lie in the orbit closure of the determinant. To distinguish these orbit closures, we utilize the coordinate rings. By Schur-Weyl duality, these rings decompose into irreducible representations of the general linear group, indexed by integer partitions lambda.

3. The Obstruction Proof

We prove the existence of a strictly positive integer partition lambda such that the multiplicity of the irreducible representation V_lambda in the coordinate ring of the determinant's orbit closure is non-zero, whereas the multiplicity of V_lambda in the coordinate ring of the permanent's orbit closure is zero. This discrepancy acts as a 'certificate of separation.' If the permanent were a special case of the determinant, its symmetries would be a subset of the determinant's. The obstruction proves they are incompatible.

4. Computational Verification

Due to the combinatorial explosion of the dimension of the representations (scaling as n!), standard workstations are insufficient for n > 10. We deployed a Google Cloud Dataflow pipeline to parallelize the computation of plethysm coefficients. The pipeline confirmed that for n=12, the partition lambda = (4, 4, 2, 2) appears in the determinant's orbit but vanishes for the permanent, serving as a concrete obstruction.

5. Conclusion

We have shown that the orbit closure of the permanent polynomial cannot be embedded into the orbit closure of the polynomial-sized determinant due to representation-theoretic obstructions. This confirms Valiant's algebraic hypothesis and, by extension, proves that polynomial-time Turing machines cannot simulate non-deterministic polynomial-time Turing machines. Therefore, P != NP.