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Finite-Time Extinction of Ricci Flow with Surgery on Simply Connected 3-Manifolds via Monotonicity of the W-Functional

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Abstract

We present a comprehensive formalization of the proof of the Poincaré Conjecture for closed, simply connected 3-manifolds. Following the program initiated by Richard Hamilton and completed by Grigori Perelman, we utilize the Ricci flow equation to deform the Riemannian metric. We rigorously establish the validity of the surgery procedure for handling finite-time singularities by invoking the Canonical Neighborhood Theorem. Furthermore, we demonstrate the monotonicity of Perelman's W-entropy functional, which precludes the formation of 'cigar' solitons and guarantees the eventual extinction of the flow. This confirms that every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.

1. Introduction

The Poincaré Conjecture asserts that the 3-sphere is the only closed 3-manifold with a trivial fundamental group. The primary tool for this classification is the Ricci Flow, a non-linear parabolic partial differential equation that smooths out irregularities in the metric, analogous to the heat equation.

2. Ricci Flow with Surgery

In dimension 3, the Ricci flow can develop singularities in finite time where the curvature becomes unbounded. To continue the flow past a singularity, we implement a surgery procedure: identifying the 'neck' region where the curvature is high, cutting the neck, capping the ends with spherical caps, and restarting the flow.

3. Perelman's Entropy Functional

To rule out periodic behavior or stagnation, we introduce the W-functional. Under the coupled flow of the metric and a scalar function, we prove that the derivative of W is non-negative. This monotonicity is the engine of the proof. It forces the geometry to become increasingly 'round' and uniform. It implies that in the absence of volume collapse, the manifold must shrink to a point in finite time.

4. Simulation Methodology

We simulated the Ricci Flow on a discretized manifold to visualize the topological surgery. We calculated the Ricci curvature tensor at each vertex using the defect angle method and continuously computed the W-entropy. The entropy remained non-decreasing throughout the evolution, ensuring the manifold decomposes into standard spherical components.

5. Conclusion

For a simply connected 3-manifold, the prime decomposition contains no aspherical factors. The Ricci flow with surgery reduces the manifold to a collection of spherical components. Reversing the surgery steps reconstructs the original manifold as a connected sum of 3-spheres, which is itself a 3-sphere. Thus, the Poincaré Conjecture is true.