2024-2025 Güz Yarıyılı

Algoritma Analizi

Ödev-1

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1.

a. $T(n) = 9T(n/4) + n^2$

$a=9$
$b=4$ $\quad 9 < 4^2$
$d=2$ $\quad$ then

$\boxed{T(n) \in \Theta(n^2)}$

b. $T(n) = 3T(n/3) + \log(n)$

$a=3$
$b=3$ $\quad n^{\log_b a} = n^1$
$f(n) = \log(n)$ $\quad \log(n) \in O(n)$

$\boxed{T(n) \in \Theta(n)}$

c. $T(n) = 3T(n/2) + n$

$a=3$
$b=2$ $\quad 3 < 2^1$
$d=1$ $\quad$ then

$\boxed{T(n) \in \Theta(n^{\log_2 3})}$

Master Theorem

$T(n) = a \cdot T(n/b) + n^d$

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

$T(n) = a \cdot T(n/b) + f(n)$

$$T(n) \in \begin{cases} \Theta(n^{\log_b a}) & \text{if } f(n) = O(n^{\log_b a - \varepsilon}) \\ \Theta(n^{\log_b a} \log n) & \text{if } f(n) = \Theta(n^{\log_b a}) \\ \Theta(f(n)) & \text{if } f(n) = \Omega(n^{\log_b a + \varepsilon}) \wedge \\ & a f(n/b) < c f(n) \end{cases}$$

$\rightarrow$ 2. Soruyu çözmek için araştırdım.

2.

int f1(n) $\longrightarrow T_1(N) = \sum_{i=0}^{N} 1 = N \Rightarrow \boxed{T_1(N) \in O(N)}$

int f2(n) $\longrightarrow$

$$T_2(n) = \sum_{i=0}^{N} \sum_{j=0}^{i} T_1(j) = \sum_{i=0}^{N} \sum_{j=0}^{i} j = \sum_{i=0}^{N} \frac{i.(i+1)}{2}$$

$$= \frac{1}{2} \sum_{i=0}^{N} i^2 + \cancel{\frac{1}{2} \sum_{i=0}^{N} i} \Rightarrow$$ Big-oh notasyonu için en yüksek mertebeli değişkene ihtiyaç var.

$$= \frac{1}{2} \cdot \frac{(N.(N+1).(2N+1))}{6} = \boxed{\frac{2N^3}{12} \in O(N^3)}$$

int p3(n) $\rightarrow$

$$T(n) = n \cdot \cancel{T(n-1)} + O(1)$$ $\rightarrow$ Çok küçük bir değer, yokmuş gibi davranacağım.

$$\cancel{T(n-1)} = (n-1) \cdot \cancel{T(n-2)}$$

$$\vdots$$

$$\cancel{T(1)} = 1 \cdot \cancel{T(0)}$$

$$\times \quad \cancel{T(0)} = 1$$

$$\boxed{T(n) = n! + O(1) \in O(n!)}$$

int p4(n) $\rightarrow$

$$T_4(n) = T_4(n/2) + T_1(n) + T_1(n) + T_1(n) + T_4(n/2)$$

$$T_4(n) = 2T_4(n/2) + 3T_1(n)$$

$T_1(n) = n$ — ilk soruda öğrendik

Backward Subs.

$$T_4(n/2) = 2T_4(n/4) + \frac{3n}{2}$$

$$T_4(n/4) = 2T_4(n/8) + 3\frac{n}{4}$$

$$T_4(n) = 2 \cdot \left(2T_4(n/4) + \frac{3n}{2}\right) + 3n$$

$$= 4T_4(n/4) + 6n$$

$$T_4(n) = 4 \cdot \left(2 \cdot T_4(n/8) + \frac{3n}{4}\right) + 6n$$

$$= 8T_4(n/8) + 9n$$

$$\vdots$$

$$T_4(n) = 2^k \cdot T_4(n/2^k) + k \cdot 3n$$

$2^k = n$ olsun $\rightarrow$ $T_4(n) = n \cdot T_4(n/n) + \log n \cdot 3n$

$\log n = k$

$$= n \cdot T_4(1) + \boxed{3n\log n \in O(n\log n)}$$

3.

1.0 2.1 3.0 4.1 5.0 6.1 7.0 8.0

4.

$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

a. $2^{n+1} + 3^{n-1} = f(n)$

if $g(n) = 3^n$

$c_1 \cdot 3^n \leq f(n)$

$c_1 = 1$ için $3^n \leq 2^{n+1} + 3^{n-1}$ $\quad n=1$ için $3 \leq 4+1$

$f(n) \leq c_2 \cdot 3^n$ $\quad n=2$ için $9 \leq 8+3$ ✓

$c_2 = 5$ için $2^{n+1} + 3^{n-1} \leq 5 \cdot 3^n$ $\quad n=2$ için $18+3 \leq 45$

$n=3$ için $16+9 \leq 135$ ✓

$\boxed{f(n) \in \Theta(3^n)}$

b. $2n \log(n+2)^2 + (n+2)^2 \log \frac{n}{2} = f(n)$

$\rightarrow$ if $g(n) = n^2 \log n$

$c_1 \cdot n^2 \log n \leq f(n)$

$c_1 = 1$ için $3n^2 \log n \leq f(n)$ $\quad n=2$ için $1,20 \leq 1,44$ ✓

$n=3$ için $4,28 \leq 7,33$ ✓

$c_2 \cdot n^2 \log n \geq f(n)$

$c_2 = 2$ için $2n^2 \log n \geq f(n)$ $\quad n=1$ için $0 \geq -2,25$ ✓

$n=2$ için $2,40 \geq 1,40$ ✓

$\boxed{f(n) \in \Theta(n^2 \log n)}$

5. $\sum_{i=1}^{n} (i+1) \cdot 2^{i-1} = \sum_{i=1}^{n} i \cdot 2^{i-1} + 2^{i-1} = \frac{1}{2} \sum_{i=1}^{n} i \cdot 2^i + \frac{1}{2} \sum_{i=1}^{n} 2^i$

$= \frac{1}{2} \cdot ((n-1) \cdot 2^{n+1} + 2) + \frac{1}{2} \cdot \cancel{(2^{n+1} - 2)}$ $\rightarrow$ Big-Oh notasyon için mertebesi yüksek elemanı kullanıyorum.

$= \frac{1}{2} \cdot (n-1) \cdot 2^{n+1}$

$= \frac{1}{2} \cdot 2^{n+1} \cdot n - \cancel{\frac{1}{2} \cdot 2^{n+1}} = 2^n \cdot n \in O(2^n \cdot n)$

6. $T(n) = T(n-2) + 2n$

$$T(n) = T(n-4) + 4n - 4$$
$$= T(n-6) + 6n - 12$$
$$= T(n-8) + 8n - 24$$

$$T(n-2) = T(n-4) + 2n - 4$$
$$T(n-4) = T(n-6) + 2n - 8$$
$$T(n-6) = T(n-8) + 2n - 12$$

$$T(n) = T(n-2k) + 2k.n - k.(2k-2)$$

$2k = n$ için

$$T(n) = T(0) + n^2 - \frac{n}{2}.(n-2)$$
$$= T(0) + n^2 - \frac{n^2 + 2n}{2}$$
$$= T(0) + \frac{n^2 + 2n}{2}$$

$$\boxed{T(n) = \frac{n^2 + 2n}{2}}$$

$k=1$	0
$k=2$	$+4$
$k=3$	$+12$
$k=4$	$+24$
$k=5$	$+40$

$k.(2k-2)$