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ABSTRACT

In this paper, a modified normalized Rortex/vortex identification method named $\widetilde{\Omega}_R$ is presented to improve the original Ω_R method and resolve the bulging phenomenon on the isosurfaces, which is caused by the original Ω_R method. Mathematical explanations and the relationship between the Q criterion and $\widetilde{\Omega}_R$ are described in detail. In addition, the new developed formula does not require two original coordinate rotations, and the calculation of $\widetilde{\Omega}_R$ is greatly simplified. The numerical results demonstrate the effectiveness of the new modified normalized Rortex/vortex identification method.

Vortex definition and vortex identification methods are still a common concern in the community of fluid mechanics research, attracting many researchers to design various vortex identification methods.[1–4] These vortex identification methods are designed to overcome the deficiencies of the vorticity method. However, these vortex identification methods are not equivalent in themselves,[5] and the physical meanings are not clear. It is also far-fetched to use these quantities to express the intensity of the vortex. First, a vortex is a rotational motion of the fluid, and thus, the rotation should have a rotating axis. However, most popular vortex identification methods do not give a clear definition of the axis of rotation. The detailed reviews on these vortex identification methods can be referred to Refs. 6–9. Recently, a new vector called the Rortex is proposed to describe the local rotational axis and rotational strength of the vortex.[10,11] The Rortex vector represents a rigid rotation extracted from the vorticity. The remainder of the vorticity is antisymmetric shear. The Rortex vector is parallel to the coherent vortex isosurfaces, and the isosurfaces of the magnitude of the Rortex can be used to identify the vortex structures.[6,10,11] Due to the clear physical meaning of the Rortex vector, it has gained a lot of research attention.[12–20] Although the isosurfaces of magnitude of the Rortex vector are very effective to capture the coherent vortex structures, the determination of the threshold is still case-dependent.[15–17] Recently, using the idea of the widely used Ω method,[6,9,16–21] a normalized Rortex/vortex identification method is developed by Dong *et al.*[22] The new method denoted by Ω_R has several important advantages, including the following: (1) Ω_R is a dimensionless relative quantity from 0 to 1, which can be used to do statistics and correlation analysis directly; (2) Ω_R can distinguish the vortex from high vorticity concentration with high shear and exclude high-shear boundary layers; (3) Ω_R is robust to the threshold change and can empirically be set as 0.52 to visualize the vortex structures; and (4) Ω_R has the capability of capturing both strong and weak vortices simultaneously.[6,22] In view of the above advantages, Ω_R quickly attracted many people to use.[16,17,20] However the vortex structure shown by the original Ω_R method is not smooth and there is a bulging phenomenon on the isosurfaces.

In this paper, a modified method named $\widetilde{\Omega}_R$ is presented to improve the original Ω_R method and solve the bulging phenomenon on the isosurfaces. This modified method was established by analyzing the original Ω method in 2D and 3D flow fields, taking into account the effects of fluid stretching and compression.

In the Rortex method, following the Δ-criterion, we believe that, in the region with rotation, the velocity gradient tensor $\nabla\vec{v}$ has complex conjugate eigenvalues $\lambda_{cr} \pm \lambda_{ci}i$, and the real unit eigenvector $\vec{r}$ is the direction of the local rotational axis. Hence, we have

$$\nabla\vec{v}\cdot\vec{r} = \lambda_r\vec{r}, \tag{1}$$

where $\vec{v} = [u, v, w]^T$ is the velocity vector and λ_r denotes the real eigenvalue which indicates the rate of change on the rotation axis. In addition, the definition of the local rotation axis means there is no cross-velocity gradient on the local rotation axis.[11] In order to obtain the rotation strength, the first coordinate rotation $\boldsymbol{Q_r}$ is used so that the rotation axis is the Z axis.[6] Then,

$$\nabla\vec{\boldsymbol{V}} = \boldsymbol{Q_r}\nabla\vec{\boldsymbol{v}}\boldsymbol{Q_r^T} = \begin{bmatrix} \frac{\partial U}{\partial X} & \frac{\partial U}{\partial Y} & 0 \\ \frac{\partial V}{\partial X} & \frac{\partial V}{\partial Y} & 0 \\ \frac{\partial W}{\partial X} & \frac{\partial W}{\partial Y} & \frac{\partial W}{\partial Z} \end{bmatrix}, \tag{2}$$

where $\vec{\boldsymbol{V}} = [U, V, W]^T$ is the velocity vector in the new XYZ frame. Hence, in the new XYZ frame, the coordinate Z-axis is the local rotation axis. From the definition of the Rortex vector, the rotation strength is obtained by a second coordinate rotation $\boldsymbol{P_r}$ in the XY plane to make $\left|\frac{\partial U}{\partial Y}\right|$ or $\left|\frac{\partial V}{\partial X}\right|$ be the minimum. After turning θ angle, the gradient tensor of velocity $\vec{\boldsymbol{V}}_{\boldsymbol{\theta}}$ will become

$$\nabla\vec{\boldsymbol{V}}_{\boldsymbol{\theta}} = \boldsymbol{P_r}\nabla\vec{\boldsymbol{V}}P_{\boldsymbol{r}}^{\boldsymbol{T}}. \tag{3}$$

Therefore, the terms in the 2×2 upper left submatrix of $\nabla\vec{\boldsymbol{V}}_{\boldsymbol{\theta}}$ are the following:

$$\begin{aligned} \left.\frac{\partial U}{\partial Y}\right|_\theta &= \alpha\sin(2\theta+\varphi) - \beta, \\ \left.\frac{\partial V}{\partial X}\right|_\theta &= \alpha\sin(2\theta+\varphi) + \beta, \\ \left.\frac{\partial U}{\partial X}\right|_\theta &= -\alpha\cos(2\theta+\varphi) + \tfrac{1}{2}\left(\tfrac{\partial U}{\partial X} + \tfrac{\partial V}{\partial Y}\right), \\ \left.\frac{\partial V}{\partial Y}\right|_\theta &= \alpha\cos(2\theta+\varphi) + \tfrac{1}{2}\left(\tfrac{\partial U}{\partial X} + \tfrac{\partial V}{\partial Y}\right), \end{aligned} \tag{4}$$

where

$$\alpha = \frac{1}{2}\sqrt{\left(\frac{\partial V}{\partial Y} - \frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)^2}, \tag{5}$$

$$\beta = \frac{1}{2}\left(\frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}\right), \tag{6}$$

In fact, β in (6) is the vorticity in the XYZ frame. The detailed production and the expression of φ can be referred to Refs. 6 and 23. Then, the rotation strength $R = 2(\beta - \alpha)$, and the Rortex vector is defined by

$$\vec{\boldsymbol{R}} = R\vec{\boldsymbol{r}}. \tag{7}$$

The original normalized Rortex/vortex identification method Ω_R is defined as[22]

$$\Omega_R = \frac{\beta^2}{\alpha^2 + \beta^2 + \epsilon}, \tag{8}$$

where ϵ is a small parameter to prohibit the computational noise. And, ϵ is proposed as $\epsilon = b_0(\beta^2 - \alpha^2)_{\max}$, where b_0 is a small positive number around 0.001–0.002. Although the original method has many advantages, careful observation will reveal that the isosurfaces formed by Ω_R are not smooth and have many bulges. Recall the definition of the Ω method,[6,21] using the denotations of the symmetric tensor $\boldsymbol{A} = \frac{1}{2}(\nabla\vec{\boldsymbol{v}} + \nabla\vec{\boldsymbol{v}}^T)$ and the antisymmetric spin tensor $\boldsymbol{B} = \frac{1}{2}(\nabla\vec{\boldsymbol{v}} - \nabla\vec{\boldsymbol{v}}^T)$. Then, we can formulate Ω method as

$$\Omega = \frac{b}{a + b + \epsilon}, \tag{9}$$

where $a = \|\boldsymbol{A}\|_F^2$, $b = \|\boldsymbol{B}\|_F^2$, and $\|\cdot\|_F$ is the Frobenius norm. For two-dimensional flow, by the Rortex eigenvalue decomposition,[11,23] and the Galilean invariance of the Ω method,[24] we can get

$$\nabla\vec{\boldsymbol{V}}_{\boldsymbol{\theta}\mathbf{min}} = \begin{bmatrix} \lambda_{cr} & -(\beta-\alpha) \\ \beta+\alpha & \lambda_{cr} \end{bmatrix} = \begin{bmatrix} \lambda_{cr} & \alpha \\ \alpha & \lambda_{cr} \end{bmatrix} + \begin{bmatrix} 0 & -\beta \\ \beta & 0 \end{bmatrix} \triangleq \boldsymbol{A} + \boldsymbol{B} \tag{10}$$

and

$$\Omega_{2\mathrm{D}} = \frac{b}{a+b+\epsilon} = \frac{\beta^2}{\beta^2 + \alpha^2 + \lambda_{cr}^2 + \epsilon}. \tag{11}$$

Similarly, for three-dimensional flow,[11]

$$\begin{aligned} \nabla\vec{\boldsymbol{V}}_{\boldsymbol{\theta}\mathbf{min}} &= \begin{bmatrix} \lambda_{cr} & -(\beta-\alpha) & 0 \\ \beta+\alpha & \lambda_{cr} & 0 \\ \xi & \eta & \lambda_r \end{bmatrix} \\ &= \begin{bmatrix} \lambda_{cr} & \alpha & \frac{1}{2}\xi \\ \alpha & \lambda_{cr} & \frac{1}{2}\eta \\ \frac{1}{2}\xi & \frac{1}{2}\eta & \lambda_r \end{bmatrix} + \begin{bmatrix} 0 & -\beta & -\frac{1}{2}\xi \\ \beta & 0 & -\frac{1}{2}\eta \\ \frac{1}{2}\xi & \frac{1}{2}\eta & 0 \end{bmatrix} \triangleq \boldsymbol{A} + \boldsymbol{B} \end{aligned} \tag{12}$$

and

$$\Omega_{3D} = \frac{\beta^2 + \frac{1}{4}(\xi^2 + \eta^2)}{\beta^2 + \frac{1}{2}(\xi^2+\eta^2) + \alpha^2 + \lambda_{cr}^2 + \frac{1}{2}\lambda_r^2 + \epsilon}. \tag{13}$$

Considering Eqs. (11) and (13), the normalized Rortex/vortex identification method Ω_R (8) is not completely matched with the Ω method. In order to reflect the original Ω method as much as possible, we recommend the following new modified normalized Rortex/vortex identification method which does not include terms of $(\xi^2 + \eta^2)$:

$$\widetilde{\Omega}_R = \frac{\beta^2}{\beta^2 + \alpha^2 + \lambda_{cr}^2 + \frac{1}{2}\lambda_r^2 + \epsilon}, \tag{14}$$

and $\widetilde{\Omega}_R$ requires a parameter greater than 0.5 which is the same as Ω or Ω_R methods. Although the choice of the thresholds for Ω or Ω_R is empirical, it has been found that the vortex structures shown by the isosurfaces from 0.51 to 0.60 are almost the same from many numerical experiments by original Ω or Ω_R methods, and the threshold 0.52 is almost suitable for all test cases.[6,9,16–22] Similarly, we empirically take $\widetilde{\Omega}_R = 0.52$ as a fixed threshold for the approximation of vortex boundaries. Here, for the convenience of explanation, α and β are used to form the modified $\widetilde{\Omega}_R$ method. We will then give an explicit formula that does not require the previous $\boldsymbol{Q_r}$ and $\boldsymbol{P_r}$ rotations to get α and β. It will greatly simplify the solution process.

At present, the Q criterion is a very popular vortex identification method used in engineering.[2] Below, we will explain the mathematical relationship between the new modified $\widetilde{\Omega}_R$ method and the popular Q criterion. In fact, neglect the small ϵ,

$$\widetilde{\Omega}_R = \frac{\beta^2}{\beta^2 + \alpha^2 + \lambda_{cr}^2 + \frac{1}{2}\lambda_r^2} > 0.5, \tag{15}$$

i.e.,

$$\beta^2 - \alpha^2 > \lambda_{cr}^2 + \frac{1}{2}\lambda_r^2. \tag{16}$$

Assume the flow is incompressible. Then, $\lambda_r = -2\lambda_{cr}$.[5] Hence, we have

$$\beta^2 - \alpha^2 > 3\lambda_{cr}^2. \tag{17}$$

From the matrix (12), $\beta^2 - \alpha^2 = \lambda_{ci}^2$.[11] Therefore, $\lambda_{ci}^2 > 3\lambda_{cr}^2$. That is,

$$\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2 < \frac{1}{3}. \tag{18}$$

For incompressible flow, the second invariant Q of velocity gradient tensor can be explicitly written as[5]

$$Q = \lambda_{ci}^2\left(1 - 3\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2\right). \tag{19}$$

The $Q > 0$ criterion requires $\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2 < \frac{1}{3}$. Therefore, the mathematical vortex boundaries for $\widetilde{\Omega}_R > 0.5$ and $Q > 0$ are equivalent completely for incompressible flow, and all regions described by them are a subset of the region defined by $\lambda_{ci} > 0$. But the new modified $\widetilde{\Omega}_R$ is a dimensionless quantity from 0 to 1. Furthermore, from Eq. (8), $\Omega_R = \frac{\beta^2}{\alpha^2+\beta^2} > 0.5$ and $\beta^2 - \alpha^2 = \lambda_{ci}^2$, and we can conclude that the mathematical vortex boundaries defined by $\Omega_R > 0.5$ and $\lambda_{ci} > 0$ are the same. Although the region defined by $\widetilde{\Omega}_R > 0.5$ is a subset defined by $\Omega_R > 0.5$, Eq. (18) will avoid regions of strong outward spiraling given by $\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2 > \frac{1}{3}$.[5] Furthermore, from our computation, the new modified $\widetilde{\Omega}_R$ method can keep smooth and get rid of the bulges on the isosurfaces.

Recently, Wang *et al.* gave an explicit formula of the Rortex vector to simplify the calculations of β and α, which can be reformulated as

$$R = \vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{r}} - \sqrt{\left(\vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{r}}\right)^2 - 4\lambda_{ci}^2}, \tag{20}$$

where $\vec{\boldsymbol{\omega}}$ represents the vorticity and $\vec{\boldsymbol{\omega}}\cdot\vec{\boldsymbol{r}}$ is always set to be positive.[25] Then, we can obtain

$$\beta = \frac{1}{2}\vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{r}}, \tag{21}$$

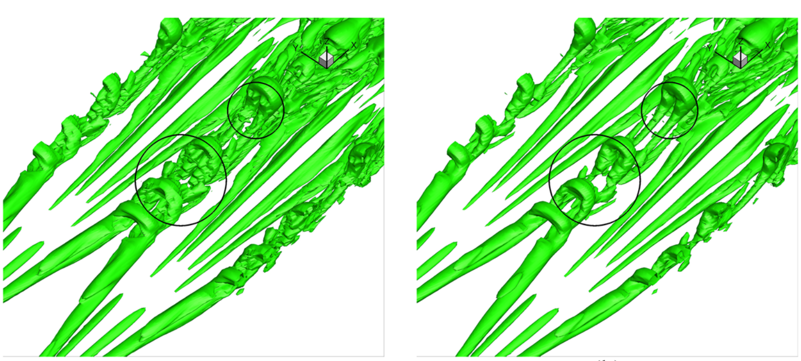
$$\alpha = \frac{1}{2}\sqrt{\left(\vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{r}}\right)^2 - 4\lambda_{ci}^2}. \tag{22}$$

Hence, the present new modified normalized Rortex/vortex identification method (14) can be rewritten as

$$\widetilde{\Omega}_R = \frac{\left(\vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{r}}\right)^2}{2\left[\left(\vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{r}}\right)^2 - 2\lambda_{ci}^2 + 2\lambda_{cr}^2 + \lambda_r^2\right] + \epsilon}. \tag{23}$$

The new formula (23) does not require the coordinate rotations $\boldsymbol{Q_r}$ and $\boldsymbol{P_r}$, and the calculation is much simplified.

In order to show that the modified normalized Rortex/vortex identification method (14) or (23) can get rid of bulging phenomenon produced by original Ω_R method, the data calculated by direct numerical simulation (DNS) of a boundary layer transition on a flat plate at a Mach number of 0.5 and Reynolds number 1000 are studied. Figure 1 shows the isosurfaces obtained by the original and modified normalized Rortex/vortex identification methods. By the original Ω_R method (8), the isosurfaces lost smoothness and show the emergence of the bulging phenomenon as depicted in Fig. 1(a). The result obtained by the new modified $\widetilde{\Omega}_R$ method is shown in Fig. 1(b), which demonstrates the effectiveness of the new method. $\widetilde{\Omega}_R$ is a dimensionless relative quantity from 0 to 1. The isosurfaces formed by the $\widetilde{\Omega}_R$ method are not sensitive to the threshold change. For different examples, we can always take $\widetilde{\Omega}_R = 0.52$, and the $\widetilde{\Omega}_R$ method has the shape-preserving feature as the threshold increases. However, for the popular Q criterion, the determination of the proper threshold in advance is very hard and needs to be adjusted case by case; sometimes the threshold could be as large as 10^8 (see Ref. 17), but the current example can only take a small value. Therefore, Q is very sensitive to the threshold selection, while $\widetilde{\Omega}_R$ is not. As a demonstration, we compare the results of the isosurfaces obtained by the Q criterion and the new modified $\widetilde{\Omega}_R$. The results obtained by the Q criterion at different Q thresholds are shown in Figs. 2(a)–2(c). The vortex structures obtained by the $\widetilde{\Omega}_R$ method at $\widetilde{\Omega}_R$ thresholds are shown in Figs. 2(d)–2(f). The graphic of the DNS data shows that as the threshold of the Q criterion increases, many vortex structures disappear in the downstream of flow, especially for the ring structure. The $\widetilde{\Omega}_R$ method still maintains the vortex structures as the threshold increases. Furthermore, the vortex intensity represented by the Q criterion has been greatly reduced in downstream, but the relative vortex strength measured by $\widetilde{\Omega}_R$ is still relatively large.



(a) (b)

FIG. 1. Isosurfaces of hairpin vortex structures by different normalized Rortex/vortex identification methods: (a) Ω_R = 0.52, (b) $\widetilde{\Omega}_R$ = 0.52.

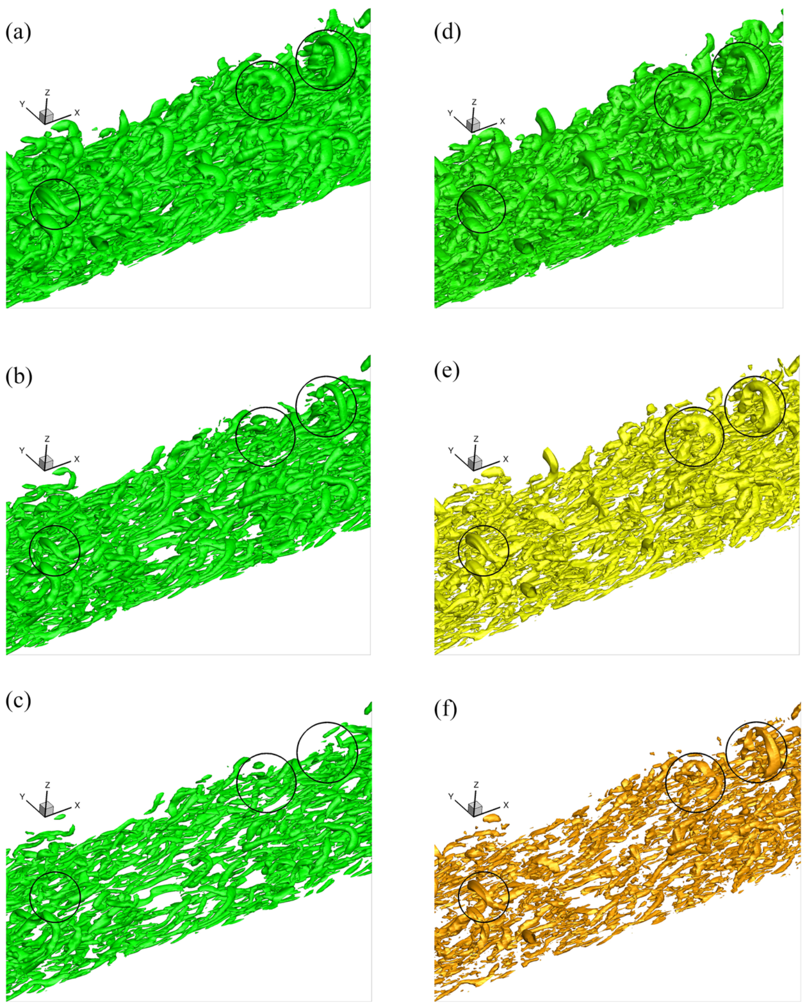


FIG. 2. Isosurfaces vortex structures obtained by Q criterion and the $\widetilde{\Omega}_R$ method at different thresholds: (a) $Q = 0.005$, (b) $Q = 0.015$, (c) $Q = 0.025$, (d) $\widetilde{\Omega}_R = 0.52$, (e) $\widetilde{\Omega}_R = 0.7$, and (f) $\widetilde{\Omega}_R = 0.8$.

In this paper, a modified normalized Rortex/vortex identification method named $\widetilde{\Omega}_R$ is presented to improve the original Ω_R method and solve the bulging phenomenon on the Ω_R isosurfaces. Detailed mathematical explanations are provided. Also, the relationship between the Q criterion and $\widetilde{\Omega}_R$ is described. The final new formula does not require original coordinate rotations $\boldsymbol{Q_r}$ and $\boldsymbol{P_r}$, and the calculation is much simplified. Furthermore, the new modified $\widetilde{\Omega}_R$ method retains the advantages of the original Ω_R method. It is a dimensionless relative quantity from 0 to 1, which can be used to do statistics and correlation analysis directly. $\widetilde{\Omega}_R$ can distinguish the vortex from high vorticity concentration with high shear and exclude high shear boundary layers. $\widetilde{\Omega}_R$ is robust and can always be set as 0.52 to visualize the vortex structures. $\widetilde{\Omega}_R$ has the capability of capturing both strong and weak vortices simultaneously. In this paper, we also compared the $\widetilde{\Omega}_R$ results with the Q criterion. The results show the $\widetilde{\Omega}_R$ method can retain the ring structures when the threshold increases. In many places, the relative strength of vortex structures is still big.

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