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Multi-objective stochastic supply chain modeling to evaluate tradeoffs between profit and quality

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ABSTRACT

Many companies struggle with justifying the cost of quality within their supply chain. Outsourcing suppliers to countries such as China has become popular in recent years due to the fact it appears to be more profitable. These outsource decisions do not effectively determine the impacts of quality defects. In this paper we demonstrate a method for evaluating the systemic supply chain risk of poor quality. We introduce a multi-objective stochastic model that uses Six Sigma measures to evaluate financial risk. Results from modeling suggest quality, profit, and customer satisfaction can be evaluated.

1. Introduction

Many companies struggle with justifying the cost of quality within their supply chain. Outsourcing suppliers to countries such as China has become popular in recent years due to the fact it appears to be more profitable. However, what many companies fail to see is the cost associated with varying quality levels from their suppliers. In order to create a quality product, the company must address all aspects of the supply chain, including individual processes and supplier selection.

A supply chain (SC) can be expressed as the sum of parts involved in fulfilling a customer request (Chopra and Meindl, 2007). By this definition, a supply chain consists of suppliers, manufacturers, warehouses, retailers, transporters, and customers. The purpose of a supply chain analysis is to maximize an organization's profit in the process of generating value for the customer, namely maximizing the difference between the final product worth and the total cost expended by the supply chain to provide the product to the customer.

Many organizations emphasize quality as a means to stay competitive in the marketplace over the long run. They view having a reputation of high quality as representing future market share for new customers and maintaining market share for existing customers over their lifetime. Further, improving quality can provide long term financial savings, such as scrap and rework reduction. We associate these quality savings as long term savings that are difficult to quantify. One method to quantify quality is the initiative known as Six Sigma. The label "Six Sigma" originates from statistical terminology, wherein sigma (σ) represents standard deviation. The probability of falling within plus or minus six sigma on a normal curve is 0.9999966, which is more commonly represented as a defect rate of 3.4 parts per million (Yang and El-Haik, 2003). This level of quality is seen as the goal in most Six Sigma initiatives, but the terminology is also used to evaluate current levels below that, such as 4 Sigma representing a plant that has 6210 defects per million. The corresponding defective rates for each Sigma level are shown in Table 1 below.

In order for a supply chain to remain profitable, quality from suppliers must be considered on the decision making process. Competing strategies of increasing profit as opposed to increasing quality will require many tradeoffs. The purpose of this article is to model the tradeoffs and to identify situations that a decision maker can use to optimize the benefits of both.

2. Background

As various quality assurance methods are being developed and discarded, total quality management (TQM) through Six Sigma is becoming popular. The goal of TQM and Six Sigma is to identify the poor quality immediately during the production process, rather than spending time to inspect the finished product. The quality of the manufacturing process determines the quality of a finished product. In the supply chain it is not always possible to control the manufacturing process for incoming materials, especially for outside suppliers. In this instance, quality can only be measured by the percentage of defective goods received from the suppliers. In order to more effectively manage the supply

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Table 1
Sigma levels for parts per million defective.

Sigma level	Parts per million defective
1	691,462
2	308,538
3	66,807
4	6210
5	233
6	3.4

chain, companies must choose suppliers that will produce quality materials without a substantial price tag.

Supply chain management (SCM) is responsible for the optimization of the flow of products between the various levels of the supply chain and for minimizing the total cost of these operations. This term SCM is a unification of a series of concepts about integrated business planning that are joined together by the advances in information technology (IT) (Shapiro, 2007). Despite the IT advances, many companies have not completely taken advantage of computing power to make cost effective decisions.

Competition between companies, more demanding customers, and reduced profit margins make the success of a company more challenging. In this context, SCM became a very important practice for companies that not only want to stay in business but also have their results optimized and meet the clients' expectations. Responsiveness in the supply chain has gained importance and it is a trend that will dictate future decisions regarding supply chain design.

It can be seen that SCM plays and will continue to play an active role in successful companies' routines. Literature about supply chain design is extensive and diverse. The common issue addressed in similar literature is to optimize the supply chain while the customer's demand is satisfied at some level.

While existing supply chain decision levels have been divided in strategic, tactical, and operational levels (Anthony, 1965), the focus of this research addressed strategic and tactical aspects of the supply chain. The main difference between these levels is that strategic decisions are concerned with the supply chain topology, such as which nodes are going to be utilized, represented by binary variables. Tactical decisions usually assume that the supply chain topology is already given and its main purpose is to optimize production rates, utilization, vendor selection, and resource allocation. Several works have been published about the optimization of supply chains. A very useful literature review and critique is presented by Meixell and Gargeya (2005).

2.1. *Stochastic optimization in supply chains*

The use of uncertainty models in supply chain management problems is a natural extension of the traditional deterministic approach. This happens due to the fact that most problems faced by companies have as a characteristic some degree of uncertainty. Thus, the assumptions that all the parameters used in modeling are deterministic is not realistic, especially when considering elements that are in most cases beyond the scope of the company, such as demand, prices, and efficiency rates. In order to address this need, some approaches introduce stochastic elements in the model. Some use system dynamics and control theory to model uncertainty. Perea et al. (2000) and Vlachos et al. (2007), for example, utilize traditional control theory, whereas others utilize model predictive control (MPC) like Perea-López et al. (2003). Haralambos et al. (2008) discuss the characteristics of this approach and give a literature review of the subject.

The largest category of uncertainty models addresses the uncertainty of variables by assigning statistical distributions. Works such as MirHassani et al. (1999) and Ahmed et al. (2000a) address the production planning problem under uncertainty. A common point in those works is that they only analyze tactical decisions and do not account for the strategic level decision formulation.

More recent works approach the tactical and strategic decision making by the use of two stage stochastic integer programming in most cases. MirHassani et al. (1999) uses this strategy and scenario analysis of solutions with Benders' decomposition in a multi-period resource allocation model. Ahmed et al. (2000) approached the capacity expansion of a supply chain problem by using a multi-stage stochastic programming formulation with a scenario tree to model the evolution of uncertainty. In Gupta and Maranas (2000, 2003) and Tsiakis et al. (2001), the authors also use two-stage stochastic programming to make tactical and strategic decisions in the supply chain under demand uncertainty. Alonso-Ayuso et al. (2003) made use of a Branch and Fix Coordination algorithm to solve the stochastic integer programming problem of a two stage supply chain problem. Blackhurst et al. (2004) uses a network-based approach to model uncertainty in a supply chain. Ryu et al. (2004) uses bi-level programming and parametric programming to solve a supply chain resource allocation problem under demand uncertainty.

2.2. *Multi-objective optimization and supplier selection in supply chains*

Most of the existing literature is focused on the optimization of only one objective function, usually cost or profit, and other important factors such as quality and supplier selection are left outside the analysis.

There are many techniques for multi-objective optimization such as the ε-constrained method, sequential optimization, weighted methods, and distance-based methods (Szidarovszky et al., 1986). Guillén et al. (2005) utilizes the ε-constrained method in order to optimize profit, demand satisfaction, and financial risk cost of a three echelon supply chain. Azaron et al. (2008) uses the goal attainment technique to optimize total cost, total cost variance, and financial risk cost of a three echelon supply chain. Chen et al. (2003) uses a two-phase fuzzy decision-making to optimize profit between the supply chain participants, customer service levels, and safe inventory level. In this paper, the utilized method is the ε-constrained in order to optimize the profit and quality objective function. This method provides a set of objectives that are Pareto efficient, thus forming a Pareto frontier (for more information on multi-objective stochastic programming, refer to Stancu-Minasian, 1984).

2.3. *The cost of poor quality*

Quality is a significant concern in the supply chain because of the historical problems that have come from supplier selection. The most common reasons quality assurance (QA) fails to adequately address these issues are: (1) the company outsourced to does not have its own QA team, and assumed that the client would complete this in-house, (2) the project had a very tight deadline and so QA testing was done rapidly or set aside to give development a priority, and (3) the vendor did not fully understand the system requirements, and so testing did not cover them (Singh, 2006). These and other reasons generate both the recall and product quality issues that make the news. It has also been shown that outsourcing poses a quality risk, on average (Gray et al., 2007).

The issue of supplier portfolio selection based on the quality of raw material is an important topic and has gained more focus with total quality management (TQM) and Six-Sigma initiatives from industry (Pike and Barnes, 1996; Adams et al., 2002). Raw material quality is usually an objective that can be achieved by sourcing material from better quality suppliers. In most cases, those higher quality suppliers are more costly due to their tighter quality specifications. By analyzing only those two separate measures of profit and quality, it can be seen that they are contradictory. One may try to give monetary values to quality but its impacts are much more extensive (e.g. companies that use better quality suppliers may charge more from customers for better quality products, thus a strategic decision that is beyond the scope of this paper) and therefore a weight factor is used in this paper to assign importance to raw material defects.

There is very little research that addresses supplier portfolio selection and supply chain modeling in a comprehensive fashion. The literature focuses on supplier selection and its impact on inventory policies. There is no research that addresses the issue of quality and supplier selection in the context of a complete stochastic supply chain, therefore, the purpose of this paper is to demonstrate a method for evaluating the systemic supply chain risk of poor quality.

2.4. Problem statement

Companies must decide on conflicting strategies of maximizing short term profits or seeking long term sustainability through high quality standards. Usually supply chain decisions are evaluated by analysts that consider the logistics costs and the quality initiatives independently. This independent decision making process does not effectively determine the impacts of quality defects and can lead to ineffective strategic decisions that do not comprehensively account for the complexity of the problem. Our objective is to address that phenomenon by creating a multi-objective model that can demonstrate the tradeoffs between quality and profitability that companies consider when making decisions. In addition, most optimization decisions are based on forecasts that by definition have some degree of uncertainty. Consequently, the model should take into account uncertainty for robustness and consistency.

The model proposed in this paper aims to help in the design of a four-echelon supply chain, consisting of suppliers, manufacturing plants, distribution centers, and customer centers. Using a multi-objective stochastic optimization approach the methodology seeks to (1) maximize a company's total profit, (2) increase quality levels by minimizing defective material from suppliers, and (3) investigate the effect of uncertainty on the model. From the result of multi-objective optimization, the model produces a set of solutions that can be used in order to find the desired configuration based on the performance level of each objective. We hypothesize that the systemic supply chain risk of poor quality will have a negative effect on profitability.

3. Methodology

3.1. Model definition

This paper introduces a multi-objective stochastic model that uses Six Sigma measures to evaluate financial risk. The model definition consists of four distinct steps to aid in the design of a four-echelon supply chain: identify objectives, establish the constraints of the model, evaluate the economic risk, and formulate the model using the multi-objective ε-constrained method.

Step 1. *Identify objective functions*

The proposed model is based on two objective functions. The first objective seeks to maximize the total profit of the SC, which is the most common objective in SC modeling, and the second objective is to increase the Sigma quality level by minimizing the total number of defects in raw material obtained from the suppliers. The second objective seeks to improve operations and quality by reduction of variability from raw materials and thus from manufacturing processes. We demonstrate an example of this using the quality indicators used by Six Sigma.

In order to solve the described model, a two-stage stochastic optimization approach is proposed. Initially, some decisions must be performed before uncertainty in demand is unveiled. These decisions are the strategic variables that decide which plants and distributions centers are candidates to open. The recourse decisions are those made after uncertainty of random variables is unveiled. These are operational decisions regarding the material flow between the nodes, capacities, and production. These two consecutive decisions make up the two-stage model.

The objective of the stochastic problem is usually the sum of the performance of the first stage decisions and the expected performance of the second stage decisions. In two-stage models the first stage decisions are not scenario dependent and therefore assume a deterministic value. The second stage variables assume different values for each random scenario considered, thus assuming a probabilistic value.

Objective 1: *Maximize profit*

The goal of the first objective (o_1) is the maximization of the expected profit. This value is given by the basic statistics as the summation of the objective of each scenario times its probability of occurring and it is shown in Eq. (1)

$$o_1 = \max E[Profit] = \max \sum_s Pb_s Profit_s, \tag{1}$$

where $E[Profit]$ is the expected profit, Pb_s is the probability of occurrence of scenario s, and $Profit_s$ is the SC profit at scenario s. The calculations for $Profit_s$ is shown in the following six equations.

$Profit_s$ is the net present value of the financial flow (Bal_{ts}) from each period t for each scenario s, which is calculated by the subtracting from the total income (Inc_{ts}) generated by the sales performed in Eq. (2), the sum of the purchasing costs (PC_{ts}) of raw materials in Eq. (3), the variable cost (VC_{ts}) in Eq. (4), and fixed costs (FC_{ts}) of plants and DCs in Eq. (5). Inc_{ts} is the summation of the volume of product p transported from DC j to customer k on period t over scenario s (Y^2_{pjkts}) multiplied by the sales price of product p to customer k (g_{pk}).

$$Inc_{ts} = \sum_p \sum_j \sum_k g_{pk} Y^2_{pjkts} \quad \forall t \in \mathcal{T}, s \in \mathcal{S}. \tag{2}$$

The purchasing costs of raw materials can be calculated by multiplying the cost of the raw material (a_{rf}) by the volume of raw material transported from supplier f to plant i on period t over scenario s (Y^0_{rfits}):

$$PC_{ts} = \sum_r \sum_f \sum_i a_{rf} Y^0_{rfits} \quad \forall t \in \mathcal{T}, s \in \mathcal{S}. \tag{3}$$

In the same fashion, the variable cost is the sum of the manufacturing cost, handling and inventory costs at the DCs, and the transportation costs between the SC nodes,

$$VC_{ts} = \sum_p \sum_i m_{pi} X_{pits} + \sum_p \sum_j \sum_k h_{pj} Y^2_{pjkts} + \sum_p \sum_j ic_{pj} I_{pjts} + \sum_r \sum_f \sum_t Y^0_{rfits} b^0_{rfi} + \sum_p \sum_i \sum_j Y^1_{pijts} b^1_{pij}$$

$$+\sum_p\sum_j\sum_k Y^2_{pjkts}b^2_{pjk} \quad \forall t \in \mathcal{T}, s \in \mathcal{S}. \tag{4}$$

where m_{pi} is the producing cost for product p on at plant i, X_{pits} is the volume of product p produced by plant i during period t on scenario s, h_{pj} is the handling cost of product p on DC j, ic_{pj} is the inventory cost of product p on DC j, I_{pjts} is the level of inventory of product p on DC j on period t over scenario s, b^0_{rfi} is the transportation cost of raw material r from supplier f to manufacturing plant i, Y^1_{pijts} is the volume of product p transported from plant i to DC j on period t over scenario s, b^1_{pij} is the transportation cost of raw material r from plant i to DC j, b^2_{pjk} is the transportation cost of product p from DC j to customer k.

The fixed cost is calculated with the help of the binary decision variables α_i and β_j that define if the manufacturing plants and DC's, respectively, are open or closed. F^1_{pi} is the fixed cost of processing product p at plant i, F^2_{pj} is the fixed cost of processing product p at DC j. This constraint is stated by (5)

$$FC=\sum_p\sum_i \alpha_i F^1_{pi}+\sum_p\sum_j \beta_j F^2_{pj} \tag{5}$$

With those elements calculated, the balance for each period t for each scenario s (Bal_{ts}) and the net present value of those period t balances ($Profit_s$) can be expressed by Eq. (6)–(7)

$$Bal_{ts}=Inc_{ts}-(PC_{ts}+VC_{ts}+FC) \quad \forall t \in \mathcal{T}, s \in \mathcal{S}, \tag{6}$$

$$Profit_s=\sum_t \frac{Bal_{ts}}{(1+ir)^{t-1}} \quad \forall s \in \mathcal{S}, \tag{7}$$

where ir is the cost of capital incurred by the company.

Objective 2: *Minimize supplier defects* (*increase quality level*)

The goal of the second objective function (o_2) is to minimize the total number of defective raw material parts and thus increase the Sigma quality level. This function is given in Eq. (8)

$$o_2=\min\sum_r\sum_f\sum_i\sum_t\sum_s Y^0_{rfits}dr_{rf}w_r \tag{8}$$

For this objective, dr_{rf} is the quality level of raw material r from supplier f, and the weight factor of the raw material w_r is an estimation of the impact of raw material defects on the manufacturing process.

Step 2: *Establish the constraints of the model*

Mass balance constraints

The mass balance constraints of the supply chain must be held in order to keep the material flowing properly between the nodes. Thus, for each supplier f the total amount of raw material r purchased in each period t is equal to the amount of raw material utilized to produce products p as expressed by constraint (9) where A_{rf} is the assignment matrix with binary elements of which raw materials r are supplied by each supplier f and U_{rp} is the usage of each raw material r to produce product p

$$\sum_f Y^0_{rfits}A_{rf}=\sum_p X_{pits}U_{rp} \quad \forall r \in \mathcal{R}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}. \tag{9}$$

For the products produced and transported for each plant i the total amount of products p produced on period of time t must be shipped to a warehouse j. This constrain can be seen in Eq. (10)

$$X_{pits}=\sum_j Y^1_{pijts} \quad \forall p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}. \tag{10}$$

Constraint (11) covers product flow from plants to warehouses, from warehouses to customers, and warehouses' inventories. This constraint states that the amount of products p shipped from plants i to warehouse j in the period of time t plus the inventory of on hand of product p of warehouse j on the previous period $t-1$ must be equal to the amount of product p shipped from warehouse j to customers k on period t plus the inventory of product p on hand of the same period of time t

$$\sum_i Y^1_{pijts}+I_{pjt-1s}=\sum_k Y^2_{pjkts}+I_{pjts} \quad \forall p \in \mathcal{P}, j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S}. \tag{11}$$

The initial conditions of the system must be stated. In this SC system case it is assumed that the values of the decision variables that represent material transportation are equal to zero in the first period. This is assumed because in this period the strategic decisions regarding the SC are made. These seven constraints are expressed in (12)

$$X_{pits}=0, \quad Y^0_{rfits}=0, \quad Y^1_{pijts}=0, \quad Y^2_{pjkts}=0 \quad t=1,$$
$$\forall p \in \mathcal{P}, i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, s \in \mathcal{S}. \tag{12}$$

Capacity constraints

The amount of raw material r that suppliers f can supply must be less than or equal to the continuous variable that represents its capacity (C^0_f) as stated in Eq. (13)

$$\sum_r Y^0_{rfits}c_{rf} \le C^0_f \quad \forall f \in \mathcal{F}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}, \tag{13}$$

where c_{rf} is the capacity consumption factor of raw material r and supplier f.

In addition, C^0_f is constrained by its lower and upper bounds (C^{0L}_f, C^{0U}_f) and must agree with constraint (14). The binary variable γ_f is the decision of whether the supplier will be utilized or not. As can be seen, if the supplier f is not utilized, the capacity C^0_f is set to zero

$$C^{0L}_f\gamma_f \le C^0_f \le C^{0U}_f\gamma_f \quad \forall f \in \mathcal{F}. \tag{14}$$

The total number of products manufactured in each plant must be less than or equal to the continuous variable that represents the manufacturing plant capacity (C^1_i) as expressed by constraint (15). The capacity consumption factor for the products i in plants p, c_{pi}, informs how much capacity each product p takes from the plant i

$$\sum_p X_{pits}c_{pi} \le C^1_i \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}. \tag{15}$$

Similarly, C^1_i is constrained by its lower and upper bounds (C^{1L}_i, C^{1U}_i) and the binary variable α_i is the decision whether the plant will be open or not. The constraint is stated in (16)

$$C^{1L}_i\alpha_i \le C^1_i \le C^{1U}_i\alpha_i \quad \forall i \in \mathcal{I}. \tag{16}$$

The storage capacity of DC j is given by the continuous variable C^2_j that must be greater than or equal to the sum of products inventory of the warehouse in a given period of time as stated by Eq. (17). The capacity consumption factor for the products i in DC j, c_{pj} informs how much capacity each product p takes from the each DC j

$$\sum_p I_{pjts}c_{pj} \le C^2_j \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S}. \tag{17}$$

As occurred with the capacity of the plants, the capacity of the DCs must also be bounded by its lower and upper bounds (C^{2L}_j, C^{2U}_j) as expressed in constraint (18). The binary variable β_j defines if the DC is to remain open or not in the supply chain configuration

$$C^{2L}_j\beta_j \le C^2_j \le C^{2U}_j\beta_j \quad \forall j \in \mathcal{J}. \tag{18}$$

Demand satisfaction

This constraint stipulates that the DC's should not ship more than the demand. A 100% demand satisfaction level means that the demand is met completely. D_{pkts} is the demand for product p by customer k on period t for scenario s. This is expressed by constraint (19)

$$\sum_j Y^2_{pjkts} \le D_{pkts} \quad \forall p \in \mathcal{P}, k \in \mathcal{K}, t \in \mathcal{T}, s \in \mathcal{S}. \tag{19}$$

The demand satisfaction level must be equal to some predetermined value otherwise the second objective of reducing the amount of defective raw material r can be achieved by not meeting the demand, which is not desirable. This constraint is expressed by Eq. (20)

$$Sat = \frac{\sum_p \sum_j \sum_k \sum_t \sum_s Y^2_{pjkts}}{\sum_p \sum_k \sum_t \sum_s D_{pkts}}. \tag{20}$$

Step 3: Evaluate the financial risk

In order to measure the risk associated with the supply chain configuration, financial risk will be utilized. This measure can be defined as the probability of a certain objective of cost or profit not meeting a determined level Λ. In the case of the two-stage stochastic optimization problem modeled, this measure is the probability of a given design x not meeting a target profit level Λ (McCary, 1975; Barbaro and Bagajewics, 2003; Barbaro and Bagajewics, 2004) as stated by Eq. (21),

$$Risk(x, \Lambda) = Pb[Profit(x) < \Lambda], \tag{21}$$

where $Profit(x)$ is the actual profit of design x after uncertainty is unveiled and the scenario is realized. Moreover, the outcome of meeting the target level Λ is either 0 or 1. Thus, the financial risk formula can be rewritten as follows with binary variables:

$$Risk(x, \Lambda) = \sum_s Pb_s z_s(x, \Lambda), \tag{22}$$

where z_s is a defined binary variable for each scenario as follows:

$$z_s(x, \Lambda) = \begin{cases} 1 & \text{if } Profit(x) < \Lambda \\ 0 & \text{otherwise} \end{cases} \quad \forall s \in \mathcal{S}. \tag{23}$$

The profit discrete probability distribution utilized in this work is the histogram shown in Fig. 1.

Step 4: Formulate the multi-objective two-stage stochastic problem

As mentioned before, the approach used in this paper to deal with the uncertainty of the model is the two stage stochastic programming that was first introduced by Danzig (1955) and Beale (1955). On this type of formulation there are the first stage decisions, which are the decisions made before uncertainty is unveiled; and the second stage decisions, which are the ones made after randomness of the variables are unveiled. In the supply chain modeling context, the first stage variables are the decisions that need to be done here-and-now. They are the strategic decisions of which nodes are to remain open that do not vary across the scenarios. The second stage variables are the wait-and-see decisions. They are all the decision variables that vary with the scenarios assuming different values in each one (for more information refer to Birge and Louveaux, 1997; Sahinidis, 2004).

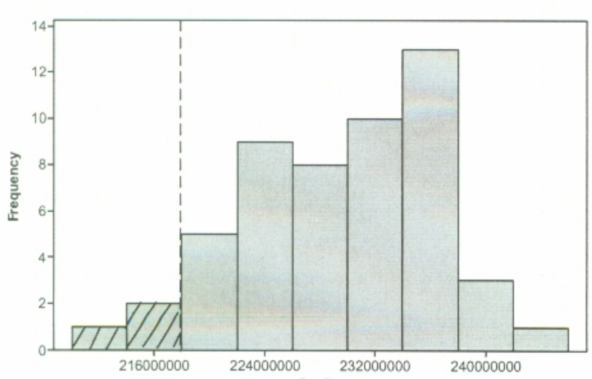

Fig. 1. Profit discrete probability distribution.

The stochastic programming includes two objectives: maximization of profit and maximizing quality. The formulation can be expressed by

o_1 and o_2 subject to Eqs. (2)–(7), (9)–(23).

The multi-objective modeling is achieved by the use of ε-constrained method. The steps for the method for solving this problem are:

(1) Set a value to the decrement ε_0;
(2) Optimize the first objective function only, set $\varepsilon_1 = o_1$;
(3) Evaluate the second objective function at the previous step, $\varepsilon_2 = o_2$;
(4) Add the additional constraint $o_2 < \varepsilon_2 - \varepsilon_0$ and re-optimize the first function;
(5) Set $\varepsilon_1 = o_1$ and $\varepsilon_2 = o_2$;
(6) Repeat 4 and 5 until you get in the infeasible region.

The collection of points ε_1 and ε_2 form the set of Pareto efficient solutions. A point $x^* \in R \subseteq \mathbb{R}^n$ is called an efficient point or a Pareto optimal solution if there does not exist any $u \in R$ with $f(u) \neq f(x^*)$ and $f(u) \le_p f(x^*)$. Where $\le_p$ establishes partial order in $\mathbb{R}^k$ since two arbitrary vectors are not necessarily comparable (Hillermeier, 2001).

This means that in a Pareto efficient state, one cannot improve the value of one of the functions without decreasing the value of one or more other functions. A collection of those points is called a Pareto curve or front. These concepts are very important, since it is necessary for the solution's points to be Pareto efficient, otherwise some of the objective functions can do better than the solution and it would not be optimal.

4. Numerical example

A numerical application of the proposed model is presented in order to illustrate its capabilities and robustness in dealing with supply chain problems. Consider a four-echelon supply chain similar to the one seen in Fig. 2.

There is a set of seven suppliers that supply raw material for two manufacturing plant candidates. There are seven raw material types that are used by the plants to produce three products. Those products are then shipped to four candidates in order to be distributed. From the DC's, the product is shipped to final customer centers. There is a total of seven customer center in our numerical example. These variables are listed in Table 2.

The demand for 50 scenarios was generated using Monte Carlo sampling that provided a set of 50 equiprobable scenarios according to a normal distribution. The expected demand of the products for the customers in the first period is assumed to be zero since during this period all the strategic decisions on the first stage of the stochastic optimization are performed. In the second period the demand is given by a normal distribution with mean given by

$$E[D_{pkts}] = \{10000, 75000, 43000, 56000, 19000, 34000, 85000\} \quad \forall p = P_1, k \in \mathcal{K}, t = 2, s \in \mathcal{S};$$

$$E[D_{pkts}] = \{15000, 55000, 18000, 70000, 54000, 29000, 45000\}, \quad \forall p = P_2, k \in \mathcal{K}, t = 2, s \in \mathcal{S};$$

The standard deviation of the demand for the second period is considered to be 15% of the mean. For the rest of the periods, it is assumed that the demand will experience a growth of 5% each period. The standard deviation will also grow by 5% each period,

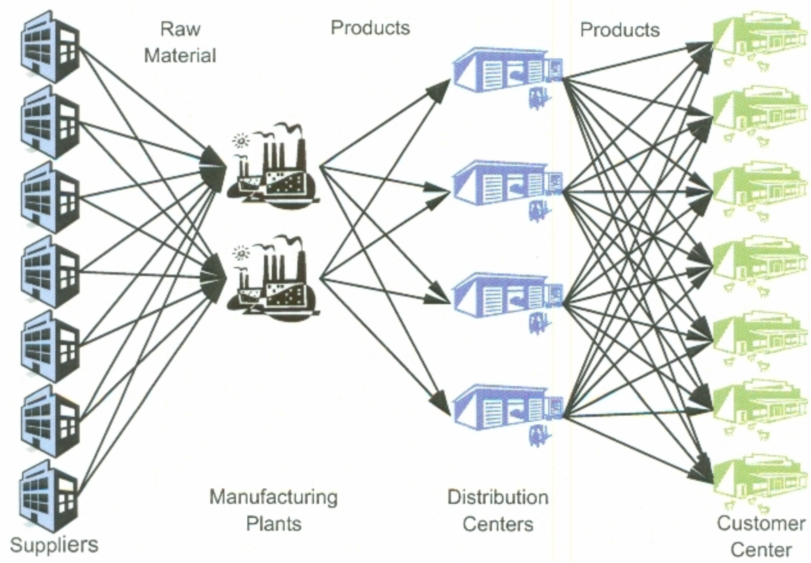

Fig. 2. 4-Echelon supply chain configuration.

Table 2
Description of variables in the supply chain configuration.

Variable	Description	Subset
F	Suppliers	Miami-FL, Detroit-MA, Atlanta-GA, Newark-NJ, Houston-TX, Omaha-NE, and Columbus-OH
I	Candidate plants	Omaha-NE, Memphis-TN
J	DC candidates	Omaha-NE, Miami-FL, Houston-TX, Indianapolis-IN
K	Customers	New York-NY, Los Angeles-CA, Houston-TX, Kansas City-MO, Miami-FL, Norfolk-VA, Seattle-WA
R	Raw material from suppliers	R1, R2, R3, R4, R5, R6, R7
P	Products manufactured in plants	P1, P2

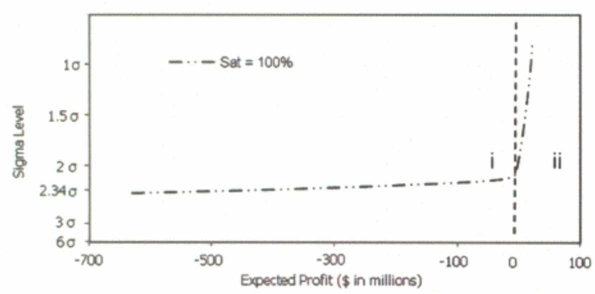

Fig. 3. Pareto front areas.

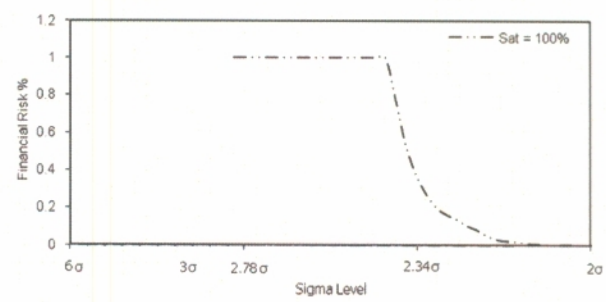

Fig. 4. Financial risk vs. sigma level.

going from 15% in the second period to 15.75% of the mean then 16.5375% mean in the third and four periods, respectively. The cost of capital chosen for this case is ir=20%, set for the case of a conservative investment (Jones et al., 2007). The results showing the Pareto efficient front for Sigma level versus profit can be seen in Fig. 3.

The plot shows that the set of results can be roughly divided in two regions that exist approximately for values lower and greater than zero for the profit. For the region to the left of zero profit, the profit has a greater weight, since for small increments in Sigma level there is a large increment in the profit. For the region to the right of zero profit the situation is the opposite, for small increments in profit there is a large increase in Sigma level. This is an important factor when analyzing the tradeoff between the objectives. While there is a little downside to improving quality to near 2.34 Sigma (100,000 defects per million), increasing quality beyond this has a highly negative effect on profit. Thus we would expect any optimization model to result in a suggested Sigma level near 2.34, and any significant deviation would require the influence of a heavily weighted factor.

We now consider how the financial risk behaves as the Sigma level varies, which can be seen in Fig. 4. In this case Λ is also set to the value of zero and x is a Pareto efficient configuration for the two objectives.

Fig. 4 confirms that the financial risk decreases as the Sigma level increases, so care should be taken in increasing the quality level by seeking the "lowest defect suppliers". The analysis of the risk is important in indicating how the solutions could be impacted if the variables were changed. The results demonstrated that the supplier channel selection does need to be considered when designing a supply chain in order to allow for a more comprehensive decision. A decision making process that does not account for this multi-objective complexity may lead to decisions that perform well for one of the objectives but perform poorly for the other and may unnecessarily increase financial risk of the supply chain design project.

5. Conclusions

Modeling a supply chain can be a challenging process due to the fact that there are a large number of factors that need to be translated into the model. The use of multi-objective stochastic optimization partially overcomes those problems since more information is used when building the model. Uncertainty on parameters and multiple objectives are ways to give more flexibility and robustness to the decision making process since the process can take into account much more information. Profit or cost is usually the classic choice when optimizing a supply chain. The problem with this formulation is that other important factors, such as quality and supplier selection, are not taken into account in the model.

In this paper we proposed a formulation with a comprehensive approach that considers the aforementioned factors. A stochastic optimization model was proposed to optimize the profit and the quality function of the supply chain. This model defines typical strategic and tactical decisions regarding the supply chain.

A set of Pareto efficient solutions is generated and the strategic binary variables take into account the randomness of the demand. This provides a useful tool for decision making since this process rarely is done based on only one objective or without considering the risks of randomness.

Numerical experiments showed the tradeoffs of the profit and quality function considered in the objectives and gave important insight to assist decision making process. We generated a set of scenarios in order to approximate a continuous distribution for the stochastic demand. The example illustrated how the model could be used in a real situation. It demonstrates the tradeoffs and the financial risk by a numeric application, thus supporting the need for the multi-objective and the stochastic approach.

Finally, there is the possibility of extending the model by analyzing the production lines and determining how the raw material interacts with the process to generate imperfect products. This formulation will require several assumptions and may be difficult to solve to proven optimality. The result is a multi-objective nonlinear stochastic programming model.

Notation

Sets

$\mathcal{F}$ set of suppliers
$\mathcal{I}$ set of plants
$\mathcal{J}$ set of DCs
$\mathcal{K}$ set of customers
$\mathcal{R}$ set of raw materials
$\mathcal{P}$ set of products
$\mathcal{S}$ set of scenarios
$\mathcal{T}$ set of time periods

Parameters

D_{pkts} demand of product p by customer k on period t at scenario s, $\forall p \in \mathcal{P}, k \in \mathcal{K}, t \in \mathcal{T}, s \in \mathcal{S}$
dr_{rf} defect rate of raw material r from supplier f, $\forall r \in \mathcal{R}, f \in \mathcal{F}$
w_r weight factors for importance of raw material r, $\forall r \in \mathcal{R}$
C_f^{0L} minimum capacity of supplier f, $\forall f \in \mathcal{F}$
C_f^{0U} maximum capacity of supplier f, $\forall f \in \mathcal{F}$
C_i^{1L} minimum capacity of plant i, $\forall i \in \mathcal{I}$
C_i^{1U} maximum capacity of plant i, $\forall i \in \mathcal{I}$
C_j^{2L} minimum capacity of DC j, $\forall j \in \mathcal{J}$
C_j^{2U} maximum capacity of DC j, $\forall j \in \mathcal{J}$
U_{rp} usage matrix of raw material r at product p, $\forall r \in \mathcal{R}, p \in \mathcal{P}$
c_{rf} capacity consumption factor of raw material r at supplier f, $\forall r \in \mathcal{R}, f \in \mathcal{F}$
c_{pi} capacity consumption factor of product p at plant i, $\forall p \in \mathcal{P}, i \in \mathcal{I}$
c_{pj} capacity consumption factor of product p at DC j, $\forall p \in \mathcal{P}, j \in \mathcal{J}$

$$A_{rf} \quad \begin{cases} 0, & \text{if supplier } f \text{ does not supplies raw material } r \\ 1, & \text{if supplier } f \text{ supplies raw material } r \end{cases}, \quad \forall r \in \mathcal{R}, f \in \mathcal{F}$$

Sat demand percentage satisfaction level
Pb_s probability of occurrence of scenario s

Cost estimators

a_{rf} cost of raw material r from supplier f, $\forall r \in \mathcal{R}, f \in \mathcal{F}$
b_{rfi}^0 transportation cost of raw material r from supplier f to manufacturing plant i, $\forall r \in \mathcal{R}, f \in \mathcal{F}, i \in \mathcal{I}$
b_{pij}^1 transportation cost of product p from plant i to DC j, $\forall p \in \mathcal{P}, i \in \mathcal{I}, j \in \mathcal{J}$
b_{pjk}^2 transportation cost of product p from DC j to customer k, $\forall p \in \mathcal{P}, j \in \mathcal{J}, k \in \mathcal{K}$
m_{pi} producing cost of product p at plant i, $\forall p \in \mathcal{P}, i \in \mathcal{I}$
h_{pj} handling cost of product p at DC j, $\forall p \in \mathcal{P}, j \in \mathcal{J}$
ic_{pj} inventory cost of product p at DC j, $\forall p \in \mathcal{P}, j \in \mathcal{J}$
F_{pi}^1 fixed cost of processing product p at plant i, $\forall p \in \mathcal{P}, i \in \mathcal{I}$
F_{pj}^2 fixed cost of processing product p at DC j, $\forall p \in \mathcal{P}, j \in \mathcal{J}$
g_{pk} sales price of product p to customer k, $\forall p \in \mathcal{P}, k \in \mathcal{K}$
Ir the cost of capital of the company or interest rate

Cost variables

Inc_s total sales income on scenario s, $\forall s \in \mathcal{S}$
PC_s purchasing cost incurred on scenario s, $\forall s \in \mathcal{S}$
VC_s variable cost incurred on scenario s, $\forall s \in \mathcal{S}$
FC_s fixed cost incurred on scenario s, $\forall s \in \mathcal{S}$

First stage strategic variables

$$\alpha_i = \begin{cases} 0, & \text{if plant } i \text{ is not selected for operating} \\ 1, & \text{if plant } i \text{ is selected for operating} \end{cases}$$

$$\beta_j = \begin{cases} 0, & \text{if DC } j \text{ is not selected for operating} \\ 1, & \text{if DC } j \text{ is selected for operating} \end{cases} \quad \forall i \in \mathcal{I}, j \in \mathcal{J}$$

Second stage operational variables

$$\gamma_f \quad \begin{cases} 0, & \text{if supplier } f \text{ is not selected for suppliying} \\ 1, & \text{if supplier } f \text{ is selected for suppliying} \end{cases}, \quad \forall f \in \mathcal{F}$$

X_{pits} volume of product p produced by plant i during period of time t over scenario s, $\forall p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}$
Y_{rfits}^0 volume of raw material r transported from supplier f to plant i during period of time t over scenario s, $\forall r \in \mathcal{R}, f \in \mathcal{F}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}$
Y_{pijts}^1 volume of product p transported from plant i to DC j during period of time t over scenario s, $\forall p \in \mathcal{P}, i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S}$
Y_{pjkts}^2 volume of product p transported from DC j to customer k during period of time t over scenario s, $\forall p \in \mathcal{P}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}, s \in \mathcal{S}$
I_{pjts} level of inventory of product p on DC j on period t over scenario s, $\forall p \in \mathcal{P}, j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S}$
C_f^0 capacity used for supplier f, $\forall f \in \mathcal{F}$
C_i^1 capacity installed for plant i, $\forall i \in \mathcal{I}$
C_j^2 capacity utilized for DC j, $\forall j \in \mathcal{J}$

Objective functions

o_1 first objective function
o_2 second objective function
Bal_{ts} cash flow for each period t at scenario s, $\forall t \in \mathcal{T}, s \in \mathcal{S}$
$Profit_s$ present value of the cash flows for each scenario s, $\forall s \in \mathcal{S}$

Other functions

$Risk(x, \Lambda)$ financial risk of design x at a target level Λ
$DRisk(x, \Lambda)$ downside risk of design x at a target level Λ
$Profit(x)$ profit of design x
$f(x, \Lambda)$ profit pdf
$z_s(x, \Lambda)$ probability of profit of design x being under target level Λ
$\delta_s(x, \Lambda)$ positive deviation from target level Λ for design x under scenario s

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