Q.no.1

(i) True:-

A square is a specific type of rectangle, so there is at least one figure (a square) that satisfies both properties.

(ii) True:-

There are many rectangle (like a rectangle with unequal sides) that are not squares.

(iii) True:-

By definition, every square is a rectangle.

Q no. 2

Let,

P : 12 divides 709,438

Q : 3 divides 709,438

R : The sum of the digits of 709,438 is divisible by 9.

The argument can be expressed as:

1. $P \to Q$
2. $R \to Q$
3. $\sim R$
4. Therefore, $\sim P$

P	R	Q	P→Q	R→Q	~R	~P
T	T	T	T	T	F	F
T	T	F	F	F	F	F
T	F	T	T	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	F	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

The argument is invalid. The truth table shows that it's possible for both premises to be true and the conclusion false (e.g. P=F, R=F, Q=F).

Q.3.

1. If P: 54,587 is a prime number, then Q: 17 is not a divisor of 54,587.

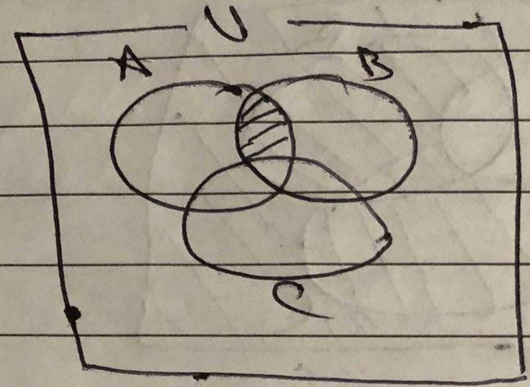
2. Q: 17 is a divisor of 54,587

3. Therefore ~P: 54,587 is not a prime number.

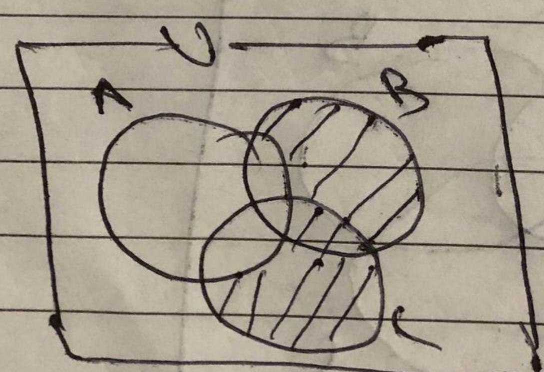
This is a classic form of the fallacy known as "denying the antecedent" which is invalid because even if second premise true it doesnot necessarily mean the first premise is false.

4)

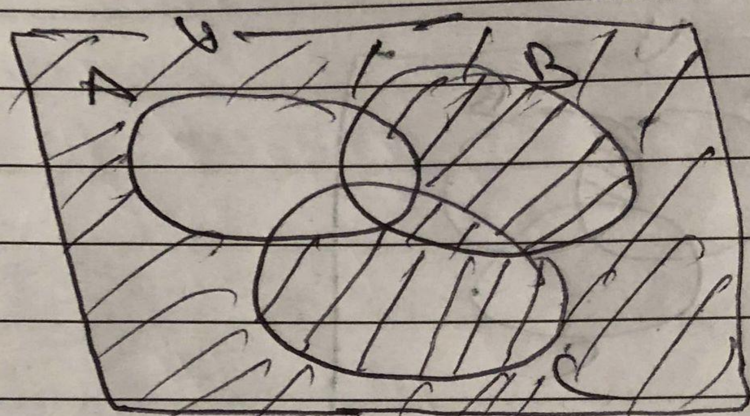
a) $A \cap B$



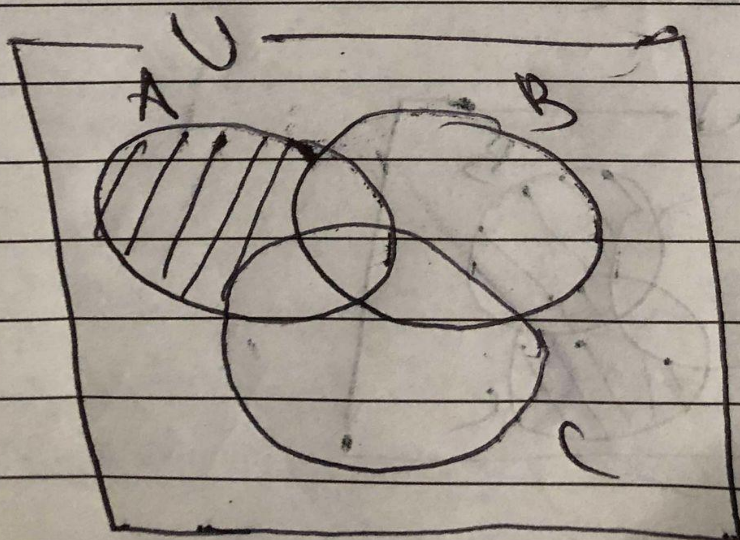
b) $B \cup C$



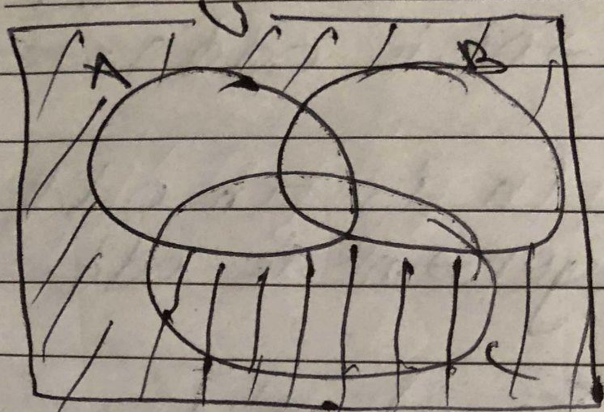
(c) A'



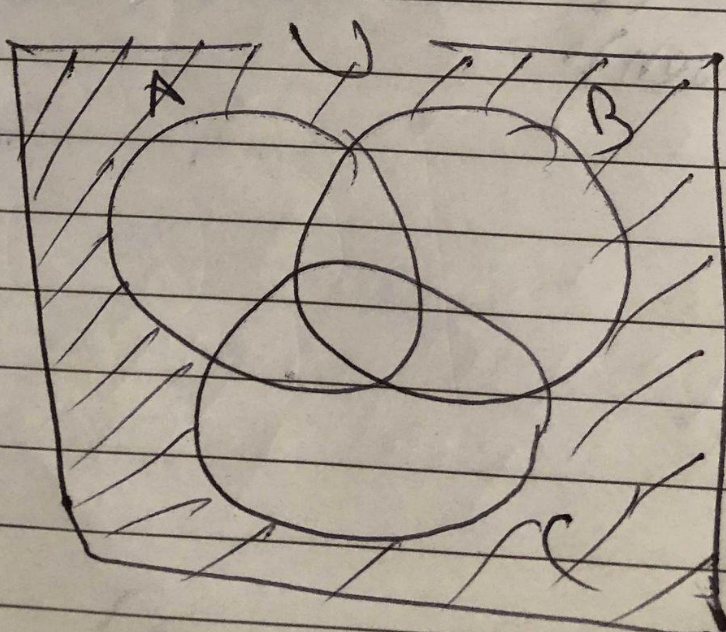
(d) $A - (B \cup C)$



6 $(A \cup B)'$



7 $A^c \cap B^c$



(3)

(i) $\forall n$ (n is an odd integer $\to n^2$ is odd).

(ii) $\forall n$ (n is an odd integer $\to n^2$ is odd.

(iii) The negation:

There exists an odd integer n such that n^2 is not odd (i.e n^2 is even).

Q.no.6.

Statement:

Every rational number can be written as a ratio of some two integer.

Formal version:-

$\forall q$ (q is a rational number $\rightarrow \exists a \exists b (a, b \in \mathbb{Z} \land b \neq 0 \land q = a/b)$)

Q.7
logical equivalence

Truth table

p	q	r	$q \to r$	$p \to (q \to r)$	$(p \to q) \to r$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	T	T	F

The statement
$p \to (q \to r)$ and $(p \to q) \to r$ are not logically equivalent

Q no. 8.

(a) $A \subseteq B$

Disprove:

Take an example, $x=4$ is in A, but $4 \bmod 18 = 4$, not in B.

(b) $B \subseteq A$:

Disprove:

Take an example, $y=2$ is in B, but y is not A ($2 \bmod 6 = 2$).

(c) $B = C$ (Disprove)

$B = y = 18b + 2$

$C = z = 18c + 16$

$b - c = \frac{14}{18} = \frac{7}{9}$

Since, $b-c$ cannot equal $\frac{7}{9}$ (as both b and c must be integer), we cannot have a common integer soln. $B \neq C$.

Q.no.9

a Ans

There exists an integer d such that 6/d is an integer and $d \neq 3$.

b Ans

There exists a function that is differentiable but not continuous.