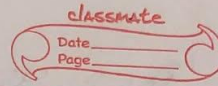


Home work - 7

Problem 53 on page 237



Given,

Population mean  $\mu = 50$ Population standard deviation  $\sigma = 12$ 

Sample sizes:

 $n = 9$  for a part (a) $n = 40$  for part (b)

we need

$$P(\bar{X} \geq 51)$$

Step 1: Standard Error of the Mean:

$$SE = \frac{\sigma}{\sqrt{n}}$$

for  $n = 9$ 

$$SE = \frac{12}{\sqrt{9}} = 0.4$$

for  $n = 40$ 

$$SE = \frac{12}{\sqrt{40}} = 0.1898$$

Step 2: Convert to standard Normal form

we have,

 $\bar{X} = 51$  using Z-score formula:

$$Z = \frac{\bar{X} - \mu}{SE}$$

for  $n = 9$ 

$$Z = \frac{51 - 50}{0.4} = 2.5$$

$$P(\bar{X} \geq 51) = P(Z \geq 2.5)$$

Standard normal table

$$P(Z \leq 2.5) = 0.9938$$

$$P(Z \geq 2.5) = 1 - 0.9938 = 0.0062$$

For  $n = 40$

$$Z = \frac{51 - 50}{0.1898} = 5.27$$

$$P(X \geq 51) = P(Z \geq 5.27)$$

from the standard table:-

$$P(Z \geq 5.27) \approx 0$$

$$Z = 5$$

So,

$$(a) \quad P(X \geq 51) \approx 0.0062$$

$$(b) \quad P(\bar{X} \geq 51) \approx 0$$



2. Problem 6 on page 285

Given,

Population standard deviation  $\sigma = 100$

Sample size  $n = 25$

Sample mean  $\bar{X} = 8439$

Confidence level 90% for part a  
and 92% for part b

Step 1: Compute Standard Error (SE)

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{25}} = 20$$

Step 2: Find the Z-Score for 90% CI  
for 90%.

The critical value  $Z_{\alpha/2}$

$$Z_{0.05} = 1.645$$

Since,

sr. the probability is in each tail  
for total of 10% outside the interval

Step 3: Compute the Confidence Interval

The 90% confidence interval.

$$\bar{X} \pm Z_{\alpha/2} \times SE$$

$$8439 \pm 1.645 \times 20$$

$$8439 \pm 32.9$$

$$(8406.1, 8471.9)$$

classmate

Date \_\_\_\_\_

Page \_\_\_\_\_

step 4:

Modify for a 92% Confidence Interval

for 92% confidence level,

from standard table:-

for 92%,  $\alpha$ 

$$Z_{\alpha/2} = Z_{0.04} = 1.75$$

$$8439 \pm 1.75 \times 20$$

$$(8404, 8474)$$

So,

92% confidence interval is

$$(8404, 8474)$$



3. Problem 17 on page 293

Given data,

Sample size  $n = 153$

Sample mean  $\bar{x} = 135.59$

Sample standard deviation  $s = 4.59$

Standard error of the mean  $SE = 0.37$

Confidence level: 99%.

Step 1: Find the  $t$ -score for 99% lower confidence bound

$t$ -distribution with  $df = n - 1$   
 $= 152$

$t_{0.01, 152} \approx 2.33$

Step 2: Compute the lower confidence bound

The 99% lower confidence bound

$$L = \bar{x} - t_{\alpha} \cdot SE$$

$$= 135.59 - (2.33 \times 0.37)$$

$$= 134.33$$

Step 3: Interpretation

With 99% confidence, true ultimate strength at least 134.33 ksi.

This means that if we repeatedly take random sample size 153 at least 99% of the time the true mean tensile strength will be greater than 134.33 ksi.

4. Problem 19 on Page 294

Given,

Sample size:  $n = 356$

Number of successes (dies that passed)  $x = 201$

Sample proportion

$$\hat{p} = \frac{x}{n} = \frac{201}{356} \approx 0.564$$

Confidence level: 95%

Standard normal critical value for 95% for CI

$$Z_{0.025} = 1.96$$

Step 1: Compute Standard Error

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ &= \sqrt{\frac{(0.564)(1-0.564)}{356}} \\ &= 0.0263 \end{aligned}$$

Step 2: Compute the Confidence Interval Margin of error

$$\begin{aligned} ME &= Z_{\alpha/2} \times SE \\ &= 1.96 \times 0.0263 \\ &= 0.0516 \end{aligned}$$

The confidence interval (CI)

$$\hat{p} \pm ME$$

$$0.564 \pm 0.0516$$

$$(0.512, 0.616)$$

Step 3: Interpretation:

With 95% confidence, true proportion of all dies bet<sup>n</sup> 51.2% and 61.6%. This means that if repeatedly take random sample of 356 dies about 95% of these sample would contain the true proportion in this range.