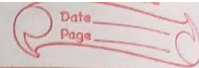Problem 59

on Page 84

Soln.

Regular gas (A_1):
$$P(A_1) = 0.40$$

Plus gas (A_2)
$$P(A_2) = 0.35$$

Premium gas (A_3):
$$P(A_3) = 0.25$$

Filling the tank :-
- Regular: $P(B|A_1) = 0.30$
- Plus: $P(B|A_2) = 0.60$
- Premium: $P(B|A_3) = 0.50$

→ Ans Probability of Selecting plus Gas and Filling the Tank :-

$$P(A_2 \cap B) = P(A_2) \times P(B|A_2)$$
$$= 0.21$$

bave Probability that the Next Customer Fill the Tank

$$P(B) = P(A_1) P(B|A_1) + P(A_2) P(B|A_2)$$
$$+ P(A_3) P(B|A_3)$$

$$= 0.12 + 0.21 + 0.125$$
$$= 0.435$$

CA4

Premium Gross!

Conditional Probabilities Given That the Tank is Filled:

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

Regular Gross:-

$$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)}$$

$$= \frac{0.40 \times 0.30}{0.435}$$

$$= 0.2759$$

Plus Gross $(A_2|B)$:

$$P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{0.21}{0.435}$$

$$= 0.4828$$

Premium Gross $(A_3|B)$:

$$P(A_3|B) = \frac{P(A_3 \cap B)}{P(B)}$$

$$= \frac{0.125}{0.435} = 0.2874$$

Problem 64 page 84

Given Data:-

Short (S) $P(S) = 0.60$
Medium (M) $P(M) = 0.30$
Long (L) $P(L) = 0.10$

Probability of format given length:-

$P(W|S) = 0.80$
$P(W|M) = 0.50$
$P(W|L) = 0.30$

a)

Probability that a randomly selected review was submitted in Word:-

$P(W) = P(W|S) P(S) + P(W|M) P(M) + P(W|L) P(L)$

$= 0.48 + 0.15 + 0.03$
$= 0.66$

b)
Posterior probabilities of the review being short, Medium or Long given that it was:-
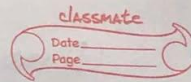
$P(S|W) = \dfrac{P(W|S) P(S)}{P(W)} = 0.727$

$P(M|W) = \dfrac{P(W|M) P(M)}{P(W)} = 0.227$

$P(L|W) = \dfrac{P(W|L) P(L)}{P(W)} = 0.045$

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Problem 74

Given,

$P(A) = 0.40$

$P(B) = 0.11$

$P(AB) = 0.04$

$P(O) = 0.45$

a) Ans

Probability that both individual have blood type O:-

$P(O \text{ and } O) = P(O) \times P(O)$

$= 0.205$

b) Ans

Probability that both individual have the same blood type.

$P(A \text{ and } A) + P(B \text{ and } B) + P(AB \text{ and } AB) + P(O \text{ and } O)$

$= 0.16 + 0.0121 + 0.0016 + 0.2025$

$= 0.3762$

Q.no. 4   Page 98

Given,
   4- digit PIN (randomly)
Digits (0-9)
   X - represents no. of nonzero
       digits.

Possible value of X

  $X \in \{0, 1, 2, 3, 4\}$

So,
   $X = 0 \rightarrow$ (0000)
   $X = 1 \rightarrow$ (0004)
   $X = 2 \rightarrow$ (0303)
   $X = 3 \rightarrow$ (9021)
   $X = 4 \rightarrow$ (5621)

Three possible outcomes

  PIN = 0000 $\rightarrow X = 0$
  PIN = 3050 $\rightarrow X = 2$
  PIN = 8742 $\rightarrow X = 4$