

Homework - 4

1 problem 2 on page 38

* Three Examples of Bernoulli Random Variable *

1. Website click-through:

- Random Variable (X) : A user clicks on an advertisement (1) or does not click (0).
- Success $(X=1)$: The user clicks the ad.
- Failure $(X=0)$: The user ignores the ad.

2. Student ~~Att~~ Attendance in class:

- Random Variable (X) : A randomly chosen student ~~and~~ attends class (1) or does not attend (0).
- Success $(X=1)$: The student is present.
- Failure $(X=0)$: The student is absent.

3. Machine Defect Check:

- Random Variable (X) : A product coming off a production line is defective (1) or not defective (0).

- Success $(X=1)$: The product is defective.
- Failure $(X=0)$: The product is in good condition.

Problem 15 on pages 107-108

a) Ans

The list of different possible outcomes:-

(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)

b) Ans

Possible values of X

$X=0$ (no defective)

$X=1$ (one defective)

$X=2$ (Both defective boards selected)

Counting Outcomes:

$X=0$

~~Possible~~ Possible Pairs (3,4), (3,5), (4,5)

$X=1$

(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)

$X=2$

(1,2)

Possible distribution of X

X	0	1	2
$P(X)$	0.3	0.6	0.1

c) Ans ~~the~~ Cumulative Distribution Function is

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.3 & 0 \leq x < 1 \\ 0.9 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

Problem 32 on page 116

Given

$$p(x) = \begin{cases} 0.2 \\ 0.5 \\ 0.3 \end{cases}$$

or we

compute $E(x)$, $E(x^2)$ and $V(x)$

$$\begin{aligned} \text{Expected Value } E(x) &= \sum x p(x) \\ &= 16 \times 0.2 + 18 \times 0.5 + 20 \times 0.3 \\ &= 3.2 + 9 + 6 \\ &= 18.2 \end{aligned}$$

$$\begin{aligned} \text{Expected Value of } x^2, \quad E(x^2) &= \sum x^2 p(x) \\ &= 333.2 \end{aligned}$$

$$\begin{aligned} \text{Variance } V(x) &= E(x^2) - [E(x)]^2 \\ &= 333.2 - (18.2)^2 \\ &= 1.96 \end{aligned}$$

or we Expected Price Paid by next customer

$$p(x) = 70x - 2650$$

$$E(p) = E(70x - 2650)$$

$$= 70E(x) - 2650$$

$$E(x) = 18.2$$

$$= 1070$$

Ans

Variance of the Price Paid by the Next Customer

$$V(ax + b) = a^2 V(x)$$

where

$$a = 70 \text{ and } b = -2650$$

$$V(P) = 70^2 V(x) \\ = 9804$$

Ans

Expected Actual Capacity of the Freezer

$$h(x) = x - 0.008x^2$$

$$E(h(x)) = E(x - 0.008x^2) \\ = E(x) - 0.008E(x^2)$$

using previous

$$E(h(x)) = 18.2 - 0.008(333.2) \\ = 15.5344 \text{ ft}^2$$

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$$p = 0.1$$

$$q = 1 - p \\ = 0.9$$

$$X \sim \text{Bin}(n=6, p=0.1)$$

Q11

Probability that ~~exactly~~ exactly one goblet

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for $k=1$

$$P(X=1) = \binom{6}{1} (0.1)^1 (0.9)^5 \\ = 0.3543$$

Q12

Probability that at least two goblets

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - 0.5314 - 0.3543 \\ = 0.1143$$

Problem 13 on Page 107

Given:

$$X = 0, 1, 2, 3, 4, 5, 6$$

$$P(X) = 0.1, 0.15, 0.20, 0.25, 0.06, 0.04$$

a) Ans

$$\begin{aligned} P(X \leq 3) &= P(0) + P(1) + P(2) + P(3) \\ &= 0.10 + 0.15 + 0.25 \\ &= 0.50 \end{aligned}$$

b) Ans

$$\begin{aligned} P(X < 3) &= P(0) + P(1) + P(2) \\ &= 0.10 + 0.15 + 0.20 \\ &= 0.45 \end{aligned}$$

c) Ans

$$\begin{aligned} P(X \geq 3) &= P(3) + P(4) + P(5) + P(6) \\ &= 0.25 + 0.20 + 0.06 + 0.04 \\ &= 0.55 \end{aligned}$$

e) Ans

$$\begin{aligned} P(2 \leq X \leq 4) &= P(2) + P(3) + P(4) \\ &= 0.20 + 0.25 + 0.20 \\ &= 0.65 \end{aligned}$$

d) Ans

$$\begin{aligned} P(2 \leq X \leq 5) &= P(2) + P(3) + P(4) + P(5) \\ &= 0.20 + 0.25 + 0.20 + 0.06 \\ &= 0.71 \end{aligned}$$

f) Ans

$$\begin{aligned} P(X \leq 2) &= P(0) + P(1) + P(2) \\ &= 0.10 + 0.15 + 0.20 \\ &= 0.45 \end{aligned}$$