Homework - 5

Problem 48 on Page 123

Soln,

$X \sim Bin(25, 0.05)$

$n = 25$ (no. of trails

$p = 0.05$ (probability of success per tri

binomial probability formula:-

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

a Ans

$$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$$

$$P(X < 3) = P(0) + P(1) + P(2)$$

b Ans

$$P(X \geq 4) = 1 - P(X \leq 3)$$

c Ans

$$P(1 \leq X \leq 3) = P(1) + P(2) + P(3)$$

d Ans

$$E(X) = np = 25 \times 0.05 = 1.25$$

$$\sigma_X = \sqrt{np(1-p)}$$
$$= \sqrt{1.1875} \simeq 1.09$$

e Ans

$$P(X=0) = (1-p)^{50}$$
$$= (0.95)^{50}$$

Problem 5 on page 146-147

Soln,

Given,

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Ans

The value of k

$$\int_0^2 kx^2\,dx = 1$$

$$k\int_0^2 x^2\,dx$$

$$= k\left[\frac{2^3}{3} - \frac{0^3}{3}\right]$$

$$= k\left[\frac{8}{3}\right] = 1$$

$$\therefore k = \frac{3}{8}$$

Now,

The pdf is

$$f(x) = \begin{cases} \frac{3}{8}x^2, & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

b. Probability that the lecture ends within 1 min

$$P(0 \le X \le 1) = \int_0^1 \frac{3}{8}x^2\,dx$$

$$= \frac{3}{8}\left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{8}$$

Thus,

$$P(0 \le X \le 1) = \frac{1}{8} = 0.125$$

c) Ans Probability that the lecture continues between 60 and 90 seconds

$$P(1 \le X \le 1.5) = \int_1^{1.5} \frac{3}{8}x^2\,dx$$

$$= \frac{3}{8}\left[\frac{x^3}{3}\right]_1^{1.5}$$

$$= 0.297$$

d) Ans Probability that the lecture continues for at least 90 seconds.

$$P(X \ge 1.5) = \int_{1.5}^2 \frac{3}{8}x^2\,dx$$

$$= \frac{3}{8}\left[\frac{x^3}{3}\right]_{1.5}^2$$

$$= 0.578$$

3. Problem 11 (a) (b) (c) on page 154

Given,

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

a Ans

$$P(X \le 1) = F(1) = \frac{1^2}{4} = \frac{1}{4}$$

$$= 0.25$$

b Ans

$$P(0.5 \le x \le 1) = F(1) - F(0.5)$$

$$= \frac{1^2}{4} - \frac{(0.5)^2}{4}$$

$$= 0.1875$$

c Ans

$$P(X > 1.5) = 1 - P(X \le 1.5)$$

$$= 1 - F(1.5)$$

$$= 0.4375$$

Problem 13 (a)(b)(c) on page 155

$$f(x) = \begin{cases} \frac{k}{x^4}, & x > 1 \\ 0 & x \leq 1 \end{cases}$$

ans

$$\int_1^\infty f(x)\,dx = 1$$

$$\int_1^\infty \frac{k}{x^4}\,dx = 1$$

$$k\int_1^\infty x^{-4}\,dx$$

$$= k\left[\frac{x^{-3}}{-3}\right]_1^\infty$$

$$= k\left[0 - \frac{1^{-3}}{-3}\right]$$

$$= k \times \frac{1}{3} = 1$$

$$\therefore k = 3$$

pdf is: $$f(x) = \begin{cases} \frac{3}{x^4}, & x > 1 \\ 0 & x \leq 1 \end{cases}$$

b) Ans CDF

$$F(x) = P(X \le x) = \int_1^x f(t)\,dt$$

$$= \int_1^x 3t^{-4}\,dt$$

$$= 3\left[\frac{t^{-3}}{-3}\right]_1^x$$

$$= 3 \times \left[-\frac{1}{3x^3} + \frac{1}{3}\right]$$

$$= 1 - \frac{1}{x^3}, \quad x > 1$$

Thus, cumulative distribution function is

$$F(x) = \begin{cases} 0 & x \le 1 \\ 1 - \frac{1}{x^3} & x > 1 \end{cases}$$

c) Ans

$$P(X > 2) = 1 - P(X \le 2) = 1 - F(2)$$

$$= 0.125$$

$$P(2 \le X \le 3)$$

$$= F(3) - F(2)$$

$$= \frac{208}{216} - \frac{189}{216}$$

$$= 0.08796$$