

EPPS problem 5.10.15 / P273

$$\textcircled{3} \quad \begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \left| \begin{array}{r} -1 \\ 2 \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \right. \begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

$$\begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \left| \begin{array}{r} -1 \\ 2 \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \right. \begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

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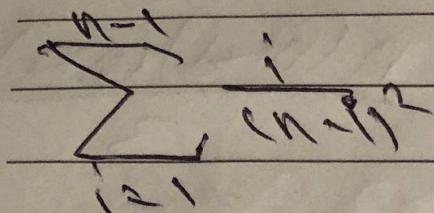
$$g_3 = \begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \left| \begin{array}{r} -1 \\ 2 \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \right. \begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

$$g_n = \begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \left| \begin{array}{r} -1 \\ 2 \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \right. \begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

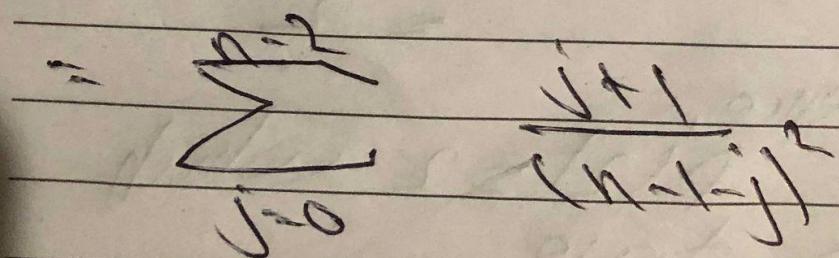
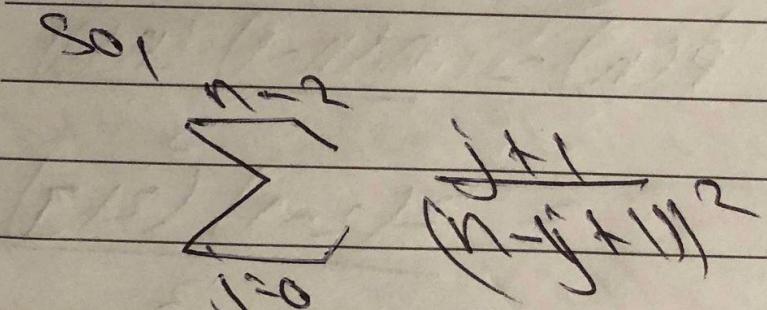
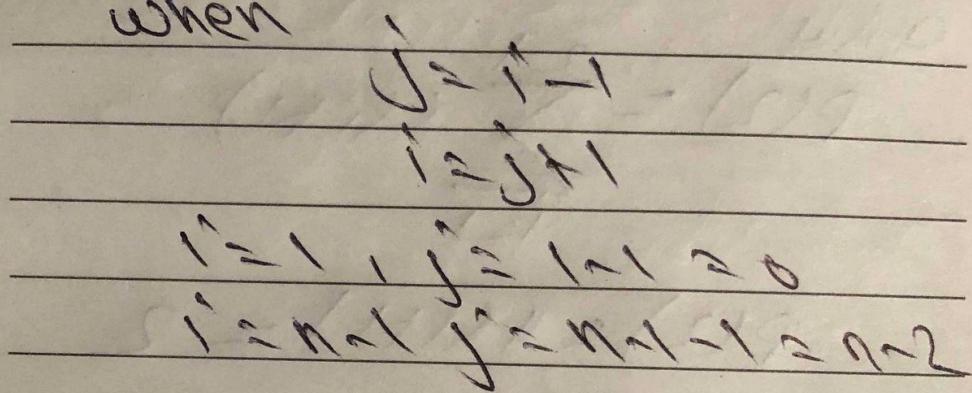
where n is an integer

$$g_n \neq 0 \quad n \geq 0$$

2. EPPS problem 5.1.57 / P274



When



3. Epps problem 5.2.4 (p28)

$$\sqrt{\frac{P(1)}{P(1)}} = \frac{n(n-1)(n-2)}{3}$$

$$P(2) = \sqrt{\frac{P(1)}{P(1)}}$$

$$P(2) = 1/(n) - 2$$

$$P(n) = \frac{n(n-1)(n-2)}{3}$$

$$= \frac{2 \cdot 1 \cdot 0}{3} \quad 3$$

Hence,

$P(2) = 2$ which

$$\textcircled{6} \quad P(K) = \frac{K_1}{[i(i_1) - K_1 K_2]} \quad \boxed{3}$$

$$\textcircled{7} \quad P(K_1)$$

$$\textcircled{8} \quad P(K_1) = \frac{K_1}{[i(i_1) - K_1 K_2]} \quad \boxed{3}$$

$$= \frac{K_1}{[i(i_1) - K_1 K_2]} \quad \boxed{3}$$

$$P(K_1) = \frac{K_1}{[i(i_1) - K_1 K_2]} \quad \boxed{3}$$

$$= \frac{K_1}{[i(i_1) - K_1 K_2]} \quad \boxed{3}$$

$$\therefore P(K_1) = \frac{K_1}{[i(i_1) - K_1 K_2]} \quad \boxed{3}$$

① Mathematical induction,

Base case:

$P(1) = 2$ true

Hypothesis:

$$P(k) \quad | \quad k \geq 2$$

$$\text{if } i \leq k \Rightarrow P(i) \quad | \quad i \leq k$$

Induction:

$$P(k+1) \quad | \quad k \geq 2$$

$$P(k+1) \quad | \quad \begin{cases} P(k) \\ P(k+1) = P(k) + P(k+2) \end{cases}$$

inductive step:

$$P(KN) \rightarrow \begin{cases} X \\ Y \\ Z \end{cases}$$

split:

$$P(KN) \rightarrow \begin{cases} X \\ Y \\ Z \end{cases} \rightarrow \begin{cases} X(K) \\ Y(K) \\ Z(K) \end{cases}$$

$$\begin{cases} X \\ Y \\ Z \end{cases} \rightarrow \begin{cases} X(K) \\ Y(K) \\ Z(K) \end{cases}$$

$$P(KN) \rightarrow \begin{cases} X(K) \\ Y(K) \\ Z(K) \end{cases}$$

$$P(KN) = X(KN) \begin{pmatrix} K \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \rightarrow \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \rightarrow \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$P(KM) = K(KM) \times KM$$

∴

$$P(KM) = (KM) KM$$

Prove by induction

EPPS problem 5.2.12/p287

(2)

$$\frac{1}{1} \frac{1}{2} \frac{1}{3} \frac{1}{4} \dots \frac{1}{n} \frac{1}{n+1} = 0$$

Base: for every integer $n \geq 1$

$$n \geq 1$$

Left hand side

$$\frac{1}{1} \frac{1}{2} \frac{1}{3}$$

Right hand side = $\frac{1}{1} \frac{1}{2} \frac{1}{3}$

Hypothesis

$$\frac{1}{1} \frac{1}{2} \frac{1}{3} \dots \frac{1}{k}$$

$$\frac{1}{1} \frac{1}{2} \frac{1}{3} \dots \frac{1}{k} + \frac{1}{k+1} = \frac{1}{1} \frac{1}{2} \frac{1}{3} \dots \frac{1}{k+1}$$

Inductive Step:

11 11 11 11 11 11 11 11 11 11 11 11

Giving hypothesis

$$\begin{array}{cccc} t_1 & t_2 & t_3 & t(t+1) \\ \text{left-hand side becomes} & & & \\ t_1 + t_2 & & & (t+1)(t+2) \end{array}$$

$$\frac{1}{(k_1 k_2)} \cdot \frac{1}{(k_1 k_2)} = \frac{1}{(k_1 k_2)^2}$$

$$= \frac{(RT_2)Y_1}{(C_1)(RT_2)}$$

$$\frac{x^2 + 2x + 1}{(x+1)(x+2)}$$

$$\frac{(x+1)^2}{(x+1)(x+2)}$$

$$\frac{x+1}{x+2}$$

Prop:

D.S. Epps problem 5.2.27/p288

$s^3 + s^4 + s^5 + \dots + s^k$ where
 k is any integer $k \geq 3$

$$s^3 + s^4 = 125$$

$$s^3 = 5$$

$$s^3 = \frac{s \times s^n - 1}{s - 1}$$

$$= 125 + \frac{s^{n-1} - 1}{s - 1}$$

The

$$s^3 + s^4 + s^5 + \dots + s^k = 125 \times \frac{s^{k-2} - 1}{s - 1}$$

E. Epps problem 5.3.17 p28

$$\textcircled{2} \quad 1 + 3n \leq 4^n \text{ for } n \geq 0$$

Base

$$n = 0$$

$$1 + 3^0 \leq 4^0$$

$$1 \leq 1$$

Hypothesis:

$$1 + 3k \leq 4^k \quad \text{for } k \geq 0$$

Inductive:

$$1 + 3(k+1) \leq 4^{k+1}$$

$$1 + 3k + 3 \leq 4^{k+1}$$

From hypothesis $\textcircled{1}$

$$4^k + 3 \leq 4^{k+1}$$

$$4^k + 3 \leq 4^k \cdot 2$$

Done.

B7. Cpp5 problem 5.3.23/p298

P. $n^3 > 2n+1$ for $n \geq 2$

Base

$$n = 2$$

$$2^3 > 2(2) + 1$$

$$8 > 5 \quad (\text{true})$$

hypothesis

$$n > k$$

$$k^3 > 2k+1$$

Inductive

$$n > k$$

$$\text{left side, } (k+1)^3 > 2(k+1)+1$$

$$(k+1)^3$$

$$= (k^3 + 3k^2 + 3k + 1)$$

$$= k^3 + 3k^2 + 3k + 1$$

$$= k^3 + 2k^2 + k + 1$$

$$= k^3 + 3k^2 + 3k + 3$$

$$k^3 + 3k^2 + 3k + 1 > 2k^3$$

$$k^3 + 3k^2 + 3k + 1 - (2k^3) > 0$$

$$k^3 + 3k^2 + k - 2 > 0$$

$$k^2$$

$$2^3 + 3 \cdot 2^2 + 2 - 2$$

$$= 26$$

$$\frac{2}{16}$$

$$26 > 0$$

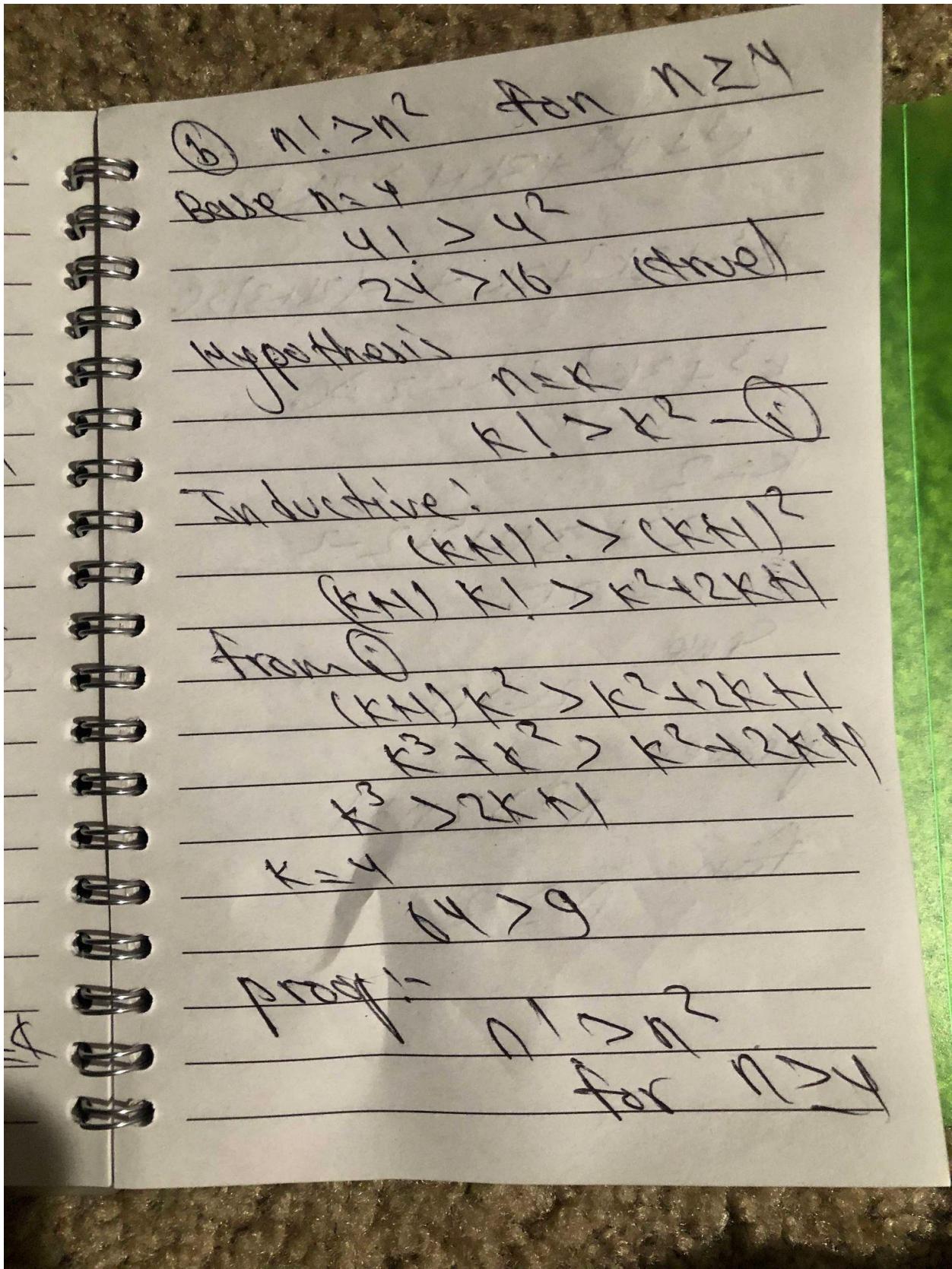
$$P_{\text{ave.}}$$

$$k^3 + 2k^2 + 1$$

$$k^2$$

$$2^3 + 2 \cdot 2^2 + 1$$

27



8. Epps problem 54.21 p31

$$g_1 = 3, g_2 = 5$$

$$g_k = 3g_{k-1} - 2g_{k-2}$$

for $k \geq 3$

$$g_k = 2g_{k-1} + 2g_{k-2}$$

for $k \geq 3$

base:

$$g_1 = ?$$

$$g_2 = 03$$

$$= 3$$

$$g_1 = ?$$

$$g_1 = 5$$

$$= 1$$

if other

$$g_k = 2^k$$

$$= k$$

strong induction!

$$g_{n+1} = 2g_n + 1$$

$$g_{n+1} = 3g_n - 2g_{n-1}$$

So, Using hypothesis

$$g_n = 2^n + 1$$

$$g_{n-1} = 2^{n-1} + 1$$

$$g_{n+1} = 3g_n - 2g_{n-1}$$

$$= 3(2^n + 1) - 2(2^{n-1} + 1)$$

$$g_{n+1} = 3 \cdot 2^n - 2 \cdot 2^{n-1} + 1$$

Prove

J. Epps problem 5.4.20/ P3R

Given

$$b_1 = 0 \quad b_2 = 3$$

$$b_k = 5.b_{k-2} + 6 \quad \text{for } k \geq 3$$

Prove

b_n is divisible by 3, $\forall n$

Base: $n=1$

$$b_1 = 0 \\ b_2 = 3$$

Hypothesis:

$$1 \leq i \leq k$$

Stay Inductive

$$b_{k+1} = 5.b_k + 6$$

$$b_k$$

$$b_{k+1} = 5.3m + 6$$

$$b_{k+1} = 15m + 6$$

$$\text{Since, } b_{k+1} = 3(5m+2)$$

Since,

$$b_{k+1} = 3 \times \text{integer}$$

Divisible by 3 $\forall n \geq 1$

10. EPPS problem 5.68/0387

$$\sqrt{r} = \sqrt{r_1} - \sqrt{r_2} \quad \text{for } r \geq 3$$

$$\sqrt{r_1} = \sqrt{16} - \sqrt{3}$$

$$\sqrt{r_2} = \sqrt{21} - \sqrt{1}$$

$$\sqrt{r_3} = \sqrt{25} - \sqrt{9}$$

$$\sqrt{r_4} = \sqrt{25} - \sqrt{13} + 1 = 8$$

first four terms:

$$\sqrt{r_1} =$$

$$\sqrt{r_2} =$$

$$\sqrt{r_3} =$$

$$\sqrt{r_4} = 9$$