1. Epps problem 7.1.16 | 436

Soln,

Given two function F and G from set's of all real number to itself.

Difference function

$F - G$ and $G - F$

- for every $x \in R$, $(F-G)(x) = F(x) - G(x)$
- for every $x \in R$, $(G-F)(x) = G(x) - F(x)$

Comparision

$$F - G = G - F$$

or, $(G-F)(x) = G(x) - F(x)$

$= -F(x) - G(x)$

$= -(F-G)(x)$

So,

$(G-F)(x) = -(F-G)(x)$ for every $x \in R$

So,

$(F-G)(x) \neq (G-F)(x)$

[unless $f(x) = G(x)$ for all x, in which case both are zero functions], the functions $F-G$ and $G-F$ are not equal.

Therefore, $F - G \neq G - F$ in general.

2. Epp5 problem 7.2.37 | 460

Soln,

Yes, f + g is onto if both f and g are onto.

Justification:-

Let

$f: R \to R$ and $g: R \to R$ be onto functions.

- $f(x_1) = y$
- $g(x_2) = y$

$(f+g)(x) = f(x) + g(x)$.

for real no. y,

$(f+g)(x) = y$

Since,

f and g are onto, for any y, there exist x_1 and x_2

- $f(x_1) = y_1$
- $g(x_2) = y_2$

Counter example

let $f(x) = x$ and $g(x) = -x$, which are both onto because the range of $f + g$ is $\{0\}$, not R

Even if f and g are both onto, $f + g$ is not necessarily onto.

3. Epp5 problem 7.3.4 | 471

$f(x) = x^5$
$G(x) = x^{1/5}$

$(G \circ f)(x) = G(f(x)) = G(x^5) = (x^5)^{1/5}$
$= x$

$(G \circ f)(x) = x$

So,
$(f \circ G) = x$
so, $G \circ f = f \circ G$

4. Epps problem 8.1.7 b, c, d | 493

b. Is 2 R 13?

$2^2 - 13^2$

$= -165$

Since,

5 divides -165 i.e $5 \times (-33)$

Yes, 2 R 13

c) Is 2 R (-8)?

$2^2 - (-8)^2 = 4 - 64$

$= -60$

$[5 \times (-12)]$

Yes,

d) Is (-8) R 2?

→ Yes, (-8) R 2

because,

$(-8)^2 - 2^2$

$= 64 - 4$

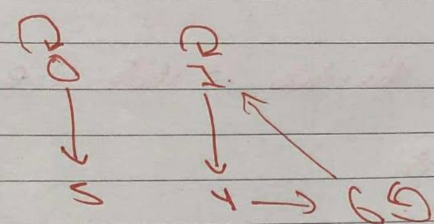
$= 60$

$= 5 \times 12$

5. Epps problem 8.1.18 1494

$A = \{0, 1, 3, 4, 5, 6\}$

$x \vee y \Leftrightarrow 5 \mid (x^2 - y^2)$



0V0
0V5
1V1
1V4
3V3
4V1
4V4
4V6
5V0
5V5
6V1
6V4
6V6

6. Epps problem 8.2.13 | S03

$m F n \Leftrightarrow 5 | (m-n)$

Reflexive: $m F m$ holds because $5 | (m-m) = 0$

Symmetric :- If $m F n$ meaning $5 | (m-n)$

then $n F m$ holds since $5 | (n-m)$

Transitive :- If $m F n$ and $n F p$ meaning $5 | (m-n)$ and $(n-p)$. then $5 | (m-p)$.

7. Epps problem 9.1.11 a,b (ii.iii) | 571

a Ans

{(H,H,H) HHT, HTH, HTT, THH, THT, TTH, TTT}

b Ans

$E_1 = \{HTT, THT, TTH\}$

$P(E_1) = \frac{\text{No. of favourable outcomes}}{\text{Total outcomes}}$

$= \frac{3}{8}$

(ii) $E_2 = \{HHH, HHT, HTH, THH\}$

$P(E_2) = \frac{\text{No. of favorable outcomes}}{\text{total outcomes}}$

$= \frac{4}{8} = \frac{1}{2}$

(iii) $P(E_3) = \frac{\text{No. of favorable outcomes}}{\text{Total outcomes}}$

$= \frac{1}{8}$

(8) Epps problem 9.5.5 c,d,e,f,g | 630

(c) $\binom{6}{2} = \frac{6!}{2!.(6-2)!} = 15$

(d) $\binom{6}{3} = \frac{6!}{3!.(6-3)!} = 20$

(e) $\binom{6}{4} = \frac{6!}{4!.(6-4)!} = 15$

(f) $\binom{6}{5} = \frac{6!}{5!.(6-5)!} = 6$

(g) $\binom{6}{6} = \frac{6!}{6!.1!} = 1$

9. Epps problem 9.5.7 (631)

(a) $\binom{13}{7} = \frac{13!}{7!(13-7)!}$

$= 1716$

(b) Choose 4 women from 7 and 3 men from 6

$\binom{7}{4} \cdot \binom{6}{3}$

$= 35 \cdot 20$

$= 700$

(i) P

$\binom{13}{7} = 1716$

$\binom{7}{7} = 1$

So,

$\binom{13}{7} - \binom{7}{7} = 1715$

(ii) $\binom{7}{0}\binom{6}{7} = 0$

$\binom{7}{1}\binom{6}{6} = 7$

$\binom{7}{2}\binom{6}{5} = 126$

$\binom{7}{3} \cdot \binom{6}{4} = 525$

total,

$0 + 7 + 126 + 525$

$= 656$

group with most three women.

men

c) $\binom{11}{5} = 462$

$\binom{13}{7} - \binom{11}{5}$

$= 1716 - 462$

$= 1254$

d) $\binom{11}{5}$

$= 462$

b. Epps problem 9.5.9.9/631

$n = 40$

$r = 6$

$\binom{n}{r}$

$= \binom{40}{6}$

$= 3838380$