CS 2200

Quiz 03

Fall 2024

Points: 20 points

Reference: Epp5 Sections: 4.1, 4.4, 4.8, 4.10, 6.2

1) Find a counterexample to show that the following statement is false:

For all nonzero real numbers, a, b, c and d,

a

$b +$

�

�

d

$=$

�

� $+ c$

$b + d$

2) Does 12 divide 72? Justify your answer.

3) Use the Euclidean algorithm to find the greatest common divisor of 284 and 168.

Show your work.

CS2200-Fall 24 Quiz 3

4) Epp5 problem 4.4.45 / 199

5) Epp5 problem 6.2.31 / 406

6) Use the element method for proving a set equals the empty set to

Prove that for all sets A and C,

$$(A - C) \cap (C - A) = \emptyset$$

7) Epp5 problem 4.1.22 / 172

8) Epp5 problem 4.4.42 b, c / 199

9) Epp5 problem 4.10.27 c / 256

Q.no.1

$$\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}$$

Let

$a=1$

$b=2$

$c=3$

$d=4$

So,

left side

$$\frac{1}{2}+\frac{3}{4}=\frac{5}{4}$$

Right side:

$$\frac{1+3}{2+4}=\frac{4}{6}=\frac{2}{3}$$

Since,

$\frac{5}{4}\neq\frac{2}{3}$, that shows

the statement is false

2. Yes, 12 does divide 72.

i.e,

$$72 \div 12 = 6$$

Since,
The result is whole no. (6).
by,

$$q = dr$$
$$72 = 12 \times 6$$

(2)

$168 \overline{)\,284\,}$ 1
168
116

$116 \overline{)\,168\,}$ 1
116
52

$52 \overline{)\,116\,}$ 2
104
12

$12 \overline{)\,52\,}$ 4
48
4

$4 \overline{)\,12\,}$ 3
12
0

GCD(284, 168) = 4

Q.4

Proof:-

Suppose n is a nonnegative integer whose decimal representation ends in 5.

$$n = 10m + 5$$

for some integer 'm'.

factor :-

$$n = 10m + 5$$
$$= 5(2m+1)$$

Here,

2m+1 is integer because m is an integer.

Since,

n ~~2m+1~~ can be expressed as 5 multiplied by (2m+1) we conclude that n is divisible by 5!

$5 | n$.

Q.5.

Let, U be a universal set.

Complement of U be U'

Now,

U' is not empty

means,

$$x \in U'$$

By definition

$$x \notin U$$

However, this leads to contradiction because x cannot be both in U' but still element of U.

Since,

This assumption leads to a contradiction, we conclude that assumption must be false.

$$U' = \phi.$$

8.

Let, x be an arbitrary element.

$x \in (A-C)$

$x \in (C-A)$

Now,

from this,

$x \in A$ and $x \notin C$

$x \in C$ and $x \notin A$

This leads to contradiction because x cannot be both in A and not in A likewise C

Since

Assume x element $(A-C) \cap (C-A)$ leads to a contradiction, we conclude that there cannot be any element x in the intersection.

$(A-C) \cap (C-A) = \phi$

proof.

Q.7. To prove $n^2 - n + 11$ = prime
where,
$1 \leq n \leq 10$

for,
$n=1 = 1^2 - 1 + 11 = 11$ (prime
$n=2 = 2^2 - 2 + 11 = 13$ (prime)
$n=3 = 3^2 - 3 + 11 = 17$ (prime)
$n=4 = 4^2 - 4 + 11 = 23$ (prime)
$n=5 = 5^2 - 5 + 11 = 31$ (prime)
$n=6 = 6^2 - 6 + 11 = 41$ "
$n=7 = 7^2 - 7 + 11 = 53$ "
$n=8 = 8^2 - 8 + 11 = 67$ "
$n=9 = 9^2 - 9 + 11 = 83$ "
$n=10 = 10^2 - 10 + 11 = 101$ "

Since,
$n^2 - n + 11$ produce prime number for each integer n from 1 to 10.
(True).

Q.P.
(b)

Standard factored form

$20! = 20 \cdot 19 \cdot 18 \cdot 17 \cdot \cdots 3 \cdot 2 \cdot 1$
which can be prime product

$20! = 2^{18} \cdot 3^{8} \cdot 5^{4} \cdot 7^{2} \cdot 11^{1} \cdot 13^{1} \cdot 17^{1} \cdot 19^{1}$

(c) So, the no. of trailing zeros in factorial is determined by the no. of times 10 is factore
So, $10 = 2 \cdot 5$ no. of trailing zeros is determined by the smaller counts of 2s and 5s in the factorization
(i) Count factor of 5 in 20!

$= \frac{20}{5} + \frac{20}{25}$

$= 4$

Q.no. 8.b

(ii) Count of factor of 2 in 20!

$= \frac{20}{2} + \frac{20}{4} + \frac{20}{8} + \frac{20}{16}$

$= 10 + 5 + 2 + 1$

$= 18$

Since,
there are more factors of 2 than of 5, the no. of trailing zeros in 20!

(d) Trailing zeros in 20! = 4

So,
Trailing zeros in $(20!)^2$

$= 2 \times 9$

$= 2 \times 4$

$= 8$

Q.no.9

lemma 4.10.3 states

$a \geq b > 0$,

$gcd(a, b) = gcd(b, a-b)$

Proof :-

① Divisibility properties :-

$d = gcd(a, b)$

mean,

$a = d \cdot k_1$ and $b = d \cdot k_2$

② Substituting second gcd

$a-b$

so, $a-b = d \cdot k_1 - d k_2$

$= d(k_1 - k_2)$

Thus,

$d' = gcd(b, a-b)$

So,

$d \leq d'$

The reverse

$d' = gcd(b, a-b)$

So, $a = (a-b) + b$

means, d' divides a.

$d' \leq d$

so, $d = d'$.

$gcd(a, b) = gcd(b, a-b)$

(b)

$a = 768, \quad b = 348$

first Iteration:

$a \geq b$

$a = a - b$

$= 768 - 348 = 420$

New,

$a = 420, \quad b = 348$

Second Iteration

$a \geq b$

$a = 420 - 348 = 72$

New,

$a = 72 \quad b = 348$

Third Iteration

$a < b$

$b = b - a$

$= 348 - 72 = 276$

New value,

$a = 72 \quad b = 276$

(5) fourth Iteration

$a < b$

$b = 276 - 72 = 204$

Newvalue

$a = 72 \quad b = 204$

(6) fifth Iteration

$a < b$

$b = 204 - 72 = 132$

New value

$a = 72, b = 132$

(7) Sixth Iteration

$a < b$

$b = 132 - 72 = 60$

New

$a = 72, b = 60$

(8) Seventh Iteration

$a \geq b$

$a = 72 - 60 = 12$

New value

$a = 12, b = 60$

9. Eighth Iteration

$a < b$

$b = 60 - 12 = 48$

So, $a = 12$, $b = 48$

(10) Ninth Iteration

$a < b$

$b = 48 - 12 = 36$

New value

$a = 12$ $b = 36$

(11) Tenth Iteration

$a < b$

$b = 36 - 12 = 24$

(12) Eleventh Iterat

$a < b$

$b = 24 - 12 = 12$

(13) Twelth Iterat

$a \geq b$

$a = 12 - 12 = 0$

$a = 0$ we stop.

gcd(768, 348) = b = 12

Iteration Number	a	b	gcd
0	768	348	
1	420	348	
2	420	72	
3	348	72	
4	276	72	
5	204	72	
6	132	72	
7	60	72	
8	60	12	
9	48	12	
10	24	12	
11	12	12	
12	0	12	12