Homework-7

Problem 53 on page 237

Given,

Population mean $\mu = 50$

Population standard deviation $\sigma = 1.2$

Sample sizes:

$n = 9$ for a part (a)

$n = 40$ for part (b)

we need

$P(\bar{X} \geq 51)$

Step I: Standard Error of the Mean:

$$SE = \frac{\sigma}{\sqrt{n}}$$

for $n = 9$

$$SE = \frac{1.2}{\sqrt{9}} = 0.4$$

for $n = 40$

$$SE = \frac{1.2}{\sqrt{40}} = 0.1898$$

Step 2: Convert to Standard Normal form

We have,

$\bar{X} = 51$ using Z-score formula:

$$Z = \frac{\bar{X} - \mu}{SE}$$

for $n = 9$

$$Z = \frac{51 - 50}{0.4} = 2.5$$

$$P(\bar{X} \geq 51) = P(Z \geq 2.5)$$

Standard normal table

$$P(Z \leq 2.5) = 0.9938$$

$$P(Z \geq 2.5) = 1 - 0.9938$$

$$= 0.0062$$

for $n = 40$

$$Z = \frac{51-50}{0.1898} = 5.27$$

$P(X \geq 51) = P(Z \geq 5.27)$

from the standard table:-

$P(Z \geq 5.27) \simeq 0$

$Z = 5$

So,

ⓐ $P(X \geq 51) = 0.0062$

ⓑ $P(\bar{X} \geq 51) \approx 0$

2. Problem 6 on page 285

Given,

Population standard deviation $\sigma = 100$

Sample size $n = 25$

Sample mean $\bar{x} = 8439$

Confidence level 90% for part a and 92% for part b

Step I: Compute Standard Error (SE)

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{25}} = 20$$

Step 2: Find the Z-Score for 90% CI

for 90%.

The critical value $Z_{\alpha/2}$

$$Z_{0.05} = 1.645$$

Since,

5% the probability is in each tail for total of 10% outside the interval

Step 3: Compute the Confidence Interval

The 90% confidence interval.

$$\bar{x} \pm Z_{\alpha/2} \times SE$$

$$8439 \pm 1.645 \times 20$$

$$8439 \pm 32.9$$

$$(8406.1, 8471.9)$$

classmate

Date

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step 4:

Modify for a 92% confidence Interval

for 92% confidence level,

from standard table:-

for 92% CI

$$Z_{\alpha/2} = Z_{0.04} = 1.75$$

$$8439 \pm 1.75 \times 20$$

$$(8404, 8474)$$

So, 92% confidence Interval is (8404, 8474)

3. Problem 17 on page 293

Given data,

Sample size $n = 153$

Sample mean $\bar{x} = 135.39$

Sample standard deviation: $s = 4.59$

Standard error of the mean $SE = 0.37$

Confidence level: 99%.

Step 1: find the t-score for 99%.
lower confidence Bound

t-distribution with $df = n - 1$
$= 152$

$t_{0.01, 152} \approx 2.33$

Step 2: Compute the lower Confidence Bound

The 99% lower confidence bound

$L = \bar{x} - t_{\alpha} - SE$
$= 135.59 - (2.33 \times 0.37)$
$= 134.33$

Step 3: Interpretation:

With 99% confidence, true ultimate strength at least 134.53 ksi.
This means that if we repeatedly take random sample size 153 at least 99% of the time the true mean tensile strength will be greater than 134.53 ksi.

4. Problem 19 on Page 294

Given,

Sample size : $n = 356$

Number of success (dies that passed) $x = 201$

sample proportion

$$\hat{p} = \frac{x}{n} = \frac{201}{356} \approx 0.564$$

Confidence level : 95%

standard normal critical value for 95% for CI

$$Z_{0.025} = 1.96$$

Step 1: Compute Standard Error

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= \sqrt{\frac{(0.564)(1-0.564)}{356}}$$

$$= 0.0263$$

Step 2: Compute the Confidence Interval

Margin of error

$$(ME) = Z_{\alpha/2} \times SE$$

$$= 1.96 \times 0.0263$$

$$= 0.0516$$

The confidence interval (CI)

$$\hat{p} \pm ME$$

$$0.564 \pm 0.0516$$

$$(0.512, 0.616)$$

Step 3: Interpretation:

with 95% confidence, true proportion of all dies betn 51.2% and 61.6%. This means that if repeatly take random samples of 356 dies, about 95% of those sample would contain the true proportion in this range.