Q.no. ~~0~~2

Epp5 problem 4.1.24 (p171

a. Is $4rs$ even?

→ Since 4 is even, $4rs$ will always be even for any integers r and s. Thus, regardless of whether r and s are odd or even, $4rs$ is even.

b. Is $6r + 4s^2 + 3$ odd?

Break down:-

$6r$ = even

$4s^2$ = even

3 = odd

Sum of even $(6r + 4s^2)$ is even. Adding an odd no. to an even no. results an odd number

Therefore,

$6r + 4s^2 + 3$ is odd

(c) Is $r^2+2rs+s^2$ composite?

This expression can be factored as

$(r+s)^2$

Since

r & s are both positive.

Therefore

$(r+s)^2$ is perfect square.

$(r+s > 1)$

Eg:-

$r=1$ & $s=1$

then,

$(1+1)^2 = 4$

which is composite.

Q.2. Epps problem 4.1.161 p172

~~Counte~~ *Counterexample*

let,

$n = 2$

n^2+1

$= 2^2+1$

$= 5$

In this case 5 is prime

Again

$n = 8$

n^2+1

$= 64+1$

$= 65$

65 is not prime.

Specially $n = 8$ we have disapprove where an even integer results in n^2+1 being composite which contradicts the claim that n^2+1 is prime for even n. This demonstrate that the original statement is false

3. Epp5 problem 4.4.13 (p197

Soln,

$$n = 4k + 3$$

so,

$$n^2 = (4k+3)^2$$
$$= 16k^2 + 24k + 9$$

Now,

$$n^2 - 1$$
$$= 16k^2 + 24k + 8$$

factor out 8!

$$n^2 - 1 = 8(2k^2 + 3k + 1)$$

Yes, 8 divides, when $n = 4k + 3$

4. Epp5 problem 4.4.21 | p198

So,

n is divisible by 16 so

$n = 16K$

n is divisible by 8

$n = 8m$

$n = 8(2K)$

shows, n is a multiple of 8. Thus, n is divisible by 16 and 8.

5. Epps problem 4.4.37 b, c / p198

b. $5733 = 3^2 \times 7^2 \times 13$

c $3675 = 3 \times 5^2 \times 7^2$

Q.6. Epps problem 6.2.20 | p405

①

$A \subseteq C$ x is in A & C

$B \subseteq C$ y in B & C

$A \cup B$ [x is in A or B]

$x \in A$

Since $A \subseteq C$ $x \in C$

$x \in B$

Since, $B \subseteq C$ $x \in C$

In either

x is in A or B we have

$x \in C$

x was an arbitrary element

of $A \cup B$ so $A \cup B$ contained in

C.

$A \cup B \subseteq C$

This statement true.

(7) Epps problem 6.2.35 | p416

so,

$A \subseteq B$ means x in A & B

$B \cap C = \emptyset$ no element common of B & C

If $x \in A \cap C$

so, $A \subseteq B$

$x \in A$ then $x \in B$

$B \cap C = \emptyset$

$x \in B$ and $x \in C$

contradicts

so,

There exist an x such that $x \in A \cap C$ must be false.

This implies that

$A \cap C = \emptyset$