1. Epps problem 5.1.6 | 273

6. $f_n = \left\lfloor \frac{n}{4} \right\rfloor \cdot 4$ for $n \geq 1$

$n=1$ $f_1 = \frac{1}{4} \cdot 4 = 0$

$n=2$ $f_2 = \frac{2}{4} \cdot 4 = 0$

$n=3 = f_3 = \frac{3}{4} \cdot 4 = 0$

$n=4 = f_4 = \frac{4}{4} \cdot 4 = 4$

The first four terms

0, 0, 0, 4

2. Epps problem 5.1.42/274

42. $$\sum_{m=1}^{n+1} m(m+1)$$

$$= \sum_{m=1}^{n} m(m+1) + (n+1)(n+2)$$

3. Epps problem 5.2.26 | 288

26. $3+3^2+3^3+\dots+3^n$ where $n \geq 1$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_n = \frac{3(3^n-1)}{3-1} = \frac{3(3^n-1)}{2}$$

4. Sumation notation,

$1^3 - 2^3 + 3^3 - 4^3 + 5^3$

Power cubes

$1^3, 2^3, 3^3, 3^3, 4^3, 5^3$

to get negative

$(-1)^{n+1}$

so,

series of sumation notation,

$$\sum_{n=1}^{5} (-1)^{n+1} n^3$$

5. Mathmetical induction

$n \geq 1$

$4+8+12+\cdots+4n = 2n^2+2n$

Base,

$n=1$

$4\times1 = 2\times1+2\times1$

$4=4$ (true)

hypothesis

$4+8+12+\cdots 4k = 2k^2+2k$

Inductive

$4+8+12+\cdots 4k+4(k+1) = 2(k+1)^2+2(k+1)$

Using hypothesis to L.H.S

$2k^2+2k+4(k+1)$

$= 2k^2+6k+4$

R.H.S

$2(k+1)^2+2(k+1)$

$2k^2+6k+4$

Hence, L.H.S = R.H.S

proved $n=k+1$

6. Mathematical induction:-

$n \geq 5$

$1+4n < 2^n$

Base $(n=5)$

$1+4\times5 < 2^5$

$= 21 < 32$ (true)

hypothesis $(n=k)$

$1+4k < 2^k$

Inductive :- $(n=k+1)$

$1+4(k+1) < 2^{k+1}$

$1+4k+4 < 2^{k+1}$

$2^k+4 < 2^{k+1}$

$2^k+4 < 2^k\times2$

So,

$4 < 2^k$

$k \geq 5$

$2^k \geq 32$

$n = k+1$

proved

$1+4n < 2^n$ for $n \geq 5$

(7) Epps problem 5.4.18 /312

(8) Soln,

	units digit
$9^0=1$	1
$9^1=9$	9
$9^2=81$	1
$9^3=729$	9
$9^4=6561$	1
$9^5=59049$	9

Observation

Units digits of 9^n

$= 1, 9, 1, 9, 1, 9 \ldots$

Conjecture:-

$$9^n = \begin{cases} 1 & \text{if } n \text{ is even} \\ 9 & \text{if } n \text{ is odd} \end{cases}$$

Strong Mathematical

Base:- $n=0$ $\quad 9^0=1$

$n=1$ $\quad 9^1=9$

Hypothesis,

$$9^k \begin{cases} k \text{ even} \\ k \text{ odd} \end{cases}$$

Inductive $n = k+1$

$$9^{k+1}$$
$$= 9^k \times 9$$

for even

$$9 \times 9 = 81 \; \{\text{Unit digit} = 1\}$$

for odd

$$9^k \times 9$$
$$= 1 \times 9 = 9 \; \{\text{unit digit} = 9\}$$

So,

proved

$$\text{unit of digit of } 9^n = \begin{cases} 1 & \text{if } n \text{ even} \\ 9 & \text{if } n \text{ odd} \end{cases}$$

8. Epp's problem 5.6.6 1337

(b) $t_k = t_{k-1} + 2t_{k-2}$ for $k \geq 2$

$t_0 = -1, \; t_1 = 2$

$t_2 = t_1 + 2t_0 = 2 + 2 \times (-1)$

$= 0$

$t_3 = t_2 + 2t_1$

$= 4$

$t_4 = t_3 + 2t_2$

$= 4$

First four sequence,

$t_0 = -1,$

$t_1 = 2$

$t_2 = 0$

$t_3 = 4$

$t_4 = 4$

9. Epps problem 5.6.14/337

Given,

$$d_n = 3^n - 2^n \text{ for } n \geq 0$$

need to show,

$$d_k = 5d_{k-1} - 6d_{k-2}$$

general term:-

$$d_n = 3^n - 2^n$$

first few term,

$$d_0 = 3^0 - 2^0 = 0$$

$$d_1 = 1$$

$$d_2 = 5$$

$$d_3 = 19$$

$$d_4 = 65$$

check,

$$d_k = 5d_{k-1} - 6d_{k-2}$$

$k = 2$

$$d_2 = 3^2 - 2^2 = 5$$

so,

$$d_2 = 5d_1 - 6d_0$$

$$= 5$$

$d_k = 3^k - 2^k$

$d_{k-1} = 3^{k-1} - 2^{k-1}$

$d_{k-2} = 3^{k-2} - 2^{k-2}$

Substitute

$d_k = 5d_{k-1} - 6d_{k-2}$

$3^k - 2^k = 5(3^{k-1} - 2^{k-1}) 6(3^{k-2} - 2^{k-2})$

expand:-

R.H.S

$6(3^{k-2} - 2^{k-2}) = 6 \cdot 3^{k-2} - 6 \cdot 2^{k-2}$

so,

$3^k - 2^k = 5 \cdot 3^{k-1} - 5 \cdot 2^{k-1} - 6 \cdot 3^{k-2}$
$+ 6 \cdot 2^{k-2}$

we have,

$3^k = 3 \cdot 3^{k-1}$

$2^k = 2 \cdot 2^{k-1}$

Rewrite,

$3 \cdot 3^{k-1} - 2 \cdot 2^{k-1} = 5 \cdot 3^{k-1} - 5 \cdot 2^{k-1}$
$- 6 \cdot 3^{k-1} + 6 \cdot 2^{k-2}$

Rearrange:

$$-2(3^{k-1} - 3^{k-2}) = -3(2^{k-1} - 2^{k-2})$$

Thus

$$d_k = 5d_{k-1} - 6d_{k-2}$$

10. Epps problem 6.2.34 1466

(34) For all A and B, if $B \subseteq A'$ then $A \cap B = \emptyset$

Given,

$A \subseteq U$, $B \subseteq U$

proof:-

$A \cap B = \emptyset$ So,

$x \in A \cap B$

means, $x \in A$ & $x \in B$

Analyze:-

$x \in B$ & $B \in A'$

So,

$x \in A'$

means,

$x \notin A$.

We have contradiction

$A \cap B = \emptyset$ must be false,

So,

Proved $B \subseteq A'$ then $A \cap B = \emptyset$