Computational Physics Assignment 3: Numerical Root-finding

Given equation

$$f(x) = 2\sin(\sqrt{x}) - x + 1 = 0 \tag{1}$$

- (1) Using the graphical method, estimate the region where the root of Eq. (1) is located.
- (2) Determine the root of Eq. (1) using the bisection method with precision 10^{-10} using a bracket with $x_1 = 0$ and $x_u = 5$.
- (3) Determine the root of Eq. (1) using the false-position method with precision 10^{-10} using a bracket with $x_l = 0$ and $x_u = 5$.
- (4) Determine the root of Eq. (1) using the fixed-point iteration method with precision 10^{-10} using an initial guess of 1.
- (5) Determine the root of Eq. (1) using the Newton-Raphson method with precision 10^{-10} using an initial guess of 1.
- (6) Make a plot of the total error in each of these cases as a function of the iteration number.
- (7) Determine weather the convergence rate in (4) matches the theoretical expectations.
- (8) What is the maximum precision you can reach with each method?
- (9) Based on your results, estimate which method is the fastest. How does that depend on the problem parameters (e.g., initial guesses or the choice of function *g*)?