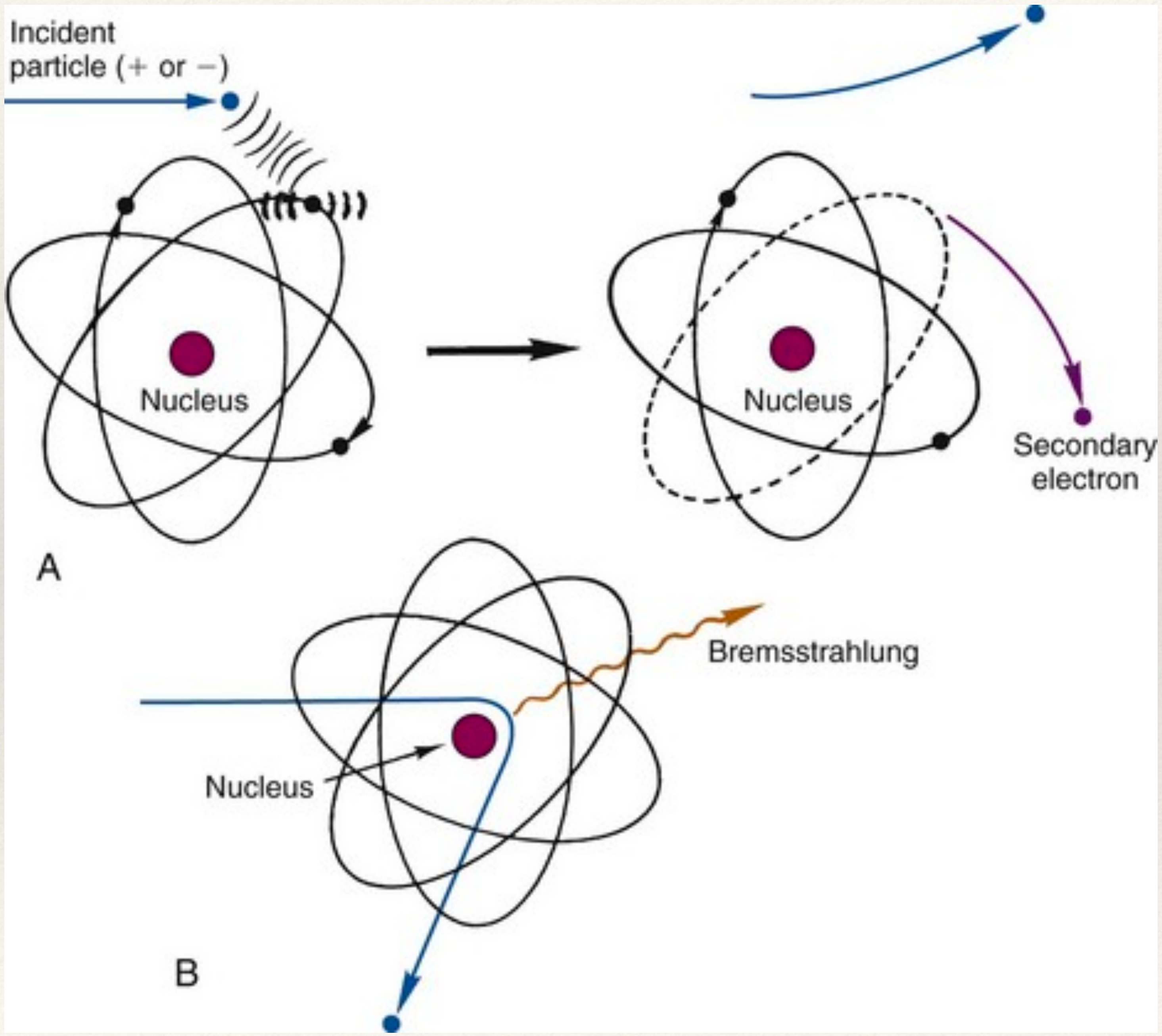
QED radiative corrections off proton and nuclei

Taisiya Mineeva 19/10/18

Lecture 1

Charged particle interaction with matter



Ionization

collision between the charged particle and an orbital electron of atom

Bremsstrahlung

interaction with nucleus

QED bremsstrahlung

- Bremsstrahlung is electromagnetic radiation produced by acceleration or deceleration of charged particle when deflected by *magnetic field* or *another charged particle*.

- From QED theory, when charged particle accelerates/ decelerates, it must radiated energy in the form of real photons

- Electron in the *magnetic field of magnets / undulatory*

 - synchrotron radiation (acceleration of relativistic e- in B field)

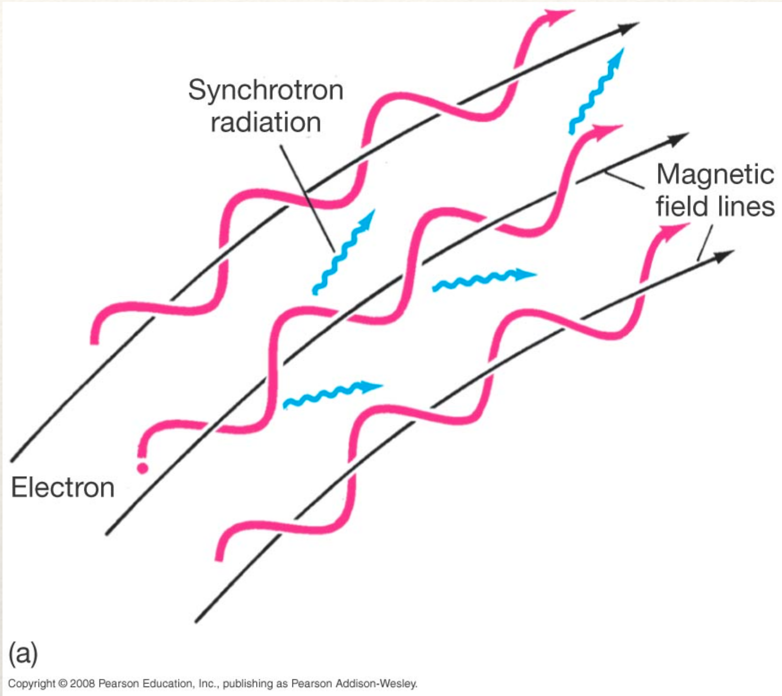
 - cyclotron radiation (non-relativistic charged particles)

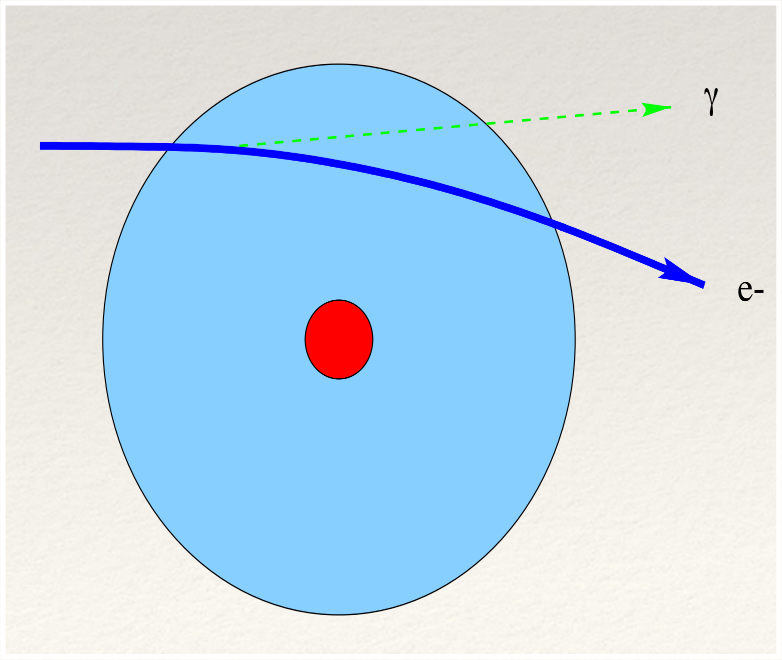
- Electron passing through the *Coulomb field of atoms*

 - here, probability of emitting a photon is ~ Z^2/M^2

 Z - charge of an atom, M is the mass of scattered particle

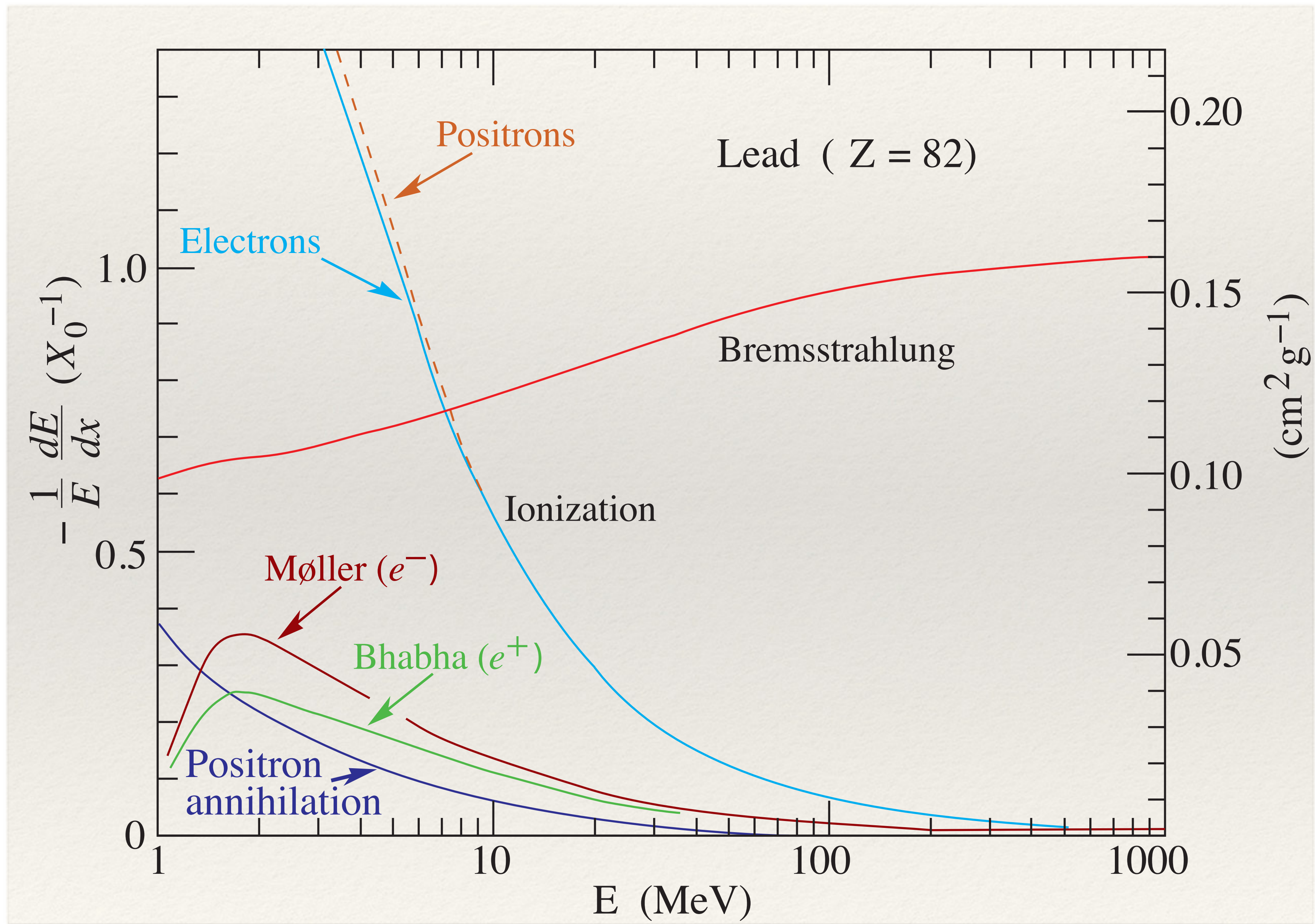
(see Andrea's lecture1 slide #28)





Ionization vs Bremsstrahlung

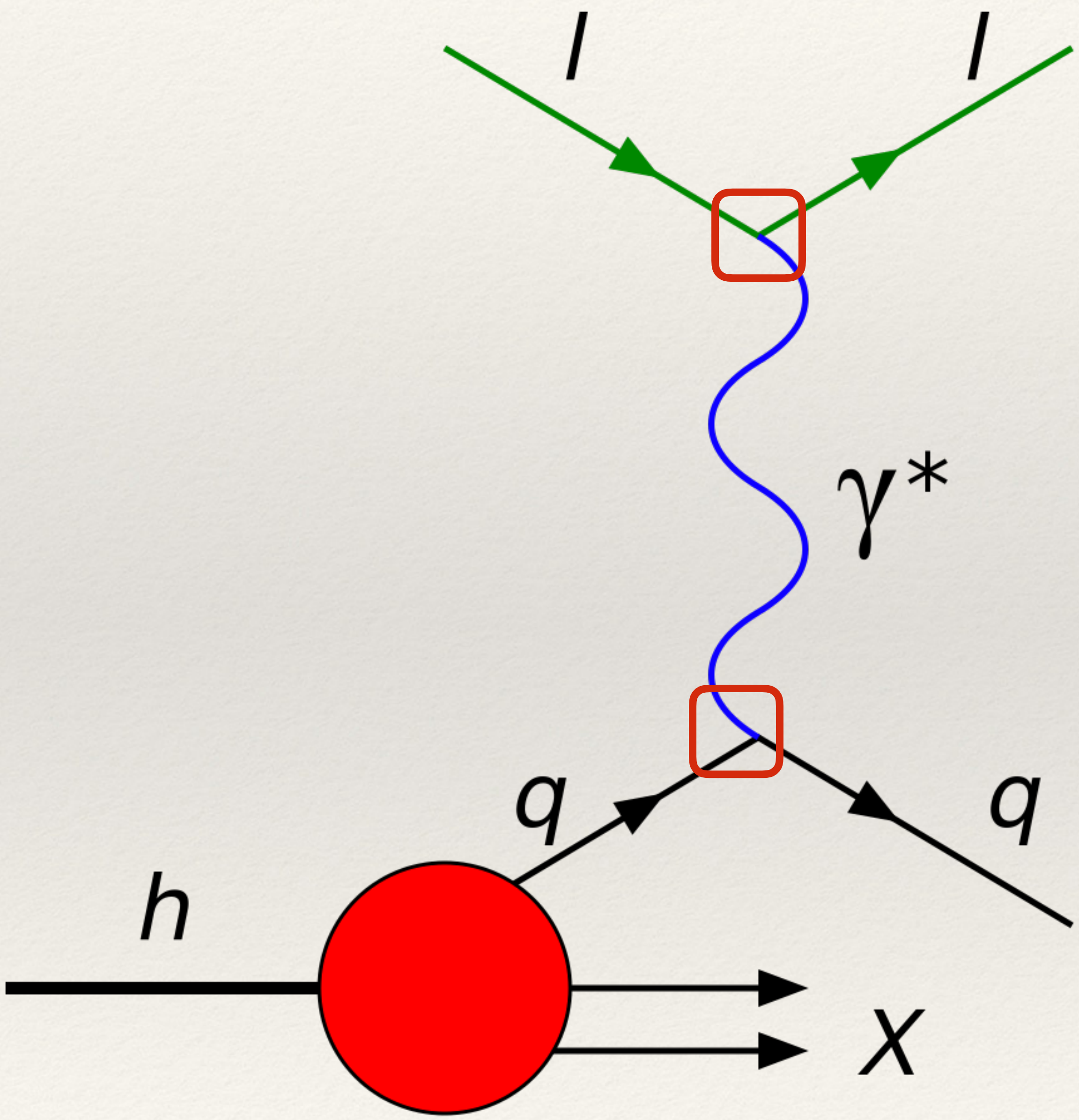
Fractional energy loss per radiation length in lead as a function of electron or positron energy.



PDG: http://pdg.lbl.gov/2018/reviews/rpp2018-rev-passage-particles-matter.pdf

Deep inelastic scattering in 1st Born approximation

Born approximation - one photon exchange between scattered lepton and quark



leptonic vertex where electron emits virtual photon is fully described by QED

hadronic vertex, where the virtual photon is absorbed by the proton, is not easy to calculate due to the structure of the p, described by QCD

What is 1st Born approximation?

$$\psi_n(\vec{r}) = \phi_{inc}(\vec{r}) - \frac{\mu}{2\pi\hbar^2}\int \frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} V(\vec{r}\,')\psi_{n-1}(\vec{r}\,')\, d^3r'$$

n=1 is valid only when the scattered wave $\Psi(r)$ is only slightly different from the incident plane wave $\phi(r)$

$$\psi_0(\vec{r}) = \phi_{inc}(\vec{r})$$

$$\phi_{inc}(\vec{r}) = Ae^{i\vec{k}_0\cdot\vec{r}}$$

scattering amplitude

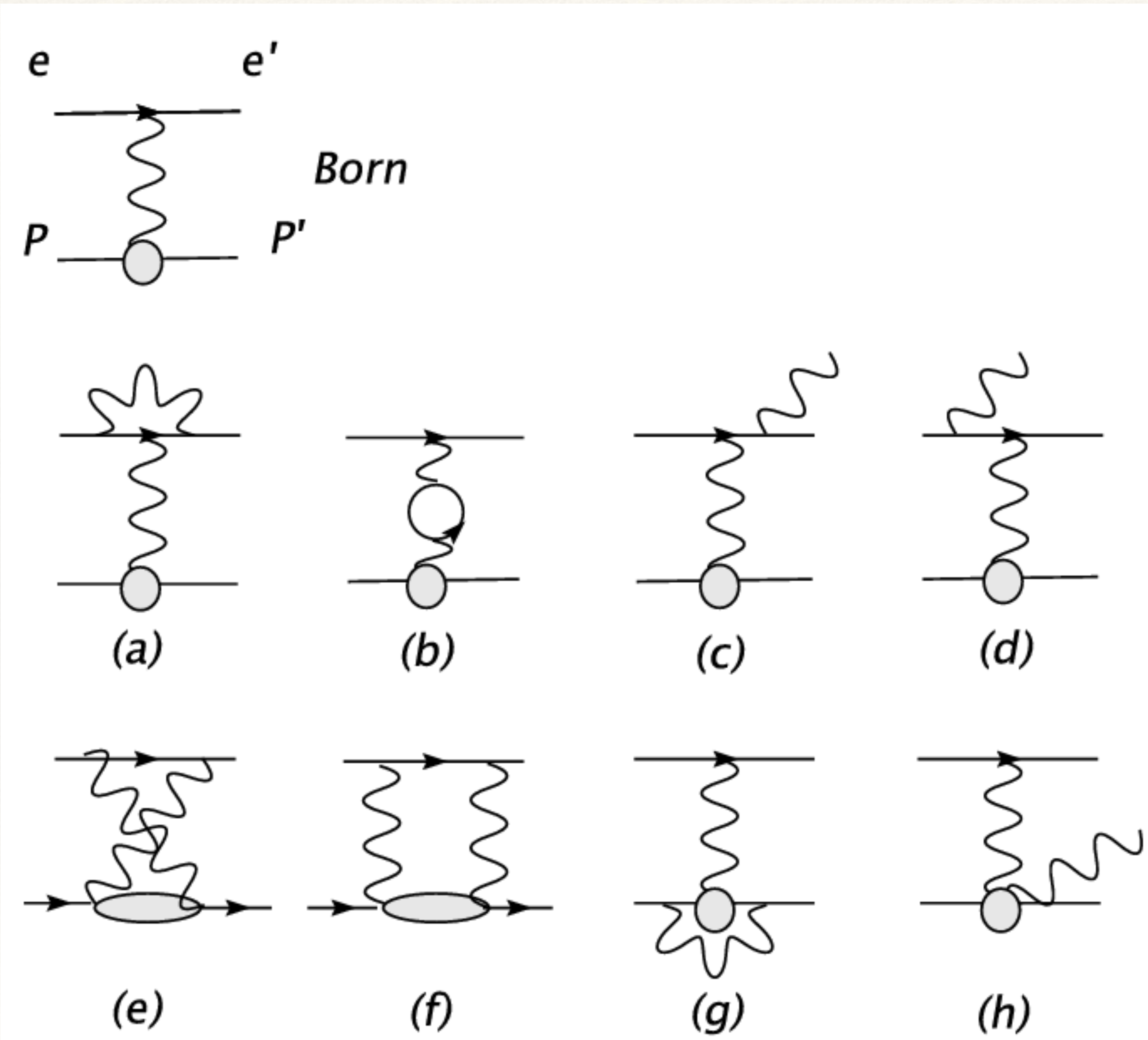
$$f(\theta,\varphi) = -\frac{\mu}{2\pi\hbar^2}\int e^{-i\vec{k}.\vec{r}\,'} V(\vec{r}\,')\psi(\vec{r}\,')\, d^3r'$$

first order Born cross section

$$\frac{d\sigma}{d\Omega} = \left| f(\theta,\varphi)\right|^2 = \frac{\mu^2}{4\pi^2\hbar^4}\left|\int e^{i\vec{q}\cdot\vec{r}\,'} V(\vec{r}\,')\, d^3r'\right|^2$$

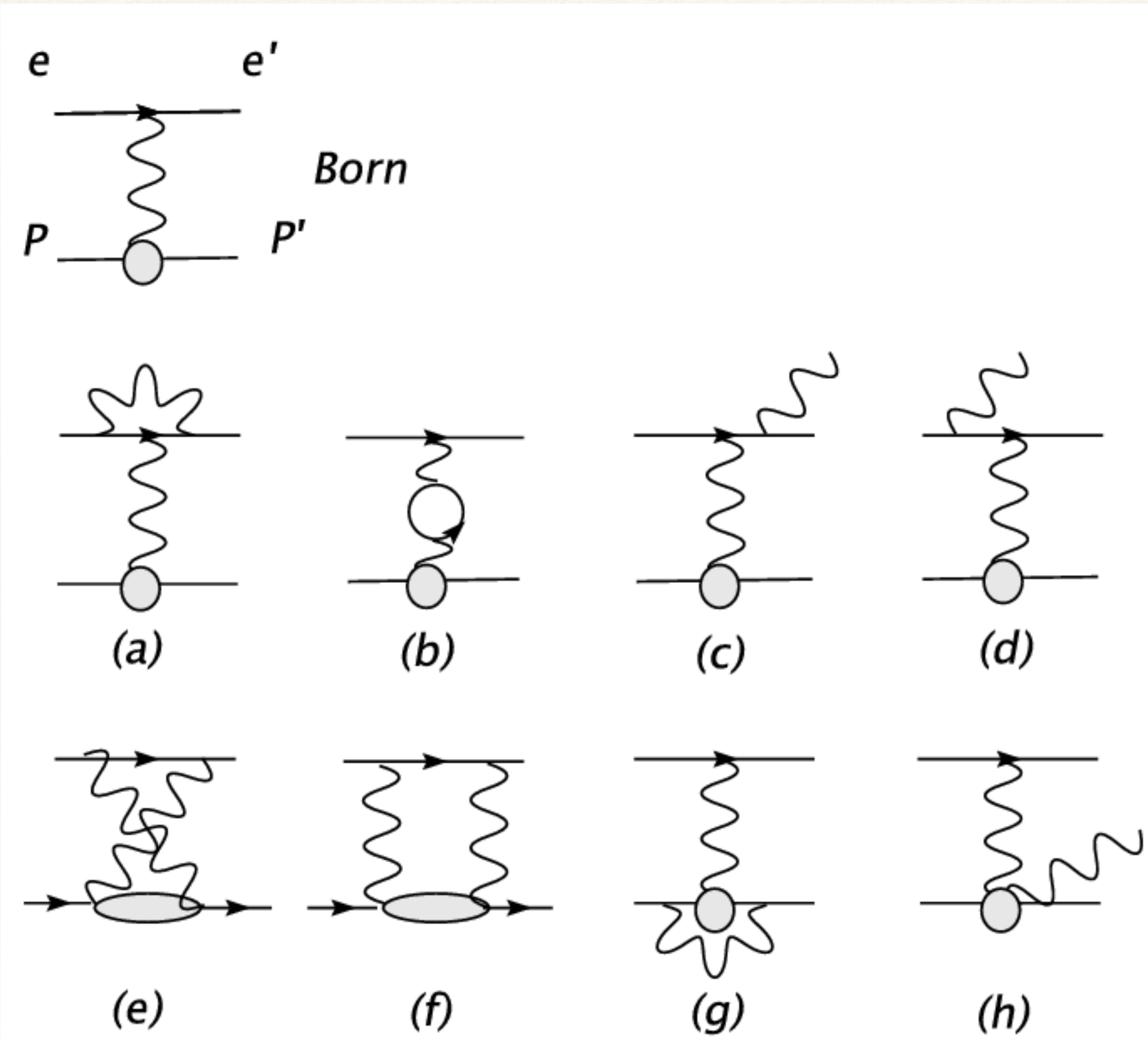
$\vec{q} = \vec{k}_0 - \vec{k}$ is momentum transferred

Deep inelastic scattering Feynman diagrams



Higher order corrections due to next to leading order Feynman diagrams

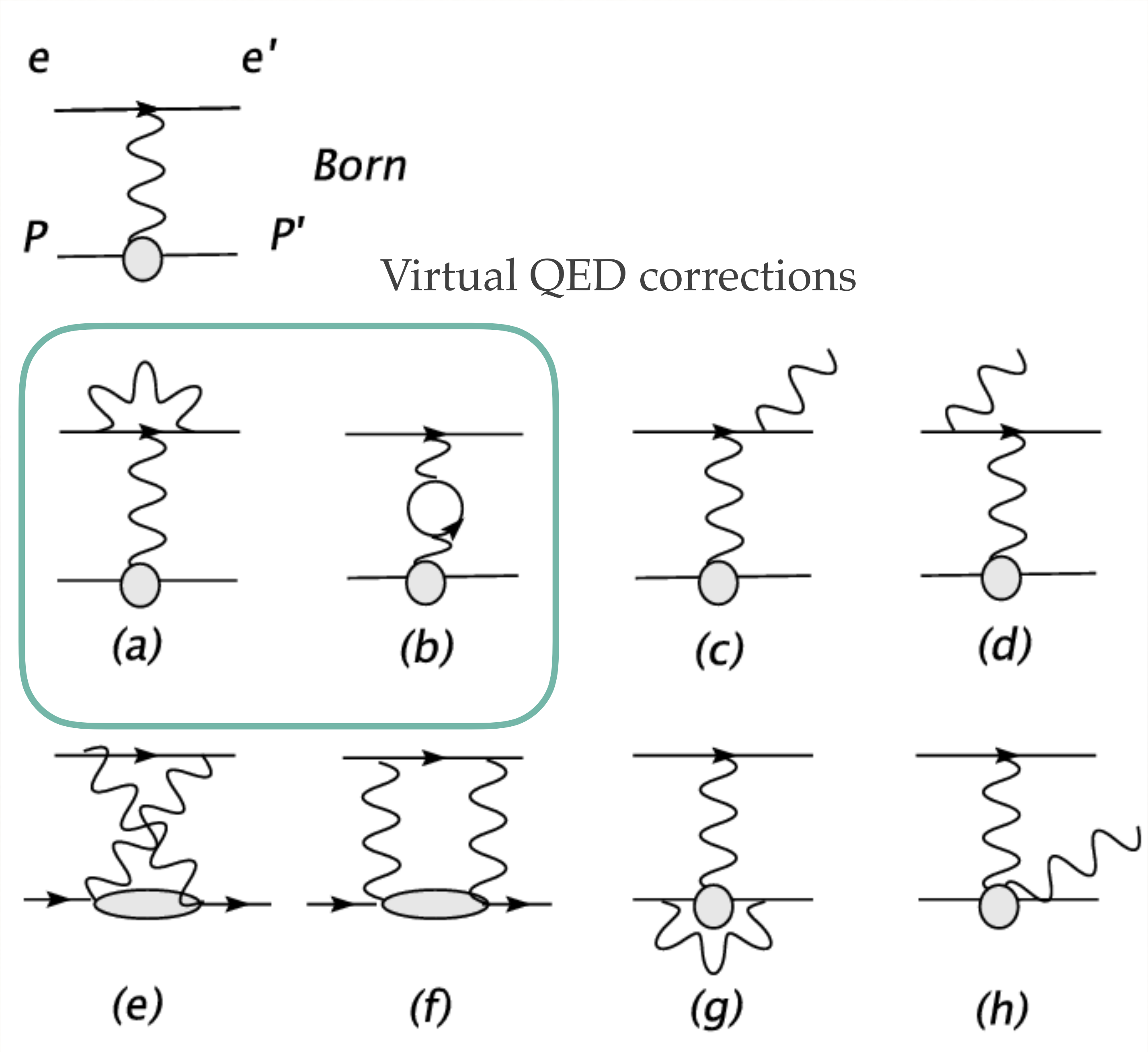
Deep inelastic scattering Feynman diagrams



Higher order corrections due to next to leading order Feynman diagrams

a) vertex correction
b) vacuum polarization
c) & d) electron bremsstrahlung
e) & f) the two-photon exchange terms
g) proton vertex correction
h) proton bremsstrahlung

Deep inelastic scattering Feynman diagrams



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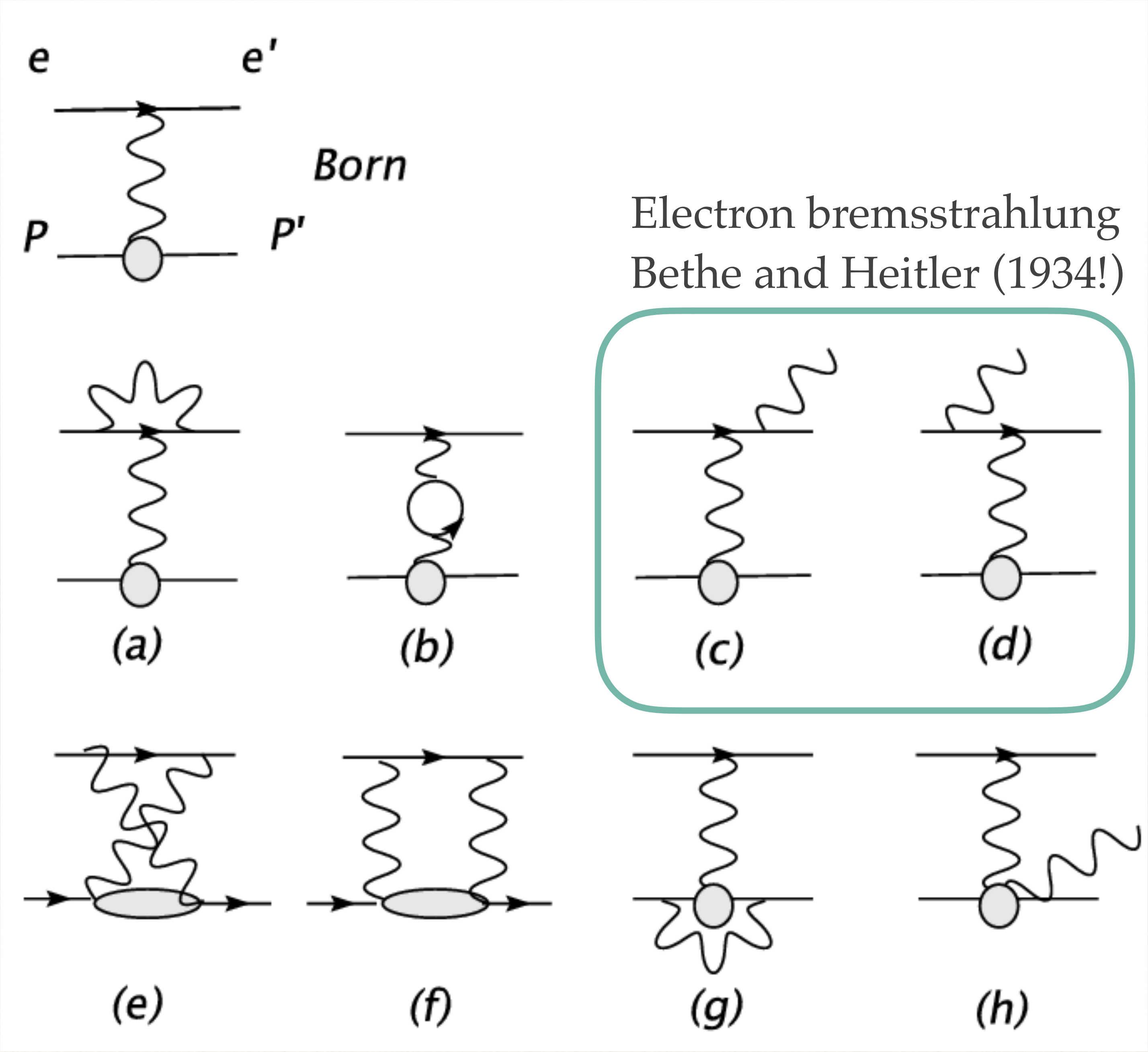

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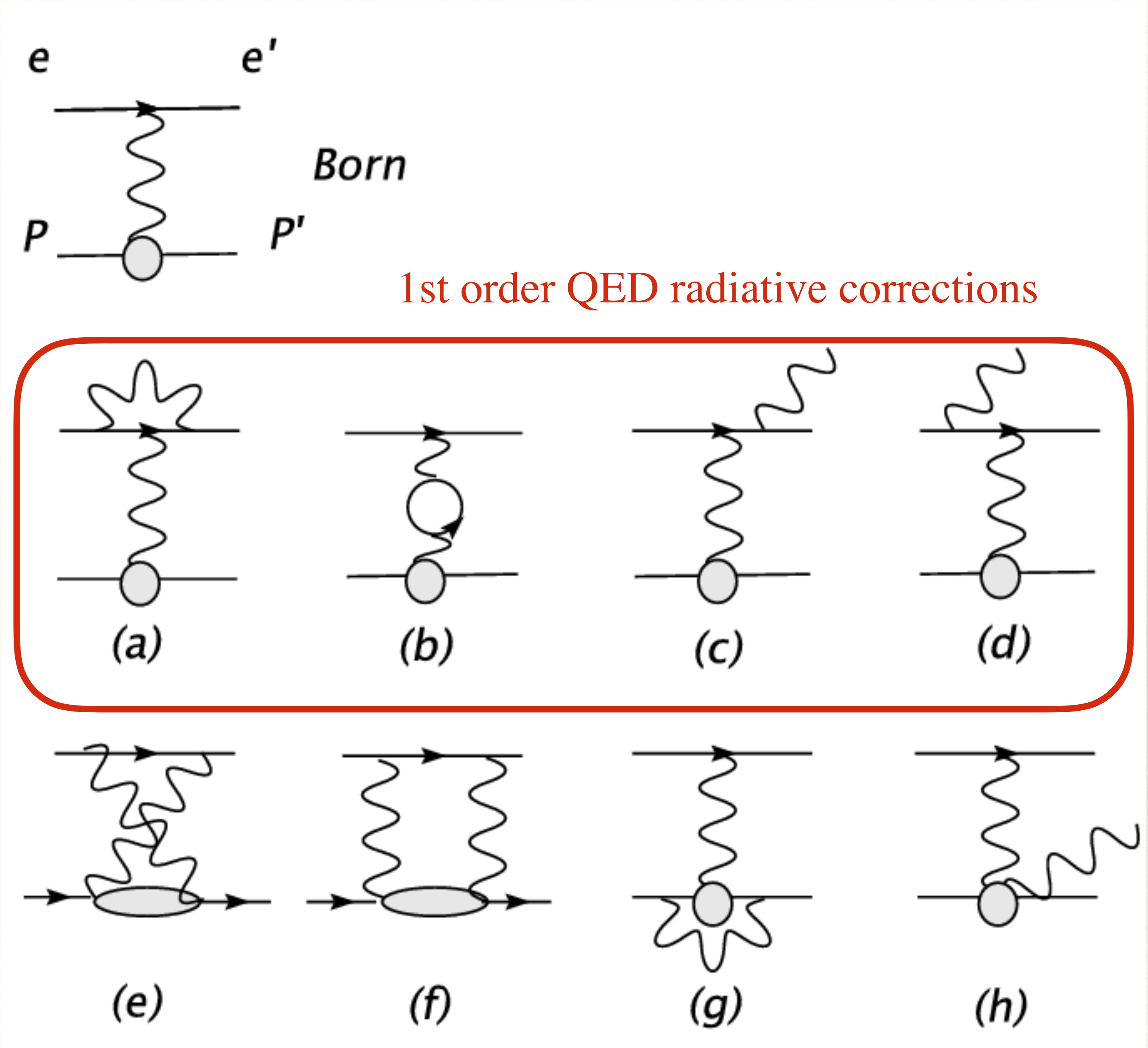
Deep inelastic scattering Feynman diagrams



Higher order corrections due to next to leading order Feynman diagrams

a) vertex correction
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c) & d) electron bremsstrahlung
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NOTE: due to those effects, electron energy measured in the detector is different from the interaction energy at the vertex

Radiated vs Born cross section

The differences between the Born cross section and the measured cross section are radiative corrections.

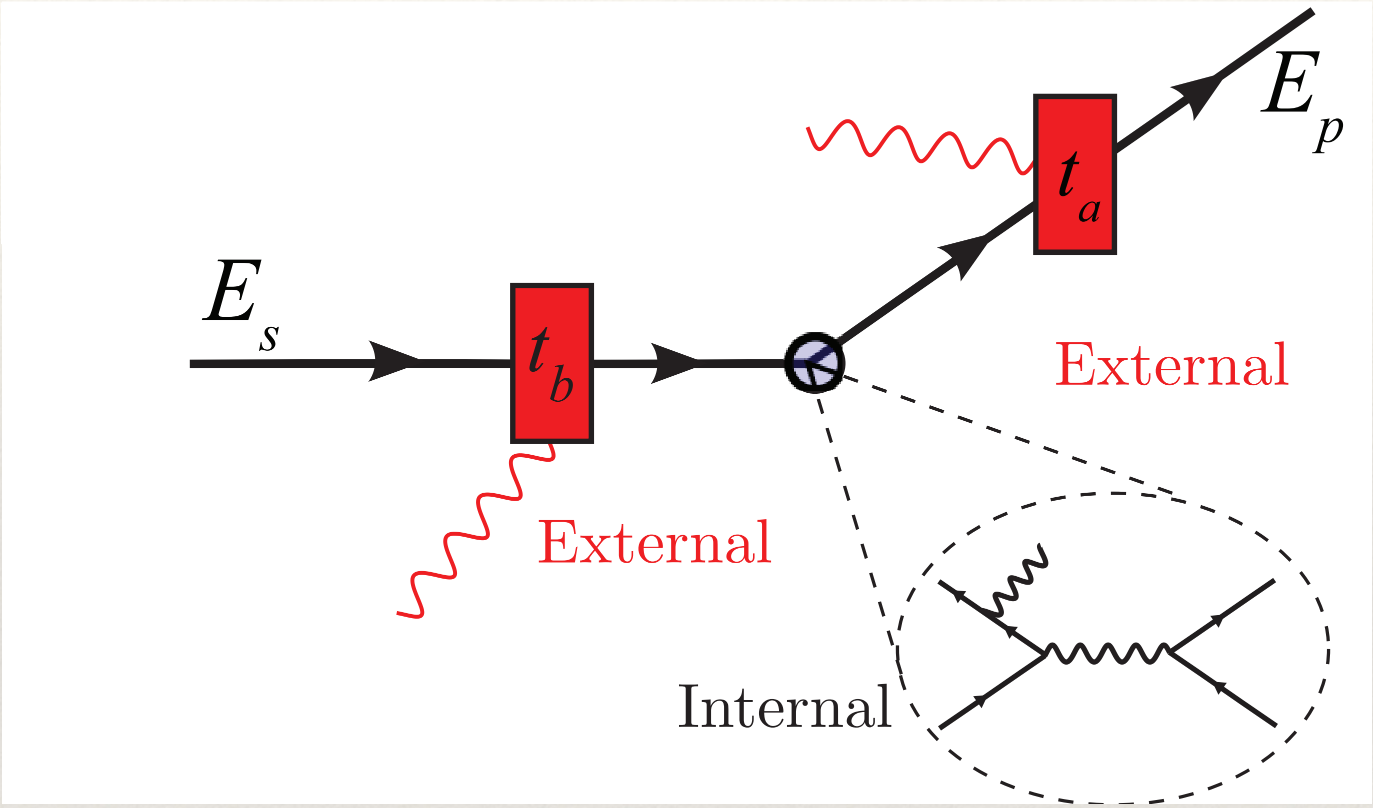
$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Meas}} = \boxed{(1+\delta_{rad})} \left.\frac{d\sigma}{d\Omega}\right|_{\text{Born}}$$

$$d\sigma_{\text{full}} \quad \propto \quad |\mathcal{M}_{\text{full}}|^2 \approx \alpha^2 \left[\mathcal{M}_{\text{Born}}^2 + 2\alpha \mathcal{M}_{\text{born}} \Re\left(\mathcal{M}_{\text{vertex}} + \mathcal{M}_{\text{vacuum}} + \mathcal{M}_{\text{self}}\right)\right] + \cdots$$

QED radiative effects

QED radiative effects

Distinguish: external vs internal radiation



External radiation
bremsstrahlung in the field of proton in the material either before or after the scattering vertex

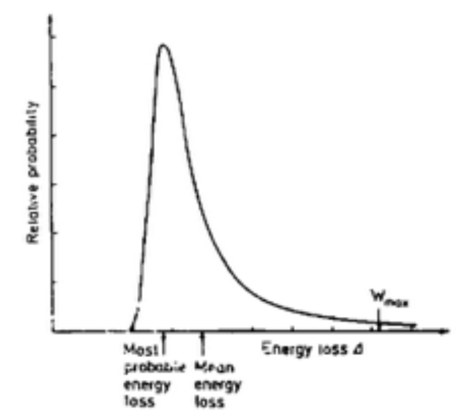
Internal radiation
bremsstrahlung emission in the field of the proton from which the scattering took place (scattering vertex)

External radiation

External radiative correction

Straggling effect

When passing through the target, electrons will lose energy due to ionization and bremsstrahlung. The amount of energy loss is not equal to the mean value but has statistical fluctuations.



$I(E_0, E, t)dE$ —— the probability of finding an electron in the energy interval between E and E+dE at a depth t (in units of radiation length)

measured cross section due to straggling effect:

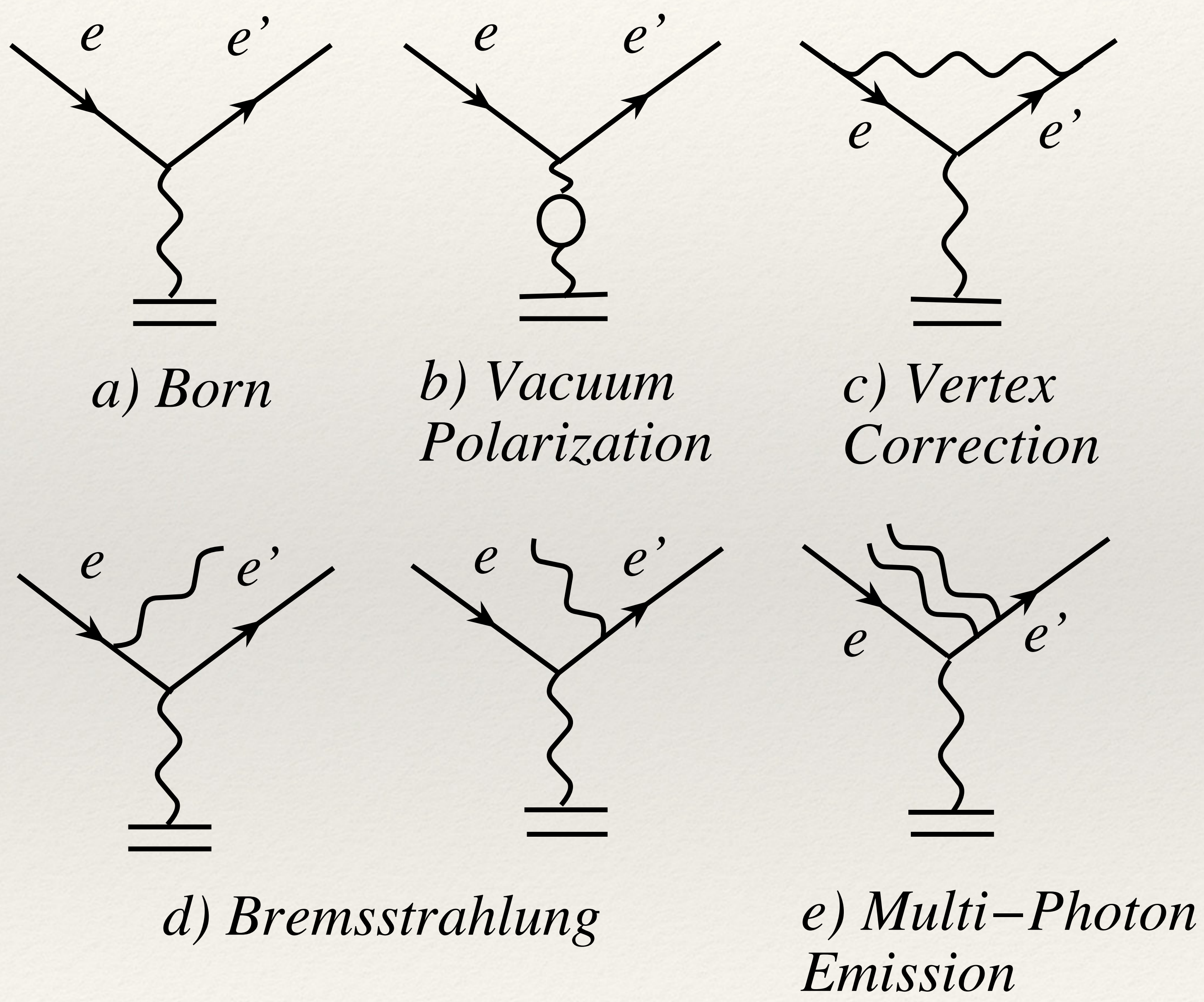
$$\sigma_{exp}(E_s, E_p) = \int_0^T \frac{dt}{T} \int_{E_{s\,min}(E_p)}^{E_s} dE'_s \int_{E_p}^{E_{p\,max}(E'_s)} dE'_p \; I(E_s, E'_s, t)\, \sigma_r(E'_s, E'_p)\, I(E'_p, E_p, T-t)$$

cross section including internal radiation

Hanjie Liu; Columbia University;
2017 Hall A&C Analysis meeting

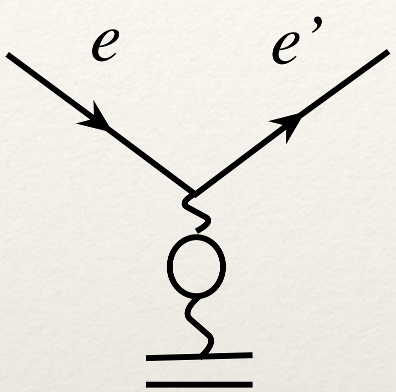
NOTE: CLAS data internal radiation is included in GEANT simulation for detector & target material

Internal radiation



P.S. two-photon exchange corrections are small and we will not consider them here

Internal radiation



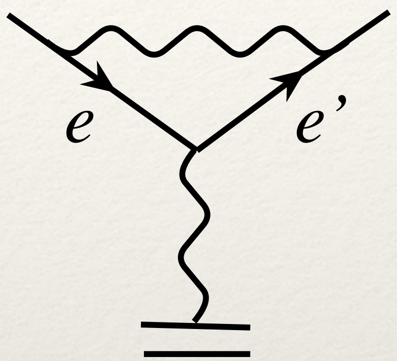
b) Vacuum Polarization

Vacuum corrections first computed by J.Schwinger in 1949!

Vacuum polarization (QED): loops for when virtual photon spontaneously splits in $e^{\pm}$, $\mu^{\pm}$, $\tau^{\pm}$ and qqbar pairs.

Those charged pairs act as electric dipole, creating a partial screening of the field - > the field is weaker as compared to the expected field of the vacuum, and that is why is it referred to as vacuum polarization.

Internal radiation



c) Vertex Correction

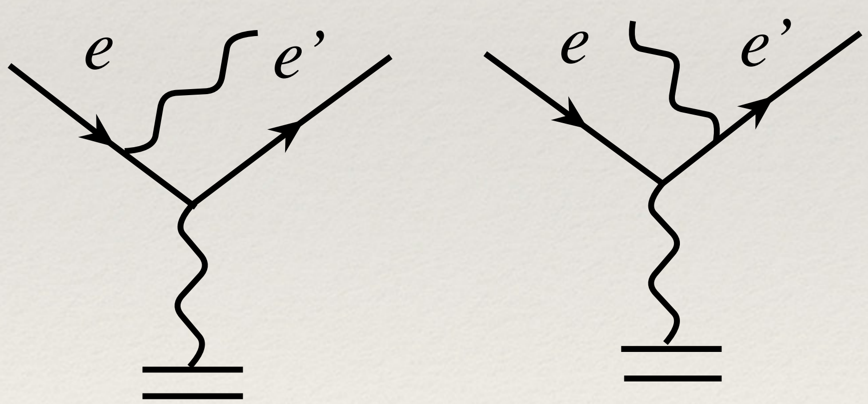
Vertex correction (QED) is correction to the classical electron magnetic moment predicted for Dirac equation. It is a small correction which is related to the anomalous electron magnetic moment.

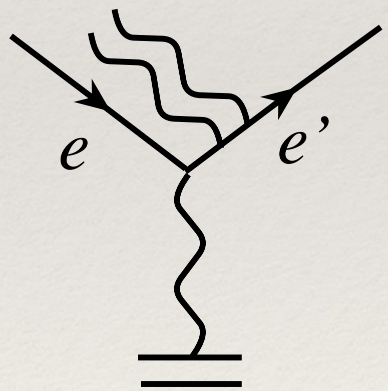
Internal radiation

Bremsstrahlung emission: interaction of the charged accelerating probe, electron, with proton.

It occurs in the Coulomb field of the target nucleus before and after interaction vertex.

Emitted photon can be: a single hard photon, which will carry away most of the radiated energy, or it can be multiple soft photons emitted that can carry arbitrary small amount of energy.





d) Bremsstrahlung

e) Multi−Photon Emission

Deep inelastic electron scattering

Inclusive electron scattering

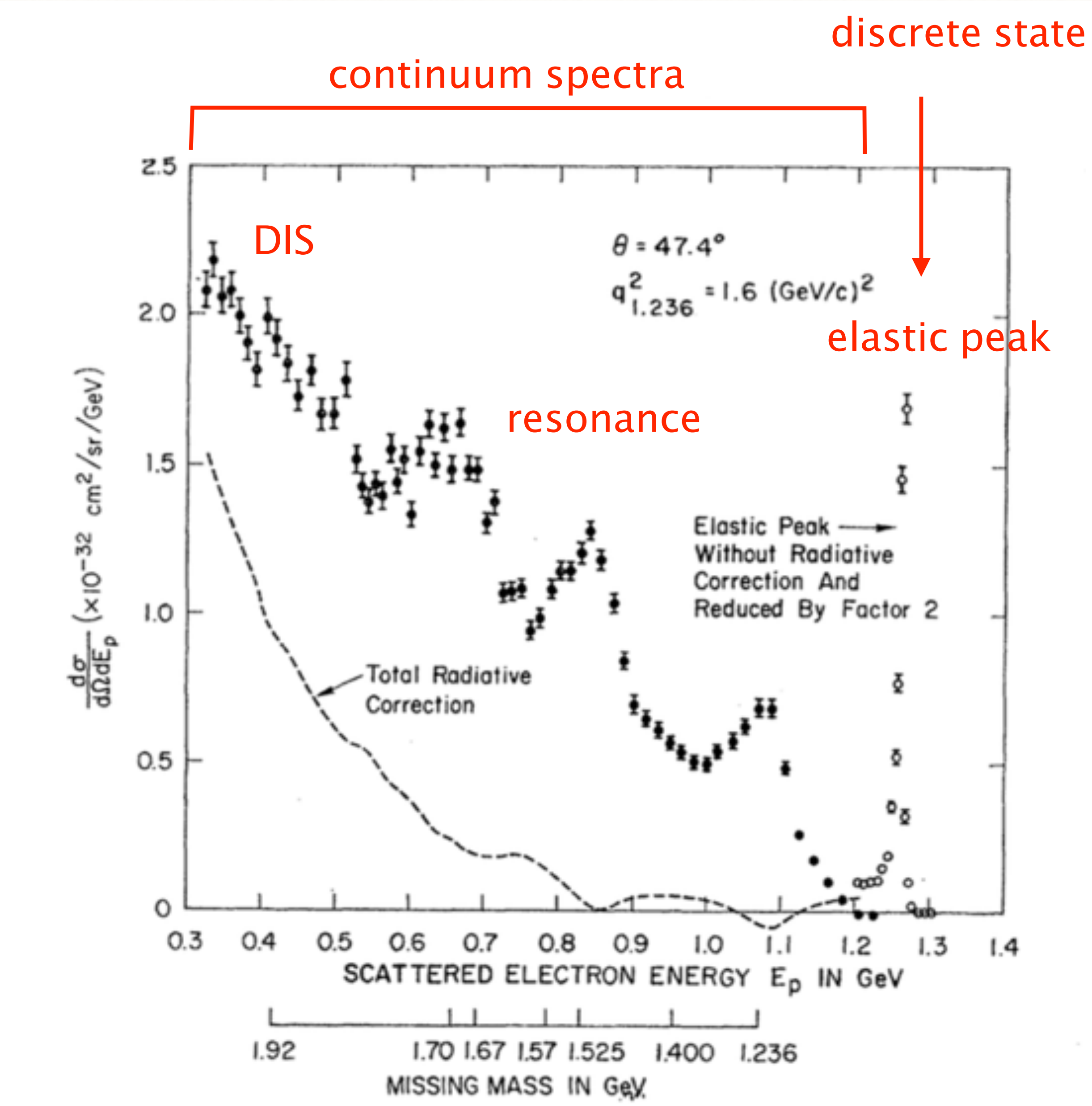
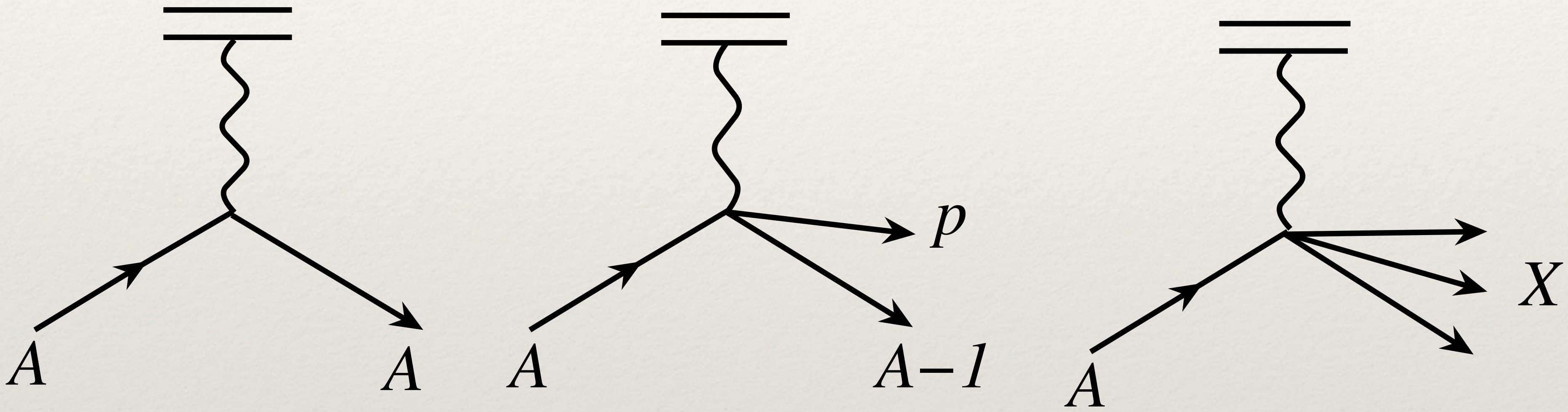


FIG. 1. A typical spectrum of inelastic *ep* scattering and the radiative corrections.

Processes contributing to DIS radiative corrections



a) Elastic   b) Quasi−Elastic   c) Inelastic

On order to calculate radiative cross sections to DIS, we need to compute radiative cross sections for elastic scattering, quasi-elastic and deep inelastic

Recap: cross sections on nucleon

Born cross section: elastic

$\sigma = \sigma_{El} + \sigma_{QE} + \sigma_{IN}$

Define linear combinations of F_1 and F_2:

$$G_E = F_1 + \frac{\kappa q^2}{4m_p^2} F_2 \qquad\qquad G_M = F_1 + \kappa F_2$$

Differential cross-section becomes:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

where $\tau = Q^2/4m_p^2$ and G_M is associated with the proton recoil

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where $\tau = Q^2/4m_p^2$ and G_M is associated with the proton recoil

proton structure: charge & magnetic moment

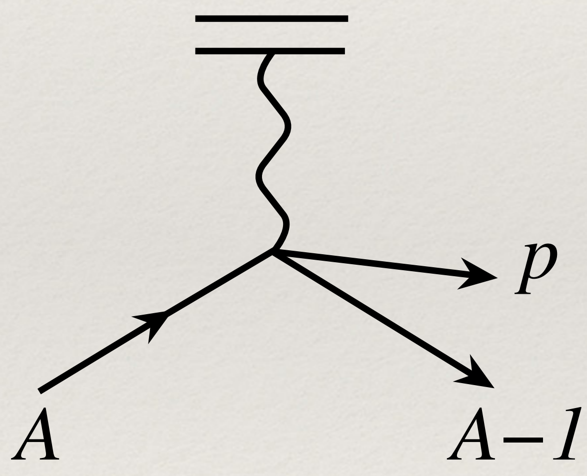
Mott cross section on point-like scattering center

Elastic: scattering off entire nucleus, after scattering it remains intact

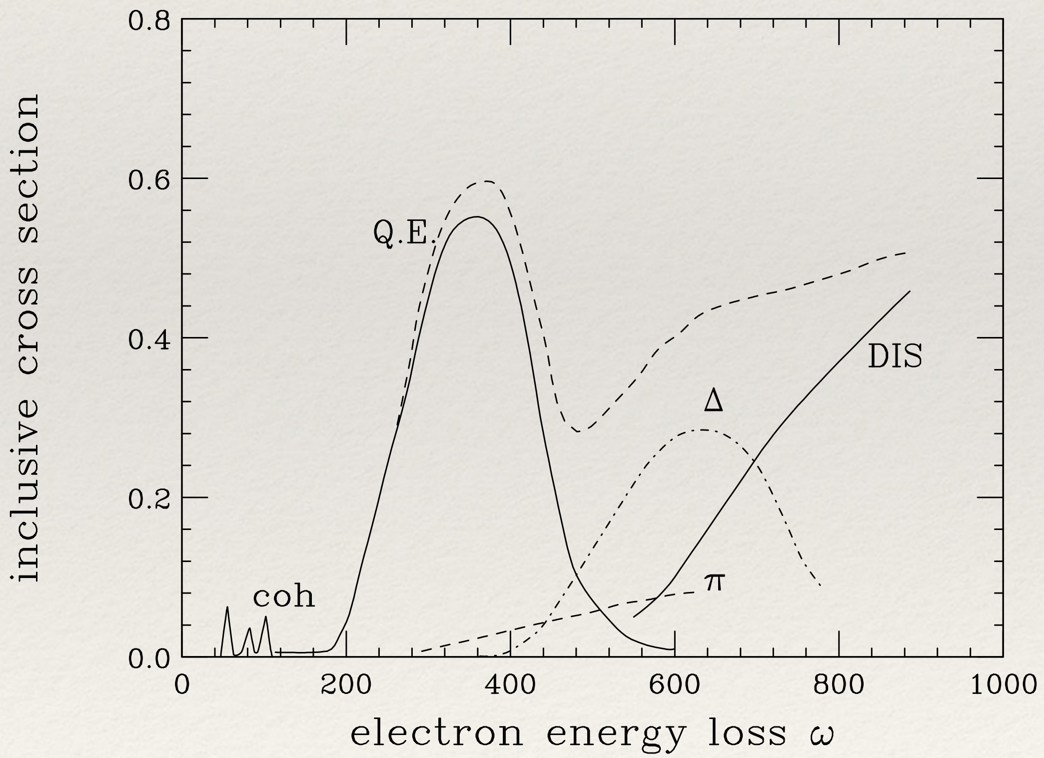
Born cross section: quasi-elastic

$\sigma = \sigma_{El} + \sigma_{QE} + \sigma_{IN}$

$$\frac{1}{\sigma_{\mathrm{Mott}}}\frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_f\,\mathrm{d}\varepsilon_f} = \left(\frac{q^2}{\boldsymbol{q}^2}\right)^2 S^{\mathrm{L}}(|\boldsymbol{q}|,\omega) + \left(-\frac{q^2}{2\boldsymbol{q}^2} + \mathrm{tg}^2\frac{\theta}{2}\right) S^{\mathrm{T}}(|\boldsymbol{q}|,\omega),$$



QE: e- scattering off moving nucleons, resulting in their knock-off



Born cross section: inelastic

$$\sigma = \sigma_{El} + \sigma_{QE} + \sigma_{IN}$$

$$\frac{d\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \left(W_2(\nu, Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, Q^2)\sin^2\frac{\theta}{2} \right)$$

In Deep Inelastic Scattering it is usual to replace the structure functions W_1, W_2 with F_1, F_2:

$$m_p W_1(\nu, Q^2) \rightarrow F_1(x) \qquad \nu W_2(\nu, Q^2) \rightarrow F_2(x)$$

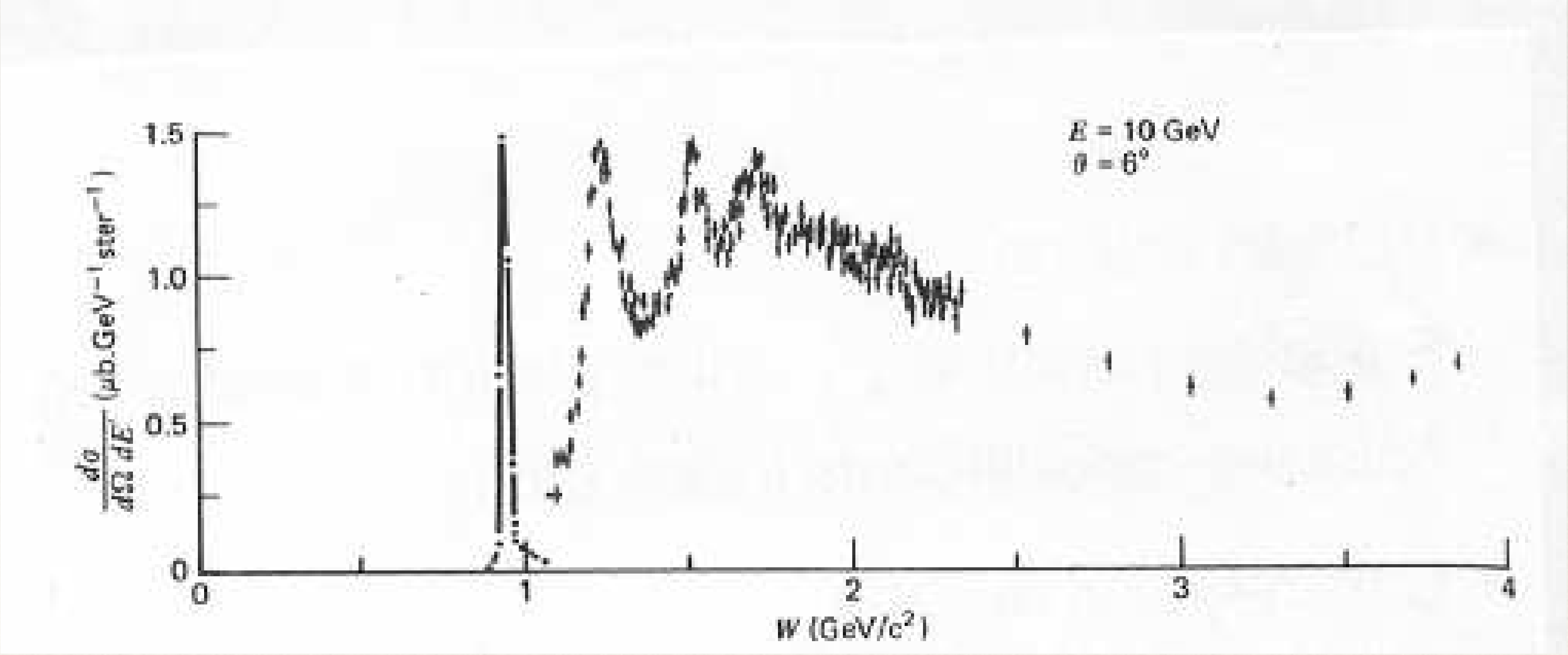
Warning - $F_{1,2}(x)$ in DIS are **not** the same as $F_{1,2}(q^2)$ in elastic scattering!

Born cross section: inelastic

$$\sigma = \sigma_{El} + \sigma_{QE} + \sigma_{IN}$$

$$\frac{d\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \left(W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} \right)$$

SLAC experiment in 1970s. Friedman, Kendall and Taylor: Nobel Prize 1990



Inelastic: scattering off point like constituents - partons

How to calculate cross sections on nuclei?

Cross section: from proton to nuclei

- Elastic: parametrization of structure functions $F_{1,2}(Q^2)$ based on the global fits to the nucleon elastic form factors which are then parametrized for nuclei
- Quasi-elastic: gaussian fit to the QE peak data; y-scaling
- Inelastic: nuclear structure function $F_{1,2}(x)$

Inelastic cross section: from proton to nuclei

The dependence of the structure functions $F_{1,2}(x)$ from the type of nuclear target is realized by multiplying the free nucleon structure function by the empirical fit function f_{EMC}

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha_F^2 \cos^2(\theta/2)}{[2E\sin^2(\theta/2)]^2}(W_2(W^2,Q^2) + 2\tan^2(\theta/2)W_1(W^2,Q^2)]$$

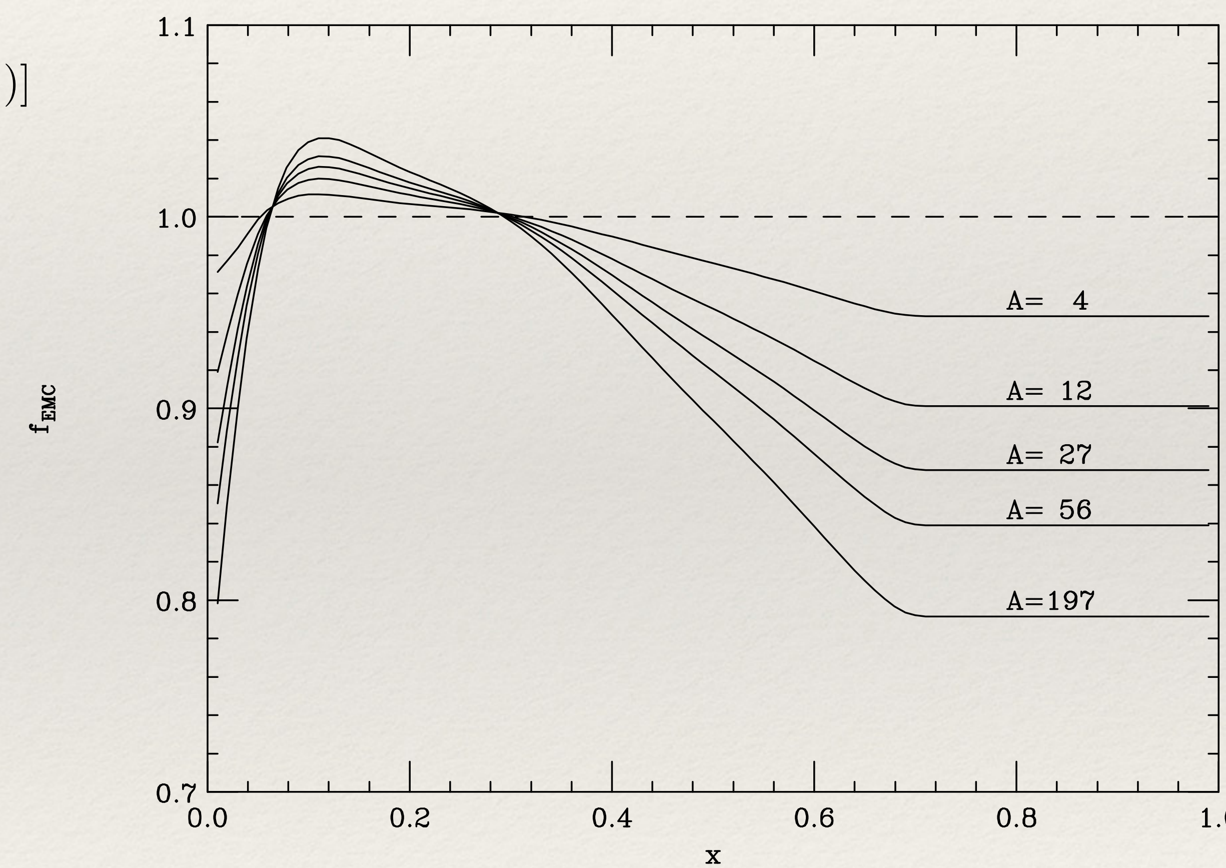
$$W_{1,2} = W^{Inelastic} + W^{QE}$$

$$W^{QE} \sim (F1,F2)$$

$$W_1^{Inel} = (W^F(W^2,Q^2) + W^{MEC}(W^2,Q^2))\_f^{EMC}(x')$$

$$W_2^{Inel} = W_1^I[1+RA(W,Q^2)]/(1+v^2/Q^2)$$

$f^{EMC}(x') = c(x')\, A^{\alpha(x')}$



P.Bosted, V.Mamyan “Empirical fit to electron-nucleus scattering” arXiv:1203.2262v2

- Data: CLAS, Hall C, SLAC (E133, E139, E140)* (arXiv:0711.0159)
  Hall C E04-001 on C, Al, Fe and Cu (arXiv:1202.1457)

  E133: eD and proton; E139 on D, He, Be, Al,C, Ca, Fe, Ag, Au and E140 is on D, Fe, Au

Calculation of radiative effects

Computing radiative effects

Virtual corrections (loop and vertex) to electron vertex are calculated exactly in QED

Two main approximations for calculating real photon emission: angular peaking
equivalent radiator

*Used for the integration and handling of infrared divergencies of the continuum spectrum

- Mo and Tsai developed formalism for computing inclusive radiative corrections:

  Mo&Tsai(1969): https://journals.aps.org/rmp/pdf/10.1103/RevModPhys.41.205

  Y.S.Tsai 1971: http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-0848.pdf

- There exist exact and complete calculations (Bardin and Shumeiko) of all higher order diagrams including two-photon exchange, but they require more computing power

Angle peaking approximation

- Radiative tail from elastic peak exact formula:

elastic form factors

$$\frac{d^2\sigma_r}{d\Omega dp} = \frac{\alpha^3 E_p}{(2\pi)^2 M E_s} \int_{-1}^{1} \frac{\omega d(\cos\theta_k)}{2q^4(u_0 - |u|\cos\theta_k)} \int_0^{2\pi} B_{\mu\nu} T_{\mu\nu}\, d\phi_k$$

Integral over ALL angles of emitted photon

- Emitted photon is most likely in the direction of incident or outgoing electron, i.e bremsstrahlung photons are collinear the initial and scattered electron
- Saves a lot of computing time when calculating angular integration
- But when $E_p < \frac{1}{3} E_{pmax}$, the peaking approximation can be in error as much as 30 to 40%. Exact formula must be used when energy loss is large.

Angle peaking approximation

- Radiative tail for *continuum* spectra: sum of many discrete states

$$(d\sigma_r/d\Omega dp)(E_s, E_p) = (d\sigma/d\Omega dp)(E_s, E_p)[1+\delta_r(\Delta)] + (d\sigma_r/d\Omega dp)(w > \Delta),$$

$$\frac{d^2\sigma_r}{d\Omega dp}(\omega > \Delta) = \frac{\alpha^3 E_p}{(2\pi)^2 M E_s} \int_{-1}^{1} d(\cos\theta_k) \int_{\Delta}^{\omega_{\max}(\cos\theta_k)} \frac{\omega d\omega}{2q^4} \int_0^{2\pi} B^c_{\mu\nu} T_{\mu\nu}\, d\phi_k$$

$$\omega = \frac{1}{2}(u^2 - M_f^2)/(u_0 - |u|\cos\theta_k)$$

contains structure functions

$F(q^2, M_f^2), G(q^2, M_f^2)$

- It's impossible to know $F(q^2, M_f^2)$,$G(q^2, M_f^2)$ for all continuum spectra
Angle peaking approx. is used to derive an approximation expression.

L. W. MO and T. S. Tsai, Rev. Mod. Phys. 41, 205 (1969), Appendix C;

Equivalent radiator method

- The effect of the internal bremsstrahlung on inelastic scattering is equivalent to placing one radiator before the scattering and another radiator of the same thickness after the scattering.
- The thickness of each radiator is equal to:

$$t = b^{-1}(\alpha/\pi)\ln(\mathrm{Q}^2/m_e^2 - 1),\ \text{with } b \text{ is close to } 4/3$$

- Takes into account multiple soft photon emission and enters as overall normalization factor applied to internal radiation

Total radiative cross section to continuum spectrum

Using angular peaking approximation and adding two radiator, before and after scattering, the radiative cross section for the continuum spectrum is

$$
\begin{aligned}
\sigma_b^c(E_s, E_p) = (1+0.5772\,bT) \Bigg[& \left(\frac{R\Delta}{E_s}\right)^{bT/2} \left(\frac{\Delta}{E_p}\right)^{bT/2} \sigma(E_s, E_p) \\
& + \int_{E_p+\Delta}^{E_{p\,max}(E_s)} dE'_p\, \sigma(E_s, E'_p) \left(\frac{E'_p - E_p}{E'_p}\right)^{bT/2} \left(\frac{(E'_p - E_p)R}{E_s}\right)^{bT/2} \frac{bT}{2(E'_p - E_p)}\, \phi\left(\frac{E'_p - E_p}{E'_p}\right) \\
& + \int_{E_{s\,min}(E_p)}^{E_s - R\Delta} dE'_s\, \sigma(E'_s, E_p) \left(\frac{E_s - E'_s}{E_p R}\right)^{bT/2} \left(\frac{E_s - E'_s}{E_s}\right)^{bT/2} \frac{bT}{2(E_s - E'_s)}\, \phi\left(\frac{E_s - E'_s}{E_s}\right) \Bigg]
\end{aligned}
$$

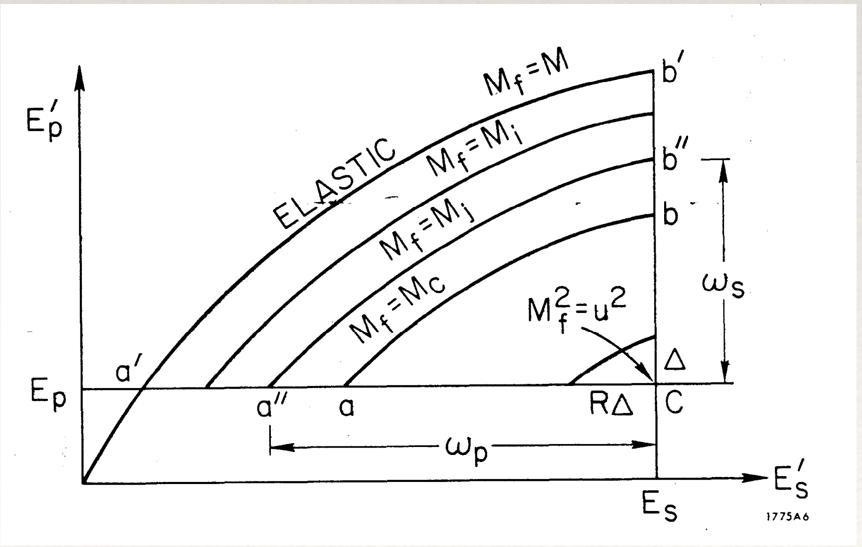
$$E_{p\,max}(E'_s) = \frac{E'_s}{1 + E_s M^{-1}(1-\cos\theta)}$$

outgoing electron after radiative core

$$E_{s\,min}(E_p) = \frac{E_p}{1 - E_p M^{-1}(1-\cos\theta)}$$

incident electron after radiative core

Integrals need to be performed over:



Δ cutoff: Δ = 10 MeV (code EXTERNALS)

Y.S.Tsai 1971: http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-0848.pdf

Coulomb corrections

$$k'_i = k_i + \varDelta k, \qquad k'_f = k_f + \varDelta k,$$

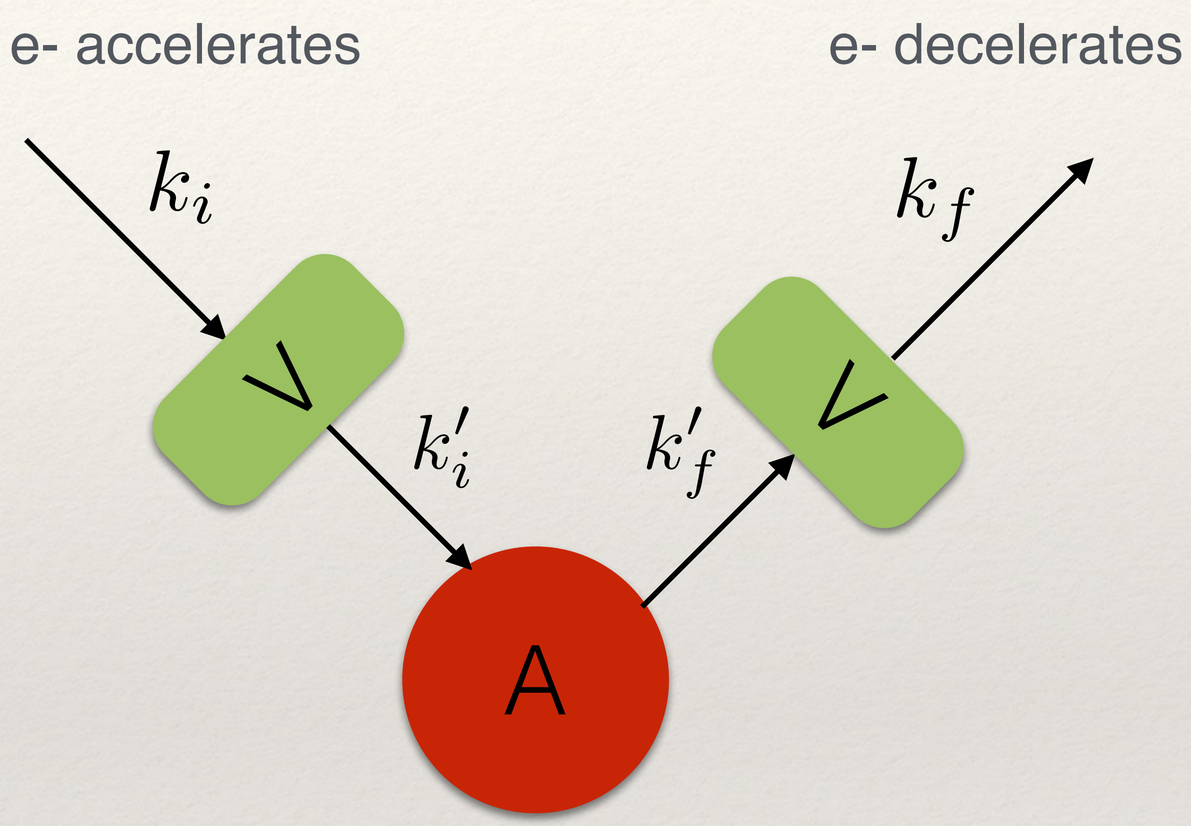
$$k'_{i,f} = |\vec{k}'_{i,f}|, \qquad \varDelta k = -V_0/c,$$

Aste et al., Eur. Phys. J. A 26, 167 (2005)

$$\Delta k = C\frac{\alpha Z}{R_e}, \quad C = 1\ldots3/2$$

$$f(0) = \left(1 - \frac{\beta}{R}\right)^{-2} \sim \left(1 - \frac{V(0)}{E}\right)^2 \sim (k'_i/k_i)^2$$

Aste et al. Nucl.Phys. A743 :259-282 (2004)



e- accelerates: $k_i' > k_i$
e- decelerates: $k_f' > k_f$

Next class:

compute radiative corrections for inclusive electron scattering with EXTERNALS

Bring your laptop!

Running EXTERNALS

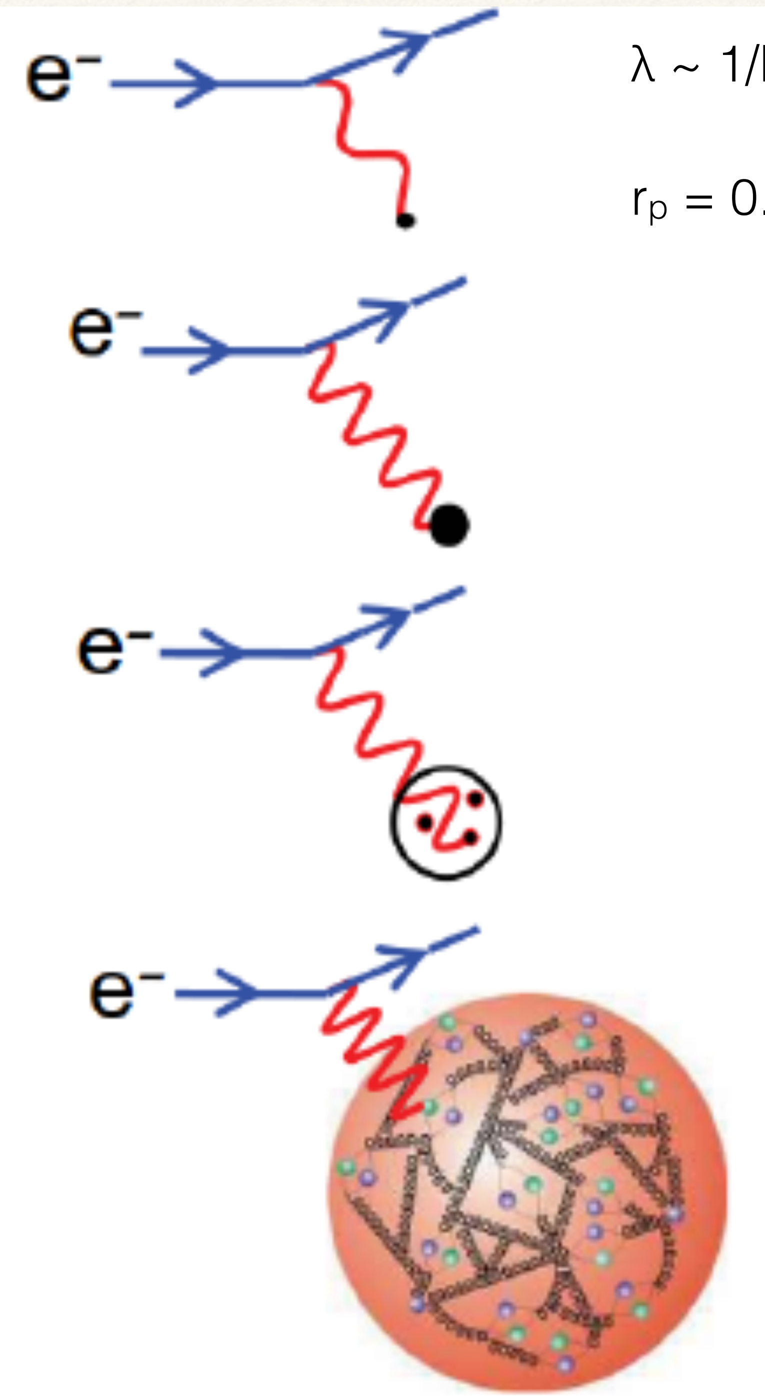
Taisiya Mineeva 21/10/18

Lecture 2

Q & A to lecture 1

Q & A: what are the structure functions?

- At very Low electron beam ($\lambda >> r_p$): scattering from a **point-like spin-less** object
- Low electron beam energies ($\lambda \approx r_p$): scattering from an **extended charged** object
- High electron beam energies ($\lambda < r_p$): scattering from **constituent quarks**
- High electron beam energies ($\lambda << r_p$): proton appears as **sea of quarks and gluons**



$\lambda \sim 1/E$

$r_p = 0.87$ fm

Alaa Dbeyssi

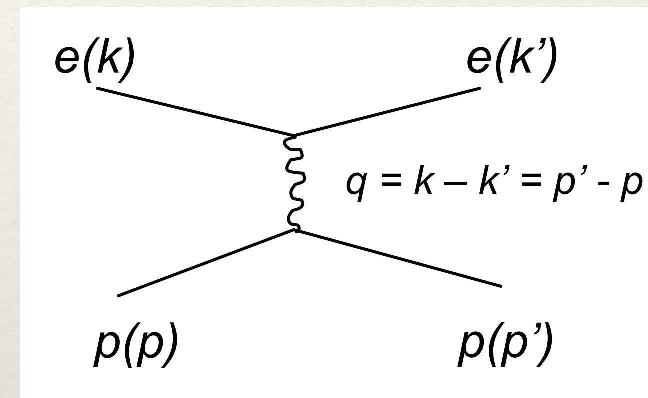
Q & A: what are the structure functions?

- Elastic ep scattering: spin 1/2 electron on on point-like particle:

Mott c.s.

$$\frac{d\sigma}{d\Omega} = \boxed{\frac{\alpha^2}{4E^2\sin^4(\theta/2)}\frac{E'}{E}\left[\cos^2(\theta/2)\right.} + \left.\frac{Q^2}{2M^2}\sin^2(\theta/2)\right]$$

Rutherford: spin0 electron spin magnetic interaction



- Elastic ep scattering: spin 1/2 electron on spin 1/2 proton:

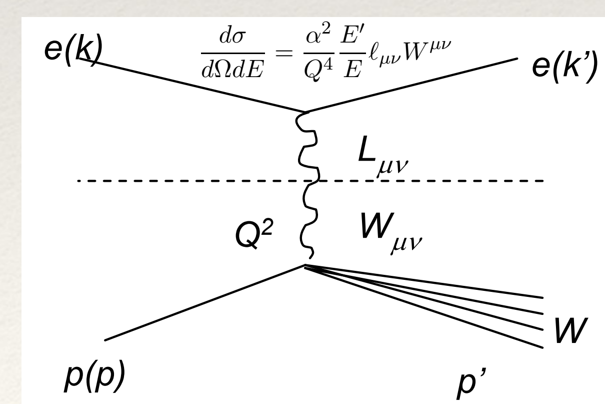
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}(Mott)\cdot\left[\frac{G_E^2 + (Q^2/4M^2)\,G_M^2}{1+Q^2/4M^2} + \frac{Q^2}{4M^2}2\,G_M^2\,tg^2(\theta/2)\right]$$

proton recoil

Here, the G_E and G_M are 'form factors' - charge and magnetic structure of proton

- Deep inelastic scattering: e- elastic scattering off spin 1/2 pointlike partons

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4(\theta/2)}\frac{E'}{E}\left[W_2(\nu,Q^2)\cos^2(\theta/2) + 2W_1(\nu,Q^2)\sin^2(\theta/2)\right]$$



W1 and W2 are proton structure functions and <u>are physical observables</u>

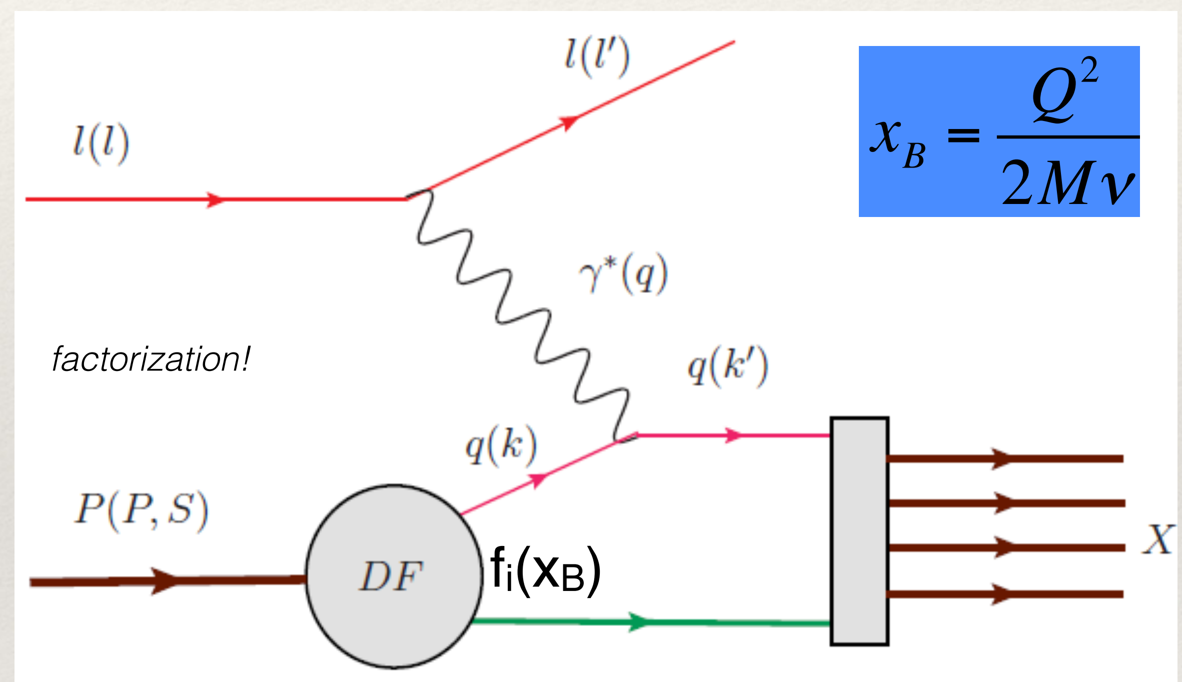
Q & A: what are the structure functions?

- Naive quark-pardon model: deep inelastic scattering is effectively an elastic scattering of a point-like parton that carries a momentum fraction x_B of the proton

$$W_1(Q^2, \nu) = \sum_i e_i^2 f_i(x_B) \frac{1}{2M}$$

$$W_2(Q^2, \nu) = \sum_i e_i^2 f_i(x_B) \frac{x_B}{\nu}$$

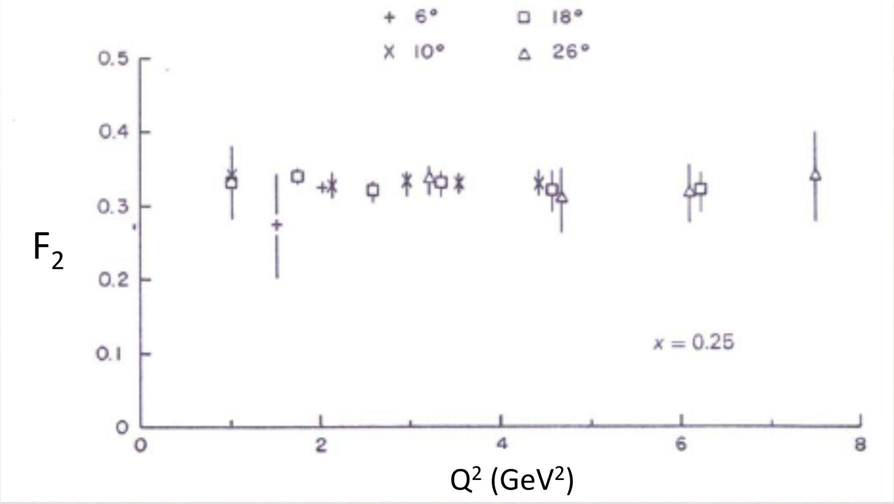
$f_i(x_B)$ is **parton distribution function** (PDF) and describes the probability to find parton of type *i* that has a fraction x_B of the proton momentum



Q & A: what are the structure functions?

- Bjorken scaling - W_1 and W_2 plotted as a function of x_B are independent of Q^2

$$F_1(x_B) = MW_1(Q^2,\nu) = \frac{1}{2}\sum_i e_i^2 f_i(x_B)$$
$$F_2(x_B) = \nu W_2(Q^2,\nu) = \sum_i e_i^2 x_B f_i(x_B)$$



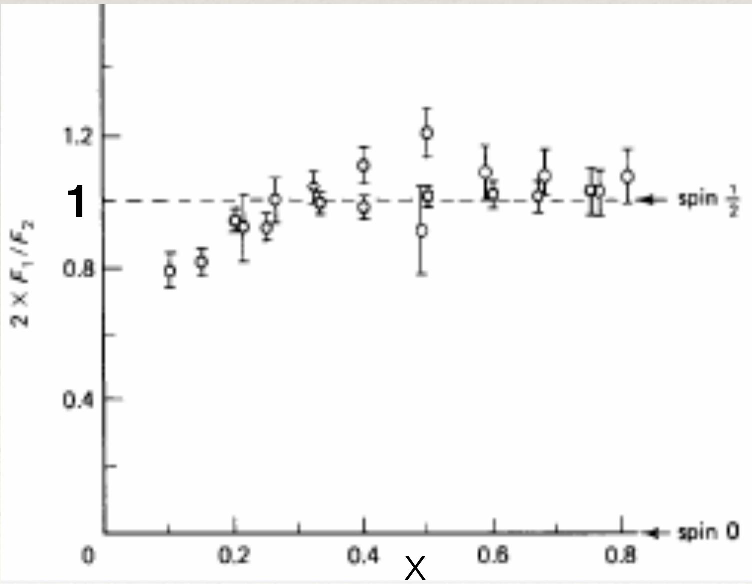
Data from SLAC experiment Ann. Rev: Nucl. Sci. 22,203 (1972)

- No explicit behavior on Q^2 : scattering off point-like constituents

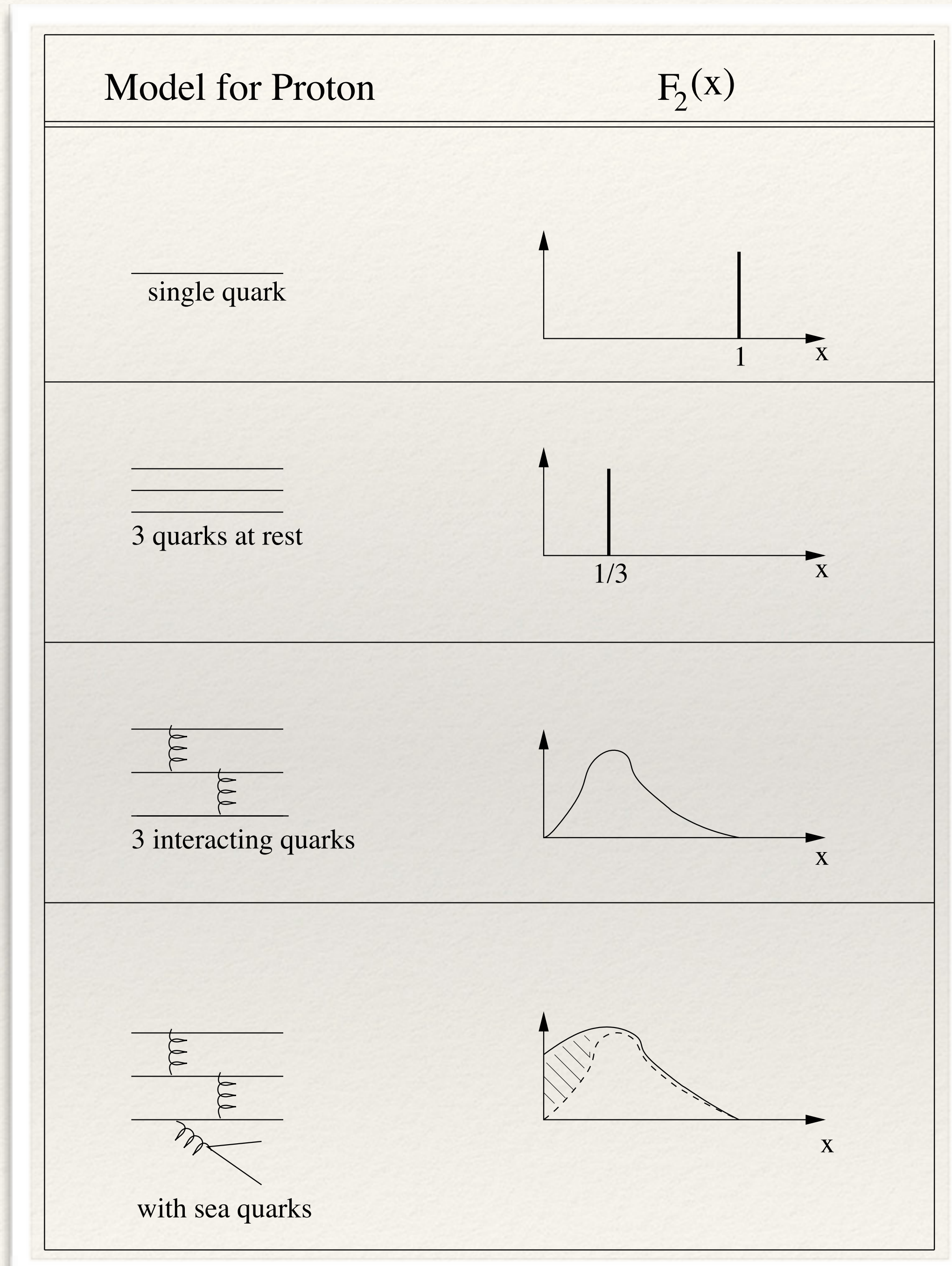
- Partons are spin 1/2 point-like constituents (Callan-Gross relation)

$$2x_B F_1(x_B) = F_2(x_B) \rightarrow 2xF_1(x_B)/F_2(x_B) = 1$$

(if partons were spin 0, the F1=0)



Q & A: what are the structure functions?



Q & A: what are the structure functions?

- In quark parton model, DIS cross section:

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

zeroth order in α_s

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

Q & A: what are the structure functions?

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[$u(x)$, $d(x)$: parton distribution functions (PDF)]

If we would only measure the deep-inelastic structure function F2, how we could separate the PDF of the u and d quarks?

Q & A: what are the structure functions?

- In quark parton model, DIS cross section:

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

Remember!

Structure functions F_2 are observables, while

parton distribution functions q(x) are 'theoretical' quantities!

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

- Assuming SU2 symmetry (isospin): u <-> d

Combine neutrons and protons pdf:

neutron (ddu) is proton (uud)

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$

$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

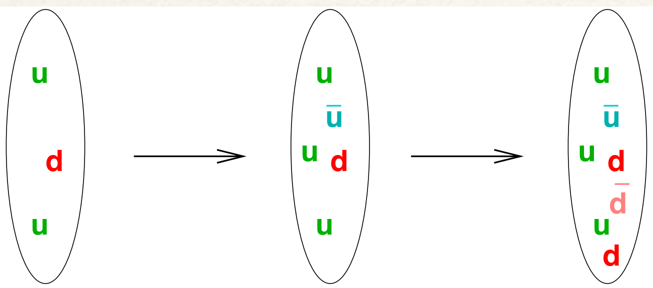
Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$

Q & A: what are the structure functions?

What about antiquarks in proton?

P.S. there is also gluon distribution functions



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge $\to$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

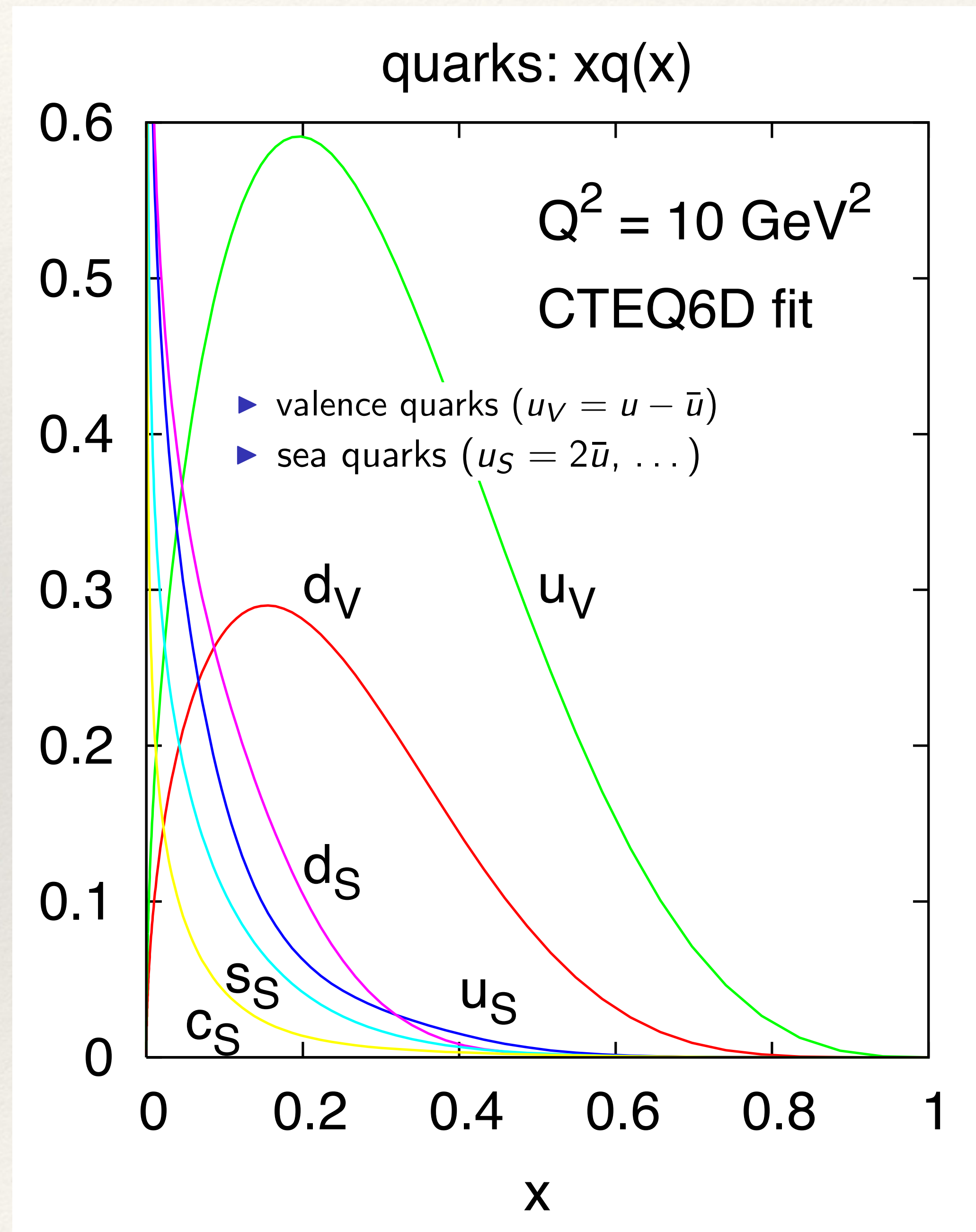
$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.



from Gavin Salam lecture at the 2009 European School of High-Energy Physics

Q & A: what is EMC effect?

- From a very naïve assumption, nuclear quark distributions = sum of protons and neutrons

$$F_2^A(x) = ZF_2^p(x) + NF_2^n(x)$$

N - number of neutrons
Z - number of protons

Q & A: what is EMC effect?

- From a very naïve assumption, nuclear quark distributions = sum of protons and neutrons

$$\xcancel{F_2^A(x) = ZF_2^p(x) + NF_2^n(x)}$$

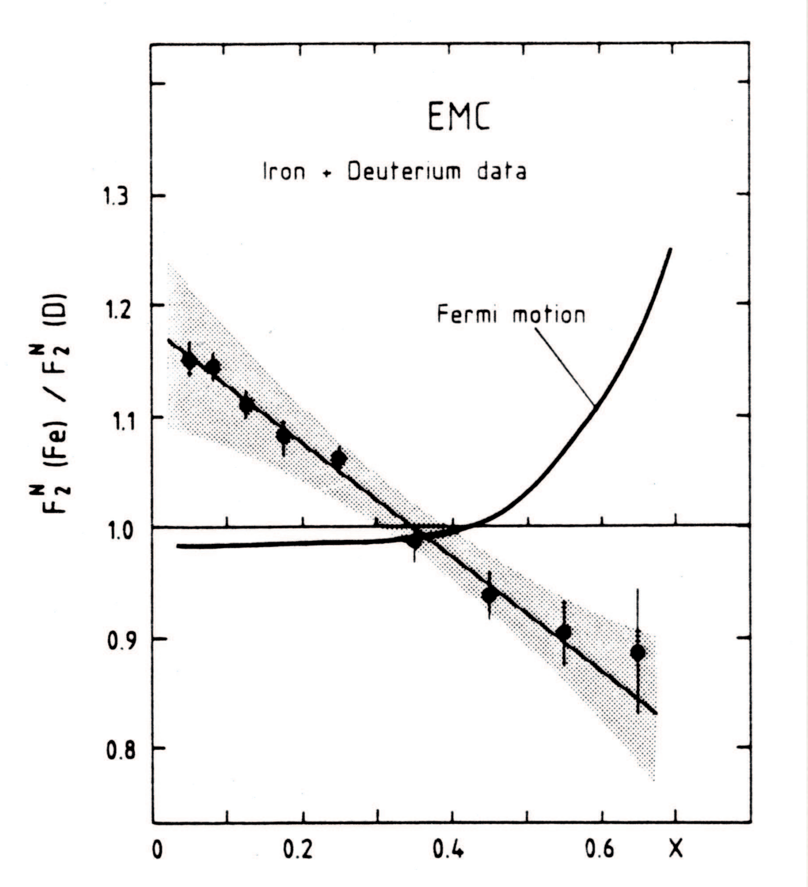
N - number of neutrons
Z - number of protons

- EMC (European Muon Collaboration) in 1983: quarks in nuclei behave differently than the quarks in free nucleon!

Quark distribution are modified inside nuclei!

Cross section ratio = Structure function ratio

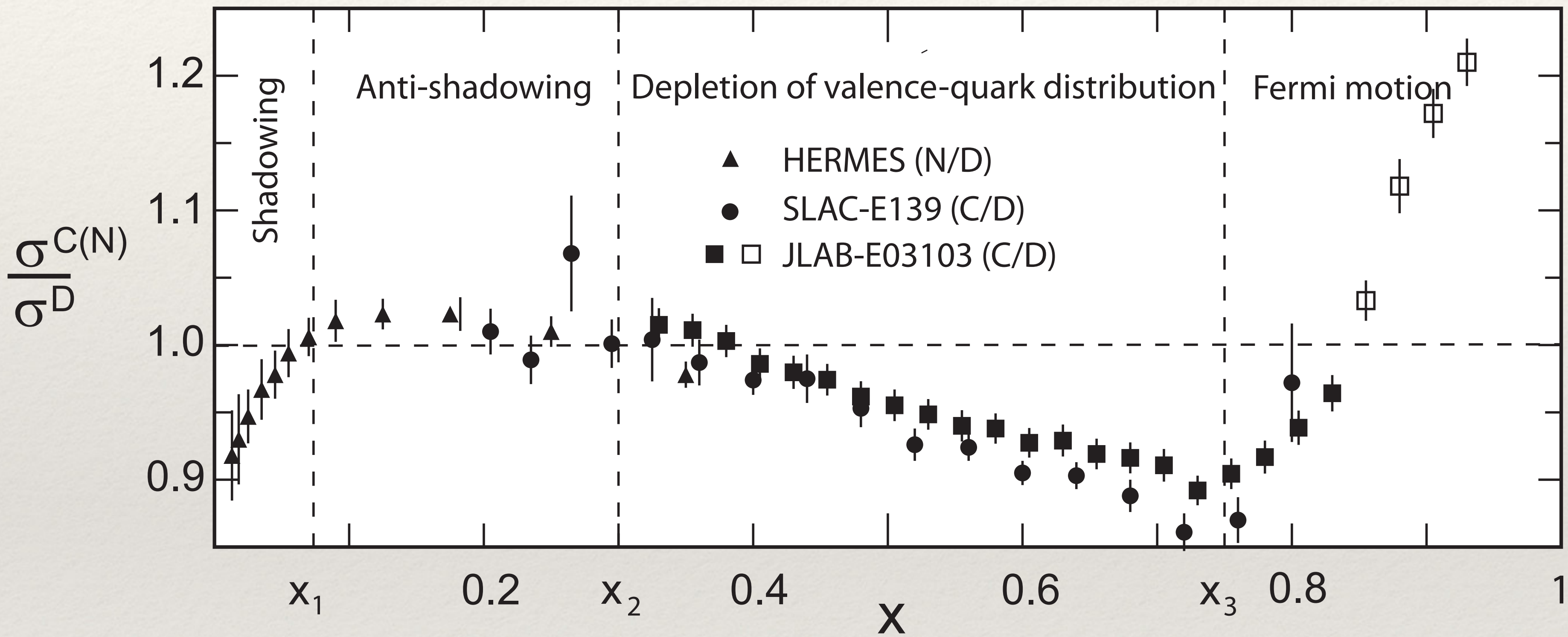
(Assuming R=σL/σT is independent of A, plus isospin corrections)



Aubert et al., Phys. Lett. B123, 275 (1983)

Q & A: what is EMC effect?

Wealth of data on the EMC effect over the past 30 years



K.Rith 'Present status of EMC effect' arXiv:1402.5000v1 [hep-ex] 20 Feb 2014

Q & A: what is EMC effect?

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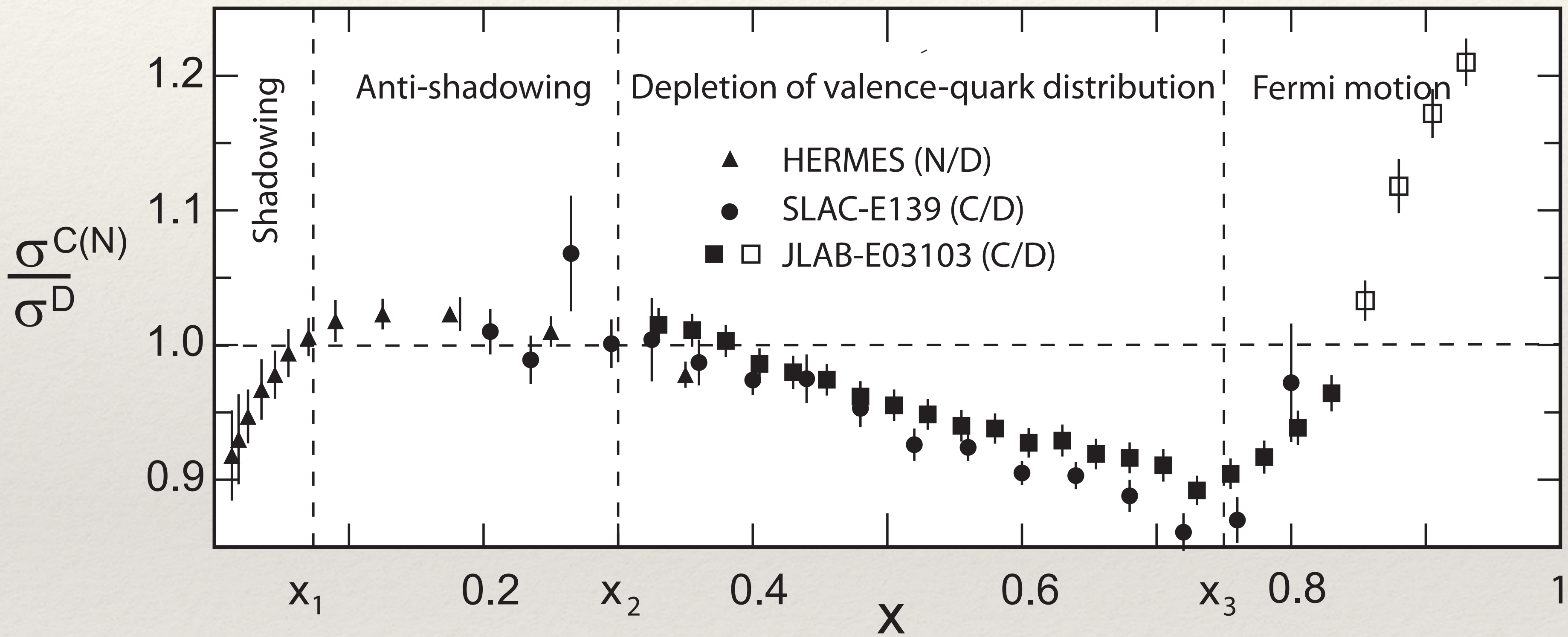


Universal shape for all nuclei in the region $0.3<x<0.7$ and $3<A<197$

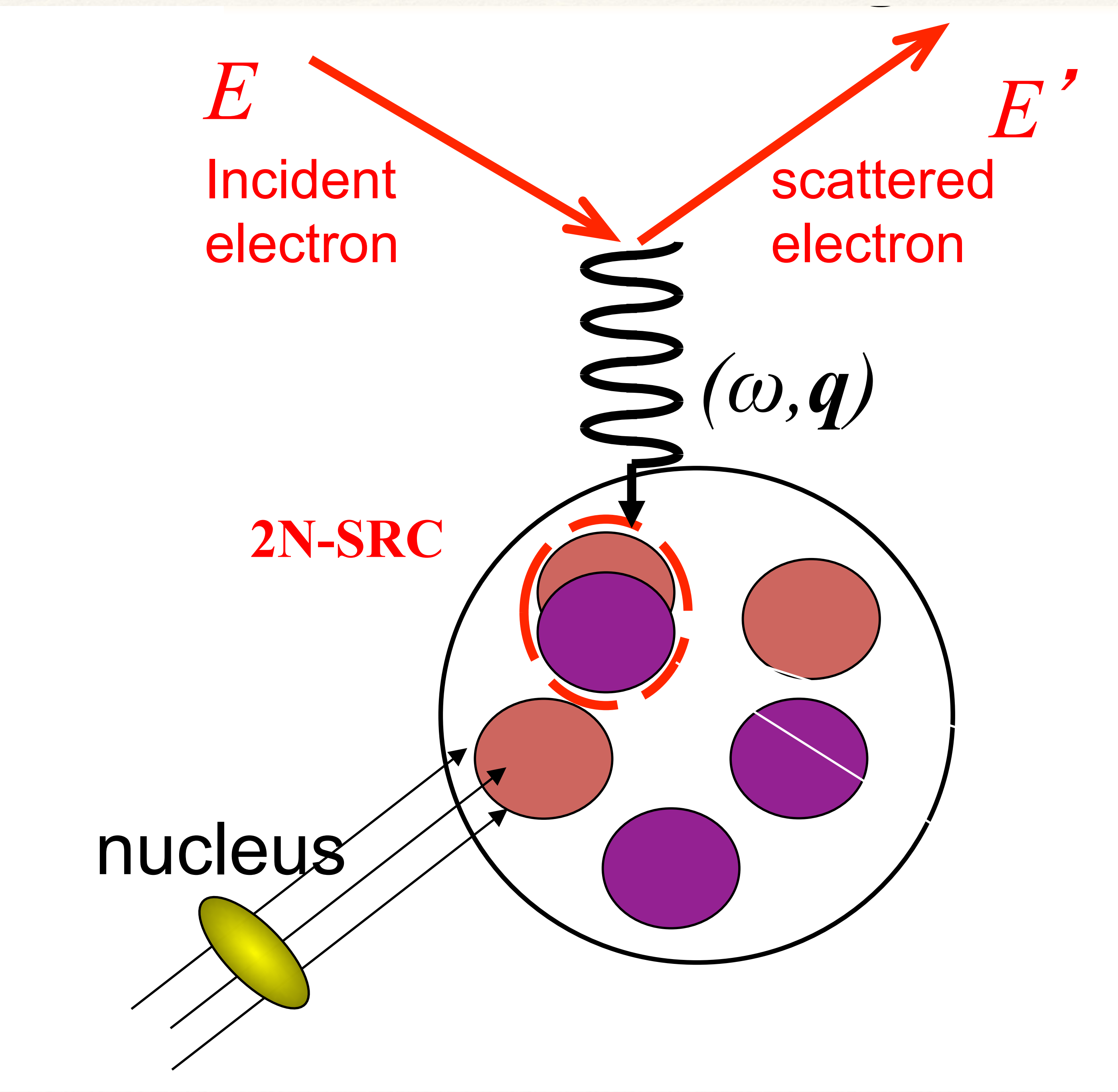
Over 1000 papers but the origin of the EMC effect is still debated upon!

K.Rith 'Present status of EMC effect' arXiv:1402.5000v1 [hep-ex] 20 Feb 2014

Q & A: what are SRC effect?

- Only about 60-70% of *nucleons in nuclei* are in single-particle mean-field orbitals.
- Some nucleons are in long-range correlated pairs and the rest of the nucleons are in short-range correlated (SRC) NN pairs.
- These SRC pairs are characterized by a large relative (to Fermi momentum) momentum and small center-of-mass momentum
- When a nucleon belongs to an SRC pair, its momentum is balanced by one other nucleon, not by the A−1 other nucleons.

Q & A: what are SRC effect?



$$Q^2 = -q_\mu q^\mu = q^2 - \omega^2$$

$$\omega = E' - E$$

$$x_B = \frac{Q^2}{2m_N\omega}$$

$$Q^2 \geq 1.5\ \mathrm{GeV}^2$$

$$0 \leq x_\mathrm{B} \leq A$$

x_B counts the number of nucleons involved :
$x_\mathrm{B} > n$ ➔ at least n+1 nucleons

from Larry Weinstein

Q & A: what are SRC effect?

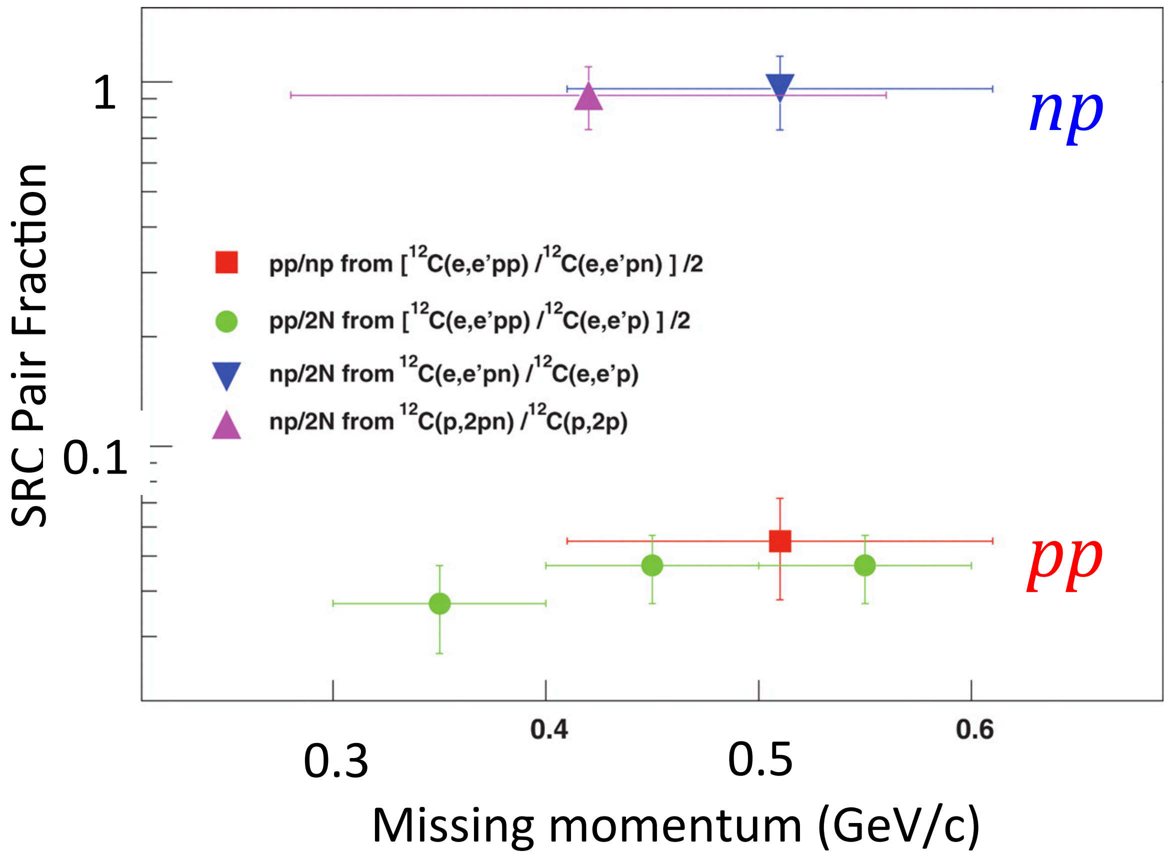
High-p nucleons have correlated partners

- Almost **all** protons with p_i > 300 MeV/c in ^{12}C(e,e'p) have a proton or neutron partner with ~opposite momentum

$$\frac{^{12}C(e,e'pn)}{^{12}C(e,e'p)} = 96^{+4}_{-23}\%$$

$$\frac{^{12}C(e,e'pp)}{^{12}C(e,e'p)} = 9.5 \pm 2\%$$

$$\frac{np}{pp} \approx 18$$



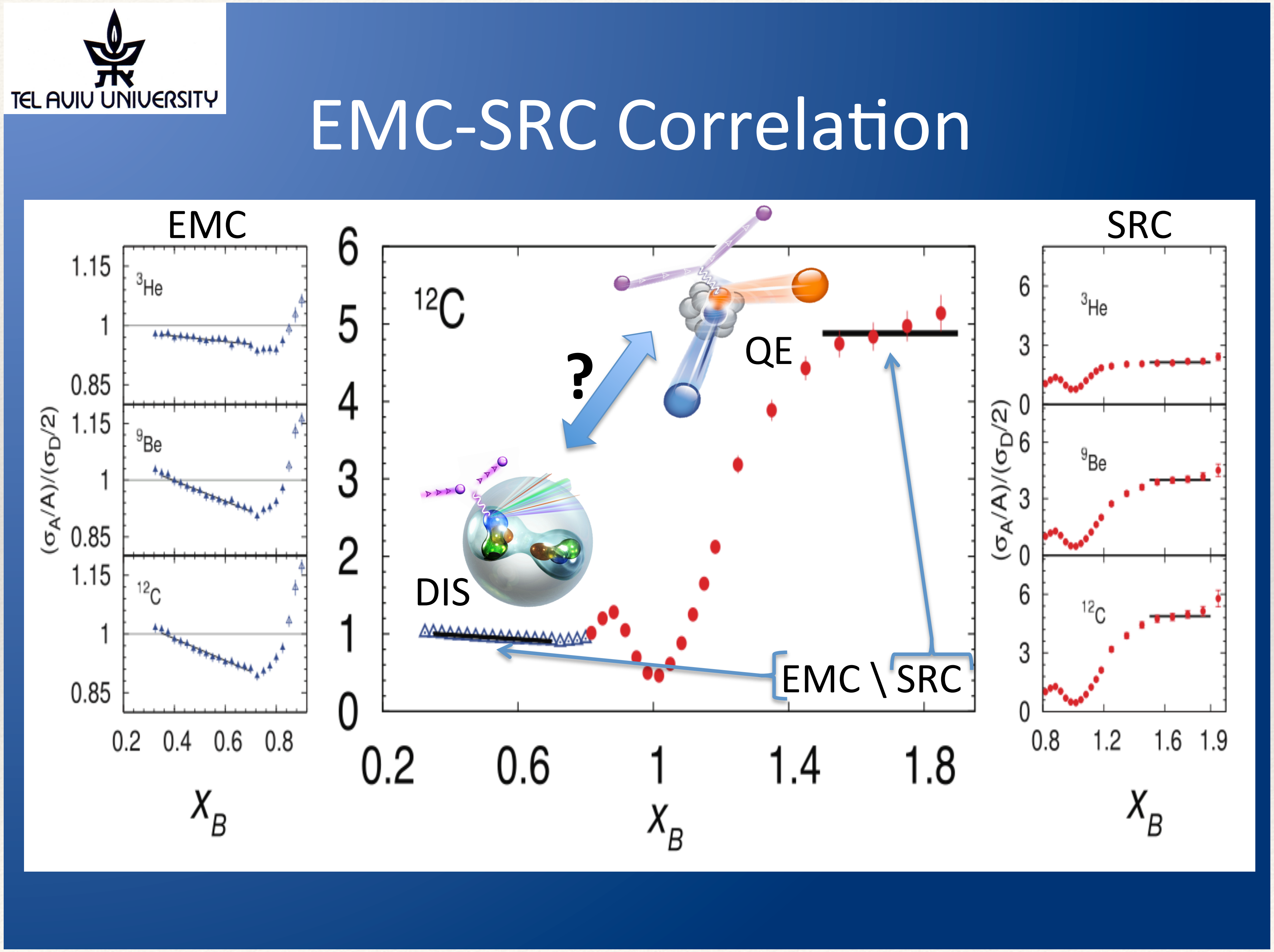
Ratios corrected for acceptance, det. efficiency and SCX

R. Subedi et al., Science **320** (5882), 1476 (2008)

R. Shneor *et al.*, PRL**99, 072501 (2007)**

from Larry Weinstein

Q & A: what are SRC effect?



from Or Hen

Lecture 2

running EXTERNALS code

Radiated vs Born cross section

The differences between the Born cross section and the measured cross section are radiative corrections.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Meas}} = \boxed{(1+\delta_{rad})} \left.\frac{d\sigma}{d\Omega}\right|_{\text{Born}}$$

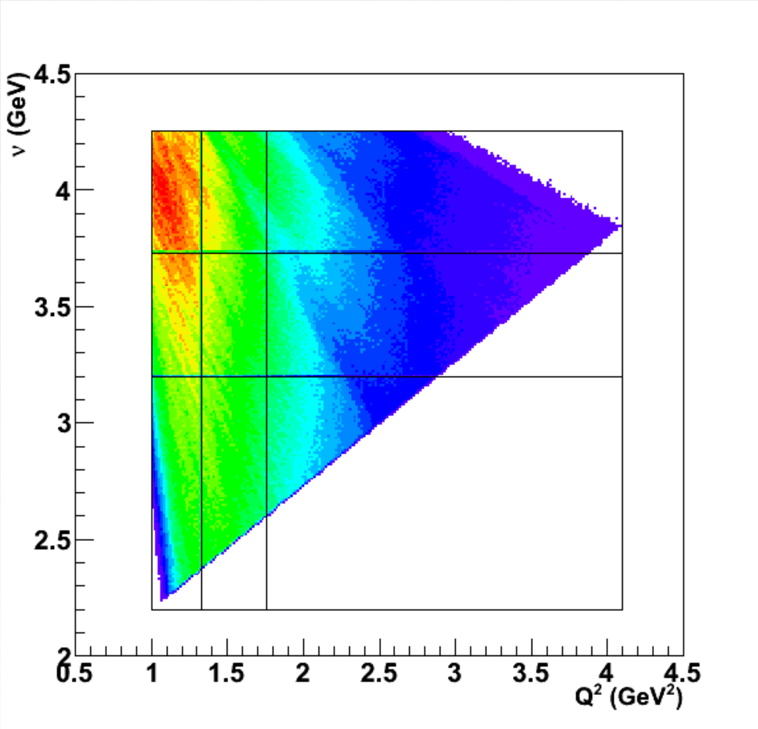
We extract RC factor with EXTERNALS code and apply this correction to data in a way:

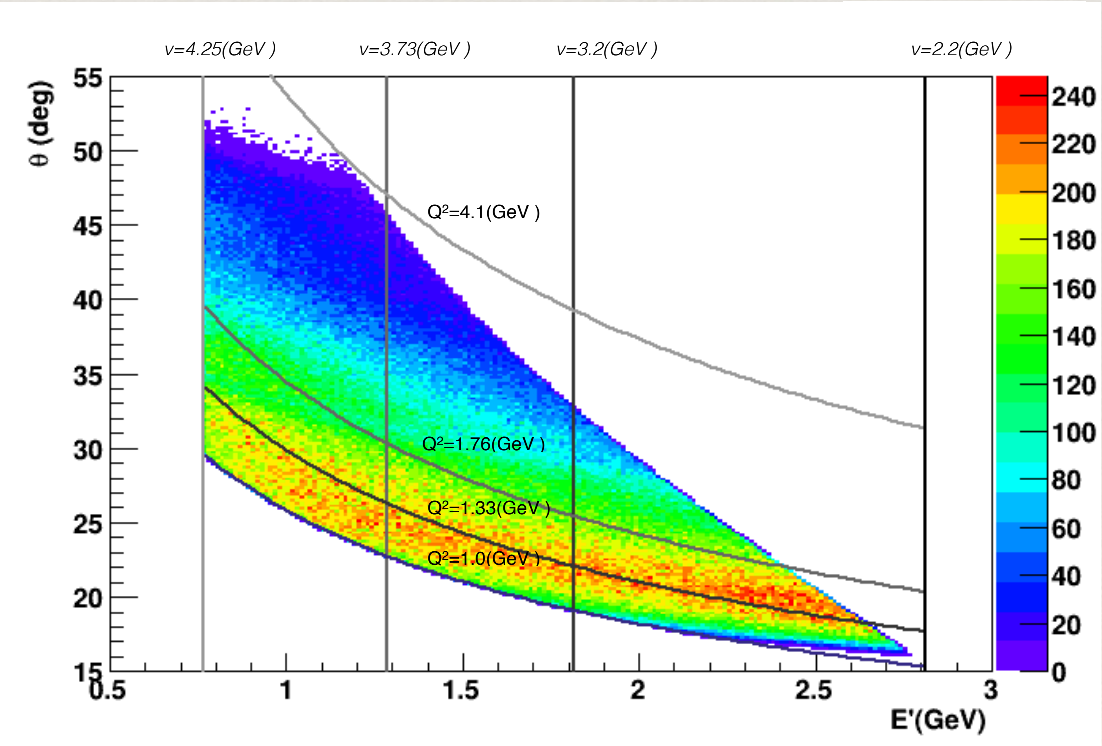
$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{meas}} = (1+\delta_{rad}) \left.\frac{d\sigma}{d\Omega}\right|_{\text{Born}} \xrightarrow{\delta_{RC}=1+\delta_{rad}} \boxed{\begin{array}{c} \delta_{RC} = \sigma_{Rad} / \sigma_{Born} \\ \sigma^{e}_{corr} = \sigma^{e}_{meas} / \delta_{RC} \end{array}}$$

What are the inputs to the EXTERNALS?

- Electron beam energy, scattering electron energy and angle θ
- Target type A (N, Z)

- If you have DIS electron in (ν, Q2) bins convert it to (E’, θ)
- The mean values <E’>, <θ> of each bin (E’, θ) will be your input





Let's runt the EXTERNALS code

- `cp -r /user/m/mineeva/EXTERNALS/externals_2017 .`
- `cd externals_2017`
- Open README file. **All instructions are there!**

READMe file

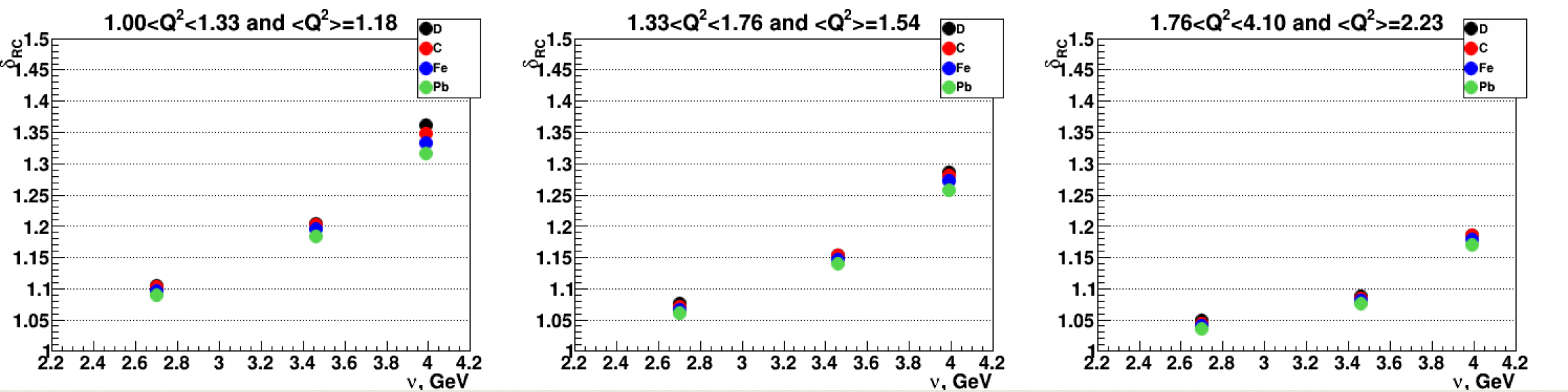
```
Structure of the EXTERNALS code:

1. INP/clasd2.inp  - this file is the "master" input file that basically just points to the other
required inputs.

2. RUNPLAN/clas_kin.inp - this file contains the kinematics at which to calculate the cross section and
RC. Note that it's pretty sensitive to formatting.

3. TARG/targ.D2tuna - this file contains a lot of info about the target (Z,A, geometry, model to use,
etc.). Right now, the only variables that matter are Z and A since the program only calculates the
internal corrections.  The model choice is also hardwired for now to use F1F209 from Peter Bosted.

4. OUT/clasd2_details.out  - this gives more detailed output than the summary table above.

5. To run, you just use the little script (run_extern) in the top level directory.
The usage is: "./run_extern <input-file>", leave the ".inp" off the master input file.

To compile on the UTFSM cluster run:
1.  rm  *.o
2.  source /user/a/alaoui/software/env_scripts/set_64bit_taya.sh --roover 6.10.02 --cerver 2005 --
softdir /user/a/alaoui/software/ --clasver ver1
3.  make

Running the code:

1. Deuterium target
  ./run_extern clasd2
2. Lead target
   ./run_extern clasPb208


Output files are in OUT/
```

Visualize the result

Radiative correction factors from EXTERNALS

$$\delta_{RC} = \sigma_{Rad} \,/\, \sigma_{Born} \quad \text{and} \quad N \sim N^{e}_{meas} / \delta_{RC}$$

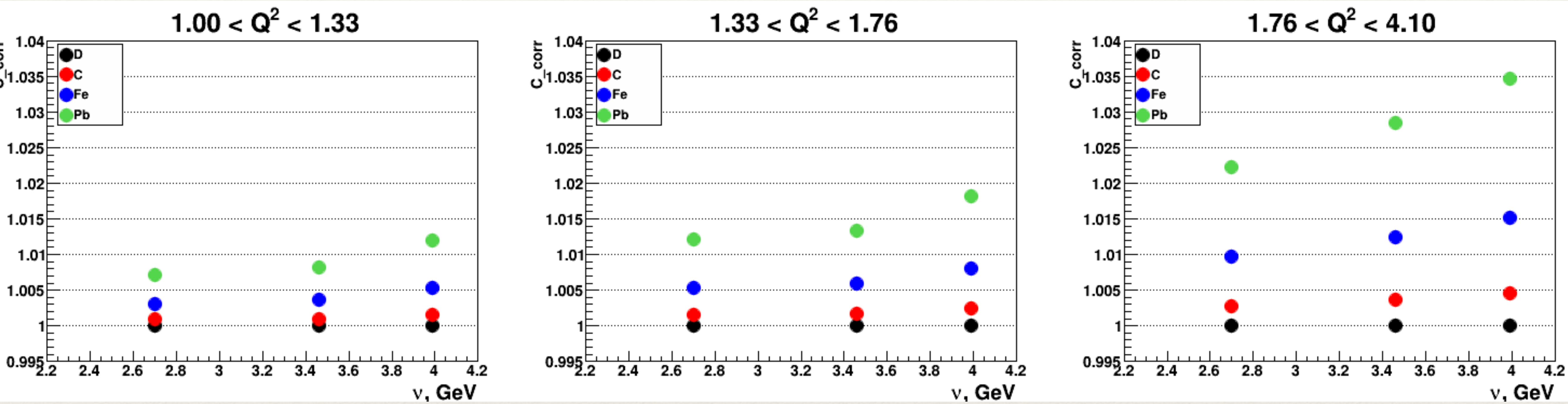
Radiative correction factors δ_{RC} from EXTERNALS for D, C, Fe, Pb in (Q2, ν) bins



Note:in this kinematics, RC factors δ_{RC} f could be as high as 30% corrections!

Coulomb corrections from EXTERNALS

$$C\_Corr = \sigma_{QE} \,/\, \sigma_{Coulomb} \text{ and } N \sim N^{e}_{meas} * C\_Corr$$



Coulomb correction factors in this kinematics are 3.5% at maximum for Pb target

Now you are all set to extract your own inclusive electron RC on nuclear targets!