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ORIE 5380/ CS 5727 Optimization Methods (Fall 2024)
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Project (Deadline: Mon, Dec 2, 2024 at 7 PM)

XXX, a company specializing in a digital platform market, is optimizing its delivery network in City A. The company has $|M|$ warehouses in the city, where M is the set of warehouses, and, assume that for a particular day, there are $|N|$ customers who need a delivery, where N is the set of customers. The company currently employs k delivery drivers.

We first assume that each customer only requests one item of product A and warehouse i has s_i items of product A. Assume that locations lie on a two-dimensional Cartesian coordinate plane and that the distance between any two pairs of locations is calculated using the Euclidean distance formula. A location can be a warehouse or a customer. Also, assume that the total supply is greater or equal to the total demand.

Question 1. Assume that each delivery driver can complete only one delivery per day. One delivery here refers to a one-way trip from one location to another. Additionally, each driver can start their route from any warehouse. For this question, the demand may not be fully satisfied for a particular day, but we must try to satisfy it as much as our capacity allows. The objective is to match the drivers to the customers minimizing the total distance traveled by the drivers. We let d_{ij} represent the Euclidean distance between warehouse $i \in M$ and customer $j \in N$. You are asked to:

1) Formulate an optimization model in general form (using d_{ij}, s_i, etc. instead of the values in the dataset) through a linear program.
2) Solve this optimization problem on the dataset we provide using Gurobi and print out the final delivery schedule you find.

Question 2. Assume now that each delivery driver can handle deliveries up to 5 customers. The company needs to determine the best way to arrange the route for each delivery person. The objective is to minimize the total traveling distance for all delivery drivers from the warehouse they originate from to the last customer they deliver to. A colleague proposes that we can model the delivery route assignment as the "flow" of items in a min-cost flow problem. For example, if we have a flow of 3 units from warehouse i to customer 1, 2, and 3 (see Figure 1), we can consider this as a delivery schedule for a driver who picks up 3 units of items from warehouse i and then travels to customer 1, 2, and 3 sequentially to deliver the items.

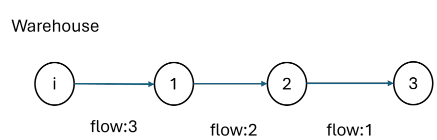

FIGURE 1. Example flow

The colleague then suggests that if we have an unlimited number of drivers, we can model the problem as a min-cost flow problem. He claims that we can view warehouses as supply nodes and customers as demand nodes with exactly one unit of demand. Then we can define the decision variable y_{ij}, representing the number of items of product A transported from node i to node j. We can minimize the total distance traveled by all items of product A subject to the usual constraints in a min-cost flow network. His idea is that items are matched with drivers, so solving a min-cost flow problem gives the best delivery schedule under the assumption that there is an unlimited number of drivers. The following is the LP that your colleague comes up with, where d_{ij} represents the Euclidean distance between location i and customer

$j \in N$:

$$
\begin{aligned}
\min \sum_{i \in M \cup N, j \in N} & y_{ij} d_{ij} \\
\text{s.t.} \sum_{j \in N} y_{ij} \leq s_i, & \quad \text{for all } i \in M \\
\sum_{i \in N \cup M} y_{ij} = \sum_{i \in N} & y_{ji} + 1, \quad \text{for all } j \in N \\
y_{ij} \geq 0, & \text{ for all } i \in M \cup N, j \in N.
\end{aligned}
$$

If we solve this LP, what would the optimal delivery schedule be like? Do you think this is the schedule that minimizes the total distance traveled by all drivers when we have an unlimited number of drivers? Why or why not?

Question 3. Reformulate the problem introduced in Question 2 by adjusting the linear program given by your colleague. This may require introducing additional integer constraints. In this problem, assume the following: 1. Drivers will start from a warehouse and will not visit any warehouse again on their delivery path. 2. Each customer will be visited by no more than one driver. 3. The demand must be satisfied. You are asked to:

1) Formulate the optimization problem as an integer program
2) Solve the resulting optimization problem on the provided dataset using Gurobi.
3) Provide the final optimal delivery schedule and briefly describe it.

Hint: Start from the "flow" discussed in the previous question. How can we define a decision variable that gives us some information about the number of drivers that we need to have and whether they are deployed on a particular route?

Question 4. The company wants to understand how the number of drivers affects the total distance traveled. Vary the number of drivers k from 4 to 10. For each value of k, solve the integer program from Question 3 and plot the trend of the total distance traveled.

Question 5. (BONUS Question) In the previous question, we assumed that drivers could start their routes directly from any warehouse. Now, the company also wants to account for the fact that drivers need to travel from their homes to a warehouse. Let D represent the set of drivers' home locations. Modify the integer program (IP) from Question 3 to minimize the total travel distance for all drivers, starting from their home locations and ending at the last customer they deliver to. You are asked to:

1) Formulate the optimization problem as an integer program
2) Solve the resulting optimization problem on the provided dataset using Gurobi.
3) Provide the final optimal delivery schedule and briefly describe it.