

Web-based Supporting Materials for “Joint analysis for
longitudinal data with nonignorable missing” by Jun Zhang,
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Table 1: Simulation results with different intermittent missing proportions.

	Naive (setting 1)			Joint (setting 1)			Naive (setting 2)			Joint (setting 2)		
	BIAS %	SD	CP	BIAS %	SD	CP	BIAS %	SD	CP	BIAS %	SD	CP
Parameters for continuous outcomes												
$a_1^{(1)} = -4$	0.475	0.053	0.959	0.125	0.051	0.967	0.525	0.055	0.950	0.175	0.055	0.967
$a_1^{(2)} = -2$	0.950	0.070	0.914	0.300	0.066	0.949	0.800	0.070	0.927	0.350	0.070	0.935
$b_1^{(1)} = 1$	0.800	0.038	0.923	0.800	0.038	0.930	1.200	0.040	0.909	0.800	0.039	0.921
$b_1^{(2)} = 1.5$	0.800	0.050	0.950	0.667	0.049	0.953	1.000	0.052	0.923	0.600	0.051	0.949
$b_2^{(1)} = 1.5$	0.867	0.031	0.923	0.067	0.031	0.921	1.467	0.034	0.905	0.133	0.033	0.949
$b_2^{(2)} = 1$	0.800	0.032	0.936	0.000	0.031	0.953	1.400	0.034	0.905	0.100	0.034	0.949
Parameters for ordinal outcomes**												
$a_{3,1} = 0.1$	-3.000	0.029	0.936	-1.000	0.029	0.935	-5.000	0.031	0.932	-2.000	0.030	0.940
$a_{4,1} = -0.1$	1.000	0.027	0.936	-1.000	0.027	0.930	1.000	0.029	0.914	-0.000	0.029	0.921
$a_{5,1} = 0.2$	3.500	0.032	0.955	1.500	0.032	0.967	4.500	0.032	0.964	1.500	0.032	0.963
$a_{6,1} = -0.2$	-1.000	0.023	0.959	0.500	0.021	0.977	-1.500	0.023	0.950	-1.000	0.023	0.953
$a_{7,1} = 0.3$	0.333	0.031	0.964	-1.000	0.031	0.963	1.333	0.033	0.955	-0.333	0.032	0.944
$a_{8,1} = -0.3$	-1.000	0.023	0.950	-0.667	0.023	0.953	-0.667	0.024	0.964	-0.333	0.024	0.958
$a_{9,1} = 0.4$	0.500	0.026	0.945	0.500	0.025	0.949	1.000	0.025	0.945	0.500	0.025	0.949
$a_{10,1} = -0.4$	1.250	0.034	0.968	0.750	0.033	0.977	1.750	0.035	0.959	0.500	0.034	0.972
$a_{1,2} = 1.5$	-0.067	0.029	0.950	-0.067	0.027	0.972	-0.067	0.029	0.950	-0.133	0.029	0.963
$a_{2,2} = 1.5$	-0.200	0.030	0.959	-0.133	0.030	0.963	-0.200	0.032	0.959	-0.067	0.032	0.963
$a_{3,2} = 1.6$	-0.188	0.033	0.955	0.125	0.032	0.967	-0.188	0.036	0.950	0.062	0.035	0.958
$a_{4,2} = 1.4$	-0.071	0.031	0.927	0.000	0.032	0.926	-0.071	0.032	0.923	0.071	0.033	0.926
$a_{5,2} = 1.7$	0.471	0.038	0.936	0.294	0.037	0.949	0.706	0.040	0.936	0.353	0.040	0.940
$a_{6,2} = 1.3$	0.231	0.026	0.968	0.000	0.026	0.972	0.385	0.027	0.964	0.231	0.027	0.963
$a_{7,2} = 1.8$	0.222	0.038	0.959	0.000	0.037	0.963	0.333	0.038	0.968	0.000	0.038	0.967
$a_{8,2} = 1.2$	0.250	0.027	0.941	0.167	0.026	0.944	0.167	0.028	0.923	0.083	0.028	0.949
$a_{9,2} = 1.9$	0.053	0.034	0.936	0.000	0.034	0.940	0.158	0.036	0.941	0.053	0.036	0.935
$a_{10,2} = 1.1$	-0.273	0.033	0.973	-0.091	0.034	0.967	-0.636	0.035	0.968	-0.182	0.035	0.953
$a_{1,3} = 2$	-0.100	0.037	0.941	-0.100	0.035	0.944	-0.100	0.036	0.945	-0.100	0.035	0.953
$a_{2,3} = 2$	-0.050	0.034	0.941	0.000	0.033	0.949	-0.150	0.036	0.945	0.000	0.036	0.940
$a_{3,3} = 2.1$	-0.095	0.037	0.950	0.095	0.037	0.949	-0.095	0.040	0.945	0.095	0.039	0.944
$a_{4,3} = 1.9$	-0.105	0.033	0.955	-0.053	0.034	0.949	-0.105	0.037	0.941	-0.053	0.038	0.935
$a_{5,3} = 2.2$	0.364	0.040	0.945	0.227	0.039	0.949	0.545	0.042	0.959	0.273	0.043	0.967
$a_{6,3} = 1.8$	0.278	0.029	0.959	0.111	0.029	0.963	0.389	0.030	0.950	0.278	0.030	0.958
$a_{7,3} = 2.3$	0.261	0.041	0.950	0.087	0.040	0.958	0.391	0.042	0.964	0.130	0.041	0.967
$a_{8,3} = 1.7$	0.176	0.029	0.941	0.176	0.029	0.935	0.118	0.030	0.936	0.059	0.030	0.940
$a_{9,3} = 2.4$	0.042	0.040	0.936	0.042	0.040	0.926	0.083	0.042	0.950	0.042	0.042	0.944
$a_{10,3} = 1.6$	-0.188	0.037	0.968	-0.062	0.038	0.958	-0.437	0.039	0.968	-0.125	0.039	0.963
$a_{1,4} = 2.5$	-0.080	0.040	0.941	-0.040	0.038	0.949	-0.080	0.039	0.950	0.000	0.038	0.972
$a_{2,4} = 2.5$	-0.040	0.041	0.945	0.040	0.040	0.958	-0.080	0.041	0.936	0.040	0.041	0.949
$a_{3,4} = 2.6$	-0.077	0.043	0.932	0.038	0.043	0.940	-0.115	0.044	0.950	0.038	0.044	0.958
$a_{4,4} = 2.4$	-0.042	0.037	0.959	-0.042	0.038	0.958	-0.125	0.041	0.909	-0.042	0.042	0.926
$a_{5,4} = 2.7$	0.296	0.044	0.950	0.185	0.044	0.949	0.481	0.047	0.959	0.296	0.047	0.958
$a_{6,4} = 2.3$	0.348	0.033	0.964	0.217	0.033	0.958	0.391	0.034	0.968	0.304	0.034	0.972
$a_{7,4} = 2.8$	0.250	0.044	0.950	0.107	0.043	0.958	0.357	0.046	0.945	0.143	0.046	0.944
$a_{8,4} = 2.2$	0.182	0.033	0.932	0.136	0.033	0.926	0.136	0.035	0.932	0.136	0.035	0.940
$a_{9,4} = 2.9$	0.103	0.045	0.950	0.103	0.045	0.958	0.138	0.046	0.955	0.138	0.046	0.958
$a_{10,4} = 2.1$	-0.143	0.040	0.964	0.000	0.041	0.977	-0.333	0.041	0.968	-0.095	0.042	0.967
$b_3^{(1)} = 0.5$	0.400	0.021	0.927	0.200	0.022	0.916	0.400	0.022	0.936	0.000	0.022	0.935
$b_4^{(1)} = 0.4$	1.000	0.018	0.955	0.750	0.018	0.935	1.000	0.019	0.950	0.750	0.019	0.953
$b_5^{(1)} = -0.4$	1.250	0.019	0.950	0.750	0.019	0.949	1.000	0.020	0.945	0.500	0.020	0.958
$b_6^{(1)} = -0.2$	1.000	0.014	0.950	0.500	0.014	0.949	1.000	0.016	0.918	0.500	0.016	0.935
$b_7^{(1)} = -0.5$	1.200	0.022	0.918	0.800	0.020	0.944	1.200	0.022	0.945	0.600	0.021	0.944
$b_8^{(1)} = 0.2$	0.000	0.015	0.932	0.000	0.015	0.935	0.500	0.016	0.927	0.500	0.016	0.930
$b_9^{(1)} = -0.3$	1.000	0.016	0.918	0.667	0.016	0.940	1.333	0.017	0.941	0.667	0.017	0.949
$b_{10}^{(1)} = 0.6$	0.833	0.023	0.936	0.333	0.022	0.958	0.667	0.024	0.964	0.167	0.023	0.967
$b_3^{(2)} = -0.3$	0.667	0.013	0.964	0.333	0.013	0.944	1.333	0.014	0.941	-0.000	0.014	0.967
$b_4^{(2)} = -0.2$	1.500	0.011	0.955	1.000	0.011	0.958	2.000	0.012	0.941	0.500	0.012	0.967
$b_5^{(2)} = 0.6$	1.167	0.017	0.927	0.500	0.017	0.949	1.667	0.019	0.927	0.500	0.018	0.940
$b_6^{(2)} = 0.2$	1.500	0.010	0.941	1.000	0.010	0.949	2.000	0.011	0.941	1.000	0.011	0.958
$b_7^{(2)} = 0.5$	1.200	0.016	0.936	0.400	0.015	0.967	1.800	0.016	0.955	0.400	0.016	0.977
$b_8^{(2)} = 0.1$	1.000	0.010	0.955	0.000	0.010	0.953	1.000	0.010	0.950	0.000	0.010	0.963
$b_9^{(2)} = 0.1$	1.000	0.011	0.945	1.000	0.011	0.944	2.000	0.012	0.927	0.000	0.012	0.930
$b_{10}^{(2)} = -0.5$	1.200	0.017	0.945	0.400	0.016	0.953	1.600	0.018	0.923	0.400	0.018	0.967

Setting 1: Total 31.27% missing, 20.73% intermittent miss.

Setting 2: Total 39.10% missing, 29.30% intermittent miss.

$a_{k,l}$: item k's level l.

Table 2: Estimated random effects.

	Mean	SD	95% CI	
$\sigma_0^{(1)}$	1.160	0.065	1.037	1.286
$\sigma_1^{(1)}$	0.246	0.021	0.206	0.287
$\sigma_0^{(2)}$	1.484	0.083	1.327	1.655
$\sigma_1^{(2)}$	0.360	0.031	0.302	0.422
$\sigma_0^{(3)}$	3.506	0.177	3.172	3.860
$\sigma_1^{(3)}$	0.576	0.042	0.500	0.662

σ_0 : random intercept.

σ_1 : random slope.

superscript (k) : k th domain.

Table 3: Estimated correlation coefficients for random effects.

	nM-EDL	M-EDL		Motor		
1.000	$-0.179_{0.077}^*$	$0.626_{0.037}$	$-0.079_{0.077}$	$0.187_{0.053}$	$0.039_{0.076}$	$nM - EDL_{sd}$
	$[-0.324, -0.027]^{**}$	$[0.553, 0.695]$	$[-0.236, 0.064]$	$[0.082, 0.288]$	$[-0.111, 0.180]$	95%CI
	1.000	$-0.017_{0.082}$	$0.665_{0.060}$	$-0.090_{0.073}$	$0.246_{0.087}$	$nM - EDL_{sd}$
		$[-0.212, 0.110]$	$[0.509, 0.751]$	$[-0.263, 0.033]$	$[0.025, 0.382]$	95%CI
		1.000	$-0.120_{0.074}$	$0.370_{0.046}$	$-0.045_{0.074}$	$M - EDL_{sd}$
			$[-0.259, 0.028]$	$[0.279, 0.460]$	$[-0.192, 0.100]$	95%CI
			1.000	$-0.105_{0.075}$	$0.669_{0.057}$	$M - EDL_{sd}$
				$[-0.245, 0.042]$	$[0.549, 0.772]$	95%CI
				1.000	$-0.387_{0.067}$	$motor_{sd}$
					$[-0.516, -0.258]$	95%CI
					1.000	$Motor$

* $-0.179_{0.077}$: correlation coefficient mean estimate= -0.179 , SD= 0.077 .

** $[-0.324, -0.027]$: 95% CI= $[-0.324, -0.027]$ for the above element ($-0.179_{0.077}$).

Table 4: Estimated random errors in three MDS-UPDRS parts.

	Mean	SD	95% CI	
$\sigma_e^{(1)}$	0.264	0.025	0.215	0.309
$\sigma_e^{(2)}$	0.532	0.030	0.477	0.596
$\sigma_e^{(3)}$	1.280	0.051	1.179	1.393

Table 5: Estimated parameters in Cox model and Logistic model.

	Mean	SD	95% CI	
w	-3.081	0.095	-3.299	-2.902
γ	0.036	0.139	-0.239	0.304

Table 6: Estimated parameters in MDS-UPDRS Part I: nM-EDL.

	Difficulty Parameters				Discrimination Parameters			
	$a_{k,1}$	$a_{k,2}$	$a_{k,3}$	$a_{k,4}$	Mean	SD	95% CI	
Cognitive Impairment	1.326	3.686	5.357	7.418	0.957	0.061	0.843	1.079
Hallucinations	3.400	5.705	6.636	8.583	0.824	0.081	0.673	0.993
Depressed Mood*	1.500	3.516	5.164	7.306	1.000	0.000	1.000	1.000
Anxious Mood	1.047	3.140	4.832	7.372	0.838	0.051	0.743	0.947
Apathy	2.103	3.912	6.038	8.393	1.201	0.073	1.065	1.345
Dopamine Dysregulation	3.934	5.692	8.394	17.407	0.778	0.091	0.605	0.958
Sleep Problem	−0.038	1.348	2.669	4.633	0.804	0.050	0.713	0.907
Daytime Sleepiness	−0.201	1.500	5.071	7.500	1.030	0.059	0.922	1.156
Pain & other sensations	−0.142	2.034	3.360	5.254	0.790	0.049	0.701	0.886
Urinary	−0.010	1.987	3.561	5.183	0.730	0.048	0.642	0.823
Constipation	0.576	2.823	4.459	9.246	0.772	0.049	0.683	0.872
Light Headedness	1.341	3.388	5.150	7.885	0.961	0.061	0.850	1.082
Fatigue	0.247	3.135	5.228	7.204	1.658	0.091	1.487	1.845

$a_{k,l}$: item k 's level l .

*: item to put constrains.

Table 7: Estimated difficulty parameters in MDS-UPDRS Part II: M-EDL.

	Difficulty Parameters				Discrimination Parameters			
	$a_{k,1}$	$a_{k,2}$	$a_{k,3}$	$a_{k,4}$	Mean	SD	95% CI	
Speech	0.721	2.291	4.657	8.769	0.789	0.046	0.703	0.884
Saliva & Drooling	0.734	1.619	2.978	4.867	0.673	0.041	0.597	0.754
Chewing & Swallowing	1.978	4.548	5.436	9.460	0.679	0.047	0.593	0.778
Eating Tasks*	0.800	3.722	7.312	17.501	1.000	0.000	1.000	1.000
Dressing	0.422	4.279	8.786	11.422	1.491	0.078	1.348	1.650
Hygiene	1.363	6.501	9.286	11.097	1.124	0.063	1.001	1.250
Handwriting	-0.602	1.402	3.186	5.395	0.717	0.040	0.641	0.799
Doing Hobbies	0.588	3.318	5.701	7.350	1.177	0.062	1.065	1.304
Turning in Bed	0.980	4.924	7.291	10.441	0.963	0.054	0.863	1.077
Tremor	-1.675	1.086	3.380	6.175	0.202	0.024	0.158	0.252
Getting out of bed	0.369	3.831	6.381	10.508	1.278	0.069	1.154	1.424
Walking & Balance	0.529	4.079	5.096	7.973	0.941	0.054	0.835	1.048
Freezing	3.743	5.827	7.765	10.113	1.159	0.076	1.010	1.311

$a_{k,l}$: item k 's level l .

* : item to put constrains.

Table 8: Estimated parameters in MDS-UPDRS Part III: Motor Examination.

	Difficulty Parameters				Discrimination Parameters			
	$a_{k,1}$	$a_{k,2}$	$a_{k,3}$	$a_{k,4}$	Mean	SD	95% CI	
Speech	-0.123	2.626	6.233	15.871	0.210	0.014	0.183	0.237
Facial Expression	-2.227	0.837	3.294	6.819	0.301	0.016	0.271	0.331
Rigidity-Neck	-0.120	1.437	4.186	6.981	0.302	0.016	0.270	0.333
Rigidity-RUE	-1.208	0.289	2.745	7.108	0.012	0.008	0.001	0.028
Rigidity-LUE	-0.567	1.656	5.093	11.130	0.618	0.026	0.568	0.669
Rigidity-RLE	-0.079	1.276	3.282	6.279	0.069	0.010	0.049	0.091
Rigidity-LLE	0.592	2.171	4.529	8.329	0.460	0.022	0.417	0.506
Finger Tapping-Right	-1.153	0.524	2.198	4.971	0.034	0.009	0.017	0.052
Finger Tapping-Left	-1.216	2.163	5.278	9.738	0.982	0.041	0.908	1.063
Hand Movement-Right	-0.718	0.984	2.716	5.964	0.048	0.009	0.029	0.067
Hand Movement-Left*	-0.500	2.894	6.149	10.839	1.000	0.000	1.000	1.000
Pronation-Right	-0.671	0.987	2.857	6.351	0.009	0.006	0.000	0.024
Pronation-Left	-0.213	2.579	5.300	9.249	0.837	0.032	0.773	0.901
Toe Tapping-Right	-0.670	1.097	2.954	5.595	0.060	0.010	0.042	0.079
Toe Tapping-Left	-0.735	1.879	4.413	8.294	0.676	0.029	0.622	0.734
Leg Agility-Right	0.160	2.141	4.183	6.517	0.098	0.011	0.078	0.120
Leg Agility-Left	0.661	3.107	5.612	8.586	0.642	0.030	0.585	0.702
Arising from chair	1.676	3.721	5.395	7.055	0.219	0.017	0.187	0.252
Gait	-0.736	2.443	4.704	6.936	0.185	0.013	0.158	0.211
Freezing of Gait	4.554	6.320	6.934	7.321	0.339	0.045	0.256	0.429
Postural Stability	2.215	3.252	3.825	6.147	0.173	0.019	0.136	0.214
Posture	-0.494	1.832	4.010	6.015	0.199	0.013	0.176	0.224
Body Bradykinesia	-2.594	0.337	2.815	7.802	0.346	0.017	0.314	0.381
Postural Tremor-Right	0.611	2.460	4.606	15.133	0.002	0.002	0.000	0.006
Postural Tremor-Left	1.065	3.184	5.508	15.839	0.239	0.015	0.210	0.269
Kinetic Tumor-Right	0.745	3.158	6.384	15.842	0.024	0.011	0.003	0.045
Kinetic Tumor-Left	0.863	3.139	6.961	9.357	0.261	0.016	0.233	0.293
Rest Tumor-RUE	0.304	0.929	2.292	8.589	0.001	0.001	0.000	0.002
Rest Tumor-LUE	0.959	1.894	3.698	15.552	0.272	0.016	0.242	0.304
Rest Tumor-RLE	1.932	3.100	5.261	15.389	0.003	0.003	0.000	0.012
Rest Tumor-LLE	2.213	3.417	6.228	16.244	0.122	0.018	0.086	0.156
Rest Tumor-Lip	2.617	4.038	6.963	16.347	0.094	0.021	0.055	0.136
Constancy of Rest	-0.806	0.200	0.927	2.222	0.019	0.009	0.003	0.036

 $a_{k,l}$: item k 's level l .

*: item to put constrains.