Macroeconomic Model of Credit Losses for Multiple Loan Portfolios

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Abstract

We propose a dynamic macroeconomic model of credit risk for multiple portfolios with two factors for each portfolio. We follow the common approach that the credit risk on a loan portfolio can be decomposed into a probability of default and a loss given default and assume that both are driven by two underlying factors: one common for all borrowers in the portfolio and one individual for each single borrower. Our model additionally to the current research estimates the interconnectedness of the portfolios through the risk factors and enhances the current research by introducing dynamics and incorporating the external (macroeconomic) influence. We estimate the model on a set of two large real estate loan portfolios (one residential and one commercial) and show how the portfolios are interconnected and how the credit risk is influenced by macroeconomic environment.

Keywords: credit risk, mortgage, loan portfolio, dynamic model, estimation, interconnectedness, cointegration

JEL Classification: G32

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1. Introduction

At the end of the last decade, when the financial crisis fully hit the US economy, losses from real estate loans in the US increased ten times, compared with the period of economic growth ending in 2007. The aim of our paper is to construct a model of credit losses, which will allow to investigate the risk factors, which drove the increase of delinquencies and charge-offs of residential and commercial real estate loans and estimate the interconnectedness of the residential and commercial mortgage portfolios.

To this end, we propose a credit risk model based on Merton's assumption that credit risk is driven by underlying risk factors (Merton, 1974). We derive a Merton-Vasicek type of the loss distribution (Vasicek, 1987) with several extensions, of which the most important is the inclusion of macroeconomic environment. Our model converts default (or delinquency) rate and a real experienced percentage loss (or loss given default) on a portfolio into underlying factors. Our proposed methodology is conceptually similar to the approaches of Frye (Frye, 2000), Pykhtin (Pykhtin, 2003), Jimenez & Mencia (Jimenez & Mencia, 2009) or Witzany (Witzany, 2011), mainly in the idea of decomposition of the credit risk into underlying factors. On the other hand, in contrary, we study the interconnectedness of all factors and take into account the relevance of external (macroeconomic) conditions. On the other hand the studies examining the relationship between the credit risk and the macroeconomic environment, e.g. Pesaran (Pesaran, Schuermann, Treutler, & Weiner, 2003), do not decompose the credit risk into a multi-factor matrix.

The recent research clearly proved that there is an obvious relationship between the state of the economy and the credit risk. Hamerle et al. (Hamerle, Dartsch, Jobst, & Plank, 2011) showed on a bond portfolio the necessity of taking into account changes in macroeconomic environment. Similarly, (Sommar & Shahnazarian, 2009) used the vector error correction model to estimate the dependency of expected default frequency of a portfolio of nonfinancial listed companies on several macroeconomic factors, from which they found the most influencing the interest rate. The mentioned results are in line with the findings of Pesaran et al. (Pesaran, Schuermann, Treutler, & Weiner, 2003) or Virolainen (Virolainen, 2004), who proved a dependency of the credit risk on the key macroeconomic variables, including interest rates.

Our approach brings a further extension of the abovementioned frameworks in three ways. First, we introduce dynamics in the underlying risk factors. Second, we use the cointegration analysis to find a relationship between common factors and the external (macroeconomic) environment. Also the approach to the estimation of the interconnectedness of multiple portfolios is an enhancement.

We apply the model on a real dataset of US nationwide residential and commercial real estate loan portfolios 30+ delinquencies (loans more than 30 days past due) and charge-off (net charge-offs of loans from books) rates. The cointegration analysis of underlying risk factors and macroeconomic variables such as GDP, unemployment, HPI, personal income, etc. clearly shows that the risk performance of a loan portfolio is linked to the macroeconomic environment. Additionally, our analysis shows that there exists a complex interconnectedness between loan portfolios, commercial and residential mortgages in our case.

The paper is organized as follows. In the following section we provide a description of the model methodology. In Section 3 we describe the data, the empirical analysis and our results. Finally, Section 4 concludes.

2. The Model

Similarly to Vasicek (Vasicek, 1987), we assume that the default of a loan happens when

$$A < B$$

where A is the value the debtor's (hypothetical) assets and B is the value of his debts. The recovery rate is, in line with Pykhtin (Pykhtin, 2003) computed as

$$R = \frac{\min(P, p)}{p} = \min(p^{-1}P, 1)$$

where p is the outstanding principle of the loan and P is the price of the collateral.

As it is usual, we assume that

$$a = \exp\{X + Z\}, \quad P = \exp\{H + E\}$$

where X, H are the common factors, and Z, E are mutually independent normally distributed individual factors, specific for each loan. For simplicity, we assume that B is common for all loans in the portfolio.

If the loan portfolio is large and homogeneous, then the default rate (sometimes imprecisely referenced as probability of default - PD) of a large homogeneous loan portfolio may be approximated as

$$Q = \frac{number\ of\ defaults}{number\ of\ loans} \doteq \mathbb{P}[A < B|Y] = \varphi(-Y), \qquad Y = \frac{X - \log B}{\rho}$$

where φ is a standard normal c.d.f. and ρ is the standard deviation of Z. The loss given default (LGD) - another quantity of usual interest - comes out as

$$G = \frac{total\ loss\ of\ porfolio}{number\ of\ defaults} \doteq \mathbb{E}[1 - R|I] = 1 - \mathbb{E}[\min(\exp\{I\ +\ E\}, 1)\ =\ \eta(I; \sigma)$$

$$I = H - \log p, \quad \eta(\iota; \sigma) = e\iota \int_{-\infty}^{-\iota} \varphi\left(\frac{x}{\rho}\right) e^x dx = \varphi(-\frac{\iota}{\sigma}) - \exp\left\{\iota\ +\ \frac{1}{2}\sigma^2\right\} \varphi(-\frac{\iota}{\sigma} - \sigma)$$

where σ is the standard deviation of E. For more detailed explanation, see Gapko and Šmíd (Gapko & Šmíd, 2012) or Šmíd and Dufek (Šmíd & Dufek, 2016).

As for the dynamics, we suppose that the vector $(X,\ B,\ H)$ follows a VAR model, i.e.

$$(X_t,\ B_t,\ H_t) = \Gamma U_t + \mathcal{E}_t$$

where Γ is unknown deterministic matrix parameter, $\mathcal{E}_t$ is a Gaussian white noise and the regressors U_t are allowed to include constants, trends, lagged values and exogenous variables. Consequently,

$$Q_t = \varphi(-Y_t) = \varphi\left(-\left[\Gamma^Q U_i + \epsilon_{1,t}\right]\right),$$

$$G_t = \eta(I_t; \sigma) = \eta\left(\Gamma^G U_i + \varepsilon_{2,t}; \sigma\right)$$

for some vector parameters Γ^Q and Γ^G and a Gaussian white noise $(\epsilon_1, \varepsilon_2)$.

Once Q_t and G_t are observable, the factors might be got by transformation

$$Y_t = -\varphi^{-1}(Q_t), \qquad I_t = \eta^{-1}(G_t; \sigma)$$

where the (unknown) value of σ might be guessed e.g. from the volatility of a house price index and the average default rate - see the Appendix for details. Consequently, the parameters $\Gamma^{\bullet}$ and the variance matrix of ϵ may be estimated by standard techniques. For the proof of strict monotonicity of $\eta(\bullet; \sigma)$, see Appendix of Gapko and Šmíd (Gapko & Šmíd, 2012).

If, instead of G_t, the charge-off rate

$$L_t = \frac{total\ credit\ loss\ in\ the\ porfolio}{number\ of\ loans\ in\ the\ portfolio}$$

is observed, G_t may be easily computed using the relation $L_t = Q_t G_t$.

3. Data

The dataset used consists of four time series, namely residential and commercial mortgage delinquency rates, which are proportions of loans more than 30 days past due (30+) on the total balance, and residential and commercial mortgage charge off rates, which are proportions of charged off loans (net of recoveries) on the average total balance. The dataset was obtained from the United States Federal Reserve System and thus includes the US nationwide statistics. The time period covered ranges from 1991 to 2016 in a quarterly granularity.

The 30+ delinquency rates were used as proxy metrics for default rates and the charge-off rates represent real losses from the unpaid balance. Table 4.1 and Figure 4.1 summarize descriptive statistics and show the time series of the input data. From the Figure 4.1 it is obvious that the time series are correlated. Also, the recent economic crisis, which started in the US in late 2007 and impacted the US mortgage and real estate markets excessively is visible, as all time series rocketed up to multiples of their preceding values between 2007 and 2010.

Statistic	30+ delinquency rate residential	Charge-off rate residential	30+ delinquency rate commercial	Charge-off rate commercial
Mean value	0.041188	0.004693	0.038408	0.009065
Median	0.023051	0.001584	0.022500	0.002884
Minimum	0.013358	0.000673	0.008500	0.000100
Maximum	0.110150	0.027057	0.120600	0.036297
Standard deviation	0.030813	0.006504	0.031300	0.010536
Variance	0.74810	1.3858	0.81495	1.1623
Skewness	1.1154	1.8399	1.1312	1.1850
Excess kurtosis	-0.332530	2.1346	0.091479	-0.082452
5% percentile	0.010670	0.001345	0.015785	0.0008
95% percentile	0.11055	0.031084	0.10596	0.02177

Table 3.1: Descriptive statistics of input data

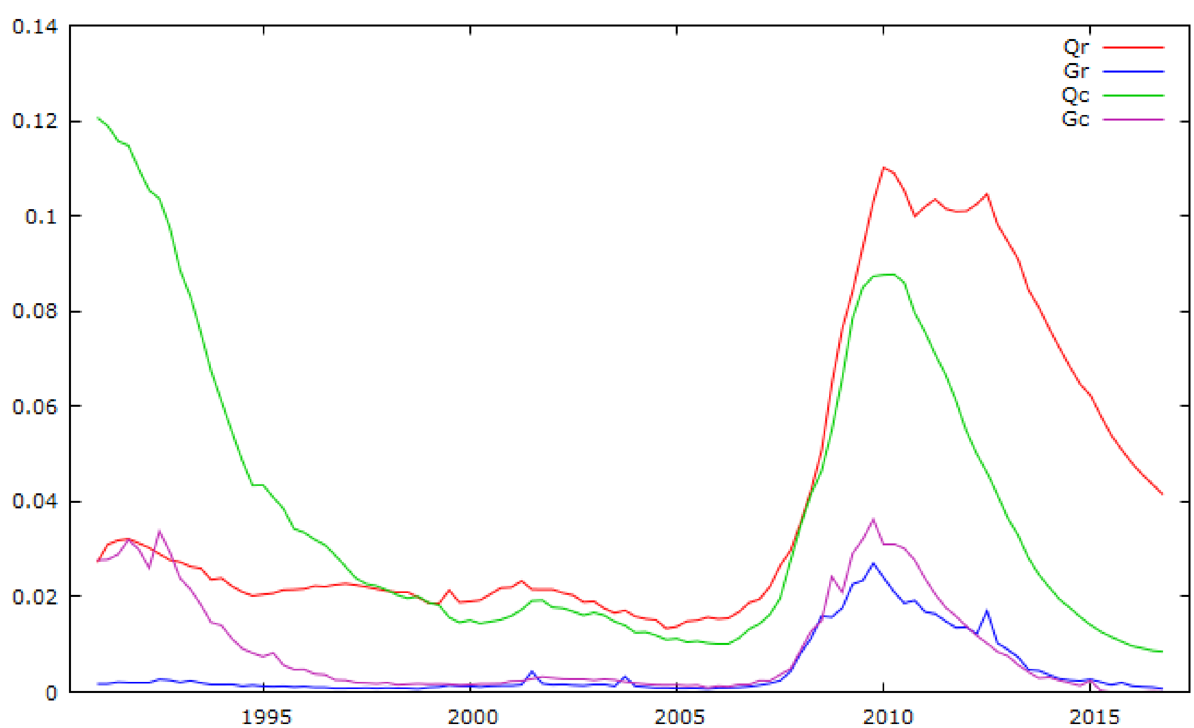

Figure 3.1: Development of the 30+ delinquency rates (Qr = residential, Qc = commercial) and charge-off rates (Gr = residential, Gc = commercial)

Before the empirical estimation of the model we extracted the underlying factors by the method described in Section 2. The resulting time series of the extracted common factors Y (default rate) and I (loss given default) for both commercial (Yc, Ic) and residential (Yr, Ir) mortgage portfolios are illustrated in Figure 3.2. Similarly to the original input dataset, there is a strong visual correlation, especially between Yr and Yc, and Ir and Ic. This provides us with the basis for the exact estimation of the interconnectedness of the four factors.

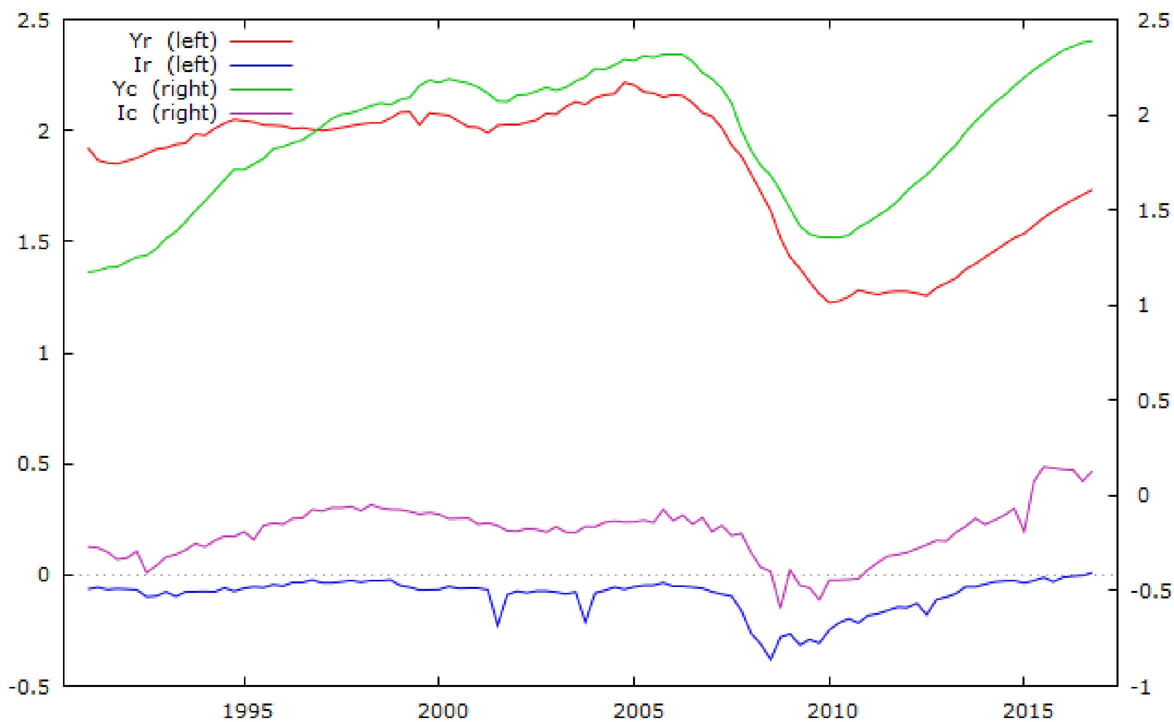

Figure 3.2: The development of the extracted common factors Yr and Ir (left axis), and Yc and Ic (right axis)

4. Results

After we extracted the underlying factors, we studied the interconnectedness of the factors and the macroeconomic environment. In our setting, the macroeconomic environment was represented by a set of variables, namely the FED base interest rate, unemployment, GDP, personal income, industrial production and the Case-Shiller HPI index. The macroeconomic variables were chosen to reflect our expectations, i.e. that the default rates are suspected to be driven by the wealth, which was represented by the GDP, unemployment, personal income and industrial production. The HPI index and the interest rate on the other hand are expected to be a driver of property prices, i.e. determine the charge-off rate. In the choice of the macroeconomic factors we were inspired by Pesaran (Pesaran, Schuermann, Treutler, & Weiner, 2003). All time series of macroeconomic variables were obtained from the FED. The estimation of the dependency was performed on logarithms of all variables and was executed in several stages.

First, to be able to estimate the correct relationships between the factors and the macroeconomic environment, we tested all variables for unit root presence. Based on the ADF unit root test we couldn't reject the unit root in all variables except the FEDR. The detailed results of the ADF test can be found in the Appendix in the table A.2.1.

The presence of unit root in all variables is a condition for cointegration. Thus, during the second step, the cointegration analysis of the set of macroeconomic variables we focused on the significance of individual variables and also on the rank of the cointegrating relationship as the analysis was used in the second step to determine the cointegration rank of all factors and macroeconomic variables jointly. The cointegrating regression, where GDP was used as the dependent variable, shows strong significance of regressors. Additionally, the Lmax test in the Johansen's test of cointegration shows a presence of just one cointegrating relationship. The details of the cointegrating regression as well as the Johansen's test of cointegration can be found in the Appendix in the section A.3.

The natural following step was to determine the potential rank of the cointegration of all variables, i.e. inclusive of all factors and all macroeconomic variables. The Engle-Granger cointegration test of individual factors and macroeconomic variables revealed that there exists a strong cointegration among macroeconomic variables and factors Yc, Ic and Ir on a 90% probability level. Only the cointegration among macroeconomic variables and the factor Yr was not confirmed as the unit root test of residuals from the cointegrating regression couldn't be accepted. The economic interpretation of the non-significant cointegration here might be that the retail mortgage default rate was one of the key triggers of particularly the 2007-2009 economic crisis in the United States, in other words, the Yr enters the system as an exogenous variable. This assumption is confirmed by the results from the Johansen's cointegration test of all variables, which shows that the system consists of three cointegrating relationships. The details of the Lmax test, which confirms the presence of the three cointegrating relationships, can be found in the Appendix in the section A.4.

In the final step, we searched for the final model, which was at the end estimated by the VECM. In the estimation procedure, we used the assumption of three cointegrating relationships (i.e. rank 3 model) and sought only for those macroeconomic variables, which proves itself to be statistically significant. In our case, the significant regressors were IP, FEDR, U and HPI. The resulting setting of the estimated VECM is summarized in the Table 4.1. EC_n represents the error correction term of the n-th VECM equation.

In the last step, we estimated the VECM model of a rank 3, which allows to include both endogenous and exogenous variables.

Variable	d_Ic	d_Ir	d_Yc	d_Yr
Constant	-1.944 ***	1.117 **	-0.719 **	-2.437 ***
d_Ic (lag1)	-0.369 ***	-	-	0.061 *
d_Ir (lag1)	-	-	-	-0.138 ***
d_Yc (lag1)	0.658 ***	0.779 ***	0.348 ***	-
d_HPI (lag1)	-1.355 ***	-	0.719 ***	-
d_U (lag1)	-	-	-0.184 ***	-0.222 ***
FEDR (lag1)	-0.006 ***	0.009 ***	-0.005 ***	-0.009 ***
d_IP (lag1)	-	-	-0.365 **	-
EC1	-0.103 ***	0.073 ***	0.072 ***	-0.039 ***
EC2	-	-0.657 ***	0.087 *	0.199 ***
EC3	0.335 ***	0.229 ***	-0.087 **	0.161 ***
Adjusted R-square	31 %	36 %	82 %	73 %

Table 4.1: Results of the VECM estimation (significance: * - 90%, ** - 95%, *** - 99%)

The resulting VECM model confirmed strong interconnectedness of the factors and the macroeconomic environment. The final VECM model can be used to estimate the future value of the factors given the macroeconomic factors and thus predict the credit risk under various macroeconomic scenarios.

We constructed a set of predictions to compare the model with the currently commonly used Vasicek's distribution suggested losses on the 95% probability level. The Vasicek's distribution is implemented in practice in the IRB formula for unexpected losses. For our comparison, we used the IRB formula and replaced the 99.9% probability level with the 95% (as 99.9% is an unrealistic quantile). The comparison was performed on one year horizon, i.e. we constructed forecasts for 4 periods/quarters by our model. The comparison shows that our model suggests higher loss on the 95% probability level. Therefore, the IRB formula might underestimate the far quantiles loss rate and suggest lower capital requirement. This shortcoming might be a reason why the CRR regulation requires banks to hold capital to cover the 99.9% quantile loss. The comparison of the two models is summarized in the table 4.2.

Segment/Model	IRB	Our
Retail	0.198%	0.374%
Commercial	0.034%	0.054%

Table 4.2: Comparison of the predictions of IRB vs. our model – 12 month loss on the 95% probability level

5. Conclusion

We constructed a multi-period multi-portfolio dynamic macroeconomic model of credit losses. We estimated the presented model on a large U.S. national portfolio of residential and commercial mortgage loans. The empirical analysis showed that there exists a clear and estimable relationship between the credit risk and the macroeconomic environment. Additionally, we proved that the default rate on the portfolio and the loss given default are not independent, as well as there exists interconnectedness between portfolios. Thus, a reasonable model of credit risk has to incorporate the interconnectedness between defaults (represented e.g. by a probability of default) and losses (or, in other words, loss given default) and among risk factors of different portfolios. Finally, we demonstrated the possibility of prediction of the credit risk.

The proposed model describes the interconnectedness of the credit risk in the loan book consisting of multiple portfolios with the macroeconomic environment and predicts the credit risk. Additionally, the model, thanks to an inclusion of macroeconomic variables, is capable of estimating potential future development of the portfolio based on different macroeconomic assumptions and therefore can be used as a tool for macroeconomic stress testing. Given all the possible applications, the model can be used as a model of economic capital within a financial institution.

The empirical comparison of the model with the Vasicek's distribution, the key element of the IRB approach in capital requirements calculation, shows that the Vasicek's distribution based IRB formula tends to underestimate the quantile loss and therefore suggests to hold lower capital amount. As a consequence, the credit risk might be underestimated.

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Appendix

A.1: Determination of σ

Assume that, at time t, the portfolio contains multiple "generations" of loans namely the loans originated at $t-1, t-2, \dots, t-k$ (the loans older than k are no longer present in the portfolio). Assume further that the inflow of fresh loans into the portfolio is constant in time. Finally, assume that all the collaterals securing loans from the generation which started at s have been bought for the same price $\exp\{H_s\}$ and that the price of each of them at t is $\exp\{H_t + (S_t - S_s)\}$ where S is a normal random walk, specific to the loan, with variance θ^2,

Denote G_t the age of a loan randomly chosen at t. Clearly, after k periods, the ratio of the generations within the portfolio is: $(1-q): \dots : (1-q)^{k-1}$, which uniquely determines $\pi_i = \mathbb{P}[G_t = i]$.

Let P_t be the price of a randomly chosen collateral. By the Law of Iterated Variance, we then get

$$\tilde{\sigma}^2 = var(\log P_t|H) = var(\mathbb{E}(\log P_t|G_t, H)|H) + \mathbb{E}(var(\log P_t|G_t, H)|H)$$

$$= var(\mathbb{E}(S_t - S_G|G, H)|H) + \mathbb{E}(var(S_t - S_G|G, H)|H) = \theta^2\, \mathbb{E}G_t = \theta^2 \textstyle\sum_{i=1}^{k} i\pi_i,$$

Even though the $\mathcal{L}(\log P_t|H)$ is a mixture of normal distributions rather than a normal distribution, it is thin tailed so it will not make a big harm to approximate it by $N(H_t, \tilde{\sigma}^2)$.

A.2: Results of the ADF tests and the Engle-Granger cointegration tests for individual factors

Variable	**P-value of the ADF unit root test**
Ic	0.87
Ir	0.52
Yc	0.19
Yr	0.11
U (unemployment)	0.97
PI (personal income)	0.79
IP (industrial production)	0.83
GDP	0.93
HPI	0.97
FEDR (FED interest rate)	0.005

Table A.2.1: Results of the ADF unit root tests

Testing for a unit root in Ic

```
Dickey-Fuller test for Ic
sample size 103
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.0497293
```

```
  test statistic: tau_ct(1) = -1.33714
  p-value 0.8729
  1st-order autocorrelation coeff. for e: -0.275
```

Testing for a unit root in Ir

```
Dickey-Fuller test for Ir
sample size 103
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.100647
  test statistic: tau_ct(1) = -2.13345
  p-value 0.5209
  1st-order autocorrelation coeff. for e: -0.170
```

Testing for a unit root in Yc

```
Augmented Dickey-Fuller test for Yc
including 2 lags of (1-L)Yc
sample size 101
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0184176
  test statistic: tau_ct(1) = -2.82117
  asymptotic p-value 0.1895
  1st-order autocorrelation coeff. for e: -0.032
  lagged differences: F(2, 96) = 134.873 [0.0000]
```

Testing for a unit root in Yr

```
Augmented Dickey-Fuller test for Yr
including 3 lags of (1-L)Yr
sample size 100
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0293095
  test statistic: tau_ct(1) = -3.07102
  asymptotic p-value 0.1133
  1st-order autocorrelation coeff. for e: -0.021
  lagged differences: F(3, 94) = 46.857 [0.0000]
```

Testing for a unit root in U

```
Dickey-Fuller test for U
sample size 103
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.0128755
  test statistic: tau_ct(1) = -0.690306
  p-value 0.9708
  1st-order autocorrelation coeff. for e: 0.675
```

Testing for a unit root in PI

```
Dickey-Fuller test for PI
sample size 103
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.0428271
  test statistic: tau_ct(1) = -1.5951
  p-value 0.7884
```

```
  1st-order autocorrelation coeff. for e: -0.148
```

Testing for a unit root in IP

```
Dickey-Fuller test for IP
sample size 103
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.0262377
  test statistic: tau_ct(1) = -1.47207
  p-value 0.833
  1st-order autocorrelation coeff. for e: 0.503
```

Testing for a unit root in GDP

```
Dickey-Fuller test for GDP
sample size 103
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.0141484
  test statistic: tau_ct(1) = -1.05409
  p-value 0.9309
  1st-order autocorrelation coeff. for e: 0.322
```

Testing for a unit root in HPIr

```
Dickey-Fuller test for HPIr
sample size 103
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.00810067
  test statistic: tau_ct(1) = -0.646854
  p-value 0.9738
  1st-order autocorrelation coeff. for e: 0.750
```

Testing for a unit root in FEDR

```
Augmented Dickey-Fuller test for FEDR
including 3 lags of (1-L)FEDR
sample size 100
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0898696
  test statistic: tau_ct(1) = -4.17634
  asymptotic p-value 0.004789
  1st-order autocorrelation coeff. for e: -0.005
```

A.3: Results of the cointegration testing of the macroeconomic variables

Engle-Granger Step 1: cointegrating regression

Dependent variable: GDP

	coefficient	std. error	t-ratio	p-value	
const	0.128411	0.621682	0.2066	0.8368	
IP	0.431848	0.0484118	8.920	1.98e-14	***
PI	0.691862	0.0881088	7.852	4.26e-12	***
U	0.0918087	0.00926205	9.912	1.27e-16	***
HPIr	0.0973953	0.0123797	7.867	3.96e-12	***
time	0.00227997	0.000506206	4.504	1.78e-05	***

Johansen's rank test:

```
Rank  Eigenvalue  Trace test [p-value]    Lmax test  [p-value]
   0  0.27947     80.345     [0.0398]     34.744     [0.0902]
   1  0.18937     45.601     [0.2688]     22.254     [0.3908]
   2  0.11800     23.347     [0.4927]     13.309     [0.6609]
   3  0.080769    10.038     [0.4845]     8.9271     [0.5106]
   4  0.010422    1.1105     [0.2920]     1.1105     [0.2920]

Corrected for degrees of freedom (df = 94)
Rank     Trace test [p-value]
   0     80.345     [0.0503]
   1     45.601     [0.2873]
   2     23.347     [0.5015]
   3     10.038     [0.4856]
   4     1.1105     [0.2980]
```

A.4: Results of the cointegration testing of all variables

Cointegrating regression -
OLS, using observations 1991:1-2016:4 (T = 104)
Dependent variable: Ic

```
             coefficient   std. error   t-ratio    p-value
  ---------------------------------------------------------
  const      30.9510       3.15466       9.811    3.43e-016 ***
  U          -0.369376     0.0651256    -5.672    1.46e-07  ***
  PI         -1.81483      0.573778     -3.163    0.0021    ***
  IP          2.11787      0.324598      6.525    3.10e-09  ***
  GDP        -2.79685      0.539503     -5.184    1.18e-06  ***
  HPIr        0.299283     0.0806026     3.713    0.0003    ***
  time        0.0298391    0.00283753   10.52     1.04e-017 ***

Mean dependent var  -0.190888   S.D. dependent var   0.144941
Sum squared resid    0.332984   S.E. of regression   0.058590
R-squared            0.846113   Adjusted R-squared   0.836594
Log-likelihood       151.1211   Akaike criterion    -288.2422
Schwarz criterion   -269.7314   Hannan-Quinn        -280.7429
rho                  0.510871   Durbin-Watson        0.977351

Testing for a unit root in uhat

Dickey-Fuller test for uhat
sample size 103
unit-root null hypothesis: a = 1

  model: (1-L)y = (a-1)*y(-1) + e
  estimated value of (a - 1): -0.489129
  test statistic: tau_ct(6) = -5.70672
  p-value 0.01346
  1st-order autocorrelation coeff. for e: -0.047
```

Cointegrating regression -
OLS, using observations 1991:1-2016:4 (T = 104)
Dependent variable: Ir

```
             coefficient   std. error   t-ratio    p-value
  ---------------------------------------------------------
  const      24.0517       2.08559      11.53     6.89e-020 ***
  U           0.0223668    0.0430554     0.5195   0.6046
  PI         -1.09478      0.379333     -2.886    0.0048    ***
  IP          2.17336      0.214596     10.13     7.11e-017 ***
  GDP        -2.89900      0.356673     -8.128    1.43e-012 ***
  HPIr        0.399876     0.0532875     7.504    2.99e-011 ***
  time        0.0243888    0.00187593   13.00     5.67e-023 ***

Mean dependent var  -0.092010   S.D. dependent var   0.079836
```

```
Sum squared resid    0.145538   S.E. of regression   0.038735
R-squared            0.778312   Adjusted R-squared   0.764600
Log-likelihood       194.1593   Akaike criterion    -374.3186
Schwarz criterion   -355.8079   Hannan-Quinn        -366.8193
rho                  0.511374   Durbin-Watson        0.971527

Testing for a unit root in uhat

Dickey-Fuller test for uhat
sample size 103
unit-root null hypothesis: a = 1

  model: (1-L)y = (a-1)*y(-1) + e
  estimated value of (a - 1): -0.488626
  test statistic: tau_ct(6) = -5.73702
  p-value 0.0124
  1st-order autocorrelation coeff. for e: -0.056
```

Cointegrating regression -
OLS, using observations 1991:1-2016:4 (T = 104)
Dependent variable: Yc

```
             coefficient   std. error   t-ratio    p-value
  ---------------------------------------------------------
  const      22.8334       5.70629       4.001    0.0001    ***
  U          -0.562029     0.0933475    -6.021    3.02e-08  ***
  PI         -4.68746      0.800614     -5.855    6.37e-08  ***
  IP          4.04688      0.452070      8.952    2.27e-014 ***
  HPIr        0.659988     0.112513      5.866    6.06e-08  ***
  time        0.0334555    0.00460100    7.271    8.78e-011 ***

Mean dependent var   1.883028   S.D. dependent var   0.353531
Sum squared resid    1.108573   S.E. of regression   0.106358
R-squared            0.913886   Adjusted R-squared   0.909493
Log-likelihood       88.57890   Akaike criterion    -165.1578
Schwarz criterion   -149.2915   Hannan-Quinn        -158.7299
rho                  0.859357   Durbin-Watson        0.256658

Step 7: testing for a unit root in uhat

Augmented Dickey-Fuller test for uhat
including 2 lags of (1-L)uhat
sample size 101
unit-root null hypothesis: a = 1

  model: (1-L)y = (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.233003
  test statistic: tau_ct(5) = -4.53415
  asymptotic p-value 0.07904
  1st-order autocorrelation coeff. for e: -0.018
  lagged differences: F(2, 98) = 8.805 [0.0003]
```

Cointegrating regression -
OLS, using observations 1991:1-2016:4 (T = 104)
Dependent variable: Yr

```
             coefficient   std. error    t-ratio     p-value
  -----------------------------------------------------------
  const      -0.190922     0.307739      -0.6204   0.5364
  U          -0.882586     0.0421590    -20.93     3.85e-038 ***
  HPIr        0.858367     0.0553383     15.51     3.00e-028 ***
  FEDR       -0.0552038    0.00661670    -8.343    4.39e-013 ***
  time       -0.0116112    0.000453633  -25.60     1.92e-045 ***

Mean dependent var   1.825992   S.D. dependent var   0.310982
Sum squared resid    0.436184   S.E. of regression   0.066377
R-squared            0.956211   Adjusted R-squared   0.954442
Log-likelihood       137.0827   Akaike criterion    -264.1654
Schwarz criterion   -250.9435   Hannan-Quinn        -258.8088
rho                  0.868036   Durbin-Watson        0.254628
```

```
Testing for a unit root in uhat

Augmented Dickey-Fuller test for uhat
including 3 lags of (1-L)uhat
sample size 100
unit-root null hypothesis: a = 1

  model: (1-L)y = (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.195126
  test statistic: tau_ct(4) = -3.64373
  asymptotic p-value 0.2673
  1st-order autocorrelation coeff. for e: 0.050
  lagged differences: F(3, 96) = 2.247 [0.0877]
```

Johansen's rank test:

Variables Yr, Yc, Ic, Ir, FEDR, U, IP, HPI

```
Number of equations = 8
Lag order = 4
Estimation period: 1992:2 - 2016:4 (T = 102)
Case 5: Unrestricted trend and constant

Log-likelihood = 2217.97 (including constant term: 2217.97)

Rank Eigenvalue Trace test  p-value  Lmax test  p-value
   0    0.47193     231.70 [0.0000]     63.214 [0.0014]
   1    0.38684     168.49 [0.0000]     48.423 [0.0242]
   2    0.32386     120.07 [0.0003]     38.744 [0.0670]
   3    0.30333     81.323 [0.0038]     35.783 [0.0253]
   4    0.23654     45.540 [0.0800]     26.720 [0.0616]
   5   0.099636     18.820 [0.5166]     10.391 [0.7124]
   6   0.056243     8.4296 [0.4279]     5.7308 [0.6527]
   7   0.026892     2.6988 [0.1004]     2.6988 [0.1004]
Corrected for sample size (df = 66)
Rank Trace test p-value
   0     231.70 [0.0000]
   1     168.49 [0.0001]
   2     120.07 [0.0013]
   3     81.323 [0.0088]
   4     45.540 [0.1088]
   5     18.820 [0.5455]
   6     8.4296 [0.4421]
   7     2.6988 [0.1072]
```