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Evaluating The Epstein-Zin Function as an ICM-PPO Optimization Target

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README

SPEC: RL INVESTOR (PPO + ICM) with Epstein-Zin Preferences + Learnable Fractional Differencing

A self-contained, implementation-ready specification. No code included. All math, shapes, distributions, rewards, targets, and losses are explicit.

0) PURPOSE & SCOPE

This document replaces CRRA with **Epstein–Zin (EZ)** recursive preferences and **adds a Learnable Fractional Differencing (FracDiff) layer** in the feature pipeline, while preserving the **PPO + ICM** training stack and environment mechanics. It is **drop-in**: policy parameterization, buffers, rollout loop, exact log-probabilities, and PPO machinery remain intact. Only the **preference aggregator** (reward/value semantics) and **feature memory module** (FracDiff) change.

What stays the same (do not touch):

- Time/indexing, assets, returns, risk-free, ~~turnover/transaction cost~~, budget identity.
- Actor heads (consumption squashed Gaussian; ~~risky weights Dirichlet/softmax~~), exact log-prob math. (No more portfolio optimization)
- Critic backbone mechanics (but we output two EZ heads; see §5).
- ICM encoder/forward (and optional inverse) and curiosity reward wiring.
- PPO: ratio, clipping, GAE, epochs/minibatching, entropy, optimizers.
- Data split, standardization, rollout collection, buffer contents.

What changes:

1. **Utility/Value:** CRRA is replaced by **Epstein–Zin** with a numerically stable target in (z)-space (§4–§6).
2. **Features:** Insert **Learnable FracDiff** over returns before feature construction (§3).

1) Core definitions (time, single risky asset, wealth, consumption)

We now consider a **single risky asset** (S&P) and remove portfolio optimization entirely.

1.1 Time and assets

- Discrete time ($t = 0, 1, 2, \dots, T - 1$).
- One risky asset with **gross return** ($R_{t+1} \in R_{>0}$) between (t) and ($t + 1$).
- No explicit risk-free asset and no portfolio weights – all *unconsumed* wealth is automatically invested in the risky asset.

1.2 Wealth, consumption, normalization

- Wealth at start of step (t): ($W_t > 0$).
- Consumption fraction (action): ($c_t \in (0, 1)$); **dollar consumption** ($C_t := c_t \cdot W_t$).
- Running max wealth ($M_t := \max_{0 \leq \tau \leq t} W_\tau$); normalized wealth ($\tilde{W}_t := W_t / M_t \in (0, 1]$).

1.3 Budget identity (wealth transition)

In the simplified world, after consuming ($C_t = c_t W_t$), the remaining wealth ($(1 - c_t)W_t$) is fully invested in the risky asset, which realizes a gross return (R_{t+1}) over ($[t, t + 1]$).

The **wealth transition** is

$$([W_{t+1} = (1 - c_t)W_t R_{t+1} .])$$

We may optionally clip (W_{t+1}) below by a small floor ($\varepsilon_W > 0$) for numerical stability. Running max wealth is updated as

$$([M_{t+1} := \max(M_t, W_{t+1}) .])$$

2) OBSERVATIONS, FEATURES, STATE (BASELINE PIPELINE)

2.1 Observables at time (t)

- (W_t) and a causal feature vector ($x_t \in R^d$) built **only** from data ($\leq t$).
- Standardize (x_t) via train-set ((μ, σ)) to (\tilde{x}_t) (store (μ, σ) from training only).

2.2 State to networks

- **State:** ($s_t := \text{concat}(\tilde{W}_t, \tilde{x}_t) \in R^{1+d+n}$) (fixed order).

At time (t) the agent observes:

- Normalized wealth ($\tilde{W}_t = W_t / M_t$).
- A standardized feature vector ($\tilde{x}_t \in R^d$), built from the FracDiff pipeline and other signals, using only information up to time (t).

The **state** fed to the policy and critic is

$$(s_t := \text{concat}(\tilde{W}_t, \tilde{x}_t) \in R^{1+d}).)$$

There is no (w_{t-1}) term in the state any more, since there is no portfolio decision.

3) Learnable Fractional Differencing (returns-domain feature module)

3.1 Goal & parameter

Learn a memory depth ($d_{\text{target}} \in [d_{\min}, d_{\max}]$) (e.g., ([0, 1])) that controls the fractional differencing of returns to **capture long memory** while promoting **stationarity**.

3.2 Placement in pipeline

- Input raw **log-returns** per asset: ($r_t \in R^n$) (or windows).
- Apply a FracDiff operator with effective exponent (d_{eff}):
 - **Mode "direct"**: apply $((1 - L)^{d_{\text{target}}})$ to returns.
 - **Mode "price_equiv"**: apply $((1 - L)^{d_{\text{target}} - 1})$ to returns (equivalent to price fracdiff of (d_{target}) without reconstructing prices).

- Truncate the kernel to length (K) (auto-chosen from (d_{eff}) and a tolerance). Outputs lose the first (K) steps.

3.3 State augmentation & alignment

- Build usual statistics **from** the FD output (lags, MAs, vol, PCA, cross-sectional transforms).
- **Shift** all time-aligned targets by (K) (drop first (K) steps) so shapes match.
- Optionally append ($\text{stop}_{\text{grad}}(d_{\text{target}})$) and (K) as scalar features so the policy/critic can adapt to memory depth.

3.4 Regularization & constraints

- Keep (d_{target}) within bounds via a sigmoid reparameterization.
- Add a small L2 penalty if (d_{target}) sticks to the bounds.
- Optional "whiteness" regularizer: penalize low-lag autocorrelation of FD residuals to avoid over-memory.

Everything backpropagates end-to-end because kernel weights are differentiable functions of (d_{eff}).

4) Epstein–Zin Preferences (replace CRRA)

Let ($\beta \in (0, 1)$) be the subjective discount, ($\gamma > 0$) risk aversion, ($\psi > 0$) intertemporal elasticity (EIS).

Define transforms:

- $(z(V) := V^{1 - \frac{1}{\psi}})$ (EIS/consumption space)
- $(y(V) := V^{1 - \gamma})$ (risk space)

4.1 EZ aggregator (Kreps–Porteus form)

For lifetime utility (V_t) and consumption (C_t):

$$(V_t = \left[(1 - \beta)C_t^{1 - \frac{1}{\psi}} + \beta \left(E_t[V_{t+1}^{1 - \gamma}] \right)^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right]^{\frac{1}{1 - \frac{1}{\psi}}}.)$$

4.2 Practical RL parameterization (stable targets)

We train in (z)-space with a two-head critic predicting ($\hat{z}_t \approx z(V_t)$) and ($\hat{y}_t \approx y(V_t)$).

- **External (shaped) reward:** $(r_t^{\text{ext}} := (1 - \beta)C_t^{1 - \frac{1}{\psi}}).$
- **One-step bootstrap target for (z):**

$$(T_t^{(z)} := (1 - \beta)C_t^{1 - \frac{1}{\psi}} + \beta \left(\hat{y}_{t+1} \right)^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}}.)$$

- **Value loss:** $(L_{\text{value}} := \frac{1}{2} (\hat{z}_t - T_t^{(z)})^2).$

- Optional **consistency** regularizer: encourage $(\hat{y}_t \approx (\hat{z}_t)^{\frac{1-\gamma}{1-\psi}})$ with a small weight.

Degeneracies: $(\psi \rightarrow 1)$ approaches additive/separable (log-like); $(\gamma \rightarrow 1)$ reduces risk curvature; recipe reduces toward CRRA smoothly.

5) ACTOR & CRITIC (Z-functions, distributions, exact log-probs)

We now have a **single action dimension**: the consumption rate ($c_t \in (0, 1)$).

Dimensions and symbols used throughout this section

- State at time (t): ($s_t \in R^{1+d+n}$) is the concatenation ($s_t := \text{concat}(\tilde{W}_t, \tilde{x}_t)$), where ($\tilde{W}_t = W_t / M_t$) and (\tilde{x}_t) is the standardized feature vector.
- Consumption fraction (action component): ($c_t \in (0, 1)$). Dollar consumption: ($C_t := c_t W_t$).
- Hyperparameters for heads: ($\sigma_{\min} > 0$) (std floor).

5.1 Actor (f_θ)

The actor takes ($s_t \in R^{1+d}$) and passes it through a shared backbone (e.g. an MLP) to produce parameters for a scalar Gaussian in a latent space:

- Pre-squash Normal parameters: ($[\mu_c(s_t) \in R, \quad \ell_c(s_t) \in R, \quad \sigma_c(s_t) := \text{softplus}(\ell_c) + \sigma_{\min} .]$)
- Sample pre-squash variable ($[y_c \sim N(\mu_c(s_t), \sigma_c(s_t)^2) .]$)
- Squash to the action space via the sigmoid: ($[c_t := \sigma(y_c) = \frac{1}{1 + e^{-y_c}} \in (0, 1) .]$)
- Deterministic (evaluation) action is given by ($[c_t^{\text{det}} := \sigma(\mu_c(s_t)) .]$)

There is no risky-weights head any more; (w_t) is implicitly equal to (1) on the single asset.

5.2 Exact log-probability of (c_t)

Let

$$([y_c = \text{logit}(c_t) = \log \frac{c_t}{1 - c_t} .])$$

The log-probability under the squashed Gaussian is

$$([\log p(c_t | s_t) = \log N(y_c, \mu_c(s_t), \sigma_c(s_t)^2) - \log(c_t(1 - c_t)) .])$$

where the first term is the Gaussian log-density of (y_c) and the second term is the log-Jacobian of the sigmoid.

This ($\log p(c_t | s_t)$) is the **only** action log-probability used in PPO here.

5.3 Critic (g_ψ) (two heads for EZ)

The critic takes (s_t) and outputs two scalars:

- ($\hat{z}_t \approx z(V_t)$) with ($z(V) := V^{1-\frac{1}{\psi}}$).
 - ($\hat{y}_t \approx y(V_t)$) with ($y(V) := V^{1-\gamma}$). These are used to build the EZ bootstrap target and TD residual below.
-

6) ENVIRONMENT STEP (FULL SEQUENCE)

At time (t), given state (s_t) and sampled consumption rate (c_t), the environment performs:

1. Consumption and wealth evolution

Dollar consumption:

$$([C_t = c_t W_t .])$$

Remaining wealth:

$$([W_t^{\text{after}} = (1 - c_t) W_t .])$$

Apply the risky asset gross return (R_{t+1}):

$$([W_{t+1} = W_t^{\text{after}} R_{t+1} = (1 - c_t) W_t R_{t+1} .])$$

Optionally clip ($W_{t+1} \geq \varepsilon_W$) if needed for numerical stability.

2. Running max and next state

$$([M_{t+1} = \max(M_t, W_{t+1}), \quad \tilde{W}_{t+1} = \frac{W_{t+1}}{M_{t+1}} .])$$

The feature pipeline (including FracDiff) produces the next standardized feature vector (\tilde{x}_{t+1}) from market data up to time ($t+1$).

The next state is

$$([s_{t+1} = \text{concat}(\tilde{W}_{t+1}, \tilde{x}_{t+1}) .])$$

3. Termination

Episodes end when ($t = T - 1$) (or when data runs out).

Wealth transition

- Gross growth factor:

$$([G_{t+1} := (1 - c_t) (R_f[t+1] + w_t^\top \tilde{R}[t+1]) - \kappa \|w_t - w_{t-1}\|_1 .])$$

- Next wealth: ($W_{t+1} := W_t \cdot G_{t+1}$). Safety floor may clip ($G_{t+1} \geq \varepsilon_g > 0$).

7) REWARDS (EXTERNAL EZ FLOW, INTRINSIC ICM)

We use the Epstein–Zin flow term for external reward and the Intrinsic Curiosity Module (ICM) to supply an intrinsic shaping signal.

EZ parameters

- Discount ($\beta \in (0, 1)$)
- Risk aversion ($\gamma > 0$)
- Elasticity of intertemporal substitution (EIS) ($\psi > 0$)
- Consumption ($C_t = c_t W_t$)

7.1 External reward (EZ flow term in (z) -space)

The **external reward** at time (t) is the EZ flow term in (z) -space, depending only on consumption:

$$([r_t^{\text{ext}} = (1 - \beta)C_t^{1 - \frac{1}{\psi}} = (1 - \beta)(c_t W_t)^{1 - \frac{1}{\psi}} .])$$

This is the main objective that encourages good consumption timing.

7.2 Intrinsic Curiosity Module (ICM) — complete specification

We define the ICM exactly and fully:

Network dimensions

Let:

- Feature dimension (d)
- State dimension ($1 + d$) since state is ($\text{concat}(\tilde{W}_t, \tilde{x}_t)$)
- State-embedding dimension (m) (e.g., 64)
- Hidden widths for ICM networks:
 - Encoder hidden width (E) (e.g., 128)
 - Forward-model hidden width (F) (e.g., 128)

Define the state encoder:

$$([\phi_\omega: R^{1+d} \rightarrow R^m .])$$

State encoder network

Given state ($s_t \in R^{1+d}$):

$$([e1 = \text{GELU}(W_{e1}s_t + b_{e1}), \quad W_{e1} \in R^{E \times (1+d)} .])$$

$$([e2 = \text{GELU}(W_{e2}e1 + b_{e2}), \quad W_{e2} \in R^{E \times E} .])$$

$$([\phi(s_t) = W_{eo}e2 + b_{eo}, \quad W_{eo} \in R^{m \times E} .])$$

Define:

$$([\phi_t := \phi(s_t), \quad \phi_{t+1} := \phi(s_{t+1}) .])$$

Action embedding (scalar action)

Because the action is **only** consumption ($c_t \in (0, 1)$), we embed it as:

$$([y_c = \text{logit}(c_t) = \log \frac{c_t}{1 - c_t} .])$$

Then define:

$$([\psi(a_t) := \psi(c_t) := y_c \in R^1 .])$$

Dimensions: action embedding is 1-dimensional.

Forward dynamics model

Maps $((\phi(s_t), \psi(a_t)))$ into a prediction of $(\phi(s_{t+1}))$.

Input dimension to forward model:

$$([m + 1.])$$

Forward model layers:

$$([u1 = \text{GELU} \left(W_{f1}, \text{concat}(\phi_t, \psi(c_t)) + b_{f1} \right), \quad W_{f1} \in R^{F \times (m+1)} .])$$

$$([u2 = \text{GELU}(W_{f2}u1 + b_{f2}), \quad W_{f2} \in R^{F \times F} .])$$

$$([\hat{\phi}_{t+1} := W_{fo}u2 + b_{fo}, \quad W_{fo} \in R^{m \times F} .])$$

There is **no inverse model** in the consumption-only version.

Intrinsic reward

Given:

- Encoded next state (ϕ_{t+1})
- Predicted next state ($\hat{\phi}_{t+1}$)

The intrinsic reward is:

$$([r_t^{\text{int}} := \eta \left| \phi_{t+1} - \hat{\phi}_{t+1} \right|_2^2,])$$

with a small scale factor ($\eta > 0$) (e.g., (10^{-3})).

ICM losses

Forward loss:

$$([L_{\text{fwd}}(\omega) := \left| \phi_{t+1} - \hat{\phi}_{t+1} \right|_2^2,])$$

Inverse loss:

$$([L_{\text{inv}} := 0])$$

since we removed portfolio weights and do not reconstruct (w_t).

The action is 1-dimensional and directly known, so inverse dynamics is unnecessary.

Total ICM loss:

$$([L_{\text{ICM}} = L_{\text{fwd}},])$$

7.3 Total reward used by PPO

$$(r_t := r_t^{\text{ext}} + r_t^{\text{int}}).$$

8) Advantages, EZ targets, and losses (consumption-only)

All variables used below are defined here or earlier sections.

8.1 EZ one-step target in (z)-space

We have two critic heads:

- $(\hat{z}_t \approx z(V_t) := V_t^{1-\frac{1}{\psi}})$
- $(\hat{y}_t \approx y(V_t) := V_t^{1-\gamma})$

The one-step EZ bootstrap target is:

$$([T_t^{(z)} = (1-\beta)C_t^{1-\frac{1}{\psi}} + \beta(\hat{y}_{t+1})^{\frac{1-\frac{1}{\psi}}{1-\gamma}},])$$

All terms are fully defined:

- $(C_t = c_t W_t)$ is consumption
 - (\hat{y}_{t+1}) comes from the critic applied to next state
 - exponents come from EZ preference structure
-

8.2 Value loss (z-head)

$$(L_{\text{value}} = \frac{1}{2} (\hat{z}_t - T_t^{(z)})^2 .])$$

8.3 TD residual in (z) -space and GAE

Define the **combined reward**:

$$(r_t = r_t^{\text{ext}} + r_t^{\text{int}}])$$

and the **EZ temporal-difference residual**:

$$([\delta_t^{\text{EZ}} := r_t + \beta (T_t^{(z)} - r_t^{\text{ext}}) - \hat{z}_t .])$$

This matches the structure of the general EZ TD residual while incorporating intrinsic reward.

Generalized Advantage Estimation (GAE)

Let $(\lambda \in [0, 1])$ be the GAE parameter.

Compute the advantages by backward recursion:

$$([\tilde{A}_t = \delta_t^{\text{EZ}} + (\beta\lambda), \delta_{t+1}^{\text{EZ}} + (\beta\lambda)^2, \delta_{t+2}^{\text{EZ}} + \dots])$$

Practical implementation uses backward iteration over a rollout.

We normalize (\tilde{A}_t) to mean 0 and variance 1 in each minibatch.

8.4 PPO clipped policy loss (consumption-only)

Let:

- $(\log\pi_{\theta_{\text{old}}}(c_t | s_t))$ be the stored behavior log-prob.
- $(\log\pi_{\theta}(c_t | s_t))$ be recomputed with the current actor.
- Importance ratio:

$$([r_t(\theta) := \exp (\log\pi_{\theta}(c_t | s_t) - \log\pi_{\theta_{\text{old}}}(c_t | s_t)) .])$$

- Clipping parameter ($\varepsilon \in (0, 1)$).

The PPO objective (to minimize) is:

$$([L_{\text{PPO}} = -E_t [\min (r_t(\theta)\tilde{A}_t, \text{clip}(r_t(\theta), 1 - \varepsilon, 1 + \varepsilon)\tilde{A}_t)] .])$$

8.5 Entropy term (Gaussian action only)

The actor samples

$$([y_c \sim N(\mu_c(s_t), \sigma_c(s_t)^2)])$$

before applying the sigmoid.

Entropy of a Normal:

$$([H_c = \frac{1}{2} \log(2\pi e \sigma_c^2) .])$$

We encourage exploration by adding the entropy term:

$$([L_{\text{ent}} := -H_c .])$$

There is no Dirichlet entropy here since we removed risky-weight allocations.

8.6 ICM loss

As defined in §7.2:

$$([L_{\text{ICM}} = L_{\text{fwd}} = \left\| \phi_{t+1} - \hat{\phi}_{t+1} \right\|_2^2 .])$$

The inverse loss is zero in consumption-only and can be enabled later if desired.

8.7 Final training loss

Define scalar weighting hyperparameters:

- ($c_v > 0$): value loss weight
- ($\beta_{\text{ent}} > 0$): entropy loss weight
- ($c_{\text{icm}} > 0$): curiosity loss weight

The full objective is:

$$([L_{\text{total}} = L_{\text{PPO}} + c_v L_{\text{value}} + \beta_{\text{ent}} L_{\text{ent}} + c_{\text{icm}} L_{\text{ICM}} .])$$

9) TRAINING PROCEDURE (COLLECT → TARGETS → PPO)

9.1 Hyperparameters (additions/changes)

- **EZ**: choose ($\gamma \in [5, 10]$), ($\psi \in [0.5, 1.0, 1.5]$), ($\beta \in [0.95, 0.999]$).
- **FracDiff**: (d_{target}) init (0.3) – (0.5) within ([0, 1]), tolerance (10^{-4}), ($K_{\text{max}} \in [1024, 4096]$) (match horizon).

- **RL:** keep PPO (λ), clip, epochs, minibatch, lrs same initially.

9.2 Rollout collection (unchanged mechanics)

- Collect tuples $(s_t, a_t = (c_t, w_t), r_t, s_{t+1}, \log\pi_{\theta_{\text{old}}})$ where (r_t) includes EZ flow + curiosity.
- Align time by dropping first (K) steps due to FracDiff.

9.3 Target building & PPO update

- For each step, compute $(T_t^{(z)}, (\delta_t^{\text{EZ}}))$, GAE, and (z) -value loss.
- Recompute current $(\log\pi_{\theta})$ exactly (§5.2); perform clipped PPO with entropy and ICM losses.
- After epochs, set $(\theta_{\text{old}} \leftarrow \theta)$.

9.4 Evaluation (deterministic)

- Use $(c_t := \sigma(\mu_c)), (w_t := \alpha / \sum_i \alpha_i)$.
 - Recover EZ value via inverse transform for reporting: $(\hat{V}_t = \hat{z}_t^{1/(1-\frac{1}{\psi})})$.
 - Report PnL, CAGR, MDD, Calmar, turnover, and (\hat{V}_0) .
-

10) DIAGNOSTICS, CHECKS, & ABALATIONS

- **EZ sanity:** as $(\psi \rightarrow 1)$ or $(\gamma \rightarrow 1)$, curves and training behavior should smoothly approach separable/CRRA.
 - **Scale hygiene:** track (\hat{z}_t, \hat{y}_t) magnitudes; clamp/normalize if exploding.
 - **FracDiff:** monitor learned (d_{target}) trajectory; inspect ACF/PACF of FD outputs; avoid non-stationary drift.
 - **Alignment:** verify all post-FD tensors drop the first (K) steps; shapes of policy/value/ICM batches match.
 - **Ablations:** (i) turn off FracDiff (identity) to test EZ alone; (ii) (ψ) grid with fixed (γ) ; (iii) compare CRRA vs EZ at matched (γ) with $(\psi \approx 1)$.
-

11) MINIMAL MIGRATION CHECKLIST

- Expose (γ, ψ, β) in config; leave PPO hypers unchanged initially.
 - Critic: switch to **two heads** $((\hat{z}, \hat{y}))$; keep shared backbone.
 - Reward pipe: compute $(r_t^{\text{ext}} = (1 - \beta)C^{1-1/\psi})$; add curiosity as before $\rightarrow (r_t)$.
 - Targets: build $(T^{(z)})$ with next-state (\hat{y}) ; compute (δ^{EZ}) and GAE in (z) -space.
 - Insert **Learnable FracDiff** before feature builder; shift by (K) .
 - Log (d_{target}, K) , ACF diagnostics, and (\hat{z}, \hat{y}) summaries.
-

12) GLOSSARY

- (C_t) : dollar consumption; (W_t) : wealth; (c_t) : consumption rate.
- (γ) : risk aversion; (ψ) : EIS; (β) : discount.
- (V_t) : EZ lifetime utility; $(z(V) = V^{1 - 1/\psi})$; $(y(V) = V^{1 - \gamma})$.
- (d_{target}) : fraccdiff memory parameter; (K) : kernel truncation length.
- ICM: Intrinsic Curiosity Module; (ϕ) : encoder; (f) : forward model.

13) TL;DR (one-screen summary)

- **Objective change:** CRRA \rightarrow Epstein–Zin with a two-head critic $((\hat{z}, \hat{y}))$, shaped external reward $((1 - \beta)C^{1 - 1/\psi})$, and a one-step (z) -target using (\hat{y}_{t+1}) .
- **Feature change:** Insert Learnable FracDiff over returns; align time by kernel length (K) .



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