Homework 5.

1]

1. (10 pts) Use truth tables (worlds) to show that the following pairs of sentences are equivalent:
   - $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$
   - $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

a) $P \Rightarrow \neg Q \quad Q \Rightarrow \neg P$

P	Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Since $P \Rightarrow \neg Q$ & $Q \Rightarrow \neg P$ have identical truth tables
∴ Both are equivalent

b) $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

A $\qquad$ B

P	Q	$P \Rightarrow \neg Q$	$\neg Q \Rightarrow P$	A	$P \wedge \neg Q$	$\neg P \wedge Q$	B
F	F	F	F	F	F	F	F
F	T	T	T	T	F	T	T
T	F	T	T	T	T	F	T
T	T	F	F	F	F	F	F

Since A & B have identical truth table values,
A & B are equival.

2]

2. (20 pts) Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:
   - (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
   - (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
   - ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Justify your answer using truth tables (worlds).

a) Smoke - S $\quad$ Heat - H
Fire - F

S	F	$S \Rightarrow F$ ($\neg S \vee F$)	$\neg S \Rightarrow \neg F$ ($\neg S \vee F$)	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
F	F	T	T	T
F	T	F	T	T
T	F	T	F	F
T	T	T	T	T

Since there is one F and 3 T.
Neither

b)

S	F	H	$S \Rightarrow F$	$S \vee H$	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$	
F	F	F	T	F	T	T
F	F	T	T	T	F	F
F	T	F	T	F	T	T
F	T	T	T	T	T	T
T	F	F	F	T	T	F
T	F	T	F	T	T	F
T	T	F	T	T	T	T
T	T	T	T	T	T	T

Rough

$\therefore$ Since there is both F & T in table

Neither

c)

S	F	H	A $(S \wedge H) \Rightarrow F$	B $S \Rightarrow F$	C $H \Rightarrow F$	$A \Leftrightarrow B \vee C$	
F	F	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T
T	F	T	F	F	F	F	T
T	T	F	T	T	T	T	T
T	T	T	T	T	T	T	T

Rough

Since there are only T.

Valid

3]

3. (30 pts) Consider the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).
(b) Convert the knowledge base into CNF.
(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

Justify your answers using resolution by providing corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations.

Let Unicorn = U   Mammel – ML
Mythical = M   Horned – H
Immortal = I   Magical – MG

a) Sentence 1 -: $M \Rightarrow I$

2 -: $[\neg M \Rightarrow \neg I] \wedge [\neg M \Rightarrow ML]$

3 -: $(I \vee ML) \Rightarrow H$

4 -: $H \Rightarrow MG$

Logic knowledge base -: $[M \Rightarrow I] \wedge [\neg M \Rightarrow \neg I] \wedge [\neg M \Rightarrow ML] \wedge [(I \vee ML) \Rightarrow H] \wedge [H \Rightarrow Mg]$

b) $[\neg M \vee I] \wedge [M \vee \neg I] \wedge [M \vee ML] \wedge [(\neg I \wedge \neg ML) \vee H] \wedge$
$[\neg H \vee Mg]$
$= [\neg M \vee I] \wedge [M \vee \neg I] \wedge [M \vee ML] \wedge [\neg I \vee H] \wedge [\neg ML \vee H] \wedge [\neg H \vee Mg]$

c) Let the above answer represent knowledge base : Δ.
"Unicorn is mythical" : $\alpha = M$, $\neg\alpha = \neg M$

$\Delta \models \alpha$ iff $\Delta \wedge \neg\alpha$ is inconsistent

By resolution :
1. $\neg M \vee I$
2. $M \vee \neg I$
3. $M \vee ML$
4. $\neg I \vee H$
5. $\neg ML \vee H$
6. $\neg H \vee MG$

α : 7. $\neg M$

- 7 & 2 — 8. $\neg I$
- 8 & 1 — 9. $\neg M$
- 9 & 3 — 10. ML
- 10 & 5 — 11. H
- 11 & 6 — 12. MG — Terminal

Since we explore all permutations before terminating, therefore there is no inconsistency with $\neg\alpha$
∴ Unicorn is not always mythical.

"Unicorn is magical" : $\alpha = Mg$
$\neg\alpha = \neg Mg$

α : 7. $\neg Mg$

- 7 & 6 | 8. $\neg H$
- 8 & 4 | 9. $\neg I$
- 9 & 2 | 10. $\neg M$
- 8 & 5 | 11. $\neg ML$
- 11 & 3 | 12. M 13. Contradiction (12 & 10)

Since $\Delta \wedge \neg\alpha$ is inconsistent (contradicted)
∴ $\Delta \models \alpha$
∴ Unicorn is magical

"Unicorn is horned" : $\alpha = h$
$\neg\alpha = \neg h$

α : 7. $\neg H$

- 7 & 4 — 8. $\neg I$
- 8 & 2 — 9. $\neg M$
- 9 & 5 — 10. $\neg ML$
- 10 & 3 — 11. M
- 12. Contradiction (9 & 11)

Since $\Delta \wedge \neg\alpha$ is inconsistent (contradicted)
∴ $\Delta \models \alpha$
∴ Unicorn is horned

4]

4. (20 pts) Consider the two NNF circuits in Figure 1 and Figure 2. Identify whether they are decomposable, deterministic, smooth and why.

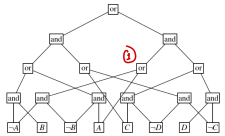
Figure 1

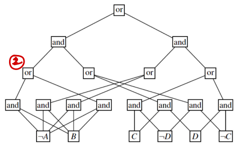
Figure 2

Figure 1 -: It is decomposible as for every "conjuncts" there are no common variables in both side of conjunct.
[vars(α) = A,B and vars(β) = C,D in every conjunct]

It is deterministic as $\alpha \wedge \beta$ for a disjunct is always inconsistent.

eg 1 in figure has $\alpha = \neg A \wedge \neg B$ & $\beta = \phi$

It is not smooth as for 1 vars$(\alpha) \neq$ vars(β)

Figure 2 : It is decomposible (same reason as above)
[vars(α) = A,B and vars(β) = C,D in every conjunct]

Non-deterministic as for every "disjunct", $\alpha \wedge \beta$ is not inconsistent always. For 2 $\alpha = \neg A \wedge B$, $\beta = \neg A \wedge B$ [clearly $\alpha \wedge \beta \neq 0$]

It is smooth as for all disjunct, vars(α) = vars(β)

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5. (20 pts) Given a propositional formula, where each literal has a weight ω in [0,1], the weight of a truth assignment is defined as the product of its literals weights. For example, ω(A, ¬B, C) = ω(A) ω(¬B) ω(C). The Weighted Model Count (WMC) of a propositional formula is defined as the added weight of its satisfying assignments (i.e., models).

Suppose we have the following literal weights: ω(A)=0.1, ω(¬A)=0.9, ω(B)=0.3, ω(¬B)=0.7, ω(C)=0.5, ω(¬C)=0.5, ω(D)=0.7, ω(¬D)=0.3.

(a) Compute the Weighted Model Count for formula (¬A ∧ B) ∨ (¬B ∧ A) by enumerating its models, computing their weights, then adding them up.

(b) Consider the decomposable, deterministic and smooth NNF circuit in Figure 3. If we assign the weights of literals to all the leaf nodes, the count of each ∧ node is computed as the product of the counts of its children, and the count of each ∨ node is computed as the sum of the counts of its children. What is the relation between the count on the root with the Weighted Model Count for the formula?

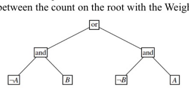


Figure 3

(c) Compute the Weighted Model Count for the formula associated with the decomposable, deterministic and smooth NNF circuit in Figure 4.

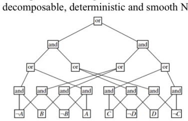


Figure 4

5] a) $(\neg A \wedge B) \vee (\neg B \wedge A)$

A	B	$\neg A \wedge B$	$\neg B \wedge A$	$(\neg A \wedge B) \vee (\neg B \wedge A)$
F	F	F	F	F
F	T	T	F	T
T	F	F	T	T
T	T	F	F	F

$\therefore$ Condition satisfied only when {A=F, B=T}, {B=F, A=T}

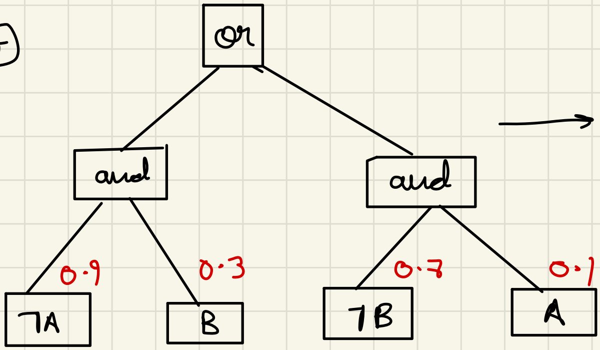
$= w(\neg A) \cdot w(B) + w(A) \cdot w(\neg B)$

$= 0.9 \cdot 0.3 + 0.1 \cdot 0.7$

$= 0.27 + 0.07$

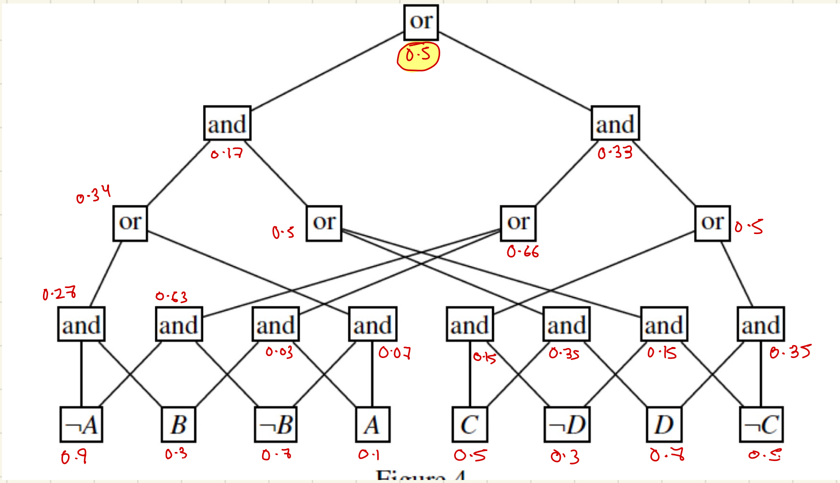
$= 0.34$

b)



$\longrightarrow (0.9 \times 0.3) + (0.7 \times 0.1) = \underline{0.34}$

c)



As seen in the figure, the model count gives <u>0.5</u> as result.