Homework 6

Problem 1

a) $P(A,A,B)$, $P(x,y,z)$

Yes it is unifiable where $\theta = \{x/A, y/A, z/B\}$

b) $Q(y, G(A,B))$, $Q(G(x,x), y)$

Not unifiable as y cannot be $G(A,B)$ & $G(x,x)$ at same time

c) $R(x,A,z)$, $R(B,y,z)$

Yes it is unifiable where $\theta = \{x/B, A/y\}$ and z is consistent in both.

d) $Older(Father(y), y)$, $Older(Father(x), John)$

Yes it is unifiable where $\theta = \{x/John, y/John\}$

e) $Knows(Father(y), y)$, $Knows(x,x)$

Not unifiable as x cannot take the value of y & Father (y) at the same time.

Problem 2

- John likes all kinds of food.
- Apples are food.
- Chicken is food.
- Anything someone eats and isn't killed by is food.
- If you are killed by something, you are not alive.
- Bill eats peanuts and is still alive. *
- Sue eats everything Bill eats.

a) Definitions -:

$L(x,y) \rightarrow x$ likes y

$F(x) \rightarrow x$ is food

$E(x,y) \rightarrow x$ eats y

$K(x,y) \rightarrow x$ is killed by y

$A(x) \rightarrow x$ is alive

First order logic statements –

1. $\forall x \; F(x) \Rightarrow L(John, x)$
   → CNF: $\neg F(x) \vee L(John, x)$

2. $F(apple)$ → CNF remains same

3. $F(chicken)$ → CNF remains same

4. $\exists x \, \forall y \; ((E(x,y) \wedge \neg K(x,y)) \Rightarrow F(y))$ [De-morgans law]
   → CNF: $\exists x \, \forall y \; \neg E(x,y) \vee K(x,y) \vee F(y)$
   $\neg E(x',y) \vee K(x',y) \vee F(y)$
   [x' is skolem constant]

5. $\exists x \, \exists y \; K(x,y) \Rightarrow \neg A(x)$
   → CNF: $\exists x \, \exists y \; \neg K(x,y) \vee \neg A(x)$
   $\neg K(x',y') \vee \neg A(x')$ [x', y' skolem constant]

6. $E(Bill, Peanuts) \wedge A(Bill)$ → CNF splits into 2 elements.

7. $\forall x \, (E(Bill, x) \Rightarrow E(Sue, x))$
   → CNF: $\neg E(Bill, x) \vee E(Sue, x)$

c) 3. Prove that John likes peanuts using resolution.

Let Δ be knowledge base
α : John likes peanuts → $L(John, Peanuts)$
$\neg\alpha$: John doesn't like peanuts. → $\neg L(John, Peanuts)$
To prove this $\Delta \models \alpha$, $\Delta \wedge \neg\alpha$ is consistent

Δ:
1. $\neg F(x) \vee L(John, x)$
2. $F(apple)$
3. $F(chicken)$
4. $\neg E(x',y) \vee K(x',y) \vee F(y)$
5. $\neg K(x',y') \vee \neg A(x')$
6. $E(Bill, Peanuts)$
7. $A(Bill)$
8. $\neg E(Bill, x) \vee E(Sue, x)$
9. – $\neg\alpha$: $\neg L(John, Peanuts)$
10. $\neg F(Peanuts)$ — 9 & 1
11. $\neg K(Bill, y')$ — 5 & 7
12. $K(Bill, Peanuts) \vee F(Peanuts)$ — 6 & 4
13. $F(Peanuts)$ — 11 & 12
14. Contradiction — 10 & 13

∴ Since arrive at a contradiction for $\neg\alpha$
∴ Proved by resolution, John likes Peanuts

d) What does Sue eat?

Consider the CNFs -:

(6) $E(Bill, Peanuts)$

(8) $\neg E(Bill, x) \vee E(Sue, x)$

Using resolution in (6) & (8)

$\rightarrow$ $E(Sue, Peanuts)$

∴ Sue eats peanuts, proved by 1-step resolution.

e)

5. Use resolution to answer (d) if, instead of the axiom marked with an asterisk above, we had:
   - If you don't eat, you die.
   - If you die, you are not alive.
   - Bill is alive.

Lets rewrite all CNF we have

(1) $\neg F(x) \vee L(John, x)$

(2) $F(apple)$

(3) $F(chicken)$

(4) $\neg E(x', y) \vee K(x', y) \vee F(y)$

(5) $\neg K(x', y') \vee \neg A(x')$

(6) $E(Bill, Peanuts)$

(7) $A(Bill)$

(9) $\neg E(Bill, x) \vee E(Sue, x)$

(6), (7) → Replaced by

($D(x)$ – x died)

(6) $\exists x \forall y \; \neg E(x, y) \Rightarrow D(x) \xrightarrow{CNF} E(x', y) \vee D(x')$

(7) $\exists x \; D(x) \Rightarrow \neg A(x) \xrightarrow{CNF} \neg D(x') \vee \neg A(x')$

(8) $A(Bill) \xrightarrow{CNF} A(Bill)$

→ What does Sue eat?

(10) $\neg D(Bill)$ — (7) & (8)

(11) $E(Bill, y)$ — (6) & (10)

(12) $E(Sue, y)$ — (11) & (9)

Since y is a variable and in the original first order logic y can be anything, there is not a specific thing that Sue eats.