

Q1

Ans

For biquadratic interpolation we have,

$$V(x, y) = \sum_{i=0}^2 \sum_{j=0}^2 a_{ij} x^i y^j$$

we have 9 entries in matrix A (coeff mat)

$$A = \begin{bmatrix} a_{00} & a_{01} & \dots \\ \vdots & \vdots & \vdots \\ a_{20} & \dots & a_{22} \end{bmatrix}$$

$$V = X A$$

$$V = \begin{bmatrix} v_1 \\ \vdots \\ v_9 \end{bmatrix} \quad A = \begin{bmatrix} a_{00} & \dots \\ \vdots & \vdots \\ a_{22} \end{bmatrix} \quad X = \begin{bmatrix} 1, y_1, y_1^2, \dots, x_1^2 \\ \vdots \\ 1, y_9, y_9^2, \dots, x_9^2 \end{bmatrix}$$

$$A = X^{-1} V$$

So if $|X| = 0$

$A = (X + \lambda I_9)^{-1} V$, for some small real number λ , such that $\det(X + \lambda I) \neq 0$

Q2

Ans

we have

$$x_i = 1/1.5 = \frac{2}{3}$$

$$y_i = 1/1.5 = \frac{2}{3}$$

So we get 4 neighbours as

$$(0,0), (0,1), (1,0), (1,1)$$

Now

$$V = xA \quad [V(x,y) = ax + by + cx + d]$$

$$\begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$A = x^{-1} V$$

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

$$, a=5, b=5, c=1, d=5$$

$$V\left(\frac{2}{3}, \frac{2}{3}\right) = \frac{5 \times 2}{3} + \frac{5 \times 2}{3} + \frac{5 \times 2 \times 2}{3 \times 3} + 5$$

$$= 13.88$$

∴ Hence → Interpolated value is 13.88