SML Assignment - 3

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Q5

Ans a) we have.

$$d_i = y_i(\beta^T x_i + \beta_0)$$

Update Rule parameter will not change by changing activation function

As we have.

$$\beta_{new} = \beta - n \frac{d}{d\beta}(d_i)$$

$$\beta_{0new} = \beta_0 - n \frac{d}{d\beta_0}(d_i)$$

Here update rule is not changing as the distance is independent ~~of~~ of the activation function.

b) $$\phi(\beta, \beta_0) = -\sum_{i=1}^{N} y_i(\beta^T x_i + \beta_0)$$

$$\Rightarrow \beta^T \beta = 1$$

$$g(\beta) = \beta^T \beta - 1$$

$$\mathcal{L}(\beta, \beta_0) = \phi(\beta, \beta_0) + \lambda g(\beta)$$
$$= -\sum_{i=1}^{N} y_i(\beta^T x_i + \beta_0) + \lambda(\beta^T \beta - 1)$$

$$\frac{d\mathcal{L}}{d\beta} = -\sum_{i=1}^{N} y_i x_i + \lambda(\beta, \beta) = 0$$

$$\frac{d\mathcal{L}}{d\beta_0} = -\sum_{i=1}^{N} y_i = 0$$

Hence $$\boxed{\beta_{new} = \beta - n \frac{d\mathcal{L}}{d\beta}}$$

$$\beta_{0new} = \beta_0 - \eta \frac{dL}{d\beta_0}$$

$\eta$ → hyperparameter.

Q6

A

$x \rightarrow y_i \rightarrow$ Perceptron $\rightarrow y_2$

$$y_1 = \sigma(\beta_{11} x + \beta_{01})$$

$$y_2 = \cancel{\sigma}\ \text{Sigm}(\beta_{12} y_i + \beta_{02})$$

$$\rightarrow d_i = -y_i(\beta_{12} y_{i1} + \beta_{02})$$

$$\frac{d d_i}{d\beta_{11}} = -y_i\left(\beta_{12} \frac{d y_{i1}}{d\beta_{11}} + 0\right)$$

$$\sigma'(x) = \sigma(x)[1 - \sigma(x)]$$

$$= -y_i(\beta_{12}\,\sigma'(\beta_{11} x_i + \beta_{01} x_i)$$

$$= -y_i x_i \beta_{12} e^{-v_i}(1 + e^{-v_i})^2$$

$$\frac{d d_i}{d\beta_{01}} = -y_i\left(\beta_{12} \frac{d y_{i1}}{d\beta_{01}}\right)$$

$$\frac{d y_{i1}}{d\beta_{01}} = \sigma'(\beta_1 x_i + \beta_{01}) = \left(\frac{e^{-v_i}}{1 + e^{-v_1}}\right)^2$$

$\partial$

$$\frac{\partial d_i}{\partial B_{01}} = \frac{-y_i \, B_{12} \, e^{-v_i}}{(1 + e^{-v_i})^2}$$

$$\cancel{\frac{\partial d_i}{\partial B_{12}}} = \cancel{-y_i \left( \sigma (B_{11}) \right)}$$

$$\frac{\partial d_i}{\partial B_{12}} = -y_i \left[ \sigma [B_{11} x + B_{01}] \right]$$

$$\frac{\partial d_i}{B_{02}} = -y_i$$

d)

$$\boxed{\begin{aligned} B_{01} &= B_{01} - n \frac{\partial d_i}{\partial B_{01}} \\ B_{12} &= B_{12} - n \frac{\partial d_i}{\partial B_{12}} \\ B_{02} &= B_{02} - n \frac{\partial d_i}{\partial B_{02}} \\ B_{11} &= B_{11} - n \frac{\partial d_i}{\partial B_{11}} \end{aligned}}$$