Optimal Economic Load Dispatch (ELD) Iterative Algorithm

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1 Introduction

Economic Load Dispatch (ELD) is a critical optimization problem in power systems that ensures electricity generation is allocated across multiple power plants to meet demand at the lowest possible fuel cost while respecting operational constraints. This project implements an automated ELD algorithm using MATLAB, focusing on scenarios where transmission losses are negligible. The algorithm dynamically adjusts generator outputs iteratively to achieve optimality, making it scalable for systems with any number of generators.

2 Economic Load Dispatch (ELD)

Economic Load Dispatch (ELD) is an optimization process in power systems that ensures the total generation cost is minimized while meeting the system load demand and adhering to generator constraints. The power balance equation forms the foundation of ELD:

$$\sum_{i=1}^{N} P_i = P_{load}$$

where P_i represents the power output of the i-th generator, P_{load} is the total load demand, and N is the number of generators.

Each generator's fuel cost is expressed as a quadratic cost function:

$$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i$$

where a_i, b_i, and c_i are cost coefficients specific to the i-th generator. The goal is to minimize the total cost:

$$C_{total} = \sum_{i=1}^{N} C_i(P_i)$$

Incremental cost is a critical concept in ELD, representing the change in generation cost per unit of power produced. It is derived as the derivative of the cost function:

$$\frac{\partial C_i(P_i)}{\partial P_i} = 2a_i P_i + b_i$$

This helps in determining how power output adjustments impact overall costs.

2.1 Lagrangian Multiplier and the Equal-λ Criterion

To achieve cost minimization, ELD uses the Lagrangian multiplier (λ) to enforce the power balance constraint. The equal-λ criterion ensures that all generators operate at the same marginal cost:

$$\frac{\partial C_i(P_i)}{\partial P_i} = \lambda \quad \forall i$$

This principle aligns the incremental costs across all generators, leading to optimal dispatch. For each generator, the power output is adjusted iteratively using the relation:

$$P_i = \frac{\lambda - b_i}{2a_i}$$

subject to generator constraints $P_{i,min} \leq P_i \leq P_{i,max}$.

2.2 Generator Limits and Penalty Factor

Generator limits ensure that the output of each unit remains within specified bounds:

$$P_{i,min} \leq P_i \leq P_{i,max}$$

When transmission losses are included, the penalty factor (F_i) adjusts the generator cost to reflect its effective contribution to load demand and losses:

$$F_i = \frac{1}{1 - \frac{\partial P_{loss}}{\partial P_i}}$$

This ensures that generators contributing more to losses are appropriately penalized.

2.3 Lossless Power Transmission

In cases of negligible transmission losses, the optimization problem simplifies. The total generated power equals the load demand, and the equal-λ criterion is directly applied. This approach is computationally efficient and widely used for systems with minimal loss impact.

2.4 Power Transmission with Losses

When transmission losses are significant, the power balance equation is modified to include losses:

$$\sum_{i=1}^{N} P_i = P_{load} + P_{loss}$$

Transmission losses (P_{loss}) are typically modeled using a loss coefficient matrix (B):

$$P_{loss} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j$$

This introduces complexity, as P_{loss} depends nonlinearly on generator outputs. The optimization process incorporates penalty factors and iterative methods to balance costs and losses.

3 Why ELD Matters

Economic Load Dispatch (ELD) plays a pivotal role in power system operations by ensuring cost-effective and reliable electricity generation. It minimizes fuel costs by optimally distributing power among generators while maintaining balance with system demand. By adhering to operational constraints, ELD enhances reliability, ensuring generators operate within safe limits and reducing the risk of overloading or equipment damage. Its scalability allows it to adapt to systems of varying sizes, from small-scale networks to large grids with numerous generators. This adaptability and efficiency make ELD essential for modern power systems, driving sustainable and economical energy management practices.

4 Algorithm Overview

Economic Load Dispatch (ELD) ensures optimal power distribution among generators while adhering to operational constraints. The iterative algorithm dynamically adjusts generator outputs to minimize costs and meet load demands.

4.1 Workflow

The ELD algorithm operates through the following steps:

1. **Initialization:** All generator outputs are initially set to zero, and generators are flagged as *free*, indicating they are adjustable for optimization.

2. - **Calculate λ (Lagrangian Multiplier):** The marginal cost of power generation for free generators is determined using:

   $$\frac{\partial C_i(P_i)}{\partial P_i} = \lambda \tag{1}$$

   - **Compute Tentative Outputs:** Each generator's output is updated iteratively using:

   $$P_i = \frac{\lambda - b_i}{2a_i} \tag{2}$$

   - **Check Limit Violations:** Outputs are checked against operational bounds $P_{i,min}$ and $P_{i,max}$. If a generator violates its limits, it is fixed at the respective limit, and the algorithm adjusts remaining free generators.
   - **Repeat:** Steps are repeated until all generator outputs are within bounds, and power balance is achieved.

3. **Termination:** The algorithm concludes when the total generation meets demand, or iteration limits are reached. If a feasible solution cannot be found due to insufficient generator capacity, a warning is triggered.

4.2 Key Features

- **Adaptive Violation Handling:** Automatically identifies and fixes generators that exceed operational limits, ensuring outputs remain within prescribed ranges.
- **Feasibility Check:** Flags issues when total generation capacity is inadequate to meet demand, providing critical insights for system planning and reliability assessment.

5 Code Explanation

5.1 Inputs

The algorithm requires the following key inputs:

- a, b, c: Vectors containing the quadratic (a), linear (b), and constant (c) cost coefficients of the generator fuel cost function $C_i(P_i) = a_i P_i^2 + b_i P_i + c_i$.
- P_{min}, P_{max}: Vectors specifying the minimum ($P_{i,min}$) and maximum ($P_{i,max}$) power output limits for each generator.
- P_{demand}: The total system power demand to be met by the generators.

5.2 Outputs

The algorithm generates the following outputs:

- $P_{optimal}$: A vector indicating the optimal power allocation for each generator after optimization.
- λ_{value}: The final value of the Lagrangian multiplier (λ) used to equalize incremental costs across generators.
- `feasibility`: A Boolean flag indicating whether the demand could be met given generator constraints.
- `total_cost`: The total generation cost, calculated as the sum of individual generator costs.

5.3 Critical Functions

1. Lambda Calculation: The algorithm iteratively adjusts the value of λ to balance total power supply and demand while ensuring marginal costs are equal for all free generators. The tentative power output for a generator is calculated as:

$$P_i = \frac{\lambda - b_i}{2a_i} \tag{3}$$

The iterations continue until the power balance equation is satisfied:

$$\sum_{i=1}^{N} P_i = P_{demand} \tag{4}$$

2. Limit Enforcement: During each iteration, the algorithm checks if a generator's calculated P_i violates its limits ($P_{i,min}$ or $P_{i,max}$). If a violation occurs:

- The generator is fixed at the violated limit.
- It is marked as no longer *free*, and the optimization continues with the remaining adjustable generators.

This dynamic handling of violations ensures that the solution respects generator constraints while optimizing power allocation.

6 Sample Calculation and Result

Consider a three-generator Economic Load Dispatch (ELD) problem where the total system load is 300 MW. The cost functions for the generators are given as:

$$C_1(P_1) = 0.05P_1^2 + 8P_1 + 300, \quad C_2(P_2) = 0.08P_2^2 + 6P_2 + 200, \quad C_3(P_3) = 0.1P_3^2 + 5P_3 + 150$$

The power output limits for the generators are:

$$50 \le P_1 \le 150, \quad 40 \le P_2 \le 120, \quad 30 \le P_3 \le 80$$

The incremental cost functions, which are the derivatives of the cost functions, are as follows:

$$\frac{dC_1}{dP_1} = 0.1P_1 + 8, \quad \frac{dC_2}{dP_2} = 0.16P_2 + 6, \quad \frac{dC_3}{dP_3} = 0.2P_3 + 5$$

At optimality, these incremental costs are equal to the Lagrangian multiplier λ. This gives:

$$0.1P_1 + 8 = \lambda, \quad 0.16P_2 + 6 = \lambda, \quad 0.2P_3 + 5 = \lambda$$

Rearranging these equations, the power outputs for each generator are expressed as:

$$P_1 = 10(\lambda - 8), \quad P_2 = 6.25(\lambda - 6), \quad P_3 = 5(\lambda - 5)$$

Using the power balance constraint:

$$P_1 + P_2 + P_3 = 300,$$

we substitute the expressions for P_1, P_2, and P_3:

$$10(\lambda - 8) + 6.25(\lambda - 6) + 5(\lambda - 5) = 300$$

Simplifying, we have:

$$\lambda = 20.82$$

Substituting $\lambda = 20.82$ into the equations, we get the optimal power outputs:

$$P_1 = 128.2\, MW, \quad P_2 = 92.63\, MW, \quad P_3 = 79.1\, MW$$

7 Limitations

- Assumes negligible transmission losses, limiting real-world applicability.
- Relies on convex, quadratic cost functions, making it unsuitable for generators with non-convex characteristics.
- Fixed iteration limits may cause premature termination in complex systems.

8 Future Scope

- Incorporating transmission losses using B-coefficients or Newton-Raphson load flow calculations.
- Addressing non-convex cost functions with metaheuristic algorithms like Genetic Algorithms or Particle Swarm Optimization.
- Extending the framework to hybrid systems integrating renewable energy sources.

9 Conclusion

This ELD algorithm provides a robust and scalable framework for minimizing generation costs in idealized power systems. However, addressing real-world complexities will enhance its relevance and effectiveness for modern power grids.