Regression

Introduction

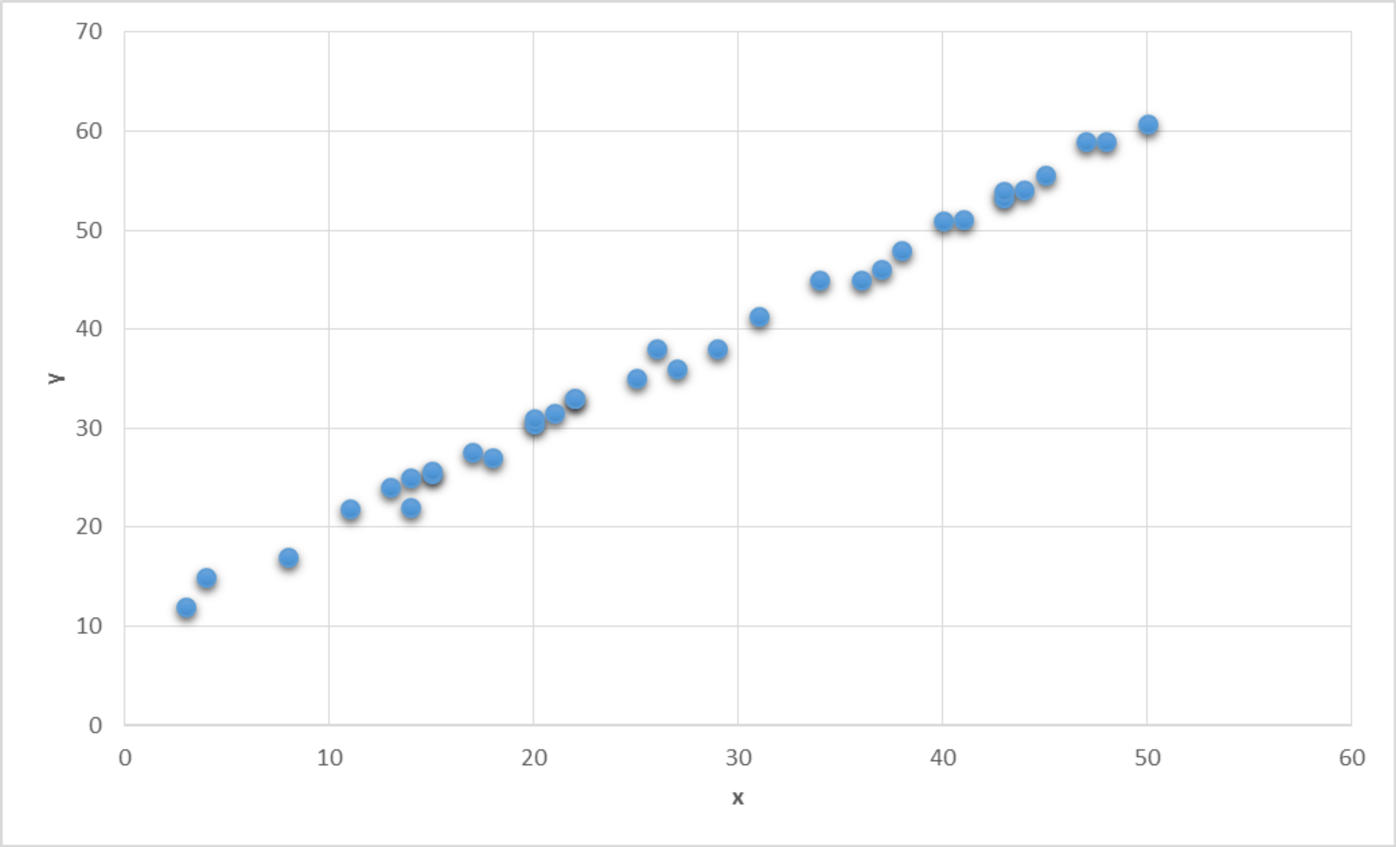
Contents

- Correlation
- Simple Regression
- R-Squared
- Multiple Regression
- Adj R-Squared
- P-value
- Multicollinearity
- Interaction terms

Correlation

What is Regression

Regression



What is Regression

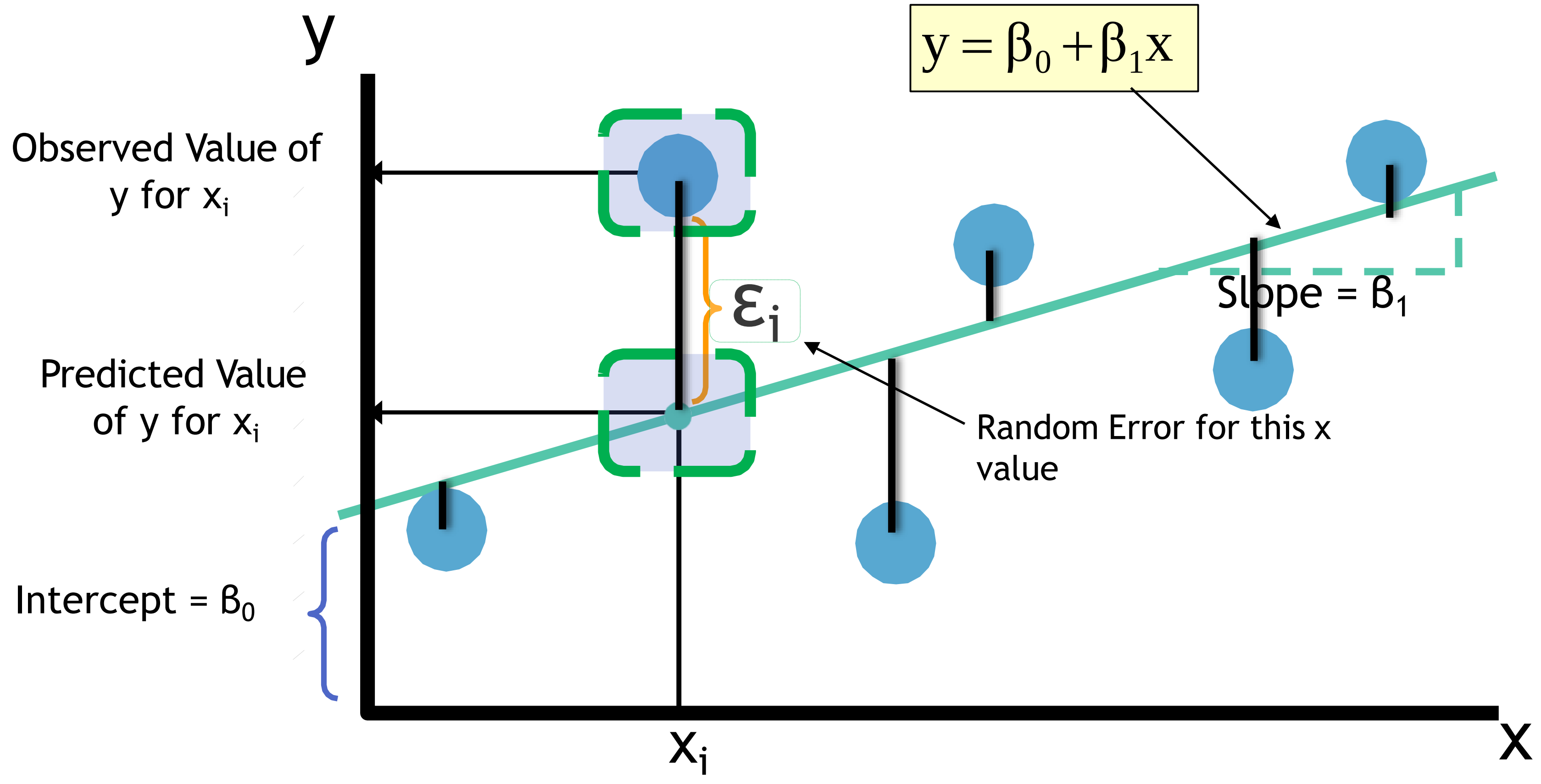
- A regression line is a mathematical formula that quantifies the general relation between a predictor/independent (or known variable x) and the target/dependent (or the unknown variable y)
- Below is the regression line. If we have the data of x and y then we can build a model to generalize their relation

- What is the best fit for our data?
- The one which goes through the core of the data
- The one which minimizes the error

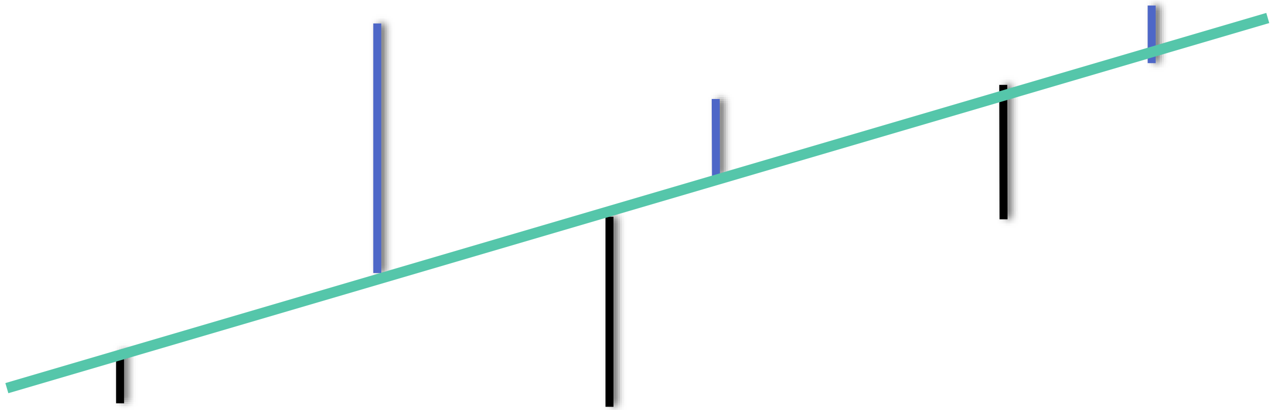
$$\boxed{y = \beta_0 + \beta_1 x}$$

Regression Line fitting-Least Squares Estimation

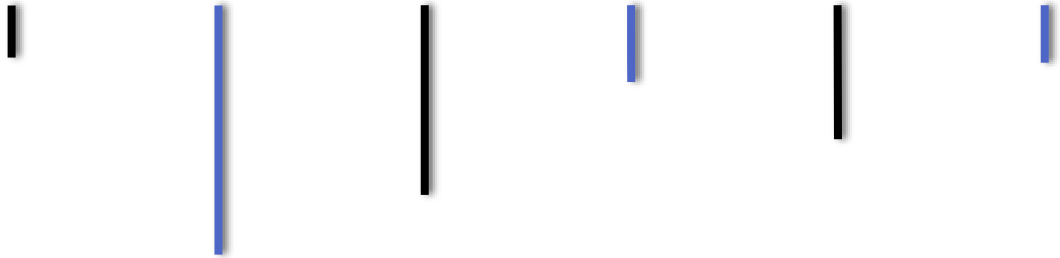
Regression Line fitting



Regression Line fitting



Minimizing the error



- The best line will have the minimum error
- Some errors are positive and some errors are negative. Taking their sum is not a good idea
- We can either minimize the squared sum of errors Or we can minimize the absolute sum of errors
- Squared sum of errors is mathematically convenient to minimize
- The method of minimizing squared sum of errors is called least squared method of regression

Least Squares Estimation

- X: x1, x2, x3, x4, x5, x6, x7,……..
- Y:y1, y2, y3, y4, y5, y6, y7…….
- Imagine a line through all the points
- Deviation from each point (residual or error)
- Square of the deviation
- Minimizing sum of squares of deviation

$$\sum e^2 = \sum (y - \hat{y})^2$$
$$= \sum (y - (\beta_0 + \beta_1 x))^2$$

β_0 and β_1 are obtained by minimize the sum of the squared residuals

LAB: Regression Line Fitting

- Dataset: Air Travel Data\Air_travel.csv
- Find the correlation between Promotion_Budget and Passengers
- Draw a scatter plot between Promotion_Budget and Passengers. Is there any pattern between Promotion_Budget and Passengers?
- Build a regression line to predict the passengers using Inter_metro_flight_ratio

Code: sklearn vs statsmodels

- Several package options for building regression lines in python
- sklearn and statsmodels are two most widely used options
- sklean is first choice. But gives limited summary statistics
- But statmodels gives well formatted (R-like) summary and model statistics.
- You can use any one of them. Use sklearn of you are not interested in model statistics. Use stastmodels when you are at learning phase.

- We will use both

How good is my regression line?

How good is my regression line?

- Take an (x,y) point from data.
- Imagine that we submitted x in the regression line, we got a prediction as y_{pred}
- If the regression line is a good fit then the we expect $y_{pred}=y$ or $(y-y_{pred})=0$
- At every point of x, if we repeat the same, then we will get multiple error values $(y-y_{pred})$ values
- Some of them might be positive, some of them may be negative, so we can take the square of all such errors

$$\boxed{SSE = \sum (y - \hat{y})^2}$$

SSE

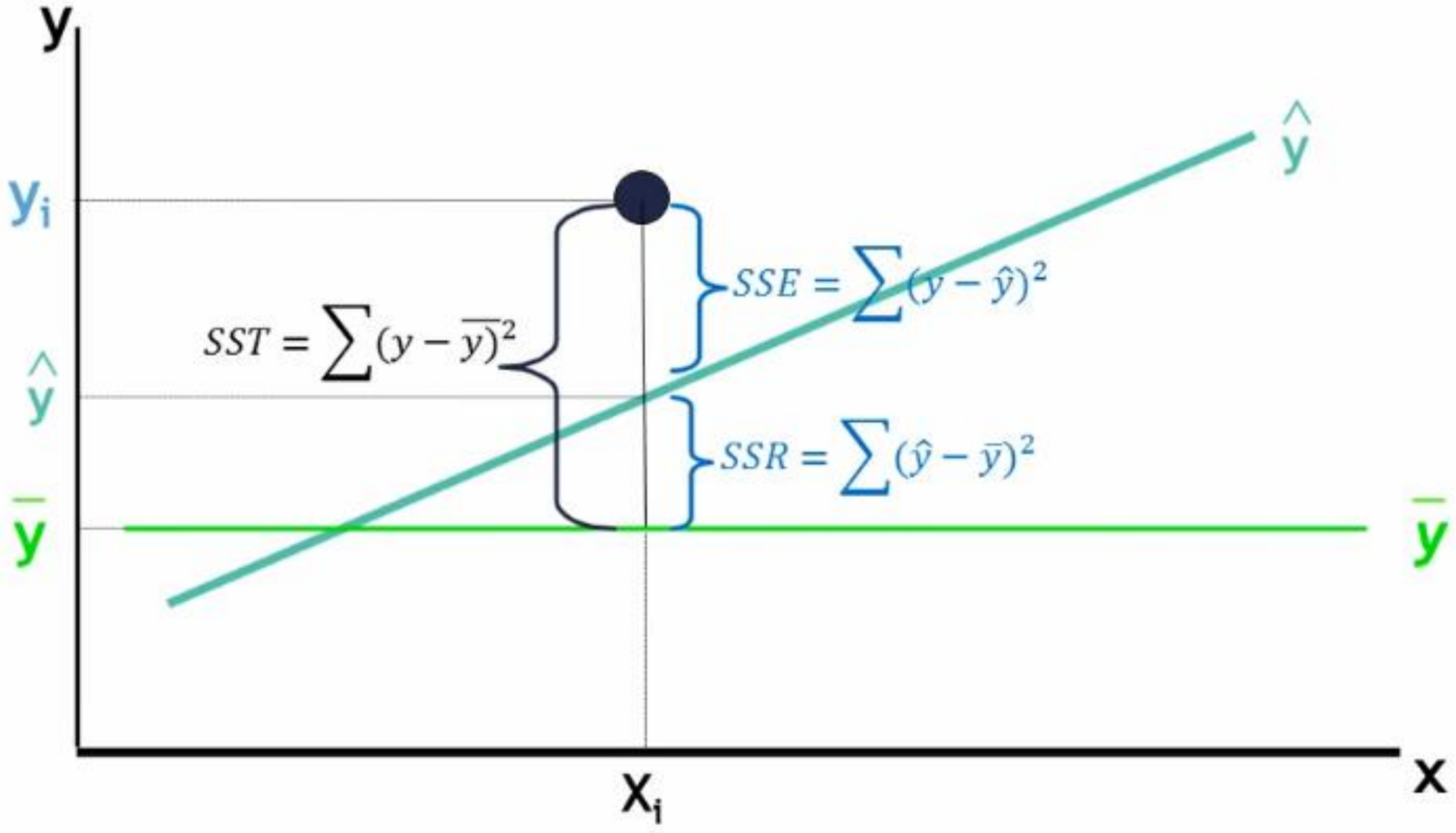
- For a good model we need SSE to be zero or near to zero
- Standalone SSE will not make any sense, For example SSE= 100, is very less when y is varying in terms of 1000’s. Same value is is very high when y is varying in terms of decimals.
- We have to consider variance of y while calculating the regression line accuracy

$$SSE = \sum (y - \hat{y})^2$$

How good is my regression line?

- Error Sum of squares (SSE- Sum of Squares of error)
  - $SSE = \sum(y - \hat{y})^2$
- Total Variance in Y (SST- Sum of Squares of Total)
  - $SST = \sum(y - \overline{y})^2$
  - $SST = \sum(y - \hat{y} + \hat{y} - \overline{y})^2$
  - $SST = \sum(y - \hat{y} + \hat{y} - \overline{y})^2$
  - $SST = \sum(y - \hat{y})^2 + \sum(\hat{y} - \bar{y})^2$
  - $\mathrm{SST} = \mathrm{SSE} + \sum(\hat{y} - \bar{y})^2$
  - $\mathrm{SST} = \mathrm{SSE} + SSR$
- So, total variance in Y is divided into two parts,
  - Variance that can't be explained by x (error)
  - Variance that can be explained by x, using regression

Explained and Unexplained Variation



How good is my regression line?

- So, total variance in Y is divided into two parts,
  - Variance that can be explained by x, using regression
  - Variance that can't be explained by x

S S T = S S E + S S R

Total sum of Squares	Sum of Squares Error	Sum of Squares Regression
$SST = \sum (y - \bar{y})^2$	$SSE = \sum (y - \hat{y})^2$	$SSR = \sum (\hat{y} - \bar{y})^2$

R-Squared

R-Squared

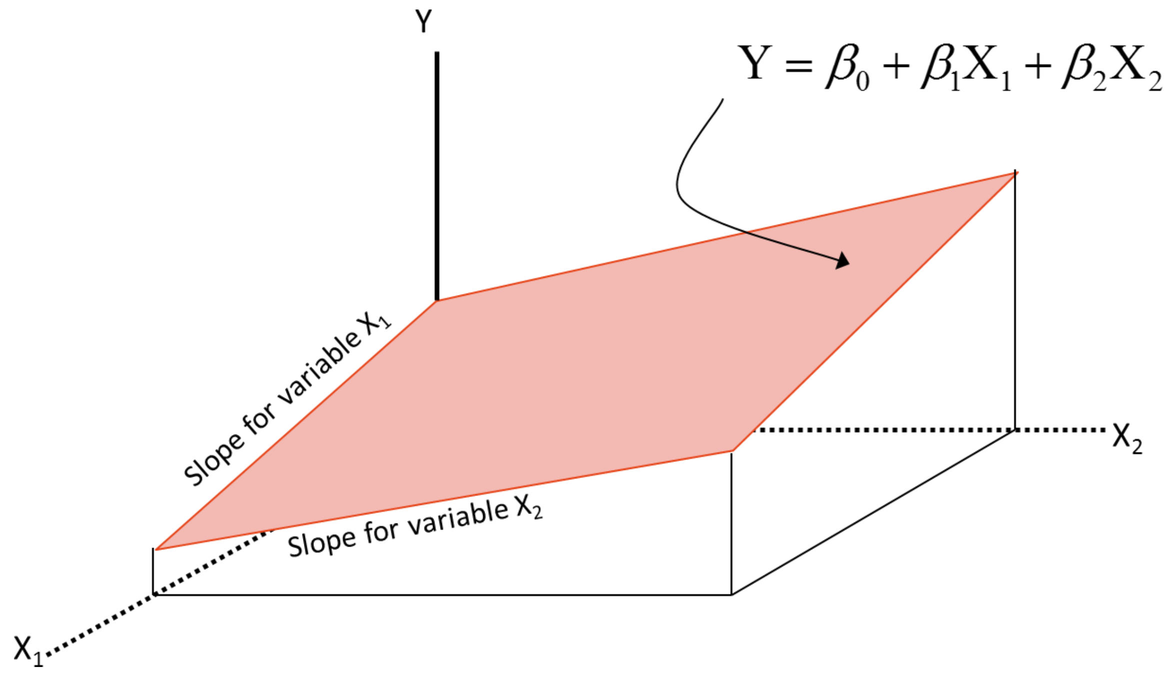
- A good fit will have
  - SSE (Minimum or Maximum?)
  - SSR (Minimum or Maximum?)
  - And we know SST= SSE + SSR
  - SSE/SST(Minimum or Maximum?)
  - SSR/SST(Minimum or Maximum?)
- The coefficient of determination is the portion of the total variation in the dependent variable that is explained by variation in the independent variable
- The coefficient of determination is also called R-squared and is denoted as R^2

$$\boxed{R^2 = \frac{SSR}{SST}} \quad \text{where} \quad \boxed{0 \leq R^2 \leq 1}$$

Multiple Regression

Multiple Regression

- Using multiple predictor variables instead of single variable
- We need to find a perfect plane here



Part-12:Individual Impact of variables

Individual Impact of variables

- Look at the P-value
- Probability of the hypothesis being right.
- Individual variable coefficient is tested for significance
- Beta coefficients follow t distribution.
- Individual P values tell us about the significance of each variable
- A variable is significant if P value is less than 5%. Lesser the P-value, better the variable
- Note it is possible all the variables in a regression to produce great individual fits, and yet very few of the variables be individually significant.

To test

$$H_0 : \beta_i = 0$$
$$H_a : \beta_i \neq 0$$

Test statistic:

$$t = \frac{b_i}{s(b_i)}$$

Reject H_0 if

$$t > t(\frac{\alpha}{2}; n - k - 1) \quad or$$
$$t < -t(\frac{\alpha}{2}; n - k - 1)$$

Adjusted R-Squared

Adjusted R-Squared

- Is it good to have as many independent variables as possible? Nope
- R-square is deceptive. R-squared never decreases when a new X variable is added to the model – True?
- We need a better measure or an adjustment to the original R-squared formula.
- Adjusted R squared
  - Its value depends on the number of explanatory variables
  - Imposes a penalty for adding additional explanatory variables
  - It is usually written as (R-bar squared)
  - Very different from R when there are too many predictors and n is less

$$\overline{R}^2 = R^2 - \frac{k-1}{n-k}(1-R^2)$$

n-number of observations, k-number of parameters

Multiple Regression- issues

Part-15: Multicollinearity

Multicollinearity

- Multiple regression is wonderful - In that it allows you to consider the effect of multiple variables simultaneously.
- Multiple regression is extremely unpleasant -Because it allows you to consider the effect of multiple variables simultaneously.
- The relationships between the explanatory variables are the key to understanding multiple regression.
- Multicollinearity (or inter correlation) exists when at least some of the predictor variables are correlated among themselves.
- The parameter estimates will have inflated variance in presence of multicollineraity
- Sometimes the signs of the parameter estimates tend to change
- If the relation between the independent variables grows really strong then the variance of parameter estimates tends to be infinity – Can you prove it?

Multicollinearity detection

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4$$

- Build a model X1 vs X2 X3 X4 find R square, say R1
- Build a model X2 vs X1 X3 X4 find R square, say R2
- Build a model X3 vs X1 X2 X4 find R square, say R3
- Build a model X4 vs X1 X2 X3 find R square, say R4

- For example if R3 is 95% then we don't really need X3 in the model
- Since it can be explained as liner combination of other three
- For each variable we find individual R square.
- $1/(1-R^2)$ is called VIF.
- VIF option in SAS automatically calculates VIF values for each of the predictor variables

R Square	40%	50%	60%	70%	75%	80%	90%
VIF	1.67	2.00	2.50	3.33	4.00	5.00	10.00

LAB: Multicollinearity

- Identify the Multicollinearity in the Final Exam Score model
- Drop the variable one by one to reduce the multicollinearity
- Identify and eliminate the Multicollinearity in the Air passengers model

Multiple Regression model building

Conclusion -Regression

Conclusion -Regression

- We discussed the basic concepts of correlation, regression
- Adjusted R-squared is a good measure of training/in time sample error. We can't be sure about the final model performance based on this. We may have to perform cross-validation to get an idea on testing error.
- Outlies can influence the regression line, we need to take care of data sanitization before building the regression line.

Thank you