॥ त्वं ज्ञानमयो विज्ञानमयोऽसि ॥

Artificial Intelligence-2 (CSL 7040)

Lecture 6

Value for Information

- What to ask???

- Information value theory □ what information to acquire □ simplified forms of sequential decision making □ only affects the agent’s belief state not their physical state

Value for Information: Example

- Oil company –proposed to purchase n no. of blocks□ $C\$$
- Price of each block = C/n \$
- Seismologist □ Yes or no for any particular block
- The probability of that particular block containing oil under it = $1/n$
- If the block contains oil truly, then profit = $C-C/n= (n-1)C/n$
- The probability of that particular block not containing oil under it = $(n-1)/n$
- Profit = $C/(n-1)-C/n = C/(n(n-1))$
- Total profit of the survey= $1/n*(n-1)C/n + (n-1)/n * C/n(n-1)=C/n$

General Formula for Computation of Perfect Information

- Exact evidence about some random variable: E_j
- Initial evidence → e
- Value of current best action → α
- $EU(\alpha|e) = \max_a \sum_{s'} P(Results\ (a) = s'|a, e)U(s')$
- $EU\left(\alpha_{e_j}\middle|e, e_j\right) = \max_a \sum_{s'} P\left(\mathrm{Res}ults(a) = s'\middle|a, e, e_j\right)U(s')$
- $VPI_e(E_j) = (\sum_k P(E_j = e_{jk}|e)EU\left(\alpha_{e_{jk}}\middle|e, E_j = e_{jk}\right))\ - EU(\alpha|e)$

Value for Information

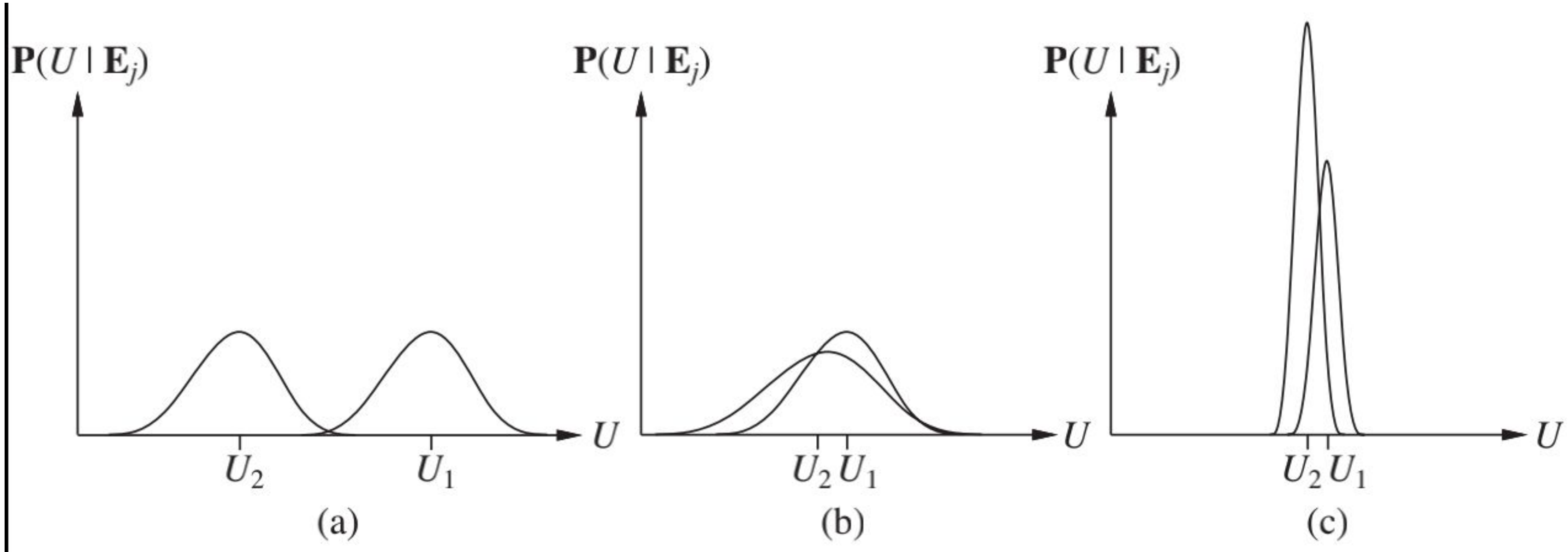
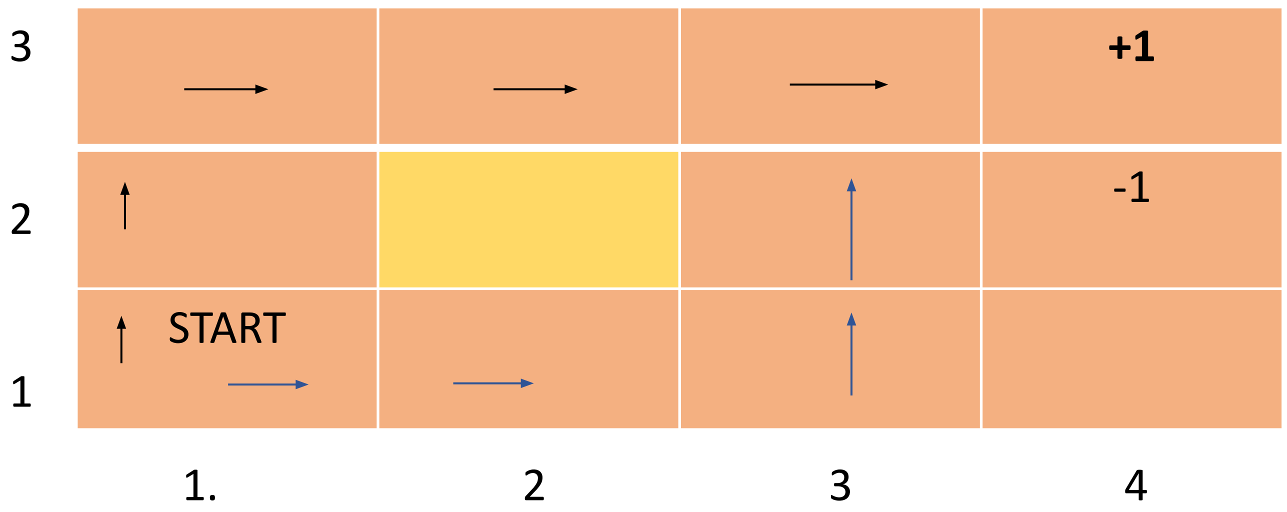
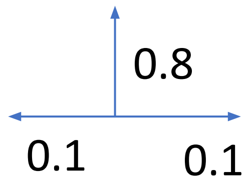


Figure 16.8 Three generic cases for the value of information. In (a), a_1 will almost certainly remain superior to a_2, so the information is not needed. In (b), the choice is unclear and the information is crucial. In (c), the choice is unclear, but because it makes little difference, the information is less valuable. (Note: The fact that U_2 has a high peak in (c) means that its expected value is known with higher certainty than U_1.)

Sequential Decision Making





R(s)= -0.04

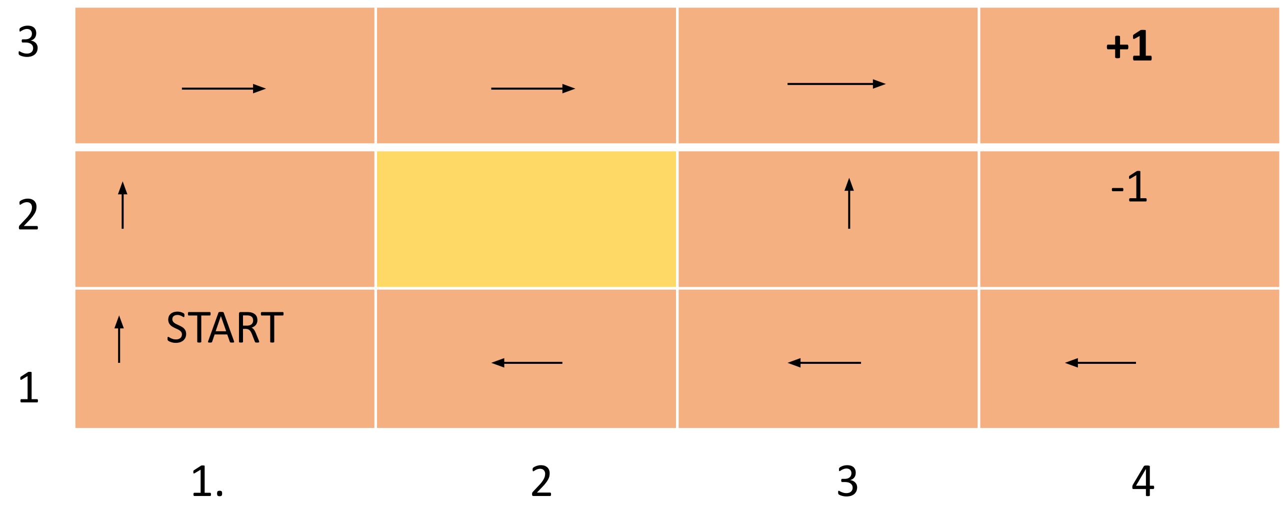
Set of actions ={UP, DOWN, RIGHT, LEFT}

Set of Intended Actions ={UP, UP, RIGHT, RIGHT, RIGHT}
{RIGHT, RIGHT, UP, UP, RIGHT}

Probability of reaching state +1 only by taking intended actions= 0.8^5=0.32768
0.1^4*0.8
Total probability of reaching +1=0.327+0.1^4*0.8=0.3277

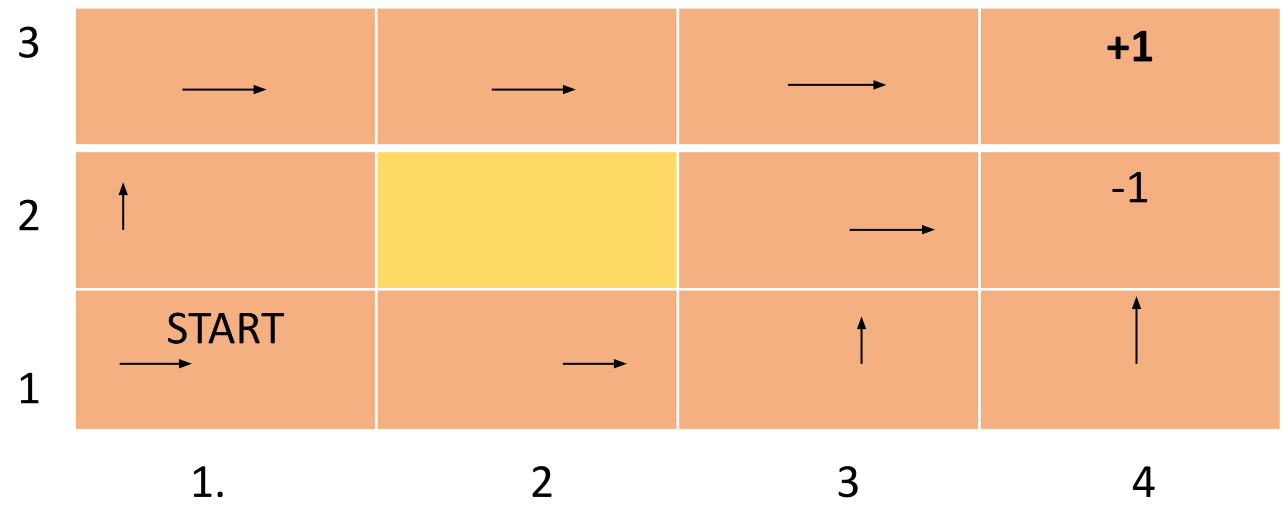
Markovian Decision Process and Policy

- A sequential decision problem in fully observable environment
  - Set of states
  - Set of ACTIONS(s) in each state
  - Transition model $P(s'|s,a)$
  - Reward function R(s)
- Policy: The solution what an agent should do in a particular state
- $\pi(s) \rightarrow Action\ recommended\ in\ state\ s$
- Quality of the Policy: EU of all the possible environment history
- Optimal Policy: The policy that generates the highest EU (π^*)



3	→	→	→	**+1**
2	↑		↑	-1
1	↑ START	←	←	←
	1.	2	3	4

R(s)= -0.04

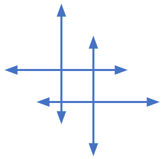


3	→	→	→	**+1**
2	↑		→	-1
1	START →	→	↑	↑
	1.	2	3	4

R(s)= -1.8

3	→	→	→	**+1**
2	↑		←	-1
1	↑ START	←	←	↓
	1.	2	3	4

$-0.02<R(s)<0$



3	↔↕	↔↕	←	**+1**
2	↔↕		←	-1
1	START ↔↕	↔↕	↔↕	↓
	1.	2	3	4

R(s)= 0.5

Utilities over Time

- $U_h([s_0, s_1, \ldots., s_{N+k}]) = U_h([s_0, s_1, \ldots., s_N]) \ \forall k > 0$
- Optimal policy in finite horizon is non-stationary
- We are dealing here with infinite horizon → don’t have any fixed deadline → MDP to have one terminal state

Stationary Preference:

The preference between$[s_0, s_1, \ldots.]\ and[s_0', s_1', \ldots.]$ if $s_0 = s_0'$ then

Is equivalent to the preference between $[s_1, s_2, \ldots.]\ and[s_1', s_2', \ldots.]$

Assigning utility to preference

- Additive Reward:

$$U_h([s_0, s_1, s_2, \dots)] = R(s_0) + R(s_1) + R(s_2) + \cdots$$

- Discounted Reward:

$$U_h([s_0, s_1, s_2, \dots)] = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \cdots$$

$$\gamma \rightarrow discount\ factor\colon 0 \leq \gamma \leq 1$$

$$discount\ factor \equiv interest\ rate\ (\frac{1}{\gamma} - 1)$$

Discount factor

- If there is no terminating state in the environment → history is going to be infinitely long → utility with additive reward = $+$ \infinity → difficult to handle
- Solution??
  1. Set $\gamma < 1$

     $U_h([s_0, s_1, \dots]) = \sum_{t=0}^{infinity} \gamma^t R(s_t) \leq \sum_{t=0}^{infinity} \gamma^t R_{max} = R\_\max/(1-\gamma)$
  2. Should chose a policy that guarantees to reach a terminal state → Proper policy
  3. Infinite sequence could be compared in terms of average reward obtained per time step.

Optimal policies and Utilities of the States

- Assume s→ Initial state; s_t→ random variable: agent reaches here at time t after executing the policy π
- EU by executing the policy π:

$$U^{\pi}(s) = E[\sum_{t=0}^{\propto} \gamma^{t} R(s_t)]$$

Expectation w.r.t. probability distribution over state sequences determined by s and π

$$\pi_s^* = \arg\max_{\pi} U^{\pi}(s)$$

Discounted utilities with finite horizon → optimal policy is independent of the starting state. Actions can’t be independent → policy function specify action for each state

- π_a^* and π_b^* those should not disagree with another optimal policy π_c^* $\rightarrow$ single policy π^*

True Utility of a State

- $U^{\pi^*}(s)$→ Expected sum of discounted rewards after executing optimal policy

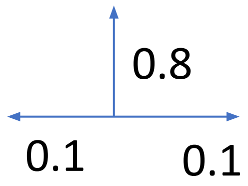
- $R(s) \rightarrow Short\ term\ reward\ for\ being\ in\ the\ sate\ s$
- $U(s) \rightarrow long - term\ total\ reward\ from\ s\ onwards$

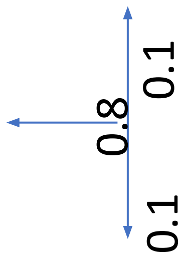
$$\pi^*(s) = argmax_{a \in \mathrm{A}(s)} \sum_{s\prime} P\left(s' \middle| s \quad ,a\right) U(s')$$

Value Iteration

- To calculate an optimal policy □ calculate utilities in each state and use the state utilities to select an optimal action in each state

3	**0.812**	**0.868**	**0.918**	**+1**
2	0.762		0.660	-1
1	START 0.705	0.655	0.611	0.338
	1	2	3	4





Bellman Equation for Utilities:

- Utility of a state = Immediate reward for the state + expected discounted utility of the next state, assuming the agent will take the optimal action
- $U(s) = R(s) + \gamma \max_{a \in \mathrm{A}(s)} \sum_{s'} P(s'|s,a) U(s')$

$U(1{,}1) = -0.04 + \gamma \mathrm{max}[\ 0.8U(1{,}2) + 0.1U(2{,}1) + 0.1U(1{,}1), \rightarrow Up,$

$0.9U(1{,}1) +$

$0.1\mathrm{U}(1{,}2), \mathrm{Left} \qquad 0.\,9\mathrm{U}(1{,}1) + 0.\,1(2{,}1) \rightarrow \mathrm{Down},$

$0.8\mathrm{U}(2{,}1) + 0.1\mathrm{U}(1{,}2) + 0.1\mathrm{U}(1{,}1) \rightarrow \mathrm{Right}]$

Value Iteration Algorithm

- $U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in \mathrm{A}(s)} \sum_{s'} P(s'|s,a) U_i(s')$

Function: Value-iteration