Question: Show that the global minimum of C(G) = max V (G, D) is achie…

Show that the global minimum of $C(G) = \max_D V(G, D)$ is achieved if and only if $p_G = p_{\text{data}}$. At that point, show that $C(G)$ achieves the value $-\log 4$.

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Expert Answer

Anonymous answered this
23 answers

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Theorem:- Show that the global minimum of $C(G) = \max_D V(G, D)$ is achieved if and only if $p_G = p_{data}$. At that point show that $C(G)$ achieves the value $-\log 4$.

Proof:-

-> First let us look at the objective function,

$$\min_{\phi} \max_{\theta} [E_{x \sim p_{data}} \log D_\theta(x) + E_{z \sim p(z)} \log(1 - D_\theta(G_\phi(z)))]$$

-> We will expand it to its integral form

$$\min_{\phi} \max_{\theta} \int_x p_{data}(x) \log D_\theta(x) + \int_z p(z) \log(1 - D_\theta(G_\phi(z)))$$

-> Let $P_G(x)$ denote the distribution of the x's generated by the generator and since x is a function of z we can replace the second integral as shown below

$$\min_{\phi} \max_{\theta} \int_x p_{data}(x) \log D_\theta(x) + \int_x p_G(x) \log(1 - D_\theta(x))$$

-> Revised objective is given by

$$\min_{\phi} \max_{\theta} \int_x (p_{data}(x) \log D_\theta(x) + p_G(x) \log(1 - D_\theta(x))) dx$$

-> To find the optima we will take the derivative of the term inside the integral w.r.t. D & set it to zero

$$\frac{d}{d(D_\theta(x))} (p_{data}(x) \log D_\theta(x) + p_G(x) \log(1 - D_\theta(x))) = 0$$

$$p_{data}(x) \frac{1}{D_\theta(x)} + p_G(x) \frac{1}{1 - D_\theta(x)} (-1) = 0$$

$$\frac{p_{data}(x)}{D_\theta(x)} = \frac{p_G(x)}{1 - D_\theta(x)}$$

$$(p_{data}(x))(1 - D_\theta(x))(D_\theta(x))$$

→ this means D^* for any given generator.

$$D^*_G(G(x)) = \frac{P_{data}(x)}{P_{data}(x) + P_G(x)}$$

→ Now the if part of the theorem says "if $P_G = P_{data}$"

→ So let us substitute $P_G = P_{data}$ into $D^*_G(G(x))$ and see what happens to the loss function:-

$$D^*_G = \frac{p_{data}}{p_{data} + p_G} = \frac{1}{2}$$

$$V(G, D^*_G) = \int_x p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))dx$$

$$= \int_x p_{data}(x) \log\frac{1}{2} + p_G(x) \log\left(1 - \frac{1}{2}\right)dx$$

$$= -\log 2 \int_x p_G(x)dx - \log 2 \int_x p_{data}(x)dx$$

$$= -2\log 2 = -\log 4$$

∴ Proved

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Comments

Anonymous posted 11 months ago

I appreciate the answer, but is very difficult to understand. Please can you rewrite the same. I am very sorry to ask like this.

Anonymous posted 11 months ago

give me some time will do it

Anonymous posted 11 months ago

Anonymous posted 10 months ago

Can I please have the answer

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Q: 6. Show that the global minimum of C(G) = maxV(G, D) is achieved if and only if PG = Pdata. At that point, show that C(G) achieves the value – log 4. D

A: See answer

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Q: 6. Show that the global minimum of C(G) = maxV(G, D) is achieved if and only if PG = Pdata. At that point, show that C(G) achieves the value – log 4. D

A: See answer

Q:
1. Let f:R" + R be a convex function that is f(txı + (1 – t)x2)

A: See answer 100% (1 rating)

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