Dual Simplex method

Dual simplex method is used to solve a LPP while allowing the coloum "b" of simplex table to have negative entries. While simplex method keeps "b" non-negative and tries to make $z_j - c_j$ non-negative ($\geq 0\ \forall j$), dual simplex method keeps $z_j - c_j$ non-negative ($\forall j$) and tries to make "b" non-negative. Note that if both "b" and "$z_j - c_j$" ($\forall j$) are non-negative then the table corresponds to optimal solution of the original problem. Although the method has same time complexity as simplex method; the method has applications to solve specific problems (like integer programming).

The detailed algorithm for Dual Simplex algorithm is given below:

1. Represent the problem as a maximization problem
2. Write all eqns as "$\leq$" type, introduce slack variables and compute the first simplex table
3. If some $z_j - c_j$ is negative, then the method is not applicable
4. If all $z_j - c_j \geq 0$ and coloum b is non-negative then the table corresponds to optimal solution to given LPP.
5. If some b_i's are negative then update the simplex table using the following strategy:

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(a) Choose most negative b_i, x_{B_i} leaves.

(b) Compute the ratios $\left\{ \frac{z_j - c_j}{a_{ij}} : a_{ij} < 0 \right\}$ i.e. ratios of $z_j - c_j$ with negative entries of ~~exiting~~ i^{th} row. If $\frac{z_{j_0} - c_{j_0}}{a_{ij_0}}$ is largest (i.e. least modulus) then x_{j_0} enters the simplex table.

⑥ Update the simplex table untill the coloum "b" is non-negative.

Example :-

$$\min \; 3x_1 + x_2$$

$$\text{s.t.} \quad x_1 + x_2 \geq 1$$

$$2x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

Writing the problem in desired form, the problem ~~becomes~~ can be written as,

$$\max \; -3x_1 - x_2$$

$$\text{s.t.} \quad -x_1 - x_2 + x_3 \leq -1$$

$$-2x_1 - 3x_2 \quad + x_4 \leq -2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The first simplex table is

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			-3	-1	0	0
C_B	B	b	a_1	a_2	a_3	a_4
0	x_3	-1	-1	-1	1	0
0	x_4	-2	-2	-3	0	1
$z_j - c_j$:			3	1	0	0

$z_j - c_j \geq 0 \;\forall j \Rightarrow$ Dual simplex method is applicable

b_2 is most negative $\Rightarrow x_4$ leaves the simplex table

$\dfrac{z_2 - c_2}{a_{22}}$ is largest (least modulus) & thus x_2 enters. Thus updated simplex table is:

			-3	-1	0	0
C_B	B	b	a_1	a_2	a_3	a_4
0	x_3	$-1/3$	$-1/3$	0	1	$-1/3$
-1	x_2	$2/3$	$2/3$	1	0	$-1/3$
$z_j - c_j$:			$7/3$	0	0	$1/3$

b_1 is most negative $\Rightarrow x_3$ leaves. Further $\dfrac{z_4 - c_4}{a_{14}}$ is largest $\Rightarrow x_4$ enters

Thus updated table is

			-3	-1	0	0
C_B	B	b	a_1	a_2	a_3	a_4
0	x_4	1	1	0	-3	1
-1	x_2	1	1	1	-1	0
$z_j - c_j$:			2	0	1	0

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