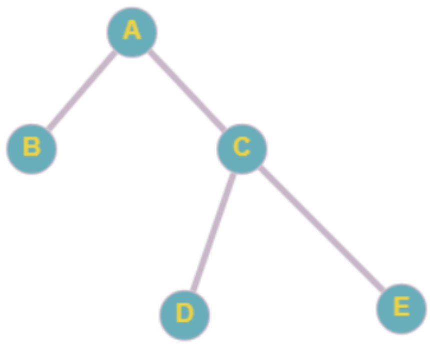
Lecture 13 and 14
Matching

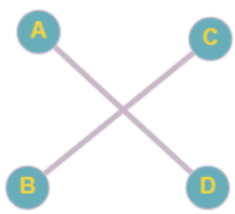
- The set of non-adjacent edges is called matching i.e independent set of edges in G such that no two edges are adjacent in the set.
- he parameter $\alpha 1$(G) = max { |M|: M is a matching in G } is called matching number of G i.e the maximum number of non-adjacent edges.
- Any matching M with |M| = $\alpha 1$(G) is called a maximum matching.

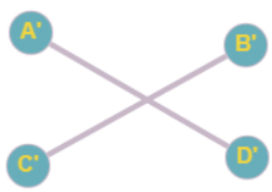
Is basically a non-loop edges with non-shared end points
Matching = = $\{AB, CD\}$,
= $\{BC\}$ = M2

M1 = $\{AB, CD\}$, M2 = $\{CD\}$
M2 = $\{AB, CD\}$, M5 = $\{\}$
M3 = $\{CD\}$



X : Boys
Y : Girls





$\{AB, CD\}$ $\{A'B', C'D'\}$
$\{AD, BC\}$ $\{A'D', B'C'\}$
$\{AC, BD\}$ $\{A'C', B'D'\}$

Proof:
By Induction
Base Condition:
A tree having n = 1 node has #PM = 0, n = 2, # PM = 1
Ind. Proof
Suppose the tree for n≤k nodes
Leaf = end nodes
Hall's Marriage Theorem
The marriage theorem, answers the following question, known as the marriage problem: if there is a finite set of girls, each of whom knows several hoys, under what conditions can all the girls marry the boys in such a way that each girl marries a hoy she knows.
Hall's Theorem
A bipartite graph G = (A ∪ B, E) has an A-perfect matching if and only if the following condition holds: ∀S ⊆ A. |N(S)| ≥ |S|,
where N(S) = {v ∈ B : ∃u ∈ S. {u, v} ∈ E.}.
Necessary condition
Proof: Suppose x-y bigraph has a matching
that saturates x. then obviously
Isl≤IN(S)|
∀ s≤x sufficient condition. if ∀ s≤x then IN(S)| ≥ |S| Then there is a matching that saturates x
if p=>q
∼q=>∼p
We shall prove the following contra positive:
if there is not such matching M that saturates x, then ∋ s≤x s. t.
s≤ N(S)|
Let u EX be a vertex
unsaturated by a
matching M.
Vertex Cover

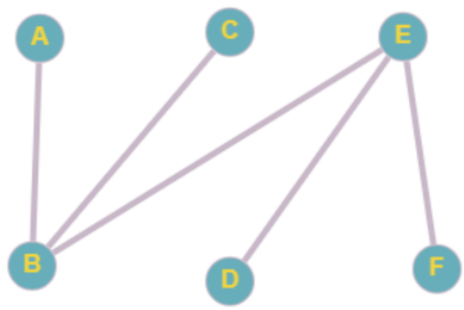
- A set of vertices K which can cover all the edges of graph G is called a vertex cover of G i.e. if every edge of G is covered by a vertex in set K.
- The parameter β0(G) = min { |K|: K is a vertex cover of G } is called vertex covering number of G i.e the minimum number of vertices which can cover all the edges.
- Any vertex cover K with |K| = β0(G) is called a minimum vertex cover.

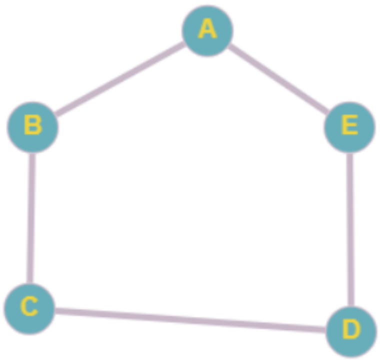
VC of graph G is set θ ⊆$v\,(G)$ that contains atleast one end point of every edge.

$$\theta = \{B, E\}$$

$$\theta 1 = \{A, B, C.D, E, F\}$$

$$\theta 2 = \{A, B, E\}$$



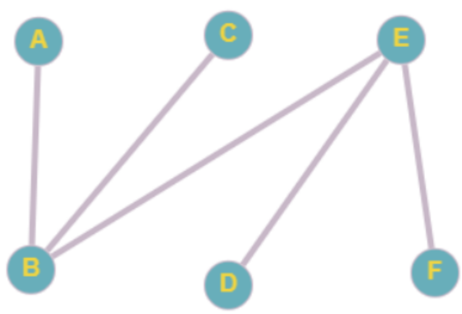


Min $VC = \{A, C, D\}$

Maximal Matching $= \{BC, DE\}$

Independent Set

- A set of vertices I is called independent set if no two vertices in set I are adjacent to each other or in other words the set of non-adjacent vertices is called independent set.
- It is also called a stable set.
- The parameter $\alpha 0$(G) = max { |I|: I is an independent set in G } is called independence number of G i.e the maximum number of non-adjacent vertices.
- Any independent set I with |I| = $\alpha 0$(G) is called a maximum independent set.



$$\propto (G) = 4$$

$$\propto' (G) = 2$$

$$\beta (G) = 4$$

$$\beta' (G) = 2$$

$$\propto (G) = Maximumsizeoftheidset$$

$$\propto'^{(G)} = Maximumsizeofthematching$$

$$\beta (G) = MinimmumsizeoftheVC$$

$$\beta'^{(G)} = Minimumsizeoftheedgecurve$$

$$\propto (G) + \beta (G) = n(G)$$

Proof:
Let S be the independent set of max then edge is incident to size at least one vertex of S'
S = $\{A, B, C\}$
S' = $\{D, E\}$
S VS' = V (G)
S' is the minimum size vertex curve

$$\beta (G) = \lfloor S' \rfloor$$

S is the max size id set

$$\propto (G) = \lfloor S \rfloor$$

$\propto (G) + \beta (G) = \lfloor S \rfloor + \lfloor S' \rfloor = \lfloor V(G) \rfloor$ = n (G)

$$\propto (G) + \beta (G) = n(G)$$

$$\propto' (G) + \beta' (G) = n(G)$$

If G is the bipartite graph with no related vertices, then,

$$\propto (G) = \beta' (G)$$

$$\propto (G) + \beta (G) = n(G)$$

$$\propto' (G) + \beta' (G) = n(G)$$

$$\propto (G) = \beta^{'(G)}$$

$$\propto = \beta'$$

Proof
Let G be the bipartite graph prove that $\propto (G) = \frac{n(G)}{2}$ if G has a perfect matching

$$\propto (G) + \beta (G) = n(G)$$

$$\propto (G) = n (G) + \beta (G)$$

$$= n (G) - \propto' (G)$$

$$if G is the PM then what will be the maximum size of the matching?$$

$$n (G) - \frac{n (G)}{2} = \frac{n(G)}{2}$$

Theorem
If the G is a simple graph, then if diam (G) $\geq$ 3then the diam (G^3) $\geq$ 3
Proof
diam (G) $\geq$ 3
$\exists U and V$, that uv $\nexists \in (G)$

$$u and v does not have a common neighbour$$

$\forall x \in$ v(G) - $\{u, V\}$ has atleast one of the $\{u, V\}$ *isnonneigbour*
diam (G) $\geq$ 3
uv $\nexists \in (G)$
uv $\in (G^2)$
ux $\in (G^{'})$, Vx $\in (G^{'})$