Week 4

Vertex Degree and Counting*

4.1 Graphical Sequence

A list of non-negative inegers that is sequence of a graph or degree sequence of a simple graph

4.1.1 What is graphica sequence? And Examples

A graphical sequence is a sequence of non-negative integers of a graph and it is the degree sequence of a given graph.

Figure 4.1: Sequence 2 2.

Above figure 4.1 shows degree of 2 2 sequence. node 1 and 2 has 2 2 sequence.

Figure 4.2: Sequence 5 3.

Above figure 4.2 shows degree of 5 3 sequence. node 1 and 2 has 5 2 sequence.

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4.2 How to identify the sequence is valid?

If graph allowed parallel and self loops then check sum of value degree is even or not. If the sum of the value degree is even then we can consider it as valid sequence. Otherway we can apply the Havel-Hakimi algorithm.

Havel-Hakimi algorithm is an algorithm to identify the given finite set of non-negative integers of degree sequence in descending order is a valid graph sequence or not. Havel-Hakimi algorithm is used for graph realization problems

pseudocode
[1]Sort the given sequence non-negative integers in descending order
[2] delete the first element and subtract 1 from the next elements,
[3] and repeat the above if all the elements become zero then sequence is valid and a simple graph exists, for all other cases invalid sequence and no simple graph exists.

Example sequence 5 5 5 2 1 1 1

4 4 3 1 0 1 1

4 4 3 1 1 1 0

3 2 0 0 1 0

3 2 1 0 0 0

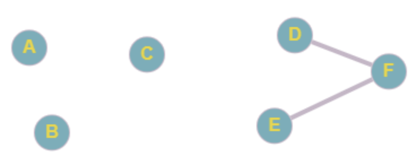

Figure 4.3: havel-hikimi algorithm.

Example sequence 5 5 4 3 2 2 2 1

4	3	2	1	1	2	1
4	3	2	2	1	1	1
2	1	1	1	1	0	
0	0	1	1	0		
1	1	0	0	0		
0	0	0				

Figure 4.4: havel-hikimi algorithm.

Proove (or) disagree If u and v are only vertices of odd degree in a graph G, then G contains u-v path.

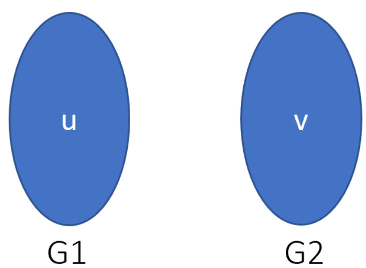

Figure 4.5: graphs G1 G2.

1 does not follow handshake lemma.

2 $\Sigma` d(v) = \gamma(G1)$

3 therefore uv have to be in connected component.

Proove (or) disagree Determine maximum number of edges in a bipartite subgraph of $\mathrm{P_n}$,$\mathrm{C_n}$ and $\mathrm{K_n}$.

Figure 4.6

maximal subgraph of $\mathrm{P_n}$ that is bipartite $\mathrm{P_n} \subseteq P_\mathrm{n}, P_\mathrm{n} = n - 1 edges.$
maximal subgraph of C_n *that is a bipartite graph.*

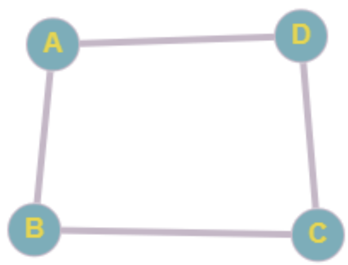

Figure 4.7: $\mathrm{C_n} \subseteq C_\mathrm{n}$

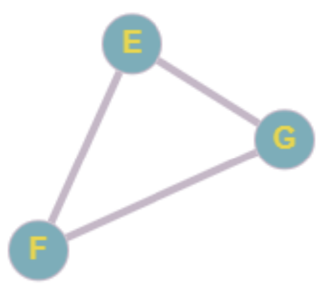

Figure 4.8: $\mathrm{P_n} \subseteq P_\mathrm{n}$

$$\begin{cases} n & \text{n even cycle,} \\ n-1 & \text{n odd cycle} \end{cases}$$

Proove (or) disagree Let l,m,n$\epsilon I_+\{0\}\cup 0$ *such that* $l+m=n$.

Find a neccessarry and sufficient condition on l,m n such that a connected simple n-vertex graph with ‘l‘ vertices of even degree and ‘m‘ vertices of odd degree.

n>=1,l=even (or) odd, m=even

Ex (2,0,2) valid sequence? not possible, graph should be simple.

Figure 4.9

4.3 Directed graphs

A Directed graph or diagraph G is a triple consisting of a vertex set v(G), an edge set E(G) and a function assigning each edge an ordered direction.

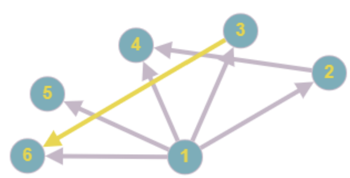

Figure 4.10: directed graph

4.3.1 Underlying graphs

Underlying graph of a directed graph D is the graph G obtained by treating each edges of D as unordered pairs.

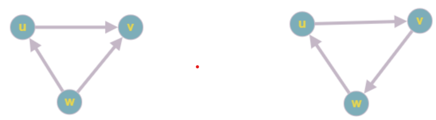

Figure 4.11: graphs under directed graph

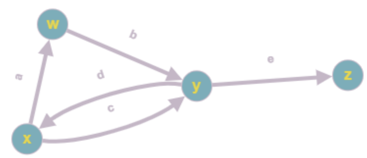

Figure 4.12: directed graph

Adj.Matrix

$$\begin{array}{c c} & \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{array}{c} w \\ x \\ y \\ z \end{array} & \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Incident Matrix

$$\begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} w \\ x \\ y \\ z \end{array} & \begin{pmatrix} -1 & +1 & 0 & 0 & 0 \\ +1 & 0 & +1 & -1 & 0 \\ 0 & -1 & -1 & +1 & +1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \end{array}$$

4.3.2 Weekly and strongly connected diagraph

A diagraph is weakly connected if its underlying graph is connected. A diagraph is strongly connected if for each of pair of vertices u and v there us a path between u and v.

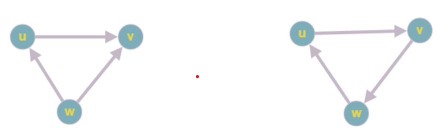

Figure 4.13: underlying graphs

Kernel of DiGraph A kernek in the digraph D is a subset of vertices S such that S induces no edges and every vertex outside S has a successor in S.

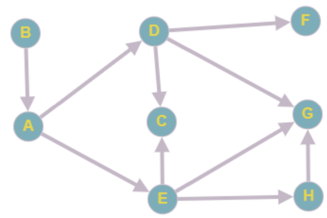

Figure 4.14: graph

{a,f,g,c}