Week 7

Matching*

7.1 Matching

Matching is a set of non-loop edges with non-shared end points.The top match on the graph is an incomprehensible match enlarged by adding hem. Size matching match of size limit between all similarities on the graph.

7.1.1 Hall's Theorem

The name appears from the establishment of a cohesive relationship between the n group of men and the group of n women. If every man fits k women and every woman fits k men, there should be a perfect match. Again edges are allowed, which increases the range of applications.

Proof: An X, Y-bigraph G has a matching that saturates X if and only if

$$|(V(S)| \geq |S| \forall S \subseteq X.$$

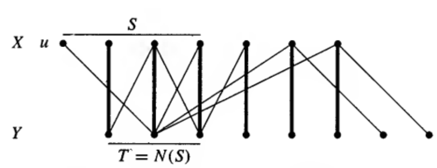

Figure 7.1: S

fficient conditions: If

$$\forall S \subseteq X, |N(S)| \geq |S|$$

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Then there is a matching that saturates X.
if

$$p = q$$

$$\sim p = \sim q$$

If M is a maximum matching in G and M does not saturate X, then we obtain a set S c X such that

$$| A(S) | \leq | S | .$$

Let

$$u \in X$$

be a vertex unsaturated by M. Among all the vertices reachable from u by M-alternating paths in G, let S consist of those in X, and let T consist of those in Y.
Following condition need to be proved:
If there is no such matching M that saturates X, then

$$\forall S \subseteq X$$

, such that

$$| S | > | N(S) |$$

Suppose 2 subsets

$$S \subseteq X$$

and

$$T \subseteq Y$$

are considered as follows:
S= End points if M-alternating paths starting from u with the last edge belonging to M
T= End points if M-alternating paths starting from u with the last edge not belonging to M
The M-alternating paths from u reach Y along edges not in M and return to X along edges in M. Hence every vertex of S - u is reached by an edge in M from a vertex in T. Since there is no M-augmenting path, every vertex of T is saturated; thus an M-alternating path reaching

$$y \subseteq T$$

extends via M to a vertex of S. Hence these edges of M yield a bijection from T to S — [u], and we have

$$| T | = | S - u |$$

$$| S = 1 + | T | = 1 + | N(S) |$$

$$| S | > | N(S) |$$

With T = N(S), we have proved that

$$| N(S) |=| T |=| S | -1 <| S |$$

for this choice of S. This completes the proof of the contrapositive.

Let's say there are 1000 students and 2000 companies are coming and each company has one position and certain eligibility criteria. The question is that everyone gets the job or not?

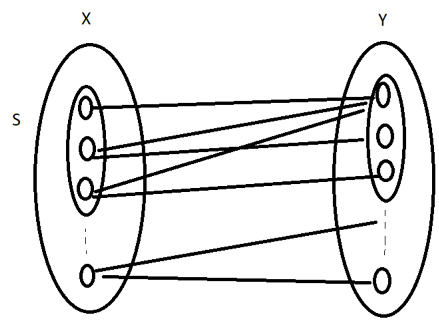

Figure 7.2:

an Hall's theorem be used for this problem?
If Hall's theorem will be applied then each and every subset of X will be considered and cardinality need to be analysed for S and N(S).
Refer to figure Assign S(subset) to 3 companies and these 3 has 3 matching and this has to be done for 2(1000) and that is the problem because it's quite high so this cannot be solved using Hall's theorem.
To solve this there is a derived feature which is called degree constraint graph.
If

$$\forall x \subseteq X, deg(x) \leq d$$

and

$$\forall y \subseteq Y, deg(y) \geq d$$

This property is called as Degree constraint.If the above properties are getting satisfied then perfect matching can be formed.

7.2 Vertex Cover

A vertex cover of a graph G is a set

$$Q \subseteq V(G)$$

that contains at least one endpoint of every edge. The vertices in Q cover E(G).

Matching and Vertex covers: In the left-hand graph below we mark the vertex size 2 cover and show the size 2 in bold.

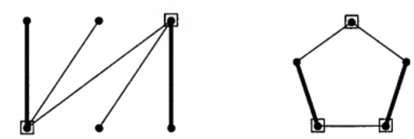

Figure 7.3:

ertex cover size 2 prohibits matching with 2 edges, too size 2 matching prevents vertex covers with less than 2 layers. As shown on the right, the correct values vary by 1 in the odd cycle.

7.2.1 Independent sets:

Independence graph number is the largest independent set of vertices.
In a graph below, independent sets are A,C.No vertex closes the two edges of the match. Similarly, no edge consists of two vertical set vertices.

Edges: An edge cover of G is a set L of edges such that every vertex of G is incident to some edge of L.

For the correct size sizes of the frames as well to cover the problems we have described, using the notice below.

$$\alpha(G) maximum size of independent set \tag{7.1}$$
$$\alpha'(G) maximum size of matching \tag{7.2}$$
$$\beta(G) minimum size of vertex cover \tag{7.3}$$
$$\beta'(G) minimum size of edge cover \tag{7.4}$$

7.2.2 Proof:

If S is an independent set, then all edges are at least one event vertex of S. Conversely, when S covers all edges, no edges intersect 5 vertices.

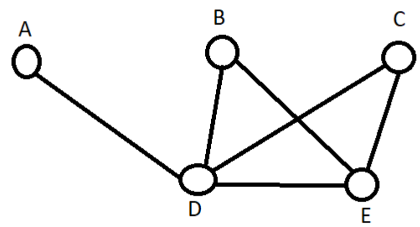

Figure 7.4: T

us the entire upper set of frames corresponds to a small vertex cover, as well as

$$\alpha(G) + \beta(G) = n(G)$$

$$\text{S} = \text{A,B,C}$$
$$\widetilde{S} = D, E$$
$$\text{S U } \widetilde{S} = V(G)$$

Here,

$$\widetilde{S}$$

covers all edges and if it covers all the edges, then there are no edges joining vertices of S. It is a minimum size vertex cover.

$$\beta(G) = \widetilde{S}$$

S is maximum size independent set

$$\alpha(G) = \mid S \mid$$

Theref0re, $\alpha(G) + \beta(G) = \mid S \mid + \mid \widetilde{S} \mid = V(G) = n(G)$
$Hence, \alpha(G) + \beta(G) = n(G)$

7.2.3 Proof:

If G is a graph without isolated vertices, then

$$\alpha'(G) + \beta'(G) = n(G)$$

If G is a bipartite graph with no isolated vertices then

$$\alpha(G) = \beta'(G)$$
$$\alpha(G) + \beta(G) = n(G)$$
$$\alpha'(G) + \beta'(G) = n(G)$$

$$\alpha'(G) = \beta'(G)$$
$$\alpha(G) + \beta(G) = \alpha'(G) + \beta'(G)$$
$$= \beta(G) + \beta'(G)$$
$$\alpha = \beta'$$

7.2.4 Theorem:

If G is a simple graph, then diam G $\geq$ 3 => $diam\widetilde{G} \leq 3$

Proof: For every
$$x \subseteq V(G) - u, v$$
has at least one of u,v as a non neighbour.

When diam G 2, there exist nonadjacent vertices
$$u, v \subseteq V(G)$$
with no common neighbour
. This makes x dependent on G to have at least one of the u, v of G. Since
$$v \subseteq E(G)$$
in all pairs x, y there are x, y-way lengths of at least 3 in G through (u, v).

Since,
$$uv \subseteq E(G)$$
$$=> uv \subseteq E(G^c)$$
for every pair x, y there is an x, y-path of length at most 3 in G through (u, v).
Hence diam G $\leq$ 3.