# Week 11

# Lecture 22 and 23*

Continuation from Lecture 21.

## 11.1 Graph Coloring

**Definition:** A k-coloring of graph G is a labelling $f : V(G) \to S$ where $S = \{C_1, C_2, ..., C_k\}$ is a set of k-colors. A k-coloring is proper if adjacent vertices have different labels.

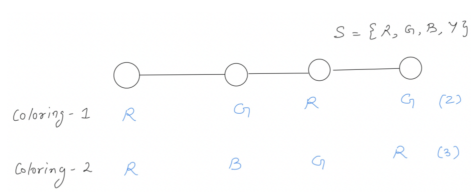


Figure 11.1: Graph coloring example.

**Chromatic Number** The least value of k such that $G$ is proper k-colourable. For the above graph chromatic number is 2.

### 11.1.1 Greedy Algos for Graph Coloring

- *Step*1: Choose any vertex and color it.
- *Step*2: Do the following for remaining $|V| - 1$ vertices.
  - Choose andy non colored vertex
  - Color it with the lowest numbered color that has not been used on any previously colored vertex adjacent to It

*Lecturer: Anand Mishra. Scribe: Utkarsh Thusoo.

- if all previously used colors appears on the adjacent vertex then assign a new vertex to color It
- $S = R, G, Y, C, B, W$

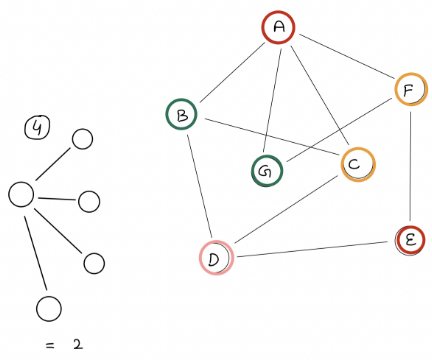


Figure 11.2: Greedy algo.

### 11.1.2 Clique Size

Clique $W(G)$ is a fully connected subgraph.

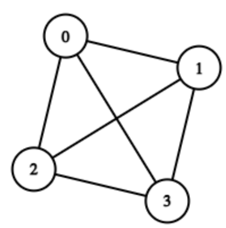


Figure 11.3: Clique of size 4.

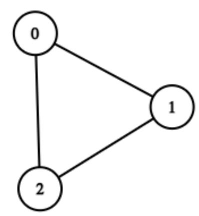


Figure 11.4: Clique of size 3.

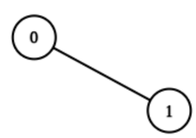


Figure 11.5: Clique of size 2.

For every graph chromatic number is greater than clique size, $X(G) > W(G)$

**Theorem 11.1.** *Prove that* $X(G)$ *of a graph* $G = max(X(G_1), X(G_2)....X(G_k))$ *where* $G_1, G_2, ..., G_k$ *are k compoenets of the graph*

*Proof.* Chromatic number of a graph will be max of chromatic number of different components of that graph. The reason is they are different components and if we use maximum colors to color thr graph with max chromatic number same colors can be used to color the other components also. $\square$

### 11.1.3 Bounds on chromatic number

$\Delta G$ is maximum degree of any of the nodes.

**Trivial upper bound** $X(G) \leq |V(G)|$. The number will be equal for the complete graph.

**Trivial lower bound** $X(G) > 0$. The number will be 0 for a null graph

**Star Graph**

- $\Delta G$ for a star graph having n nodes $= n - 1$
- $X(G) \leq n$

**Complete Graph**

- $\Delta G$ for a complete graph having n nodes $= n$
- $X(G) \leq n + 1$

**Cycle Graph**

- $\Delta G$ for a cycle graph is 2.
- $X(G) = 2$ for even
- $X(G) = 3$ for odd

**Wheel Graph**

- $\Delta G$ for a wheel graph is $n - 1$ becasue entire node will have the maxium degree.
- $X(G) \leq n$

### 11.1.4 Welbh powel bound

If $G$ has a degree sequence $d_1 \geq d_2 ... \geq d_n$ then, $X(G) \leq 1 + maxmind_1, i - 1$

You have a degree sequence $d_1 tod_n$ then you can compute $X(G)$