Question: 1. Let f:R" + R be a convex function that is f(txı + (1 – t)x2)

1. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function that is $f(t\mathbf{x}_1 + (1-t)\mathbf{x}_2) \le tf(\mathbf{x}_1 + (1-t)f(\mathbf{x}_2)$. Let $\mathbf{x}$ be a random vector with joint PDF $p(\mathbf{x})$. If f is a convex function, then show that

$$\mathbb{E}_{\mathbf{x}\sim p}[f(\mathbf{x})] \ge f(\mathbb{E}_{\mathbf{x}\sim p}[\mathbf{x}]).$$

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Expert Answer

Amitakshar Biswas answered this
476 answers

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We know that f(X) is a convex function.

Now let, L(X)=a+bX is a linear function tangential to f(X) at point E[X].

Hence, since f is convex and L is tangential to f, we know by definition that:

$$f(x) \ge L(x)\ \forall x \in X$$

Thus,

$$E[f(X)] \ge E[L(X)] \ge E[A + bX] \ge a + bE[X] \ge L(E[X]) \ge f(E[X])$$

The proof is quite straightforward. If one function is always greater than or equal to another function, then the unconditional expectation of the first should be at least as large as that of the second. The interior of the proof follow from the definition of L, the linearity of expectations, and another application of the definition of L.

The finally, by the definition of the straight line L, we know that L[E[X]] is tangential with f at E[E[X]]=E[X]=f(E[X]).

Proved!

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Q: 2. Consider that p= N(H. 2) and q = N(0,1). Here I = diag(01.02. ...,ok). Then, shown that the KL divergence between p and q is defined as below. Dk(plg = }§ 67+? (c +k? - 1 - loge(?))

A: See answer 100% (1 rating)