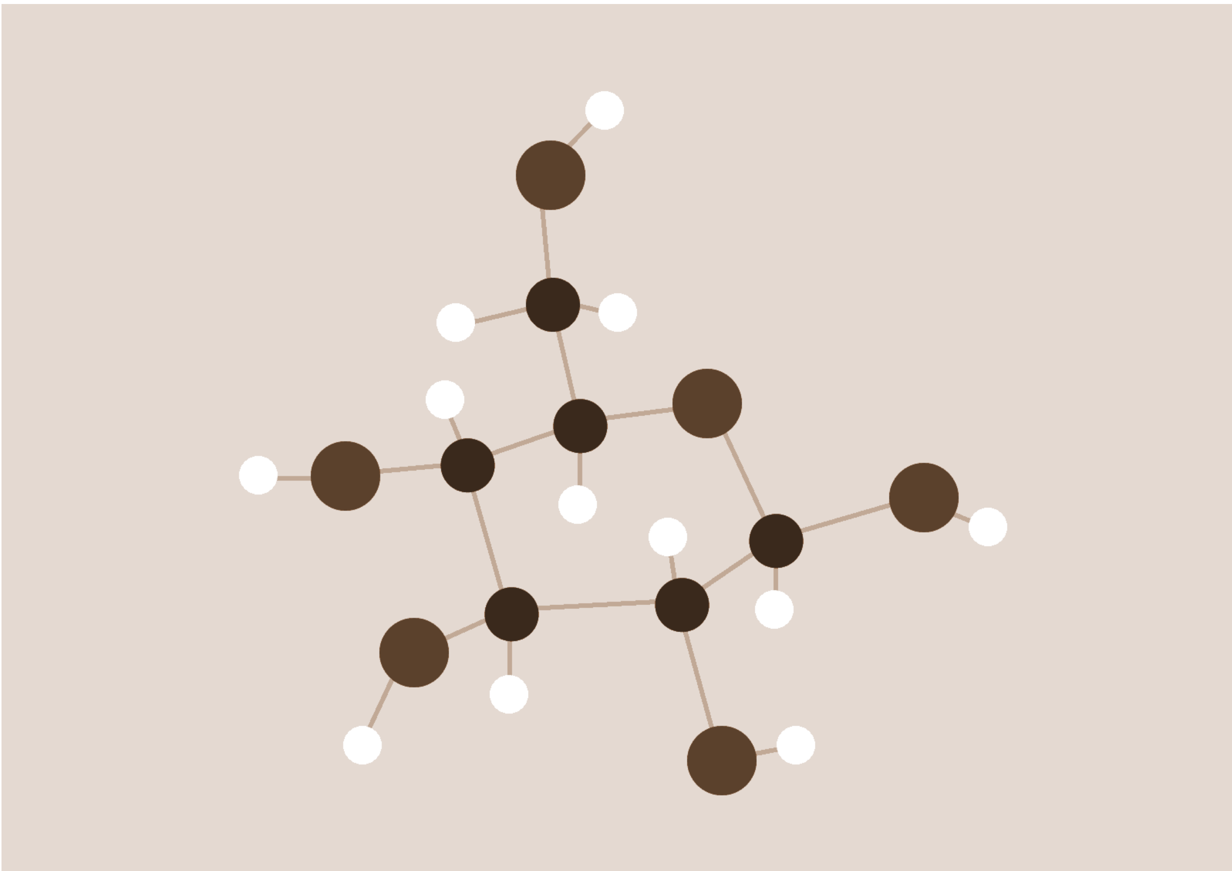
Graph Theory And Application

Lecture Scribing Week - 3



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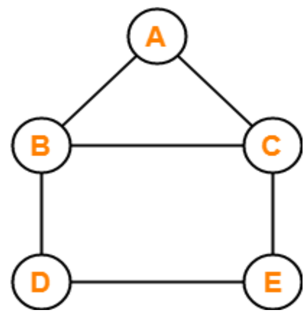
WALK AND CYCLE:

- A walk is an alternating series of vertices and edges with a limited length.
- Length of the walk is equal to the total number of edges covered.



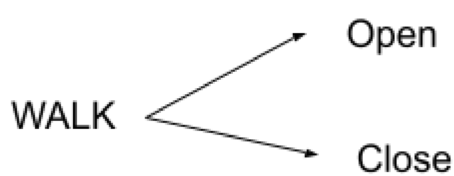
- In above graph length of walk is following:

a , b , c , e , d (Length = 4)

d , b , a , c , e , d , e , c (Length = 7)

e , c , b , a , c , e , d (Length = 6)

- Two types of walk in graph theory:

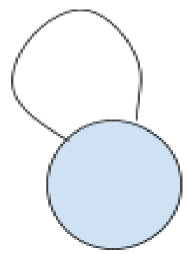


- **Open Walk:** Walk starts and ends with different- different vertices.
- **Close Walk:** In this, Walks starts and ends with same vertices
- **"If the total length of any walk is equal to zero, then it is known as Trivial Walk".**
- **Cycle:** A cycle is a close path in which neither vertex nor edges are allowed to repeat.

True or False:

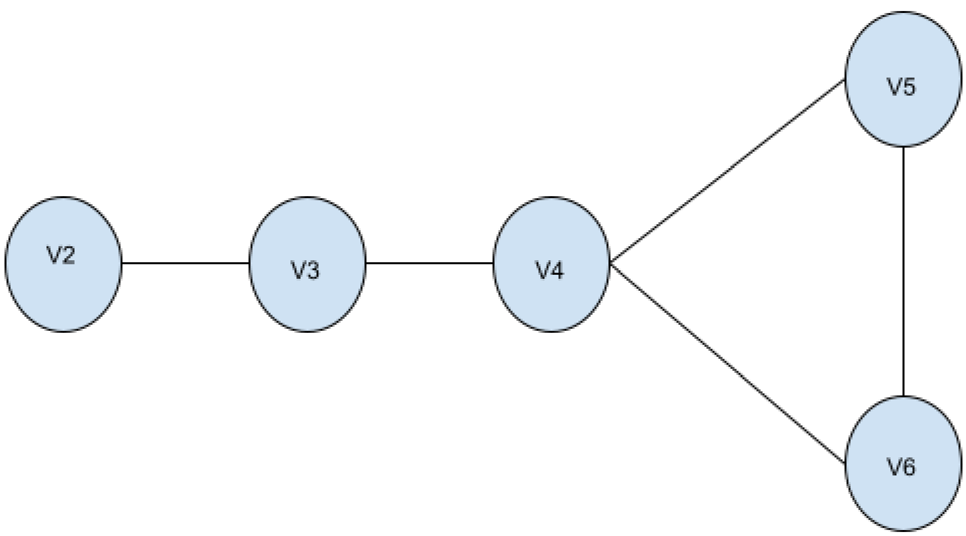
Lemma 1 : Every closed odd walk contains an odd cycle.

CASE 1: .If there is no repetition of vertex in walk, then a closed walk = a closed cycle.



IF Length (L) is equal to 1 (H.P)

CASE 2 : If there is repetition in vertex in the walk, Then consider following Example



To prove lemma let consider above graph :

Close walk for graph (G) is following:

W1 = V2 ,V3, V4, V5,V6,V4,V3,V2

W2 = V4,V5,V6,V4

|W1| + **|W2|** = **|W|** = **ODD**

|W1| IS ODD AND |W2| IS ODD

|W2| < W = L (:: assumption that length of W is L)

|w2| Contains Odd Cycle.

Using Induction Hypothesis W contains an ODD cycle. (H.P)

Bipartite Graph:

A bipartite graph has vertices that can be separated into two distinct and independent sets U and V, with each edge connecting a vertex in U to a vertex in V.

Theorem: A graph is bipartite iff it has no odd cycle.

Necessary Condition:

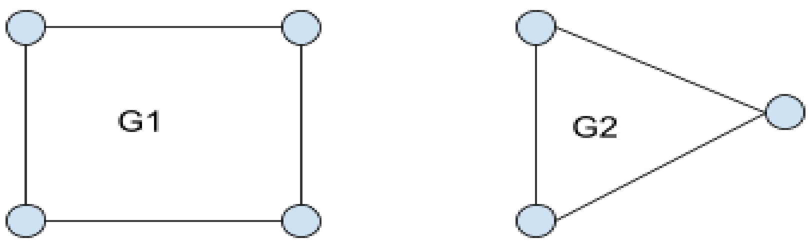
Assume the graph is bipartite and X and Y are two independent sets. To have a cycle, one has to traverse X to Y to X OR Y to X to Y one or more times. Therefore, a bipartite graph can not have an odd cycle.

Sufficient Condition:

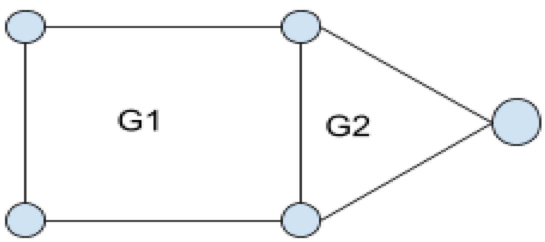
If G has no odd cycle => it does not contain a cycle OR it contains an even cycle. It does not contain a cycle => take one vertex in X and next vertex in Y. It contains an even cycle => Partition the graph such that each even length cycle is one subgraph. We know Cn is bipartite for an even length cycle. If G has no odd cycle => it does not contain a cycle OR it contains an even cycle.

Union of Graph:

A procedure for combining two or more graphs to create a bigger graph.



Union

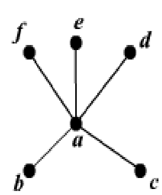


Eulerian Graph:

If a graph has a closed path that contains all of the edges, it is said to be Eulerian. When we don't define the initial vertex but keep the list in cyclic order, we call it a circuit. In a graph, an Eulerian circuit is a circuit that contains all of the edges.

Vertex Degree and Counting:

The number of edges linking a vertex determines its degree.Vertex a has a degree of 5 in the example below, whereas the others have a degree of 1.



Order and Size of Graph:

The number of nodes in a graph determines its order. Keeping in mind that the graph is actually just a collection of nodes and edges, the order is V's cardinality (N).

The size of the graph is defined as the maximum number of edges that might exist if everyone in the network has a relationship with everyone else. For an undirected network, it is computed as [n(n-1)]/2, whereas for a directed network, it is calculated as [n(n-1)].