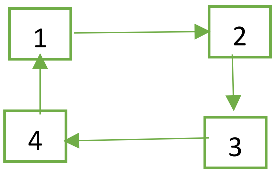
Week 5

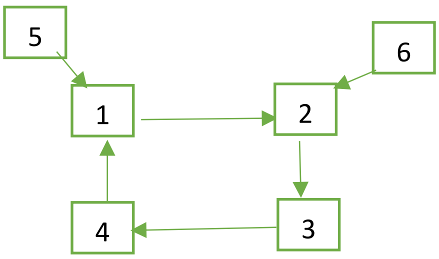
Graph Theory 101

1.Kernel: In a directed graph D = (V, A), a stable set S ⊆ V is said to be a kernel if from every node of V \S there is an arc to S. Kernels have several applications in combinatorics and game theory, and there has been extensive work on the characterization of digraphs that have kernels. S induces no edges and every vertex outside of S has successor in S.



S1= {1,3}

S2= {2,4}

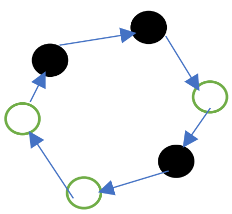


S1= { 1,3,6}

S2= {2,4,5}

Set S1 and S2 vertex are independent to each other.

Supposed T= {1,3} is totally wrong, because S1 is set of those vertices who have not connected with each other.



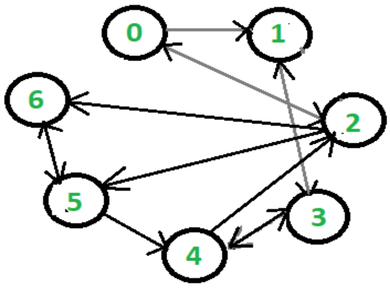
Cn where n is odd, let k is the kernel of the Cn. Then any u,v €k. Should have following property.

1: They should be independent.

2: w not element of k. Then w have must have successor in k.

2: Out Degree and Indegree:

For a vertex, the number of head ends adjacent to a vertex is called the indegree of the vertex and the number of tail ends adjacent to a vertex is its outdegree (called branching factor in trees).



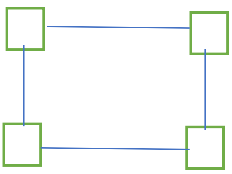
Vertex	In Degree	Out Degree
0	1	2
1	2	1
2	2	3
3	2	2
4	2	2
5	2	2
6	2	1

In a directed graph

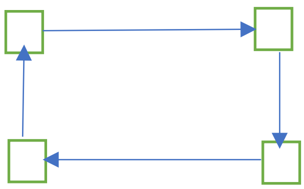
$\sum d+(v)=\sum d-(v)=|E(G)|$

3: Orientation of Graph:

An orientation of Graph G is directed graph D obtained from G by choosing an orientation (x-->y or x←y)



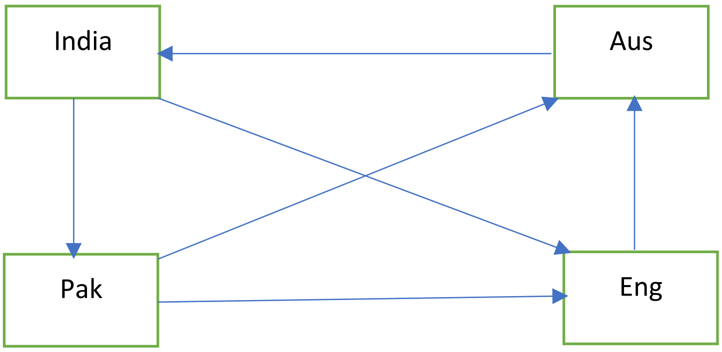
G



D

4: Tournament:

Tournament is an orientation graph of complete graph

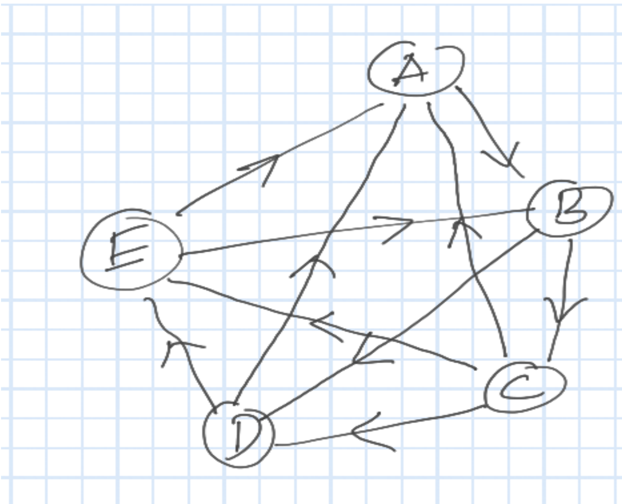


X→Y

X wins against Y

5: King of Tournament:

Let T be a tournament. Say that a vertex u is *reachable in* k *steps* from a vertex v if there is a path of length k from v to u. A vertex v in T is a *king* if each other vertex of T is reachable from v in at most 2 steps. A vertex v of T is a *serf* if v is reachable in at most 2 steps *from* each other vertex of T. For distinct vertices v and u of T let b(v,u) be the number of paths of length 2 from v to u. (Equivalently, b(v,u) is the number of vertices w distinct from both u and v such that vwu is a path in T.) A vertex v of T is a *strong king* if b(v,u)>b(u,v) for every vertex u such that uv is an edge of T. (Note that this does imply that v is a king.)



B→C

B→D→A

B→D

B→D→E

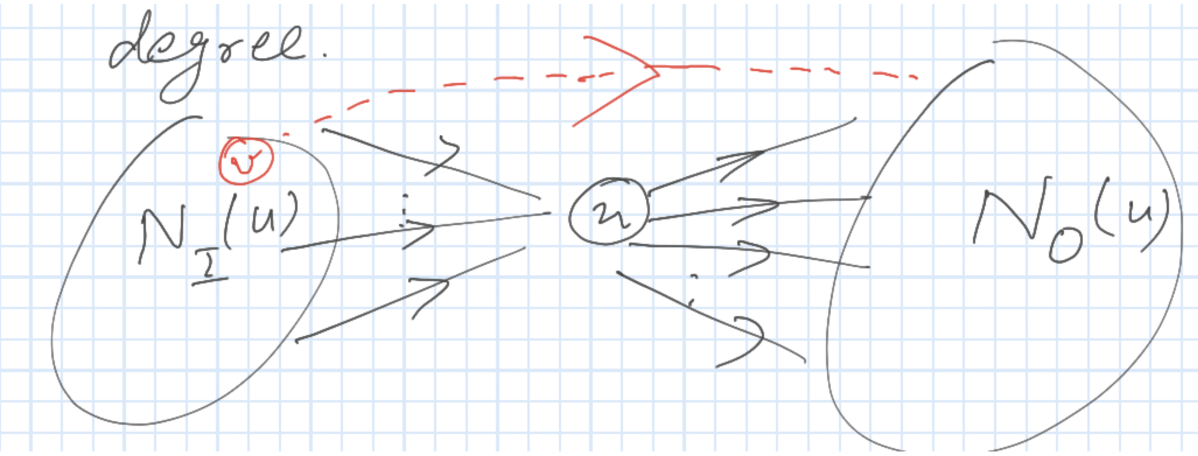
C→A

C→E→B

C—>D

C→E

Every Tournament has a king, suppose u is a node with highest outgoing degree.



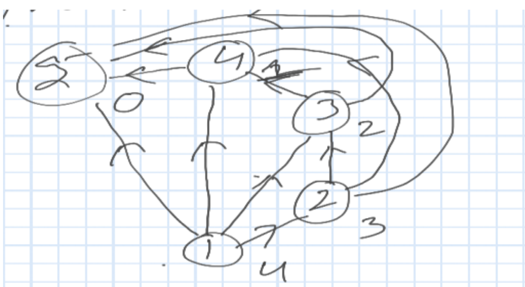
|N1(G)|+|N1(G)|+1=|V(G)|

u is not a king

number of outgoing edge of node v=1+|N0(G)|>no of outgoing edge of u

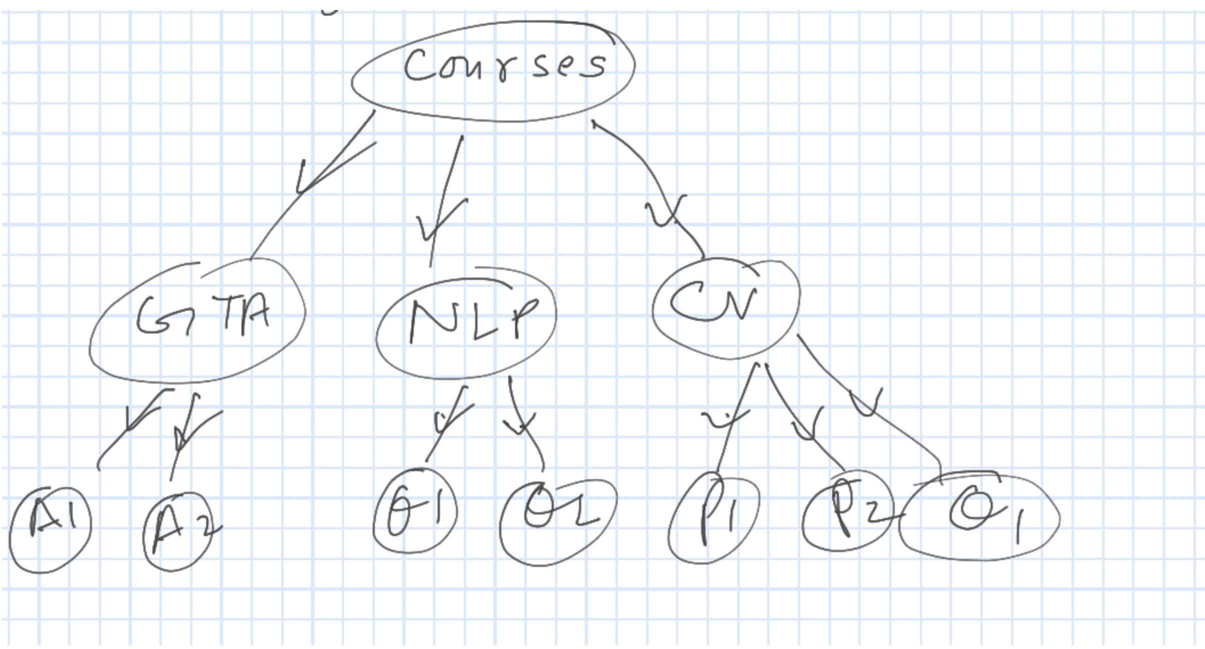
6:Prove of Disprove

if D is an orientation of a simple graph with 5 vertex then the vertices of D cannot have distinct outdegree



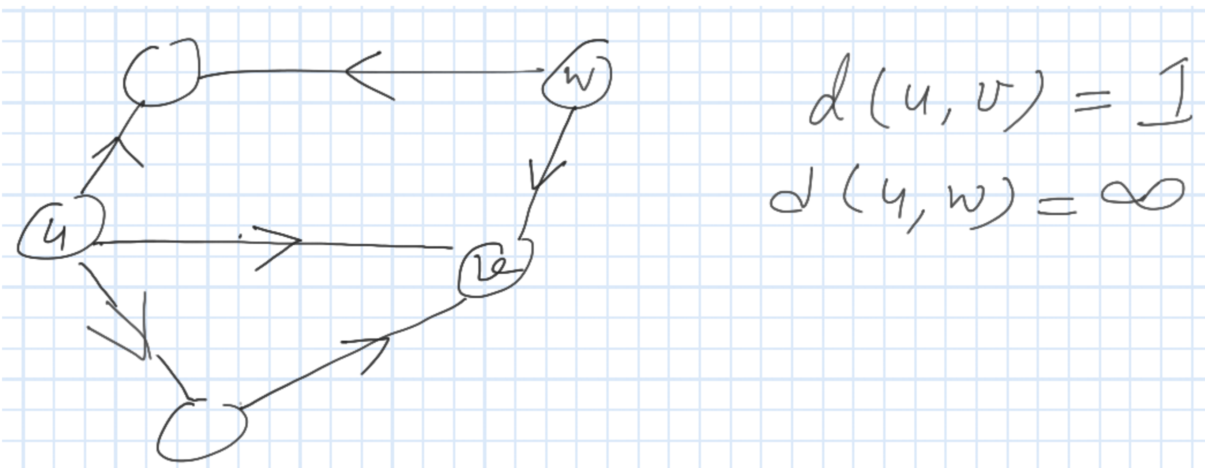
7: Give Example of one real word relation whose digraph have no cycle.

Example:



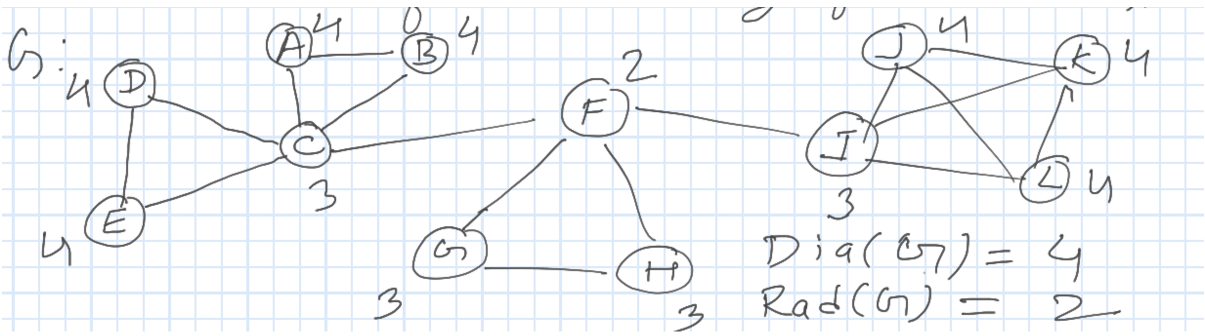
8: Distance of a Graph:

If a graph G have a u-v path, then the distance between u to v is written as d(u,v) is the least length of u-v path

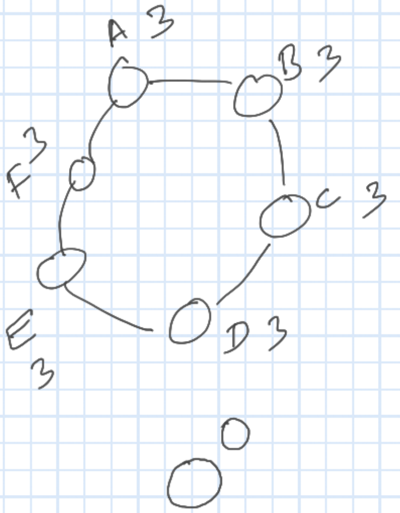


9: Diameter: is maximum of distances between any two pairs of vertices.

10: Eccentricity: Of a vertex u is the max of distances it has with any node in the graph and radius rad(G) is the minimum of eccentricity of all nodes.



Dia(G) = 4
Rad(G) = 2



A	B	1
A	C	2
A	D	3
A	E	2
A	F	1

Diam$(C_6) = 3$
Rad$(C_6) = 3$

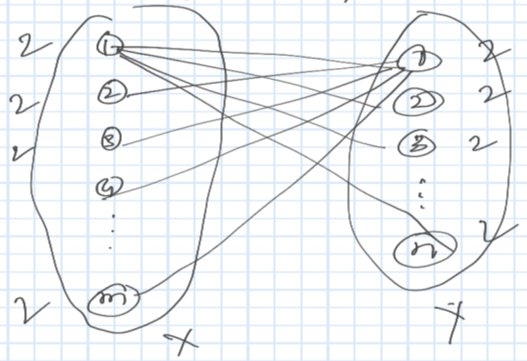
Theorem: If G is simple graph then diam (G) greater than equal to 3 implies diam (G') less than equal to 3

Proof: Diam(G) greater than equal to 3

∃ u,v € V(G)

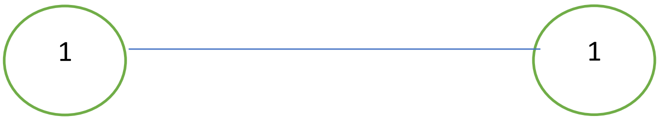
They do not have common neighbour in G and u and v are non adj.

Problem ① Compute diameter and radius of $K_{m,n}$.

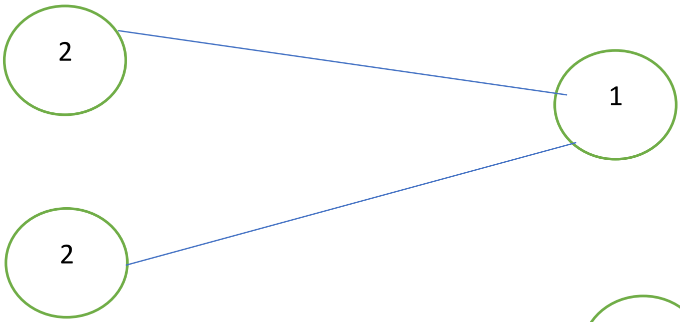


$x_{①}$ $x_{②}$

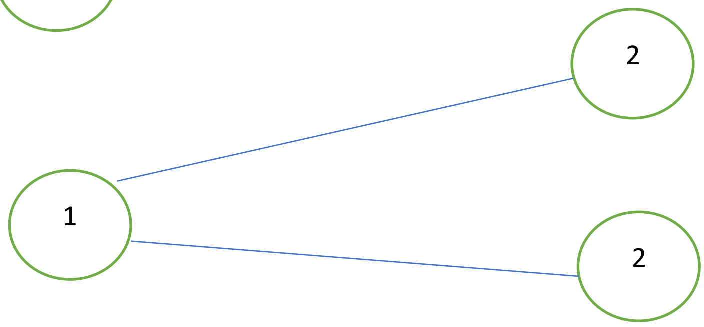
diam$(K_{m,n})$
= rad$(K_{m,n})$
= 2



M=N=1

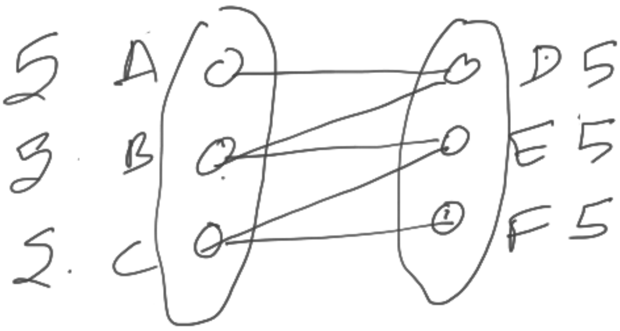


M=2, N=1



M=1, N=2

M is greater than equal to 2 and N is greater than equal to 2



AC = 4

km/n

AF = 5