ML 1 Fractal 3 Assignment Mudit Singh
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Problem 1

x_1	x_2	Class
1	1	+1
-1	-1	-1
0	0.5	-1
0.1	0.5	-1
0.2	0.2	+1
0.9	0.5	+1

Table 1: Sample data

Given initial $w^T x = 0$
Decision Boundry as $[1,1]$

Applying Perceptron learning Algorithm

let for class +1, $y=1$

$$\sum_{i=0}^{n} w_i x_i > 0$$

for class -1, $y=0$

$$\sum_{i=0}^{n} w_i x_i < 0$$

$mean(x_1) = 0.2$ $mean(x_2) = 0.28$

	x_1	x_2	$x_1 - mean(x_1)$	$x_2 - mean(x_2)$	y
1	1	1	0.8	0.72	1
2	-1	-1	-1.2	-1.28	0
3	0	0.5	-0.2	0.22	0
4	0.1	0.5	-0.1	0.22	0
5	0.2	0.2	0	-0.08	1
6	0.9	0.5	0.7	0.22	1

let y' be our prediction

⇒ Step 1 $w[1,1]$ initialization

1. data $= w^T x$
   $= 1.52 > 0$ so $y' = 1$ (correct classified)
2. data $= w^T x$
   $= -2.48 < 0$ so $y' = 0$ (correct classification)
3. data $= w^T x$
   $= 0.02 > 0$ so $y' = 1$ (incorrect)
   $w = w - x$
   so new weights are $w[1.2, 0.78]$

→ Step 2 $w[1.2, 0.78]$

1. data $= w^T x$
   $= 1.52 > 0$ so $y' = 1$ (correct classified)
2. data $= w^T x$
   $= -2.44 < 0$ so $y' = 0$ (correct)
3. data $= w^T x$
   $= -0.07 < 0$ so $y' = 0$ (correct)
4. data $= w^T x$
   $= 0.049 > 0$ so $y' = 1$ (incorrect)
   $w = w - x$
   so new weight $w[1.3, 0.56]$

→ Step 3 $w[1.3, 0.56]$

1. data $= w^T x > 0$ (1.44) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-2.28) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.13) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.007) $y' = 0$ (correct)
5. data $= w^T x < 0$ (-0.046) $y' = 0$ (incorrect)
   New $(w) = w + x$
   $= [1.3, 0.48]$

→ Step 4 $w[1.3, 0.48]$

1. data $= w^T x > 0$ (1.3) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-2.1) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.1) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.02) $y' = 0$ (correct)
5. data $= w^T x < 0$ (-0.04) $y' = 0$ (incorrect)
   new $w = w + x$
   $= [1.3, 0.4]$

→ Step 5 $w[1.3, 0.4]$

1. data $= w^T x > 0$ (1.32) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-2.07) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.17) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.04) $y' = 0$ (correct)
5. data $= w^T x < 0$ (-0.03) $y' = 0$ (incorrect)
   new $w = w + x$
   $= [1.3, 0.32]$

→ Step 6 $w[1.3, 0.32]$

1. data $= w^T x > 0$ (1.26) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-1.9) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.19) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.06) $y' = 0$ (correct)
5. data $= w^T x < 0$ (-0.026) $y' = 0$ (incorrect)
   new $w = w + x$
   $= [1.3, 0.24]$

→ Step 7 $w[1.3, 0.24]$

1. data $= w^T x > 0$ (1.2) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-1.8) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.2) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.07) $y' = 0$ (correct)
5. data $= w^T x < 0$ (-0.02) $y' = 0$ (incorrect)
   new $w = w + x$
   $= [1.3, 0.16]$

→ Step 8 $w[1.3, 0.16]$

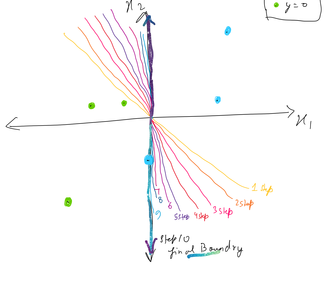
1. data $= w^T x > 0$ (1.15) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-1.7) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.22) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.9) $y' = 0$ (correct)
5. data $= w^T x < 0$ (-0.013) $y' = 0$ (incorrect)
   new $w = w + x$
   $= [1.3, 0.08]$

→ Step 9 $w[1.3, 0.08]$

1. data $= w^T x > 0$ (1.09) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-1.66) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.24) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.11) $y' = 0$ (correct)
5. data $= w^T x < 0$ (-0.006) $y' = 0$ (incorrect)
   new $w = w + x$
   $= [1.3, 0]$

→ Step 10 $w[1.3, 0]$

1. data $= w^T x > 0$ (1.04) $y' = 1$ (correct)
2. data $= w^T x < 0$ (-1.56) $y' = 0$ (correct)
3. data $= w^T x < 0$ (-0.26) $y' = 0$ (correct)
4. data $= w^T x < 0$ (-0.13) $y' = 0$ (correct)
5. data $= w^T x = 0$ (0) $y' = 1$ (correct)
6. data $= w^T x > 0$ (0.91) $y' = 1$ (correct)

All the points are correctly classified with $w[1.3, 0]$

1. The Perceptron learning Algorithm converge in 10 steps
2. Final decision boundry