Week 6

Lecture subscribing*

Theorem 6.1. *If G is a simple graph then diam(G)≥ 3* $\implies$ *diam(G^c)* ≤ 3

Proof.

diam(G)$\geq$ 3

$\Rightarrow \exists u, v \in V(G)$ s.t.

(i) u and v have not adjacent

(ii) u and v have no common neighbours

$\Rightarrow$ uv does not exist in G.

$\forall x \in V(G) - \{u, v\}$, ux or vx does not exist in G.

uv exist in G^c and ux or vx will also exist in G^c .

$\Rightarrow$ diam(G^c) $\leq$ 3 □

Theorem 6.2. *If G is a simple graph then diam(G)≥ 4* $\implies$ *diam(G^c)* ≤ 2

Proof.

we have to prove that

P $\Rightarrow$ Q

where P $\equiv$ diam(G)$\geq$ 4

Q $\equiv$ diam(G^c) $\leq$ 2

we will have equivalent to above

$\sim$ Q $\Rightarrow \sim$ P

$\Rightarrow \sim$ Q $\equiv$ diam(G^c) $\geq$ 3

$\sim$ P $\equiv$ diam(G^c) $\leq$ 3

$\therefore$ diam(G)$\geq$ 3 $\implies$ diam(G^c) $\leq$ 3

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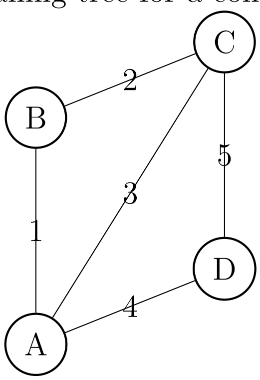
$\therefore$ diam(G^c) ≥ 3

$\Rightarrow$ diam($(G^c)^c$) ≤ 3

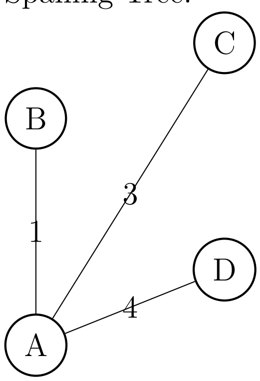
$\Rightarrow$ diam(G^c) ≤ 3 $\square$

Spaning Tree

Spaning tree for a connected graph G is a subgraph of G that is a tree.



Spaning Tree:



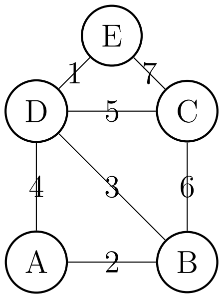
spaning tree cost = 1+3+4=8

Krushkal Algorithm

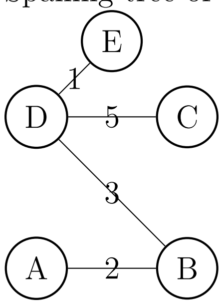
Steps:

- Sort the graph edges with respect to their weights.
- Start adding edges to the MST from the edge with the smallest weight until the edge of the largest weight.
- Only add edges which doesn't form a cycle , edges whcih connect only disconnected components.

Let us consider a graph G.



Spaning tree of Graph G using Krushkal Algorithm.



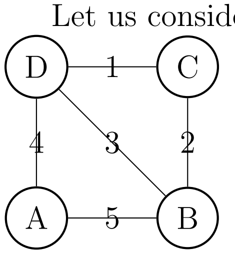
total cost = 1+2+3+5 = 11

Prims Algorithm

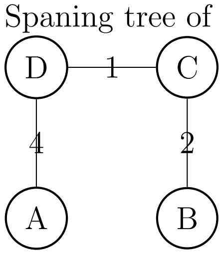
Steps:

- Maintain two disjoint sets of vertices. One containing that are in growing spaning tree and other are not in the growing spanning tree.
- Select the cheapest vertex that is connected to the growing spanning tree.
- Check for cycles.

Let us consider a graph G.



Spaning tree of Graph G using Prims Algorithm.



total cost = 1+2+4 = 7

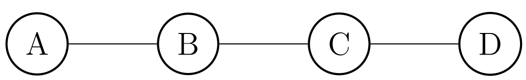
Matching

It is a set of non-loop edges with non-shared end points.

Matching = $\{AB, CD\}$ = $\{BC\}$

Maximal Matching

A matching M of a graph G is said to be maximal if no other edges of G can be added to M.



Maximal Matching = $\{AB, CD\}$
Maximal Matching = $\{BC\}$

Maximum Matching

Maximum matching is defined as the maximal matching with maximum number of edges.

Maximal Matching = $\{AB, CD\}$
Maximal Matching = $\{BC\}$
So, Maximum matching = $\{AB, CD\}$

Perfect Matching

A matching M of graph G is said to be a perfect match, if every vertex of graph G is incident to exactly one edge of the mathcing M.
Note: perfect matching of graph is also a maximum matching of graph.

Theorem 6.3. *Prove that every tree have atleast one perfect matching*

Proof.

we are going to prove this using induction technique.

for n = 1 , tree having one node , PM = 0

for n = 2 , tree having two nodes, PM = 1

suppose that theorem is true for n $\leq$ k nodes.

PM in T1 $\leq$ 1

PM in T2 $\leq$ 1

PM in T = PM in T1 * PM in T2

PM in T $\leq$ 1

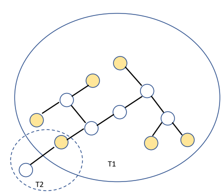


Figure 6.1

□

Hall's Marriage Theorem

An X-Y bigraph G has a matching that saturates X iff
$|N(S)| \geqslant |S| \forall S \subseteq X$
Here N(S) $\subseteq Y$ is set of neighbours of elements in S.