How many maximum nodes you can connect using k-edge's so that number of connected component is 1.

If number of connected components is '1' then How many nodes we can connect?

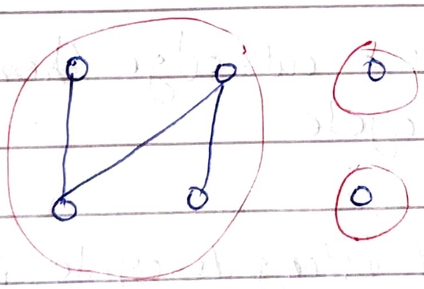
$K+1$

number of vertices 'n'

remaining vertices with '1' connected component $= n-(K+1)$

$$CC = n-(K+1)$$

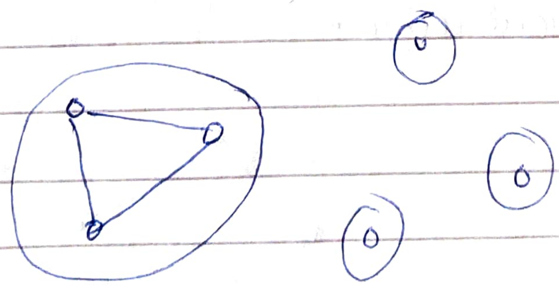
Total minimum $CC = n-(K+1)+1 = n-K$



$n=6$

$K=3$

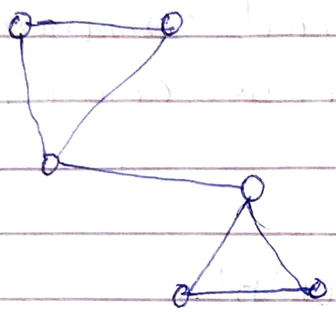
~~Total~~ $CC = n-K = 3$



– 4 connected components

cut-edge (or) cut-vertex

A cut-edge (or) cut-vertex of a graph is an edge (or) vertex whose deletion increases the number of components.



Can there be multiple cut-edge & cut-vertex YES

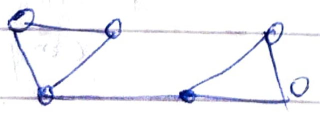
Theorem

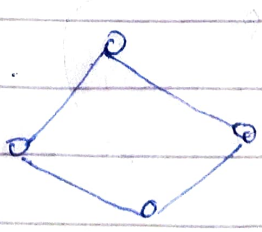
Statement - An edge is a cut-edge if and only if it belongs to no cycle.

In other words

① Let an edge is a cut-edge then it does not belong to cycle.

② Let it does not belong to cycle then it is a cut-edge.



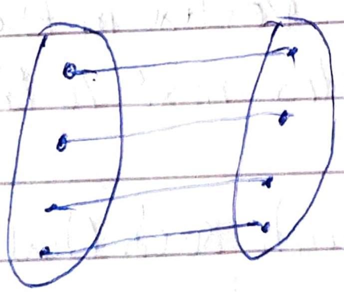


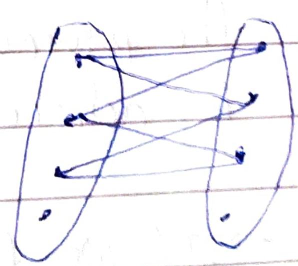
Bipartition graph - A bipartition of a graph is a specification of two disjoint independent sets in 'G' whose union is V(G). The statement "G is a bipartite graph with bipartition X & Y" specifies one such partition.

? disjoint set Vs independent disjoint set.

Theorem

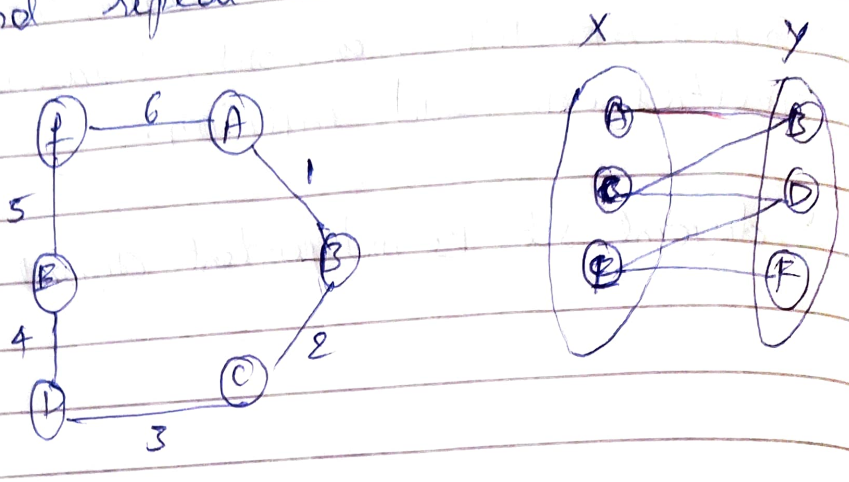
statement - A graph is bipartite iff it has no odd cycle.





X to Y
Y to X cycle

Suppose 'G' has even length cycle then we can put first node in x and second node in y and repeat



proposition

① A simple path is bipartite graph.
② C_n is bipartite iff 'n' is even.

Path Vs simple path

→ A path is a sequence of vertices with the property that each vertex in the sequence is adjacent to the vertex next to it.

→ If we traverse a graph such that we do not repeat a vertex and nor we repeat an edge.

→ A path that doesnot repeat vertices is called a simple path.

Simple graph

→ A simple graph is a graph that does not have more than one edge between any two vertices and no edge starts and end's at the same vertex.

In other words a simple graph is a graph without loops and multiple edges.

① True

② True - dividing into 'x' & 'y' with odd is problem.

* Lecture 7 *

Vertex degree and counting

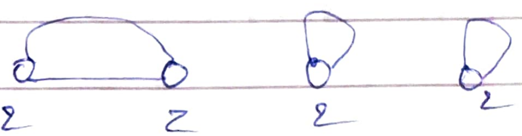
Degree of a vertex -

Lecture.

Graphical sequence - A list of non-negative integers, ask if it is a sequence of degrees for a graph

Ex -

degree of 2 2 sequence



sequence 5, 3

Note :- If graph parallel & self loops allowed then check sum of value degree is even or not.

Havel - Hakimi algorithm

	5	5	5	2	1	1	1
remove one	4	4	3	1	0	1	1
sort	4	4	3	1	1	1	0
remove one		3	2	0	0	1	0
		3	2	1	0	0	0

zero degree

(5) 5 4 3 2 2 2 1

4 3 2 1 1 2 1

sort 4 3 2 2 1 1 1

2 1 1 0 1 1

sort 2 1 1 1 1 0

0 0 1 1 0

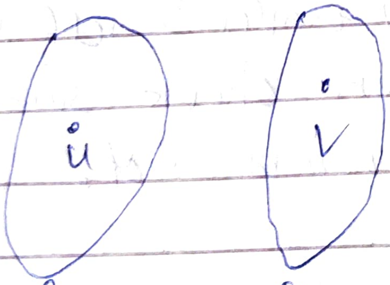
sort 1 1 0 0 0

0 0 0 0

0 0

0 0

Proof on disagree :- If u and v are only vertices of odd degree in a graph 'G', then 'G' contains u-v path.



G_1 G_2

→ G_1 is also graph.

→ G_1 has only one node with odd degree.

→ does not follow hand shake lemma

→ $\Sigma' d(v) = v \in V(G_1)$.

→ therefore u & v have to be in connected component.

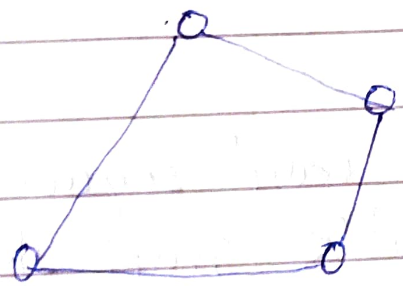
Determine maximum no. of edges in a bipartite subgraph of P_n, C_n and K_n.

maximal subgraph of P_n that is bipartite

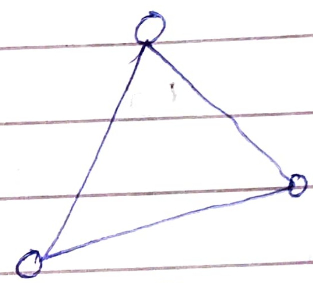
$P_n \subseteq P_n$

P_n — $n-1$ edges

maximal subgraph of C_n that is a bipartite graph.



$C_n \subseteq C_n$



$P_n \subseteq C_n$

$$\begin{cases} n & - n \text{ even cycle} \\ n-1 & - n \text{ odd cycle} \end{cases}$$

Let $l, m, n \in I^+ \cup \{0\}$ such that $l+m=n$

Find a necessary & sufficient condition on l, m & n such that $\exists$ a connected simple n-vertex graph with 'l' vertices of even degree and 'm' vertices of odd degree.

$n \geq 1$ $l =$ even or odd

$m =$ even.

Ex $(2, 0, 2)$

(l, m, n) valid sequence?

not possible, graph should be simple