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Artificial Intelligence-2 (CSL 7040)

Lecture 4 : Utility Theory

Utility Function

- Function to map lotteries to real number
- Agents should follow the axioms
- An agent can have any preference

Utility scale

- Preference Elicitatoin:
- Normalized Utilities:
- Standard Lottery: A choice and a lottery: $[p, u_T, (1-p)u_\perp]$
- Micromort: Chance of death is one in a million

Micromort

- Russian Barrel: Micromort= $10000
- Car pricing: $50/ micromort
- Qualy: Quality adjusted life

The utility of money

- Monotonic preference: More money will be preferred over less money

- Expected Money Value (EVM):
  1. \$10000 Reject
  2. A lottery with equal possibilities of winning \$25000 or nothing Accept

EVM(L)= (25000+0)/2=12500

$EU(Accept)=(U(S_k)+U(S_{k+25000}))/2= 9+5/2=6.5$
$EU(Reject) = U(S_{k+10000})=8$
$U(S_k)=5, U(S_{k+25000})=9; U(S_{k+10000})=8$

Value of a lottery

A. $400 or B. L: 1000$ or 0 with equiprobability

EVM(L)=(1000+0)/2=500

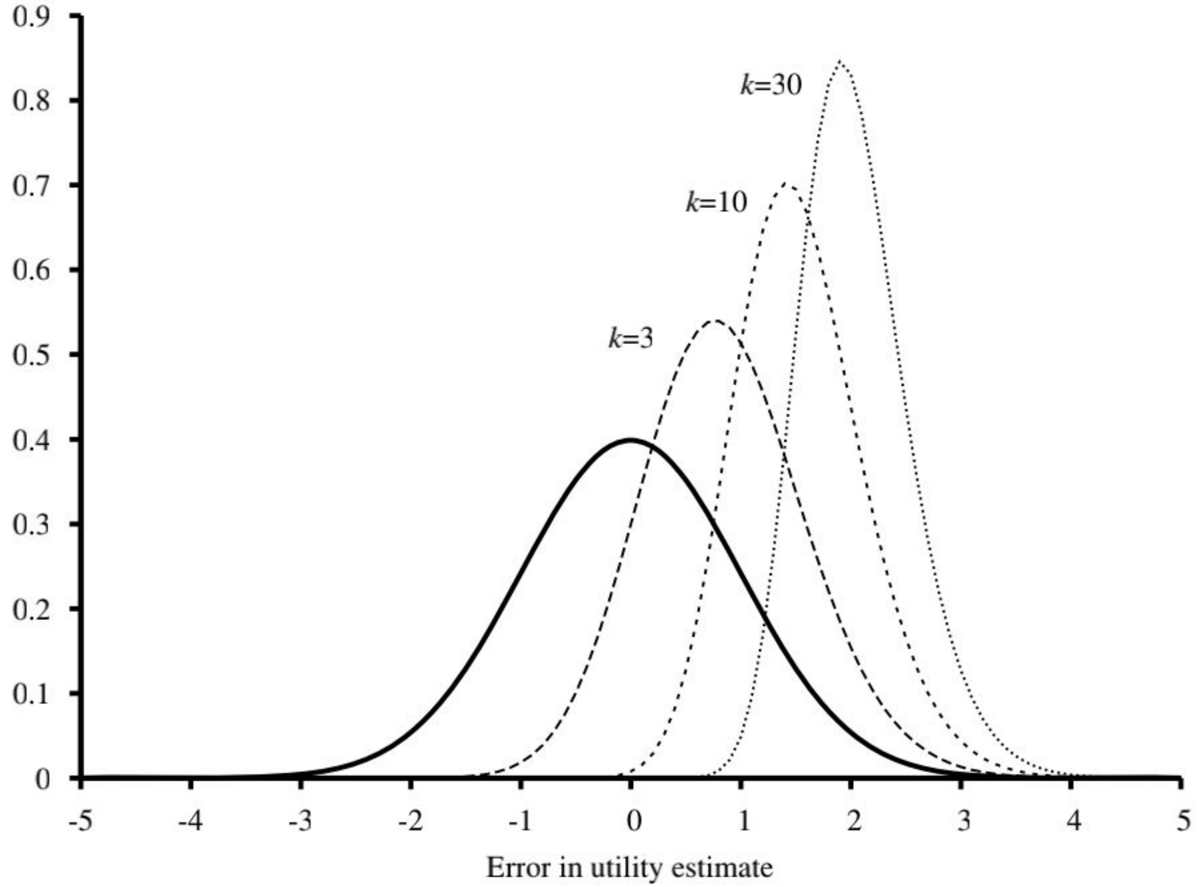
Value for the lottery: 500-400=100= EVM(L)-Certainty equivalent □ Insurance premium

Expected Utility and Post-Decision Disappointment

- $a^* = argmax_a EU(a|e)$

- Best possible action:
  - EU is calculated correctly w.r.t. probability model
  - Probability model should correctly reflect the underlying stochastic process
  - EU will be correct iff the whole process is repeated many times
- Issues: i) oversimplified models, ii) computing true EU is too difficult
- If estimation is correct : $E\left(\widehat{EU}(a|e) - EU(a|e)\right) = 0$

Post Decision Disappointment



gure 16.3 Plot of the error in each of k utility estimates and of the distribution of the aximum of k estimates for $k = 3$, 10, and 30.

Human judgment & Irrationality

- Normative Theory: Describes how agent should act

- Descriptive Theory: How human actually act

A: 80% chance of winning \$4000	C: 25% chance of \$4000
B: 100% chance of winning \$3000	D: 30% chance of \$3000
U(3000)>U(4000)	U(4000)>U(3000)

Certainty Effect□ People are strongly attracted to the gains with certainty

Human judgment & Irrationality

1/3: Red balls 2/3: Black or yellow balls

A: $100 for a red ball C: $100 a red and a yellow ball

B: $100 for a black ball D: $100 a black and yellow

Ambiguity aversion □ People prefer known probability than unknown

Human judgment & Irrationality

- Framing effect:

- Anchoring Effect: Sale

Multiattribute utility function

- Decision making in public policy → money+lives

- Multiattribute utility theory
- Attributes: $X = X_1, \dots, X_n$: Higher value~ Higher utility

Dominance

- Selection of airport site→ attributes: {cost, noise pollution, safety}
- Let there be three choices of sites: $S_1, S_2\ and\ S_3$
- If one choice has higher utilities for all the attributes → Strictly dominant
- If an action is stochastially dominated by all the attributes it can be discarded

Preference structure and multiatribute utility

- n no. of d-dimentional attributed→ to have a complete utility fn.: d^n no. of values
- $U(x_1, x_2, \dots, x_n) = F(f(x_1), \dots, f_n(x_n))$ → Representation theorem
- Preference without uncertainty:
- Cost= 4 billion, noise= 3.7; Safety level 0.12 or 0.03
- {noise, cost and death}→ mutual preference independent (MPI)

Gerard Debru Theorem

- If there are n no. MPI attributes, agent's preference behavior can be described by maximizing the following fn:

- $V(x_1, \ldots, x_n) = \sum_i V_i x_i$

- $V(noise, cost, death) = -noise \times 10^4 - cost - deaths \times 10^{12}$ → additive value function

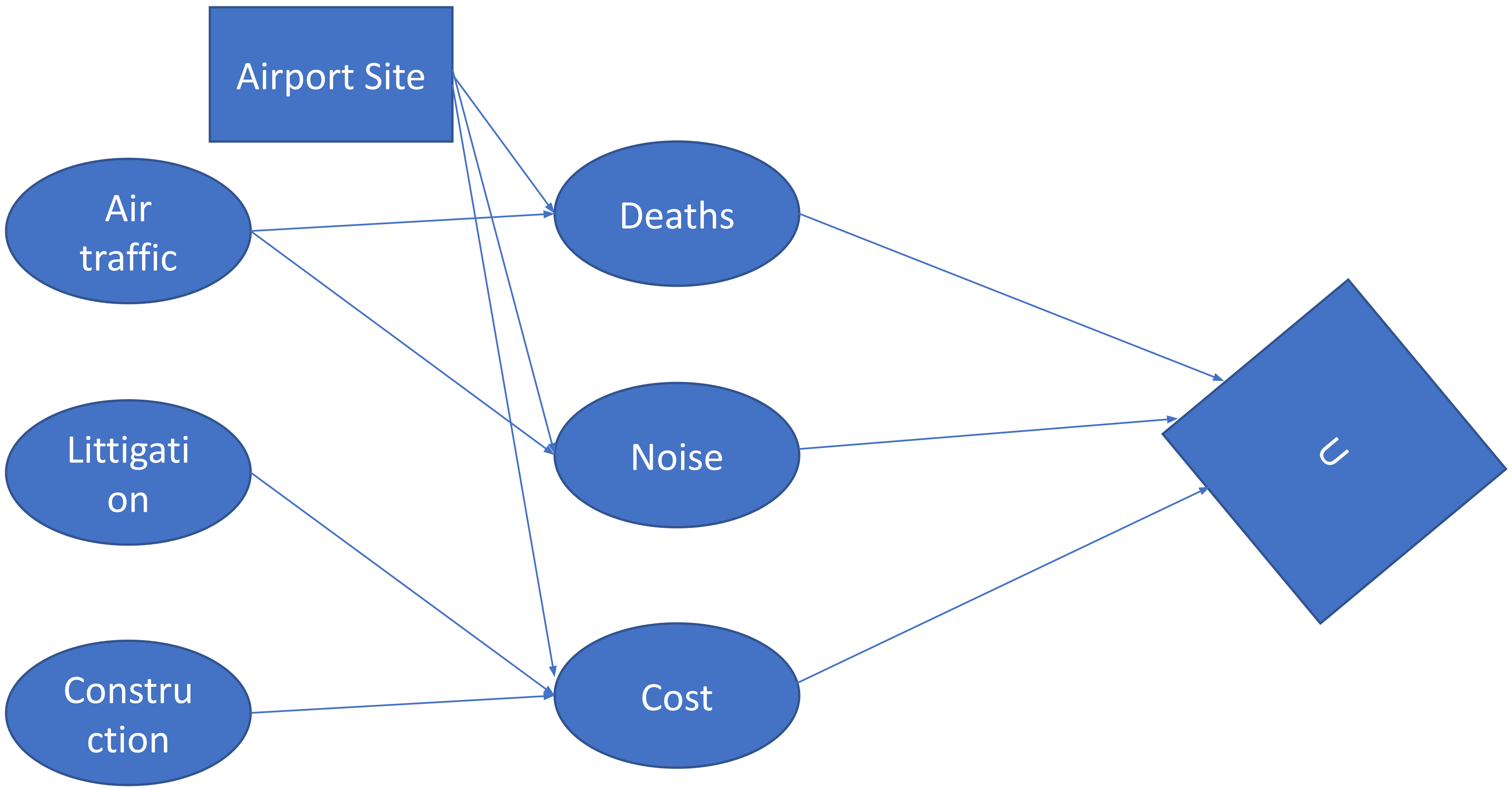
Decision Networks

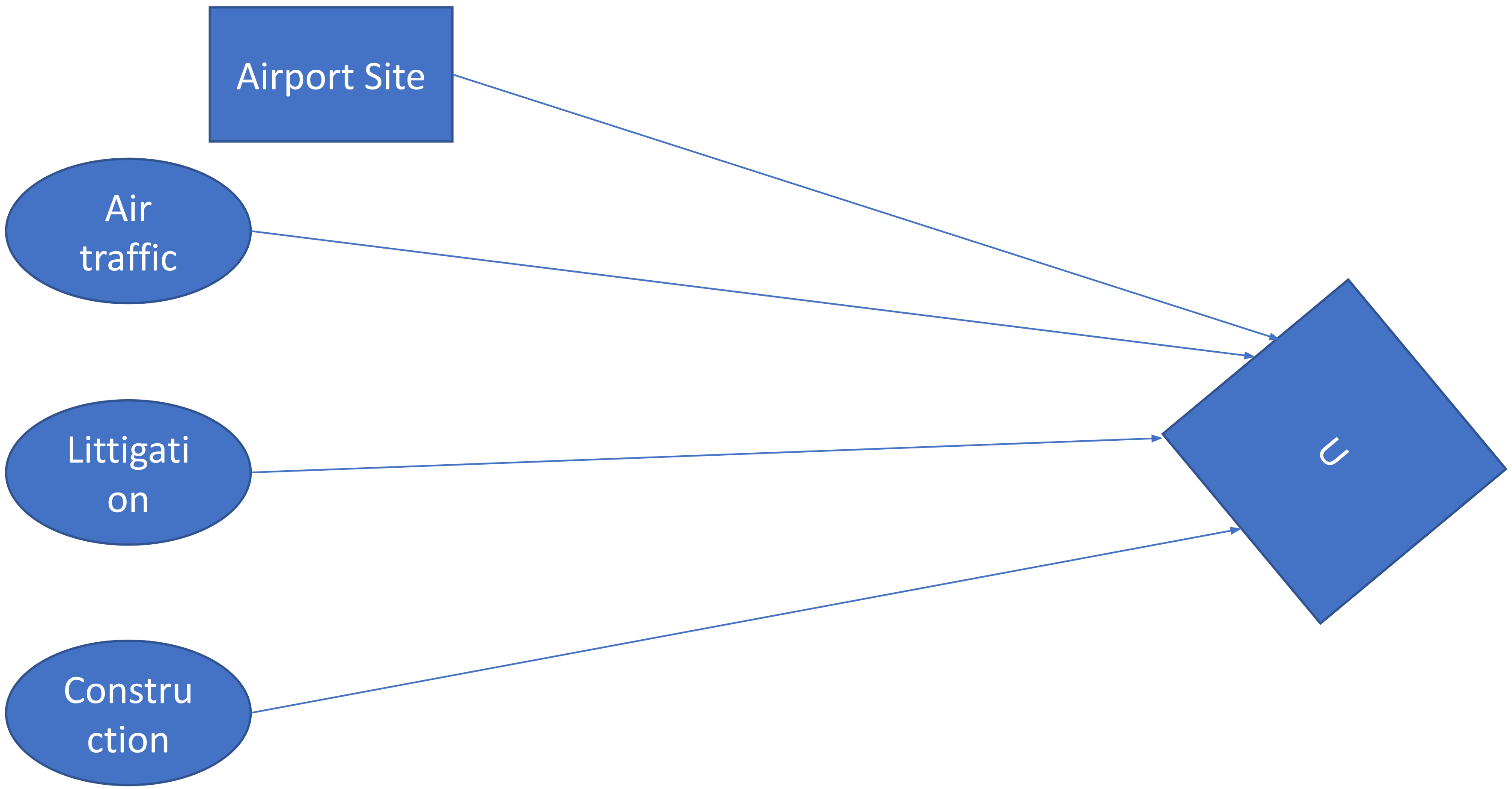
- It combines baye’s net for action and utility
- Agent’s current state, its possible actions, the resultant state, and utility of the state

 Chance node (oval): Random variables about which the agent is uncertain associated with a conditional distribution

 Decision Node (rectangle) Represents the point where the choice of action is given

 Utility Node (diamond) Represents agent’s utility fn.


Airport Site
Air traffic
Deaths
Littigati on
Noise
U
Constru ction
Cost


Airport Site
Air traffic
Littigati on
Constru ction
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Evaluation of a decision n/w:

- Set evidence variable for the current state
- For each possible value of decision node do:
    - Set decision node to the value
    - Compute posterior probability of parent nodes of utility nodes
    - Compute resulting utility of the action
- Return action with the highest utility