Week 7

Lecture Scribing*

7.1 Hall's Theorem

Proof:-

Suppose XY bigraph has matching that saturates X.Then:-

$$|S| \leq |N(s)|$$

The sufficient condition thereby would be

$$\text{if } \forall S \subseteq X, |N(s)| \leq |S|$$

Then there is a matching that saturates X.

We will prove this using contra-positive,that is to prove if $p \Rightarrow q$ we will prove $\sim q \Rightarrow \sim p$

.

We shall prove this by, if there is not matching M that saturates X then

$$\exists S \subseteq X, such that\ |S| \geq |N(s)|$$

.

Let $u \in X$ be a vertex unsaturated by a matching M.

Suppose two subsets

$$S \subseteq X and, T \subseteq Y$$

are considered , definition of subsets are:-

S=Starting from u, end points of M- alternating points with last edge belonging to M

T=Starting from u, end points of M- alternating points with last edge not belonging to M

We find that

$$|S| = 1 + |T| = 1 + |N(s)|$$
$$\Rightarrow |S| \geq |N(s)|$$

Hence Proved

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7.1.1 Vertex Cover

A vertex cover of a graph G is a set $Q \subseteq V(G)$ that contains at least one end point of every edge. This can be understood as ,let us suppose all the edges of graphs as roads and one wants to light corner of every road .

So one of the vertex cover is

$$Q_1 = [B, E]$$

$$Q_2 = [A, B, C, D, E, F]$$

$$\alpha(G) = \text{maximum size of independent set}$$

$$\alpha'(G) = \text{ maximum size of matching set}$$

$$\beta(G) = \text{minimum size of vertex cover}$$

$$\beta'(G) = \text{minimum size of edge cover}$$

To Prove

$$\alpha(G) + \beta(G) = n(G)$$

Proof:-

Let S be an indpendent set then every edge is incident to atleast one vertex of S'

$$S \cup S' = V(G)$$

S' is min. size vertex cover

$$\beta(G) = |S'|$$

S is maximum size independent set

$$\alpha(G) = |S|$$

$$\therefore \alpha(G) + \beta(G) = |S| + |S'| = |V(G)| = n(G)$$

$$\therefore \alpha(G) + \beta(G) = n(G)$$

$$\alpha'(G) + \beta'(G) = n(G)$$

To Prove If G is bigraph with no isolated vertices then

$$\alpha(G) = \beta'(G)$$

$$\alpha(G) + \beta(G) = n(G)$$

$$\alpha'(G) + \beta'(G) = n(G)$$

$$\alpha'(G) = \beta(G)$$

$$\alpha(G) + \beta(G) = \alpha'(G) + \beta'(G)$$

$$= \beta(G) + \beta'(G)$$

$$\alpha(G) = \beta'(G)$$

To Prove Let G be a bigraph.Prove that

$$\alpha(G) = n(G)/2$$

if G has perfect matching.

$$\alpha(G) + \beta(G) = n(G)$$

$$\alpha(G) = n(G) - \beta(G)$$

$$= n(G) - \alpha'(G)$$

for perfect matching we know that $\alpha'(G) = n(G)/2$

$$= n(G) - n(G)/2 = n(G)/2$$