Lecture - 7
Vertex Degree and Counting
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1 Havel-Hakimi Algorithm

For a given sequence of non-negative integers, the task is to check if there exists a simple graph corresponding to this degree sequence.

- Sort the sequence of non-negative integers in decreasing order.
- Delete the first element (V) and subtract 1 from the next (V) elements.
- Repeat 1 and 2 until some of the stopping conditions is met.
- Stop when encountered a negative value or not enough elements left for subtraction or come up with all zeros.
- All zeros, there exists a simple graph otherwise none.

2 Prove or Disprove

If u and v are the only vertices of odd degree in a graph G, then G contains a u-v path. It is **true** because there is a contradiction that the sum of degrees of vertices is odd if it is false, which is not possible by statements of graph properties.

3 Maximum Number of edges in bipartite subgraphs, P_n, C_n, K_n

- The maximal bipartite sub-graph of Pn is $P_n \subseteq P_n$, maximum edges are n-1.
- The maximal bipartite sub-graph of Cn is $C_n \subseteq C_n$, if n is even then maximum edges are n, otherwise if n is odd then maximum edges will be n-1.
- The maximal bipartite sub-graph of Kn is $K_n \subseteq K_n$, if there are k vertices in one set and n-k in the other, a complete bipartite graph has k(n-k) edges. But maximum edges possible when n is even (k=n/2) is given by

  $$n^2/4$$

  When n is odd i.e. k=(n-1)/2 or k=(n+1)/2, then maximum edges are

  $$(n^2-1)/4$$

4 Condition for a connected n-vertices simple graph

Let l, m, n $\in I^+ \cup \{0\}$ such that l+m=n.

Find a necessary and sufficient condition on l, m, n such that there exit a connected simple n-vertex graph with l vertices of even degree and m vertices of odd degree.

Condition-1: $n \geq 1$

Condition-2: l may be even or odd and m should be even.