Indian Institute of Technology- Jodhpur

Graph Theory and Applications(GTA-2)
COURSE CODE: CSL7410

Lecture Scribing Assignment: Week 2

Done by:
Dheeraj Kumar Gehlot
M20MA070

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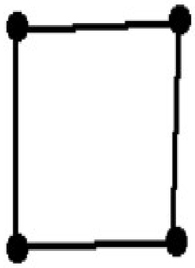
॥ त्वं ज्ञानमयो विज्ञानमयोऽसि ॥

Contents

1 Subgraph

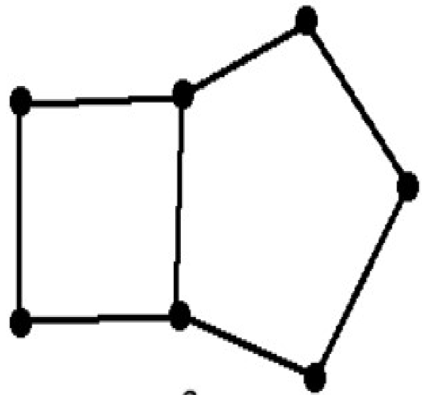
Definition: A Subgraph S of a graph G is a graph whose vertex set $V(S)$ is a subset of the vertex set $V(G)$, that is $V(S) \subseteq V(G)$, and whose edge set $E(S)$ is a subset of the edge set $E(G)$, that is $E(S) \subseteq E(G)$.

For example, the following graph S is a subgraph of G,



S

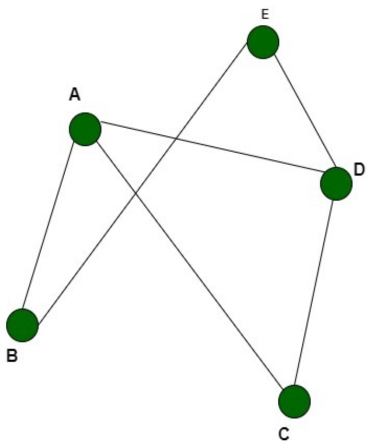
(Subgraph of G)



G

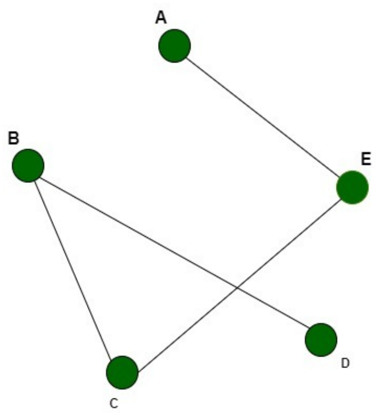
2 Independent Set and Clique

Definition: A set of vertices is called *independent* if no two vertices in the set are adjacent.
A set of vertices is called a *clique* if every two vertices in the set are adjacent.



G ≅

Clique of size 3 {A C D}

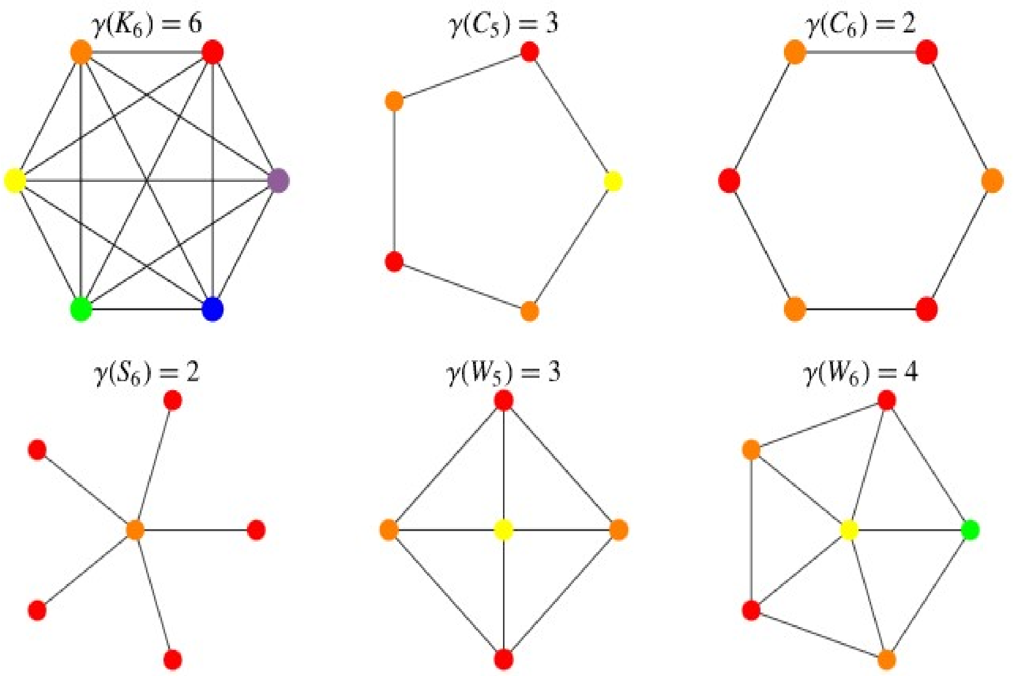


G' ≅

Independent Set of size 3 {A C D}

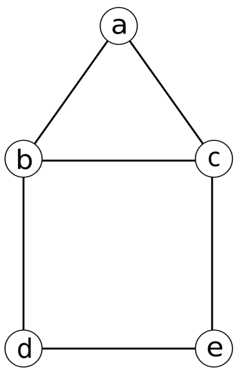
3 Chromatic Number

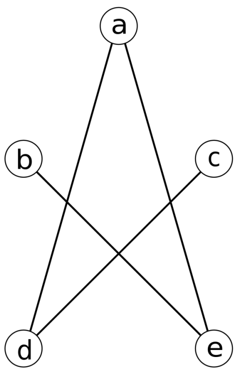
Definition: The chromatic number of a graph G is the smallest number of colors needed to color the vertices of G so that no two adjacent vertices share the same color. Consider the following examples,(γ denotes the chromatic number of the graph).



4 Complement of a Graph

Definition: The complement of a graph G, sometimes called the edge-complement , is the graph G', sometimes denoted G or G^c , with the same vertex set but whose edge set consists of the edges not present in G (i.e., the complement of the edge set of G with respect to all possible edges on the vertex set of G).Consider the following example:



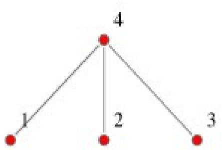


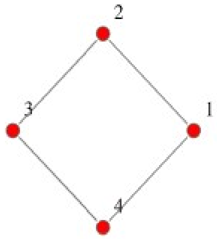
Both graphs are complement to each other and adding these two results in a complete graph of 5-vertices.

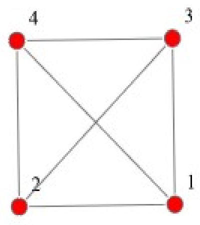
5 Representing graphs

5.1 Adjacency Matrix

Definition: The adjacency matrix, sometimes also called the connection matrix, of a simple labeled graph is a matrix with rows and columns labeled by graph vertices, with a 1 or 0 in position (v_i, v_j) according to whether v_i and v_j are adjacent or not. For a simple graph with no self-loops, the adjacency matrix must have 0s on the diagonal. For an undirected graph, the adjacency matrix is symmetric. Following are some examples of Adjacency matrices for Claw graph, Cycle graph $C - 4$, and Complete graph $K - 4$.



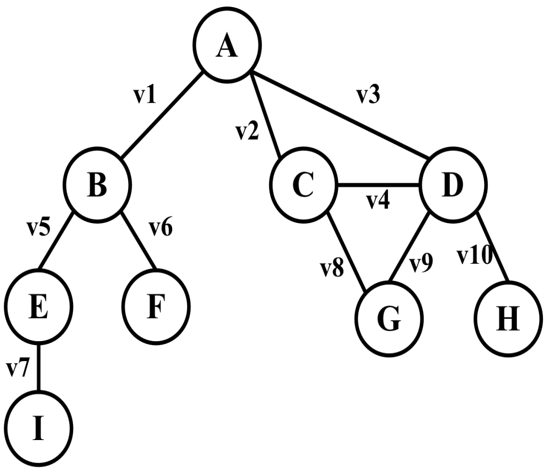




$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

5.2 Incident Matrix

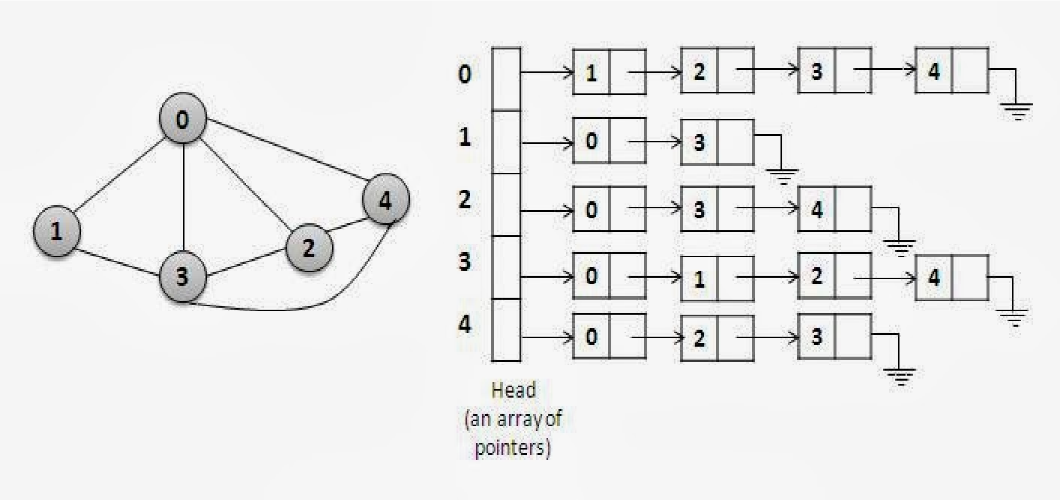
Definition: Incidence matrix is that matrix which represents the graph such that with the help of that matrix we can draw a graph. This matrix can be denoted as $[A_C]$ As in every matrix, there are also rows and columns in incidence matrix $[A_C]$.
The rows of the matrix $[A_C]$ represent the number of nodes and the column of the matrix $[A_C]$ represent the number of branches in the given graph. If there are 'n' number of rows in a given incidence matrix, that means in a graph there are 'n' number of nodes. Similarly, if there are 'm' number of columns in that given incidence matrix, that means in that graph there are 'm' number of branches. Following is the example of an incidence matrix :



	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10
A	1	1	1	0	0	0	0	0	0	0
B	1	0	0	0	1	1	0	0	0	0
C	0	1	0	1	0	0	0	1	0	0
D	0	0	1	1	0	0	0	0	1	1
E	0	0	0	0	1	0	1	0	0	0
F	0	0	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	1	1	0
H	0	0	0	0	0	0	0	0	0	1
I	0	0	0	0	0	0	1	0	0	0

5.3 List

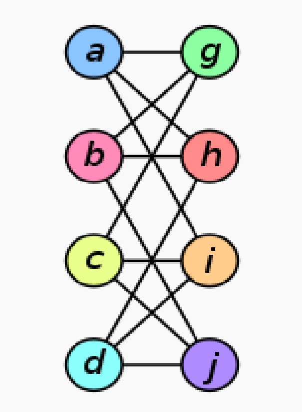
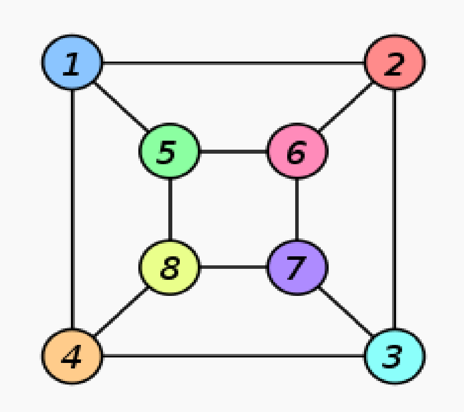
Definition: An adjacency list represents a graph as an array of linked lists. The index of the array represents a vertex and each element in its linked list represents the other vertices that form an edge with the vertex.Following is an illustration:



Adjacency List Representation of Graph

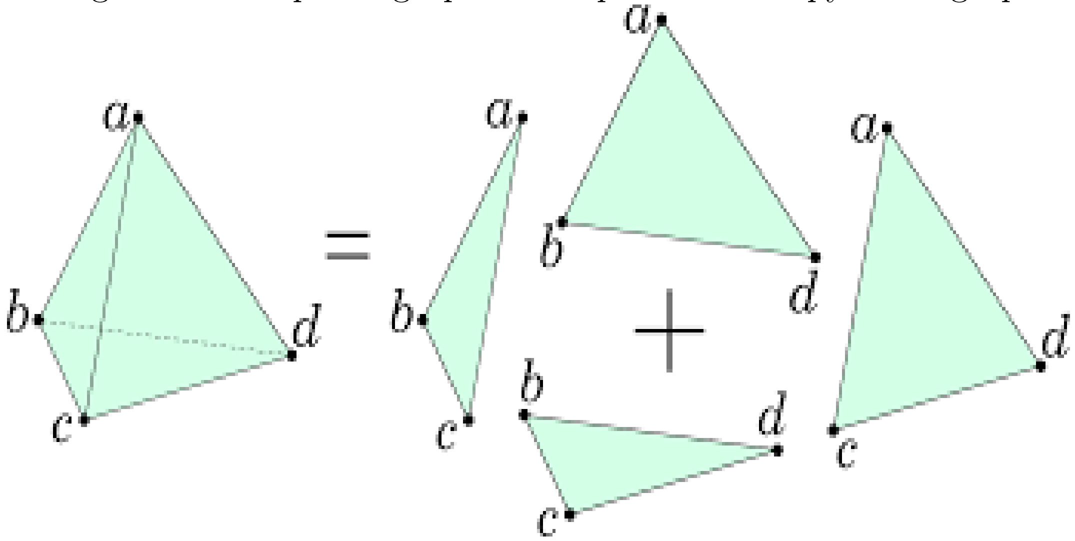
6 Isomorphism

Definition: A graph G_1 is said to be isomorphic to G_2 if there exists a function f from vertices of G_1 to vertices of G_2 [$f : V(G1) \Rightarrow V(G2)$], such that
Case (i): f is a bijection (both one-one and onto)
Case (ii): f preserves adjacency of vertices, i.e., if the edge $U, V \in G1$, then the edge $f(U), f(V) \in G2$, then $G1 \equiv G2$.

Graph G	Graph H	An isomorphism between G and H
		$f(a) = 1$ $f(b) = 6$ $f(c) = 8$ $f(d) = 3$ $f(g) = 5$ $f(h) = 2$ $f(i) = 4$ $f(j) = 7$

7 Decomposition of Graphs

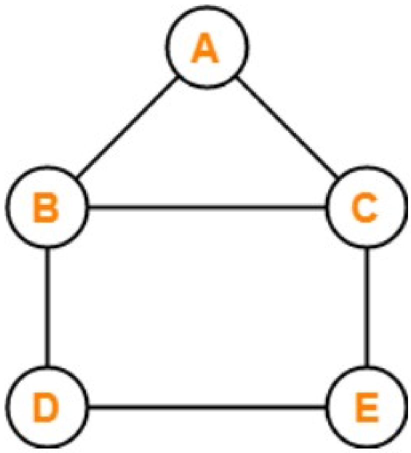
Definition: A decomposition of a graph G is a collection of edge-disjoint subgraphs $H_1, H_2, ..., H_r$ of such that every edge of belongs to exactly one H_i . Following is an example of graph decomposition of a pyramid graph.



8 Connectivity in Graphs

8.1 Walk

In graph theory,
• A walk is defined as a finite length alternating sequence of vertices and edges.
• The total number of edges covered in a walk is called as Length of the Walk.
Consider the following example :



In this graph, few examples of walk are-

- a , b , c , e , d (Length = 4)
- d , b , a , c , e , d , e , c (Length = 7)
- e , c , b , a , c , e , d (Length = 6)

8.1.1 Open Walk

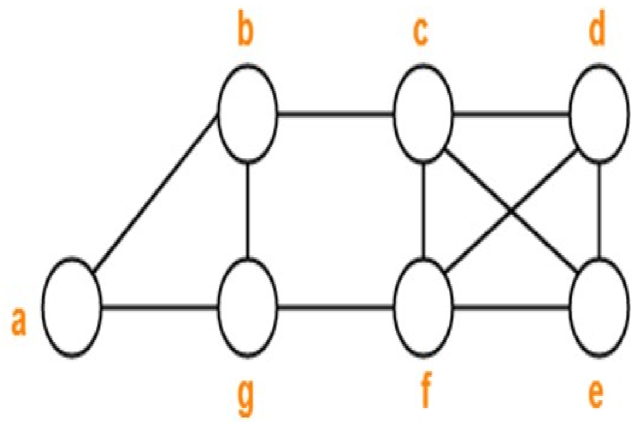
A walk is called as an Open walk if-
•Length of the walk is greater than zero
•And the vertices at which the walk starts and ends are different.

8.1.2 Closed Walk

A walk is called as a Closed walk if-
•Length of the walk is greater than zero.
•And the vertices at which the walk starts and ends are same.

8.2 Path

A path is defined as an open walk in which-
•Neither vertices (except possibly the starting and ending vertices) are allowed to repeat.
•Nor edges are allowed to repeat.
Example: Consider the following graph–>

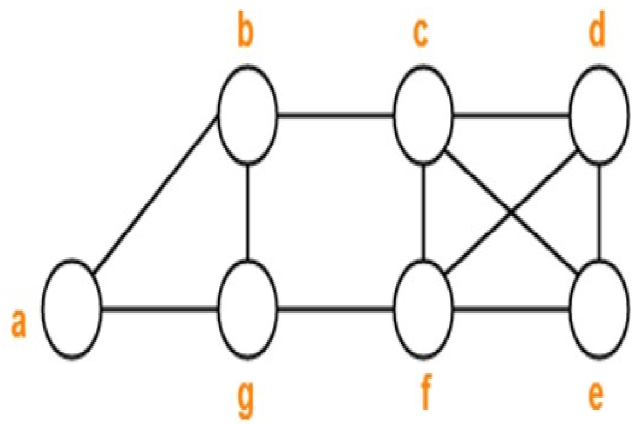


The example of a path is :
$a \longrightarrow g \longrightarrow b \longrightarrow c \longrightarrow f \longrightarrow e$

8.3 Cycle

A cycle is defined as a closed walk in which-
• Neither vertices (except possibly the starting and ending vertices) are allowed to repeat.
• Nor edges are allowed to repeat.

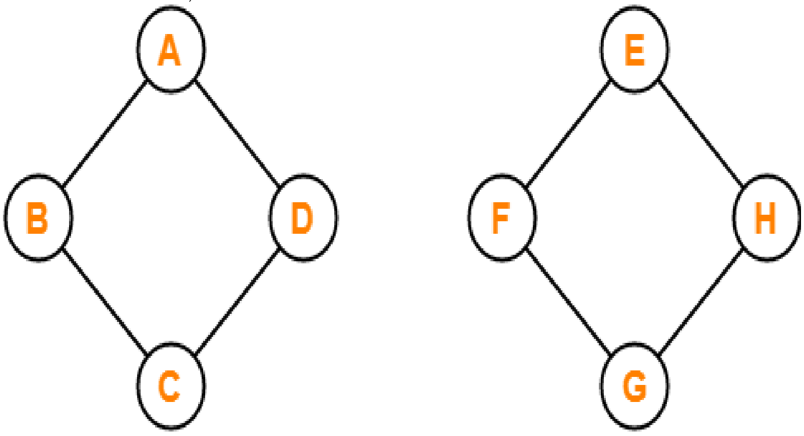
Example: Consider the graph as shown:



The example of a cycle is :
$a \longrightarrow g \longrightarrow f \longrightarrow e \longrightarrow d \longrightarrow c \longrightarrow b \longrightarrow a$

9 Connected and Disconnected Graphs :

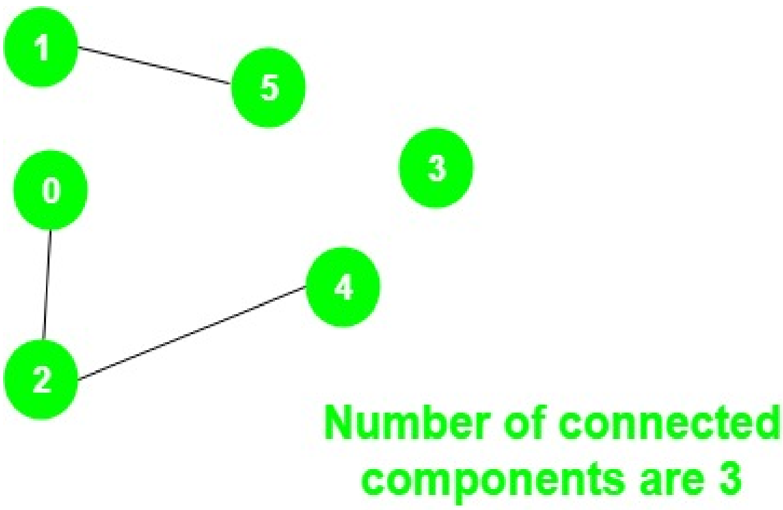
Definition: A graph is said to be connected if there exists a path connecting all the vertices, else it is disconnected.



Example of Disconnected Graph

10 Connected Components

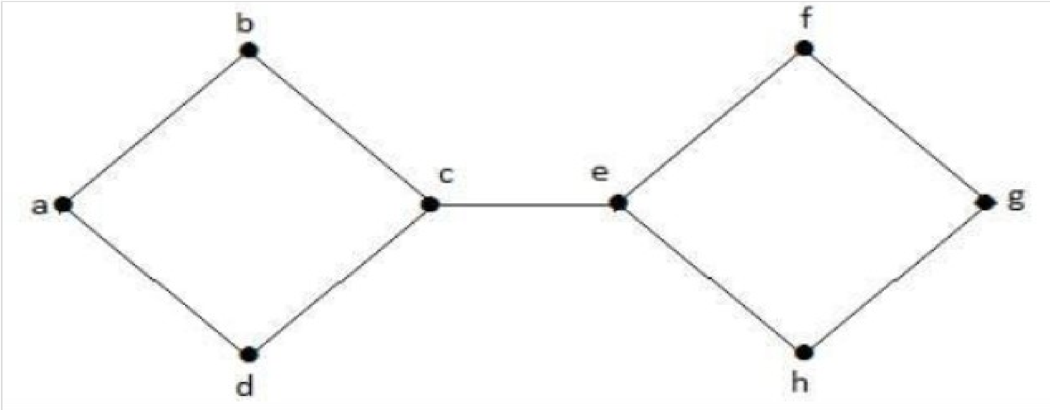
A connected component or simply component of an undirected graph is a subgraph in which each pair of nodes is connected with each other via a path. Connected component of Graph G is maximal connected subgraphs of G.



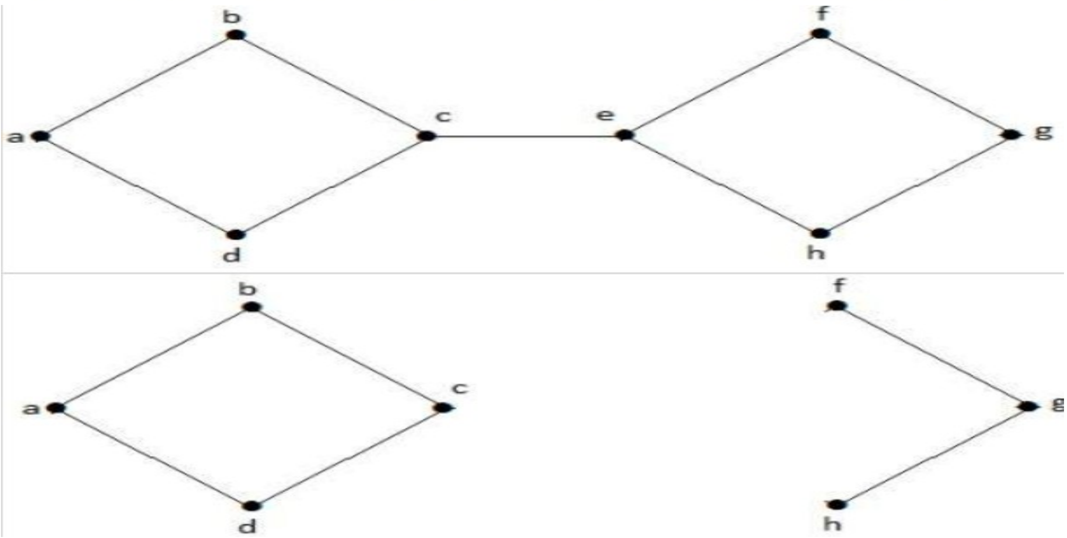
10.1 Cut-Edge And Cut-Vertex

A cut-edge or cut-vertex of a graph is an edge or vertex whose deletion increases the number of components.

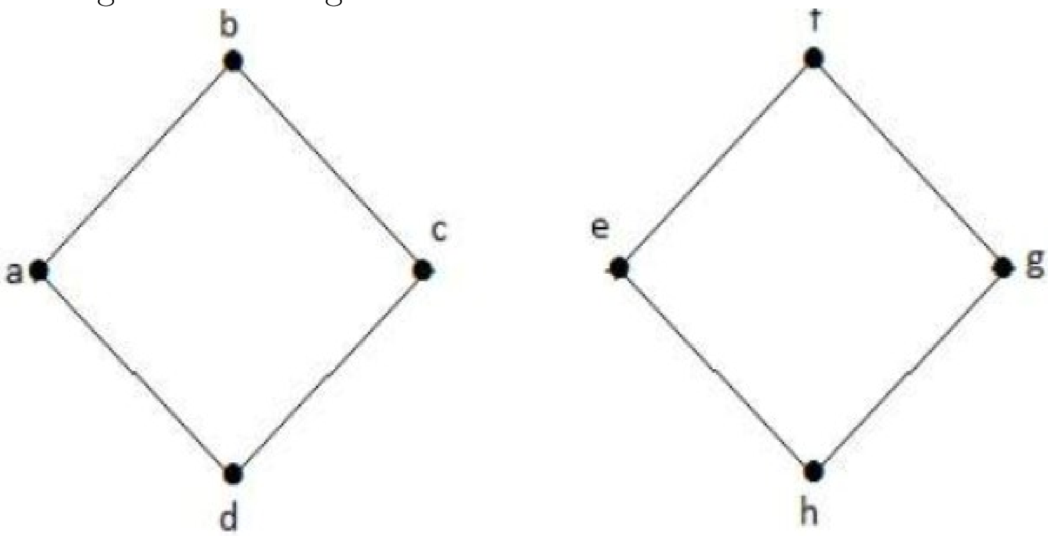
In the following graph, vertices 'e' and 'c' are the cut vertices.



By removing 'e' or 'c', the graph will become a disconnected graph.

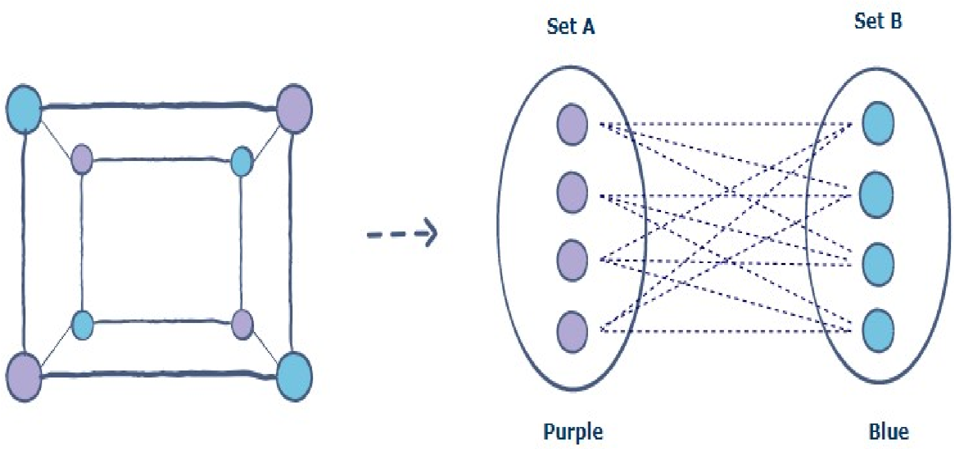


The edge ce is cut-edge



11 Bipartite Graph

Definition :A bipartite graph also called a bi-graph, is a set of graph vertices, i.e, points where multiple lines meet, decomposed into two disjoint sets, meaning they have no element in common, such that no two graph vertices within the same set are adjacent.
Following is an example of a bipartite graph



Each color is connected to a different color.