# Arithmetic Progression

## Definition

An **arithmetic progression (AP) or arithmetic sequence [2]** is a sequence of numbers such that the difference between the consecutive terms is constant. For instance, the sequence 3, 6, 9, 12, 15 is an arithmetic progression with common difference of 3.
If the initial term of an arithmetic progression is $a_1$ and the common difference of successive members is $d$, then the nth term of the sequence $a_n$ is given by:

$$a_n = a_1 + (n-1)d,$$

and in general

$$a_n = a_m + (n-m)d,$$

## Sum

The sum of the members of a finite arithmetic progression is called an **arithmetic series**. For example, consider the sum: 3 + 6 + 9 + 12 + 15 + 18. This sum can be found quickly by taking the number $n$ of terms being added (here 6), multiplying by the sum of the first and last number in the progression (here 3 + 18 = 21), and dividing by 2:

$$\frac{n(a_1 + a_n)}{2}$$

In the case above, this gives the equation:

$$3+6+9+12+15+18 = \frac{6(3+18)}{2} = \frac{6 \times 21}{2} = 40$$

## Derivation

To derive the above formula, begin by expressing the arithmetic series in two different ways:

$$S_n = a_1 + (a_1+d) + (a_1+2d) + \cdots + (a_1+(n-2)d) + (a_1+(n-1)d)$$
$$S_n = (a_n-(n-1)d) + (a_n-(n-2)d) + \cdots + (a_n-2d) + (a_n-d) + a_n$$

Adding both sides of the two equations, all terms involving $d$ cancel:

$$2S_n = n(a_1 + a_n)$$

Dividing both sides by 2 produces a common form of the equation:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

An alternate form results from re-inserting the substitution: $a_n = a_1 + (n-1)d$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

# Geometric Progression

## Definition

A **geometric progression** or **geometric sequence** [1], is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the **common ratio**.
For example, the sequence 2, 4, 8, 16, ... is a geometric progression with common ratio 2.
Examples of a geometric sequence are powers $r^k$ of a fixed number $r$, such as $2^k$ and $3^k$.
The n-th term of a geometric sequence with initial value $a$ and common ratio $r$ is given by

$$a_n = a\,r^{n-1}$$

Such a geometric sequence also follows the recursive relation

$$a_n = r\,a_{n-1}$$

for every integer $n \geq 1$.

## Sum

The sum of the members of a geometric progression is called an **geometric series**. For example, consider the sum: 2 + 4 + 8 + 16 + 32. The sum can be found out by

$$\frac{a(1-r^m)}{1-r}$$

where $a$ is the first term (here 2), $m$ is the number of terms (here 5), and $r$ is the common ratio (here 2). In the example above, this gives: $2 + 4 + 8 + 16 + 32 = \frac{2(1-2^5)}{1-2} = \frac{-62}{-1} = 62$.

## Derivation

To derive this formula, first write a general geometric series as:

$$\sum_{k=1}^{n} ar^{k-1} = ar^0 + ar^1 + ar^2 + ar^3 + \cdots + ar^{n-1}$$

We can find a simpler formula for this sum by multiplying both sides of the above equation by $1-r$, and we'll see that

$$\begin{aligned}(1-r)\sum_{k=1}^{n} ar^{k-1} &= (1-r)(ar^0 + ar^1 + ar^2 + ar^3 + \cdots + ar^{n-1}) \\ &= ar^0 + ar^1 + ar^2 + ar^3 + \cdots + ar^{n-1} \\ &\qquad - ar^1 - ar^2 - ar^3 - \cdots - ar^{n-1} - ar^n \\ &= a - ar^n\end{aligned}$$

since all the other terms cancel. If $r \neq 1$, we can rearrange the above to get the convenient formula for a geometric series that computes the sum of n terms:

$$\sum_{k=1}^{n} ar^{k-1} = \frac{a(1-r^n)}{1-r}$$

## References


[1] Arithmetic progression. `http://en.wikipedia.org/wiki/Arithmetic_progression`.

[2] Geometric progression. `http://en.wikipedia.org/wiki/Geometric_progression`.