###### IV. Approximation of Function, Curve Fitting, Interpolation

1. The following data have been measured in an experiment:

***k*** ***xk*** ***yk*** ***zk***

1 23.000 22.000 10800.000

2 35.000 21.999 162010.797

3 71.000 22.012 831492.000

4 103.000 22.078 2234520.000

5 111.000 22.622 4062960.000

6 109.000 25.536 5918854.000

7 100.000 36.094 7510450.000

8 86.000 57.113 8512614.000

9 71.000 76.565 8764492.000

10 59.000 85.632 8416764.000

11 47.000 86.572 7701761.000

12 39.000 82.884 6800436.000

13 32.000 76.928 5841266.500

14 28.000 70.121 4901137.000

15 24.000 63.270 4022114.000

16 22.000 56.796 3222201.250

17 22.000 50.913 2534144.000

18 22.000 45.663 1966323.250

19 22.000 41.076 1504742.000

20 22.000 37.144 1135166.000

It is proposed to approximate the data by an expression of the form



where (^) denotes “estimated”. Determine a least-squares approximation of the form given by the above equation. (note that there are two independent variables, *x* and *y*)

2. Consider the approximation of the function  in the interval [-2, 2]. First map the *t*-domain to the *x*-domain in such a way that [-2, 2] (in *t*-domain) maps into [-1,1] (in *x*-domain). Approximate the function by employing a Legendre basis . Graphically compare the function to be approximated with the resulting approximants.

3. The water level in the North Sea is mainly determined by the so-called M2-tide, whose period is about 12 hours and thus has the form , where *t* is in hours. One has made the following measurements:

***t*, hours** 0 2 4 6 8 10

***H*(*t*), meters** 1.0 1.6 1.4 0.6 0.2 0.8

Fit H(t) to the series of measurements using the method of least squares and determine *h*0, *a*1 and *a*2.

4. Estimate the value of the function at *x* = 4 from the table of data given below, using, (a) Lagrange interpolating polynomial of 2nd order; (b) Newton’s interpolating polynomial.

***x f(x)***

1 1

2 12

3 54

5 375

6 756