Gaussian Process V0.2

May 14, 2021

1 System Setup

[1]:
```
import sys
# !{sys.executable} -m pip install --upgrade pip
# !{sys.executable} -m pip install GPy
# !{sys.executable} -m pip install seaborn
```

[2]:
```
import numpy as np
import os
import matplotlib.pyplot
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib as mpl
import matplotlib.ticker as mticker
import matplotlib.dates as mdates
import datetime
from tqdm import tqdm
import GPy
from collections import defaultdict
from pathlib import Path
import seaborn as sns
import scipy.stats as stats

mpl.rcParams['legend.frameon'] = False
mpl.rcParams['figure.autolayout'] = True
mpl.rcParams['figure.dpi'] = 300
# mpl.rcParams['axes.spines.right'] = False
# mpl.rcParams['axes.spines.top'] = False


plt.rcParams.update({
    "text.usetex": True,
    "font.family": "sans-serif",
    "font.sans-serif": ["Helvetica"]})

plt.rcParams.update({
    "text.usetex": True,
```

```
    "font.family": "serif",
    "font.serif": ["Palatino"],
})


def utkarshGrid():
    plt.minorticks_on()
    plt.grid(color='grey',
             which='minor',
             linestyle=":",
             linewidth='0.1',
             )
    plt.grid(color='black',
             which='major',
             linestyle=":",
             linewidth='0.1',
             )
```

2 Data Wrangling

[3]:
```
class AllData:
    """Load the data from the set path and prepare it in a useable format.
    """
    def __init__(self):
        self.folder_name = "/bns_m3_3comp" # Change folder name of data as␣
↪required.

    def load_path(self, path_to_dir):
        """ User defined path
        """
        self.folder_path = path_to_dir
        self.path = path_to_dir + self.folder_name
        return None

    def load_raw_data(self):
        """ Loads raw data from given path. Implimentation may different for␣
↪windows and mac/linux users.
        >>> data = AllData()
        >>> data.load_path("/Users/utkarsh/PycharmProjects/SURP2021")
        >>> data.load_raw_data()
        >>> print(data.raw_data.file_name.iloc[0])
        nph1.0e+06_mejdyn0.001_mejwind0.130_phi45.txt
        >>> data.raw_data.file_name.iloc[192] == "nph1.0e+06_mejdyn0.
↪001_mejwind0.090_phi0.txt"
        True
```

```
        >>> data.raw_data.file_name.iloc[192] == "nph1.0e+06_mejdyn0.
↪005_mejwind0.110_phi0.txt"
        False
        >>> data.raw_data.file_name.iloc[192] == "nph1.0e+06_mejdyn0.
↪005_mejwind0.110_phi0.txt"
        False
        """
        resd = defaultdict(list)
        folder_path = Path(self.path)
        for file in folder_path.iterdir():
            with open(file, "r") as file_open:
                resd["file_name"].append(file.name)
        temp_df = pd.DataFrame(resd)
        self.raw_data = temp_df[temp_df.file_name != ".DS_Store"].
↪reset_index(drop=True)
        return None


    def process(self):
        """ Processes the data to a readable reference dataframe.
        >>> data = AllData()
        >>> data.load_path("/Users/utkarsh/PycharmProjects/SURP2021")
        >>> data.load_raw_data()
        >>> data.process()
        >>> print(data.reference_data.mejwind.iloc[68])
        0.03
        >>> print(data.reference_data.mejdyn.iloc[173])
        0.02
        >>> data.reference_data.phi.iloc[55] == 75
        False
        >>> data.reference_data.phi.iloc[57] == 75
        False
        >>> data.reference_data.phi.iloc[56] == 75
        True
        """
        split_series = self.raw_data.file_name.apply(lambda x: x.split('_'))
        temp_df = split_series.apply(pd.Series)
        temp_df["file_name"] = self.raw_data.file_name
        temp_df.columns = ["nph", "mejdyn", "mejwind", "phi", "filename"]
        temp_df["mejdyn"] = temp_df["mejdyn"].str.extract("(\d*\.?\d+)",␣
↪expand=True)
        temp_df["mejwind"] = temp_df["mejwind"].str.extract("(\d*\.?\d+)",␣
↪expand=True)
        temp_df["phi"] = temp_df["phi"].str.extract("(\d*\.?\d+)", expand=True)
        temp_df["nph"] = temp_df["nph"].apply(lambda x: float(x[3:]))
        temp_df[["mejdyn", "mejwind", "phi"]] = temp_df[["mejdyn", "mejwind",␣
↪"phi"]].apply(pd.to_numeric)
```

```
        self.reference_data = temp_df.reset_index(drop=True)
        return None

    def save_reference(self):
        """ Saves the reference data into a file for future use.
        """
        try:
            self.reference_data.to_csv("reference.csv", index = False)
            print("[STATUS] Reference Saved")
        except Exception:
            print("[ERROR] Reference Unsaved")

    def load_reference(self, name):
        """ Loads the saved dataframe to save on computing time.
        >>> data = AllData()
        >>> data.load_reference("reference.csv")
        >>> print(data.reference_data.mejwind.iloc[68])
        0.03
        >>> print(data.reference_data.mejdyn.iloc[173])
        0.02
        >>> data.reference_data.phi.iloc[55] == 75
        False
        >>> data.reference_data.phi.iloc[57] == 75
        False
        >>> data.reference_data.phi.iloc[56] == 75
        True
        """
        self.reference_data = pd.read_csv(name)
```

3 Light Curve Selection

[4]:
```
class LightCurve():
    """ The information regarding KNe light curves and data corresponding to␣
↪KNe light curves.
    """

    def __init__(self, referenceName):
        """ Initializes class, reference is all the light curves, and selected␣
↪represents ones of interest to be narrowed.
        """
        self.reference = pd.read_csv(referenceName)
        self.selected = self.reference.copy()

    def _slice(self, typ, Min, Max):
```

```
        sliced = self.selected[self.selected[typ] >= Min]
        sliced2 = sliced[sliced[typ] <= Max]
        return sliced2

    def select_curve(self, phiRange, mejdynRange, mejwindRange, nphRange =␣
↪[1e6]):
        """ Select a measurment based on the physics limits required.
        >>> data = LightCurve("reference.csv")
        >>> phi_range = [30]
        >>> mejdyn_range = [0.01]
        >>> mejwind_range = [0.11]
        >>> data.select_curve(phiRange = phi_range, mejdynRange = mejdyn_range,␣
↪mejwindRange = mejwind_range)
        >>> print(data.selected.filename.iloc[0])
        nph1.0e+06_mejdyn0.010_mejwind0.110_phi30.txt
        """
        self.selected = self._slice("nph", min(nphRange), max(nphRange))
        self.selected = self._slice("phi", min(phiRange), max(phiRange))
        self.selected = self._slice("mejdyn", min(mejdynRange),␣
↪max(mejdynRange))
        self.selected = self._slice("mejwind", min(mejwindRange),␣
↪max(mejwindRange))
        return None

    def _set_path(self):
        """ Sets the path to the file to be extracted. Chooses first file if␣
↪there are many.
        """
        self.folder_path = os.getcwd() + "/bns_m3_3comp/"
        self.path = self.folder_path + self.selected.filename.iloc[0]
        if len(self.selected.filename) > 1:
            print(f"[WARNING] Many curves in data: First curve has been␣
↪selected. \n[CURVE] {self.selected.filename.iloc[0]}")

        return None

    def extract_curve(self):
        """ Extracts curve based on selected data and converts it into a␣
↪readable format.
        >>> data = LightCurve("reference.csv")
        >>> phi_range = [60]
        >>> mejdyn_range = [0.02]
        >>> mejwind_range = [0.11]
        >>> data.select_curve(phiRange = phi_range, mejdynRange = mejdyn_range,␣
↪mejwindRange = mejwind_range)
        >>> data.extract_curve()
```

```
        >>> data.curve.shape
        (11, 500)
        >>> zBand = 910
        >>> plotDf = data.curve.loc[:, [zBand]]
        >>> print(plotDf.loc[1,zBand][3])
        0.0028678
        >>> print(data.selected.filename.iloc[0])
        nph1.0e+06_mejdyn0.020_mejwind0.110_phi60.txt
        """
        # Obtain path to read curve from.
        self._set_path()

        # Read txt file containig light curve information
        temp0 = pd.read_csv(self.path, header = None, names = ["data"])

        # Set parameters for viewing angles, numbers of wavelengths, and time␣
↪step.
        self.Nobs = int(temp0.data.iloc[0])
        self.Nwave = float(temp0.data.iloc[1])
        self.Ntime = list(map(float, temp0.data.iloc[2].split()))

        # Drop information header and reset index.
        temp1 = temp0.iloc[3:].reset_index(drop = True)

        # Convert data from string to float
        temp1["data"] = temp1["data"].apply(lambda x: list(map(float, x.
↪split())))

        # Obtain wavelength from messy data list. Convert to nm
        temp1.loc[:, 'wavelength'] = temp1.data.map(lambda x: x[0]/10)

        # Remove wavelengths from data vector.
        temp1["data"] = temp1["data"].apply(lambda x: x[1:])

        # Pivot to order the table by wavelengths
        temp1 = temp1.pivot(columns = "wavelength", values = "data")

        # Concatenate all rows to remove NA values to get a neat, readable␣
↪dataframe.
        final = pd.concat([temp1[col].dropna().reset_index(drop=True) for col␣
↪in temp1], axis=1)

        # Rename axis titles.
        final.index.name = "iobs"
        final.columns.name = "wavelength"
        self.curve = final
```

```
        return None

    def _odd(self,x):
        """Rounds to nearest odd numbers
        >>> data = LightCurve("reference.csv")
        >>> data._odd(3)
        3
        >>> data._odd(2.5)
        3
        >>> data._odd(2)
        3
        >>> data._odd(1.999)
        1
        """
        return 2 * int(x/2) + 1

    def simple_plot(self, wv):
        print("[STATUS] Plotting...")
        self.time_arr = np.linspace(int(self.Ntime[1]), int(self.Ntime[2]),␣
↪int(self.Ntime[0]), endpoint = True)
        self.wavelength = 10*self._odd(wv/10)

        viewing_angles = np.linspace(0, 1, data.Nobs, endpoint = True)
        plt.figure()
        plt.gca().set_prop_cycle("color", sns.color_palette("coolwarm_r",self.
↪Nobs))
        for i,j in self.curve.loc[:, [self.wavelength]].iterrows():
            ang = round(np.degrees(np.arccos(viewing_angles[i])), 2)
            plt.plot(self.time_arr, j.values[0], label = f"{ang}"r"$^o$",␣
↪linewidth = 1)
        plt.xlabel("Time (Days)")
        plt.ylabel(r"Flux $Erg s^{-1} cm^{-2}A^{-1}$")
        plt.title(f"Lights curves for {self.Nobs} viewing angles at {self.
↪wavelength}nm")
        utkarshGrid()
        plt.legend(title = r"$\Phi$")
        return None
```

4 Script

[5]:
```
# if __name__ == "__main__":
import doctest
doctest.testmod()
```

[5]:
```
TestResults(failed=0, attempted=45)
```

[6]:
```
# Parse Data
initial = AllData()

# Only needs to be done the first time
initial.load_path("/Users/utkarsh/PycharmProjects/SURP2021")
initial.load_raw_data()
initial.process()
initial.save_reference()
initial.reference_data
```

```
[STATUS] Reference Saved
```

[6]:

	nph	mejdyn	mejwind	phi	\
0	1000000.0	0.001	0.13	45	
1	1000000.0	0.010	0.05	15	
2	1000000.0	0.001	0.01	75	
3	1000000.0	0.005	0.09	75	
4	1000000.0	0.020	0.11	60	
..	…	…	…	…	
191	1000000.0	0.005	0.11	0	
192	1000000.0	0.001	0.09	0	
193	1000000.0	0.020	0.07	0	
194	1000000.0	0.010	0.05	30	
195	1000000.0	0.001	0.13	60	

	filename
0	nph1.0e+06_mejdyn0.001_mejwind0.130_phi45.txt
1	nph1.0e+06_mejdyn0.010_mejwind0.050_phi15.txt
2	nph1.0e+06_mejdyn0.001_mejwind0.010_phi75.txt
3	nph1.0e+06_mejdyn0.005_mejwind0.090_phi75.txt
4	nph1.0e+06_mejdyn0.020_mejwind0.110_phi60.txt
..	…
191	nph1.0e+06_mejdyn0.005_mejwind0.110_phi0.txt
192	nph1.0e+06_mejdyn0.001_mejwind0.090_phi0.txt
193	nph1.0e+06_mejdyn0.020_mejwind0.070_phi0.txt
194	nph1.0e+06_mejdyn0.010_mejwind0.050_phi30.txt
195	nph1.0e+06_mejdyn0.001_mejwind0.130_phi60.txt

```
[196 rows x 5 columns]
```

[7]:
```
# Selecting a singular light curve

data = LightCurve("reference.csv")
phi_range = [30,60]
mejdyn_range = [0.01, 0.02]
```

```
mejwind_range = [0.11]
data.select_curve(phiRange = phi_range,
                  mejdynRange = mejdyn_range,
                  mejwindRange = mejwind_range)
data.selected
```

[7]:

	nph	mejdyn	mejwind	phi	filename
4	1000000.0	0.02	0.11	60	nph1.0e+06_mejdyn0.020_mejwind0.110_phi60.txt
88	1000000.0	0.01	0.11	60	nph1.0e+06_mejdyn0.010_mejwind0.110_phi60.txt
106	1000000.0	0.01	0.11	45	nph1.0e+06_mejdyn0.010_mejwind0.110_phi45.txt
123	1000000.0	0.02	0.11	30	nph1.0e+06_mejdyn0.020_mejwind0.110_phi30.txt
159	1000000.0	0.01	0.11	30	nph1.0e+06_mejdyn0.010_mejwind0.110_phi30.txt
186	1000000.0	0.02	0.11	45	nph1.0e+06_mejdyn0.020_mejwind0.110_phi45.txt

[8]:
```
data.extract_curve()
data.curve.head(2) # iobs goes from 0 --> 10 (11 rows)
```

```
[WARNING] Many curves in data: First curve has been selected.
[CURVE] nph1.0e+06_mejdyn0.020_mejwind0.110_phi60.txt
```

[8]:

wavelength	10.0
iobs	
0	[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
1	[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …

wavelength	30.0
iobs	
0	[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
1	[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …

wavelength	50.0
iobs	
0	[4.0387e-15, 1.2203e-06, 2.4324e-07, 7.9271e-0…
1	[2.9736e-15, 1.3601e-06, 2.3303e-07, 8.1683e-0…

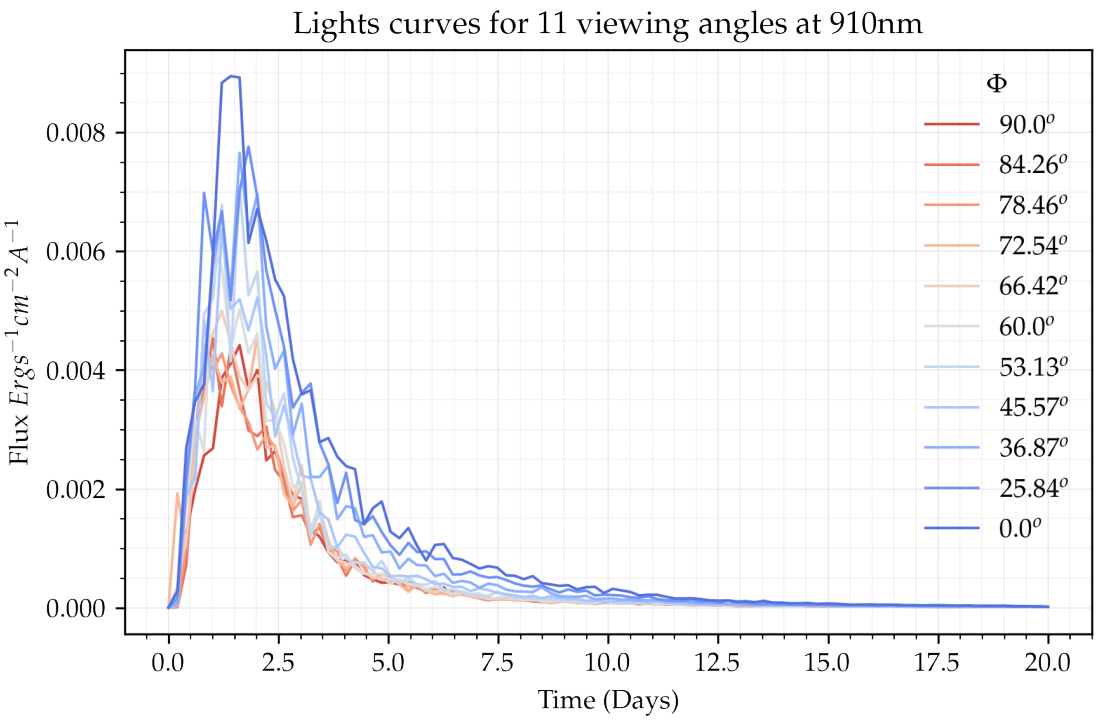
wavelength	70.0
iobs	
0	[1.4257e-12, 3.9016e-05, 1.2179e-05, 3.1122e-0…
1	[2.43e-12, 3.9696e-05, 1.2017e-05, 3.0895e-06,…

```
wavelength                                                 90.0    \
iobs
0           [3.2954e-10, 0.00017416, 0.00010168, 3.2372e-0…
1           [5.3195e-10, 0.00017769, 0.00010233, 3.2907e-0…

wavelength                                                110.0   \
iobs
0           [3.4215e-08, 0.0027166, 0.00045816, 0.00013985…
1           [5.1259e-08, 0.0027128, 0.00046453, 0.00014821…

wavelength                                                130.0   \
iobs
0           [1.2376e-06, 0.00091114, 0.00071721, 0.0004897…
1           [1.769e-06, 0.00090998, 0.00073516, 0.00041236…

wavelength                                                150.0   \
iobs
0           [2.0326e-05, 0.0029708, 0.0013356, 0.0011571, …
1           [2.2727e-05, 0.0030483, 0.0013946, 0.0011474, …

wavelength                                                170.0   \
iobs
0           [8.5044e-05, 0.0047244, 0.0021553, 0.0011496, …
1           [8.6299e-05, 0.0048463, 0.0020631, 0.0011407, …

wavelength                                                190.0   …  \
iobs                                                               …
0           [0.00015567, 0.0015238, 0.0059957, 0.0013954, …  …
1           [0.00014803, 0.0015737, 0.0059852, 0.0017754, …  …

wavelength                                                9810.0  \
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 2.4802e-08, 0.0, 0.0…
1           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …

wavelength                                                9830.0  \
iobs
0           [0.0, 0.0, 0.0, 0.0, 2.9992e-08, 0.0, 0.0, 0.0…
1           [0.0, 0.0, 0.0, 0.0, 0.0, 1.2769e-08, 0.0, 0.0…

wavelength                                                9850.0  \
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
1           [0.0, 0.0, 0.0, 0.0, 4.4778e-05, 0.0, 0.0, 0.0…

wavelength                                                9870.0  \
```

```
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
1           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …

wavelength                                                9890.0  \
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.8119e-08…
1           [0.0, 0.0, 0.0, 0.0, 2.8966e-08, 0.0, 0.0, 0.0…

wavelength                                                9910.0  \
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
1           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.7845e-08…

wavelength                                                9930.0  \
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
1           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …

wavelength                                                9950.0  \
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
1           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …

wavelength                                                9970.0  \
iobs
0           [0.0, 0.0, 0.0, 3.1167e-08, 0.0, 0.0, 0.0, 0.0…
1           [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …

wavelength                                                9990.0
iobs
0           [0.0, 0.0, 0.0, 0.0, 0.0, 3.827e-05, 0.0, 0.0,…
1           [0.0, 0.0, 0.0, 3.1445e-08, 0.0, 0.0, 0.0, 0.0…

[2 rows x 500 columns]
```

[9]:
```
zBand = 900.0
lBand = 3450
data.simple_plot(zBand)
```

```
[STATUS] Plotting…
```



Plot is based on viewing angle. The plot is also general for any wavelength.

5 Gaussian Process

=== NOT PROPER YET ===

[10]:
```
gp = LightCurve("reference.csv")
phi_range = [45]
mejdyn_range = [0.01]
mejwind_range = [0.11]
gp.select_curve(phiRange = phi_range,
                  mejdynRange = mejdyn_range,
                  mejwindRange = mejwind_range)
gp.extract_curve()
z = gp.curve.T[gp.curve.T.index == 910]
z = z.reset_index(drop = True)
z = z.apply(pd.Series.explode).reset_index(drop = True)
time_arr = np.linspace(gp.Ntime[1], gp.Ntime[2], int(gp.Ntime[0]), endpoint =␣
↪True)
z["time"] = time_arr
z.index.name = "time_step"
print(z.shape)
z
```

(100, 12)

[10]:
iobs	0	1	2	3	4	5
time_step						
0	0.000008	0.000075	0.000068	0.000074	0.000002	0.000002
1	0.000821	0.000162	0.0002	0.00032	0.000109	0.000424
2	0.001215	0.000896	0.0011	0.001182	0.001163	0.000646
3	0.002716	0.002916	0.001421	0.001165	0.002129	0.003567
4	0.001977	0.002169	0.001102	0.002042	0.003423	0.002785
…	…	…	…	…	…	…
95	0.000024	0.000023	0.000023	0.000023	0.000022	0.000023
96	0.000018	0.000019	0.000019	0.000019	0.000019	0.000019
97	0.000016	0.000014	0.000015	0.000014	0.000016	0.000018
98	0.000014	0.000014	0.000014	0.000016	0.000015	0.000016
99	0.000013	0.000012	0.000013	0.000013	0.000013	0.000014

iobs	6	7	8	9	10	time
time_step						
0	0.000002	0.000001	0.000001	0.000001	0.000001	0.000000
1	0.000436	0.00078	0.000874	0.002856	0.000297	0.202020
2	0.001896	0.001691	0.002209	0.001234	0.00303	0.404040
3	0.003088	0.003888	0.005563	0.004848	0.004608	0.606061
4	0.00437	0.005405	0.00737	0.005397	0.003681	0.808081
…	…	…	…	…	…	…
95	0.000024	0.000027	0.000025	0.000027	0.00003	19.191919
96	0.000019	0.000021	0.000025	0.000028	0.000025	19.393939
97	0.000021	0.000023	0.000022	0.000022	0.000023	19.595960
98	0.000015	0.000014	0.000018	0.000022	0.000026	19.797980
99	0.000016	0.000021	0.000019	0.000017	0.000021	20.000000

[100 rows x 12 columns]

[11]:
```
# Q5
# Get time closest to one day.
time_ind = np.argmin(np.abs(time_arr-1)) # Not sure what to do with "one day"

delta = 0
one_day = z.iloc[time_ind - delta: time_ind + delta + 1] # select around one day
del one_day["time"] # dont need time after choosing our time frame
one_day
```

[11]:
iobs	0	1	2	3	4	5
time_step						
5	0.002709	0.002237	0.003472	0.003301	0.004151	0.005073

iobs	6	7	8	9	10
time_step					

```
5          0.004041  0.005399  0.005223  0.005627  0.007347
```

[12]:
```
# Q6, Q7
# Median normalization
def med_norm(df):
    med = np.median(df)
    return df.divide(med) - 1

def med_norm_arr(arr):
    med = np.median(arr)
    return arr/med - 1

normed = med_norm(one_day)
normed
```

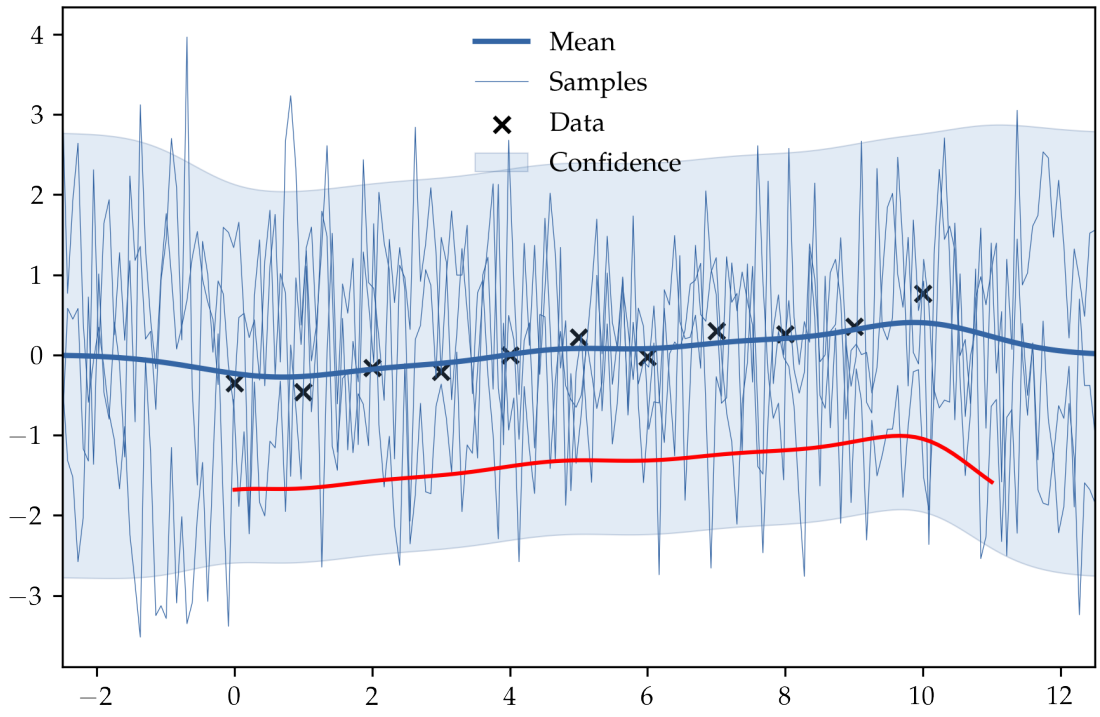
[12]:
```
iobs             0        1         2         3    4         5         6   \
time_step
5         -0.347282 -0.46114 -0.163462 -0.204852  0.0  0.222102 -0.02638

iobs             7         8        9         10
time_step
5          0.300713  0.258215  0.35557  0.770141
```

[13]:
```
# Q8, Q9
X = np.arange(0, gp.Nobs, 1)
Y = np.array(normed.iloc[0])
X = X.reshape(len(X), 1)
Y = Y.reshape(len(Y), 1)
kernel = GPy.kern.RBF(input_dim=1, variance = 1, lengthscale=1) # Variance
m = GPy.models.GPRegression(X, Y, kernel)
```

[14]:
```
# Q10
predX = np.linspace(0,11,100).reshape(100, 1)
predY = m.predict(predX)[0] - m.predict(predX)[1] # What does it mean to plot a␣
 ↪"random draw"
# predY = med_norm_arr(predY)
m.plot(samples = 3)
plt.plot(predX,predY, color = "red", label = "prior")
# plt.ylim(-1,1)
# Y-Axis: Fractional distance of median flux

predY_mean, predY_var = m.predict(predX,  full_cov=True)

print(predY_var) # plot covariance

# Multidmensional version of scipy normal
# Inject predY_var
```

```
# Random Draw from normal

mu = 0
sigma = 1
x = np.linspace(mu - 3*sigma, mu + 3*sigma, 100)
# plt.plot(x, stats.norm.pdf(x, mu, sigma), label = "Unit Gaussian")
```

```
[[ 1.44892176e+00  4.34477924e-01  4.13754835e-01 …  2.29965413e-08
   1.00237570e-08 -1.41542169e-10]
 [ 4.34477924e-01  1.42961917e+00  4.18108979e-01 …  4.68015794e-08
   3.12689179e-08  1.83009471e-08]
 [ 4.13754835e-01  4.18108979e-01  1.41580009e+00 …  7.49629893e-08
   5.74064464e-08  4.18363546e-08]
 …
 [ 2.29965413e-08  4.68015794e-08  7.49629893e-08 …  1.72645119e+00
   7.44319169e-01  7.50809591e-01]
 [ 1.00237570e-08  3.12689179e-08  5.74064464e-08 …  7.44319169e-01
   1.77216774e+00  7.88414213e-01]
 [-1.41542169e-10  1.83009471e-08  4.18363546e-08 …  7.50809591e-01
   7.88414213e-01  1.81462434e+00]]
```



[15]:
```
# Q11, Q12
display(m)
```

```
# m.optimize(messages=True)
m.optimize_restarts(num_restarts = 10)
```

```
<GPy.models.gp_regression.GPRegression at 0x7f8c9f1aa130>

Optimization restart 1/10, f = -1.3297537699107762
Optimization restart 2/10, f = -1.3297537699106217
Optimization restart 3/10, f = -1.3297537699096056
Optimization restart 4/10, f = -1.3297537698897681
Optimization restart 5/10, f = -1.3297537699114832
Optimization restart 6/10, f = -1.329753769908617
Optimization restart 7/10, f = -1.3297537699114308
Optimization restart 8/10, f = -1.329753769784281
Optimization restart 9/10, f = -1.3297537699114939
Optimization restart 10/10, f = -1.32975376990992
```

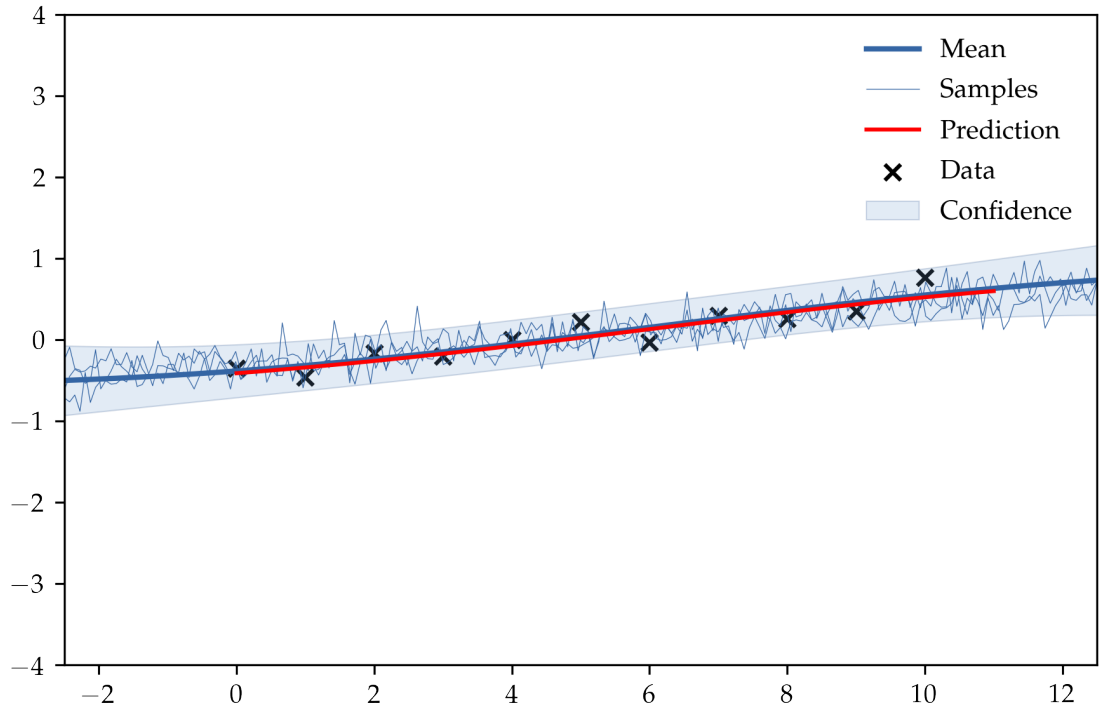
[15]:
```
[<paramz.optimization.optimization.opt_lbfgsb at 0x7f8ca16d22e0>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e3ca280>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e3c4b20>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e409e50>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e2365e0>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e3d26d0>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e3caac0>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8ca16d2370>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e409e80>,
 <paramz.optimization.optimization.opt_lbfgsb at 0x7f8c9e236d60>]
```

[16]:
```
predX = np.linspace(0,11,100).reshape(100, 1)
predY = m.predict(predX)[0] - m.predict(predX)[1] # Should I be applying a␣
 ↪random variance in pred iteratively
# predY = med_norm_arr(predY) # Why does turning this off make a difference?

print(m)

m.plot(samples = 3)
plt.plot(predX,predY, color = "red", label = "Prediction")
plt.ylim(-4,4)
plt.legend()
```

```
Name : GP regression
Objective : -1.3297537699114939
Number of Parameters : 3
Number of Optimization Parameters : 3
Updates : True
Parameters:
  GP_regression.          |                value  |  constraints  |
priors
```

```
  rbf.variance              |    0.5048698656337448  |      +ve       |
  rbf.lengthscale           |    10.237695206905478  |      +ve       |
  Gaussian_noise.variance   | 0.019581530190045226   |      +ve       |
```

[16]: `<matplotlib.legend.Legend at 0x7f8cb711da60>`



Q13) Comments about fidelity of the emulator prediction.

[17]:
```python
#Q14
hist = []
vec = np.array(normed.iloc[0])
# print("Vector:",vec)

for i in range(11):
# for i in range(1):
    loo = np.delete(vec, i)
    trueY = vec[i:i+1]
    if trueY in loo:
        print("[ERROR] Element left out is in your training data")
        break


    X = np.arange(0, gp.Nobs - 1, 1)
    Y = loo
```

```
    X = X.reshape(len(X), 1)
    Y = Y.reshape(len(Y), 1)
    kernel = GPy.kern.RBF(input_dim=1, variance = 1, lengthscale=1)
    mLoo = GPy.models.GPRegression(X, Y, kernel)

    mLoo.optimize #Maybe optimize researts is the correct thing to use here.

    predX = np.linspace(0,10,100).reshape(100, 1)
    predY_mean, predY_var = mLoo.predict(predX)
    predY_mean = predY_mean.reshape(1, len(predY_mean))[0]
    predY_var = predY_var.reshape(1, len(predY_var))[0]
    print(predY_var) # Variances are the same?
    hist.append((predY_mean - trueY)/np.sqrt(predY_var))

hist
```

```
[1.44892176 1.43113568 1.41794277 1.40868383 1.40258386 1.39885663
 1.39678959 1.395798   1.3954451  1.39543293 1.39557423 1.39575774
 1.39591789 1.39601503 1.39602734 1.39595075 1.39580081 1.39561131
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[18]:
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plt.figure()
plt.hist(hist, bins = 2)
mu = 0
sigma = 1
x = np.linspace(mu - 3*sigma, mu + 3*sigma, 100)
plt.plot(x, stats.norm.pdf(x, mu, sigma), label = "Unit Gaussian")
plt.legend()
plt.show()
```