**Q1 - 2024 (01 Feb Shift 1)**

Let f : $\mathbf{R} \rightarrow \mathbf{R}$ and g : $\mathbf{R} \rightarrow \mathbf{R}$ be defined as

$f(x)=\begin{cases}\log_e x & , \quad x>0 \\ e^{-x} & , \quad x \leq 0\end{cases}$ and

$g(x)=\begin{cases}x & , \quad x \geq 0 \\ e^x & , \quad x<0\end{cases}$. Then, gof : $\mathbf{R} \rightarrow \mathbf{R}$ is :

(1) one-one but not onto

(2) neither one-one nor onto

(3) onto but not one-one

(4) both one-one and onto

**Q2 - 2024 (01 Feb Shift 2)**

If the domain of the function $f(x)=\frac{\sqrt{x^2-25}}{(4-x^2)}+\log_{10}\left(x^2+2x-15\right)$ is $(-\infty, \alpha) U[\beta, \infty)$, then $\alpha^2+\beta^3$ is equal to :

(1) 140

(2) 175

(3) 150

(4) 125

**Q3 - 2024 (27 Jan Shift 1)**

The function f : N $-\{1\} \rightarrow$ N; defined by f(n) $=$ the highest prime factor of $n$, is :

(1) both one-one and onto

(2) one-one only

(3) onto only

(4) neither one-one nor onto

**Q4 - 2024 (27 Jan Shift 2)**

Let f : R $-\left\{\frac{-1}{2}\right\}$ $\rightarrow$ R and g : R $-\left\{\frac{-5}{2}\right\}$ $\rightarrow$ R be defined as $f(x)=\frac{2x+3}{2x+1}$ and $g(x)=\frac{|x|+1}{2x+5}$. Then the domain of the function fog is :

(1) R $-\left\{-\frac{5}{2}\right\}$

(2) R

(3) R $-\left\{-\frac{7}{4}\right\}$

(4) R $-\left\{-\frac{5}{2},-\frac{7}{4}\right\}$

**Q5 - 2024 (29 Jan Shift 1)**

If $f(x)=\begin{cases}2+2x, -1\leq x<0\\ 1-\frac{x}{3}, 0\leq x\leq 3\end{cases}; g(x)=\begin{cases}-x, -3\leq x\leq 0\\ x, 0<x\leq 1\end{cases}$,

then range of (fog(x)) is

(1) (0, 1]

(2) [0, 3)

(3) [0, 1]

(4) [0, 1)

**Q6 - 2024 (29 Jan Shift 1)**

Consider the function f : $\left[\frac{1}{2},1\right]$ $\rightarrow$ R defined by $f(x)=4\sqrt{2}x^3-3\sqrt{2}x-1$. Consider the statements

(I) The curve $y=f(x)$ intersects the $x$-axis exactly at one point

(II) The curve $y=f(x)$ intersects the $x$-axis at x $=\cos\frac{\pi}{12}$

Then

(1) Only (II) is correct

(2) Both (I) and (II) are incorrect

(3) Only (I) is correct

(4) Both (I) and (II) are correct

**Q7 - 2024 (29 Jan Shift 1)**

Let $\mathrm{f(x)=2^x-x^2, x\in R}$. If m and n are respectively the number of points at which the curves $y=f(x)$ and $y=f'(x)$ intersects the $x$-axis, then the value of $m+n$ is

**Q8 - 2024 (30 Jan Shift 1)**

If the domain of the function $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\left(\log_e(3-x)\right)^{-1}$ is $[-\alpha,\beta)-\{y\}$, then $\alpha+\beta+\gamma$ is equal to :

(1) 12

(2) 9

(3) 11

(4) 8

**Q9 - 2024 (30 Jan Shift 1)**

Let $A=\{1,2,3,\ldots 7\}$ and let $P(1)$ denote the power set of $A$. If the number of functions $\mathrm{f: A\rightarrow P(A)}$ such that $\mathrm{a\in f(a), \forall a\in A}$ is $\mathrm{m^n}$, m and $\mathrm{n\in N}$ and m is least, then $\mathrm{m+n}$ is equal to _______

**Q10 - 2024 (30 Jan Shift 2)**

If the domain of the function $f(x)=\log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha,\beta]$, then the value of $5\beta-4\alpha$ is equal to

(1) 10

(2) 12

(3) 11

(4) 9

**Q11 - 2024 (30 Jan Shift 2)**

Let $f: R\rightarrow R$ be a function defined $f(x)=\frac{x}{(1+x^4)^{1/4}}$ and $g(x)=f(f(f(f(x))))$ then $18\int_0^{\sqrt{2\sqrt{5}}}x^2g(x)dx$

(1) 33

(2) 36

(3) 42

(4) 39

**Q12 - 2024 (31 Jan Shift 1)**

If $f(x)=\frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ and $(f \circ f)(x)=g(x)$, where $g: \mathbb{R}-\left\{\frac{2}{3}\right\} \rightarrow \mathbb{R}-\left\{\frac{2}{3}\right\}$, then (gogog) (4) is equal to

(1) $-\frac{19}{20}$

(2) $\frac{19}{20}$

(3) -4

(4) 4

**Q13 - 2024 (31 Jan Shift 2)**

If the function $f:(-\infty,-1] \rightarrow(a, b]$ defined by $f(x)=e^{x^3-3x+1}$ is one-one and onto, then the distance of the point $\mathrm{P}(2\,\mathrm{b}+4, \mathrm{a}+2)$ from the line $x+e^{-3} y=4$ is :

(1) $2\sqrt{1+\mathrm{e}^6}$

(2) $4\sqrt{1+\mathrm{e}^6}$

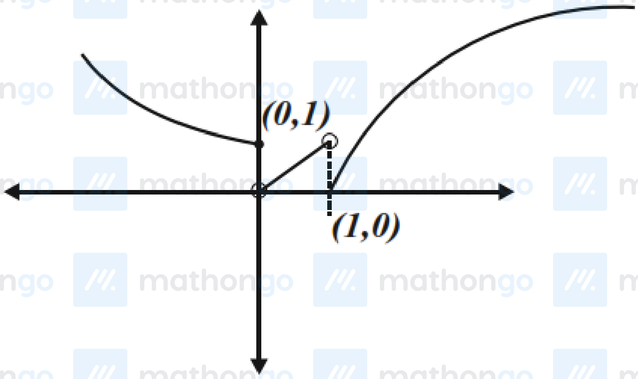
(3) $3\sqrt{1+\mathrm{e}^6}$

(4) $\sqrt{1+\mathrm{e}^6}$

# Answer Key

| | | | |
|---|---|---|---|
| **Q1** (2) | **Q2** (3) | **Q3** (4) | **Q4** (1) |
| **Q5** (3) | **Q6** (4) | **Q7** (5) | **Q8** (3) |
| **Q9** (44) | **Q10** (2) | **Q11** (4) | **Q12** (4) |
| **Q13** (1) | | | |

**Q1**




$$g(f(x)) = \begin{cases} f(x), f(x) \geq 0 \\ e^{f(x)}, f(x) < 0 \end{cases}$$

$$g(f(x)) = \begin{cases} e^{-x}, (-\infty, 0] \\ e^{\ln x}, (0,1) \\ \ln x, [1, \infty) \end{cases}$$

Graph of $g(f(x))$

$g(f(x)) \Rightarrow$ Many one into

**Q2**

$$f(x) = \frac{\sqrt{x^2 - 25}}{4 - x^2} + \log_{10}(x^2 + 2x - 15)$$

Domain : $x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$

$x^2 + 2x - 15 > 0 \Rightarrow (x+5)(x-3) > 0$

$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$

$\therefore x \in (-\infty, -5) \cup [5, \infty)$

$\alpha = -5; \beta = 5$

$\therefore \alpha^2 + \beta^3 = 150$

**Q3**

$f : N - \{1\} \rightarrow N$

$f(n) =$ The highest prime factor of n.

$f(2) = 2$

$f(4) = 2$

$\Rightarrow$ many one

4 is not image of any element

$\Rightarrow$ into

Hence many one and into

Neither one-one nor onto.

**Q4**

$$f(x) = \frac{2x+3}{2x+1}; x \neq -\frac{1}{2}$$

$$g(x) = \frac{|x|+1}{2x+5}, x \neq -\frac{5}{2}$$

Domain of $f(g(x))$

$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$

$x \neq -\frac{5}{2}$ and $\frac{|x|+1}{2x+5} \neq -\frac{1}{2}$

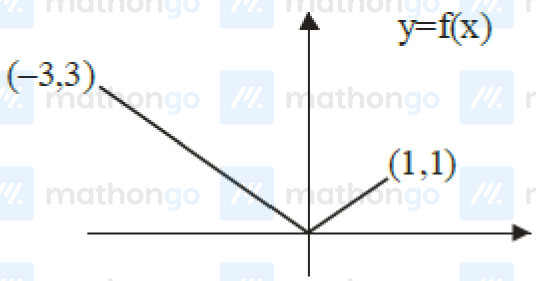
$x \in R - \left\{-\frac{5}{2}\right\}$ and $x \in R$

$\therefore$ Domain will be $R - \left\{-\frac{5}{2}\right\}$

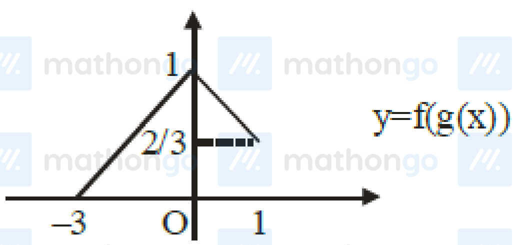
**Q5**

$$f(g(x)) = \begin{cases} 2+2g(x) & , \quad -1 \le g(x) < 0 \quad .....(1) \\ 1-\frac{g(x)}{3} & , \quad 0 \le g(x) \le 3 \quad .....(2) \end{cases}$$

By (1) $x \in \phi$




And by (2) $x \in [-3, 0]$ and $x \in [0, 1]$



Range of f(g(x)) is [0, 1]

**Q6**

$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \geq 0$ for $\left[\frac{1}{2}, 1\right]$

$f\left(\frac{1}{2}\right) < 0$

$f(1) > 0 \Rightarrow$ (A) is correct.

$f(x) = \sqrt{2}\left(4x^3 - 3x\right) - 1 = 0$

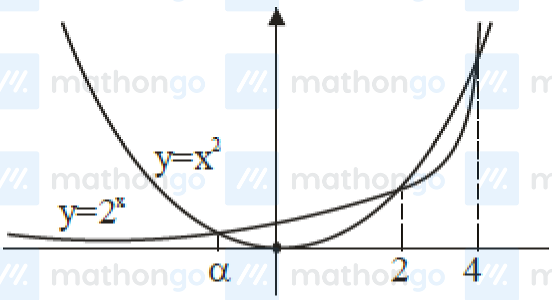
Let $\cos\alpha = x$,

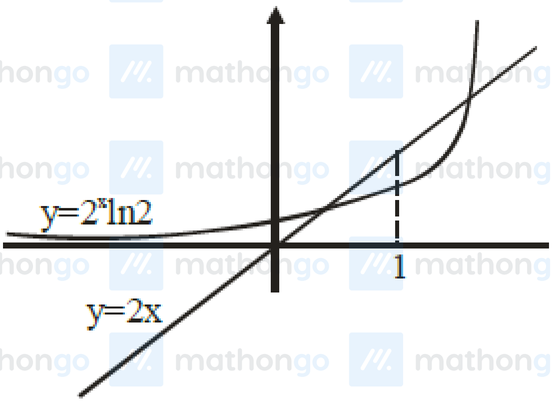
$\cos 3\alpha = \cos\frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$

$x = \cos\frac{\pi}{12}$

(4) is correct.

**Q7**




$\therefore m = 3$

$f'(x) = 2^x \ln 2 - 2x = 0$

$2^x \ln 2 = 2x$

$\therefore \mathrm{n}=2$
$\Rightarrow \mathrm{m}+\mathrm{n}=5$

**Q8**

$-1 \leq\left|\frac{2-|x|}{4}\right| \leq 1$

$\Rightarrow\left|\frac{2-|x|}{4}\right| \leq 1$
$-4 \leq 2-|x| \leq 4$
$-6 \leq-|x| \leq 2$
$-2 \leq|x| \leq 6$
$|x| \leq 6$

$\Rightarrow x \in[-6,6]$ ...(1)

Now, $3-x \neq 1$

And $x \neq 2$ ...(2)

and $3-x>0$

$x<3$ ...(3)

From (1), (2) and (3)
$\Rightarrow \quad x \in[-6,3)-\{2\}$
$\alpha=6$
$\beta=3$
$\gamma=2$
$\alpha+\beta+\gamma=11$

**Q9**

$\mathrm{f}: \mathrm{A} \rightarrow \mathrm{P}(\mathrm{A})$

$a \in f(a)$

That means 'a' will connect with subset which contain element ' $a$ '

Total options for 1 will be $2^6$. (Because $2^6$ subsets contains 1)

Similarly, for every other element

Hence, total is $2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6=2^{42}$

Ans. $2+42=44$

**Q10**

$$\frac{2x+3}{4x^2+x-3} > 0 \text{ and } -1 \le \frac{2x-1}{x+2} \le 1$$

$$\frac{2\times+3}{(4x-3)(x+1)} > 0 \quad \frac{3x+1}{x+2} \ge 0 \& \frac{x-3}{x+2} \le 0$$

$(-\infty/2+-1$

$$(-2,3] \quad \ldots \left[\frac{-1}{3},\infty\right) \quad \ldots\ldots(1)$$

$$\left[\frac{-1}{3},3\right]\ldots\ldots(3) \quad (1)\cap(2)\cap(3)$$

$$\left(\frac{3}{4},3\right]$$

$$\alpha=\frac{3}{4}\beta=3$$

$5\beta-4\alpha=15-3=12$

**Q11**

$$f(x)=\frac{x}{\left(1+x^4\right)^{1/4}}$$

$$f\circ f(x)=\frac{f(x)}{\left(1+f(x)^4\right)^{1/4}}=\frac{\frac{x}{\left(1+x^4\right)^{1/4}}}{\left(1+\frac{x^4}{1+x^4}\right)^{1/4}}=\frac{x}{\left(1+2x^4\right)^{1/4}}$$

$$f(f(f(f(x))))=\frac{x}{\left(1+4x^4\right)^{1/4}}$$

$$18\int_0^{\sqrt{2\sqrt{5}}}\frac{x^3}{\left(1+4x^4\right)^{1/4}}dx$$

Let $1+4x^4=t^4$

$16x^3dx=4t^3dt$

$$\frac{18}{4}\int_1^3\frac{t^3dt}{t}$$

$$=\frac{9}{2}\left(\frac{t^3}{3}\right)_1^3$$

$$=\frac{3}{2}[26]=39$$

**Q12**

$$f(x)=\frac{4x+3}{6x-4}$$

$$g(x)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{34x}{34}=x$$

$$g(x)=x \therefore g(g(g(4)))=4$$

**Q13**

$$f(x)=e^{x^3-3x+1}$$

$$f'(x)=e^{x^3-3x+1}\cdot\left(3x^2-3\right)$$

$$=e^{x^3-3x+1}\cdot 3(x-1)(x+1)$$

For $f'(x)\geq 0$

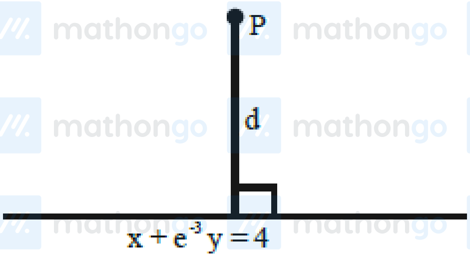
$\therefore$ f(x) is increasing function

$$\therefore a=e^{-\infty}=0=f(-\infty)$$

$$b=e^{-1+3+1}=e^3=f(-1)$$

$$P(2b+4,a+2)$$

$$\therefore P\left(2e^3+4,2\right)$$




$$\mathrm{d}=\frac{\left(2\mathrm{e}^3+4\right)+2\mathrm{e}^{-3}-4}{\sqrt{1+\mathrm{e}^{-6}}}=2\sqrt{1+\mathrm{e}^6}$$