HW 8 — AI

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Q1. Knowledge Base:

$$p \Rightarrow (q \Rightarrow r)$$
$$= \neg p \vee (\neg q \vee r)$$
$$= \neg p \vee \neg q \vee r$$

Query:

$$(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$$
$$= \neg(\neg p \vee q) \vee (\neg p \vee r)$$
$$= (p \wedge \neg q) \vee (\neg p \vee r)$$
$$= (p \vee \neg p \vee r) \wedge (\neg q \vee \neg p \vee r)$$
$$= \neg q \vee \neg p \vee r$$

$\neg$Query:

$$\neg(\neg q \vee \neg p \vee r)$$
$$= q \wedge p \wedge \neg r$$

Resolution:

1. $\neg p \vee \neg q \vee r$
2. q
3. p
4. $\neg r$

Step 1: Resolve 1 and 2

$(\neg p \vee \neg q \vee r), q \Rightarrow \neg p \vee r$

Step 2: Resolve above and 3

$\neg p \vee r, p \Rightarrow r$

Step 3: Resolve above and 4 $\quad r, \neg r \Rightarrow$ null

Since the resolution resulted in null, the query is entailed by the knowledge base.

Q2. 1) a) Everything that can jump jumps higher than a building

b) There are 100 politicians
They are either honest or liars
Taking any two, atleast one is a liar
There exists a politician that is honest

2) a) Variable's x and y which have a domain of all things

$x \in$ All things , $y \in$ All things

b) Variables P_1, P_2 which represents politicians
The domain of P_1, P_2 are the 100 politicians.
$P_1 \in \{P1, P2, \ldots, P100\}$, $P_2 \in \{P1, P2, \ldots, P100\}$

3) a) Predicates:

CanJump(x) returns true if x can jump
returns false if x can't jump

Building(x) returns true if x is a building
returns false if x is not a building

Higher(x,y) returns true if x can jump higher than y
returns false if x can't jump higher than y

b) Honest(p) returns true if p is honest
returns false if p is not honest

Liar(p) returns true if p is a liar
returns false if p is not a liar

4. a) $\forall x \forall y \; (CanJump(x) \wedge Building(y) \Rightarrow Higher(x,y))$

b) $\forall p_1 \; Honest(p_1) \vee Liar(p_1)$

$\forall p_1 \forall p_2 \; (\neg(p_1 = p_2)) \Rightarrow (Liar(p_1) \vee Liar(p_2))$

$\exists p_1 \; Honest(p_1)$

Q3 1.

	Madison	Seattle	Boston	Vancouver	Winnipeg	Montreal
Madison	0	1,617	931	1,654	597	800
Seattle	1,617	0	2,486	121	1,154	2,286
Boston	931	2,486	0	2,501	1,344	250
Vancouver	1,654	121	2,501	0	1,159	2,293
Winnipeg	597	1,154	1,344	1,159	0	1,132
Montreal	800	2,286	250	2,293	1,132	0

2. Iteration 1:

a) Vancouver and Seattle are the closest clusters

b) The distance between them is 121 miles

c) The clusters are {Vancouver, Seattle}, Madison, Boston, Winnipeg, Montreal

Iteration 2

a) Boston and Mont real

b) 250 miles

c) {Vancouver, Seattle}, {Boston, Montreal}, Madison, Winnipeg

Iteration 3

a) Madison and Winnipeg

b) 597 miles

c) {Vancover, Seattle}, {Boston, Montreal}, {Madison, Winnipeg}

Iteration 4

a) {Boston, Montreal} and {Madison, Winnipeg}

b) 1344 miles

c) {Boston, Montreal, Madison, Winnipeg}, {Vancouver, Seattle}

The two clusters are

{Boston, Montreal, Madison, Winnipeg} and

{Vancouver, Seattle}

3. Iteration 1

a) Boston and Madison

b) 931 miles

c) {Boston, Madison}, Seattle, Vancouver, Winnipeg, Montreal

Iteration 2

a) Winnipeg and Montreal

b) 1,132 miles

c) {Boston, Madison}, {Winnipeg, Montreal}, Seattle, Vancouver

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Iteration 3

a) {Winnipeg, Montreal} and Vancouver

b) 2,293 miles

c) {Winnipeg, Montreal, Vancouver}, {Boston, Madison}, Seattle

Iteration 4

a) {Boston, Madison} and Seattle

b) 2486 miles

c) {Winnipeg, Montreal, Vancouver} and {Boston, Madison, Seattle}

Q4. 1. Iteration 1

a) $y_1 = 1$ (1st cluster center, $c_1 = 1$)

$y_2 = 1$

$y_3 = 1$

$y_4 = 2$

$y_5 = 2$

$y_6 = 2$

b) $c_1 = \frac{0+2+4}{3} = 2$

$c_2 = \frac{6+7+8}{3} = 7$

c) Energy $= (0-2)^2 + (2-2)^2 + (4-2)^2 + (6-7)^2 + (7-7)^2 + (8-7)^2$

$\text{Energy} = 4 + 0 + 4 + 1 + 0 + 1 = 10$

Iteration 2

$y_1 = 1$

$y_2 = 1$

$y_3 = 1$

$y_4 = 2$

$y_5 = 2$

$y_6 = 2$

$\Rightarrow$ Since the assignments are the same as before, cluster centers will stay at the same place and energy will be same and we end the algorithm.

2. Iteration 1

a) $y_1 = 1$

$y_2 = 2$

$y_3 = 2$

$y_4 = 2$

$y_5 = 2$

$y_6 = 2$

b) $c_1 = \frac{0}{1} = 0$

$$c_2 = \frac{2+4+6+7+8}{5} = \frac{27}{5} = 5.4$$

c) $\text{Energy} = (1-0)^2 + (2-5.4)^2 + (4-5.4)^2 + (6-5.4)^2 + (7-5.4)^2 + (8-5.4)^2 = 1 + 11.56 + 1.96 + 0.36 + 2.56 + 6.76$

$$\text{Energy} = \underline{\underline{24.2}}$$

Iteration 2

a) $y_1 = 1$

$y_2 = 1$

$y_3 = 2$

$y_4 = 2$

$y_5 = 2$

$y_6 = 2$

b) $c_1 = \frac{0+2}{2} = 1$

$$c_2 = \frac{4+6+7+8}{4} = \frac{25}{4} = 6.25$$

c) $$\text{Energy} = (0-1)^2 + (2-1)^2 + (4-6.25)^2 + (6-6.25)^2 + (7-6.25)^2 + (8-6.25)^2$$

$$= 1 + 1 + 5.06 + 0.06 + 0.56 + 3.06$$

$$= 10.74$$

Iteration 3

$y_1 = 1$

$y_2 = 1$

$y_3 = 2$

$y_4 = 2$

$y_5 = 2$

$y_6 = 2$

$\Rightarrow$ As the values of $y_1, \dots, y_6$ are the same, the centers and energy is the same and we end the algorithm.

3. The K-means solution for the first part where the initial centers are $c_1 = 1$, $c_2 = 10$ is better since we get a lower energy of 10 thus more closely packed clusters than in the second solution where the energy is 10.74.