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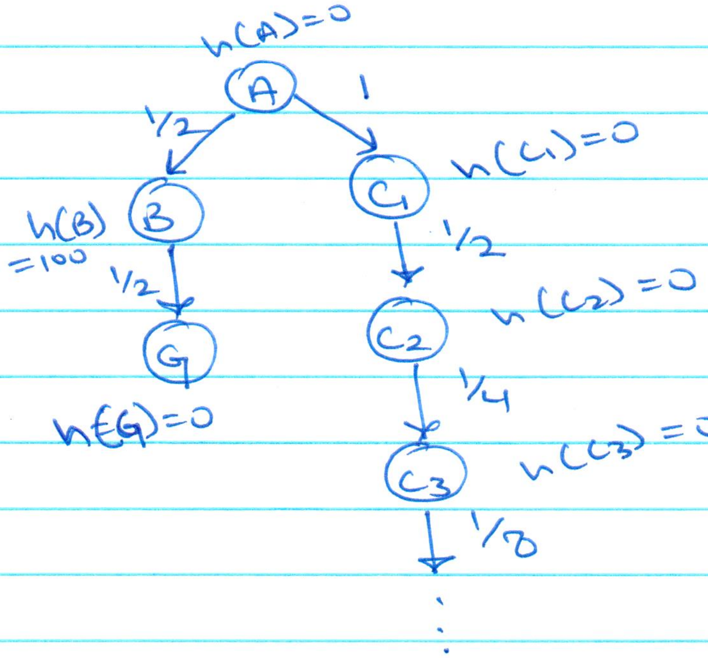
HW4

Q1.

1) As the cost of going from B → G is equal to 1/2, the range of h(B) that would be admissible would be [0, 1/2].

2)



Iteration 1

States in Closed = {A}

States in Open = {B, C1}

A: ~~f(A)=0~~ $g(A)=0$, $h(A)=0$, $f(A)=0$

B: $g(B)=1/2$, $h(B)=100$, $f(B)=100\frac{1}{2}$, Backpointer to A

C_1: $g(C_1)=1$, $h(C_1)=0$, $f(C_1)=1$, Backpointer to A

Iteration 2.

States in ~~c~~Closed = $\{A, C_1\}$
States in Open = $\{B, C_2\}$

A: $g(A)=0$, $h(A)=0$, $f(A)=0$
B: $g(B) = 1/2$, $h(B)=100$, $f(B)=100\,1/2$, Backpointer to A

C_1: $g(C_1)=1$, $h(C_1)=0$, $f(C_1)=1$, Backpointer to A

C_2: $g(C_2) = 1\,1/2$, $h(C_2)=0$, $f(C_2)=1\,1/2$, Backpointer to C_1

Iteration 3

States in Closed = $\{A, C_1, C_2\}$
States in Open = $\{B, C_3\}$

A: $g(A)=0$, $h(A)=0$, $f(A)=0$
B: $g(B)=1/2$, $h(B)=100$, $f(B)=100\,1/2$, Backpointer to A
C_1: $g(C_1)=1$, $h(C_1)=0$, $f(C_1)=1$, Backpointer to A
C_2: $g(C_2)=1\,1/2$, $h(C_2)=0$, $f(C_2)=1\,1/2$, Backpointer to C_1
C_3: $g(C_3)=1\,3/4$, $h(C_3)=0$, $f(C_3)=1\,3/4$, Backpointer to C_2

Iteration 4

States in Closed = $\{A, C_1, C_2, C_3\}$

States in Open = $\{B, C_4\}$

A: $g(A)=0$, $h(A)=0$, $f(A)=0$

B: $g(B)=\frac{1}{2}$, $h(B)=100$, $f(B)=100\frac{1}{2}$, Backpointer to A

C_1: $g(C_1)=1$, $h(C_1)=0$, $f(C_1)=1$, Backpointer to A

C_2: $g(C_2)=1\frac{1}{2}$, $h(C_2)=0$, $f(C_2)=1\frac{1}{2}$, Back pointer to C_1

C_3: $g(C_3)=1\frac{3}{4}$, $h(C_3)=0$, $f(C_3)=1\frac{3}{4}$, Backpointer to C_2

C_4: $g(C_4)=1\frac{7}{8}$, $h(C_4)=0$, $f(C_4)=1\frac{7}{8}$, Backpointer to C_3

Iteration 5

States in Closed = $\{A, C_1, C_2, C_3, C_4\}$

States in Open = $\{B, C_5\}$

A: $g(A)=0$, $h(A)=0$, $f(A)=0$

B: $g(B)=\frac{1}{2}$, $h(B)=100$, $f(B)=100\frac{1}{2}$, Backpointer to A

C_1: $g(C_1)=1$, $h(C_1)=0$, $f(C_1)=1$, Backpointer to A

C_2: $g(C_2)=1\frac{1}{2}$, $h(C_2)=0$, $f(C_2)=1\frac{1}{2}$, Backpointer to C_1

C_3: $g(C_3)=1\frac{3}{4}$, $h(C_3)=0$, $f(C_3)=1\frac{3}{4}$, Backpointer to C_2

C_4: $g(C_4)=1\frac{7}{8}$, $h(C_4)=0$, $f(C_4)=1\frac{7}{8}$, Backpointer to C_3

C_5: $g(C_5)=1\frac{15}{16}$, $h(C_5)=0$, $f(C_5)=1\frac{15}{16}$, Backpointer to C_4

3) $f(C_i) = \dfrac{(1-(1/2)^i)}{1-1/2}$ [Sum of a Geometric Progression with $r = 1/2$]

$$\lim_{i \to \infty} \frac{1-(1/2)^i}{1-1/2}$$

$$= \lim_{i \to \infty} \left(1 - \frac{1}{2^i}\right) \times 2$$

$$= \underline{\underline{2}}$$

$$\left[\lim_{i \to \infty} \frac{1}{2^i} \approx \frac{1}{\infty} \approx 0\right]$$

4) The search will never find G because to get to G, we would have to expand node B which will only happen if B is popped out of open. B will never be popped out of open as it's f value is $100\frac{1}{2}$ and the f value of all C_i will never even exceed 2. So nodes in the right branch will keep on being popped out of open and B will never be popped out.

5) For the A^* search to find G,

$$f(B) = h(B) + 1/2 < 2$$

$$h(B) < 1\tfrac{1}{2}$$

and for $h(B)$ to be inadmissable,

$$h(B) > 1/2$$

So, range for $h(B)$ is

$$\boxed{1/2 < h(B) < 1\tfrac{1}{2}}$$

6) An admissable h is a sufficient condition for A^* search to find the optimal goal. Like in this example, if $1/2 < h(B) < 1\tfrac{1}{2}$, then h will be inadmissable but still find the optimal goal. Hence it is not a necessary condition.

Q2 Iteration 1

Current point = 2 , Successor = 3

$T(1) = 2(0.9)^1 = 1.8$

$$p = \exp\left\{ - \frac{|f(x) - f(y)|}{T} \right\}$$

$f(x) = \max\{4 - |x|, 2 - |x-6|, 2 - |x+6|\}$

$f(x) = \max\{4-2, 2-|2-6|, 2-|2+6|\}$

$= \max\{2, -2, -6\}$

$= 2$

$f(y) = \max\{4-3, 2-|3-6|, 2-|3+6|\}$

$= \max\{1, -1, -7\}$

$= 1$

$$p = \exp\left\{ - \frac{|2-1|}{1.8} \right\}$$

$= \exp\left\{ \frac{-1}{1.8} \right\} = 0.5737$

As, $0.102 < 0.5737$, we move to point 3

Iteration 2

Current point = 3 , Successor = 1

$T(2) = 2(0.9)^2 = 2 \times 0.81 = 1.62$

$f(x) = 1$

$f(y) = f(1) = \max\{4 - |1|, 2 - |1-6|, 2 - |1+6|\}$
$= \max\{3, -3, -5\}$
$= 3.$

As $f(y) > f(x)$,
we move to point 1. Thus, $p = 1$

Iteration 3

Current point = 1 , Successor = 1

$T(3) = 2(0.9)^3 = 2 \times 0.729 = 1.458$

$f(x) = f(y) = 3$

$P = \exp\left\{\frac{-0}{1.458}\right\} = e^{-0} = \frac{1}{e^0} = 1$

Iteration 4

Current point = 1, Successor = 4

$T(4) = 2(0.9)^4 = 1.3122$

$f(x) = 3$

$$f(y) = \max\{4\cancel{8} - |4|,\ 2 - |4-6|,\ 2 - |4+6|\}$$
$$= \max\{0, 0, -8\}$$
$$= 0.$$

$$p = \exp\left\{\frac{-|3-0|}{1.3122}\right\}$$

$$= \exp\left\{\frac{-3}{1.3122}\right\} = 0.101$$

As $0.493 > 0.101$, we do not move.

~~Iteration~~

Iteration 5

Current point = 1, Successor = 2

$T(5) = 2(0.9)^5 = 1.181$

$f(x) = 3$

$f(y) = f(2) = \max\{4-|2|, 2-|2-6|, 2-|2+6|\}$

$= \max\{2, -2, -6\}$

$= 2.$

$p = \exp\left\{\dfrac{-|3-2|}{1.181}\right\}$

$= \exp\left\{\dfrac{-1}{1.181}\right\} = 0.4288$

As $0.312 < 0.4288$,

we move to 2.

Iteration 6

Current = 2, Successor = 3

$T(6) = 1.063$

$f(x) = 2$

$f(y) = f(3) = 1$

$$p = \exp\left\{-\frac{|2-1|}{1.063}\right\} = \exp\left\{\frac{-1}{1.063}\right\} = 0.3903$$

Iteration 7.

Current = 2, Successor = 4

$T(7) = 0.957$

$f(x) = 2$

$f(y) = f(4) = 0$

$$p = \exp\left\{\frac{-|2-0|}{0.957}\right\} = \exp\left\{\frac{-2}{0.957}\right\} = 0.1237$$

Iteration 8

Current = 2 , Successor = 3

$T(8) = 0.861$

$f(x) = 2$

$f(y) = f(3) = 1$

$$p = \exp\left\{ \frac{-|2-1|}{0.861} \right\} = \exp\left\{ \frac{-1}{0.861} \right\} = 0.313$$

As $0.313 < 0.887$,

we stay at point 2.

Q3.

1) The number of states for n trees is the same as the number of permutation a string with n characters which is $\boxed{n!}$

2) There will be n-1 possible successors as there are n-1 values for j and each swap gives a unique successor. The fraction of the state space covered is,

$$\frac{n-1}{n!} = \frac{n-1}{n(n-1)(n-2)!} = \boxed{\frac{1}{n(n-2)!}}$$

3) $n = 112,511$

No. of states $= (112511)! = 1.04 \times 10^{519455}$

$= \boxed{1 \times 10^{519455}}$

4) Total distance $= d(0, t_1) + \sum_{i=1}^{n-1} d(t_i, t_{i+1}) + d(t_n, 0$

$= 10 + (112,511 - 1) \times 10 + 10$

$= 10 + (112,510) \times 10 + 10$

$= 20 + 1125100$

$= 1125120$ Km

1 LD $= 384,400$ Km

Total distance $= \frac{1125120}{384400}$ LD $= 2.92$ LD

$\approx \boxed{3 \text{ LD}}$

5) Best case $= 10 + (112,510) \times 10 + 10$

$= 1125120$ m

$= \frac{1,125,120}{1000}$ Km $= \boxed{1125.12 \text{ Km}}$

6) 1125.12 Km $= 699.12$ miles.

$\frac{699.12}{25} = 27.96$ hours.

→ Even in the best case, the inspector cannot finish in one day.