

MTH 210

Assignment 1

Problem Statement 3

Generate 100 observations from a normal distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$. Test whether the generated observations are really “random” or not.

Methodology: using the run test

1. Data generation:

$$X \sim \mathcal{N}(0, 1), \quad \text{for } n = 100$$

The data was generated using the `rnorm()` function in R.

2. **Runs Test:** - Null Hypothesis (H_0): The sequence of observations is random. - Alternative Hypothesis (H_a): The sequence of observations is not random. - P-value threshold: $\alpha = 0.05$.

3. **Recursive Approach:** - The dataset was recursively divided into subsets, and the runs test was applied to each subset. - A data frame recorded the subset size, p-values, and randomness results.

4. **Overall Assessment:** - The mean p-value across all subsets was computed. - If the mean p-value > 0.05 , the data is concluded to be random; otherwise, it is not random.

Results

1. **Runs Test Results:** The following table summarizes the results of the runs test for subsets:

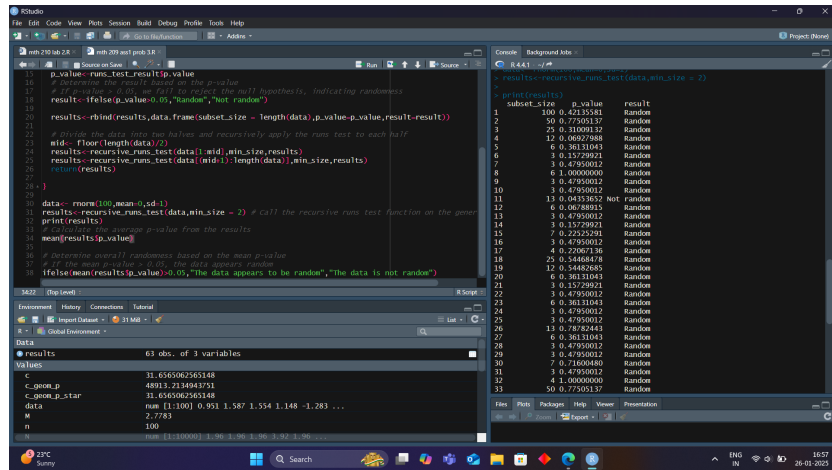


Figure 1: this for results.

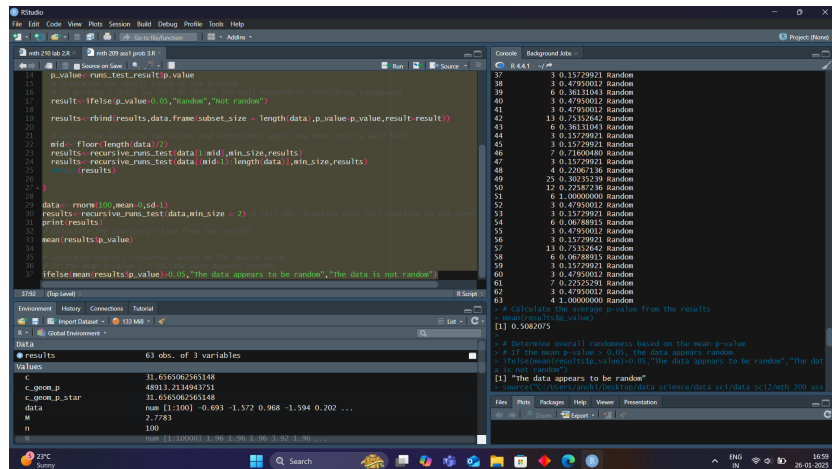


Figure 2: this for PS3 output.

Subset Size	P-Value	Result
100	0.421	Random
50	0.775	Random
25	0.311	Random
...

2. Mean P-Value:

Mean P-Value = 0.5083

3. **Final Conclusion:** Since the mean p-value > 0.05 , we fail to reject the null hypothesis. Thus, the data appears to be random.

Conclusion

The runs test indicates that the generated observations from a normal distribution ($\mu = 0, \sigma^2 = 1$) exhibit randomness. Recursive testing confirms this conclusion across different data subsets.