Lab 3 Solution

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Problem 1 Use the Acceptance–Rejection method to generate samples from Binomial(n, p) setting Geometric as the proposal distribution.

(1) For $(n, p) = (10, 0.3)$ use Geometric(p) as the proposal.
(2) Estimate the value of c.
(3) For $(n, p) = (100, 0.3)$ use Geometric(p^*) as the proposal with appropriate choice of p^*.
(4) Estimate the value of c and compare it with the value of c when the proposal is set as Geometric(p).

Solution:

```
## Function to generate binomial distribution from geometric distribution
## using Acceptance-rejection Method

generate_binomial_geometric <- function(n, p, c, p_star){

  # Count to store the number of times required for one acceptance
  count <- 0

  # Checking for acceptance
  while(TRUE){

    # Generating a random sample from Geometric(p*)
    geom_sample <- rgeom(1, p_star)

    # Generating a random sample from Uniform(0,1)
    u <- runif(1)

    # Incrementing the number of times the loop ran
    count <- count + 1

    # Checking for condition
```

```r
    if (u <= dbinom(geom_sample, n, p)/(c * dgeom(geom_sample, p_star))){
      # Returning the corresponding sample and no. of times the loop ran
      return(c(geom_sample, count))
    }
  }
}
```

The function defined in the above code takes four parameters namely n required for generating the binomial sample, p the probability of success in case of the Binomial distribution, c the expected number of times the values are generated before it is accepted and p^* (p_star) the probability of success of the proposed Geometric distribution. In this case the target distribution is Binomial (n, p) and the proposed distribution is Geometric (p^*).

```r
# Function to estimate the value of c theoretically
estimate_c <- function(n, p){
  k_vals <- 0:n

  binomial_pmf <- dbinom(k_vals, n, p)
  geometric_pmf <- dgeom(k_vals, p)

  # Calculating the theoretical value of c required in the ARM method
  c <- max(binomial_pmf/geometric_pmf)

  # Returning the theoretically obtained value
  return(c)
}
```

The function defined in the above code takes two parameters namely n and p and returns the theoretically expected value of c for the given parameters n and p.

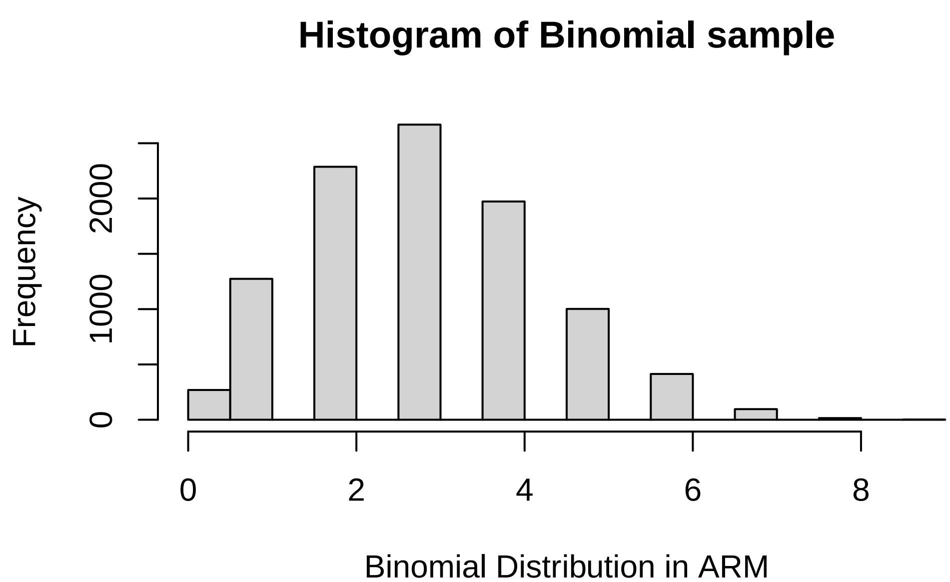
```r
## Part (1) Geometric(10,0.3)
n1 <- 10
p1 <- 0.3

# Theoretically estimating value of c for n = 10 and p = 0.3
c1 <- estimate_c(n1,p1)

# Theoretically estimated value of c for n = 10 and p = 0.3
c1
```

```
[1] 2.7783
```

```r
# Array to store the generated samples
binom_samples <- numeric(1e4)

# Array to store the c values obtained
estimate_c_array <- numeric(1e4)

# Loop to call the function iteratively to generate samples
for (i in 1:1e4){
  ele <- generate_binomial_geometric(n1, p1, c1, p1)
  binom_samples[i] <- ele[1]
  estimate_c_array[i] <- ele[2]
}

mean(binom_samples)
```

```
[1] 3.0048
```

```r
hist(binom_samples,xlab = "Binomial Distribution in ARM",
     main = "Histogram of Binomial sample")
```



The above code calls the required functions to get the theoretically expected value of c and generated 10^4 samples from Binomial$(10, 0.3)$ with Geometric(0.3) as the proposed the distribution. On checking the mean of the generated sample, we get the mean of the generated sample close to the theoretically expected mean of Binomial$(10, 0.3)$ which is $n * p = 10 * 0.3 = 3$.

```
## Part (2) Estimating the value of c

# Finding the mean of the array
mean(estimate_c_array)
```

```
[1] 2.766
```

Since, the estimate_c_array stored the experimentally obtained values of c , taking the mean of the array gives us the experimentaly estimated value of c, which is quite close to its theoretically obtained value.

```
## Part (3) Geometric(100,0.3) from Geometric(p*)

n2 = 100
p2 = 0.3

# Value of p* = 1/(1 + np)
p_star = 1/(n2*p2 + 1)

# Theoretically estimating value of c for n = 100 and p = p*
c_star <- estimate_c(n2, p_star)

# Theoretically estimated value of c for n = 100 and p = p*
c_star
```

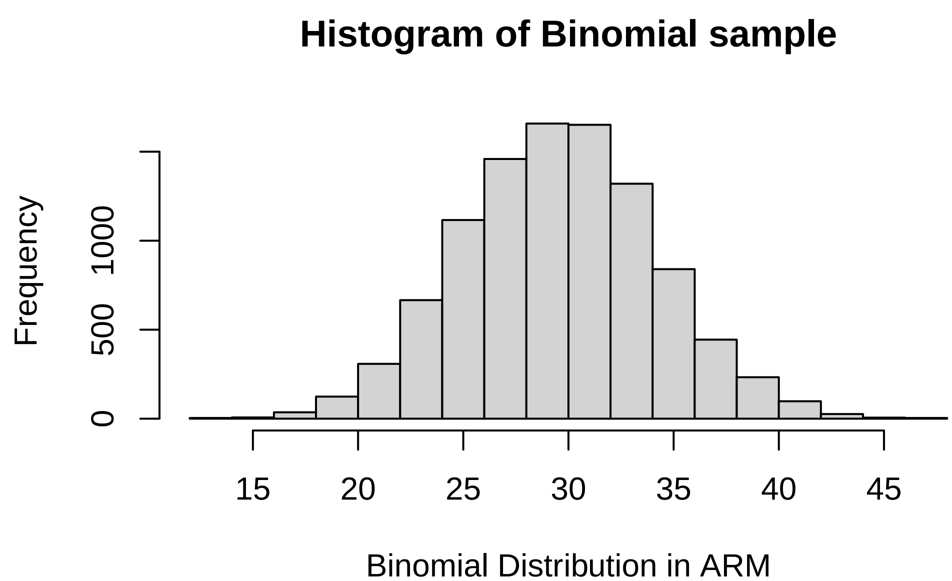
```
[1] 7.715878
```

```
# Array to store the generated samples
binomial_samples <- numeric(1e4)

# Array to store the c values obtained
estimate_c_star_array <- numeric(1e4)

# Loop to call the function iteratively to generate samples
for (i in 1:1e4){
  ele <- generate_binomial_geometric(n2, p2, c_star, p_star)
  binomial_samples[i] <- ele[1]
  estimate_c_star_array[i] <- ele[2]
```

```r
}

mean(binomial_samples)
```

```
[1] 30.0531
```

```r
hist(binomial_samples,xlab = "Binomial Distribution in ARM",
     main = "Histogram of Binomial sample")
```



The above code calls the required functions to get the theoretically expected value of c and generated 10^4 samples from Binomial$(100, 0.3)$ with Geometric(p^*) as the proposed the distribution. On checking the mean of the generated sample, we get the mean of the generated sample close to the theoretically expected mean of Binomial$(100, 0.3)$ which is $n * p = 100 * 0.3 = 30$.

```r
## Part (4) Estimating the value of c* thus obtained and
## comparing it when samples are generated using Geometric(p)

# Finding the mean of the array
mean(estimate_c_star_array)
```

```
[1] 7.6801
```

```r
# Theoretically estimating the value of c when
# success probability of Geometric distribution. is p = 0.3
c2 <- estimate_c(n2, p2)
c2
```

```
[1] 48913.21
```

From the values obtained above we can see that if the success probability of Geometric (p) is significant , then it takes a large amount of tries for the acceptance of even a single value and hence scaling the success probability from p to p^* which helps reduce the computations significantly.