CS315: DATABASE SYSTEMS RELATIONAL ALGEBRA

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> 2nd semester, 2019-20 Tue, Wed 12:00-13:15

Relational Algebra

- Procedural language to specify database queries for relational data model
- Operators are functions from one or two input relations to an output relation
 - **1** Select: σ
 - Project: П
 - **3** Union: ∪
 - Set Difference: –
 - Cartesian Product: ×
 - **1** Rename: ρ

Relational Algebra

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 - **1** Rename: ρ
- Uses propositional calculus formed by expressions connected by
 - and: ∧
 - ② or: ∨
 - one of the one of
- Each term is of the form

```
<attr/const> comparator <attr/const> where comparator is one of =, \neq, >, \geq, <, \leq
```

Select

- $\sigma_p(r) = \{t | t \in r \text{ and } p(t)\}$
- p is called the selection predicate
- Select all tuples from r that satisfies the predicate p
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- Applying $\sigma_{A=B \land D>5}$ on

Α	В	С	D
1	1	2	7
1	2	5	7
2	2	9	3
2	2	8	6

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		Α	В	С	D
retur	ns	1	1	2	7
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- $\bullet \ \Pi_{A_1,\ldots,A_k}(r)$
- A_i, etc. are attributes of r
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	Α	E	3		С		
	1	1			5	_	
	1	2	2		5		
	2	3	3		5		
	2	4	1		8		
			A	١		С	
returns			1			5	
E	turris	>	2	2		5	
			2	2		8	

Set Union

- $r \cup s = \{t | t \in r \text{ or } t \in s\}$
- Relations r and s must have the same arity (i.e., number of attributes)
- They must have same type of attribute in each column as well, i.e., attribute domains must be compatible
- If attribute names are not same, renaming should be used
- Schema is

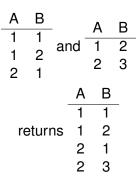
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Cartesian Product

- $r \times s = \{\langle u, v \rangle | u \in r \text{ and } v \in s\}$
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Α		В			С	D	Е
	١	1		- ad	1	2	7
1		1	aı	nd	2	6	8
2	•	2			5	7	9
			Α	В	С	D	Ε
			1	1	1	2	7
			1	1	2	6	8
retu	rns	3	1	1	5	7	9
			2	2	1	2	7
			2	2	2	6	8
			2	2	5	7	9

Rename

- $\rho_N(E)$ returns E, but under the new name N
- For *n*-ary relations, $\rho_{N(A_1,...,A_n)}(E)$ returns result of expression E, but under the new name N and attributes renamed to A_1 , etc.
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- $\rho_{s(B_1,B_2,\dots,B_k)}(r) = \{\langle t.B_1,t.B_2,\dots,t.B_k \rangle | \langle t.A_1,t.A_2,\dots,t.A_k \rangle \text{ and } t \in r \}$
- Applying $\rho_{s(C,D)}$ on r(A,B)

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Α	В	
1	1	-
1	2	
2	3	
2	4	
	С	D
·	1	1
returns	1	2
	2	3
	2	4

Additional Operations

- Additional operators have been defined
 - Set Intersection: ∩
 - 2 Join: ⋈
 - Division: ÷
 - Assignment: ←
- These do not add any power to the basic relational algebra
 - They can be defined using the six basic operators
- However, they simplify queries

- $r \cap s = \{t | t \in r \text{ and } t \in s\}$
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$$r \cap s = r - (r - s)$$

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- Natural join: If two relations share an attribute (also its name), equality join on that common attribute
 - Denoted by * or simply ⋈ without any predicate
 - Changes schema by retaining only one copy of common attribute
 - $r * s = r \bowtie s = r \bowtie_{r.A=s.A} s$
- Applying ⋈ on

Α	В		Δ	\sim	
1	1	-	$\overline{}$	O	_
•	•	and	1	2	returns
1	2	ana	•	_	TOTALLIS
•	_		2	3	
2	1		_	U	

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Α	В		Δ	С		Α	В	С
1	1	- and		_	- -	1	1	2
1	2	anu	ı	2	returns	1	2	2
2	1		2	3		2	1	3

- $r \div s = \{t | t \in \Pi_{R-S}(r) \text{ and } \forall u \in s(tu \in r)\}$
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- Schema is changed to R S
- Applying ÷ on

Α	В			
1	5	-		
1	6			
1	7		В	
2	5	and	5	returns
2	6		6	
3	5			
3	7			
4	5			

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Α	В				
1	5	-			
1	6				
1	7		В		Α
2	5	and	5	returns	1
2	6		6		2
3	5				
3	7				
4	5				

Division (contd.)

Applying ÷ on

```
A B C D

1 5 2 7

1 5 3 7

1 6 3 7

2 6 2 7 and 2 7 returns
2 6 3 7
3 6 2 7
3 6 3 7
3 5 3 7
```

Division (contd.)

Applying ÷ on

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Applying ÷ on

•
$$r \div s = \Pi_{R-S}(r) - \Pi_{R-S}((\Pi_{R-s}(r) \times s) - \Pi_{R-S,S}(r))$$

Assignment

- $s \leftarrow E(r)$ assigns the relation resulting from applying E on r to s
- Useful in complex queries to hold intermediate values
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	r	
Α	В	С
1	1	7
2	2	8
5	7	9

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Composition of Operators

- Expressions can be built using multiple operators
- Applying $\sigma_{A=C}(r \times s)$ on

Precedence and Associativity

Precedence is generally assumed to be

Precedence	Operators
Highest	σ, Π, ρ
Medium	\bowtie , \bowtie_{θ} , \times
Lowest	∪, ∩, −

- Associativity is assumed to be left-to-right
- Not part of definition
- Therefore, best to use explicit brackets

Example Schema

- course (<u>code</u>, title, *ctype*, webpage)
- coursetype (ctype, dept)
- faculty (<u>fid</u>, name, dept, designation)
- department (deptid, name)
- semester (yr, half)
- offering (coursecode, yr, half, instructor)
- student (roll, name, dept, cpi)
- program (roll, ptype)
- registration (coursecode, roll, yr, half, gradecode)
- grade (gradecode, value)

• Find all courses offered in the year 2018

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 m yr=2018}({
 m offering})$
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• Find the years when all the courses of type 5 were offered

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ct
$$\leftarrow \rho_{\text{coursecode}}(\Pi_{\text{code}}(\sigma_{\text{ctype}=5}(\text{course})))$$

 $(\Pi_{coursecode,yr}(\text{offering})) \div \text{ct}$

Extended Relational Algebra

- The power of relational algebra can be enhanced by
 - Generalized Projection
 - Aggregation and Grouping
 - Outer Join

Generalized Projection

- Extends project operator by allowing arbitrary arithmetic functions in attribute list
- $\bullet \Pi_{F_1,\ldots,F_k}(E)$
- F_i, etc. are arithmetic expressions involving constants and attributes in schema of E
- Applying $\Pi_{B-A,2C}$ on r

Α	В	С
1	1	5
1	2	5
2	3	5
2	4	8

returns

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	2	4	8	
		B-	Α	2C
retui	urns	0		10
		1		10
		2		16

- Aggregate functions that can be used are avg, min, max, sum, count
- Can be applied on groups of tuples as well
- Aggregate operation is of the form $G_1,...,G_k$ $G_{F_1(A_1),...,F_n(A_n)}(E)$ where
 - G_1, \ldots, G_k is the list of attributes on which to group (may be empty)
 - Each F_i is an aggregate function that operates on the attribute A_i
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1	1	5	_
1	2	5	returns
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Α	В	С		
1	1	5	-	cum(C)
1	2	5	returns	sum(C) 23
2	3	5		23
2	4	8		

- First, the tuples are grouped according to G_1, \ldots, G_k
- Then, aggregate functions $F_1(A_1), ..., F_n(A_n)$ are applied on each group
- Schema changes to $(G_1, \ldots, G_k, F_1(A_1), \ldots, F_n(A_n))$
- Applying ${}_{A}G_{sum(C)}$ on r

Α	В	С	
1	1	5	-
1	2	5	roturno
2	3	5	returns
2	4	8	
3	4	8	

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Α	В	С			
1	1	5	-	Α	sum(C)
1	2	5	roturno	1	10
2	3	5	returns	2	13
2	4	8		3	8
3	4	8			

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• Applying $_{A,B}G_{sum(C)}$ on r

Α	В	С	
1	1	5	roturno
1	2	5	returns
1	2	4	

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Α	В	С	-	Δ	R	sum(C)
1	1	5				Surri(O)
•	•	J	returns	1	1	5
1	2	5	returns	•		9
•	_	J		1	2	q
1	2	4		•	_	3

Outer Join

- Extension of the join to retain more information
- Computes join and then adds tuples to result that do not match
- Requires use of null values
- Left outer join $r \bowtie_{\theta} s$ retains *every* tuple from left or first relation
 - If no matching tuple is found in right or second relation, values are padded with null
- Right outer join $r \bowtie_{\theta} s$ is defined analogously
- Full outer join $r \Rightarrow c_\theta s$ retains all tuples from both relations
 - Non-matching fields are filled with null values

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- Consequently, ordinary join is sometimes called inner join
- "Outer" word is sometimes dropped from join yielding left join, right join and full join
- When no θ condition is specified, it is natural outer join

Α	В		Α	С	
1	5		1	7	-
2	6	M	2	8	=
3	7		4	9	

$$\frac{A \quad B}{1 \quad 5} \bowtie \frac{A \quad C}{1 \quad 7} = \frac{A \quad B \quad C}{1 \quad 5 \quad 7} \\
2 \quad 6 \quad 3 \quad 7$$

$$\frac{A \quad B}{3 \quad 7} \bowtie \frac{A \quad C}{4 \quad 9} = \frac{A \quad B \quad C}{1 \quad 5 \quad 7} \\
2 \quad 6 \quad 8$$

$$\frac{A \quad B}{1 \quad 5} \bowtie \frac{A \quad C}{1 \quad 7} \\
2 \quad 6 \quad 8$$

$$\frac{A \quad B}{3 \quad 7} \bowtie \frac{A \quad C}{1 \quad 7} \\
2 \quad 8 \quad 3 \quad 7 \quad \text{null}$$

$$\frac{A \quad B}{1 \quad 5} \bowtie \frac{A \quad C}{1 \quad 7} \\
2 \quad 8 \quad 4 \quad 9$$

$$\frac{A \quad B}{1 \quad 5} \bowtie \frac{A \quad C}{1 \quad 7} = \frac{A \quad B \quad C}{1 \quad 5 \quad 7} \\
2 \quad 6 \quad 8$$

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$$\frac{A \quad B}{3 \quad 7} \bowtie \frac{A \quad C}{2 \quad 8} = \frac{A \quad B \quad C}{1 \quad 5 \quad 7} \\
2 \quad 6 \quad 8$$

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3 \quad 7 \quad \text{null} \quad 9$$

Α	В		Α	С	
1	5		1	7	-
2	6	M	2	8	=
3	7		4	9	

• Find the total number of courses offered in the year 2018

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$$\mathcal{G}_{count(coursecode)}(\sigma_{yr=2018}(offering))$$

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 For each instructor, find the total number of courses offered by her in the year 2018

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$$\mathcal{G}_{\textit{count}(coursecode)}(\sigma_{\textit{yr}=2018}(\textit{offering}))$$

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$$_{\text{instructor}}\mathcal{G}_{count}(_{\text{coursecode}})(\sigma_{\text{yr}=2018}(_{\text{offering}}))$$

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$$instructor, yrG_{count(coursecode)}(offering)$$

For each course, indicate the most recent year it was offered

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instructor
$$\mathcal{G}_{count(coursecode)}(\sigma_{yr=2018}(offering))$$

 For each instructor, find the total number of courses offered by her per year

$$_{ ext{instructor,yr}}\mathcal{G}_{count(ext{coursecode})}(ext{offering})$$

For each course, indicate the most recent year it was offered

course-offering
$$\leftarrow_{\text{coursecode}} \mathcal{G}_{max(yr)}(\text{offering})$$

course-year $\leftarrow \rho_{(\text{code},yr)}(\Pi_{\text{coursecode},max(yr)}(\text{course-offering}))$
course \Rightarrow course-year

Null Values

- Null denotes an unknown or missing value
- Arithmetic expressions involving null evaluate to null
- Aggregate functions (except count) ignore null
- Duplicate elimination and grouping treats null as any other value, i.e., two null values are same
 - null = null evaluates to true

Truth Tables with Null Values

- Comparison with null otherwise returns unknown, which is neither true nor false
- If false is used, consider two expressions not(A < 5) and $A \ge 5$ when attribute A is null
 - They will not be the same
- Three-valued logic with unknown
 - Or
 - unknown or true = true
 - unknown or false = unknown
 - unknown or unknown = unknown
 - And
 - unknown and true = unknown
 - unknown and false = false
 - unknown and unknown = unknown
 - Not
 - not unknown = unknown
- Select operation treats unknown as false

Database Modification

- Contents of a database may be modified by
 - Deletion
 - Insertion
 - Updating
- Assignment operator is used to express these operations

Deletion

- r ← r − E deletes tuples in the result set of the query E from the relation r
- Only whole tuples can be deleted, not some attributes
- Applying $r \leftarrow r \sigma_{A=1}(r)$ on

Α	В	С	
1	1	5	-
1	2	5	returns
2	3	5	
2	4	8	

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Α	В	С				
1	1	5	-	Α	В	С
1	2	5	returns	2	3	5
2	3	5		2	4	8
2	4	8				

Insertion

- $r \leftarrow r \cup E$ inserts tuples in the result set of the query E into the relation r
- Only whole tuples can be inserted, not some attributes
- If a specific tuple needs to be inserted, E is specified as a relation containing only that tuple
- Applying $r \leftarrow r \cup \{(1, 2, 5)\}$ on

	Α	В	С	
•	1	1	5	roturno
	2	3	5	returns
	2	4	8	

Insertion

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Δ	В	С		Α	В	С
			=	1	1	5
1	1	5	roturno	2	2	_
2	3	5	returns	2	J	5
	4	-		2	4	8
2	4	8		4	2	5
					_	J

Updation

- Updates allow values of only some attributes to change
- $r \leftarrow \Pi_{F_1,...,F_n}(r)$ where each F_i is
 - Either the *i*th attribute of *r* if it is not to be changed
 - Or the result of the expression F_i involving constants and attributes resulting in the new value of the ith attribute
- Applying $r \leftarrow \Pi_{A,2*B,C}(r)$ on

	С	В	Α
roturno	5	2	1
returns	5	1	1
	8	4	2

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	С	В	Α
roturno	5	2	1
returns	5	1	1
	8	4	2

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Α	В	С		Δ	В	\sim
1	2	5	- -	^	_	U
•	_	J	returns	1	4	5
1	1	5	TOTALLIO	•	_	_
_		_		1	2	5
2	4	8				

Create a new department "Astronomy" with id 18

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department ← department ∪ {18, "Astronomy"}

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 course $-\sigma_{code="CS200"}(course)$

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$$\leftarrow$$
 course $-\sigma_{code="CS200"}(course)$

Update the title of the course "DBMS" to "Database Systems"

old-course
$$\leftarrow \sigma_{\text{title}=\text{"DBMS"}}(\text{course})$$

$$updated\text{-}course \leftarrow \Pi_{code, "Database \ Systems", ctype, webpage}(old\text{-}course)$$

Integrity Constraints Violations

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 - Schema integrity: If the inserted tuple does not conform to the schema
 - Should be restricted
- Updation may violate

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 - Referential integrity: If a primary key is deleted, the corresponding foreign referencing key becomes orphan
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 - Domain constraint: Value is outside the domain
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 - Key constraint: If an insertion violates the property of being a key
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 - Entity integrity: If the primary key of the inserted tuple is null
 - Should be restricted
 - Schema integrity: If the inserted tuple does not conform to the schema
 - Should be restricted
- Updation may violate all of the above

- First-order propositional logic
- Do not support recursive closure operations
 - Find supervisors of A at all levels
 - Needs specifying multiple queries, each solving only one level at a time

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- Requires strictly structured data that conforms to the schema
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Multiset Variant

- Multiset variant of relational algebra
- Relations are multisets or bags of tuples
- Multisets
 - Example: {A, A, B}
 - It is distinct from {A, B} but equivalent to {A, B, A}
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- Multisets for relational algebra
 - Duplicate tuples are allowed
 - Select, project, set operations change
- Distinct or duplicate elimination operator: δ
 - Removes duplicate tuples
 - Reduces relation to sets