# Artificial Intelligence

# Predicate Logic

First Order Logic (FOL)

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- Why? (Use)
- Comparison with propositional logic

For knowledge representation and reasoning

Propositional logic is **declarative**: pieces of syntax correspond to facts

Propositional logic has very limited expressive power (unlike natural language)
E.g., cannot say “all kids are naughty” except by writing one sentence for each kid

Whereas propositional logic assumes world contains facts,

first-order logic (like natural language) assumes the world contains

- **Objects:** people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .
- **Relations:** red, round, bogus, prime, multistoried . . ., brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .
- **Functions:** father of, best friend, third inning of, one more than, end of

# Syntax of FOL: Basic elements

- Constants: KingJohn, 2, UCB, . . .
- Predicates: Brother, >, . . .
- Functions: Sqrt, LeftLegOf, . . .
- Variables: x, y, a, b, . . .
- Connectives: $\wedge$ $\vee$ $\neg$ $\Rightarrow$ $\Leftrightarrow$
- Equality: $=$
- Quantifiers: $\forall$ $\exists$

**Sentence** → AtomicSentence | ComplexSentence

**AtomicSentence** → Predicate | Predicate(Term,...) | Term = Term

**ComplexSentence** → ( Sentence )

| ¬ Sentence | Sentence ∧ Sentence | Sentence ∨ Sentence

| Sentence ⇒ Sentence | Sentence ⇔ Sentence

| Quantifier Variable,... Sentence

**Term** → Function(Term,...)

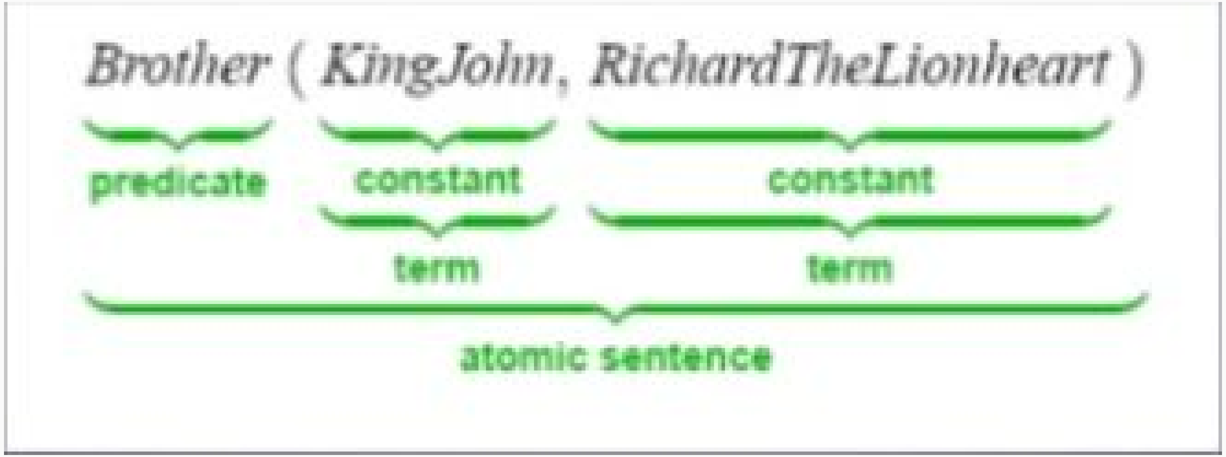
| Constant

| Variable

# Examples

- King John and Richard The Lionheart are brothers
  Brother(KingJohn, RichardTheLionheart)
- The length of left leg of Richard is greater than the length of left leg of King John
  > (Length(LeftLegOf(Richard)),Length(LeftLegOf(KingJohn)))

# Atomic Sentences



# Atomic Sentences

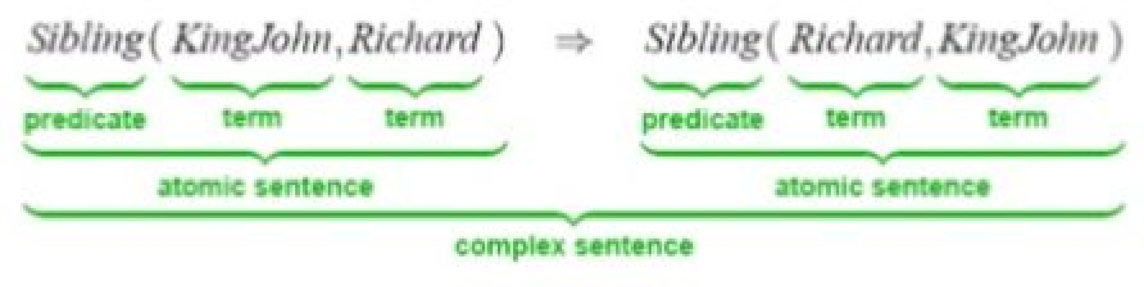
$$\underbrace{\underbrace{>}_{\text{predicate}}\ (\underbrace{\underbrace{Length}_{\text{function}}(\underbrace{LeftLegOf}_{\text{function}}(\underbrace{Richard}_{\text{constant}}))}_{\text{term}},\ \underbrace{\underbrace{Length}_{\text{function}}(\underbrace{LeftLegOf}_{\text{function}}(\underbrace{KingJohn}_{\text{constant}}))}_{\text{term}})}_{\text{atomic sentence}}$$

# Complex Sentences

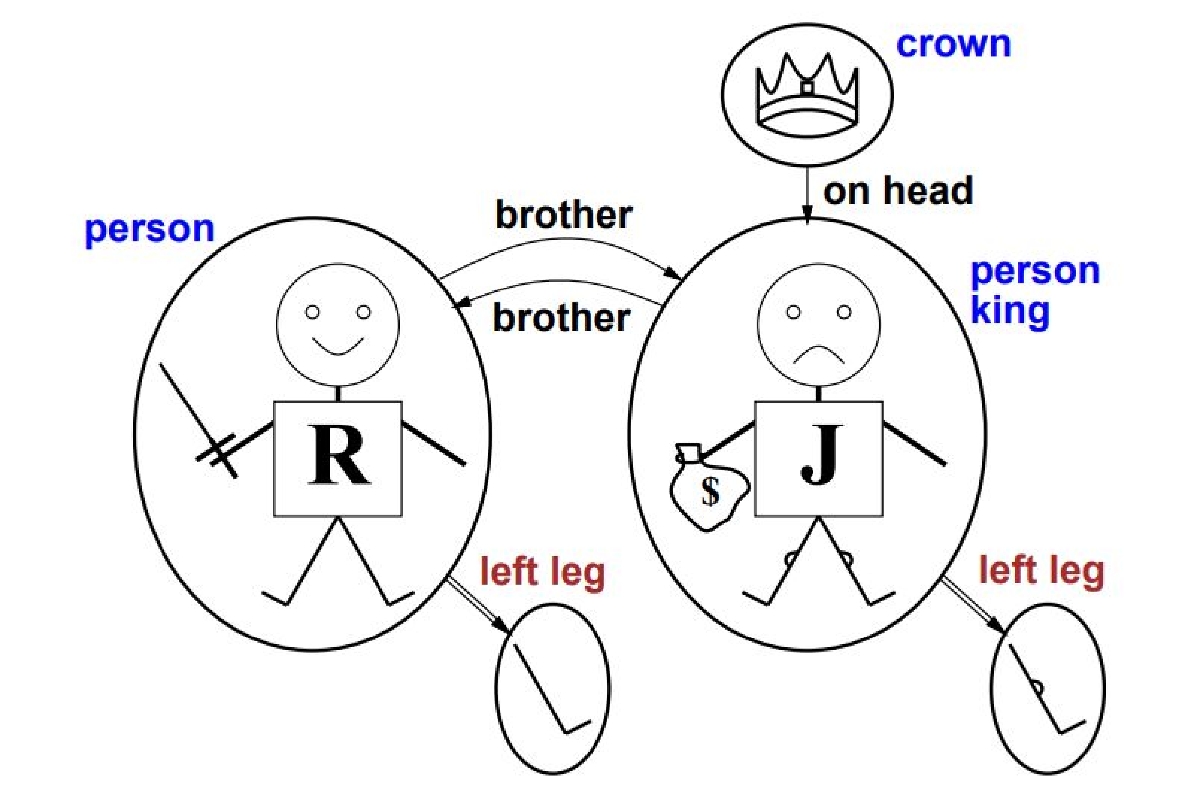
Complex sentences are made from atomic sentences using connectives

$\neg S$, $S1 \land S2$, $S1 \lor S2$, $S1 \Rightarrow S2$, $S1 \Leftrightarrow S2$

E.g. Sibling(KingJohn, Richard) ⇒ Sibling(Richard, KingJohn)



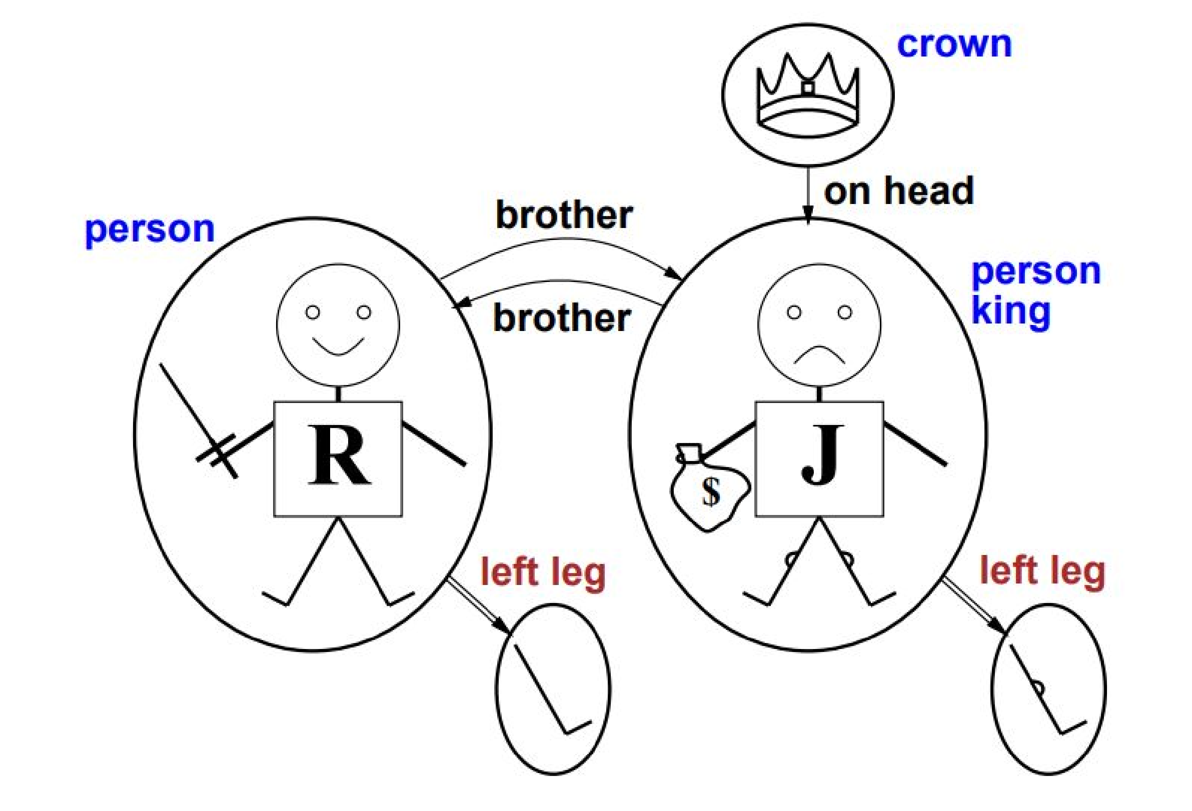
# FOL Illustration




Five objects:-

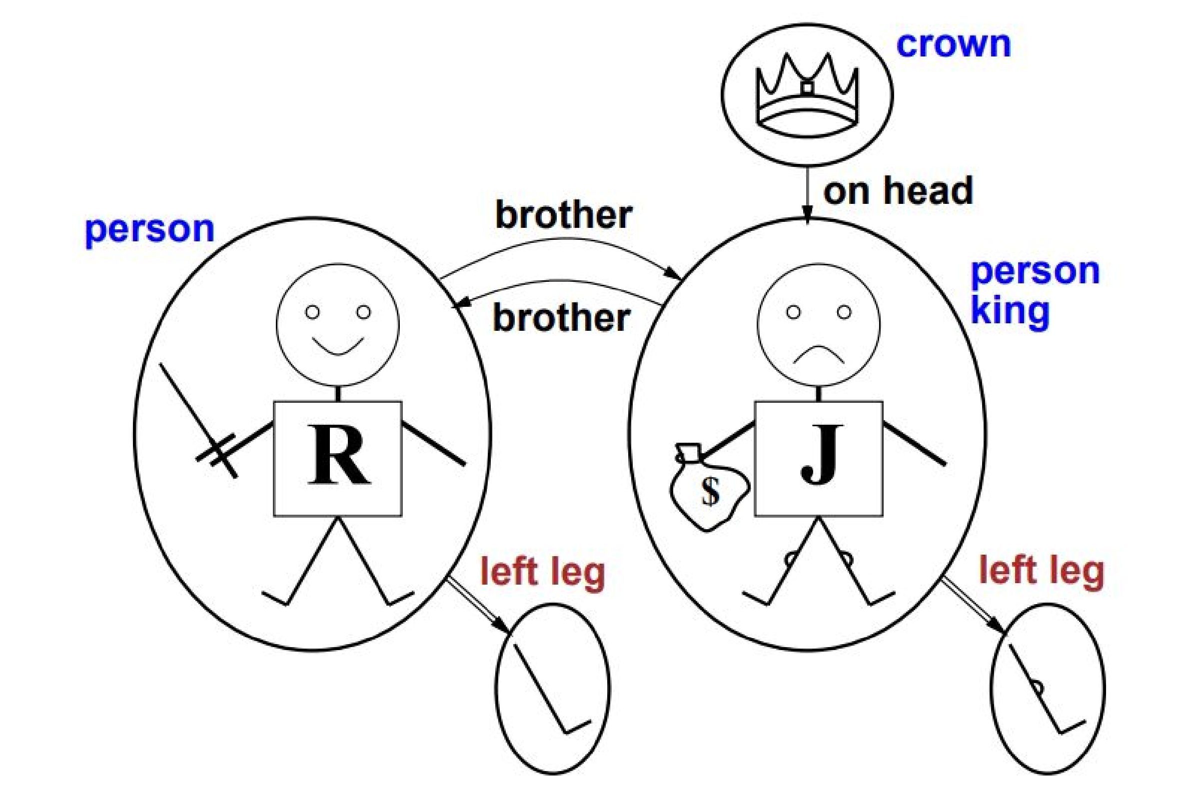
1. Richard
2. John
3. Crown
4. John’s Left leg
5. Richard’s left leg

# FOL Illustration



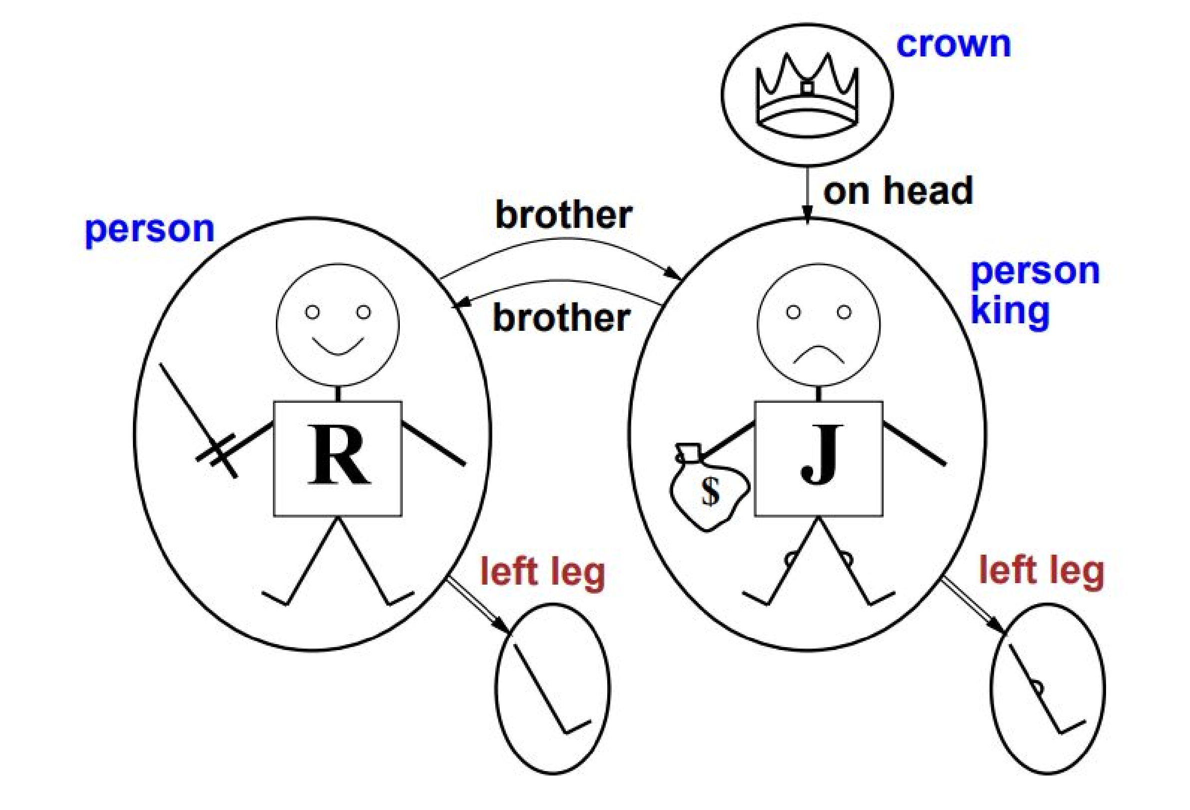

1. Objects are related with Relations
2. For example, John and Richard are related with Brother relationship
3. This relationship can be denoted as Brother (Richard, John) or Brother (John, Richard)
4. Crown and John are related with OnHead relation

# FOL Illustration




1. Properties are relations that are unary
2. Person (Richard)
   Person (John)
3. King (John)

# FOL Illustration




1. Certain relationships are expressed as funtions
2. One object is related with exactly one object
3. Richard is related to Richard’s left leg

# Universal quantification

- ∀ <variables> <sentence>

- $\forall$ x P(x)
  Translated into the English language, the expression is understood as:
  - "For all x, P(x) holds"

- "All cars have wheels" could be transformed into the propositional form,
  $\forall$ x P(x)
  - P(x) is the predicate denoting: **x has wheels**
  - the universe of discourse is only populated by cars

# Existential quantification

- ∃ <variables> <sentence>

- ∃ x P(x)
  Translated into the English language, the expression is understood as:
  - "There exists an x such that P(x) holds"

- "Someone loves you" could be transformed into the propositional form,
  ∃ x P(x)
  - P(x) is the predicate denoting: **x loves you**
  - the universe of discourse contains living creatures

- If all the elements in the universe of discourse can be listed then the universal quantification $\forall$ x P(x) is equivalent to the conjunction:

  $P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots \wedge P(x_n)$

- If all the elements in the universe of discourse can be listed then the existential quantification $\exists$ x P(x) is equivalent to the disjunction:

  $P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots \vee P(x_n)$

# Order of application of quantifiers

- When more than one variable is quantified such as $\exists y \forall x P(x, y)$, they are applied from the inside, that is, the one closest to the atomic formula is applied first.
- Thus, $\exists y \forall x P(x, y)$ read $\exists y [ \forall x P(x, y) ]$ and we say "there exists a y such that for every x, P(x, y) holds" or "for some y, P(x, y) holds for every x"
- $\exists y \forall x \, x < y$
  There is a number that is greater than every (any) number
- $\forall x \exists y \, x < y$
  For every number x, there is a number y that is greater than x

# Example knowledge base

- The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.
- Prove that Col. West is a criminal.

... it is a crime for an American to sell weapons to hostile nations:

$American(x) \land Weapon(y) \land Sells(x,y,z) \land Hostile(z) \Rightarrow Criminal(x)$

Nono … has some missiles, i.e., $\exists x \; Owns(Nono,x) \land Missile(x)$:

$Owns(Nono,M1)$ and $Missile(M1)$

… all of its missiles were sold to it by Colonel West

$Missile(x) \land Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile“:

$Enemy(x,America) \Rightarrow Hostile(x)$

West, who is American …

$American(West)$

The country Nono, an enemy of America …

$Enemy(Nono,America)$

# Inference in FOL

- Forward Chaining Algorithm

- **Unification:** finding substitutions that make different logical expressions look identical.

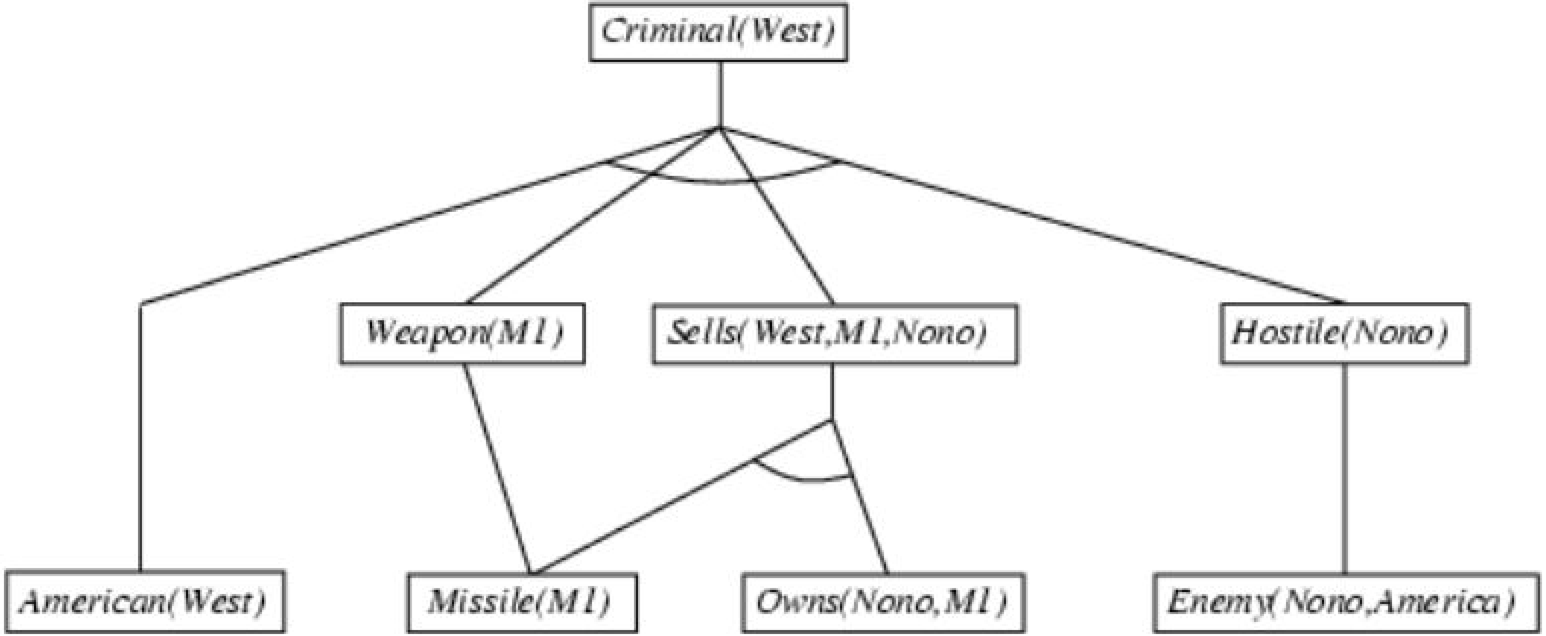
  **Examples:**

  unify(expr('Cat(x) & Dog(Dobby)'), expr('Cat(Bella) & Dog(y)'))

  **{x: Bella, y: Dobby}**

# Forward Chaining Algorithm

We look at each rule in the knowledge base and see if the premises can be satisfied. This is done by finding a substitution which unifies the each of the premise with a clause in the KB. If we are able to unify the premises the conclusion (with the corresponding substitution) is added to the KB. This inferencing process is repeated until either the query can be answered or till no new sentences can be added. We test if the newly added clause unifies with the query in which case the substitution yielded by unify is an answer to the query. If we run out of sentences to infer, this means the query was a failure.


Criminal(West)
Weapon(M1)
Sells(West,M1,Nono)
Hostile(Nono)
American(West)
Missile(M1)
Owns(Nono,M1)
Enemy(Nono,America)

# References


1. Russell, Stuart, Peter Norvig, and Artificial Intelligence. "A modern approach." Artificial Intelligence. Prentice-Hall, Egnlewood Cliffs 25 (1995): 27.
2. Kulkarni, Parag, and Prachi Joshi. ARTIFICIAL INTELLIGENCE: Building Intelligent Systems. PHI Learning Pvt. Ltd., 2015.